Joint Charging Tour Planning and Depot Positioning for Wireless Sensor Networks Using Mobile Chargers

Guiyuan Jiang, Siew-Kei Lam, Yidan Sun, Lijia Tu, and Jigang Wu

***Abstract*— Recent breakthrough in wireless energy transfer technology has enabled wireless sensor networks (WSNs) to operate with zero-downtime through the use of mobile energy chargers (MCs), that periodically replenish the energy supply of the sensor nodes. Due to the limited battery capacity of the MCs, a significant number of MCs and charging depots are required to guarantee perpetual operations in large scale networks. Existing methods for reducing the number of MCs and charging depots treat the charging tour planning and depot positioning problems separately even though they are inter-dependent. This paper is the first to jointly consider charging tour planning and MC depot positioning for large-scale WSNs. The proposed method solves the problem through the following three stages: charging tour planning, candidate depot identification and reduction, and depot deployment and charging tour assignment. The proposed charging scheme also considers the association between the MC charging cycle and the operational lifetime of the sensor nodes, in order to maximize the energy efficiency of the MCs. This overcomes the limitations of existing approaches, wherein MCs with small battery capacity ends up charging sensor nodes more frequently than necessary, while MCs with large battery capacity return to the depots to replenish themselves before they have fully transferred their energy to the sensor nodes. Compared with existing approaches, the proposed method leads to an average reduction in the number of MCs by 64%, and an average increase of 19.7 times on the ratio of total charging time over total traveling time.**

***Index Terms*— Wireless sensor networks, mobile chargers, tour planning, MC depot positioning, energy efficiency, joint design.**

I. Introduction

Sensor nodes in conventional wireless sensor networks (WSNs) are usually powered by batteries and hence, they have limited energy resources. When the operational lifetime of some sensors in a WSN expires due to energy depletion in their batteries, the network will become fragmented as data from certain sensing fields is no longer available. As such, energy efficiency has become a fundamental design objective in WSNs since the performance of WSNs is highly constrained by the limited energy resources of the battery-powered sensor nodes.

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Many energy conservation solutions for WSNs have been reported [1], such as power-efficient wireless communications [2], dynamic routing techniques [3], [4], efficient multicast tree construction [5], [6], mobile data gathering [7]–[9], and low-power hardware architecture [10]. Although these solutions are able to extend the operational lifetime of WSNs, they cannot guarantee perpetual operations with zero-downtime. Approaches utilizing energy harvesting techniques have also been proposed [11], [12], whereby sensor nodes can be recharged with ambient environmental energy through mechanical, thermal, photovoltaic or electromagnetic energy transmission. The energy harvesting technique enables the sensors to obtain renewable energy when required, but suffers from high uncertainty in the power supply as energy recharging is subjected to environmental conditions. The low efficiency and dynamics of energy harvesting impose significant difficulty on the design of protocols that leads to undetermistic network performance.

The recent technology breakthrough in wireless energy transfer based on magnetic resonant coupling provides a promising solution for perpetual sensing, communication and computation in future WSNs. This technology enables energy to be transferred from a source coil to a receiver coil wirelessly [13] by using magnetic resonant coils that operate at the same resonant frequency. The potential of applying non-radiative energy transmission over midrange has also been demonstrated, and the efficiency of transferring energy over a distance in excess of 2 meters is as high as 40%. The new technology is immune to the dynamics of neighboring environment, and does not require line-of-sight (LOS) between the charging and receiving devices (omnidirectional) [14]. In order to take advantage of this novel technology, recent studies [15]–[19] have proposed the use of a mobile charger (MC), which is equipped with a powerful transceiver and a high-capacity battery, to replenish the energy in sensor nodes before they run out of energy. The MC periodically travels around the network in a charging tour, and stops at each node for a short duration to recharge them (typically to the maximum battery capacity) via coupled magnetic resonance. The MC then returns to the depot to replenish its energy before performing the next charging tour.

While the use of MC for wireless energy transfer has been shown to work well for small-scale networks, a single MC is not sufficient for serving all the sensors in a relatively large scale network [20]. This is due to the fact that an MC may not carry sufficient energy to recharge every sensor in a large network, or a sensor's operational lifetime cannot last until the MC has charged all the other sensors before returning to it again. In addition, since the MC needs to return to the depot to replenish itself, most of its energy will be spent on traveling, and very little or no energy is left for recharging a sensor that is located too far away from the depot. In this case, multiple depots must be deployed not only to guarantee perpetual network operations but also to improve the overall energy efficiency.

Our work aims to design efficient techniques for enabling perpetual operations in rechargeable WSNs using MCs, with the objective of minimizing the required MCs as well as improving the energy efficiency of the MCs. The latter is achieved by maximizing the ratio of time for charging over the time spent on traveling. We propose novel periodic charging algorithms that jointly consider charging tour planning and MC depot positioning. The proposed algorithms comprises of the following three stages: 1) charging tour planning, 2) candidate depot identification and reduction, and 3) depot deployment and charging tour assignment. Unlike previous works, the proposed algorithms take into account the association between the MCs' charging cycle and the operational lifetime of the corresponding sensor nodes. Specifically, the charging cycle of the MC should be less than or equal to the operational lifetime of the sensor nodes in order to guarantee continuous operation. Efficient mobile charging schemes are devised for two scenarios: 1) MCs with small battery capacity, and 2) MCs with large battery capacity.

The contributions of this paper can be summarized as follows:

1) A novel periodic mobile charging approach is proposed to minimize the required MCs, as well as to improve their energy efficiency in large scale rechargable WSNs. To the best of our knowledge, this is the first work that jointly considers charging tour planning and depot positioning.
2) To the best of our knowledge, our proposed periodic charging schemes are the first to take into account the association between the MC charging cycle and operational lifetime of sensor nodes. In order to achieve this, we characterize the mobile charging tours in terms of both charging cycle and energy consumption. We demonstrate that the proposed schemes can achieve a good trade-off between the energy efficiency and the number of required MCs/depots for two scenarios, i.e. when the MCs have large battery capacity and when the MCs have small battery capacity.
3) For MCs with large battery capacity, we proposed a periodic charging scheme that limits each MC to serve only a single charging tour but allows it to repeatedly charge the sensor nodes of the tour before returning to the depot for energy replenishment. This overcomes the limitation of existing approaches whereby MCs with large battery capacity return to the depots to replenish themselves before having fully transfered their energy to the sensor nodes, leading to inefficient energy utilization. The proposed scheme first partitions the network into minimal number of un-rooted charging tours, without considering the locations of depots. Depots' locations are then determined based on the charging tour distribution. Finally, multiple charging tours/MCs are assigned to the same depot. We consider both capacitied and uncapacitied depots in this work.
4) We extend the proposed periodic charging scheme for MCs with small battery capacity to allow each MC to serve multiple charging tours in order to maximize the ratio of time spent on charging over the time spent on traveling. Specifically, an MC will spend a longer time to recharge the sensor nodes in a tour. After the MC has recharged the sensor nodes in a particular tour, it will return to the depot and serve a different tour. This overcomes the limitation of existing approaches where the MC with low battery capacity spends only a short duration for charging each sensor node in a particular tour, but needs to make frequent trips which results in poor energy utilization.
5) We provide extensive experimental results to demonstrate the effectiveness of the proposed charging schemes, which not only achieves perpetual operations in WSNs with high network utility but also leads to significant improvement in energy efficiency.

The remainder of the paper is organized as follows. Section II reviews some related works. Section III introduces the background and formulates the energy capacity and lifetime constraint for charging tours. Section IV presents the proposed three-stage approach for mobile charging using multiple MCs with large battery capacity, and Section V extends the approach for the case of using multiple MCs with small battery capacity. Section VI evaluates the performance of the proposed solutions and Section VII concludes the paper.

II. Related Works

In this section, we briefly review some related works on energy replenishment for rechargeable WSNs. Many existing works have focused on energy harvesting techniques that extract environmental energy (e.g. solar, wind, vibration) to replenish the energy in sensor nodes. Even though the existing works on energy harvesting [22]–[24] have shown potential in enabling perpetual operations in WSN, they cannot be readily deployed in practical scenarios due to the reliance of the methods on environmental conditions. This is because, the uncertainty in the energy availability provided by ambient sources raises severe challenges in developing reliable and energy-efficient power-management solutions. Recently there has been a surge of interest in using MCs (mobile chargers) traveling in the network to replenish energy of sensor nodes through strongly-coupled magnetic resonances [13] or radio frequency (RF) signals [25]. In this paper, we focus on energy replenishment solutions using reliable energy sources, i.e., MCs equiped with charging devices, to achieve the perpetual operation of WSNs.

Most of the reported approaches employ optimization algorithms to solve the wireless charging problem. A single MC can sufficiently cater to small scale WSNs while multiple

MCs are required for large scale WSNs due to the limitation on the MC energy capacity and sensor battery. Prototypes of such mobile charging systems are implemented in [17]. Generally, there are two types of charging approaches: periodic charging [15]–[19] and on-demand charging [26]–[28]. The former sends MC(s) to periodically tour the sensor network to recharge sensor nodes, while the latter initiates the charging tour only when requests from sensors are received.

Early research [16], [17] focused on periodic charging strategy that employed a MC, which is equipped with a powerful transceiver and a high-capacity battery, to periodically replenish the energy in sensor nodes. Typically, the MC periodically travels in the WSN and stays near each node for a short period to recharge it. Zhang *et al.* [29] employed multiple energy-constrained MCs with the objective of maximizing the ratio of the amount of payload energy to the overhead energy. However, they only focused on one-dimensional (1D) WSNs. Dai *et al.* [20] extended the problem into 2D WSNs and investigated the minimum number of energy-constrained MCs. Since this work aimed at guaranteeing performance ratio for worst case, its average performance is far from optimal. Hu and Wang [21] investigated the problem of minimizing the number of MCs to ensure that each sensor node operates continuously. Since the resulting charging tours have different charging cycles, charging schedules must be carefully designed in order to avoid conflict when recharging multiple MCs at the depot.

The second approach, i.e., on-demand charging, faces major challenges in the scheduling and coordination of multiple MCs for achieving high charging efficiency. This is due to the fact that the MC trajectories cannot be globally planned as the charging process depends on the dynamic variations of the sensor energy consumption. Madhja *et al.* [30] addressed two main issues in their work: what are the good coordination procedures and good trajectories for the MCs. However, they only addressed the problem for a single charging cycle without taking into account the fact that MCs can repeatedly return to their depots for energy replenishment. The work in [31] considered coordinating multiple MCs with the objective of minimizing the total traveling cost of multiple MCs while ensuring that no nodes will fail. In their work, MCs can return to their respective depots for energy replenishment. The work in [32] modeled the on-demand charging problem as scheduling mobile charging vehicles to charge life-critical sensors in the network. The objective is to minimize the number of mobile charging vehicles deployed, subject to the energy capacity constraint on each mobile charging vehicle. In addition, instead of assuming that a MC must charge a sensor to its full energy capacity before moving to the next sensor, the work in [33] assumed that each sensor can be partially charged. This enabled more sensors to be charged by the MC before its energy depletes.

While the existing on-demand charging approaches adapt better to the variations in sensor energy consumptions compared to the periodic charging methods, they inevitably require large number of MCs to accommodate to the worst case scenario where all the sensors request to be charged. On the other hand, the periodic charging approach can significantly reduce the required number of MCs through judicious charging tour planning and scheduling. Moreover, the solution of periodic charging approach can be regarded as the lower bound of resources (e.g. MCs) required for on-demand charging. However existing implementations suffer from low energy efficiency. In this paper, we focus on developing energy efficient periodic charging planning approaches, by considering the association between the MC charging cycle and the operational lifetime of the sensor nodes. Our approach also jointly considers charging tour planning and depot positioning, which will be discussed in detail in Section III (C).

Most of above mentioned existing approaches assumed that an MC can only charge a single sensor at a time, and no energy transfer is allowed between MCs, or between sensors. There also exists many works that adopted new assumptions of charging models: (1) the approaches in [14] and [34] allow energy to be transferred to multiple receiving nodes simultaneously, (2) the approaches in [35] and [36] assume the availability of multi-hop wireless charging (sensor nodes can relay energy to their neighbors), (3) the work in [37] assumes that MCs are allowed to intentionally transfer energy between themselves. However, these new charging models significantly increase the hardware overhead of the WSNs and decrease the overall energy efficiency. This is because, charging multiple nodes simultaneously requires the MCs to be equiped with complex energy transfer devices. In addition, multi-hop wireless charging requires each relay node to be equiped with charging device and the indirect energy transfer leads to high energy loss. Thus, in order to limit the hardware overhead and loss of wireless energy transfer, we assume that each MC can only charge one sensor at a time via single-hop (direct) wireless energy transfer.

There also exists many works which jointly consider mobile data collection and energy replenishment. Guo *et al.* [38] proposed efficient schemes in which the routing and charging procedures are jointly performed. Moreover, MCs are not only used as energy transmitters to recharge sensor nodes, but also as data collectors to maximize the network lifetime. Their schemes take into consideration data rates at sensors, link scheduling and flow routing, and various sources of energy consumption [39]–[42]. On the other hand, Deng *et al.* [43] jointly considered sampling rate and battery level by tackling the spatiotemporally coupled link and battery capacity constraints in order to maximize network utility for static-routing rechargeable sensor networks. The work in [44] proposed a joint energy replenishment and operation (active/sleep) scheduling mechanism for rechargeable sensor networks using a MC with given charging capacity, with the objective of maximizing the network lifetime while meeting strict sensing guarantees. However, Liang *et al.* [27] advocated that sensor energy charging and data collection should be considered separately, so that data collection protocol designers can concentrate on scheduling the mobile data collectors or routing protocol functionalities for specific applications, without taking energy recharging constraints into account. In this paper, we focus on energy replenishment using MCs instead of jointly considering it with data gathering problems.

TABLE I
LIST OF IMPORTANT NOTATIONS

n	The total number of sensors;
S	$S = \{s_1, s_2, ..., s_n\}$ is the set of sensors;
E_{min}	Minimum energy for sensors to be operational;
E_{max}	Battery capacity of each sensor;
P	The set of all recharging tours;
P_i	The i-th charging tour;
S_i	The set of sensors in the tour P_i;
$Len(P_i)$	The length of the tour P_i;
E_i	The total energy consumption during a recharge cycle for charging P_i;
T_i	Time period of recharge cycle for tour P_i;
t_0	Minimum required time for MCs to stay at its depot;
t_j^i	The charging time allocated to node s_j in tour P_i;
$dist(u, v)$	Euclidean distance between sensors u and v;
$hop(u, v)$	The hop-distance between sensors u and v;
B	Battery capacity of MCs;
p_w	Energy consumption rate of each sensor;
p_r	Energy receive rate of sensors from MCs;
p_c	Working power of MCs for charging;
p_t	Working power of MCs for traveling;
ν	Travel speed of MCs;
D	The set of depots;
d_i	The i-th depot;
$D(P_j)$	The set of depots that can serve tour P_j;
$P(d_i)$	The set of charging tours that depot d_i can serve;

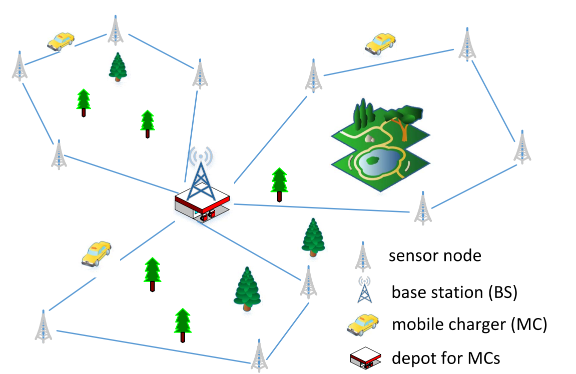

Fig. 1. Illustration of the network model.

III. Network Model and Charging Scheme

The important notations used throughout the paper are listed in TABLE I.

A. Network Model

Fig. 1 illustrates the network model used in our work. We consider n stationary sensor nodes distributed over a two-dimensional region, where the i-th node, denoted s_i, is located at (x_i, y_i). Each sensor node is powered by a rechargeable battery with energy capacity of E_{max}, and needs a minimum energy E_{min} to be operational. Each node consumes energy for data sensing, processing, reception, and transmission. As in many similar scenarios [45]–[47], we assume that the average energy consumption rate, denoted as p_w, is constant and uniform for all sensor nodes. Our approach can also be applied to networks with heterogeneous energy consumption rate by partitioning the sensors into multiple sets such that sensors in the same set have constant and uniform energy consumption rate. Specifically, let $\tau_i = \frac{(E_{max}-E_{min})}{p_i}$ be the lifetime of sensor s_i, where $\tau_1 \leq \tau_2 \leq \tau_3 \leq \cdots \leq \tau_n$. The sensors are partitioned into $\xi+1$ disjointed subsets $V_0, V_1, \cdots, V_\xi$, where ξ is calculated as $\lceil \log_2 \frac{\tau_n}{\tau_1} \rceil$, and sensor s_i with lifetime τ_i is contained in V_k if $2^k\tau_1 \leq \tau_i < 2^{k+1}\tau_1$ ($k = \lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor$) [48]. The charging cycle of sensors in V_k is set to be $2^k\tau_1$. The partitioning strategy can be changed accordingly, e.g. $\xi = \lfloor \frac{\tau_n}{\tau_1} \rfloor$ while sensor s_i with lifetime τ_i is contained in V_k if $k\tau_1 \leq \tau_i < (k+1)\tau_1$ ($k = \lfloor \frac{\tau_i}{\tau_1} \rfloor$).

The MC depots serve as data sinks as well as energy source of the network by periodically dispatching MCs to charge the sensor nodes. The charging tours of MCs start from the respective depots with full battery capacity; when the MCs have completed charging the sensor nodes in the tour, they will return to their respective depots to replenish their energy (i.e. recharging their own batteries).

B. Charging Model

We assume that each MC is energy-constrained by their battery capacity, moves at a speed that is subjected to a certain limit, and the charging time is non-negligible. We assume MCs are homogeneous, i.e. all MCs have the same battery capacity B, the travelling speed is ν, energy consumed by traveling one unit time is p_t, and the energy consumed per unit time when the MC stops and charges sensor nodes is p_c. The battery capacity of the MC is used for both traveling and wireless charging.

Instead of bounding the charging tour of each MC by a constant threshold like most previous works, we characterize the charging tour constraints in terms of energy capacity and charging cycle, where the charging cycle of an MC, denoted as T_i, is defined as the period from the time that a fully charged MC leaves the depot to the time that the MC has been replenished to its full capacity for the next trip (after completing the tour and returning to the depot). Assume that P is the set of all tours for recharging all the sensor nodes in the WSN. Let P_k ($P_k \in P$) be the k-th charging tour, which starts from its respective depot and visits each sensor node in the tour exactly once and recharges them. Formally, let $P_k =< s_0, s_1, \cdots, s_{|S_k|+1} >$ where $s_0 = s_{|S_k|+1}$ is the depot, and S_k be the set of sensors in P_k (not including the depot s_0). The MC needs to spend t_i^k time to recharge each sensor s_i ($s_i \in S_k$) and at least t_0 time at the depot for each trip. The charging cycle of P_k can be calculated as:

$$T_k = t_0 + \sum_{s_i \in S_k} t_i^k + \sum_{i=0}^{|S_k|} \frac{dist(s_i, s_{i+1})}{\nu}, \quad (P_k \in P) \qquad (1)$$

where $dist(s_i, s_{i+1})$ is the distance between nodes s_i and s_{i+1}, and ν is the travel speed of the MC.

Let p_r ($p_r > p_w$) be the energy transfer rate of an MC during charging. To ensure that each sensor node maintains continuous operation and the charging cost of an MC is minimized, the renewable energy cycle of each sensor node should be guaranteed [14]. Therefore, for each sensor node s_i ($s_i \in S_k$), we have the following equation:

$$p_w \times T_k = t_i^k \times p_r, \quad (s_i \in S_k) \tag{2}$$

For more details of the renewable energy cycle, we refer the reader to [34].

Note that since the MCs need to handle different charging tours (with different number of sensors and tour length), the MCs usually have different recharging cycles, i.e., unequal recharging time among sensors affiliated to different MCs. The charging scheme in [29] requires all MCs to have uniform schedule periods. This imposes that the MCs which complete their tours faster will have to wait at the depot for a certain period before starting the next trip. In this case, the charging cycle can be calculated as follows:

$$T_k = t_0 + \sum_{s_i \in S_k} t_i^k + \sum_{i=0}^{|S_k|} \frac{dist(s_i, s_{i+1})}{\nu} + t_k, \quad (P_k \in P) \tag{3}$$

where t_k is the waiting time at the depot for the k-th charging tour and T_k can be predefined. By adding t_k, the time t_i^k for charging a sensor also changes while t_0 and the time for traveling remains the same. In this work, for the ease of managing the charging tours, we also require all MCs to have uniform charging cycle, which is set to be the maximum charging cycle of obtained tours, by properly setting t_k. We refer the readers to reference [16] for approaches related to this issue.

By Formula (1) and (2), we can derive the following:

$$t_i^k = \frac{p_w}{p_r} \cdot T_k, \quad (P_k \in P, s_i \in S_k) \tag{4}$$

$$T_k = \frac{p_r}{p_r - p_w \times |S_k|} \cdot \frac{Len(P_k) + \nu \cdot t_0}{\nu}, \quad (P_k \in P) \tag{5}$$

where $Len(P_k) = \sum_{i=0}^{|S_k|} dist(s_i, s_{i+1})$ is the length of tour P_k and $|S_k|$ is the number of sensors in S_k.

As mentioned before, an MC needs to consume energy for traveling the tour and for wireless charging of the sensor nodes. Hence the total energy consumption in a charging cycle can be calculated as follows:

$$E_k = p_t \cdot \frac{Len(P_k)}{\nu} + p_c \cdot \sum_{s_i \in S_k} t_i^k, \quad (P_k \in P) \tag{6}$$

C. Motivational Example

We first illustrate two motivational examples before presenting the proposed charging scheme. Previous works have assumed that the distance between each sensor and the depot is upper-bounded by a threshold to ensure that the sensor nodes are recharged before the MC runs out of energy due to the limitation of its travel speed and battery capacity. However, this assumption is not applicable to large scale WSNs, and hence multiple MC depots should be utilized. An example is illustrated in Fig. 2 to show the benefit of employing multiple depots. Existing works considering multiple depots assume that the locations of depots are known [28], [48]. Based on the predefined depot locations, the existing works attempt to tour or schedule multiple MCs with different objectives, e.g. minimizing the travel distance, maximizing the charging throughput etc. To the best of our knowledge, our work is the first that jointly considers depot positioning as well as charging tour planning for multiple MCs.

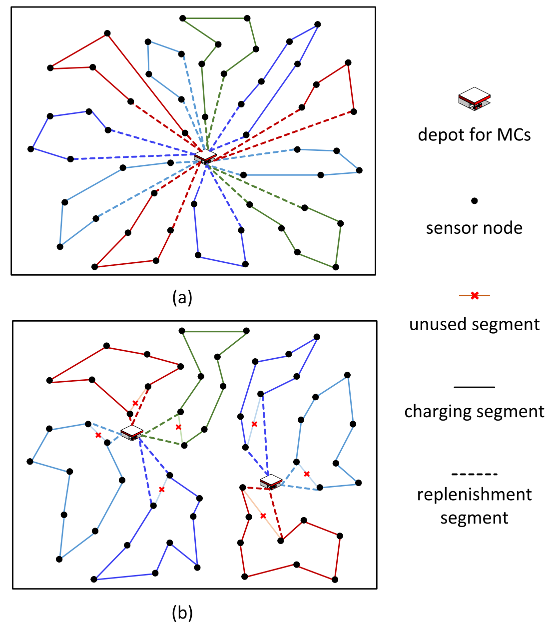

Fig. 2. Comparison of single and multiple depots, (a) one depot can cover the entire network, but requires 11 charging tours with total length of over 34KM due to high ratio of replenishment segment and inappropriate partition of tours, (b) two depots can significantly reduce the replenishment segment which leads to the overall tour length of 24 km. In this case, unrooted tours are first constructed and are assigned to nearby depots by replacing one link of the tour with two links that connects the tour with the depot.

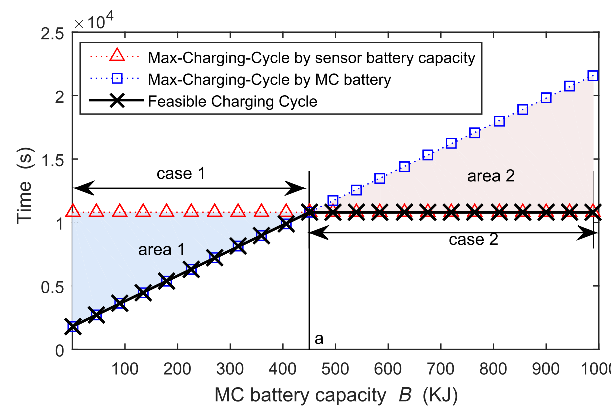

Fig. 3. Analysis on the charging cycle according to battery capacity of sensors and MCs.

Previous works also failed to consider the association between the MC charging cycle and the operational lifetime of the sensors (time that the sensor can remain in operation before the battery completely depletes) during charging tour planning. Fig. 3 illustrates the need to take into account the association between the MC battery capacity and the operational lifetime of the sensor nodes in determining the charging cycle. The sensor operational lifetime is set to a fixed value, i.e., 10260s, while the dotted blue line with square markers in Fig. 3 shows the maximum charging cycle of the MC which can vary from

1800s (time spent at the depot) to 21800s, depending on the MC battery capacity. The solid line with 'X' markers shows the feasible charging cycles of the MC. It can be observed that the feasible charging cycle is constrained by the maximum charging cycle that can be supported by the MC, when its battery capacity $B \leq a$ (case 1) (where a is the sensor operational lifetime). On the other hand, the feasible charging cycle of the MC matches the operational lifetime of sensors, when the battery capacity of the MC, $B > a$ (case 2).

If the MC has small battery capacity, its maximum charging cycle will be smaller than the operational lifetime of the sensors (i.e., case 1 in Fig. 3). Since an MC can only recharge the sensors for a short duration due to its limited battery capacity, only a small amount of energy can be transfered to the sensors per tour. This results in more frequent charging of the sensors which incurs high energy inefficiency in the MCs as most of its energy will be spent on traveling instead of charging. Conversely, if the MC has large battery capacity, its maximum allowable charging cycle is larger than the operational lifetime of the sensors (i.e., case 2 in Fig. 3), so the feasible charging cycle will depend on the operational lifetime of sensors. Thus when the MC's battery capacity is larger than the operational lifetime of the sensor nodes, the MCs return to the depots to replenish themselves before having fully transfered their energy to the sensor nodes, leading to low energy utilization of the MC battery capacity.

The observations above reveal that MCs with small and large battery capacity must adopt different charging strategies. In this paper, we develop novel charging schemes to deal with both situations. For case 1, we propose a scheme that allows each MC to take different (multiple) charging tours. In this way, the ratio of charging time to traveling time is maximized, since each sensor can be recharged from almost an empty state to its full capacity. Moreover, the number of required MCs will be significantly reduced as one MC can serve multiple tours. For case 2, each MC will travel its charging tour multiple times before going back to its depot to replenish its own energy. This ensures that the MC battery capacity is more efficiently utilized for charging the sensor nodes. The proposed charging schemes will jointly consider charging tour planning and depot positioning. Note that our approach does not simply attempt to minimize the number of depots, as this could significantly decrease the energy efficiency due to the increased proportion of replenishment segment (as shown in Fig. 2).

The proposed charging scheme comprises of the following three major stages:

- Construct minimum number of un-rooted charging tours with the objective of minimizing the required MCs while mximizing their energy efficiency.
- Identify and reduce the candidate locations for deploying MC depots.
- Determine locations of depots and assign charging tours/MCs to depots with the objective of maximizing energy efficiency.

In the following sections, we first propose the three-stage approach to address the problem in case 2 of Fig. 3 (i.e. the MCs are equipped with batteries of large capacity, hence they have long maximum charging cycles), then adapt the approach for case 1 (i.e. the MCs are equipped with batteries of small capacity, hence they have short maximum charging cycles).

IV. Charging Scheme for MCs With Large Battery Capacity

In existing approaches, the charging cycle of the MC must be bounded by the operational lifetime of the sensor nodes. Otherwise, perpetual operations of the sensor node cannot be guaranteed. This restriction leads to low energy efficiency as we have previously discussed. In order to improve the energy utilization of an MC with large battery capacity, we propose that each MC visits its tour multiple times before returning to the depot for energy replenishment. In this way, the actual charging cycle of an MC could be much longer than the operational lifetime of the sensor nodes, even if the time T_k of a single trip is shorter than the sensor lifetime. This scheme aims to reduce the frequency of returns to the depot that the MC with large battery capacity makes, hence increasing the ratio of energy for recharging over the energy spent on traveling.

A. Stage 1: Charging Tour Planning for MCs Without Considering Depots

The proposed charging tour planning algorithm does not assume that the depots' locations are known. In order to address the tour planning problem for this stage, we introduce a new parameter, i.e., the maximum allowable distance $dist$ from the return point of the charging tour to the depot.

Problem S1: Given a set of sensors S (with parameters p_w, E_{max}, and E_{min}) for which distances between all sensors formed a metric on the set, find the minimum number of required charging tours (with constraints of MC parameters $p_r, B, p_t, p_c, t_0, dist$), such that each sensor is included in one tour, each MC serves a single tour, and none of the sensors will run out of energy before being recharged.

$$\textbf{Min} \quad |P| \tag{7}$$

Subject to

$$T_k + \frac{2 \cdot dist}{\nu} \leq (E_{max} - E_{min})/p_w, \quad (\forall P_k \in P) \tag{8}$$

$$E_k + \frac{2 \cdot dist}{\nu} \cdot p_t \leq B, \quad (\forall P_k \in P) \tag{9}$$

$$S_k \cap S_j = \phi, \quad (P_k, P_j \in P, k \neq j) \tag{10}$$

$$\bigcup_{P_k \in P} S_k = S \tag{11}$$

where T_k is the total time for the tour P_k, including both charging time and traveling time, while E_k is the total energy required by performing tour P_k. Constraint (8) reserves a certain amount of time for an MC to return to its depot for energy replenishment, while constraint (9) reserves a certain amount of energy, i.e., $B_0 = p_t \cdot \frac{2 \cdot dist}{\nu}$. Constraint (10) and (11) ensure each sensor is included into exactly one tour. The number of trips that an MC takes before returning to its depot is set to $\lfloor \frac{B - B_0}{E_k} \rfloor$.

In our proposed algorithm, a minimum spanning tree (MST) T rooted at $root$ is first constructed on all sensors. Note that

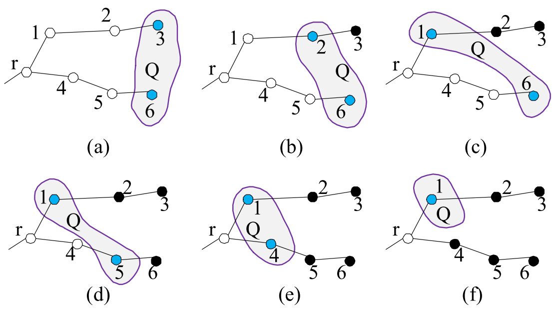

Fig. 4. Illustrative example of Algorithm 1.

any sensor that is located close to the centre of the network can be the $root$. The leaf nodes of T are then inserted into a queue Q. Next, the algorithm iteratively determines charging tours by finding a subset of sensor nodes from the MST T, where each subset of sensor nodes forms a charging tour. The start node of each charging tour, v, is chosen as the farthest node from the $root$. Formally,

$$v \leftarrow \arg\max_{w \in Q}\{dist(w, root)\}. \tag{12}$$

The node v is then treated as an initial (partial) tour with tour length of 0 and v is removed from tree T. If v's parent node, x, becomes a leaf node in T, x will be inserted into set Q.

The algorithm then repeatedly expands the current tour, by moving new nodes into current tour P_k from candidate set Q, until the constraint on energy or charging cycle is violated. Let S_k be the set of sensors in current tour P_k. The best candidate for inclusion into P_k is the one that is closest to the set S_k, which can be calculated as

$$u \leftarrow \arg\min_{v \in Q}\{\sum_{w \in S_k} dist(v, w)/|S_k|\ \}. \tag{13}$$

The algorithm can successfully add u into tour P_k if the resultant tour does not violate the constraints on sensor lifetime and MC energy capacity, and u will be added into set S_k, removed from Q and from tree T. In addition, if u's parent node, x, turns into a leaf node due to the deletion of u from T, then node x will be included into Q. If it cannot include u without violating constraints on sensor lifetime or MC energy capacity, a new tour will be created. The algorithm terminates when Q becomes empty.

Fig. 4 illustrates an example of Algorithm 1. Fig. 4 (a) shows the minimum spanning tree T, and current $Q = \{3, 6\}$, where 3 is the farthest node from root r. In Fig. 4 (b), node 3 is moved from Q into current tour set S_k. Node 2 becomes a leaf node due to the deletion of node 3 from T, and hence node 2 is added into Q. At this point, node 2 is the best candidate for inclusion into set S_k of the current tour. This process is repeated in Fig. 4 (c) - (e). In the next iteration as shown in Fig. 4 (f), $Q = \{1\}$ which implies that node 1 is the only candidate node that can be moved into S_k. The above process repeats until the current tour violates the constraints on sensor lifetime or MC energy capacity, or when there are no more unvisited nodes.

Constructing MST T using Prim algorithm runs in $O(n^2)$. Next, finding the farthest node from root of T runs in $O(n)$ and it repeats at most n times. The purpose of the while-loops in Algorithm 1 is to include sensors into MC tours, where each iteration attempts to include one sensor into a moving tour. Most sensor nodes can be included into a tour after one iteration, while a small number of nodes require another iteration when the resultant tour violates the constraints on sensor lifetime or MC energy capacity. The number of such sensors equals to $|P|$, where $|P| < n$ is the number of the resultant tours. As such, the while-loop repeats at most $O(n+|P|) = O(n)$ times. In addition, in each iteration of the while-loop, finding the nearest node has a time complexity of $O(n)$ and finding a TSP solution runs in $O(\delta^2)$ [49], where δ is the maximum number of sensor nodes in any tour. Thus, finding TSP solutions for all tours requires $O(n \cdot \delta^2))$ time. The runtime complexity of Algorithm 1 is therefore, $O(n^2) + O(n \cdot\ (n + \delta^2)) = O(n^2 + n \cdot \delta^2)$. Note that the maximum number of sensor nodes in any charging tour, δ, bounded by $B/(E_{max} - E_{min})$.

Algorithm 1 Charging Tour Planning

Input: sensor set S, Parameters $E_{max}, E_{min}, p_w,$
$B, p_r, p_c, p_t,\ t_0, \nu$
Output: set P of minimum number of unrooted charging tours.
$T \leftarrow$ $\texttt{Prim}(S, root)$; /* minimum spanning tree */
Put all leaf nodes into queue Q; $k \leftarrow 0$;
while $(Q \neq \phi)$ **do**
  $k \leftarrow k + 1$; /* Enable a new tour */
  $v \leftarrow \arg\max_{w \in Q}\{dist(w, root)\}$; /* farthest node from $root$ */
  $P_0 \leftarrow < v >$; $P_k \leftarrow P_0$; $S_k \leftarrow$ set of sensors in P_k;
  $T \leftarrow T - \{v\}$; $Q \leftarrow Q - \{v\}$;
  Put v's parent p into Q if p becomes a leaf node;
  while (tour P_0 is valid and $Q \neq \phi$) **do**
    $u \leftarrow \arg\min_{v \in Q}\{dist(v, S_k)\}$; /* nearest node */
    $P_0 \leftarrow$ $\texttt{TSP}(S_k \cup \{v\})$ [49];
    if (tour P_0 is valid) **then**
      $P_k \leftarrow P_0$; $S_k \leftarrow S_k \cup \{u\}$; $T \leftarrow T - \{u\}$;
      $Q \leftarrow Q - \{u\}$;
      Put u's parent p into Q if p is a leaf node;
  $P \leftarrow P \cup P_k$;

B. Stage 2: Identifying and Reducing the Candidate Locations for Depots

In the second stage, we will identify a minimal set of candidate locations for depots. We assume that the location of each sensor (or its nearby area) is suitable for deploying MC depot and hence, the locations of all sensors are included as the initial candidate set. It is noteworthy that the proposed approach is applicable for any given set of candidate depot locations.

Let $D = \{d_1, d_2, \cdots, d_n\}$ be the set of candidate locations for potential MC depots. We will not distinguish between location d_i and depot d_i. The Algorithm 2 for identifying and reducing the candidate locations for depots is as follows.

1) For each d_i, we create a set $P(d_i)$ consisting of tours that the depot d_i can serve. That is, tour $P_k \in P(d_i)$ if there exists at least one tour segment (u, v) $(u, v \in P_k)$ such that $dist(u, d_i) + dist(v, d_i) - dist(u, v) < dist$. This means that if the MC returns to the depot to replenish its battery from point u or v, no sensors in tour P_k will run out energy according to Eq. (8).
2) Among the initial depot candidates, a candidate location d_i can be removed if all the tours it serves can be served by another candidate location d_j, i.e., $P(d_i) \subseteq P(d_j)$.

Clearly, step 1) of Algorithm 2 repeats $O(|P| \cdot |D| \cdot \delta^2)$ times, where δ is the maximum number of sensors in any tour. The time complexity for Step 2) is $O(|D|^3)$ as it repeats for $|D|^2$ times and takes $O(|D|)$ for checking whether $P(d_i) \subset P(d_j)$. Therefore, the time complexity of Algorithm 2 is $O(|P| \cdot |D| \cdot \delta + |D|^3)$.

C. Stage 3: Depot Deployment and Charging Tour Assignment

To determine the number of MC depots needed to serve all the charging tours, we formulate an optimization problem, which must satisfy the following: 1) there exists at least one MC depot for each charging tour, and 2) each charging tour can only be assigned to a single depot. In addition, we consider two scenarios where the depot is unbounded (uncapacitied) and bounded (capacitied). A capacitied depot can only serve a certain number of MCs per charging cycle, whereas an uncapacitied depot can serve any number of MCs per charging cycle. In this paper, uncapacitied depots indicate that the depots could be heterogeneous, (i.e., a depot can be equipped with as many charging-piles as required), while each of the capacitied depots can only serve charging tours up to a given number.

1) Uncapacitied Depots:

Problem: Given a set of charging tours P and a set of candidate depot locations D, find the minimum number of required locations for deploying depots with unbounded service ability and assign charging tours to the selected depots such that each charging tour is served only by one MC depot without violating the energy and operational lifetime constraints.

Let binary $x_i = 1$ if location d_i is selected as an MC depot, and $x_i = 0$ otherwise. Let binary $y_{j,i} = 1$ if charging tour P_j is assigned to depot d_i, and $y_{j,i} = 0$ otherwise. Formally,

$$\textbf{Min} \quad \sum_{i=1}^{|D|} x_i \tag{14}$$

Subject to

$$y_{j,i} \leq x_i, \quad (\forall P_j \in P, d_i \in D) \tag{15}$$

$$\sum_{i=1}^{|D|} y_{j,i} = 1, \quad (\forall P_j \in P) \tag{16}$$

$$x_i, y_{j,i} \in \{0, 1\}, \quad (\forall P_j \in P, d_i \in D) \tag{17}$$

Constraint (15) ensures that an MC depot has to be deployed if any charging tour chooses it as its designated depot, and constraint (16) ensures that each charging tour can only be assigned to one depot. Since the above binary linear programming problem is NP-hard, we develop a heuristic algorithm with polynomial time complexity. The reduced set D of candidate locations d_i $(d_i \in D)$ and the sets of potential charging tours for each d_i, can be represented in TABLE II.

TABLE II
ILLUSTRATION OF COVERAGE DISTRIBUTION OF CANDIDATE DEPOTS

	d_1	d_2	d_3	d_4	d_5	$\cdots$	$d_{\lvert D\rvert}$	$\sum$
P_1	0	1	1	0	0		0	≥ 1
P_2	1	0	0	1	1		0	≥ 1
P_3	1	0	1	0	0		1	≥ 1
P_4	0	1	0	0	0		0	≥ 1
P_5	0	0	1	1	1		1	≥ 1
$\cdots$								
$P_{\lvert P\rvert}$	0	1	0	0	1		0	≥ 1

Since a depot is uncapacitied, whenever a depot, d_i, is selected, we can assume that any charging tour in $P(d_i)$ has been covered (assigned to at least one depot), even though a tour P_j $(P_j \in P(d_i))$ may not necessarily be assigned to depot d_i due to load balancing or energy efficiency considerations. In other words, the greedy selection manner can always produce feasible solutions for uncapacitied depot deployment. Our greedy algorithm consists of two phases: 1) greedy column selection and 2) greedy assignment of charging tours. The first phase selects the minimum number of columns such that the sum of each row is greater than or equal to 1, which indicates that the charging tour can be served by at least one depot. Then in the second phase, each charging tour is assigned to a desired depot.

In the first phase, for each column, we add up all the elements and the sum is called the weight of the column. This weight indicates the number of charging tours that can be covered by the depot d_i, and the higher the weight is, the stronger the coverage/service ability of the depot. We then choose the column with the largest weight and remove this column as well as rows that are covered by this column. The weights are then updated for the remaining columns and rows, and another column with the largest weight is selected. This process repeats until all the rows are removed which means all the charging tours can be served by at least one depot. Finally, these selected columns are the locations of the depots that should be deployed.

In the second phase, we assign charging tours to the depots, with consideration of the fact that the charging/service capacity of a depots is not continuous, meaning they are discrete values. For example, a depot with x charging-piles can serve at most $x \cdot Y$ tours (assume each charging-pile can serve Y tours at most). The number of charging tours affiliated to depot d_i, denoted as $Load(d_i)$, is utilized to indicate the load of the depot. It is desired that the $Load(d_i)$ is in the form of $x \cdot Y$. Thus, when a tour P_j can be served by multiple depots, a priority mechanism is required to select the best depot. Suppose tour P_j can be covered by depot d_i whose current load is $Load(d_i)$, then $w_i = (Y - Load(d_i)\%Y)\%Y$ is utilized to indicate the priority of d_i for being selected as P_j's depot. In this stage, among all the charging tours, the one that should be considered with first priority is the tour with smallest $|D(P_j)|$, where $D(P_j)$ is the set of depots that can serve P_j, because P_j has the least chance of being covered by depots. When assigning P_j to a depot, the depot with the highest w_i from all P_j's candidate depots is selected. If two depots are of the same priorities, P_j will be assigned to the closest one.

Algorithm 3 Deploying Uncapacitied Depots

Input: tour set P, set of depots D, $P(d_i)$, $D(P_j)$.
Output: Final depot locations D', assignment of tours to depots.

Step 1: greedy column selection.
$P^{temp} \leftarrow P$; $D^{temp} \leftarrow D$; $D' = \phi$;
while $(P^{temp} \neq \phi)$ **do**
  $D_{max} \leftarrow \arg\max_{D_i \in D^{temp}} \{|D_i|\}$; remove D_{max} from D^{temp};
  $D' \leftarrow D' \cup \{depot(D_{max})\}$;
  for each $P_j \in D_{max}$ **do**
    remove P_j from P^{temp};
  update each $D_i \in D^{temp}$ according to P^{temp};
Step 2: charging tour assignment.
$P^{temp} \leftarrow P$;
while $(P^{temp} \neq \phi)$ **do**
  $P_j \leftarrow \arg\min_{P_j \in P^{temp}} \{\ |DP(P_j)|\ \}$;
  $d_i \leftarrow \arg\max_{d_k \in DP(P_j)} \{(Y - Load(d_i)\%Y)\%Y\}$;
  assign P_j to d_i; update $Load(d_i)$;
  remove P_j from P^{temp};

It is easy to prove the approximation ratio of Algorithm 3 by showing that the problem is equivalent to the set cover problem in the following way. For a given set P of charging tours, set D of depots, and $P(d_i)$ for $1 \leq i \leq |D|$, we create an universe U of $|P|$ elements such that each element corresponds to a tour in P, and create $|D|$ sub-sets, $U_1, U_2, \ldots, U_{|D|}$, such that sub-set U_i ($1 \leq i \leq |D|$) contains elements that correspond to the charging tours in $P(d_i)$. Thus, it is easy to verify that there is a feasible solution for our problem if and only if there is a corresponding solution for the obtained set cover problem. In addition, our column selection algorithm adopts the same greedy strategy as the greedy algorithm for solving set cover in [50]. Thus, our column selection algorithm can guarantee performance ratio of $H(|D_{max}|)$ where $D_{max} = \arg\max_{P_j \in P}\{|D(P_j)|\}$ and H is the harmonic function, defined as $H(a) = \sum_{i=1}^{a} 1/i$.

The time complexity of Algorithm 3 is derived as follows. In each iteration of the while-loop for greedy column selection, finding the depot of maximum coverage ability runs in $O(|D|)$, the for-loop runs in $O(|P|)$, and the update operation runs in $O(|D| \cdot |P|)$. Thus the first phase has a time complexity of $O(|D|^2 \cdot |P|)$. The second phase has a time complexity of $O(|D| \cdot |P|)$. Therefore, the time complexity of Algorithm 3 is $O(|D|^2 \cdot |P|)$.

2) Capacitied Depots: The problem of deploying capacitied depots and charging tour assignment can be formulated as follows.

$$\textbf{Min} \quad \sum_{i=1}^{|D|} x_i \tag{18}$$

Subject to

$$y_{j,i} \leq x_i, \quad (\forall P_j \in P, d_i \in D) \tag{19}$$

$$\sum_{i=1}^{|D|} y_{j,i} = 1, \quad (\forall P_j \in P) \tag{20}$$

$$\sum_{P_j \in P} y_{j,i} \leq Y, \quad (\forall d_i \in D) \tag{21}$$

$$x_i, y_{j,i} \in \{0,1\}, \quad (\forall P_j \in P, d_i \in D) \tag{22}$$

where constraint (21) ensures that any depot serves at most Y charging tours.

For deploying capacitied depots, we cannot simply perform the greedy algorithm for column selection, since a column may cover more charging tours than its service capacity. As a result, the column selection and charging tour assignment must be jointly considered. In the proposed algorithm, a priority mechanism is used to evaluate both depots and charging tours.

The general idea of the proposed Algorithm 4 can be summarised as follows. The algorithm first identifies the charging tour with highest priority, say P_j. Then it finds a best candidate depot, d_i, as the one with highest priority among all depots that can cover P_j to be P_j's designated depot. Next, more charging tours from $P(d_i)$ are assigned to d_i based on tours' priority until d_i's capacity is reached or none of the tours in $P(d_i)$ is left unserved/uncovered, where $P(d_i)$ is the set of tours that can be covered by depot d_i. The above process repeats until all tours are assigned to their depots. Note Algorithm 4 takes the original set of candidate depots as input, instead of the reduced one by Algorithm 2. We next explain the mechanisms for deciding the priority of charging tours as well as candidate depots.

(1) Priority for choosing a charging tour to be assigned to depot is determined as follows. As defined earlier, $|D(P_j)|$ is the number of candidate depots that can serve as P_j's depot, thus small $|D(P_j)|$ indicates that tour P_j has lesser chance of being covered than other tours. Therefore, the tour P_j with smallest $|D(P_j)|$ will be given highest priority than other tours.

(2) When selecting the charging depot for a given tour P_j, the depot with higher coverage ability, i.e. larger $|P(d_i)|$, will be given higher preference as they tend to minimize the number of required depots. In addition, a depot that can serve a charging tours with small $|D(P_j)|$ should also be given higher priority, because this tour has lesser chance of being covered. Based on the above analysis, $w_i = \frac{|P(d_i)|^2}{\sum_{P_j \in P(d_i)} |D(P_j)|}$ is used to indicate the priority of different candidate depots, where $\frac{\sum_{P_j \in P(d_i)} |D(P_j)|}{|P(d_i)|}$ indicates the average $|D(P_j)|$ among all tours in $P(d_i)$. Candidate depot with larger w_i will be considered with higher priority.

The time complexity of Algorithm 4 can be analyzed as follows. Outside the while-loop, computing w_i for each depot d_i runs in $O(|P| \cdot |D|)$. Within the while-loop, there are four operations that are repeatedly performed: 1) find P_j with

Algorithm 4 Deploying Capacitied Depots

Input: Tour set P, depot set D, $P(d_i)$, $D(P_j)$, depot capacity Y.
Output: final depot set D', assignment of tours to depot.
$P^{temp} \leftarrow P$; $D^{temp} \leftarrow D$; $D' = \phi$;
$w_i \leftarrow \frac{|P(d_i)|^2}{\sum_{P_j \in P(d_i)} |D(P_j)|}$ for each $d_i \in D$;
while $(P^{temp} \neq \phi)$ **do**
  Step 1: select a new depot according to the tour of highest priority.
  $P_j \leftarrow \arg \min_{P_j \in P^{temp}} \{ |D(P_j)| \}$;
  $d_i \leftarrow \arg \max_{d_k:(P_j \in P(d_k) \cap d_k \in (D^{temp} - D'))} \{ w_k \}$;
  Remove P_j from P^{temp}; Remove d_i from D^{temp};
  Assign P_j to d_i; $D' \leftarrow D' \cup \{d_i\}$;
  Update $P(d_k)$ and w_k for any d_k that $P_j \in P(d_k)$ due to remove of P_j;
  Step 2: fill up depot d_i to full capacity.
  while $P^{temp} \cap P(d_i) \neq \phi$ and d_i not overloaded **do**
    $P_j \leftarrow \arg \min_{P_j \in P^{temp} \cap P(d_i)} \{ |D(P_j)| \}$;
    Remove P_j from P^{temp}; Assign P_j to depot d_i;
    Update $P(d_k)$ and w_k for any d_k that $P_j \in P(d_k)$;
  Update $D(P_j)$ for any $P_j \in P(d_i)$ due to remove of d_i;

highest priority, 2) find d_i with highest priority, 3) update w_i for any d_i where $P_j \in P(d_i)$, 4) update $D(P_j)$ for any $P_j \in P(d_i)$. Operation 1) runs in $O(|P|)$ and repeats at most $O(|P|)$ times. Operation 2) runs in $O(|D|)$ and repeats at most $O(|D|)$ times. Operation 3) runs in $O(|D|)$ and repeats at most $O(|P|)$ times and finally, operation 4) runs in $O(|P|)$ and repeats at most $O(|D|)$ times. Therefore, the time complexity of Algorithm 4 is $O(|P|^2 + |D| \cdot |P| + |D|^2) = O(|P|^2)$, since $|D| \leq |P|$.

V. MCs With Small Battery Capacity

As in the previous section, a tour planning solution can be obtained by: (1) obtain the charging tours using Algorithm 1, (2) identify and refine the set of candidate depots using Algorithm 2, and 3) allocate depots to appropriate locations and assign charging tours to depots using Algorithm 3 or Algorithm 4. The approach works well for the case of MCs with large batteries, but leads to poor quality for the case of MCs with small batteries. This is because when MCs with small batteries are utilized, the charging cycle of tours obtained from Algorithm 1 is lesser than the lifetime of sensors since each tour must satisfy the battery constraint of MCs. As such, during each small charging cycle, each sensor can only consume a part of its energy capacity, and a lot of energy is left unused. This indicates that an MC can only transfer a small amount of energy to the sensors while the energy spent on traveling remains the same, thus leading to low energy efficiency. In addition, the above approach will not produce the minimum number of required MCs and depots as it employs Algorithm 1 for charging tour planning, which does not suitably address the situation where an MC with small battery can actually serve multiple charging tours. In this section, we adapt the approach in the previous section to cater to the case of using MCs with small batteries by proposing Algorithm 5 to replace Algorithm 1.

In order to guarantee that each MC can serve as many charging tours as possible, Algorithm 5 iteratively finds a large charging tour based on sensor lifetime constraint and then partitions the large tour into several sub-tours such that each sub-tour satisfies the MC battery constraint while the sum of charging cycles of all the sub-tours does not exceed the sensor lifetime constraint. The sub-tours from the same charging tour will be served by one MC.

Let US be the set of sensors that has not been covered by a charging tour. The same strategy employed in Algorithm 1 is used for charging tour planning on the set US. After a large tour P' is constructed, Algorithm 5 partitions P' into multiple sub-tours that are rooted at center node s_c, where s_c is the location that has the minimum distance to all other sensors in P', calculated as

$$s_c \leftarrow \arg \min_{s_i \in P'} \sum_{s_j \in P', i \neq j} dist(s_i, s_j). \tag{23}$$

s_c will be the initial return point of the sub-tours to their depot, and each sub-tour will refine its return points accordingly after the tour to depot assignment. Suppose that an MC can at most serve q sub-tours, which can be estimated based on the parameters, it is possible that q sub-tours cannot include all the sensors in P'. The sensors that have not been included into the q sub-tours will be moved back into US so that they can be used for forming other tours. The above process repeats until all sensors are included in a sub-tour. Algorithm 5 enhances the likelihood that an MC has exactly q sub-tours.

Algorithm 5 Tour Planning Algorithm Based on Partitioning

Input: Sensor set S, E_{max}, E_{min}, p_w, $B, p_r, p_c, p_t, t_0, \nu$.
Output: the set of obtained rooted sub-tours
$k \leftarrow 0$; $US \leftarrow$ set of uncovered sensors;
while $US \neq \phi$ **do**
  Construct a sensor lifetime based big tour, P', on set US based on Algorithm 1;
  $s_c \leftarrow \arg \min_{s_i \in P'} \sum_{s_j \in P', i \neq j} dist(s_i, s_j)$;
  Partition the P' into sub-tours rooted at s_c using Algorithm 6;
  Uncovered sensors of P' will be put back into US;

The process of partitioning P' into multiple sub-tours that are rooted at s_c is described as follows. First, a graph $G' = (V, E)$ is constructed, where V is the set of all sensors on P' and E is the edge set such that there is an edge (u, v) in E if $dist(u, v) < TH$ for $u, v \in V$, $u \neq v$. TH is set to the minimum length that makes G' a connected graph and TH can be obtained in linear time. Using the auxiliary graph G' prevents the network from being splitted into fragments when constructing sub-tours, which will lead to significant reduction in travel distance. Note that in this partitioning step, we only consider sensors in tour P'.

Algorithm 6, for creating a sub-tour P_i based on a large tour P', works in the following way. Let P_i be a sub-tour of P', then its required energy is calculated as

$$E_i = p_t \cdot \frac{Len(P_i)}{\nu} + p_c \cdot \frac{E_{max} - E_{min}}{p_r} \cdot |S_i|, \quad (24)$$

where $\frac{E_{max}-E_{min}}{p_r}$ is slightly larger than the charging time required by each sensor in tour P'. This can avoid producing invalid tours while adjusting all the sub-tours to have uniform charging cycle. Assume that sub-tour $P_1, P_2, \ldots, P_{k-1}$ have been constructed on P', it now tries to construct the kth sub-tour P_k. We use S_k to denote the set of sensors in P_k except s_c. NS is used to denote the uncovered neighbor set of all covered sensors in sensor set S before P' is created, i.e., each sensor in NS is uncovered and it connects to at least one covered node. Similarly, we define NSP as the uncovered neighbor set of covered sensors in P', i.e., each sensor in NSP is an uncovered sensor in P' and it connects to at least one covered node in $(\bigcup_{i=1}^{k-1} P_i) \setminus \{s_c\}$. NSP is initialized as P'. The first node to be included into P_k is selected in the following way. If $NS \cap NSP$ is non-empty, it selects the sensor s_i in $NS \cap NSP$ that is the farthest sensor away from s_c. Otherwise, it selects the sensor s_i in NSP that is the farthest sensor away from s_c. Then the sub-tour P_k is initialized as $< s_c, s_i, s_c >$ with tour length $2 * dist(s_i, s_c)$. Next, it iteratively adds nodes into sub-tour P_k while ensuring that the energy constraint of MC is not violated. The strategy for extending P_k to a sub-tour by adding more sensors is defined as follows. When choosing the best candidate for inclusion into the current tour P_k, (1) If $A = NS \cap NSP \neq \phi$, choose the closest node to s_c in set A, (2) Else if $NSP \neq \phi$, choose the node closest to s_c in set NSP, (3) Otherwise, choose the uncovered node closest to s_c in set P'. We omit the pseudo-code of Algorithm 6 for constructing the sub-tour.

Fig. 5 shows an example of using Algorithm 6 to form a sub-tour P_k via partitioning the large tour P'. Fig. 5 (a) illustrates the covered area, the neighbor set NS of the covered area, and the large tour P' constructed based on Algorithm 1. Since $NS \cap NSP = \{1, 2\}$ (NSP is initialized as P') and $dist(1, s_c) > dist(2, s_c)$, thus 1 is selected as the first node to be included into sub-path P_k, as shown in Fig. 5 (b). At this stage, $NS \cap NSP = \{2\} \neq \phi$, thus node 2 will be included into P_k as shown in Fig. 5 (c), where $NSP \cap NS$ becomes empty and $NSP = \{3, 4, 5\}$. In the next iteration, node 3 of NSP, which is closest to set S_k, will be included into tour P_k, where $NSP \cap NS = \Phi$ and $NSP = \{4, 5, 6\}$. This process repeats until no more nodes can be added into P_k without violating the MC battery capacity constraint.

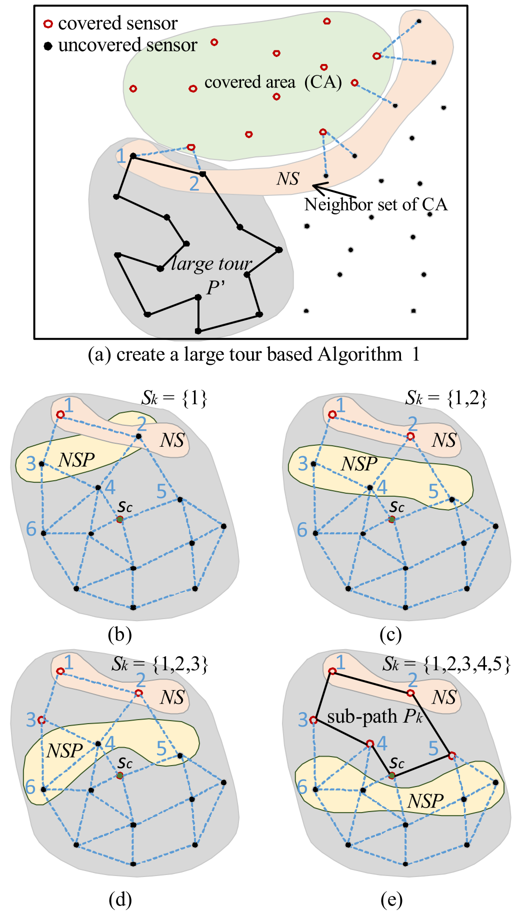

Fig. 5. Create a sub-tour based on a large tour P'.

VI. Results and Analysis

The performance metrics include the number of required charging tours n_t, number of MCs n_{mc}, charging depots n_d, the average ratio τ_c/τ_t of total charging time to total traveling time of all tours, and γ_l which is the average ratio of replenishment segment over the charging tour. Formally,

$$\tau_c/\tau_t = \frac{\sum_{P_k \in P} \sum_{i \in P_k} t_i}{\sum_{P_k \in P} \frac{Len(P_k)}{\nu}}, \quad (25)$$

$$\gamma_l = 1/|P| \cdot \sum_{P_k \in P, (a,d_k),(b,d_k) \in P_k} \frac{dist(a, d_k) + dist(b, d_k)}{Len(P_k)}, \quad (26)$$

where P is the set of all charging tours, $Len(P_k)$ is the length of tour P_k, and ν is the travel speed of the MC, $(a, d_k), (b, d_k)$ are the two links adjacent to depot d_k in tour P_k, and $dist(a, d_k)$ is the distance from a to d_k.

A. Baseline Approach

To the best of our knowledge, the algorithm reported in [20], denoted as `Alg-14`, is the most similar to ours, and hence we use it as one of the baseline algorithms to evaluate our approach. This paper investigated the problem of minimizing the number of MCs where only a single MC depot (located at the center of the WSN) is available. In order to make a fair comparison with `Alg-14`, when collecting results of our proposed algorithms, we considered both single depot scenarios and multi-depot scenarios.

We also developed an approximation algorithm as a baseline for charging tour planning that can guarantee performance ratio of 5. The approximation algorithm, denoted as `5-approx`, works in the following way: (1) For a given k, the algorithm first constructs a forest F consisting of k connected component, i.e., $C_1, C_2, \ldots, C_k$. (2) For each i ($1 \leq i \leq k$), it constructs a TSP solution as $P_i \leftarrow \texttt{TSP}(C_i)$ [49] and deletes one link so that P_i becomes a path. (3) For each i, it iteratively partitions a sub-path from P_i as long as total weight of P_i is larger than $L/2$, where L is the upper bound (i.e., charging cycle or energy consumption) on each tour. (4) Link the first node and the last node of each sub-path to form a closed tour. The algorithm tries all the possibilities of k and chooses the best one. Detailed description and the proof of the performance ratio is found in the appendix.

For depot deployment and charging tour assignment, we developed two modified algorithms for comparison. In the case of uncapacited depot, the algorithm randomly selects an uncovered charging tour, P_i, and identifies the best candidate depot, d_j (depot that can cover more uncovered charging tours), to cover the tour P_i. All the tours that can be covered by d_j will be updated as covered. The processs repeats until all charging tours are covered. In the case of capacited depot, similar operation is performed except that each depot can only serve a certain number of charging tours. The two baseline algorithms are denoted as `base_uncap` and `base_cap` respectively.

B. Simulation Environment

In the simulations, we consider a generic sensor network with n sensors randomly distributed over an $l \times l$ square area. In order to compare with the existing work, we limit the interest area to 2000×2000 for the first three group of experiments, so that `Alg-14` can have feasible solutions. Unless otherwise specified, we use the following parameter settings as in [16] and [20]: $\nu = 5\ m/s$, $p_t = 55w$, $p_c = 50w$, $p_r = 20\ w$, $t_0 = 3600\ s$. Considering the randomness of the network topology, the result of each case is averaged over 20 random simulation instances. For the first three groups of comparisons, we only collect results for depots with unlimited capacity based on the assumption in [20]. For the other two groups of comparisons, we illustrate results for both uncapacitied and capacitied depots, where the number of MCs that a capacitied depot can serve is set to 10 in the experiments.

C. MC Battery Capacity Matches Sensor Lifetime

Fig. 6 shows the performance comparison of baselines `Alg-14`, `5-approx` and our algorithms using both single (`new-single depot`) and multiple depots (`new-multi depot`), in terms of number of depots n_d, number of charging tours n_t, τ_c/τ_t which reflects the energy efficiency, and the ratio γ_l of replenishment segment over its complete charging tour. In general, it is easy to understand that the increase in the number of sensors leads to more number of charging tours as well as depots (for the multi-depot case). For our proposed approach, we collect results for the case when only single depots are deployed as well as for the case where multiple depots are deployed,

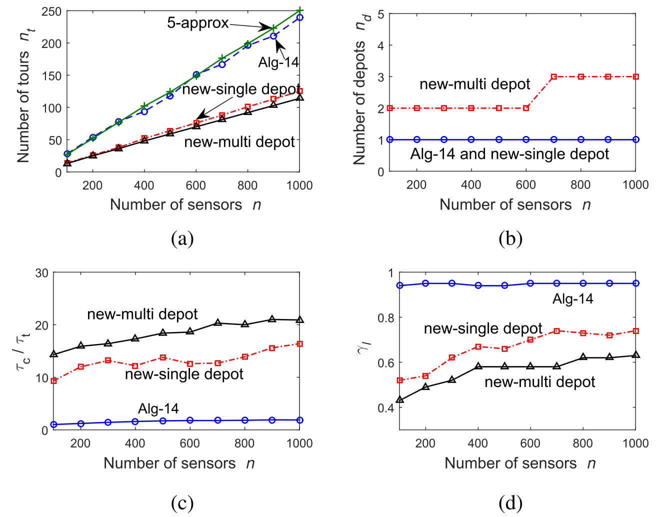

Fig. 6. When MC battery matches sensor lifetime, on WSNs with varying number of sensors n, $l \times l = 2km \times 2km$, $B = 160KJ$, $p_w = 0.8w$, $E_{max} = 5.6\ KJ$, $E_{min} = 0.54\ KJ$, $n_{mc} = n_t$ for all algorithms in this experiments. (a) averaged tours n_t. (b) averaged depots n_d. (c) averaged τ_c/τ_t. (d) averaged γ_l.

by changing the parameter B_0. The number of utilized depots is illustrated in Fig. 6(b). It can be observed in Fig. 6(a) that the proposed algorithm requires much lesser charging tours (and MCs) even when only a single charging depot is used. The improvement is more significant when more depots are deployed, because an MC can reach a depot in a shorter distance so that more energy can be used for charging sensors. The average improvements of `new-single depot` over `Alg-14` and `5-approx` are 49% and 50.7%, while the average improvements of `new-multi depot` over `Alg-14` and `5-approx` are 53.6% and 55.3%, respectively,

It can be observed that our algorithm achieves much higher τ_c/τ_t even when a single depot is deployed, as shown in Fig. 6(c). This is because each charging tour obtained using our algorithm contains more sensors with nearly the same travel distance. If more depots are available, significant improvement in energy efficiency can be achieved as illustrated in Fig. 6(c). On average, the gains in τ_c/τ_t of `new-single depot` and `new-multi depot` are as high as 8.4 times and 11.6 times respectively when compared to `Alg-14` for the cases considered. This is due to the fact that with more available depots, an MC can return to its depot for energy replenishment within short distance, i.e., the ratio of replenishment segment over the complete tour γ_l is smaller, as shown in Fig. 6(d).

D. Using MCs With Large Battery Capacity

Fig. 7 shows the performance comparison of `Alg-14`, `5-approx` and the proposed algorithms `new-single depot` and `new-multi depot`. Similar to the case shown in Fig. 6, large number of sensors leads to more number of charging tours as well as depots (for multi-depot case). The proposed algorithms significantly outperformed `Alg-14` and `5-approx` in obtaining n_t. The average improvements of `new-single depot` and `new-multi depot` on

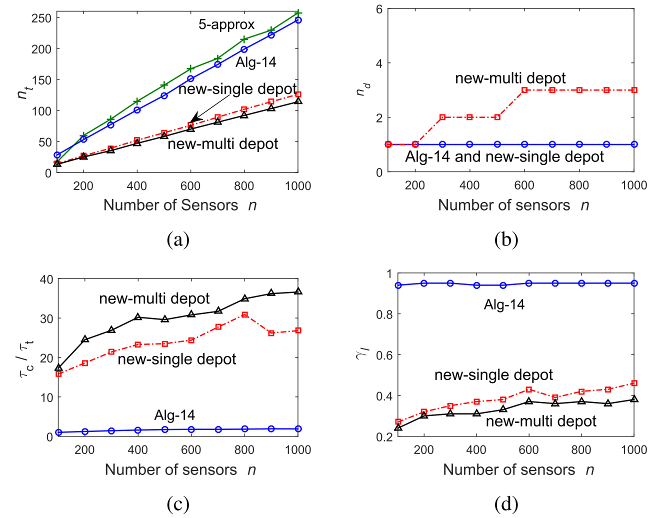

Fig. 7. For MCs with relatively large batteries, on WSNs with varying number of sensors n, $l \times l = 2km \times 2km$, $B = 500KJ$, $p_w = 0.8w$, $E_{max} = 5.6\ KJ$, $E_{min} = 0.54\ KJ$, $n_{mc} = n_t$ for all algorithms in this experiments. (a) averaged tours n_t. (b) averaged depots n_d. (c) averaged τ_c/τ_t. (d) averaged γ_l.

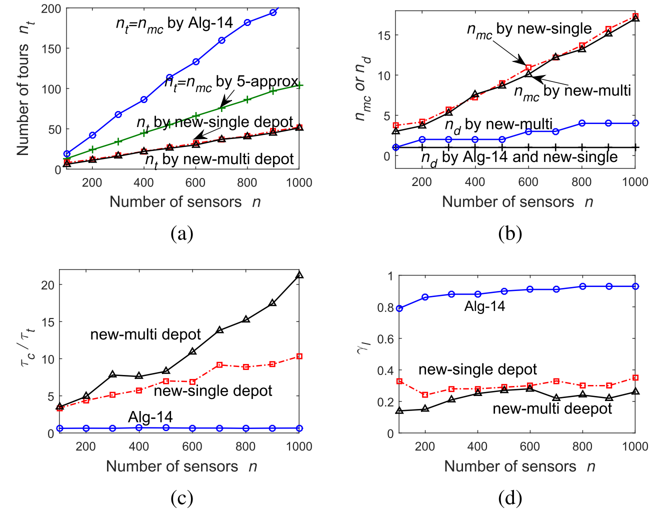

Fig. 8. For MCs with relatively small batteries, on WSNs with varying number of sensors n, $l \times l = 2km \times 2km$, $B = 160KJ$, $p_w = 0.25w$, $E_{max} = 5.6\ KJ$, $E_{min} = 0.54\ KJ$. (a) averaged tours n_t. (b) averaged depots n_d. (c) averaged τ_c/τ_t. (d) averaged γ_l.

n_t (and n_{mc}) over `Alg-14` are 47.8% and 52.5%, while the average improvements ove `5-approx` are 49% and 53.6%, respectively.

It is evident from Fig. 7(c) that our approach outperforms `Alg-14` in terms of energy efficiency, i.e. τ_c/τ_t, for both cases of single depot and multiple depots. On average, for all the cases considered, τ_c/τ_t of `Alg-single` and `Alg-multi` are 10.8 times and 12.9 times higher than `Alg-14` respectively. This is due to the fact that the previous work did not take into consideration the discrepancy between the MC battery capacity and the operational lifetime of sensors. Since each charging tour is constructed under the sensor lifetime and the MC battery capacity constraints, the feasible tour length that is obtained is much shorter than that an MC can support. Therefore, the energy spent on traveling is fixed while energy transfered to sensors is much lesser than the MC's capacity, which leads to low efficiency. On the other hand, our approach considers the association between sensor lifetime and MC battery capacity when planning the charging tours by enabling the MC to visit a charging tour multiple times before returning to its depot for energy replenishment. The improvement in energy efficiency is more significant when multiple depots are available. The γ_l that is achieved using our algorithms are much shorter compared to `Alg-14` as our approach allows an MC to serve a charging tour multiple times before returning to its depot for energy replenishment. This leads to overall reduction in γ_l as shown in Fig. 7(d).

E. Using MCs With Small Battery Capacity

Fig. 8 shows the performance comparison of `Alg-14`, `5-approx`, `new-single depot` and `new-multi depot`. Similar to the case shown in Fig. 6, large number of sensors leads to higher number of charging tours as well as depots (for multi-depot case). In addition, the proposed algorithms require much less number of charging tours n_t compared to `Alg-14` with average reductions of 53% and 58%, when using single depot and multi-depot respectively, while the average reductions over `5-approx` are 43% and 49%, respectively. In addition, our approach only needs 18% of the total MCs required by `Alg-14` in the case of a single depot. This reduces to 16% when multiple depots are allowed. This is possible as our methods produce less charging tours and each MC can serve multiple charging tours.

For the scenario where MCs have relatively small battery capacity compared to the sensor lifetime, the previous approaches achieve low energy efficiency because an MC visits its tour too frequently. Specifically, since the lifetime of sensors is much longer than the charging cycle, a sensor usually has a lot of energy left when an MC arrives to recharge it. Thus the MC can only transfer limited amount energy to the sensors, which decreases the ratio of τ_c/τ_t as the τ_t remains the same for each charging tour. On the other hand, our approach considers the association between sensor lifetime and MC battery capacity when planning the charging tours so that a single MC can serve multiple charging tours. As such, each tour can have much longer charging cycle than an MC can support, which significantly improve the energy efficiency (Fig. 8(c)). Simulation results show that an average of 16 times improvement in energy efficiency can be achieved compared to `Alg-14`. The energy efficiency increases to 25 times when multiple depots are available. Furthermore, our algorithm achieves lower γ_l than `Alg-14` even when a single depot is deployed with an average improvement of 34%, and the reduction on γ_l is more significant for multi-depot scenario with an average improvement of 58%, as shown in Fig. 8(d).

F. Impact of the Efficiency of Wireless Energy Transfering

Fig. 9 shows the impact of p_r, which indicates the efficiency of wireless energy transfer, on the proposed joint charging tour planning and depot deployment. In this scenario, the lifetime

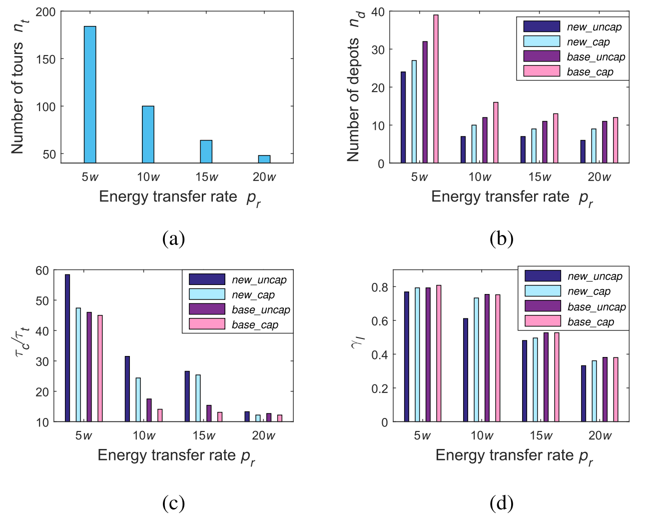

Fig. 9. Impact of p_r on the results, $l \times l = 5km \times 5km$, $B = 300KJ$, $B_0 = 20KJ$, $p_w = 1w$, $E_{max} = 10.8KJ$, $E_{min} = 0.54KJ$. (a) averaged tours n_t. (b) averaged n_d. (c) averaged τ_c/τ_t. (d) averaged γ_l.

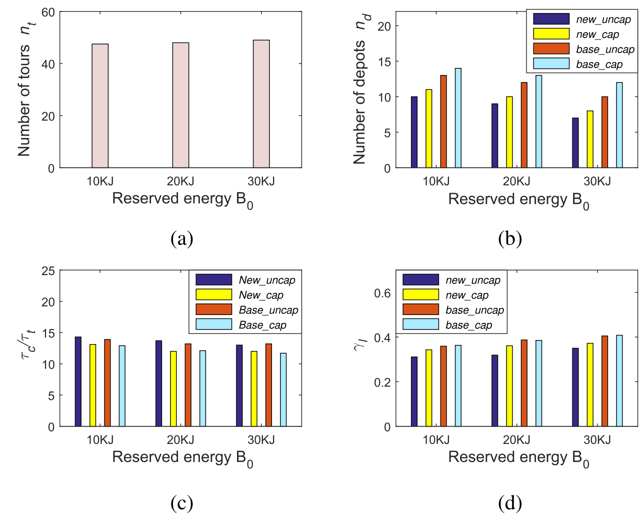

Fig. 10. Impact of parameter B_0 on the results, $l \times l = 5km \times 5km$, $B = 300KJ$, $p_w = 1w$, $E_{max} = 10.8KJ$, $E_{min} = 0.54KJ$, $p_r = 20w$. (a) averaged tours n_t. (b) averaged n_d. (c) averaged τ_c/τ_t. (d) averaged γ_l.

of sensors approximately matches the MC battery capacity, thus n_{mc} equals n_t. It is evident from Fig. 9(a) that the n_t (n_{mc}) decreases with increasing p_r, because the increase in energy transfer efficiency enable an MC to serve more sensors in a single charging trip. As such, reduction in n_t as well as n_{mc} is achieved.

As in Fig. 9(b), n_d decreases with increasing p_r because there are less charging tours to serve when p_r is large. The increase in p_r also leads to decreased τ_c/τ_t as shown in Fig. 9(c), due to the increased efficiency of wireless energy transfer. Specifically, for any charging tour, the amount of energy required by sensors and travel are fixed, so the charging time for sensors, τ_c, decreases with the increasing energy transfer efficiency, i.e., p_r. This leads to reduction in τ_c/τ_t as τ_t remains the same. Moreover, since charging tours obtained with higher p_r are relatively longer than those obtained with lower p_r, this implies that each charging tour with higher p_r can serve more sensors. Hence, γ_l is indirectly reduced when p_r increases. Therefore, γ_l decreases with increasing p_r, as shown in Fig. 9(d). The capacitied scenario requires more depots due to the limited charging ability of each depot. The larger γ_l of capacitied scenario (Fig. 9(d)) compared to uncapacitied scenario is due to the fact that the algorithm tries to obtain $load(i)$ in the form of $x \cdot Y$ for each depot d_i, in order to minimize the n_d. This results in higher γ_l for some charging tours. The increased γ_l further leads to a slight decrease in τ_c/τ_t (Fig. 9(c)). Similar results between uncapacitied and capacitied scenarios is also observed in Fig. 10.

G. Impact of the Parameter B_0

Fig. 10 shows the impact of B_0, which indicates the maximum energy allowed to be spent on the replenishment segment of a charging tour. In this scenario, the number of required MCs n_{mc} equals to n_t. It is evident from Fig. 10(a) that the n_t (n_{mc}) slightly increases with the increasing B_0, because more energy is reserved for replenishment segment, which implies that less energy can be used for charging sensors. However, since B_0 is much smaller compared to MC battery capacity, the increment in n_t (n_{mc}) is not significant. The number of required depots n_d decreases with increasing B_0 as in Fig. 10(b), as the coverage ability of a depot increases quickly when B_0 increases. The increase in B_0 leads to increment of γ_l, as in Fig. 10(d), since more energy is reserved for traveling on replenishment segment. For the same reason, larger B_0 also leads to lower τ_c/τ_t since more energy are allowed for traveling, as in Fig. 10(c). From Fig. 9 and 10, it is clear that our proposed algorithms significantly outperform the baseline algorithms, i.e., `base_uncap` and `base_cap`, for MC depot deployment and charging tour assignment.

VII. Conclusions

In this paper, we jointly considered the problem of charging tour planning and depot positioning for rechargeable WSN using MCs. We identified and addressed a situation that is neglected in previous works, i.e., the MC charging cycle does not match the sensor lifetime due to constraints on the MC battery capacity. The mismatched situation leads to low energy efficiency because an MC with small battery capacity charges sensors too frequently than necessary while an MC with large battery cannot transfer all its energy to sensors and needs to return to depot to replenish its energy. We proposed efficient approaches to deal with both situations i.e. when MCs are equipped with batteries of large capacity and batteries of small capacity. The proposed method is decomposed into three optimization problems: charging tour planning, candidate depot identification and reduction, and depot deployment and charging tour assignment. Novel algorithms have been presented to solve the three problems in order to reduce the number of MCs, and improve the energy efficiency of MCs by maximizing the ratio of time spent on charging over time spent on travelling. Experimental results clearly demonstrate the effectiveness of our approach in terms of energy efficiency as well as the number of required charging tours, MCs and charging depots.

APPENDIX
PROOF OF APPROXIMATION ALGORITHM

The following approximation algorithm works on a complete graph, G, consisting of all the sensors of S. Each node v of G is associated with a cost $c(v)$, and each link (u, v) is associated with a cost $c(u, v)$. In the case of MCs with big batteries, sensor lifetime is a stronger constraint than MC battery, thus $c(v)$ indicates the recharging time of a sensor while $c(u, v)$ indicates the travel time between nodes u and v. On the other hand, in the case of MCs with small batteries, MC battery capacity is a stronger constraint than sensor lifetime, thus $c(v)$ indicates the required energy for recharging a sensor while $c(u, v)$ indicates energy consumption for traveling between nodes u and v. Due to the constraints on MC battery capacity and sensor lifetime, there is a upper bound L on the cost of a charging tour, i.e., $L = (E_{max} - E_{min})/p_w - t_0$ or $L = B - t_0 \cdot p_w$.

The algorithm tries all the possibilities of k and chooses the best one. For each k, the algorithm first tries to cover all sensors with minimum number of paths such that the length of each path is no longer than $L/2$. The first node and last node of each path is then connected to obtain the required charging tour so that the weight of each tour is less than L.

Baseline 5-Approximation Charging Tour Planning

Input: Sensor set S, L is the cost upper bound of a charging tour.
Output: the set of obtained sub-tours
for $k \leftarrow 1$ to n **do**
  $SOL_k \leftarrow \Phi$;
  $F \leftarrow$ minimum spanning forest on S;
  $C_1, C_2, \ldots, C_k \leftarrow$ components of F;
  for $i \leftarrow 1$ to k **do**
    $N_{k,i} \leftarrow 0$;
    $P_i \leftarrow$ a path by deleting an edge from a TSP solution [49] on component C_i;
    while `Length(`P_i`)` $> L/2$ **do**
      Let $< s_1, s_2, \ldots, s_u, \ldots, s_{|P_i|} >$ represent P_i;
      $CL \leftarrow \sum_{i=1}^{u} c(s_i) + \sum_{i=1}^{u-1} cost(s_i, s_{i+1})$;
      if $CL \leq L/2 \leq CL + c(s_u, s_{u+1})$ **then**
        $SOL_k \leftarrow SOL_k \cup < s_1, s_2, \ldots, s_u >$;
        $P_i \leftarrow P_i - < s_1, s_2, \ldots, s_u >$;
        $N_{k,i} \leftarrow N_{k,i} + 1$;
      if $CL + c(s_u, s_{u+1}) \leq L/2 \leq CL + c(s_u, s_{u+1}) + c(s_{u+1})$ **then**
        $SOL_k \leftarrow SOL_k \cup < s_1, s_2, \ldots, s_u >$;
        $P_i \leftarrow P_i - < s_1, s_2, \ldots, s_{u+1} >$;
        $P_i \leftarrow < s_{u+1} > \cup P_i$, $N_{k,i} \leftarrow N_{k,i} + 1$;
    $SOL_k \leftarrow SOL_k \cup P_i$; $N_{k,i} \leftarrow N_{k,i} + 1$;
  $k \leftarrow \arg\min_{1 \leq j \leq n} \sum_{i=1}^{j} N_{j,i}$;
  return SOL_k;

It is easy to verify that the above algorithm runs in polynomial time. We next prove that the above algorithm can guarantee approximation ratio of 5. The algorithm tries all the possibilities of k and chooses the best one. Consider the iteration in which $k = k^*$.

The optimal solution is composed of k charging tours each of cost at most L. Thus the optimal total cost $OPT < kL$. The optimal solution has at least n edges and n nodes, and since forest F is a minimum forest with $n - k$ edges, thus $cost(F) = \sum_{i=1}^{n} c(s_i) + \sum_{(u,v) \in F} cost(u, v) < OPT < kL$. The cost $cost(P_i)$ of each path P_i consists of two parts, i.e., $cost(P_i) = Ncost(P_i) + Lcost(P_i)$ where $Ncost(P_i)$ is the node cost and $Lcost(P_i)$ is the link cost.

In the algorithm, for each case of $CL + c(s_u, s_{u+1}) \leq L/2 \leq CL + c(s_u, s_{u+1}) + c(s_{u+1})$, we add a new node s_{u+1} into path P_i. Assume that set S'_i contains all nodes added into path P_i $(1 \leq i \leq k)$, we have $\sum_{s \in S'_i} c(s) \leq Ncost(P_i)$ and $\sum_{i=1}^{k} \sum_{s \in S_i} c(s) < \sum_{i=1}^{n} c(s_i)$. Therefore, the total cost of all paths including the added nodes, $\sum_{i=1}^{k} \sum_{s \in S'_i} c(s) + \sum_{j=1}^{k} cost(P_j) < 2 \cdot \sum_{i=1}^{n} c(s_i) + 2 \cdot \sum_{(u,v) \in F} cost(u, v) = 2 \cdot cost(F) < 2kL$.

In the algorithm, we repeatedly create a sub-path from path P_i until the $cost(P_i)$ is no larger than $L/2$ and whenever a sub-path is created, the $cost(P_i)$ decreases by at least $L/2$. Thus, for every i, the following is satisfied ($N_{k,i}$ is the number of sub-tours created from P_i by partitioning):

$$N_{k,i} \leq \frac{\lceil \sum_{s \in S_i} c(s) + cost(P_i) \rceil}{L/2} < \frac{\sum_{s \in S_i} c(s) + cost(P_i)}{L/2} + 1.$$

Therefore,

$$\sum_{i=1}^{k} N_{k,i} \leq \sum_{i=1}^{k} \frac{\sum_{s \in S_i} c(s) + cost(P_i)}{L/2} + k \leq \frac{2kL}{L/2} + k.$$

and the performance ratio is

$$\frac{\sum_{i=1}^{k} N_{k,i}}{OPT} \leq \frac{\frac{2kL}{L/2} + k}{k} = 5.$$

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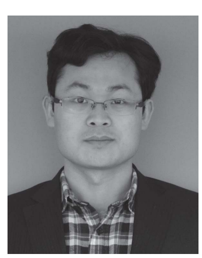

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