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An optimal charging station location model with the consideration of electric vehicle's driving range

Jia He[a], Hai Yang[b], Tie-Qiao Tang[c], Hai-Jun Huang[a,*]

[a] School of Economics and Management, Beihang University, Beijing 100191, China
[b] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
[c] School of Transportation Science and Engineering, Beihang University, Beijing 100191, China

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ABSTRACT

Reasonable charging station positions are critical to prompt the widespread use of electric vehicles (EVs). This paper proposes a bi-level programming model with the consideration of EV's driving range, for finding the optimal locations of charging stations. In this model, the upper level is to optimize the position of charging stations so as to maximize the path flows that use the charging stations, while the user equilibrium of route choice with the EV's driving range constraint is formulated in the lower level. In order to find the optimal solution of the model efficiently, we reformulate the proposed model as a single-level mathematical program and further linearize it in designing the heuristic algorithm. The model validity is demonstrated with numerical examples on two test networks. It is shown that the vehicle's driving range has a great influence on the optimal charging station locations.

1. Introduction

Traffic emission, especially road-based transport emission, has caused serious environmental pollution, which is harmful to our health. The automobile exhaust may result in atmospheric pollution, such as fog and haze, which has attracted the attention of most people. A promising way to reduce automobile exhaust is using EVs instead of internal combustion vehicles. However, the EVs' limited driving range and the insufficient charging stations have restricted the widespread adoption of EVs. It is imperative to construct sufficient number of charging stations for attracting private vehicle drivers to use EVs. A limited number of studies are carried out to investigate the optimal location of charging stations. With the development of new technology, more data are available easily and accurately. Cai et al. (2014), Shahraki et al. (2015) and Tu et al. (2016) used the trajectory data, especially the taxi's Global Position System (GPS) data, to locate the charging stations. Dong et al. (2014) applied the GPS data from the conventional gasoline vehicles to represent the real world EV travel activities, proposed an activity-based assessment method to evaluate EV feasibility in the real world driving context, and applied the genetic algorithm to find optimal locations for public charging stations. Huang and Zhou (2015) utilized the average national data to study the allocation of charging resources to satisfy charging demand for a workplace. Yang et al. (2016) designed and implemented a Stated Preference (SP) Survey to analyze the battery EV drivers' charging and rout choice behaviors. He et al. (2016) used aggregate data from municipal statistical yearbooks and the national census to estimate the EV demand, and compared the optimal locations from three different facility location models. Yang et al. (2017) applied a large-scale GPS trajectory data collected from the taxi fleet to allocate chargers for battery EV taxis, and investigated the tradeoff

* Corresponding author.
E-mail address: haijunhuang@buaa.edu.cn (H.-J. Huang).

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between installing more chargers versus providing more waiting spaces. Xu et al. (2017) utilized the GPS location information and charging information of battery EVs in Japan to investigate the choice for charging mode and location, identified an appropriate instrumental variable to solve the endogeneity issue in the mixed logit model, and provided some useful insights in the operation strategy for public charging stations. Xylia et al. (2017) used the data of 143 routes and 403 existing bus stops in Stockholm to analyze the electrification of the bus network, and revealed that the lower operational costs and fuel prices for electric buses can balance the high investment costs for charging infrastructure. The above data-driven charging station location models are suitable for electric taxi and electric bus, but the models are not applicable for private vehicles due to the limited private vehicles' GPS data. Therefore, researchers use traffic demand data between origin-destination (OD) pairs as the basic resource of the charging demand and formulate mathematical programming models to study the impacts of some factors on optimal charging station locations. Jung et al. (2014) studied the influences of queuing delay on the optimal location of charging stations. He et al. (2015) found that, if drivers simultaneously determine the whole trip chain paths and the charging plans, they might save time compared with determining the minimum path time and charging plan separately. Riemann et al. (2015) combined the flow-capturing location model with the stochastic user equilibrium model to deploy the wireless charging infrastructure. Wang et al. (2016b) investigated the efficient solution methods for a distance-constrained traffic assignment problem with trip chains embedded in equilibrium network flows. Li et al. (2016) proposed a multi-period multipath refueling location model to determine the cost-effective public EV charging station rollout scheme on both spatial and temporal dimensions. Guo et al. (2016) analyzed business-driven EV charging infrastructure investment planning, and proposed a network-based multi-agent optimization modeling framework to study the competition among different public charging suppliers. Ghamami et al. (2016) used a general corridor model to minimize the total system cost in configuring plug-in EV charging infrastructure, and proposed a modified model with consideration of the flow-dependent charging delay induced by congestion. Fuller (2016) proposed an optimization model to deploy the dynamic charging infrastructure, and found that dynamic charging can be a cost effective approach to extending driving range. Chen et al. (2017) investigated the deployment of charging lanes and charging stations along a long traffic corridor, discussed the tradeoff between the charging delay caused by stopping at charging stations and the higher charging price caused by additional devices to enable charging-while-driving, and found that charging lanes were economically viable and competitive for attracting drivers. Liu and Wang (2017) developed a model with multiple types of EV recharging facilities to minimize the public social cost, and used the efficient surface response approximation algorithm to solve the complex tri-level programming problem. The above models can deploy the locations of charging stations or wireless charging lanes, but they do not precisely compute the benefit of charging stations since charging at home and charging en-route are not simultaneously considered in these models. An optimal location of charging stations should be able to capture the maximum flow using the stations. This means after a long term development of charging stations, the optimal location of charging stations should attract sufficient flows to use the charging facilities. However, the flows captured by charging stations do not mean these flows need to use the public charging facilities because many EVs may be charged at home as well. Therefore, we use the maximum flow that can utilize charging stations en-route as the optimization objective in this paper.

Some investigations are made on the influences of limited driving range on optimal location of refueling facilities for alternative-fuel vehicles. Kuby and Lim (2005) formulated a flow refueling location model on a network so as to maximize the total flow volume refueled. Because of the limited driving range, the network vertices do not constitute a finite dominating set, Kuby and Lim (2007) then considered the arc segments where a single facility could refuel a path that would otherwise need two facilities at vertices to refuel. Upchurch et al. (2009) studied the impacts of refueling station capacity on station location. To improve the computational efficiency on large networks, Lim and Kuby (2010) developed three heuristic algorithms for locating alternative-fuel stations. What's more, based on the covered links, Capar et al. (2013) presented a new formulation of the flow refueling location model and a more computationally efficient algorithm. Kim and Kuby (2013) observed that drivers may change from a shortest path to others containing refueling facilities and, proposed an artificial feasible network-based algorithm to enhance the computational efficiency. Li and Huang (2014) developed two heuristic methods to solve the multipath refueling location model in which each OD pair has at least one feasible path. The link congestion effect was not considered in the above models, so they cannot deal with the charging station location problem in urban network. Zhang et al. (2017) extended the works by Upchurch et al. (2009) and Capar et al. (2013), through incorporating demand dynamics to the multi-period, capacitated, flow refueling model for optimizing the location of charging stations. The previous researches can decide relatively reasonable locations of charging stations, but they seldom discuss the impacts of charging stations on driver's route choice behavior. An exception is the research in Riemann et al. (2015), but this approach only deals with wireless charging without considering charging time and the model cannot ensure that there are no EV flows on infeasible paths (i.e., the lengths of these paths exceed the driving range). When a charging station is constructed, the set of feasible paths might change accordingly and immediately due to the driving range limitation, then some drivers may change their route choices which may finally change the flow pattern and in turn influence the location of other charging stations to be constructed. In this paper, we propose a bi-level model to find the optimal positions of all charging stations, through taking into account the driving range limitation and the charging time required in deploying these stations. We then reformulate the proposed model as a single-level model and linearize it in designing an efficient solution algorithm. Furthermore, we consider the situation in which some EV drivers probably charge at home, thus the maximum flows that charge en-route is regarded as the model objective. Compared with the existing studies, this paper's innovative work mainly manifests in the following aspects: (1) the complex relationship between the feasibility of a travel path and the distribution of charging stations is simplified under some reasonable assumptions, so that the linear equalities and inequalities are useful for describing this relationship; (2) the interaction between charging station constructors and travelers is explicitly formulated by a bi-level model in which flow-refueling location, route choice, en-route charging time and charging at home are all taken into account; (3) the proposed bi-level model is reformulated as a single-level model, in which a new Log model is used to significantly reduce the number of binary variables so as to enhance the computational efficiency of solution methods.

The remainder of this paper is organized as flows. In Section 2, the bi-level programming model and the reformulated single-level programming model are presented. Section 3 linearizes the single-level programming model. Two sets of numerical experiments are conducted in Section 4. Section 5 concludes the paper.

2. The model

In this study, the following notations are used.

$G(N,A)$ = Road network
N = Set of nodes
A = Set of links
W = Set of OD pairs
R^w = Set of routes between OD pair w
ψ_r^w = Set of nodes which can make route r between OD pair w feasible for constructing charging stations
D = Driving range of the EVs
l_r^w = Length of route r between OD pair w
x_k = Binary variable, which equals 1 if a charging station is located in node k and 0 otherwise
d_r^w = Binary parameter, which equals 1 if the length of route r is less than the driving range D and 0 otherwise
χ_r^w = Proportion of path flow choosing to be charged on route r
ξ_r^w = Charging time required when drivers choose route r
y_r^w = Binary variable, which equals 1 if there is a station on the feasible position of the route r and 0 otherwise
u_r^w = Binary variable, which equals 1 if the route r is feasible for EVs and 0 otherwise
q^w = Travel demand between OD pair w
f_r^w = Traffic flow on route r connecting OD pair w
n = Number of charging stations to be located
v_a = Traffic flow on link a
δ_{ar}^w = Link-path incidence, which equals 1 if link a is on path r and 0 otherwise
t_a = Travel time of link a
t_a^0 = Free-flow travel time of link a
c_a = Capacity of link a
t_r^w = Travel time of route r
M = A sufficiently large positive number

To simplify the problem to be study, the following reasonable assumptions are made:

I. Every EV driver has a private charger, and she or he can charge the car at night or leisure time. This makes sure that all EVs can start at full power.
II. The charging stations to be constructed are mainly aimed at satisfying the demands of having long distance journeys and managing unexpected occasions (e.g., some drivers forgot charging their vehicles at night or leisure time).
III. Every driver can be served only once for charging her or his vehicle en-route.[1] This requires the route length cannot exceed two times the driving range ($2D$). Otherwise, drivers may choose the internal combustion vehicles instead of EVs.
IV. The energy consumption in each EV is distance-dependent only, not flow-dependent.

Based on the above assumptions, a feasible position of the charging station on route r should generally be within the region $[l_r^w-D,D]$, as shown in Fig. 1. If $l_r^w \leqslant D$, there is no need to build the fast charging station on this route because the EV can accomplish this journey without need of en-route charging. Nevertheless, if there is already a station on this route, which is designed for flows of other routes, the flow of route r might use the station in abnormal situations. For example, a fraction of people might forget charging at home, then, they can well use this station. We set the proportion of this abnormal situation is α.[2] If $D < l_r^w \leqslant 2D$, the feasible position of a charging station on route r should be between l_r^w-D and D from the origin, and any other position cannot make this route feasible. If the station is constructed very closer to the origin (the distance from origin is less than l_r^w-D), the vehicle cannot finish the journey even it is charged at the station.

According to Assumption III, in this paper we do not investigate the case where the route length is twice over the driving range. As long as the station is constructed in the feasible region, the necessity for charging electricity so as to finish the journey is equal for all

[1] This assumption is based on the fact that charging en-route is time-consuming and that the drivers seldom choose to be serviced twice per journey at charging stations. A recent real-world survey about the charging behavior was conducted in Japan, and the statistical data show that the average (maximum) public fast charging number is only 0.06 (1.02) per day, which means that the driver seldom charges her or his vehicle more than once en-route (Xu et al., 2017).

[2] A field trial in Japan showed that 47.3% of charging events point to fast charging facilities (Sun et al., 2015). Thus, many drivers are indeed using the public charging stations, not only because of the longer journey length but also other abnormal factors. We then set this parameter α to reflect the over-expected use of public stations. One can of course ignore the abnormal situation by simply setting $\alpha = 0$.

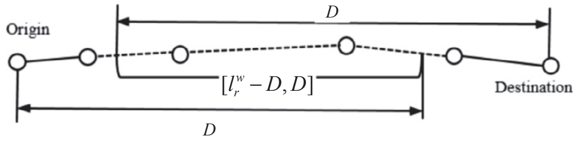

Fig. 1. The feasible position of a charging station on a route.

possible positions in the region. Observe that the cost of charging at public station is higher than that at home, thus the rational behavior for drivers is charging electricity at public station as few as possible. In addition, the optimal positions of charging stations should be in nodes for saving construction cost, because these nodes connect more paths than links. So, we let nodes constitute a finite dominating set.

Let χ_r^w be the proportion of path flow choosing to be charged on route r. It should equal α if the length of the route r is less than the driving range D and 1 otherwise, i.e.,

$$\chi_r^w = \begin{cases} \alpha, & \text{if } l_r^w \leqslant D \\ 1, & \text{otherwise} \end{cases} \tag{1}$$

Let ξ_r^w be the charging time required when drivers choose route r. This time should include a fixed part and a route length-dependent part, as follows:

$$\xi_r^w = \begin{cases} 0, & \text{if } l_r^w \leqslant D \\ \xi_0 + \xi_1(l_r^w - D), & \text{otherwise} \end{cases} \tag{2}$$

where ξ_0 is the fixed part and ξ_1 is a parameter scaling the route length-dependent part. Clearly, the longer the journey remainder, the more the time required for charging (more electric energy is demanded).

For a given network, the parameters d_r^w, χ_r^w and ξ_r^w are independent of the positions of charging stations, but depend upon the driving range limitation and the route length. Define two variables as follows: the binary variable x_k takes 1 if a charging station is located in node k and 0 otherwise; the binary variable y_r^w equals 1 if there is a station on the feasible position of the route r and 0 otherwise. Let ψ_r^w be the set of nodes which can make route r between OD pair w feasible for constructing charging stations. It is obvious that if the route length is larger than D and less than $2D$, the set ψ_r^w contains all the nodes within the region $[l_r^w - D, D]$ of route r. If the route length is less than D, the set contains all nodes of the route. We then have

$$\begin{cases} y_r^w \leqslant \sum_{k \in \psi_r^w} x_k \\ y_r^w \geqslant \sum_{k \in \psi_r^w} \frac{x_k}{n} \\ y_r^w \in \{0,1\} \end{cases} \tag{3}$$

where n is the number of charging stations to be located. In Eq. (3), the second inequality insures $y_r^w = 1$ holds if $x_k = 1$ at some node $k \in \psi_r^w$.

If the length of the route r is less than the driving range D (i.e., $d_r^w = 1$), the route r is certainly feasible for traveling from origin to destination. If there is a station on the route r (i.e., $y_r^w = 1$), the route is also feasible according to the assumptions and definitions made previously. If the route length is larger than the driving range D (i.e., $d_r^w = 0$) and there is no charging station on the route (i.e., $y_r^w = 0$), the route is infeasible. Concerning the feasibility of a route defined by a binary variable, we then have

$$\begin{cases} u_r^w \leqslant d_r^w + y_r^w \\ u_r^w \geqslant d_r^w \\ u_r^w \geqslant y_r^w \\ u_r^w \leqslant 1 \end{cases} \tag{4}$$

An interesting thing is that Eq. (4) has successively converted the binary variable u_r^w to a continuous variable. This is explained as follows:

(i) If $d_r^w = 0$ and $y_r^w = 0$ which means this route is infeasible, then Eq. (4) requires four inequalities $u_r^w \leqslant 0, u_r^w \geqslant 0, u_r^w \geqslant 0$ and $u_r^w \leqslant 1$ simultaneously hold, which leads to $u_r^w = 0$.
(ii) If $d_r^w = 1$ and $y_r^w = 0$ or $d_r^w = 0$ and $y_r^w = 1$ which means this route is feasible, then Eq. (4) lets $u_r^w \leqslant 1, u_r^w \geqslant 1, u_r^w \geqslant 0$ and $u_r^w \leqslant 1$ hold, which leads to $u_r^w = 1$.
(iii) If $d_r^w = 1$ and $y_r^w = 1$, then Eq. (4) lets $u_r^w \leqslant 2, u_r^w \geqslant 1, u_r^w \geqslant 1$ and $u_r^w \leqslant 1$ hold, which leads to $u_r^w = 1$.

Let b be the total number of paths connecting an OD pair. For each OD pair, we rank all paths according to their lengths. Without loss of generality, assume that $l_1 \leqslant l_2 \leqslant \cdots \leqslant l_b$ holds. Totally, the following three cases may occur.

Case I, $l_b \leqslant D$. In this case, the length of each path doesn't exceed the driving range. If constructing charging station is needed, we should place the charging station on a path which has the most path flow. With the consideration of abnormal demand, the flow using this charging station is αq^w.

Case II, $D \leqslant l_1$. In this case, the length of each path exceeds the driving range, for saving the cost of constructing stations and maximizing the charged electricity, we should locate the charging station on the longest path which also has maximum flow. However, the optimal location of charging stations depends upon the flow pattern which should be the result of a traffic assignment with given station distribution.

Case III, $l_1 \leqslant \cdots \leqslant l_r \leqslant D \leqslant l_{r+1} \leqslant \cdots \leqslant l_b$. In this case, there is no need to locate charging station on paths 1,2,..., and r. Nevertheless, if there already exist charging stations on these paths, the maximum flow using these charging stations is αq^w. If the charging stations are placed on paths $r+1, r+2, \ldots,$ and b, the flow using these charging stations might exceed αq^w. Since the location of charging stations affects the traffic assignment, so the optimal location of the stations depends on the travel time function, the traffic demand and the structure of the network.

For a network with general structure, one charging facility can serve more than one path. We formulate the following bi-level programming model to determine the optimal location of charging stations for EVs with the consideration of driving range.

$$\max_{x_k, y_r^w} Z = \sum_{w \in W} \sum_{r \in R^w} \chi_r^w f_r^w y_r^w \tag{5}$$

subject to

$$\sum_{k \in \psi_r^w} x_k \geqslant y_r^w,\ r \in R^w,\ w \in W \tag{6}$$

$$y_r^w \geqslant \sum_{k \in \psi_r^w} \frac{x_k}{n},\ r \in R^w,\ w \in W \tag{7}$$

$$\sum_{k \in K} x_k = n \tag{8}$$

$$x_k, y_r^w \in \{0,1\},\ k \in K,\ r \in R^w,\ w \in W \tag{9}$$

where f_r^w is the path flow solution of the following user equilibrium traffic assignment problem with the consideration of driving range, charging time and given decision variables x_k and y_r^w,

$$\min_{f_r^w} \sum_{a \in A} \int_0^{v_a} t_a(z)\mathrm{d}z + \sum_{w \in W} \sum_{r \in R^w} \xi_r^w f_r^w \tag{10}$$

subject to

$$\sum_{r \in R^w} f_r^w = q^w,\ w \in W \tag{11}$$

$$(D\cdot(1+y_r^w) - l_r^w) f_r^w \geqslant 0,\ r \in R^w,\ w \in W \tag{12}$$

$$v_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{ar}^w, a \in A \tag{13}$$

$$v_a \geqslant 0,\ f_r^w \geqslant 0,\ a \in A,\ r \in R^w,\ w \in W \tag{14}$$

In the above bi-level programming model, the constructor of charging facilities determines the location of charging stations for serving the maximum number of EVs, based on the user-equilibrium (UE) flow pattern, and the travelers choose routes based on the minimal individual disutility which is influenced by the location of charging stations. If the positions of charging stations are given, i.e., x_k and then y_r^w are known by solving the upper-level problem, the lower-level becomes a path-based user equilibrium traffic assignment problem which can be solved by the algorithms proposed in (Jiang et al. 2012, Jiang et al., 2013), Jiang and Xie (2014), Xie and Jiang (2016) and He et al. (2014). If the lengths of the shortest paths between some OD pairs exceed the driving range and n is too small, the lower-level problem might have no feasible solution. However, if we increase the number of charging stations to be constructed, the feasible solution always exists. Thus, we make such an assumption that there at least is one feasible path between each OD pair, i.e., the length of the shortest path doesn't exceed the driving range, the model would then always have feasible solutions.

The model is NP-hard due to its binary-type decision variables and also the bi-level structure. It is not possible to find the exact (global) solution of this model in large scale networks. As a result, inevitably, the non-exact approaches must be applied to obtain a rather 'good' solution in a reasonable amount of time. For example, the following heuristic algorithm can be used to approximately solve the bi-level programming model presented in this paper. On the base of flow pattern given by traditional user equilibrium assignment, the initial position of charging stations can be determined. Then, we can solve the lower level model (10)–(14) and the upper level model (5)–(9) iteratively until the change of upper-level objective function is sufficient small. The heuristic algorithm is an iterative method which may not be convergent, and the computational efficiency is very low. We now introduce some new variables and constraints to reformulate the bi-level problem as a single-level mathematical program. This program is presented below.

$$\max_{x_k, y_r^w} Z = \sum_{w \in W} \sum_{r \in R^w} \chi_r^w f_r^w y_r^w \tag{15}$$

subject to (4), (6)–(9), (11), (13) and (14), and

$$\begin{cases} 0 \leqslant f_r^w \leqslant M\sigma_r^w \\ 0 \leqslant t_r^w - \pi^w \leqslant 2M(1-\sigma_r^w) \\ \sigma_r^w \leqslant u_r^w \\ \sigma_r^w \in \{0,1\} \\ r \in R^w, w \in W \end{cases} \tag{16}$$

where the path time is computed by

$$t_r^w = \sum_{a \in A} \delta_{ar}^w t_a + \xi_r^w + M(1-u_r^w),\ r \in R^w,\ w \in W \tag{17}$$

In Eq. (17), the link travel time function is $t_a = t_a^0(1 + 0.15(v_a/c_a)^4)$ which considers the congestion effect. Constraint (16) indicates that if there is traffic flow on path r (i.e., $\sigma_r^w = 1$ and $f_r^w > 0$), the time of this route must be minimum (i.e., π^w) among all the paths connecting OD pair w, if there is no traffic flow on path r (i.e., $\sigma_r^w = 0$ and $f_r^w = 0$), the time of this route must exceed or equal the minimum path travel time (i.e., π^w). If a path is infeasible for EVs (i.e., $u_r^w = 0$) in this model, this path would not be chosen (i.e., $\sigma_r^w = 0$), so we add the constraint $\sigma_r^w \leqslant u_r^w$. Note that u_r^w is defined by Eq. (4). Wang and Lo (2010) and Wang et al. (2016a) had also used these constraints to substitute the UE conditions.

Eq. (16) calculates the time of a route, which consists of the travel time and the time spent in electricity charging. If the path is feasible (i.e., $u_r^w = 1$), the route time is the sum of all link times on that path and the charging time ξ_r^w. If the path is infeasible (i.e., $u_r^w = 0$), the route time should be added with a sufficiently large positive number, which makes the infeasible path's time infinite, and no drivers would choose this path.

The above single-level programming model is path flow-based, nonlinear and contains binary variables. More troublesome is that the minimum time between each OD pair π^w, should be endogenous (determined by the user equilibrium traffic assignment (16)). Clearly, this model is still difficult to solve. In the next section, we linearize the model and then approximately solve it efficiently.

3. Linearization of the proposed single-level model

Because the parameters d_r^w, χ_r^w and ξ_r^w are independent of the location of charging stations and the purpose of the proposed model is to find the optimal location, these parameters can be regarded as constants in the linearization process. In the single-level model (15)–(17), there are only two nonlinear parts, i.e., the objective function and the travel time function.

3.1. Linearization of the objective function

Using the similar method by Sherali and Adams (1994), we introduce a new variable φ_r^w and four linear constraints to substitute the objective function of the model (15)–(17). The new variable is defined as $\varphi_r^w = f_r^w y_r^w$, and the four linear constraints are as follows.

$$\begin{cases} \varphi_r^w - \underline{f}_r^w y_r^w \geqslant 0 \\ \varphi_r^w - \overline{f}_r^w y_r^w \leqslant 0 \\ \varphi_r^w - f_r^w + \underline{f}_r^w (1-y_r^w) \leqslant 0 \\ \varphi_r^w - f_r^w + \overline{f}_r^w (1-y_r^w) \geqslant 0 \end{cases} \tag{18}$$

where $\overline{f}_r^w$ and $\underline{f}_r^w$ are the upper and lower bounds of the path flow f_r^w, respectively. They respectively can be a sufficiently large positive number and a sufficiently small negative number. When $y_r^w = 0$, the inequalities in Eq. (18) become $0 \leqslant \varphi_r^w \leqslant 0$ and $f_r^w - \overline{f}_r^w \leqslant \varphi_r^w \leqslant f_r^w - \underline{f}_r^w$, then $\varphi_r^w = 0$. When $y_r^w = 1$, the inequalities become $\underline{f}_r^w \leqslant \varphi_r^w \leqslant \overline{f}_r^w$ and $f_r^w \leqslant \varphi_r^w \leqslant f_r^w$, then $\varphi_r^w = f_r^w$. Hence, the continuous variable φ_r^w can take the same roles as the binary parameter y_r^w and the path flow f_r^w.

Therefore, the objective function (15) $Z = \sum_{w \in W} \sum_{r \in R^w} \chi_r^w f_r^w y_r^w$, is linearized as $Z = \sum_{w \in W} \sum_{r \in R^w} \chi_r^w \varphi_r^w$ together with constraints (18). Note here again that the predetermined parameter χ_r^w is the proportion of path flow choosing to be charged on route r.

3.2. Linearization of the link travel time function

Wang and Lo (2010), Riemann et al. (2015) and Wang et al. (2016a) developed the so-called mixed linearization methods for handling the nonlinear link travel time function. The method proposed in Wang et al. (2016a) used less binary variables, so we in this paper adopt the similar method to linearize the link travel time. This method is outlined below.

For a link a, the flow region $[0, v_a]$ is partitioned into m intervals (see Fig. 2). If m is large enough, the error of linearizing the quartic item $(v_a/c_a)^4$ in arbitrary internal can be negligible. For a specific v_a, it can only fall into one internal. Thus, if $\sum_{\tau \in V(P)} \alpha_a^\tau \cdot v_a^\tau = v_a$, where P is the set of the partitioned intervals and $V(P)$ is the set of breakpoints, at most two elements of the

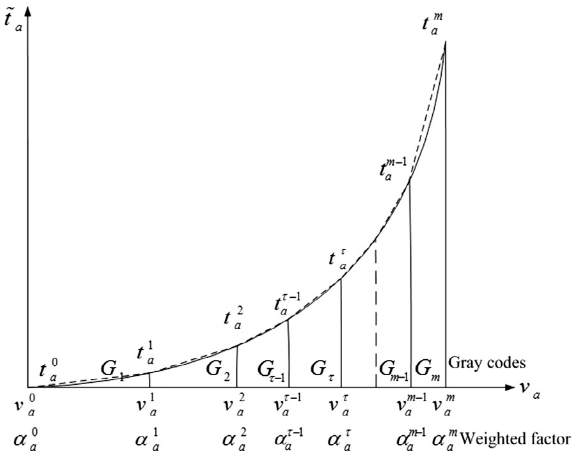

Fig. 2. Piecewise-linear approximation of $(v_a/c_a)^4$.

weighted factors $(\alpha_a^0,\alpha_a^1,...,\alpha_a^m)$, are non-zero; if the two elements are non-zero, they must be adjacent, i.e., the weighted factors are the SOS Type 2 (Beale and Forrest, 1976; CPLEX, 2014).

Generally, m binary variables are needed to clearly formulate the SOS Type 2. Wang et al. (2016a) used $h_{\max} = \lceil \log_2(m) \rceil$ binary variables in their method, where $\lceil x \rceil$ is the minimum positive integer greater than or equal to x. Wang et al. (2016a) employed m Gray codes, each Gray code contains $h_{\max}$ binary variables. For each interval in Fig. 2, a Gray code G is pre-determined in order that arbitrary two adjacent codes differ by only one bit. For example, if G_2 is $G_2 = \{0,0,0,1\}$, then one of its neighbors G_3 should be as $G_3 = \{0,0,1,1\}$ which is different from G_2 only in the third bit. Through introducing binary variables λ_a^h, Wang et al. (2016a) formulated a set of constraints for α_a^τ as follows:

$$\begin{cases} \sum\limits_{\tau \in L_h} \alpha_a^\tau \leqslant \lambda_a^h \\ \sum\limits_{\tau \in R_h} \alpha_a^\tau \leqslant 1-\lambda_a^h \\ \lambda_a^h \in \{0,1\},\ h \in H \end{cases} \tag{19}$$

where H is the index set of the binary variables, L_h and R_h are two index sets that are defined as follows (Wang et al., 2016a):

$$L_h = \{\tau \in V | (G_\tau^h = 1 \text{ and } G_{\tau+1}^h = 1) \cup (\tau = 0 \text{ and } G_1^h = 1) \cup (\tau = m \text{ and } G_m^h = 1)\} \tag{20}$$

$$R_h = \{\tau \in V | (G_\tau^h = 0 \text{ and } G_{\tau+1}^h = 0) \cup (\tau = 0 \text{ and } G_1^h = 0) \cup (\tau = m \text{ and } G_m^h = 0)\} \tag{21}$$

where G_τ^h represents the hth bit of the τth Gray code. For example, from Fig. 3, we can find that $G_5^1 = 0$ and $G_6^3 = 1$.

Based on above discussions, if the quartic item $(v_a/c_a)^4$ is partitioned into 9 intervals, the method needs 9 Gray codes and each of them contains four elements (see Fig. 3). Then, Eq. (19) can be rewritten as

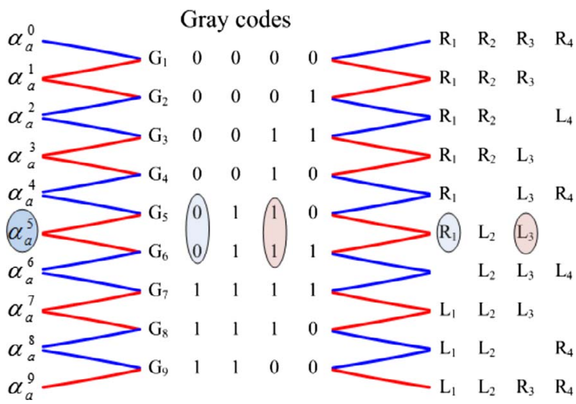

Fig. 3. Example of Gray codes and definition of sets L_h and R_h.

$$\begin{cases} \alpha_a^7+\alpha_a^8+\alpha_a^9 \leqslant \lambda_a^1 \\ \alpha_a^0+\alpha_a^1+\alpha_a^2+\alpha_a^3+\alpha_a^4+\alpha_a^5 \leqslant 1-\lambda_a^1 \\ \alpha_a^5+\alpha_a^6+\alpha_a^7+\alpha_a^8+\alpha_a^9 \leqslant \lambda_a^2 \\ \alpha_a^0+\alpha_a^1+\alpha_a^2+\alpha_a^3 \leqslant 1-\lambda_a^2 \\ \alpha_a^3+\alpha_a^4+\alpha_a^5+\alpha_a^6+\alpha_a^7 \leqslant \lambda_a^3 \\ \alpha_a^0+\alpha_a^1+\alpha_a^9 \leqslant 1-\lambda_a^3 \\ \alpha_a^2+\alpha_a^6 \leqslant \lambda_a^4 \\ \alpha_a^0+\alpha_a^4+\alpha_a^8+\alpha_a^9 \leqslant 1-\lambda_a^4 \end{cases} \tag{22}$$

Thus, the linearization of link travel time function becomes

$$t_a = t_a^0(1+0.15\widetilde{t}_a) \tag{23}$$

$$\sum_{\tau\in V(P)} \alpha_a^\tau v_a^\tau = v_a \tag{24}$$

$$\sum_{\tau\in V(P)} \alpha_a^\tau (v_a^\tau/c_a)^4 = \widetilde{t}_a \tag{25}$$

$$\sum_{\tau\in V(P)} \alpha_a^\tau = 1 \tag{26}$$

$$\sum_{\tau\in L_h} \alpha_a^\tau \leqslant \lambda_a^h \tag{27}$$

$$\sum_{\tau\in R_h} \alpha_a^\tau \leqslant 1-\lambda_a^h \tag{28}$$

$$\alpha_a^\tau \geqslant 0,\ \lambda_a^h \in \{0,1\},\ \tau \in V(P),\ a \in A,\ h \in H \tag{29}$$

Finally, the model proposed in Section 2 can be linearized as

$$\max_{x_k, y_r^w} Z = \sum_{w\in W}\sum_{r\in R^w} \chi_r^w \varphi_r^w \tag{30}$$

subject to constraints (4), (6)–(9), (11)–(14), (16)–(18), and (23)–(29).

4. Numerical examples

In this section, two numerical examples are presented to demonstrate the effectiveness of the proposed model and solution algorithm. The first example is the medium-size network from Nguyen and Dupuis (1984) and the second example is the Sioux Falls network. The commercial optimization solver, CPLEX optimization studio 12.4, is used to solve the MILP model (30).

4.1. Nguyen–Dupuis network

The Nguyen–Dupuis network is shown in Fig. 4. The values of parameters used in link travel time functions are taken from

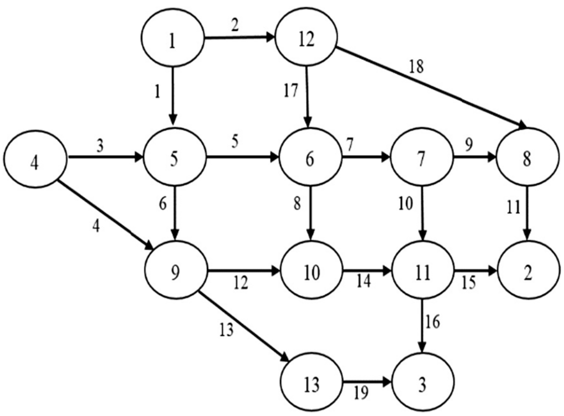

Fig. 4. Nguyen–Dupuis network.

Table 1
Link-related input parameters for Nguyen-Dupuis network.

Link	Free flow travel time (min)	Link capacity (veh/h)	Link	Free flow travel time (min)	Link capacity (veh/h)
1	7	300	11	9	500
2	9	200	12	10	550
3	9	200	13	9	200
4	12	200	14	6	400
5	3	350	15	9	300
6	9	400	16	8	300
7	5	500	17	7	200
8	13	250	18	14	300
9	5	250	19	11	200
10	9	300			

Table 2
Demand-related input parameters for Nguyen-Dupuis network.

OD pair	OD demand
(1,2)	200
(1,3)	400
(4,2)	300
(4,3)	100

Table 3
Feasible route sets.

OD pair	Node sequence of path	Distance from origin
(1,2)	1-5-6-7-8-2	0-14-20-30-40-58
	1-12-8-2	0-18-46-64
	1-12-6-7-8-2	0-18-32-42-52-70
(1,3)	1-5-6-7-11-3	0-14-20-30-48-64
	1-5-9-13-3	0-14-32-50-72
	1-12-6-7-11-3	0-18-32-42-60-76
(4,2)	4-5-6-7-8-2	0-18-24-34-44-62
	4-5-6-7-11-2	0-18-24-34-52-70
	4-9-10-11-2	0-24-44-56-74
(4,3)	4-9-13-3	0-24-42-64
	4-5-6-7-11-3	0-18-24-34-52-68
	4-5-9-13-3	0-18-36-54-76

Table 4
The set of reasonable locations (D = 50).

OD pair	Feasible nodes having station on path	Set of feasible nodes which may have charging stations
(1,2)	5, 6, 7, 8	5, 6, 7, 8, 12
	12, 8	
	6, 7	
(1,3)	5, 6, 7, 11	5, 6, 7, 9, 11, 13
	9, 13	
	6, 7	
(4,2)	5, 6, 7, 8	5, 6, 7, 8, 10, 11
	6, 7	
	9, 10	
(4,3)	9, 13	5, 6, 7, 9, 13
	5, 6, 7	
	9	

Table 5
Optimal nodes having charging stations for different driving ranges and station number allowed.

D	Node when $n = 1$	Max flow ($n = 1$)	Node when $n = 2$	Max flow ($n = 2$)	Node when $n = 3$	Max flow ($n = 3$)
58	6	976.04	5, 12	1000	–	–
59	6	976.04	5, 12	1000	–	–
60	6	982.77	5, 12	1000	–	–
61	6	986.14	5, 12	1000	–	–
62	6	707.80	8, 11	816.61	–	–
63	11	710.36	8, 11	820.56	–	–
64	5	69.95	6, 9	84.06	6, 9, 12	94.06
65	6	73.58	5, 9	89.30	6, 9, 12	99.30
66	6	77.22	5, 9	94.54	8, 9, 11	104.54
67	5	81.60	5, 12	99.77	2, 9, 11	109.77
68	5	85.99	5, 9	105.18	8, 9, 11	115.18
69	5	90.38	6, 9	111.04	5, 9, 12	121.04
70	5	94.77	5, 9	116.89	8, 9, 11	126.89
71	5	99.01	6, 9	122.74	8, 9, 11	132.74
72	11	36.75	6, 9	60.78	8, 9, 11	70.78
73	11	40.42	5, 9	64.22	8, 9, 11	74.22
74	5	31.30	2, 3	50	*	*
75	5	31.30	2, 3	50	*	*
76	5	31.30	2, 3	50	*	*

Table 6
The path flows and set of reasonable locations for $D = 80$.

OD pair	Path Flow	Set of reasonable locations on path
(1,2)	0	No
	200	1, 12, 8, 2
	0	No
(1,3)	310.177	1, 5, 6, 7, 11, 3
	89.823	1, 5, 9, 13, 3
	0	No
(4,2)	225.927	4, 5, 6, 7, 8, 2
	0	No
	74.073	4, 9, 10, 11, 2
(4,3)	100	4, 9, 13, 3
	0	No
	0	No

Riemann et al. (2015) and listed in Table 1. The distance of each link is assumed to be twice the free-flow travel time in number. The OD demand is twice that in Riemann et al. (2015) and listed in Table 2. Table 3 shows the node sequences of three alternative paths for each OD pair, the distances from origins to each node are also shown in this table. The shortest path length is 58, and the longest is 76. The fast charging efficiency is 0.67 min/kWh and the energy consumption rate is 0.29 kWh/mile, which were suggested in He et al. (2014). In this study, we assume that the fixed charging time is 5 min and the variable charging time is 0.5 min/mile (i.e., $\xi_0 = 5$ min and $\xi_1 = 0.5$ min/mile). Nine pieces of line segments are used to approximate the link travel time function.

If the driving range is less than 58, all the flows have to choose charging on their paths for avoiding running out of energy ($\chi_r^w = 1$). The objective function value of the model is 1000, only one fast charging station is sufficient to capture all the flows and the best position is 5 or 6 or 7. From Table 4, we can easily find that every node of 5, 6 and 7 can make all flows captured.

If the driving range is between 58 and 76, the lengths of some paths would exceed the driving range, others would not. Table 5 shows the optimal nodes having charging stations for different driving ranges $D \in [58,76]$ and different number of charging stations allowed $n \in [1,2,3]$. The maximum flows served by these charging stations are also given in this table. If all flows can be served by n charging stations, we certainly do not need to consider the case of constructing $n + 1$ charging stations (i.e., the cases marked by '–' in the table). With the further increase of driving range, more routes have lengths less than the driving range and the flows on these routes do not need to use the fast charging station. This may lead to station relocation. Consequently, as shown in Table 5, when $D \geqslant 64$, the maximum flows served by en-route charging station decrease greatly, far less than the OD demand. When $D \geqslant 74$, two charging stations can serve few flows only, so constructing three stations becomes meaningless (i.e., the cases marked by '*' in the table).

If the driving range is larger than 76, any location of charging stations would not affect the driver's route choices, and only proportion of path flows need to be charged, i.e., $\chi_r^w = \alpha = 0.05$. The path flow pattern is fixed, and the problem becomes a maximum-coverage rate problem. As shown in Table 6, node 5 can make 62.593% of all flows (i.e., 625.93 = 310.177 + 89.825 + 225.927) captured, and node sets [1,4] or [2,3] can make all the flows captured, and the flows that

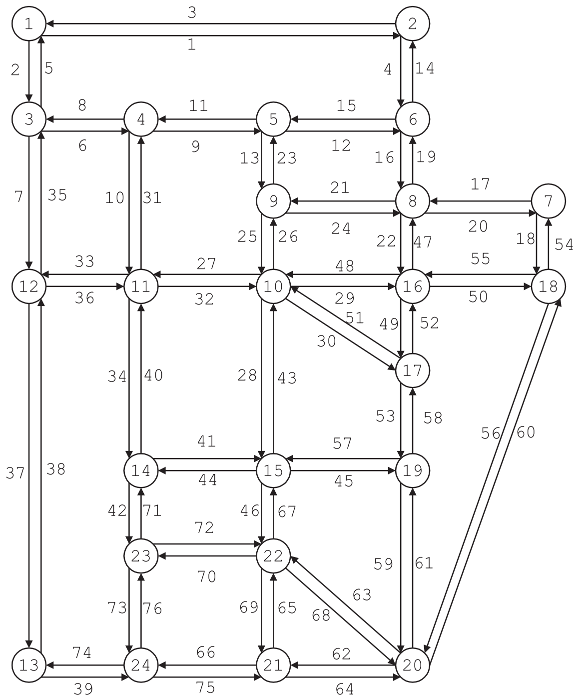

Fig. 5. Sioux Falls network.

Table 7
Link-related input parameters for Sioux Falls network.

Link	Free flow travel time	Capacity	Link	Free flow travel time	Capacity	Link	Free flow travel time	Capacity
1	6	25,900	27	5	10,000	52	2	5230
2	4	23,403	28	6	13,512	53	2	4824
3	6	25,900	29	4	4855	54	2	23,403
4	5	4958	30	8	4994	55	3	19,680
5	4	23,403	31	6	4909	56	4	23,403
6	4	17,111	32	5	10,000	57	3	14,565
7	4	23,403	33	6	4909	58	2	4824
8	4	17,111	34	4	4877	59	4	5003
9	2	17783	35	4	23,403	60	4	23,403
10	6	4909	36	6	4909	61	4	5003
11	2	17,783	37	3	25,900	62	6	5060
12	4	4948	38	3	25,900	63	5	5076
13	5	10,000	39	4	5091	64	6	5060
14	5	4958	40	4	4877	65	2	5230
15	4	4948	41	5	5128	66	3	4885
16	2	4899	42	4	4925	67	3	9599
17	3	7842	43	6	13,512	68	5	5076
18	2	23,403	44	5	5128	69	2	5230
19	2	4899	45:	3	14,565	70	4	5000
20	3	7842	46	3	9599	71	4	4925
21	10	5050	47	5	5046	72	4	5000
22	5	5046	48	4	4855	73	2	5079
23	5	10,000	49	2	5230	74	4	5091
24	10	5050	50	3	19,680	75	3	4885
25	3	13,916	51	8	4994	76	2	5079
26	3	13,916						

use fast charging stations are 31.30 (i.e., $31.30 = 625.93 * 0.05$) when $n = 1$, and 50 when $n = 2$.

4.2. Sioux Falls network

The Sioux Falls network is displayed in Fig. 5. The parameters of free-flow travel time and link capacity are listed in Table 7 and the OD demands in Table 8, all from Bar-Gera (2016). The link distances are assumed to be the same as the link free-flow travel times

Table 8
OD demand for Sioux Falls network.

O/D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	100	100	500	200	300	500	800	500	1300	500	200	500	300	500	500	400	100	300	300	100	400	300	100
2	100	0	100	200	100	400	200	400	200	600	200	100	300	100	100	400	200	0	100	100	0	100	0	0
3	100	100	0	200	100	300	100	200	100	300	300	200	100	100	100	200	100	0	0	0	0	100	100	0
4	500	200	200	0	500	400	400	700	700	1200	1500	600	600	500	500	800	500	100	200	300	200	400	500	200
5	200	100	100	500	0	200	200	500	800	1000	500	200	200	100	200	500	200	0	100	100	100	200	100	0
6	300	400	300	400	200	0	400	800	400	800	400	200	200	100	200	900	500	100	200	300	100	200	100	100
7	500	200	100	400	200	400	0	1000	600	1900	500	700	400	200	500	1400	1000	200	400	500	200	500	200	100
8	800	400	200	700	500	800	1000	0	800	1600	800	600	600	400	600	2200	1400	300	700	900	400	500	300	200
9	500	200	100	700	800	400	600	800	0	2800	1400	600	600	600	1000	1400	900	200	400	600	300	700	500	200
10	1300	600	300	1200	1000	800	1900	1600	2800	0	3900	2000	1900	2100	4000	4400	3900	700	1800	2500	1200	2600	1800	800
11	500	200	300	1400	500	400	500	800	1400	4000	0	1400	1000	1600	1400	1400	1000	200	400	600	400	1100	1300	600
12	200	100	200	600	200	200	700	600	600	2000	1400	0	1300	700	700	700	600	200	300	500	300	700	700	500
13	500	300	100	600	200	200	400	600	600	1900	1000	1300	0	600	700	600	500	100	300	600	600	1300	800	700
14	300	100	100	500	100	100	200	400	600	2100	1600	700	600	0	1300	700	700	100	300	500	400	1200	1100	400
15	500	100	100	500	200	200	500	600	900	4000	1400	700	700	1300	0	1200	1500	200	800	1100	800	2600	1000	400
16	500	400	200	800	500	900	1400	2200	1400	4400	1400	700	600	700	1200	0	2800	500	1300	1600	600	1200	500	300
17	400	200	100	500	200	500	1000	1400	900	3900	1000	600	500	700	1500	2800	0	600	1700	1700	600	1700	600	300
18	100	0	0	100	0	100	200	300	200	700	100	200	100	100	200	500	600	0	300	400	100	300	100	0
19	300	100	0	200	100	200	400	700	400	1800	400	300	300	300	800	1300	1700	300	0	1200	400	1200	300	100
20	300	100	0	300	100	300	500	900	600	2500	600	400	600	500	1100	1600	1700	400	1200	0	1200	2400	700	400
21	100	0	0	200	100	100	200	400	300	1200	400	300	600	400	800	600	600	100	400	1200	0	1800	700	500
22	400	100	100	400	200	200	500	500	700	2600	1100	700	1300	1200	2600	1200	1700	300	1200	2400	1800	0	2100	1100
23	300	0	100	500	100	100	200	300	500	1800	1300	700	800	1100	1000	500	600	100	300	700	700	2100	0	700
24	100	0	0	200	0	100	100	200	200	800	600	500	800	400	400	300	300	0	100	400	500	1100	700	0

Table 9
Set of feasible long routes for Sioux Falls network.

OD pair	Path number	Demand	OD pair	Path number	Demand
(16,1)	3	500	(6,14)	2	100
(20,1)	3	300	(1,16)	4	500
(21,1)	2	100	(2,16)	3	400
(7,2)	2	200	(11,16)	4	1400
(8,2)	2	400	(12,16)	8	700
(14,2)	5	100	(13,16)	6	600
(15,2)	8	100	(2,17)	4	200
(16,2)	3	400	(13,18)	4	100
(17,2)	4	200	(2,19)	4	100
(19,2)	7	100	(1,20)	3	300
(20,2)	2	100	(2,20)	3	100
(21,6)	4	100	(12,20)	4	500
(2,7)	2	200	(13,20)	4	600
(13,7)	4	400	(1,21)	3	100
(2,8)	2	400	(4,21)	3	200
(16,11)	5	1400	(6,21)	5	100
(16,12)	8	700	(12,21)	2	300
(20,12)	4	400	(1,22)	4	400
(21,12)	2	300	(2,22)	4	100
(7,13)	4	400	(3,22)	4	100
(16,13)	6	600	(4,22)	4	400
(18,13)	4	100	(6,22)	5	200
(20,13)	4	600	(17,24)	7	300

Table 10
Optimal locations for different number of charging stations allowed.

Number of stations	Node with station	Flows served
$n = 1$	16	384.7041
$n = 2$	16, 21	625.2193
$n = 3$	6, 10, 24	790
$n = 4$	4, 8, 10, 24	795

in number. We employ the Frank-Wolfe algorithm to solve the traditional user equilibrium traffic assignment problem, which offers all usable paths for every OD pair at each iteration. As a result, 1401 usable paths are found in total. The longest distance among all OD pairs' shortest paths is 23, and the longest path of all usable paths is 45.

When the driving range is 30, there are 46 OD pairs connected by paths longer the driving range. The number of these paths for all OD pairs and their demands are shown in Table 9. We just consider the traffic assignment between these 46 OD pairs. Other parameters are the same as these in Nguyen-Dupuis network. The outputs given by the single-level model developed in this paper are presented in Table 10. It can be found that with the increase of the number of charging stations to be constructed, the flows that use fast charging stations en-route increase at a decreasing increase rate.

5. Conclusion and discussion

Different from the internal combustion vehicles, battery EVs can be replenished with energy at home. If possible, all drivers would choose charging at home. However, as the number of EVs increases, a proportion of drivers might have to charge at the fast charging stations en-route, even though they travel on the short routes (i.e., the route length does not exceed the driving range of EVs). Obviously, the location of charging stations will influence the drivers' route choice behavior. In this paper, we develop a bi-level mathematical model to optimize the location of charging stations for EVs with the consideration of driving range. The upper-level is to maximize the flows served by charging stations, while the lower-level depicts the route choice behavior given the location of charging station. We then reformulate the bi-level problem as a single-level mathematical program and further linearize it for efficiently solving the problem. The numerical examples have verified effectiveness of the reformulated single-level model.

The model and algorithm can be further improved. Firstly, the maximum flows using the charging stations en-route are unstable if the path flows are not unique. Secondly, if letting the energy consumption of EVs be dependent upon traffic congestion, the optimal location of charging stations and the resultant flow pattern will be different somewhat. Thirdly, the computational efficiency of solving the single-level problem needs to be further improved. All these are our on-going researches.

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References

Bar-Gera, H., 2016. < https://github.com/bstabler/TransportationNetworks > (accessed 24.05.16).
Beale, E.M.L., Forrest, J.J.H., 1976. Global optimization using special ordered sets. Math. Program. 10 (1), 52–69.
Cai, H., Jia, X., Chiu, A.S.F., Hu, X.J., Xu, M., 2014. Siting public electric vehicle charging stations in Beijing using big-data informed travel patterns of the taxi fleet. Transport. Res. Part D 33, 39–46.
Capar, I., Kuby, M., Leon, V.J., Tsai, Y.J., 2013. An arc cover–path-cover formulation and strategic analysis of alternative-fuel station locations. Eur. J. Oper. Res. 227 (1), 142–151.
Chen, Z., Liu, W., Yin, Y., 2017. Deployment of stationary and dynamic charging infrastructure for electric vehicles along traffic corridors. Transport. Res. Part C 77, 185–206.
CPLEX, ILOG CPLEX Optimization Studio V12.6.0, 2014. User's Manual for CPLEX.
Dong, J., Liu, C., Lin, Z., 2014. Charging infrastructure planning for promoting battery electric vehicles: an activity-based approach using multiday travel data. Transport. Res. Part C 38 (1), 44–55.
Fuller, M., 2016. Wireless charging in California: range, recharge, and vehicle electrification. Transport. Res. Part C 67, 343–356.
Ghamami, M., Zockaie, A., Yu, N., 2016. A general corridor model for designing plug-in electric vehicle charging infrastructure to support intercity travel. Transport. Res. Part C 68, 389–402.
Guo, Z., Deride, J., Fan, Y., 2016. Infrastructure planning for fast charging stations in a competitive market. Transport. Res. Part C 68, 215–227.
He, F., Yin, Y., Lawphongpanich, S., 2014. Network equilibrium models with battery electric vehicles. Transport. Res. Part B 67 (3), 306–319.
He, F., Yin, Y., Zhou, J., 2015. Deploying public charging stations for electric vehicles on urban road networks. Transport. Res. Part C 60, 227–240.
He, S.Y., Kuo, Y.H., Wu, D., 2016. Incorporating institutional and spatial factors in the selection of the optimal locations of public electric vehicle charging facilities: a case study of Beijing, China. Transport. Res. Part C 67, 131–148.
Huang, Y., Zhou, Y., 2015. An optimization framework for workplace charging strategies. Transport. Res. Part C 52, 144–155.
Jiang, N., Xie, C., Waller, S.T., 2012. Path-constrained traffic assignment. Transport. Res. Rec J. Transport. Res. Board 2283, 25–33.
Jiang, N., Xie, C., Duthie, J.C., Waller, S.T., 2013. A network equilibrium analysis on destination, route and parking choices with mixed gasoline and electric vehicular flows. Eur. J. Transport. Logist. 3 (1), 55–92.
Jiang, N., Xie, C., 2014. Computing and analyzing mixed equilibrium network flows with gasoline and electric vehicles. Comput. Civ. Infrastruct. Eng. 29 (8), 626–641.
Jung, J., Chow, J.Y.J., Jayakrishnan, R., Park, J.Y., 2014. Stochastic dynamic itinerary interception refueling location problem with queue delay for electric taxi charging stations. Transport. Res. Part C 40, 123–142.
Kim, J.G., Kuby, M., 2013. A network transformation heuristic approach for the deviation flow refueling location model. Comput. Oper. Res. 40 (4), 1122–1131.
Kuby, M., Lim, S., 2005. The flow-refueling location problem for alternative-fuel vehicles. Soc. Econ. Plan. Sci. 39 (2), 125–145.
Kuby, M., Lim, S., 2007. Location of alternative-fuel stations using the flow-refueling location model and dispersion of candidate sites on arcs. Netw. Spatial Econ. 7 (2), 129–152.
Li, S., Huang, Y., 2014. Heuristic approaches for the flow-based set covering problem with deviation paths. Transport. Res. Part E 72, 144–158.
Li, S., Huang, Y., Mason, S.J., 2016. A multi-period optimization model for the deployment of public electric vehicle charging stations on network. Transport. Res. Part C 65, 128–143.
Lim, S., Kuby, M., 2010. Heuristic algorithms for siting alternative-fuel stations using the Flow-Refueling Location Model. Eur. J. Oper. Res. 204 (1), 51–61.
Liu, H., Wang, D.Z.W., 2017. Locating multiple types of charging facilities for battery electric vehicles. Transport. Res. Part B. http://dx.doi.org/10.1016/j.trb.2017.01.005.
Nguyen, S., Dupuis, C., 1984. An efficient method for computing traffic equilibria in networks with asymmetric transportation costs. Transport. Sci. 18 (2), 185–202.
Riemann, R., Wang, D.Z.W., Busch, F., 2015. Optimal location of wireless charging facilities for electric vehicles: flow-capturing location model with stochastic user equilibrium. Transport. Res. Part C 58, 1–12.
Shahraki, N., Cai, H., Turkay, M., Xu, M., 2015. Optimal locations of electric public charging stations using real world vehicle travel patterns. Transport. Res. Part D 41, 165–176.
Sherali, H.D., Adams, W.P., 1994. A hierarchy of relaxations and convex hull characterizations for mixed-integer zero—one programming problems. Discrete Appl. Math. 52 (1), 83–106.
Sun, X.H., Yamamoto, T., Morikawa, T., 2015. Charge timing choice behavior of battery electric vehicle users. Transport. Res. Part D 37, 97–107.
Tu, W., Li, Q.Q., Fang, Z.X., Shaw, S.L., Zhou, B.D., Chang, X.M., 2016. Optimizing the locations of electric taxi charging stations: a spatial-temporal demand coverage approach. Transport. Res. Part C 65, 172–189.
Upchurch, C., Kuby, M., Lim, S., 2009. A model for location of capacitated alternative-fuel stations. Geogr. Anal. 41 (1), 85–106.
Wang, D.Z.W., Lo, H.K., 2010. Global optimum of the linearized network design problem with equilibrium flows. Transport. Res. Part B 44 (4), 482–492.
Wang, D.Z.W., Liu, H.X., Szeto, W.Y., Chow, A.H.F., 2016a. Identification of critical combination of vulnerable links in transportation networks–a global optimization approach. Transportmetrica A 12 (4), 346–365.
Wang, T.G., Xie, C., Xie, J., Waller, T., 2016b. Path-constrained traffic assignment: a trip chain analysis under range anxiety. Transp. Res. Part C 68, 447–461.
Xie, C., Jiang, N., 2016. Relay requirement and Traffic Assignment of electric vehicles. Comput. Civ. Infrastruct. Eng. 31 580 365.
Xu, M., Meng, Q., Liu, K., Yamamoto, T., 2017. Joint charging mode and location choice model for battery electric vehicle users. Transport. Res. Part B. http://dx.doi.org/10.1016/j.trb.2017.03.004.
Xylia, M., Leduc, S., Patrizio, P., Kraxner, F., Silveira, S., 2017. Locating charging infrastructure for electric buses in Stockholm. Transport. Res. Part C 78, 183–200.
Yang, J., Dong, J., Hu, L., 2017. A data-driven optimization-based approach for siting and sizing of electric taxi charging stations. Transport. Res. Part C 77, 462–477.
Yang, Y., Yao, E., Yang, Z., Zhang, R., 2016. Modeling the charging and route choice behavior of BEV drivers. Transp. Res. Part C 65, 190–204.
Zhang, A., Kang, J.E., Kwon, C., 2017. Incorporating demand dynamics in multi-period capacitated fast-charging location planning for electric vehicles. Transport. Res. Part B. http://dx.doi.org/10.1016/j.trb.2017.04.016.