Minimizing the Number of Mobile Chargers in a Large-Scale Wireless Rechargeable Sensor Network

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***Abstract*—*Wireless Rechargeable Sensor Networks* (WRSNs) have attracted wide attention in recent years due to their potential to eliminate energy and lifetime bottlenecks. In a typical WRSN, a *Mobile Charger* ($\mathcal{MC}$) is employed to conduct charging on rechargeable sensor nodes, so that they can work continuously. However, since charging power, moving speed and total energy of one $\mathcal{MC}$ are limited, scalability becomes a major problem in such WRSNs. Therefore, multiple $\mathcal{MC}$s are required in serving large-scale WRSNs. This paper tackles the problem of deciding the minimum number of $\mathcal{MC}$s to keep every sensor node working continuously. We solve the problem in two steps. Firstly, we propose a greedy method for a *Tour Construction Problem* to construct a set of tours to 1-cover the WRSN. Energy of the sensor nodes in each of these tours can be timely replenished by one $\mathcal{MC}$ according to a decision condition, which is derived from a *Greedy Charging Scheme* (GCS). Secondly, we develop a heuristic algorithm for a *Tour Assignment Problem* to assign these tours to minimum number of $\mathcal{MC}$s. The applied heuristic rules minimize the conflict ratio of each $\mathcal{MC}$, so that it can charge along maximum number of tours. Simulation results show that, on average, the ratio of our results achieved by the two-step solution over a derived lower bound is less than 1.1.**

I. Introduction

Wireless sensor networks as powerful monitoring tools have been gaining wide attention from the research community in recent years. However, energy and lifetime bottlenecks due to limited battery pose a barrier for their large-scale deployment. Inspired by recent advances of wireless charging based on magnetic resonant coupling [1], a new paradigm of *Wireless Rechargeable Sensor Networks* (WRSNs) has shown its potential to solve the energy problem. In WRSNs, *Mobile Chargers* ($\mathcal{MC}$s) with high volume batteries are employed to replenish the depleted energy of rechargeable sensor nodes. These $\mathcal{MC}$s can be quickly replenished at a *Service Station* ($\mathcal{S}$). Applications include sustainable WRSNs in smart grids [2], underground sensor networks powered by UAVs [3], etc.

Most existing works on WRSNs only use one $\mathcal{MC}$ to conduct charging [4], [5]. However, an open issue in these works is their poor scalability. Even when the $\mathcal{MC}$ can simultaneously charge all sensor nodes within its charging area [6], the maximum network size is still limited by its charging power, moving speed and total energy. Therefore, multiple $\mathcal{MC}$s are employed to address the problem [7].

Since an $\mathcal{MC}$ can be very expensive, a first and foremost issue in designing a WRSN with multiple $\mathcal{MC}$s is to decide the minimum number of $\mathcal{MC}$s required, so that no sensor node will die. This problem, named *Minimum Mobile Chargers Problem* (MMCP) [7], is closely related to the *Distance-constrained Vehicle Routing Problem* (DVRP) [8]. Given a network $G = (E, V)$ and a distance bound D, the DVRP [8] is to find a minimum cardinality set of tours originating from a depot, that covers all the vertices in V. Each tour is required to have length at most D. Since $\mathcal{MC}$s are also energy constrained, researchers reduce MMCP to DVRP, so that solutions for DVRP can be applied to address MMCP [7]. We, however, argue that DVRP is only a sub-problem of MMCP. This is because an $\mathcal{MC}$ can conduct charging on several tours. For instance, suppose that, in a WRSN, each $\mathcal{MC}$ is not able to charge all the sensor nodes in one tour. At least k tours are needed to cover the whole WRSN, which is the answer to the varied DVRP. Suppose the time of charging along each tour is at most t, while the lifetime of any sensor node is at least $2t$, then at most $\frac{k}{2}$ $\mathcal{MC}$s are required, because each $\mathcal{MC}$ can charge along at least two tours. In this case, solving the varied DVRP only provides the first step to solving the MMCP.

Therefore, in this paper, we divide the MMCP into two sub-problems, i.e., a *Tour Construction Problem* (TCP) and a *Tour Assignment Problem* (TAP). These two problems are closely coupled. The TCP is to construct minimum number of schedulable tours to cover a WRSN. To address TCP, we construct a shortest Hamilton cycle through $\mathcal{S}$ and all the sensor nodes in a WRSN. Then based on a decision condition, we greedily split the cycle into several schedulable sub-tours. The TAP is to assign the obtained tours to minimum number of $\mathcal{MC}$s, so that all sensor nodes can be charged before their batteries are used up. We prove that TAP is NP-hard, and propose a heuristic algorithm to solve it. Following these two steps, we finally solve the MMCP.

The objectives of this paper are to decide the minimum number of $\mathcal{MC}$s and their charging schedules to keep all the sensor nodes in a WRSN working continuously. The contributions of the paper are the following. (a) We find the essential difference between the MMCP and the DVRP, and divide the MMCP into two NP-hard sub-problems. (b) We derive an efficient decision condition of whether a sequence of sensor nodes can be charged by one $\mathcal{MC}$ in one tour. The decision condition can be used to construct a set of schedulable tours to cover a WRSN. Charging schedule of charging along each tour can be easily obtained by applying the *Greedy Charging Scheme* (GCS). (c) We propose a heuristic algorithm to assign the tours to minimum number of $\mathcal{MC}$s. The developed heuristic rules can minimize the conflict ratio of an $\mathcal{MC}$, so that it can charge along maximum number of tours. (d) We develop a lower bound for MMCP. The lower bound is used to evaluate the performance of our solution.

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II. System Model and Problem Formulation

Consider a 2-D WRSN with homologous rechargeable sensor nodes $\mathcal{N} = \{N_1, N_2, \cdots N_n\}$ and $\mathcal{MC}s$ $\mathcal{M} = \{M_1, M_2, \cdots M_m\}$. Each sensor node N_i $(1 \leq i \leq n)$ has a battery with capacity E_S, and consumes energy at a constant rate p_i. The sensor nodes are equipped with wireless energy receivers, which can receive and store energy transferred from an $\mathcal{MC}$. Assume that charging and communication use different frequencies [4], then a sensor node can communicate while being charged. Suppose that the $\mathcal{MC}$s can move freely in the WRSN at a constant speed v. The energy consumed by each $\mathcal{MC}$ for moving per meter is a constant q_m. The $\mathcal{MC}$s record the positions of all sensor nodes, and can wirelessly charge the sensor nodes at a power rate q_c with efficiency η. Since η drops rapidly as the charging distance increases, we let the $\mathcal{MC}$s move to each sensor node to conduct charging. Consequently, q_c and η are viewed as constants. Charging and moving activities of each $\mathcal{MC}$ are powered by a same battery with volume E_M, which can be renewed at a service station $\mathcal{S}$ in an infinitely short period. $\mathcal{S}$ is also denoted by N_0.

Charging activities of an $\mathcal{MC}$ include 3 aspects: (a) charging a sensor node; (b) moving from one place to another; (c) resting at $\mathcal{S}$. The *charging mission* in a WRSN is to schedule the charging activities of the $\mathcal{MC}$s to ensure that all sensor nodes will never exhaust their energy. To accomplish a charging mission, the $\mathcal{MC}$s charge sensor nodes along a set of *tours* $\mathcal{R} = \{R_1, R_2, \cdots, R_z\}$. The union of the tours in $\mathcal{R}$ should fully cover the WRSN. Tour $R_i =< N_0, N_1, \cdots, N_{r_i}, N_0 >$ determines an $\mathcal{MC}$'s charging sequence of the r_i sensor nodes. An $\mathcal{MC}$ can charge along a set of tours if the $\mathcal{MC}$ can ensure liveness of all the sensor nodes in these tours, then each of the tours is called *schedulable*, and these tours can be *assigned to* the $\mathcal{MC}$. Suppose that all tours are independently charged, i.e., each $\mathcal{MC}$ must finish charging along one tour before it charges along the next. A *charging schedule* determines when do the $\mathcal{MC}$s charge which sensor nodes in what sequence and for how long. A charging schedule is called *valid* if it can be applied to accomplish the charging mission.

An $\mathcal{MC}$ periodically charges along tour R_i with *charging period* T_i $(1 \leq i \leq z)$. As will be proved in Section III, T_i of tour R_i ranges in a feasible region $[T_i^l, T_i^u]$. Assume that T_i is rational. Within each charging period T_i, the total time of an $\mathcal{MC}$ spends in moving and charging is called the *working time* of R_i, denoted by T_i^w $(T_i^w \leq T_i)$. Consequently, we also use a tuple (T_i, T_i^w) to substitute for R_i. However, if the charging period of R_i has not been determined, the tour can be denoted by a triple (T_i^l, T_i^u, ω_i), where ω_i is a working time function depended on T_i. The three representations of R_i will be used interchangeably in the rest of the paper, where no ambiguity occurs. Suppose k tours $\{R_1, R_2, \cdots, R_k\}$ are assigned to M_i, then we define the *element period* of M_i, denoted by G_i, as the greatest common divisor of charging periods of these k tours. Define the *conflict ratio* of M_i as $\delta_i = \frac{\sum_{j=1}^{k} T_j^w}{G_i}$. We will prove that if δ_i of M_i is no more than 1, a valid charging schedule of charging along these k tours can be easily constructed. For instance, in Fig. 1, $R_1 = (7.5, 1)$ and $R_2 = (5, 1.5)$ can be assigned to one $\mathcal{MC}$. Because the element period of the $\mathcal{MC}$ is $G = \text{GCD}(7.5, 5) = 2.5$, and the conflict ratio of the $\mathcal{MC}$ is $\delta = \frac{T_1^w + T_2^w}{G} = \frac{1+1.5}{2.5} = 1$, where function GCD$(\cdot)$ returns the greatest common divisor of the given numbers.

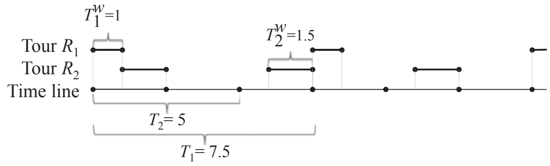

Fig. 1. Assigning R_1 an R_2 to one $\mathcal{MC}$.

The problem in this paper can be formulated as follows.

Minimum Mobile Chargers Problem (MMCP): *Given all parameters of $\mathcal{N}$ and each $\mathcal{MC}$ in a WRSN, decide the minimum size of $\mathcal{M}$ to accomplish the charging mission.*

As we obtaining the minimum number of $\mathcal{MC}$s, a valid charging schedule should be constructed at the same time. To this end, we solve the MMCP in two steps. By addressing the TCP, we answer how to charge along each tour; by tackling the TAP, we answer which $\mathcal{MC}$s charge along which tours.

III. The Tour Construction Problem

Tours in $\mathcal{R}$ should fully cover the WRSN; we consider a scenario that these tours form a exact 1-cover of the WRSN. In other words, intersection of any two tours is empty. Then we propose and address the following problem.

Tour Construction Problem (TCP): *Given all parameters of $\mathcal{N}$ and each $\mathcal{MC}$ in a WRSN, construct minimum number of schedulable tours $\mathcal{R}$ to exactly 1-cover the WRSN.*

The decision form of the TCP is NP-hard, since TCP contains DVRP as a sub-problem [7]. A simple transformation is to let $n \cdot E_S \rightarrow 0$, then the TCP can be reduced to a DVRP with distance constraint $\frac{E_M}{q_m}$. To address TCP, we firstly propose a sufficient condition for an $\mathcal{MC}$ to be able to charge along a tour. The condition is derived from a *Greedy Charging Scheme* (GCS). Then based on the decision condition, we greedily split the shortest Hamilton cycle through $\mathcal{S}$ and $\mathcal{N}$ into minimum number of schedulable sub-tours. The procedure is elaborated below.

A. The Greedy Charging Scheme

Each $M \in \mathcal{M}$ applies the GCS as follows. In each period, to charge along a tour $R =< N_0, N_1, \cdots, N_r, N_0 >$ $(1 \leq r \leq n)$, M starts from N_0, sequentially fully charges each sensor node. Then it returns to N_0, quickly renews its battery, and rests until next period. Denote that M arrives N_i at time t_i, and charges N_i for time τ_i, then we have

$$t_i = \sum_{j=0}^{i-1} \tau_j + \sum_{j=1}^{i} \frac{d_{j-1,j}}{v}. \quad 1 \leq i \leq r \tag{1}$$

In (1), $d_{i,j}$ is the distance between N_i and N_j, $\tau_0 \geq 0$ is the duration that M rests at $\mathcal{S}$, and let $t_0 = 0$ to simplify the presentation. Then M can periodically charge along R with charging period

$$T = \sum_{j=0}^{r} \tau_j + \frac{D}{v}, \tag{2}$$

where $D = \sum_{j=1}^{r} d_{j-1,j} + d_{r,0}$ is the total traveling distance of the tour. Then, within each charging period, the energy consumed by N_i can be fully replenished by M, i.e.,

$$\tau_i \cdot q_c \cdot \eta = p_i \cdot T. \quad 1 \le i \le r \tag{3}$$

To ensure the sensor nodes working continuously, their energy levels should never fall below 0 during T, i.e.,

$$E_S - p_i \cdot (T - \tau_i) \ge 0. \quad 1 \le i \le r \tag{4}$$

Meanwhile, the total energy of M should be enough to conduct charging, i.e.,

$$D \cdot q_m + \sum_{j=1}^{r} \cdot q_c \cdot \tau_j \le E_M. \tag{5}$$

Therefore, M can charge along tour R applying the GCS if and only if it satisfies (1)–(5).

B. The Decision Condition

We will show that, even when $\mathcal{MC}$s have infinite energy, the charging period of R can still range in a feasible region.

Proposition 1 *Apply the GCS to charge along tour* $R =< N_0, N_1, \cdots, N_r, N_0 >$*, then feasible charging period* T *satisfies that* $\frac{D \cdot q_c \cdot \eta}{v \cdot (q_c \cdot \eta - p_{sum})} \le T \le \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$*, where* $p_{sum} = \sum_{i=1}^{r} p_i$*, and* $p_{mx} = \max_{1 \le i \le r} p_i \cdot (q_c \cdot \eta - p_i)$.

Proof: From constraint (3) we have

$$\tau_i = \frac{p_i \cdot T}{q_c \cdot \eta}. \quad 1 \le i \le r \tag{6}$$

Since $\tau_0 \ge 0$, constraint (2) can be rewritten as $T \ge \sum_{j=1}^{r} \tau_j + \frac{D}{v}$. Combine it with (6) we have $T \ge \frac{D \cdot q_c \cdot \eta}{v \cdot (q_c \cdot \eta - p_{sum})}$, where $q_c \cdot \eta > p_{sum}$. On the other hand, by combining (4) with (6) we have $T \le \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$, where $p_{mx} = \max_{1 \le i \le r} p_i \cdot (q_c \cdot \eta - p_i)$. Therefore, the proposition follows. ■

Let $T^l = \frac{D \cdot q_c \cdot \eta}{v \cdot (q_c \cdot \eta - p_{sum})}$, and $T^u = \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$, then Proposition 1 means that the charging period of tour R cannot be less than T^l, otherwise M does not have enough time to charge all sensor nodes in R. Meanwhile, the charging period cannot exceed T^u, or some sensor node will run out of energy before it is charged. On the other hand, the charging period T of tour R is actually depended on τ_0: T increases as τ_0 increases. Also, the working time T^w of R increases as τ_0 increases, since $T^w = \frac{p_{sum}}{q_c \cdot \eta} \cdot T + \frac{D}{v}$. Based on Proposition 1, we have the following sufficient condition for M to charge along R.

Proposition 2 An $\mathcal{MC}$ applying the GCS can charge along tour $R =< N_0, N_1, \cdots, N_r, N_0 >$ if (a) $q_c \cdot \eta > p_{sum}$; (b) $v \ge \frac{D \cdot p_{mx}}{E_S \cdot (q_c \cdot \eta - p_{sum})}$; and (c) $E_M \ge D \cdot q_m + \frac{p_{sum} \cdot E_S \cdot q_c}{p_{mx}}$.

Proof: We only sketch the proof due to space limit, please refer [9] for more details. If (a) and (b) are met, it can be easily derived that $\frac{D \cdot q_c \cdot \eta}{v \cdot (q_c \cdot \eta - p_{sum})} \le \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$. Then according to the proof of Proposition 1, one can verify that (1)–(4) are satisfied. Let $T = T^u = \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$, then (c) equals to $E_M \ge D \cdot q_m + \frac{p_{sum} \cdot T}{\eta}$, which is equivalent with (5). Therefore, if the conditions in Proposition 2 are satisfied, (1)–(5) are satisfied, and the proposition follows. ■

Algorithm 1 Decide whether an $\mathcal{MC}$ can charge along R; decide T^w and T of R; construct a valid charging schedule.

Require:
  $R =< N_0, N_1, \cdots, N_r, N_0 >$, $\{p_i\}$, E_S, q_m, q_c, v, E_M, η;
Ensure:
  $\{t_i\}$, $\{\tau_i\}$ $(0 \le i \le r)$, T^w, T;
1: $t_i = 0,\ 0 \le i \le r$;
2: calculate path length D of R;
3: calculate p_{sum} and p_{mx};
4: **if** the conditions in Proposition 2 are not satisfied **then**
5:     **return** failure;
6: **end if**
7: set $T = T^u = \frac{E_s \cdot q_c \cdot \eta}{p_{mx}}$, $\quad T^w = \frac{p_{sum}}{q_c \cdot \eta} \cdot T + \frac{D}{v}$;
8: set $\tau_0 = T - T^w$, $\quad \tau_i = \frac{T \cdot p_i}{q_c \cdot \eta}$, $1 \le i \le r$;
9: set $t_i = t_{i-1} + \tau_{i-1} + \frac{d_{i-1,i}}{v}$, $1 \le i \le r$;
10: **loop**
11:     Set $t_0 = 0$;
12:     The $\mathcal{MC}$ leaves $\mathcal{S}$ at time τ_0;
13:     **for** $i = 1$ **to** r **do**
14:         The $\mathcal{MC}$ moves to N_i at t_i, and charges for τ_i;
15:     **end for**
16:     The $\mathcal{MC}$ returns to $\mathcal{S}$ at time T;
17: **end loop**

Algorithm 1 shows the procedure of deciding whether an $\mathcal{MC}$ applying the GCS can charge along R (lines 2–6), and constructing a valid charging schedule if possible (lines 7–16). The reason to choose $T = T^u$ in line 7 will be explained in Section IV. It can be proved that the computation complexity of Algorithm 1 is O(r), where r is the number of sensor nodes in tour R.

Although Proposition 2 is not necessary for a tour to be schedulable [9], $q_c \cdot \eta > p_{sum}$ is a hard condition for any $\mathcal{MC}$ to be able to charge along any tour. Furthermore, if the total charging power of $\mathcal{M}$ is not larger than the total energy consumption rate of $\mathcal{N}$ in the WRSN, the charging mission cannot be accomplished. Therefore, we obtain a lower bound of the number of $\mathcal{MC}$s needed to solve the MMCP.

Lower Bound m^***:** *Given parameters of* $\mathcal{N}$ *and each* $\mathcal{MC}$ *in a WRSN, the minimum number of* $\mathcal{MC}$*s needed to accomplish the charging mission is not less than* $m^* = \lceil \frac{\sum_{i=1}^{n} p_i}{q_c \cdot \eta} \rceil$.

The lower bound m^* is not tight, since it only considers the influence of charging. Obviously, an $\mathcal{MC}$ also spends time and energy on moving. Therefore, a more rigorous lower bound should be not less than m^*. However, determining m^* is very simple and efficient, thus we use m^* for performance evaluation in Section V.

C. Obtaining Schedulable Sub-Tours

Since TCP is NP-hard, based on the derived decision condition, we propose a greedy solution, as shown in Algorithm 2. Firstly, we construct a Hamilton tour H through $\mathcal{S}$ and $\mathcal{N}$ by applying LKH algorithm [10] (lines 2–3). Then we replace edges $e_{i,i+1}$ and $e_{j,j+1}$ by $e_{0,i+1}$ and $e_{j,0}$ (lines 5,7), and decide whether an $\mathcal{MC}$ can charge along tour $< N_0, N_{i+1}, \cdots, N_j, N_0 >$ using Algorithm 1 (line 8). A sub-tour is constructed if no more sensor node can be inserted (lines 9–13). Repeat the procedure until all sensor nodes are covered (line 4). In this way, we greedily split H into z schedulable sub-tours. Computation complexity of verifying Proposition 2 is O(r). If variables such as p_{sum} and p_{mx} can be saved in the algorithm, the complexity of updating these information is O(1). Then lines 3–15 can be executed with complexity O(n). Therefore, the computation complexity of Algorithm 2 is depended on that of LKH, which is O($n^{2.2}$) [10]. Note that one can reduce the complexity by using a simpler TSP solution when constructing H, at the cost of reducing energy efficiencies of $\mathcal{MC}$s.

Algorithm 2 Construct a Hamilton tour H through $\mathcal{S}$ and $\mathcal{N}$; split H into z schedulable sub-tours.

Require:
  $\mathcal{N} = \{N_i\}$ $(1 \le i \le n)$;
Ensure:
  $\mathcal{R} = \{R_i\}$ $(1 \le i \le z)$;
1: $z = 1$, $i = 1$;
2: apply LKH on $\mathcal{S} \cup \mathcal{N}$ to obtain H;
3: renumber sensor nodes in H as $N_0,\ N_1, \cdots, N_n$ $(N_0 = \mathcal{S})$;
4: **while** $i \le n$ **do**
5:   push N_0 into R_z;
6:   **for** $j = i$ **to** n **do**
7:     push N_j into R_z;
8:     apply Algorithm 1 on R_z;
9:     **if** Algorithm 1 returns failure **then**
10:       pop N_j from R_z;
11:       $i = j,\ z = z + 1$;
12:       **break**;
13:     **end if**
14:   **end for**
15: **end while**

IV. The Tour Assignment Problem

After obtaining schedulable sub-tours that 1-cover all the sensor nodes, we further assign these sub-tours to minimum number of $\mathcal{MC}$s. To this end, we firstly formulate the *Tour Assignment Problem* (TAP) and prove its NP-hardness. Then we propose a heuristic algorithm to solve the problem. Every tour discussed in this section is schedulable.

A. The Tour Assignment Problem

Feasible charging period T_i of tour R_i ranges from T_i^l to T_i^u according to Proposition 1, which also decides the working time T_i^w of R_i in each period: $T_i^w = \omega_i(T_i) = \frac{p_{sum}^i}{q_c \cdot \eta} \cdot T_i + \frac{D_i}{v}$. Then the assignment problem can be formulated as follows.

Tour Assignment Problem (TAP): *Given z tours $\mathcal{R} = \{R_i = (T_i^l, T_i^u, \omega_i)\}$ $(1 \le i \le z)$ and enough $\mathcal{MC}$s (q_c, q_m, v, E_M), assign tours $\mathcal{R}$ to minimum number of $\mathcal{MC}$s.*

To prove the NP-hardness of the TAP, we firstly consider r tours with equal charging period T, and investigate the sufficient and necessary condition of assigning these r tours to one $\mathcal{MC}$.

Proposition 3 *Given r tours $\{R_i = (T, T_i^w)\}$ $(1 \le i \le r)$, they can be assigned to one $\mathcal{MC}$ if and only if $\sum_{i=1}^{r} T_i^w \le T$.*

Proof: Suppose these k tours can be assigned to a same $\mathcal{MC}$, then $\sum_{i=1}^{r} T_i^w \le T$. Otherwise the $\mathcal{MC}$ can't finish charging along all tours within one charging period T, causing conflicts in the next period. On the other hand, if $\sum_{i=1}^{r} T_i^w \le T$, one can construct a valid schedule as follows. The $\mathcal{MC}$ charges sensor nodes in R_i during time $[k \cdot T + \sum_{j=1}^{i-1} T_j^w,\ k \cdot T + \sum_{j=1}^{i} T_j^w]$, where $k = 0, 1, 2, \cdots$. Since $\sum_{i=1}^{r} T_i^w \le T$, this schedule is valid. Therefore, the proposition follows. ∎

Based on Proposition 3, we reduce the TAP to a set-partition problem, and consequently prove that the decision form of the problem is NP-hard.

Proposition 4 *Decide whether z tours can be assigned to m $\mathcal{MC}$s is NP-hard.*

Proof: Consider z tours $\{R_i = (T, T_i^w)\}$ $(1 \le i \le z)$, and suppose $\sum_{i=1}^{z} T_i^w = 2T$. Let $m = 2$, then the decision problem can be reformulated as whether $\{R_i\}$ can be assigned to 2 $\mathcal{MC}$s. We now construct a set-partition problem as follows. Given z numbers $\{a_1, a_2, \cdots, a_z\}$, in which $a_i = \frac{T_i^w}{T}$ $(1 \le i \le z)$. Then according to Proposition 3, the above question is equivalent to whether the z numbers can be evenly partitioned into two sets. Since the set-partition problem is NP-hard, the original problem is also NP-hard. ∎

B. The Heuristic Solution

We propose a heuristic method to decide (a) the charging period of each tour, and (b) whether a set of tours with fixed charging periods can be assigned to one $\mathcal{MC}$.

To this end, we generalize the assumptions in Proposition 3, and assume that tours have different charging periods. Then we derive a sufficient condition for these tours to be assigned to a same $\mathcal{MC}$. Suppose that the charging periods of all the tours are rational numbers, and their greatest common divisor is G (*G can be a decimal*), then we have the following proposition.

Proposition 5 *Given r tours $\{R_i = (a_i \cdot G, T_i^w)\}$ $(1 \le i \le r)$, where $\{a_i\}$ are coprime integers, they can be assigned to one $\mathcal{MC}$ if $\sum_{i=1}^{r} T_i^w \le G$.*

Proof: Consider r tours $\{R_i' = (G, T_i^w)\}$ $(1 \le i \le r)$, if they can be assigned to one $\mathcal{MC}$, obviously $\{R_i\}$ can be assigned to a same $\mathcal{MC}$. Since $\sum_{i=1}^{r} T_i^w \le G$, according to Proposition 3, $\{R_i'\}$ can be assigned to one $\mathcal{MC}$. Therefore, $\{R_i\}$ can be assigned to one $\mathcal{MC}$. ∎

Note that Proposition 5 is not necessary. For instance, consider tour $R^A = (T, \frac{T}{2})$ and r tours $\{R_i^B = (r \cdot T, \frac{T}{2}\}$ $(r > 1,\ 1 \le i \le r)$. Since GCD$(T, r \cdot T) = T$, and the total working time of the $r+1$ tours is $\frac{(r+1)}{2} \cdot T > T$, the condition in Proposition 5 is not met. However, one can still assign them to one $\mathcal{MC}$: let the $\mathcal{MC}$ charge along tour R^A during time $[j \cdot T,\ (j + \frac{1}{2}) \cdot T]$, and charge along tour R_i^B during time $[(j \cdot r + i - \frac{1}{2}) \cdot T,\ (j \cdot r + i) \cdot T]$, where $j = 0, 1, 2, \cdots$.

Proposition 5 inspires us to minimize the conflict ratio of each $\mathcal{MC}$: $\delta = \sum_{i=1}^{r} \frac{T_i^w}{G}$. Therefore, informally speaking, δ indicates the work load of an $\mathcal{MC}$, and when it is no more than 1, valid charging schedule exists. Based on this idea, we propose the following heuristic rule to determine the charging period of each tour.

Proposition 6 *Suppose tours $R_1 = (T_1^l, T_1^u, \omega_1)$ and $R_2 = (T_2^l, T_2^u, \omega_2)$ are assigned to M, where $(k-1) \cdot T_1^u < T_2^l \le k \cdot T_1^u \le T_2^u$ $(k \ge 1)$, then the conflict ratio δ of M is minimized when $T_1 = T_1^u$ and $T_2 = k \cdot T_1^u$.*

Proof: For each tour R_i, $\omega(T_i) = \frac{p^i_{sum}}{q_c \cdot \eta} \cdot T_i + \frac{D_i}{v}$. To simplify the presentation, denote that $\omega(T_i) = a_i \cdot T_i + b_i$.

Consider δ when T_2 varies within time interval $[i \cdot T_1, (i+1) \cdot T_1]$ $(i = 1, 2, \cdots)$:

$$\begin{cases} \delta_1 = a_1 + i \cdot a_2 + \frac{b_1+b_2}{T_1}, & T_2 = i \cdot T_1 \\ \delta_2 = a_1 + (i+1) \cdot a_2 + \frac{b_1+b_2}{T_1}, & T_2 = (i+1) \cdot T_1, \\ \delta_3 = \frac{a_1 \cdot T_1 + a_2 \cdot T_2}{G} + \frac{b_1+b_2}{G}, & i \cdot T_1 < T_2 < (i+1) \cdot T_1 \end{cases}$$

where $G = \text{GCD}(T_1, T_2)$. When $i \cdot T_1 < T_2 < (i+1) \cdot T_1$, $G \leq \frac{T_1}{2}$, we hence have $\delta_3 = \frac{a_1 \cdot T_1 + a_2 \cdot T_2}{G} + \frac{b_1+b_2}{G} \geq 2a_1 + 2i \cdot a_2 + \frac{2(b_1+b_2)}{T_1}$. In other words, $\delta_1 < \delta_2 < \delta_3$. Therefore, in this scenario, δ is minimized when $T_2 = i \cdot T_1$. Then $\delta = a_1 + i \cdot a_2 + \frac{b_1+b_2}{T_1}$. On the other hand, δ decreases as T_1 increases. To minimize δ, we have $T_1 = T_1^u$. Since $(k-1) \cdot T_1^u < T_2^l \leq k \cdot T_1^u \leq T_2^u$, we have $i = k$, i.e., $T_2 = k \cdot T_1^u$. ∎

In fact, Proposition 6 requires the element period G of M to be maximized. This is the main reason that we set $T = T^u$ when deriving the decision condition in Proposition 2. Note that one can set $T = T'$ $(T^l \leq T' < T^u)$ in deriving the decision condition. In this way, the number of sub-tours may be reduced. Nevertheless, these tours are harder to be assigned to one $\mathcal{MC}$ according to Proposition 6. From this viewpoint, the two steps of the solution are tightly coupled; it is actually a tradeoff between energy efficiency of $\mathcal{MC}$s and flexibility of tour assignment when generating the schedulable sub-tours.

Algorithm 3 Sort tours $\mathcal{R}$ by T^u; greedily assign the tours to minimum $\mathcal{MC}$s based on Proposition 5 and Proposition 6.

Require:
  $\mathcal{R} = \{R_i\}$ $(1 \leq i \leq z)$;
Ensure:
  $\mathcal{M} = \{M_i\}$ $(1 \leq i \leq m)$, tour assignment of each $\mathcal{MC}$;
1: $m = 0$;
2: sort $\{R_i\}$ by T_i^u in ascending order;
3: **for** $i = 1$ **to** z **do**
4:   **if** R_i has not been assigned **then**
5:     $m+=1$;
6:     assign R_i to M_m;
7:     $G_m = T_i = T_i^u$, $\delta_m = \frac{\omega_i(T_i)}{G_m}$;
8:     **for** $j = i+1$ **to** z **do**
9:       **if** R_j has not been assigned **then**
10:         calculate the k described in Proposition 6;
11:         **if** k exists **and** $\delta_m + \frac{\omega_j(k \cdot T_i^u)}{G_m} \leq 1$ **then**
12:           $T_j = k \cdot T_i^u$, $\delta_m += \frac{\omega_j(T_j)}{G_m}$;
13:           assign R_j to M_m;
14:         **end if**
15:       **end if**
16:     **end for**
17:   **end if**
18: **end for**

Based on heuristic rules formulated in Proposition 5 and Proposition 6, we propose Algorithm 3 to greedily assign maximum number of tours to each $\mathcal{MC}$. Firstly, the tours obtained by Algorithm 2 are sorted by T^u in ascending order (line 2). Then for each $\mathcal{MC}$, sequentially consider if tour R_i can be assigned to it (lines 3–18). According to Proposition 6, the element period G_k of M_k is set to T_i^u of its first assigned tour R_i (line 7). For any subsequent tour R_j, it is assigned to M_k (lines 6,13) if (a) R_j has not been assigned to any $\mathcal{MC}$ (lines 4,9), (b) feasible period of R_j contains a multiple of G_k (lines 10–11), and (c) after assigning, the conflict ratio δ_k of M_k is still no more than 1 (line 11). Then charging period T_j of R_j is determined according to Proposition 6 (lines 7,12). It can be readily proved that the worst case computation complexity of Algorithm 3 is O(z^2), where z is the number of tours.

C. The Complete Procedure of the Solution

To summarize, the complete procedure of the two-step solution is presented as follows. Firstly, apply Algorithm 2 to obtain z schedulable tours. During this step, Algorithm 1 is constantly called to decide whether a tour of sensor nodes can be timely charged by one $\mathcal{MC}$. Secondly, apply Algorithm 3 to decide the charging period of each tour, and assign these tours to m $\mathcal{MC}$s. Then the $\mathcal{MC}$s apply the GCS to conduct charging along their assigned tours. The computation complexity of the solution is depended on Algorithm 2, which is O($n^{2.2}$).

Suppose r tours $\{R_i = (a_i \cdot G_x, T_i^w)\}$ are assigned to M_x $(1 \leq i \leq r \leq n,\ 1 \leq x \leq m)$, then a valid charging schedule is that M_x charges R_i during time $[k \cdot a_i \cdot G_x + \sum_{j=1}^{i-1} T_j^w,\ k \cdot a_i \cdot G_x + \sum_{j=1}^{i} T_j^w]$ following Algorithm 1, and rests at $\mathcal{S}$ at other times, where $k = 0, 1, 2, \cdots$. However, when M_x charges R_i for the first time, since the sensor nodes are initially fully charged, the actual working time of tour R_i is less than T_i^w. This may influence subsequent periods. To address this issue, one can make M_x stay at sensor node N_j in R_i for time $\frac{p_j \cdot T_i}{q_c \cdot \eta}$, and leave with N_j fully charged. In this way, the actual working time of R_i is T_i^w afterwards.

V. Numerical Results

In this section, we evaluate the performance of the proposed solution. To this end, WRSNs were randomly generated over a 1000 m × 1000 m square area. The sink node and service station was located at the center of the area. The sensor nodes generated data at a random rate within [10,100] kbps, and their communication radius was 200 m. The energy consumption model was adopted from [5], and the routing scheme was the well known GPSR [11], which is well scalable and also based on location information. By default, we set $E_S = 10$ kJ, $q_m = 5$ J/m, $v = 5$ m/s, $E_M = 40$ kJ, $\eta = 1$.

To better demonstrate the procedures of the solution, we randomly generated a 100-node WRSN, and set $q_c = 5$ W. According to Algorithm 2, we firstly applied LKH algorithm to construct a Hamilton tour H through $\mathcal{S}$ and $\mathcal{N}$, as shown in Fig. 2. Then based on the decision results returned by Algorithm 1, we greedily split H into 10 schedulable sub-tours, and called Algorithm 3 to assign these tours to 4 $\mathcal{MC}$s, as shown in Fig. 3. Fig. 4 depicts the energy consumption of each $\mathcal{MC}$ on charging along each tour. From the figure, we can see that the energy consumed in charging along each tour varies greatly. This is because in the assigning procedure, we selected T of $R_1 \sim R_6$ nearly the same as their T^u, while selected T of $R_7 \sim R_{10}$ much smaller than their T^u. In consequence, in charging along $R_7 \sim R_{10}$, the energy consumption of the $\mathcal{MC}$ was much less than E_M. In turn, these tours contributed less conflict ratio of the $\mathcal{MC}$, as shown in

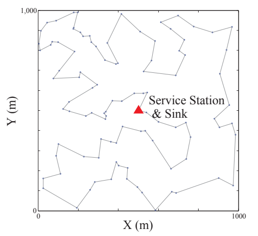

Fig. 2. TSP solution of a 100-node WRSN

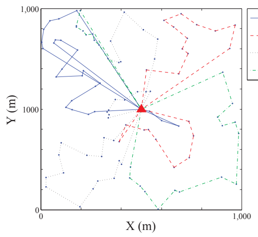

Fig. 3. 10 Sub-tours and their assignment to 4 $\mathcal{MC}$s

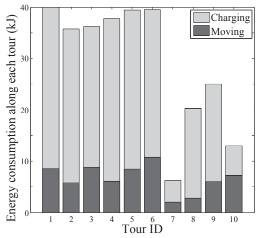

Fig. 4. Energy consumption of charging each tour

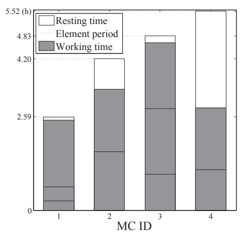

Fig. 5. Conflict ratios of the $\mathcal{MC}$s

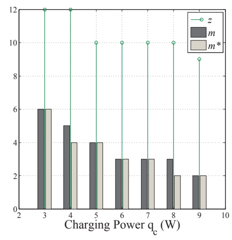

Fig. 6. Comparison between m and m^*

Fig. 5, so that the $\mathcal{MC}$ could charge more tours. The ratio of the grey area in a bar is the conflict ratio of an $\mathcal{MC}$. It can be seen that M_4 achieved a relatively low conflict ratio. This was caused by the greedy nature of our solution. Balanced energy consumption of $\mathcal{MC}$s,however, is not a concern of this paper.

We then compare the number of $\mathcal{MC}$s m obtained by our solution with the lower bound m^* to evaluate the performance. Specifically, we evaluate the approximation ratio of our results over the lower bound: $\alpha = \frac{m}{m^*}$.

The total energy consumption rate of the WRSN shown in Fig. 2 was $p_{sum} = 15.24$ W, and the corresponding $m^* = \lceil \frac{15.24}{5} \rceil = 4$. Therefore, we actually gave an optimum solution in the demonstration, regarding number of $\mathcal{MC}$s used. We further ranged charging power of the $\mathcal{MC}$s from 3 W to 9 W, and the comparison between m and m^* is shown in Fig. 6. It is clear that, in most cases, our method achieved optimum solutions of MMCP. However, when $q_c = 4$ W and $q_c = 8$ W, our solutions used one more $\mathcal{MC}$ than m^*. This is because the lower bound m^* only relates to total energy consumption rate of a WRSN and charging power of each $\mathcal{MC}$. In fact, the $\mathcal{MC}$s also spend energy and time in moving. Therefore, if the total charging power of $\mathcal{MC}$s is not significantly larger than the total energy consumption rate of the WRSN, m^* may not be achievable.

TABLE I. APPROXIMATION RATIOS VARYING NETWORK SIZE

n	p_{sum} (W)	m^*	m	α
50	[6.29, 8.08, 10.14]	[2, 2.1, 3]	[2, 2.0, 3]	1.05
100	[13.34, 15.32, 17.66]	[3, 3.6, 4]	[3, 3.9, 4]	1.08
150	[20.73, 23.32, 24.80]	[5, 5.0, 5]	[5, 5.5, 6]	1.10
200	[27.90, 31.44, 36.50]	[6, 6.8, 8]	[6, 7.0, 8]	1.03
250	[36.70, 40.19, 44.16]	[8, 8.5, 9]	[8, 9.1, 10]	1.07

To experimentally analyze the approximation ratio of our solution over the lower bound m^*, we set $q_c = 5$ W, and ranged n from 50 to 250. With each size n, we randomly generated 10 WRSNs. Table I depicts a summary of the results, where only minimum values, average values and maximum values of p_{sum}, m^* and m are listed. In fact, in most cases, our method achieved optimum solutions. In other situations, $m^* \cdot q_c \cdot \eta$ was not significantly larger than p_{sum}, and we used one more $\mathcal{MC}$ than m^*. Moreover, such "significance" enlarged when n increases. This is because as n increases, the total traveling distance of the $\mathcal{MC}$s also increases. Consequently, the movement makes larger influence, and m^* deviates more from the rigorous lower bound. Even though, according to the results, the average approximation ratio of our solution was below 1.1 compared with the lower bound m^*.

VI. Conclusion and Future Work

In this paper, we exploit the *Minimum Mobile Chargers Problem* (MMCP) in *Wireless Rechargeable Sensor Networks*. We divide the problem into two NP-hard sub-problems: the *Tour Construction Problem* (TCP) and the *Tour Assignment Problem* (TAP). We propose a greedy method to solve the TCP, and develop a heuristic algorithm to address the TAP. Simulations are conducted to evaluate the performance of our solution. Numerical results show that, in most scenarios, our method generates optimum solutions for the MMCP. Compared with the *Lower Bound* m^*, our method achieves an approximation ratio less than 1.1. Future work includes the following aspects. (a) Develop a more rigorous lower bound of the MMCP. (b) Improve the energy efficiency of $\mathcal{MC}$s by carefully merging traveling paths of some tours. (c) Consider load balance between $\mathcal{MC}$s.

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