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例 6.4、 $\hat{\theta}_1 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}$ 、 $\hat{\theta}_2 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$

Sol: $E(X_i)=\mu$, $V(X_i)=\sigma^2=E(X_i^2)-\mu^2$

則 $E(\bar{X})=\mu$, $V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-\mu^2$

$$\Rightarrow E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\sigma^2.$$

$$\Rightarrow E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right).$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2).=\sigma^2.$$

$\therefore$ $\hat{\theta}_2$ 為母體變異數 σ^2 之不偏估計量.