Topic 1: Constrained Optimization, Preference

Terms for Optimization

Objective Function: The function that we want to maximise or minimise
- Example of Objective Function: $f(x,y) = -x^2 + 2x - y^2 + 4y + 5$

First Order Condition: Conditions imposed on the First Derivative to maximise / minimise the objective function
- **Maximum/Minimum**: First Derivative $\frac{dy}{dx} = 0$
- Note that the number of first order conditions depends on the number of variables that we have. (i.e. If there are 2 variables, we will have 2 conditions and taking partial derivative on the different variables)

Second Order Condition: Conditions imposed on the Second Derivative to maximise / minimise the objective function
- **Maximum**: Second Derivative $\frac{d^2y}{dx^2} \leq 0$ (Decreasing Slope)
- **Minimum**: Second Derivative $\frac{d^2y}{dx^2} \geq 0$ (Increasing Slope)
- Note that the number of second order conditions depends on the number of variables that we have. (i.e. If there are 2 variables, we will have 2 conditions and taking derivative on the different variables

Unconstrained Optimization Problem: When there are no constrains on the objective function and we just maximise it using the first order condition
- **Objective Function**: We may want to maximise or minimise the function depending on the problem

Constraint: Just another function that we need to ensure is true while we maximise / minimise the objective function
- Example of Constraint: $x + y = 1$

Constrained Optimization Problem: When there is another equation that we must satisfy while maximising / minimising our objective function
- **Objective Function**: We may want to maximise or minimise the function depending on the problem
- **Constraint**: Need to incorporate this while maximising or minimising objective function
- Use Lagrange Multiplier Method to solve constrained optimization problem

Conversion Into Unconstrained Problem

Lagrange Multiplier Method:
Suppose constrained optimization problem is:

$$\max_{x,y} f(x,y) \ s.t. \ g(x,y) = 0$$

Where $f(x,y)$ – Objective Function, $g(x,y)$ – Constraint
1) Rewrite the constraint by putting all into one side so that it is = 0 $(g(x,y) = 0)$
2) Construct the Lagrangian Function:
$$\Lambda(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$
3) Solve the first order conditions for all the variables including the λ
   a. $\frac{\partial \Lambda}{\partial x} = \frac{\partial f(x,y)}{\partial x} + \lambda \frac{\partial g(x,y)}{\partial x} = 0$
   b. $\frac{\partial \Lambda}{\partial y} = \frac{\partial f(x,y)}{\partial y} + \lambda \frac{\partial g(x,y)}{\partial y} = 0$
   c. $\frac{\partial \Lambda}{\partial \lambda} = g(x,y) = 0$

Assumptions for Consumer Theory

1) Consumers are rational – They will maximise utility
2) Consumers face budget constraints – Budget constraints by their income
3) Consumers are fully informed – They have perfect information about all the prices of all products

Ranking

Strictly Prefer:

Definition 1.1: A consumer (strictly) prefers A to B if the consumer is more satisfied with A than with B
- Notation: $A \succ B$

Indifferent:

Definition 1.2: A consumer is indifferent between A and B if the consumer is equally satisfied with A or B
- Notation: $A \approx B$

Consumption Basket: Combination of units of goods that the consumer can consume
- Main comparison for the preference of consumers

	Food	Clothing	All others
Basket 1	40 units	20 units	10 units
Basket 2	50 units	10 units	20 units
Basket 3	30 units	30 units	15 units

Figure 1 Example of Consumption Basket

Assumptions for Preference

Completeness: Ensures that the consumer always knows how to rank 2 baskets, so that consumer preference exists between any 2 baskets
For any two baskets A and B:
- Either $A \succ B$
- Or $B \succ A$
- Or $A \approx B$

Transitivity: Ensures that the ranking does not loop
- If $A \succ B \text{ and } B \succ C, \text{ then } A \succ C$
- If $A \succ B \text{ and } B \approx C, \text{then } A \succ C$

"More is Better" Assumption:
- Consumer likes the good
- Consuming more increases the satisfaction level

Note that the "more is better" assumption may not always hold, it could also be that the consumer does not like the good and consuming less is better

If we have more of one good and the amount for the other good is constant, we will have higher satisfaction level

North-East Point: Increase in Satisfaction
South-West Point: Decrease in Satisfaction

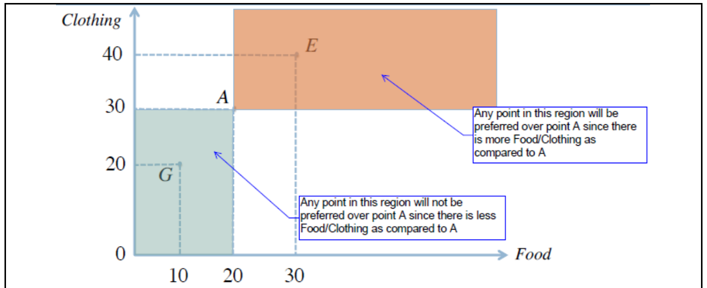


Figure 2 Graphical Representation

How to check whether "More is better" is satisfied:
- Check the marginal utilities and if they are a positive for both then more is better is satisfied for both. If only one is positive then "more is better" is satisfied only for that

Indifference Curves:

Definition 1.3: An indifference curve of a consumer connects all consumption baskets that give the consumer the same level of satisfaction

Note that any combination of the same indifference curve gives the same level of satisfaction to the consumer

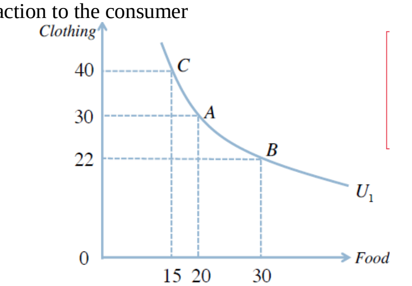


Figure 3 Example of Indifference Curve

Features: Indifference Curves do not cross because of transitivity
Since $A \approx B \ and \ A \approx C$. By transitivity, $B \approx C$. However, this will violate the fact that they are on different indifference curves

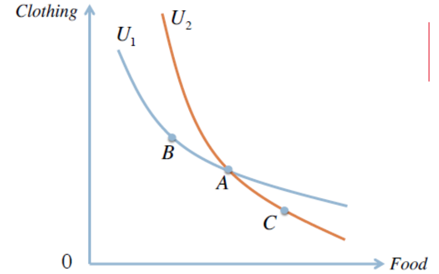


Figure 4 Graph for Crossing of Indifference Curves

Features: Indifference Curves are Downward Sloping when Consumer likes both goods under "More is Better" Assumption

If more is better is satisfied for both goods, E should be preferred to A but since A and E lie on the same indifference curve, they should have the same level of satisfaction which is a contradiction. Therefore, it should be downward sloping

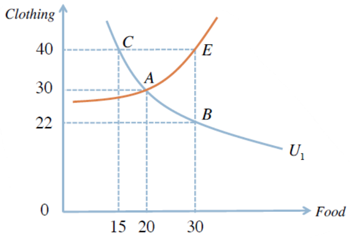


Figure 5 Graph for Upward and Downward Sloping Curve

Features: Direction of Preference when Consumer likes both goods

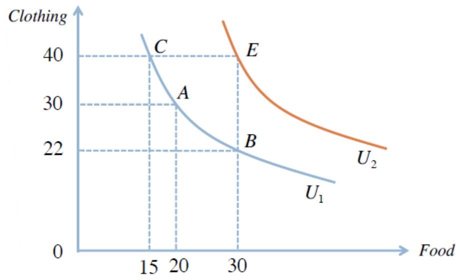


Figure 6 Graphical Representation of Indifference Curves

The level of satisfaction of the indifference curves increases in the north-east direction

- E is preferred to A
- E is preferred to any point on U_1 by transitivity
- E indifferent to any point on U_2
- Any point on U_2 preferred to any point on U_1, by transitivity

Topic 2: Preference, Budget Constraint, Consumer Choice

Marginal Rate of Substitution (MRS):

Definition 2.1: Marginal rate of substitution of x for y is the rate at which the consumer is willing to give up y to get more of x, maintaining the same level of satisfaction

$$MRS_{x,y} = -\left.\frac{dy}{dx}\right|_{Same\ U} = -\left.\frac{\Delta y}{\Delta x}\right|_{Same\ U} = \frac{MU_x}{MU_y}$$

y – Consumption of good y
x – Consumption of good x
MU_x – Marginal Utility of x
MU_y – Marginal Utility of y
Note that Δx is extremely small

Connection with Marginal Utility:
The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the ratio of the marginal utilities of the goods

- **Example**: Consumer willing to give up 2 units of y for 1 unit of x without a change in utility → MRS = 2 → If consumer is willing to give up 2 units of y for 1 unit of x → Marginal Utility of x must be 2 times more than y since 1 unit of x will give the same level of utility as 2 units of y → $MU_x = 2, MU_y = 1$

Note:
- $MRS_{x,y}$ is the negative of the indifference curve (with x on the horizontal axis and y on the vertical axis)
- It tells us how much the consumer is willing to substitute between the 2 goods without changing the level of satisfaction
- It is defined as negative so that we can get a positive number since most of the time "more is better" holds and it will be downward sloping

Diminishing Marginal Rate of Substitution

Meaning: $MRS_{x,y}$ decreases as the consumer gets more x and less y along the same indifference curve. Holding satisfaction level fixed, as the consumer gets more of x, the willingness to give up y and get additional x reduces

Intuition: Since the consumer already gets more x, it will become less valuable as compared to y. As the consumer consumes more x, they may not want to give up that many y in the process

Convex Shape of Indifference Curve: Indifference curves are convex to the origin. Because at the origin is where the consumption is balanced and it will be halfway through the decrease in MRS. Note that the follow must hold

1) Diminishing Marginal Rate of Substitution Holds
2) Completeness Holds
3) Transitivity Holds
4) More is better Holds

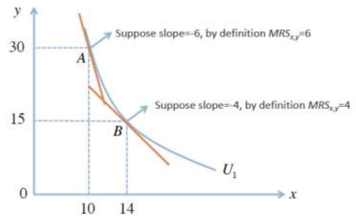


Figure 7 Example of a Convex Shape

Utility Function

Utility: Numeric value indicating the consumer's level of satisfaction

Definition 2.2: Utility function assigns a level of utility to each consumption basket so that $A \succ B$ is equivalent to $U(A) > U(B)$

Note:
- Utility function represents preference
- Higher utility = Higher level of satisfaction

Plotting Indifference Curves:
1) Get the Utility Function
2) Fix a level of Utility (i.e. U = 2, U =3), find a few points that corresponds to the level of utility
3) Connect the points that leads to same level of Utility
4) Repeat for a few Utilities and we will get different indifference curves

Marginal Utility

Definition 2.3: Marginal utility is the rate at which utility changes as the level of consumption of a good changes

For 1 Good:

$$MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$$

U – Utility Function
x – Consumption of good x
Note that Δx is extremely small

For 2 Goods:
Marginal Utility of x :

$$MU_x = \frac{\partial U}{\partial x}$$

Marginal Utility of y:

$$MU_y = \frac{\partial U}{\partial y}$$

U – Utility Function
x – Consumption of good x
y – Consumption of good y

Note:
- MU is the slope of the utility function

Whether "More is Better" is Satisfied (Look at the Sign of the Marginal Utility)	○ *Positive* – Utility is increasing with consumption (**Satisfied**) ○ *Negative* – Utility is decreasing with consumption (**Not Satisfied**)

Principle of Diminishing Marginal Utility:

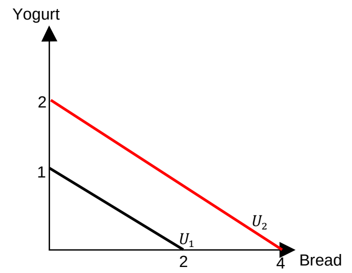
- Marginal utility *decreases* as consumption level *rises*
- Utility increases *slower* as consumption level *rises*
- Utility function becomes *flatter* as consumption level *rises*

For 2 goods:
1) MU_x decreases with x, holding y constant
2) MU_y decreases with y, holding x constant

To check for Diminishing Marginal Utility:	• Compute the marginal utilities • Keep the other variable constant (if applicable) see if the Marginal Utility is decreasing or increasing. (Can take the Second Derivative)

Perfect Substitutes

Definition 2.4: Two goods are perfect substitutes if MRS is constant

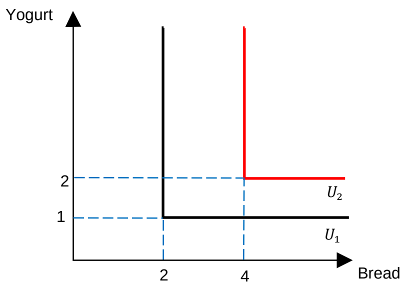


Example of Indifference Curve for Perfect Substitutes

Features	• This happens when the consumer is willing to give up each good at a constant rate for the other good (The utility if we exchange for more does not change)
	• Indifference curves will be linear
	• MRS is *constant* → Utility functions are linear

Perfect Complements

Definition 2.5: Two goods are perfect complements if MRS is 0 or infinity



Example of Indifference Curve for Perfect Complements

Features	• This happens when the ratio in which the 2 goods are consumed is constant throughout as the consumer will only consume them in that ratio
	• Indifference curves are L-shaped
	• MRS is 0 or ∞ → Utility Functions are min functions ○ $\mathbf{min}(x, y)$ = The smaller of x and y, just the minimum values they can take
	• Optimal Consumption point is at the kink point • $x = y$ depending on what is the min function, just equate x with y

Budget Constraint

$$P_F F + P_C C \leq I$$

F – Units of Good x (In this case it is Food)
C – Units of Good y (In this case it is Clothing)
P_F – Price of food
P_C – Price of clothing
I – Limited income of the consumer

Budget Set & Budget Line

Budget Set:

Definition: The set of all baskets that the consumer can afford, that is, all baskets that satisfy the budget constraint (i.e. Total Expenditure ≤ Total Income)

Budget Line:

Definition: The set of all baskets consumer can afford by spending all income (i.e. Total Expenditure = Total Income)

$$P_F F + P_C C = I$$
$$C = \frac{I}{P_C} - \frac{P_F}{P_C} F$$

F – Units of Good x (In this case it is Food)
C – Units of Good y (In this case it is Clothing)
P_F – Price of food
P_C – Price of clothing
I – Limited income of the consumer

Features	**Slope of Budget Line**: $-\frac{P_F}{P_C}$ • Rate at which the two goods can be substituted in the market, based on prices • Give up 1 unit of Food → Use the money that we save to see how many units of Clothing we can get • Note that this exchange is not done at a rate where the utility is constant, it is only based on the price ratio • This is **only dependant on the relative price ratio** • May not always be a constant as it could be that if we buy more the price ratio will change (Discount etc)
	y – intercept : $\frac{I}{P_C}$ • It is the number of units of clothing we can get if we spend all our money on clothing
	x – intercept : $\frac{I}{P_F}$ • It is the number of units of clothing we can get if we spend all our money on food

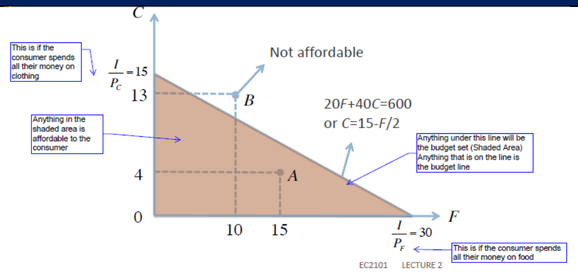


Figure 8 Graphical Representation of the Budget Line and Budget Set

Optimal Consumption:

Optimal Basket: Consumer should choose the basket that gives him/her the highest utility given the budget constraint

$$\max_{F,C} U(F, C)$$
$$s.t. P_F F + P_C C = I$$
$$F \geq 0$$
$$C \geq 0$$

F – Units of Food (**Choice variable**)
C – Units of Clothing (**Choice variable**)
$U(F, C)$ – Utility function of Food and Clothing
P_F – Price of food (**Parameter**)
P_C – Price of clothing (**Parameter**)
I – Income of the consumer (**Parameter**)

Note:
- Choice variable is something that the consumer can choose and can vary
- Parameters are fixed to the consumer and they have no control over it
- Optimal basket is not always a point of tangency
- At the optimal basket, it is not always true that both (all) goods are consumed, we could consume only one of the good
- For corner solutions, Indifference curve may not be tangent to the budget line

Conditions for basket chosen by consumer	1) On the budget line → So that the consumer maximises their income $P_F F + P_C C = I$ (**Budget constraint holds with equality**)
	2) On the highest indifference curve → So that the consumer maximises their utility *General Case*: **Budget line tangent to indifference curve** $-\frac{P_F}{P_C} = -MRS_{F,C} \Rightarrow \frac{P_F}{P_C} = MRS_{F,C}$ *Note that the optimal basket may not always be a point of tangency*

Definition 3.1: Corner solution is an optimal basket at which the consumption of at least one good is 0

Definition 3.2: An optimal basket in which both goods are consumed is an interior solution

When does Corner Solution happen?

- When the optimal consumption is negative for one good. This happens due to the **per dollar marginal utility of one good being more than the other for the whole consumption level**
- This will cause the constraint of $C \geq 0$ or $F \geq 0$ to bind where the condition binds means that it holds with equality
- The inequality only binds if there is a corner solution
- When one of the consumption is negative, then we just take the next best consumption which is where it is 0 and find the consumption of the other good at the point

Tangency Condition

Definition: The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the rate at which the two goods are exchanged in the market. Else the consumer is not maximising utility

$$MRS_{F,C} = \frac{P_F}{P_C}$$

P_F – Price of food
P_C – Price of clothing
$MRS_{F,C}$ – Marginal rate of substitution of food for clothing

Note:
- $MRS_{F,C}$ is the willingness of the consumer to exchange C for F without changing the utility. Therefore, we should find the point on the budget line where our budget is able to ensure that the goods are traded at that price level and utility is not compromised

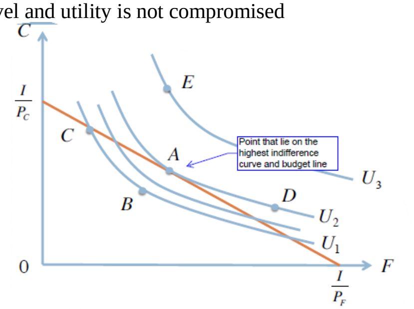


Figure 9 Example of the tangent point of the Indifference curve and the budget line

Equal Marginal Principle

Definition: To maximise utility, consumer sets marginal utility per dollar of expenditure equals for both goods

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

MU_F – Marginal utility of Food
MU_C – Marginal utility of Clothing
P_F – Price of food
P_C – Price of clothing

Note:
- Extra utility per dollar spent on food is the same as the extra utility per dollar spent on clothing
- This is just another way of writing the tangency condition

If *MU* per dollar is not the same:

Suppose $\frac{MU_F}{P_F} < \frac{MU_C}{P_C}$ → The utility gained from for every \$1 spent on C is higher → We will want to spend more on C to gain more utility instead rather than spend on F

If the slope of Indifference Curve is different from Budget Line:

For example: $MRS_{F,C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{MU_C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} > \frac{MU_C}{P_C}$
We can make use of the comparison between the slopes to determine which one has a higher per dollar marginal utility

- $MRS_{F,C}$ steeper, F has higher per dollar marginal utility
- $MRS_{F,C}$ gentler, F has higher per dollar marginal utility

Solving Optimal Basket

Given the Utility Function $U(x, y)$, P_x, P_y, I

Method 1:
Solve for:
1) *Budget Line*: $P_x x + P_y y = I$
2) *Tangency Condition*: $MRS_{x,y} = \frac{P_x}{P_y}$

Method 2:
Note that for this method we will be able to find out the λ which is the per dollar Marginal Utility

Solve for:
Lagrangian Function:

$$\Lambda(x, y, \lambda) = U(x, y) + \lambda(I - P_x x - P_y y)$$

First Order Conditions:

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda P_x = 0$$
$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda P_y = 0$$
$$\frac{\partial \Lambda}{\partial x} = I - P_x x - P_y y = 0$$

Lagrange Multiplier for Consumer Theory

$$\max_{x,y} U(x, y)$$
$$s.t. I - P_x x - P_y y = 0$$

x – Units of good x (**Choice variable**)
y – Units of good y (**Choice variable**)
$U(x, y)$ – Utility function of goods x and y
P_x – Price of good x (**Parameter**)
P_y – Price of good y (**Parameter**)
I – Income of the consumer (**Parameter**)

Lagrangian Function:

$$\Lambda(x, y, \lambda) = U(x, y) + \lambda(I - P_x x - P_y y)$$

First Order Conditions:

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda P_x = 0$$
$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda P_y = 0$$
$$\frac{\partial \Lambda}{\partial x} = I - P_x x - P_y y = 0$$

Solving Lagrangian Method:
Note that we will still be solving the 2 conditions
Tangency Condition: $\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \Rightarrow MRS_{x,y} = \frac{P_x}{P_y}$
Budget Line: $I - P_x x - P_y y = 0$

Meaning of Lagrange Multiplier λ:
- Extra utility of one extra dollar of consumption (Per dollar Marginal Utility)

Note that if we arrange the Lagrangian Function as $\Lambda(x, y, \lambda) = U(x, y) + \lambda(P_x x + P_y y - I)$ instead, we will be getting:

$$\frac{\partial \Lambda}{\partial x} = MU_x + \lambda P_x = 0$$
$$\frac{\partial \Lambda}{\partial y} = MU_y + \lambda P_y = 0$$

Whereby $\lambda = -\frac{MU_x}{P_x} = -\frac{MU_y}{P_y}$ which in this case it will be the negative of the per dollar marginal utility

Meaning of P_x & P_y:

$$P_x = \frac{MU_x}{\lambda} \;\&\; P_y = \frac{MU_y}{\lambda}$$

- This is the price that the consumer is willing to pay for one more unit of the good
- Right hand side of the equation is the extra utility from consuming one more unit of the good divided by the extra utility of one dollar spending → From one more unit of the good, the consumer will gain a level of utility and dividing by the amount of utility gained from each dollar will tell us how much the consumer is willing to pay for one more unit of the good

Topic 3: Consumer Choice, Revealed Preference, Individual Demand

Revealed Preference

Definition: Revealed Preference is the analysis that enable us to infer preference based on observed prices and choices

Strictly Prefer:

Definition 1.1: A consumer (strictly) prefers A to B if the consumer is more satisfied with A than with B
Notation: $A \succ B$

Note:
No other affordable basket is strictly preferred to A if A is the optimal basket. If basket B if strictly preferred to A, it must be that it is unaffordable and more expensive than A.

$$P_x x_B + P_y y_B > P_x x_A + P_y y_A = I$$

Indifferent:

Definition 1.2: A consumer is indifferent between A and B if the consumer is equally satisfied with A or B
Notation: $A \approx B$

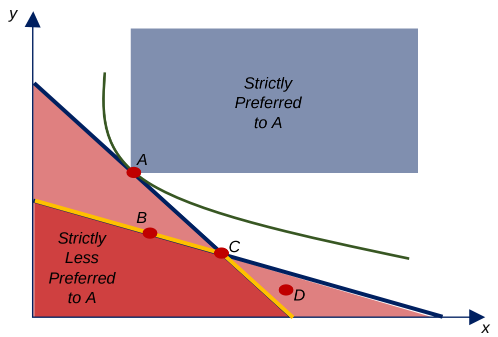
Weakly Preferred:

Definition 3.3: A is weakly preferred to B is either A is strictly preferred to B or A is indifferent to B

$$A \succ B \lor A \approx B$$

Notation: $A \succeq B$

Revealed Preferences from Optimal Choices:



Suppose A is the optimal basket for the first budget line. Suppose B is the optimal basket for the second budget line.

Note that most of the revealed preference when we have 2 budget line comes from using Transitivity and More is Better

Strictly Preferred Points
- "More is Better" tells us that any point that is in the North-East Direction is strictly preferred to the current point
- Denoted by **blue** shaded area

Weakly Preferred Points
- A is revealed to be weakly preferred to any other basket on the budget line
- Since B lies in the budget set of A, it is strictly less preferred to A and we can make use of transitivity to bring about more of the relations
- $A \succ B \to B \succ C, B \geq D \to A \succ C, A \succ D \to \ (By\ transitivity)$
- Denoted by **dark blue** line

Strictly Less Preferred Points
- A is revealed to be strictly preferred to any other basket in the budget set (but not on the budget line)
- Any basket in the budget set of A is strictly less preferred to A
- We can find another point directly above which is on the budget line. If the point in the budget set is indifferent to A, it means that the point on the budget set will be strictly preferred to A then it should be the optimal point instead. Therefore, it must be strictly less preferred
- Denoted by **red** shaded area (noting that those **dark blue** lines are not included)

Optimal Choice for Revealed Preference:

- So long as we don't know anything about the optimal choice, any point on the budget line cannot be compared with other points. The points on the budget line could be strictly less preferred to points in the budget set. They could have any kind of relations.
- We can only rely on "more is better" for any comparison if required.

Demand Curve

Definition 3.4: A consumer's demand curve for a good is the optimal consumption of the good as a function of its price. (Holding all other factors fixed)

Law of Demand: Demand Curve is downward sloping. Higher price, lower quantity demanded

Variables Required:
- Price of the good
- Quantity demanded (optimal consumption) for the good

Example of Demand Curve

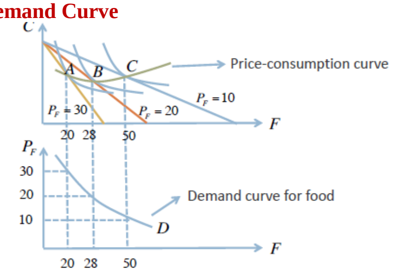


Figure 10 Example of Demand Curve

Price Consumption Curve

Definition: Tells us how the consumption of both goods changes with the price of one good

- There is kind of a mapping between the demand curve and the price consumption curve since the demand curve is telling us how the consumption of a good changes with its own price. We can map it back to our utility function to see the consumption of the other good and we can get a curve to show the relation

Example of Price-Consumption Curve

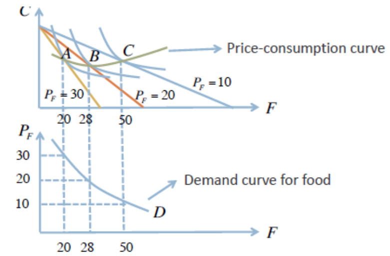
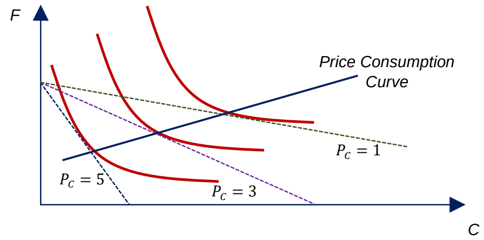


Figure 11 Price Consumption Curve with Demand Curve

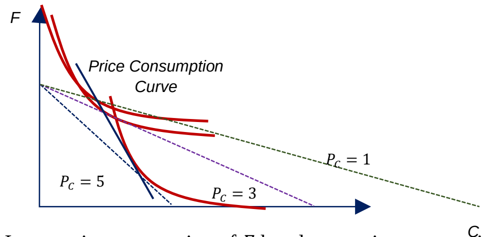
Slope of Price Consumption Curve

Upward Sloping:



- $P_C \downarrow \to$ Increase in both consumption of F and C.
- Note that if C turns out to be an inferior good, it will not be a Giffen Good because when price of the good decreases, the consumption increases
- Note that the demand curve for C will be downwards sloping here

Downward Sloping:



- $P_C \uparrow \to$ Increase in consumption of F but decrease in consumption of C.
- Note that C will be a Giffen good because when rice decreases, the consumption decreases as well
- Note that the demand curve for C will be upwards sloping here

Engel Curve

Definition 3.5: A consumer's Engel curve of a good is the curve that shows the relationship between income and optimal consumption (Holding all other factors fixed)

Variables Required:
- Income
- Quantity demanded (optimal consumption) for the good

Example of Engel Curve:

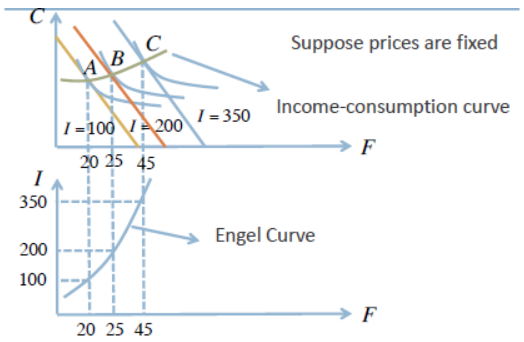


Figure 12 Example of Engel Curve

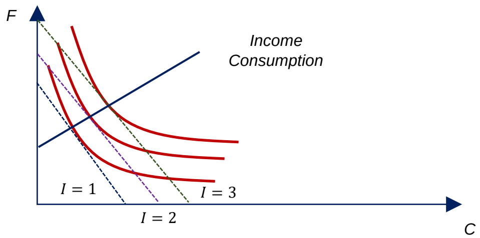
Income-Consumption Curve

Definition: Tells us how the consumption of both goods changes with a change in Income (Prices are constant here)

Gives us information on whether the goods are normal or inferior goods

Slope of Income-Consumption Curve

Positive Slope:



- $I\uparrow \rightarrow$ Increase in consumption of both goods→ Both goods are normal goods since when income increases, consumption increases as well
- Engel curve is upward sloping for both goods

Negative Slope:

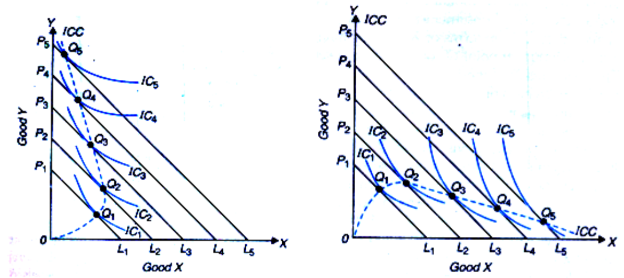


Fig. 8.29. *Income Consumption Curve in Case of Good X being Inferior Good*

Fig. 8.30. *Income Consumption Curve in Case of Good Y being Inferior Good*

- $\uparrow Y \rightarrow$ Increase in consumption of one good and decrease in consumption of the other good.
- The good that has the consumption decreased when the income increase is an inferior good → Engel Curve is downward sloping
- The good that has the consumption increased when the income increase is a normal good → Engel Curve is upward sloping

Normal vs. Inferior Good

Normal Goods:

Definition 3.6: If the good is a normal good, Engel curve is upward sloping

When income increases, consumption of the good increases

Inferior Goods:

Definition 3.7: If the good is an inferior good, Engel curve is downward sloping

When income increases, consumption of the good decreases

Demand Function

Definition 3.8: A consumer's demand function for a good is quantity demanded as a function of income and all prices

Variables that are Required:
- Price of the good
- Price of other goods
- Income
- Quantity demanded (optimal consumption) for the good

Note:
We can derive the Demand Curve and Engel Curve from the Demand Function

Suppose x is the good that we are looking at and the only other good is good y

Demand Curve: Take P_y and I to be constant

Engel Curve: Take P_x and P_y to be constant

To find the Demand Function:

Make use of the Tangency Condition and Budget Constraint:

$$\boxed{\begin{gathered}\frac{MU_x}{MU_y} = \frac{P_x}{P_y}\\ P_x x + P_y y = I\end{gathered}}$$

Write the budget constraint in terms of the optimal consumption for the other good and substitute it into the tangency condition so that we will have it only in terms of the variables that are required

Note that this method can be used to find the Demand Curve and Engel Curve so long as we have the income value or prices values to make them constant

Use Demand Function to see the relation for consumption changes due to other variables

Cobb-Douglas Utility Function:

Definition 3.9: A utility function is called a Cobb-Douglas utility function if it takes the following form

$$U(x,y) = Ax^{\alpha}y^{\beta}$$

Note that: $A > 0, \alpha > 0, \beta > 0$

Features

More is Better always satisfied & Downward Sloping Indifference Curves

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$
$$MU_y = A\beta x^{\alpha}y^{\beta-1}$$

- Marginal Utilities are always positive
- "More is Better" always satisfied for both goods
- Indifference curves are downward sloping

Diminishing MRS & Convex Indifference Curves

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1}y^{\beta}}{A\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

- Diminishing MRS. When the consumer gets more x and less y along the same indifference curve, MRS decreases
- Indifference Curves are Convex

Demand does not depend on the price of other good

Demand Function for x:

$$x = \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x}$$

Demand Function for y:

$$y = \frac{\beta}{\alpha+\beta} \times \frac{I}{P_y}$$

- Demand function does not contain the price of the other good, therefore, the demand for the good is not affected by price changes of the other good
- When the price of the other good changes, consumption of the good does not change

Fixed Proportion of Income is spent on each good

Total Expenditure on x:

$$P_x x = P_x \times \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x} = \frac{\alpha I}{\alpha+\beta}$$

Total Expenditure on y:

$$P_y y = P_y \times \frac{\beta}{\alpha+\beta} \times \frac{I}{P_y} = \frac{\beta I}{\alpha+\beta}$$

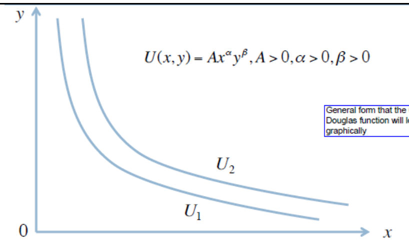


Figure 13 Typical Graphical Representation for Cobbs Douglas

Quasi Linear Function

Definition: A utility function is called a quasi-linear utility function if it takes the following form

$$U(x,y) = v(x) + ay$$

$v(x)$ is a function of x

It is only linear in one variable which is y in this case

Features	At any interior solution: • Demand for x does not depend on I → When price of x changes, there is no Income effect → CV is the same as EV

Topic 4: Voucher vs. Cash, Income and Substitution Effects, Consumer Welfare

Vouchers vs. Cash Subsidy

Effect of Voucher & Cash:
- Consumer's Choice
- Consumer's Utility

Vouchers:
- Give voucher for a specific good so that the purchasing power for that good for the consumer increases

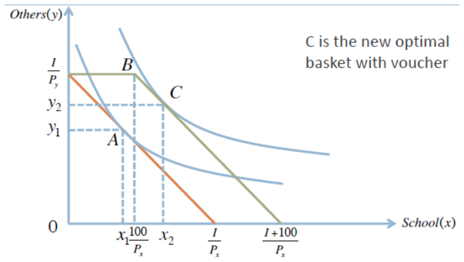


Figure 14 Impact of Voucher

Effect of Voucher:
- With the voucher on good x, the consumer can afford $100/P_x$ more good x
- The increase is denoted by a horizontal shift of the budget line with an addition of that horizontal line
- With the increase in purchasing power for good x, we see that the consumer can reach a higher indifference curve and gain more utility

Why we use Vouchers instead of Cash:
- We want to increase the consumption of one good as giving cash will only increase the consumption of the good by a little amount and the consumer will spend more on the other good

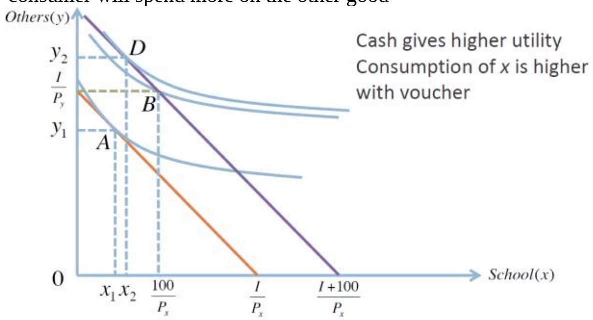


Figure 15 Example that giving cash increases the consumption of x lesser

Cash:
- Give cash to the consumer so that the purchasing power for the consumer increases

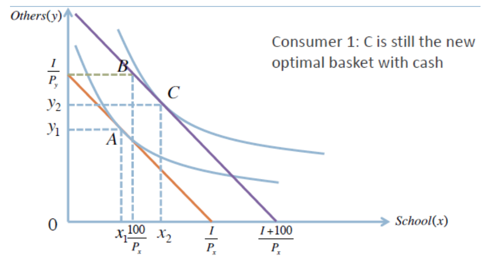


Figure 16 Example where impact of Cash Subsidy is same as Voucher

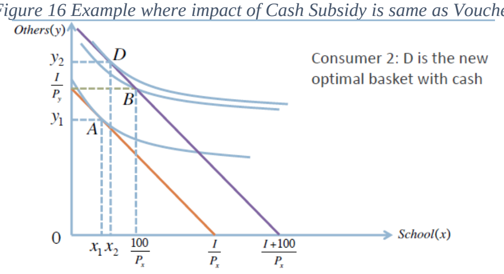


Figure 17 Example where Cash Voucher gives higher utility than Voucher

Effect of Cash:
- With the cash subsidy, it will cause a rightwards shift of the budget line which denotes an increase in the purchasing power
- Note that the cash subsidy will always either be better than or have the same effect as the voucher
- Cash subsidy will be better when the optimal consumption is at the kink of the voucher as the consumer will want to consume more of the other good
- Cash subsidy will be the same when the optimal consumption is at a tangency point below the kink because it means that with or without the cash voucher, the consumer will still use more on good x, therefore, it will not go behind the kink

Substitution Effect

Definition 4.1: Substitution effect is the change in consumption of one good associated with a change in its price, holding the level of utility and other prices constant

What we look for:
- Changing the slope of the budget line such that the optimal consumption is still on the same indifference curve

Income Effect

Definition 4.2: Income effect is the change in consumption of a good associated with a change in purchasing power, holding all prices constant

What we look for:
- From the changed slope of the budget line, we want to do a rightward shift of the budget line to encapsulate the increase in purchasing power

Analysis of Substitution & Income Effect

Decomposition of Income and Substitution Effect:

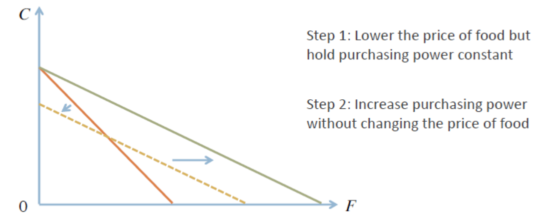


Figure 18 Decomposing the Change in Budget Line

Step 1: Changing the slope of the budget line, keeping it on the same Indifference Curve (**Substitution Effect**). Draw the intermediary budget line in yellow to encapsulate this change in relative price but keeping utility the same by ensuring the optimal point is still on the indifference curve.

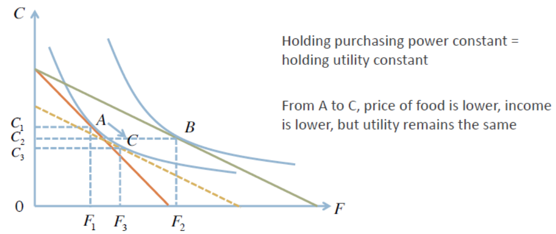


Figure 19 Example of the Movement for Substitution Effect

Step 2: Keeping the slope of the budget line after Substitution the same and shifting up to attain a higher indifference curve (**Income Effect**). Note that this is an example relative to the change in price of food so the income did not actually change so the y-intercept does not change and only the budget line changes.

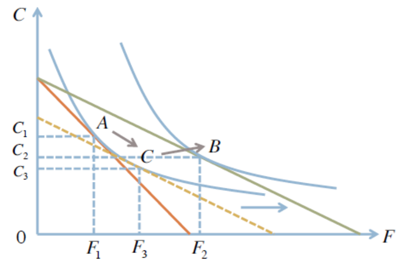


Figure 20 Example of Movement for Income Effect

Computing Substitution and Income Effect:

Effect of Price Change = Substitution Effect + Income Effect

Steps:
Suppose we have a change in price of good x

1) Compute the original optimal basket and the utility at the optimal basket
2) **Computing Substitution Effect:**
    a. Use the new prices for the budget line and use the original utility at optimal basket
    b. Solve for the consumption of both goods using the 2 equations
    c. The change in consumption of good x from the original optimal basket to the new consumption is the substitution effect
3) **Computing Income Effect:**
    a. Use the new prices for the budget line and solve for the optimal basket
    b. The change in consumption of good x from the intermediate basket (computed in Substitution Effect) to the final optimal basket is the Income Effect

Direction of Substitution Effect:

Note that this conclusion can be done through looking at the graph and by revealed preference

Decrease in price of good:
- Substitution Effect is always non-negative

Increase in price of good:
- Substitution Effect is always non-positive

Direction of Income Effect

Change in prices of Normal Goods:
- Income effect has the same sign as substitution effect
- Note:
    - Price of good ↓ → Purchasing power ↑ → Consumer buys more of the good
    - Price of good ↑→ Purchasing power ↓ → Consumer buys less of the good

Change in prices of Inferior Goods:
- Income effect has an opposite sign from substitution effect

Giffen Good

Definition 4.3: A good is a Giffen good if
- As price decreases, quantity demanded for the good drops
- A price increases, quantity demanded for the good goes up
- Holding other factors constant

Requirement
- A Giffen good needs to be an inferior good and when there is a price change, Income Effect > Substitution Effect

Note:
- Demand Curve is upward sloping for Giffen good (Contrast to the Law of Demand)
- All Giffen goods are inferior goods but not all inferior goods are Giffen goods

Reason for this happening:
- Suppose there is a price decrease for the inferior good, it causes their purchasing power to increase through income effect and they will want to buy more normal good as compared to the inferior good. When the desire to buy normal good is more as compared to the substitution effect, we will observe this Giffen behaviour.

Consumer Surplus

Definition 4.4: Consumer surplus (CS) for an individual consumer is the difference between the consumer's willingness to pay for a good and the cost of purchasing the good

Note:
- One of the ways to measure consumer welfare
- Area below the demand curve and above the price

Graphical Example:

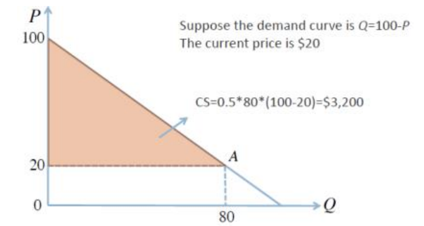


Figure 21 Example of Computation for Consumer Surplus

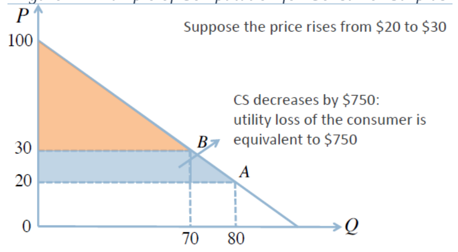


Figure 22 Example of Change in Consumer Surplus

Compensating Variation

Definition 4.5: Compensating Variation (CV) measures the amount of money (income) the consumer is willing to give up after the price drop to be just as well off as before the price drop (Note that this value will be +ve)

Price Increase: Measures the additional amount of money the consumer needs after the price increase to be as well off as before the price increase (Note that this value will be $-ve$)

Computation of CV:
- This is using a similar idea to substitution effect, we want to see after the change in price drop
- Find the consumption at the intermediate basket (Intermediate budget line where the slope is the same as the final price change but the utility is kept constant).
- Find the income at the consumption of the intermediate basket using the equation of the budget line

$$CV = Income\ needed\ at\ Original\ Optimal\ Basket - Income\ needed\ at\ Intermediate\ Basket$$

Graphical Representation of CV:

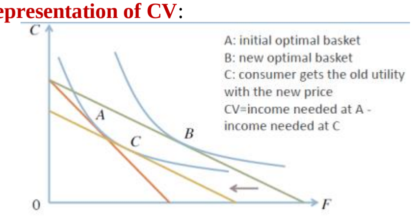


Figure 23 Graphical Representation of Compensating Variation

Solving for the CV:

Solve for the 2 equations:

$$U(x,y) = Utility\ at\ A$$
$$MRS_{x,y} = \frac{New\ P_x}{New\ P_y}$$

After we solve for the consumption, find the income required using the new prices and compute the CV using the formula above

Equivalent Variation

Definition 4.6: Equivalent Variation (EV) measures the additional amount of money (income) the consumer needs before the price drop to be as well off as after the price drop (Note that this value will be +ve)

Price Increase: Measures the money the consumer the willing to give up before the price increase to be as well as after the price increase (Note that this value will be -ve)

Computation of EV:

- For this, we will create another intermediate budget line but with the same slope as the original budget line and doing a rightward shift under it reaches the indifference curve of the new optimal consumption
- Find the consumption of the intermediate basket at the new indifference curve using the tangency condition and budget line (Noting that the utility is the same as the final optimal consumption)
- Find the income at the consumption of the new intermediate basket

$$EV = Income\ needed\ at\ New\ Intermediate\ Basket$$
$$-Income\ needed\ at\ New\ Optimal\ Basket$$

Graphical Representation of Equivalent Variation:

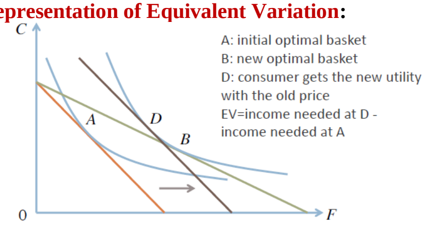


Figure 24 Graphical Representation of Equivalent Variation

Solving for the EV:

Solve for the 2 equations:

$$U(x, y) = Utility\ at\ D$$
$$MRS_{x,y} = \frac{Old\ P_x}{Old\ P_y}$$

After we solve for the consumption, find the income required using old prices and compute the EV using the formula above

Topic 5: Market Demand and Exchange Economy

Market Demand

Definition: Market Demand curves is the horizontal summation of all individual demand curves

Constructing the Market Demand:

- Look at the individual demand curves
- Identify the points where the prices have intersection
- For those points, sum up the demand and construct out the market demand curve from there and it could be a piecewise function

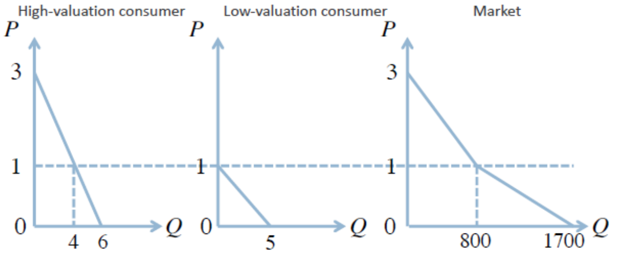


Figure 25 Graphical Example of Constructing Market Demand Curve

Analysis:

When $P > 1$:
- Only high-valuation consumers will buy
- Market demand curve: $Q = 200(6 - 2P)$

When $P \leq 1$:
- Both types of consumers will buy
- Add up the demand curves of both consumers to find the total demand at those prices
- Market demand curve: $Q = 200(6 - 2P) + 100(5 - 5P)$

Combining:

$$Q = \begin{cases} 1700 - 900P, & if\ P \leq 1 \\ 1200 - 400P, & if\ P > 1 \end{cases}$$

Partial vs. General Equilibrium:

Partial Equilibrium Analysis:
- Finding the equilibrium prices and quantity in a single market
- Holding prices in all other markets fixed

General Equilibrium Analysis:
- Finding the equilibrium prices and quantities in more than one market simultaneously

Exchange Economy

Set Up:

- Two consumers in the economy, A and B
- Two goods in the economy, 1 and 2

Consumer A's consumption basket is denoted by:

$$(x_1^A, x_2^A)$$

Consumer B's consumption basket is denoted by:

$$(x_1^B, x_2^B)$$

There is no money/income:
- Consumers can only trade their goods with each other

Allocation:

Definition: A pair of consumption basket is denoted by

$$(x_1^A, x_2^A, x_1^B, x_2^B)$$

Endowment Allocation:

Definition: The allocation of goods that the consumer starts with is denoted by

$$(\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B)$$

- Each consumer has some amount of each good to start with

Feasible Allocation

Definition 5.1: An allocation is feasible if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

- The total amount of each good consumed equals to the total amount available (Determined through the endowment)

Edgeworth Box

Definition: An Edgeworth box is used to graphically show all feasible allocations of the two goods between the two consumers. Every point in the box, including those on the boundaries, represents a feasible allocation

Set up of Edgeworth Box:

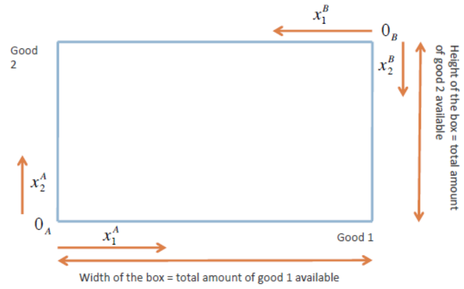


Figure 26 Set up of Edgeworth Box

- Note that the consumption of consumer A and B are kind of a mirror reflection of each other
- Take note of the direction of the axis for consumer A and consumer B. Can just flip the paper if it is too confusing but the origin is just at both corners

Feasible Allocation on Edgeworth Box:

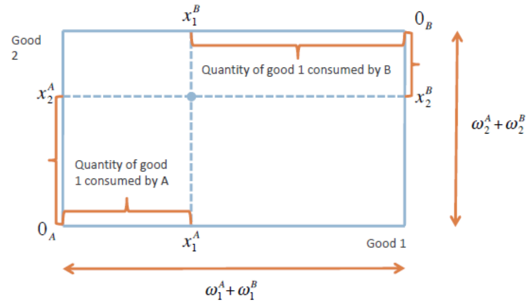


Figure 27 Feasible Allocation on Edgeworth Box

- Any feasible allocation is just a point in or on the boundaries of the Edgeworth box. The total consumption of the goods is the total endowment and can be captured by the reflecting axis since the other side will be total endowment – consumption from consumer A which adds up to the total endowment

Pareto Improvement

Definition 5.2: From allocation X to some other allocation Y is a Pareto improvement if from X to Y, at least one consumer is better off and no one else is worse off

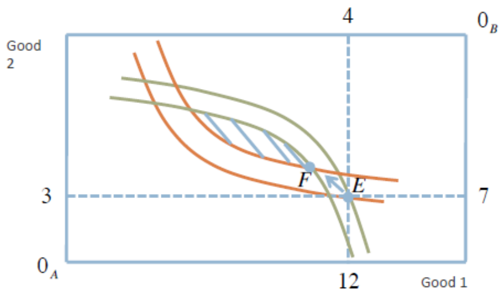


Figure 28 Example of Pareto Improvement

- We see that moving from E to F, both consumer A and B are on a higher indifference curve
- Moving from F to any point in the shaded area will result in both consumer A and B being on a higher indifference curve as well

Movements and whether it is a Pareto Improvement	**Pareto Efficient to Pareto Efficient:** No Pareto Improvement
	Pareto Efficient to Non-Pareto Efficient: No Pareto Improvement
	Non-Pareto Efficient to Pareto Efficient: Depends
	Non-Pareto Efficient to Non-Pareto Efficient: Depends
Any room for Pareto Improvement	**Currently at Pareto Efficient Point:** No
	Currently at Non-Pareto Efficient Point: Yes

Pareto Efficiency

Definition 5.3: An allocation is Pareto efficient if there is no way to make one consumer better off without making someone else worse off

A Pareto efficient allocation can be described as an allocation where there is no way to make some individual better off without making someone else worse off

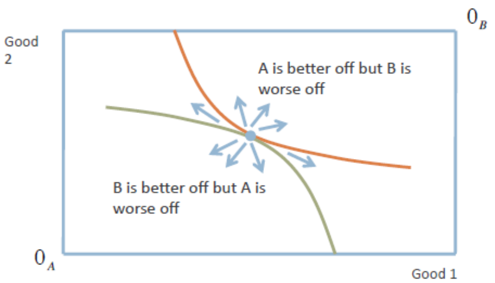


Figure 29 Example of Pareto Efficient Allocation

- We see that this point is Pareto Efficient since there is no way to move to any other point without one person worse off.

Contract Curve

Definition 5.4: The contract curve is the set of all Pareto efficient allocations

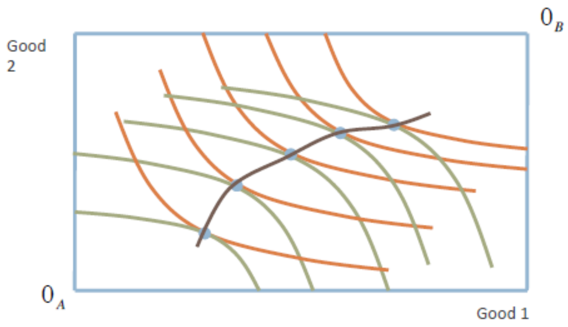


Figure 30 Example of Contract Curve

Way to find Contract Curve Equation

Tangency Condition:

$$MRS_{1,2}^A = MRS_{1,2}^B \cdots (1)$$

Feasibility of Allocation:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \cdots (2)$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B \cdots (3)$$

Substitute (2) & (3) into (1) to express in terms of

1) x_1^A and x_2^A OR
2) x_1^B and x_2^B

Budget Set

Suppose P_1 is the price of good 1 and P_2 is the price of good 2

Budget Constraint:

Consumer A: $P_1x_1^A + P_2x_2^A \leq P_1\omega_1^A + P_1\omega_2^A$

Consumer B: $P_1x_1^A + P_2x_2^A \leq P_1\omega_1^A + P_1\omega_2^A$

Budget Line:

Consumer A: $P_1x_1^A + P_2x_2^A = P_1\omega_1^A + P_1\omega_2^A$

Consumer B: $P_1x_1^A + P_2x_2^A = P_1\omega_1^A + P_1\omega_2^A$

- Note that the income for the consumer we can take it as the total worth of the endowment that they have

Budget Constraint in Graph:

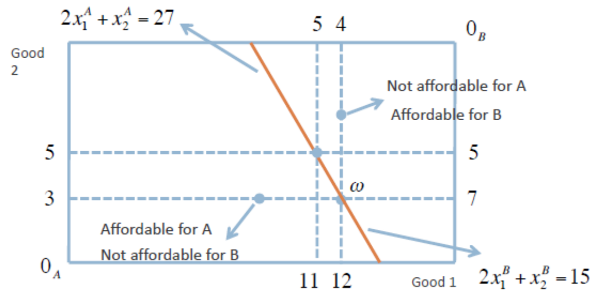


Figure 31 Budget Constraint in Graph

Slope of Budget line:

$$-\frac{P_1}{P_2}$$

- Endowment allocation is on the budget line
- Note that the budget line for consumer A and consumer B will be the same line

General Competitive Equilibrium

Definition 5.5: A (general) competitive equilibrium consists of a pair of prices and an allocation $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ such that

(**Condition 1**) Each consumer maximises his/her utility at the allocation

$$\boxed{(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})}$$

Given the equilibrium prices

(**Condition 2**) Markets for both goods clear (**Market Clearing Condition**)

$$\boxed{\begin{aligned} x_1^{*A} + x_1^{*B} &= \omega_1^A + \omega_1^B \\ x_2^{*A} + x_2^{*B} &= \omega_2^A + \omega_2^B \end{aligned}}$$

Example that the Market does not clear at the current prices:

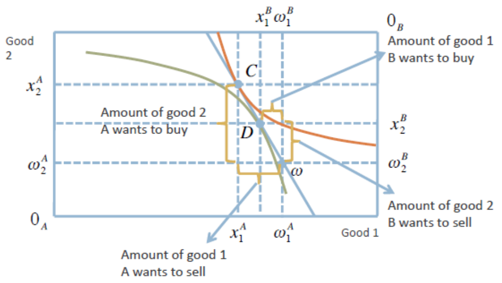


Figure 32 Market not clearing at current prices

Sum of demand for each good does not equal to the total quantity available

$$x_1^A + x_2^B < \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B > \omega_2^A + \omega_2^B$$

Excess Supply for good 1 – Amount B wants to buy is less than the amount A wants to sell

Excess Demand for good 2 – Amount A wants to buy is more than the amount B wants to sell

What does the equilibrium price tell us?

- It tells us the relative scarcity of the two goods – how the goods can be exchanged in the market.
- The prices just need to be a ratio of the equilibrium prices. Suppose the relative price is 2, the price can be \$2 to \$1 or \$4 to \$2.

Solving for Competitive Equilibrium:

We want to find the equilibrium allocation $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ and equilibrium prices P_1 and P_2

Therefore, we will have 6 equations to solve 6 unknowns.

Consumer A's optimal choice (Utility Maximising)

$$\boxed{MU_{1,2}^A = \frac{P_1}{P_2} \cdots (1)}$$

$$\boxed{P_1 x_1^A + P_2 x_2^A = P_1 \omega_1^A + P_2 \omega_2^A \cdots (2)}$$

Consumer B's optimal choice (Utility Maximizing)

$$\boxed{MU_{1,2}^B = \frac{P_1}{P_2} \cdots (3)}$$

$$\boxed{P_1 x_1^B + P_2 x_2^B = P_1 \omega_1^B + P_2 \omega_2^A \cdots (4)}$$

Market clearing

$$\boxed{x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B \cdots (5)}$$

$$\boxed{x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B \cdots (6)}$$

Note that we can set any good as the numeraire

Suppose x is the numeraire, set $P_x = 1$

1) Equate (1) and (3) together to get (7)
2) Sub (5) and (6) into (7) to put in terms of consumption for one consumer and write in terms of good 1 and 2 to get (8)
3) Substitute (8) into (1) to get the ratio of the price ratio and solve for P_2 since P_1 will be 1
4) Substitute the values of P_2 & P_1 into (2) with (8) to solve for the optimal consumption for consumer A
5) Use the optimal consumption value of consumer A to solve for consumer B using (5) & (6)

Graphical Representation of Competitive Equilibrium

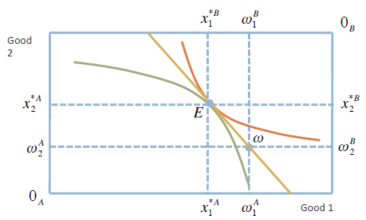


Figure 33 Competitive Equilibrium in Graph

- Note that the two indifference curves and the budget line are tangent to each other at the competitive equilibrium
- The change in budget line is a change in the relative prices but the line must still cut through the endowment allocation since it always feasible

Topic 6: First Welfare Theorem, Walras' Law

First Welfare Theorem

Definition 6.1: The First Fundamental Theorem of Welfare Economics states that a competitive equilibrium allocation is Pareto efficient

Suppose the equilibrium prices, (P_1, P_2) and the allocation $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ is the equilibrium allocation. Then $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ is Pareto efficient

Proof of First Welfare Theorem:

Proof by Contradiction:

Suppose the equilibrium allocation $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ is *not Pareto Efficient*

This means that there must be another feasible allocation $(y_1^A, y_2^A, y_1^B, y_2^B)$ such that it makes one consumer better off without making someone else worse off as compared to the equilibrium allocation.

Without loss of generality, we assume that consumer A is the one that is better off and consumer B is not worse off. Therefore, this means that consumer A strictly prefers (y_1^A, y_2^A) to (x_1^{*A}, x_2^{*A}) while consumer B weakly prefers (y_1^B, y_2^B) to (x_1^{*B}, x_2^{*B}).

By definition, the equilibrium allocation is the utility-maximising basket for each consumer given the budget constraint.
Therefore, by revealed preferences:

$$P_1 y_1^A + P_2 y_2^A > P_1 \omega_1^A + P_2 \omega_2^A \cdots (1)$$

$$P_1 y_1^B + P_2 y_2^B \geq P_1 \omega_1^B + P_2 \omega_2^B \cdots (2)$$

For consumer A, basket y must not be affordable since if it is affordable, then the consumer would have chosen it as the utility-maximising basket since the equilibrium allocation is the utility-maximising basket given the budget constraint

For consumer B, basket y must either be on the budget line or above the budget line because it is weakly preferred to the equilibrium allocation. It cannot lie below the budget line because if basket y is below the budget line, there will be another point on the budget line that is strictly preferred to y and therefore, strictly preferred to the equilibrium allocation which violates the definition of the equilibrium allocation

Summing up (1) & (2):

$$P_1(y_1^A + y_1^B) + P_2(y_2^A + y_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) \cdots (3)$$

We note that the allocation of $(y_1^A, y_2^A, y_1^B, y_2^B)$ must also be feasible, therefore, it must satisfy:

$$y_1^A + y_1^B = \omega_1^A + \omega_1^B \cdots (4)$$
$$y_2^A + y_2^B = \omega_2^A + \omega_2^B \cdots (5)$$

Substitute (4) and (5) into (3), we have a contradiction

$$P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B)$$

We see that this is a contradiction since the condition that we have achieved from the revealed preference is different. Therefore, the initial assumption is wrong and therefore, the equilibrium allocation $(x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B})$ is Pareto Efficient.

Comments on First Welfare Theorem:

Only holds in competitive markets:
- Not true if consumers or firms have price setting power (Monopoly, Oligopoly)
- Not true if there is externality
- Not true if there is asymmetric information

Efficiency does not mean equity:
- A pareto efficient allocation may or may not be an equitable allocation
- When one consumer has everything and the other consumer has nothing, it can be Pareto efficient but extreme inequality does not do any good for the economy

Allows for a mechanism to allocate resources efficient:
- First Welfare Theorem tells us that we just need to create a competitive market and the market will allocate resources efficiently

Comparison Between Pareto Efficiency and Competitive Equilibrium

Pareto Efficiency:

An allocation where it is impossible to make someone better off without making someone else worse off. If the preference for the consumer is the same, changes in prices and endowment does not change whether it is a Pareto Efficient Point.

Note that a Pareto efficient allocation may not be an equilibrium allocation, but an equilibrium allocation is a Pareto efficient allocation

Dependant:
- Preference (Utility Function)

Independent:
- Prices
- Endowment

Competitive Equilibrium

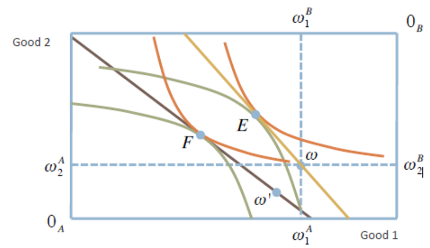
A pair of prices such that markets clear, everyone maximises utility given budget constraint. Changes in prices or endowment allocation changes the competitive equilibrium

Dependant:
- Prices
- Endowment allocation (Determines Budget Constraint)

Second Welfare Theorem

Definition: The Second Fundamental Theorem of Welfare Economics states that any Pareto efficient allocation be achieved in a competitive equilibrium through redistribution of endowments

By redistributing the endowment, we can change any Pareto efficient allocation to be the competitive equilibrium allocation



F will be an equilibrium allocation if the endowment is w'

Figure 34 Example of Changing Endowment Allocation to shift competitive equilibrium

Gross Demand

Definition: Utility maximizing quantity of each good for each consumer at the given prices. (This may not be the competitive equilibrium allocation and not the equilibrium prices)

Let P_1, P_2 be any 2 any pair of prices

Gross Demand for A: (x_1^A, x_2^A)

Gross Demand for B: (x_1^B, x_2^B)

Note that since P_1, P_2 may not be the equilibrium prices, therefore this could occur:

$$x_1^A + x_2^A \neq \omega_1^A + \omega_2^A$$
$$x_1^B + x_2^B \neq \omega_1^B + \omega_2^B$$

Net Demand

Definition 6.2: The net demand of a consumer for a good is the difference between the gross demand for that good and his/her endowment for that good

Net Demand of A for good 1: $x_1^A - \omega_1^A$

Net Demand of A for good 2: $x_2^A - \omega_2^A$

Net Demand of B for good 1: $x_1^B - \omega_1^B$

Net Demand of B for good 2: $x_2^B - \omega_2^B$

Aggregate Net Demand

Definition 6.3: The aggregate net demand for a good is the sum of the net demand for that good for the two consumers

Good 1:

$$\boxed{x_1^A + x_1^B - \omega_1^A - \omega_1^B}$$

Good 2:

$$\boxed{x_2^A + x_2^B - \omega_2^A - \omega_2^B}$$

Positive aggregate net demand: Excess Demand

Negative aggregate net demand: Excess Supply

Walras' Law

Definition 6.4: The total value of the aggregate net demand for the two goods is 0

$$\boxed{P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0}$$

Note that the value of the aggregate net demand is just multiplying the price of the good with its aggregate net demand

Implication 1: In the two-good exchange economy, if one market is in equilibrium, the other market must also be in equilibrium

Suppose that market for good 1 clears, the market for good 2 clears as well

Implication 2: In the two-good exchange economy, an excess supply in one market implies an excess demand in the other market

Suppose that there is excess supply of good 1, there will be excess demand for good 2

Holds for any Prices

Note that Walras' law holds for ANY prices, not just equilibrium prices

- At equilibrium prices, the aggregate net demand for each good is 0
- At non-equilibrium prices, the aggregate net demand for each good is not 0, there will be excess demand or excess supply

Topic 7: Production

Production Functions with 1 Input

Definition 7.1: Production function tells us the maximum quantity (Q) of output the firm can produce given the amount of L and K

$$\boxed{Q = F(L, K)}$$

Note that the quantity produced will be a function of labour (L) and capital (K)

Analogy: Similar to Utility Function in Consumer Theory

Short Run vs. Long Run:

Short Run:
- At least one input is fixed
- Normally in our case it is just capital, firms can only adjust labour
- Therefore, the production function will only be a function of labour

$$\boxed{Q = F(L)}$$

Long Run:
- All inputs are variable
- Therefore, capital and labour can be varied

Technically Efficient and Feasible

Can be seen from the production function:

- Above the production function: Not feasible
- On the production function: Technically feasible and technically efficient
- Below the production function: Technically feasible but not efficient

Marginal Products in Production:

Definition 7.2: Marginal product of labour measures the rate at which output level changes as quantity of labour changes

$$\boxed{MP_L = \frac{dQ}{dL} = \frac{\Delta Q}{\Delta L}}$$

Note that ΔL should be extremely small

Definition 7.3: Increasing marginal returns is when $\uparrow L \rightarrow \uparrow MP_L$. When we have more labour, the productivity of the workers increases

Definition 7.4: Diminishing marginal returns is when $\uparrow L \rightarrow \downarrow MP_L$. When we have more labour, the productivity of the workers decrease

Definition: Law of diminishing marginal returns

Suppose capital is fixed, marginal product of labour will eventually decline as the quantity of labour increases

Definition 7.5: Diminishing total returns is when $\uparrow L \rightarrow \downarrow Q$. We note that MP_L is negative here.

Note that as compared to marginal product, we are looking at total returns where if the total quantity produced is lesser, it must mean that with an additional unit of worker, they will be producing lesser as before the addition which means MP_L is negative

Analogy:
Marginal Product: Similar to Marginal Utility in Consumer Theory
Diminishing Marginal Returns: Similar to Diminishing Marginal Utility in Consumer Theory

Shape of Production Function:

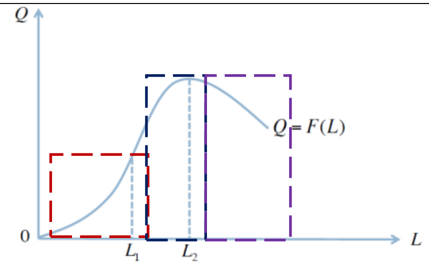


Figure 35 Typical Shape of a Production Function

Analysis from the number of workers hired:

From 0 to L_1:
- Increasing marginal returns
- At the starting of the increase of labour, we can have them specialise in other work rather than do everything. Therefore, the amount of output will increase
- Therefore, we have increasing marginal product of labour which also means that slope of production is positive and increasing

From L_1 to L_2:
- Decreasing marginal returns
- At this point, we could have all workers already specialising. So the gain more specialisation may not be as much but still gives a positive benefit to output
- Therefore, we have decreasing marginal product of labour which also means that slope of production is positive and decreasing (*Note that the slope is still positive but just that it is increasing at a slower rate so the rate of increase of the slope will be negative)*

From L_2 onwards:
- Diminishing total returns
- If we do not increase capital and just keep adding labour, having more workers may reduce the space that they can have to work so it much decrease the output level
- Therefore, we will have diminishing total returns and MP_L will be negative. Because with any additional unit of worker, it will cause a decrease in quantity produced instead

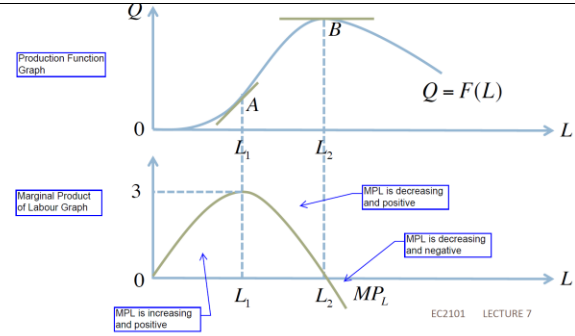


Figure 36 Corresponding Graphs of Production Functions and MP

Average Product:

Definition 7.6: Average product of labour measures the output per unit of labour

$$\boxed{AP_L = \frac{Q}{L}}$$

Graphically: We look at the slope of the ray connecting the origin and the point $(L, F(L))$

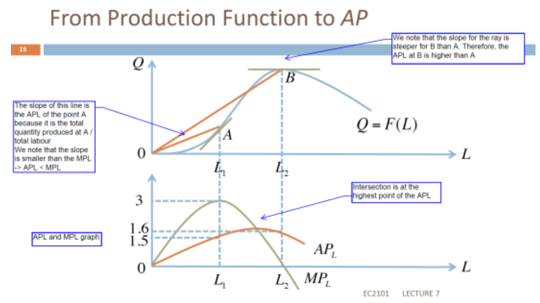


Figure 37 AP wrt Production Function

Relation of Average Product and Marginal Product:

Note: MP crosses AP at its highest point
Intuition: Note that MP tells us how much more productivity the next unit of labour will be whereas AP tells us what the average productivity is.

Rising AP_L when L increases
- As quantity of labour increases, average product of labour goes up
- Output generated by an extra unit of labour is pulling up the average
- $MP_L > AP_L$

Falling AP_L when L increases
- As quantity of labour decreases, average product of labour goes down
- Output generated by an extra unit of labour is pulling down the average
- $MP_L < AP_L$

Derivation of relation between AP and MP:

Main Idea: Find the first order derivative and note that when L increases, we can consider AP increasing and decreasing and we can see the relation between AP and MP from there

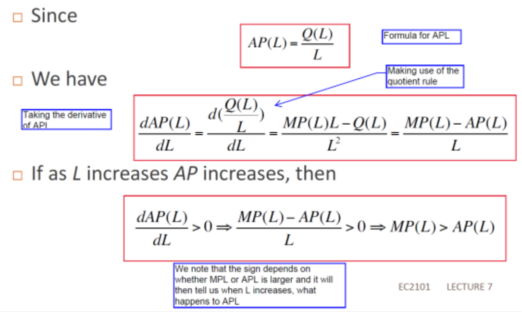


Figure 38 Mathematical Explanation

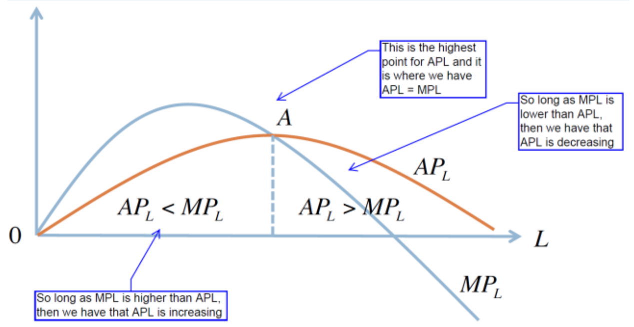


Figure 39 Graph between MP and AP

Production with 2 Inputs

Suppose that the firm can adjust both labour and capital

Production Function:

$$Q = F(L, K)$$

Marginal Products:

Marginal Product of Labour (MPL):

$$MP_L = \frac{\partial Q}{\partial L}$$

Marginal Product of Capital (MPK):

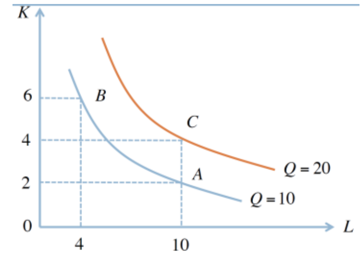
$$MP_K = \frac{\partial Q}{\partial K}$$

Isoquants

Definition 7.7: An isoquant is a curve that connects all combinations of labour and capital that generate the same level of output

Note: We will look at different levels of output and what are the combinations of labour and capital can be used to produce that

Graphically:



Common Isoquants:

Cobb-Douglas Production Function:

$$Q = AL^{\alpha}K^{\beta}, \quad A > 0, \alpha > 0, \beta > 0$$

- Downward sloping isoquants
- Isoquants are convex to the origin

Returns to scale:
$\alpha + \beta > 1 \rightarrow$ Increasing returns to scale
$\alpha + \beta = 1 \rightarrow$ Constant returns to scale
$\alpha + \beta < 1 \rightarrow$ Decreasing returns to scale

MRTS:
Note that the $MRTS$ for a Cobb-Douglas Production Function will always be a ratio of K and L

$$MRTS_{L,K} = b\left(\frac{K}{L}\right)$$

$MRTS$ along any ray will always be the same. Therefore, we can just compare the $MRTS$ for analysis of technological progress.

Linear Production Function:

$$Q = aL + bK$$

- Linear isoquants
- Two inputs are perfect substitutes

Fixed proportion Production Function

$$Q = \min(aL, bK)$$

- L-shaped isoquants
- Two inputs are perfect complements

Marginal Rate of Technical Substitution

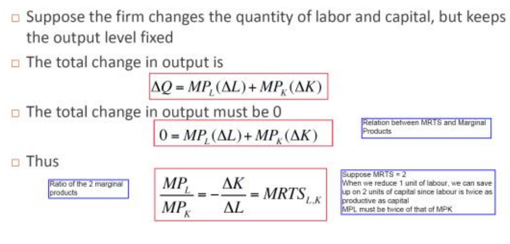
Definition 7.8: Marginal rate of technical substitution of labour for capital is the rate at which each firm can reduce the quantity of capital for more labour, holding the output level fixed

$$MRTS_{L,K} = -\left.\frac{dK}{dL}\right|_{Same\ Q} = -\left.\frac{\Delta K}{\Delta L}\right|_{Same\ Q}$$

where ΔL is extremely small

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L}$$

Derivation of ratio of marginal product:



Graphically:
It is the negative of the slope of the isoquant

Intuition:
- When the firm increases the labour by a small amount, how much capital they can save
- Analogous to MRS: Where it is how much y the consumer is willing to give up for a small unit of x

Diminishing MRTS:

- Convex Isoquants shows diminishing MRTS
- We will see that the slope is getting flatter and MRTS is getting smaller

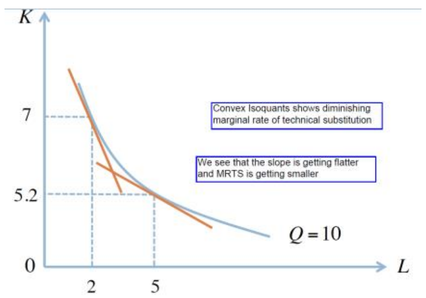


Figure 40 Graph illustrating Diminishing MRTS

Note that this has a similar idea to diminishing MRS

Uneconomic Region of Production

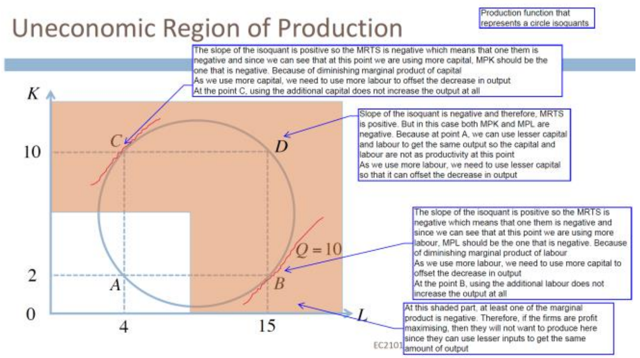
Definition 7.9: In the uneconomic region of production

- At least one of the marginal product is negative

Note:

- Cost-minimizing firms never produce in the uneconomic region of production
- Doesn't mean that if the slope of the isoquant is positive means that it is the uneconomic region of production, it could also be that both marginal product is negative and therefore, the slope of the MRTS is negative

Graphically:



Note that this is just a hypothetical graph to illustrate the uneconomic regions of production which is shaded in orange

Returns to Scale and Technological Progress

Definition: Returns to scale measures the rate at which output increases when all inputs increase proportionately

Note that firms do not have to increase all inputs proportionately, they can increase only one or increase at differing proportion but when we consider returns to scale, it is when all inputs increase proportionately

Suppose when L increases to aL and K increases to aK $(a > 1)$
Output increases to bQ

Definition 7.10: Increasing returns to scale: $b > a$

- Output increases in higher proportion than the increase in proportion of inputs

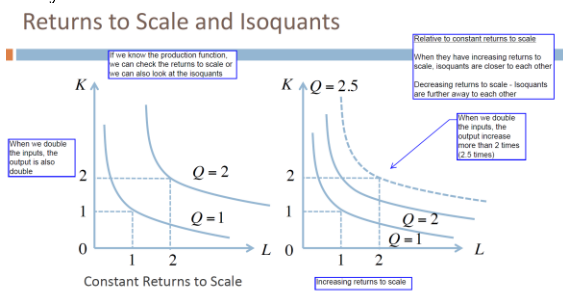
Definition 7.11: Constant returns to scale: $b = a$

- Output increases at the same proportion as inputs

Definition 7.12: Decreasing returns to scale: $b < a$

- Output increases in lower proportion than the increase in proportion of inputs

Graphically:



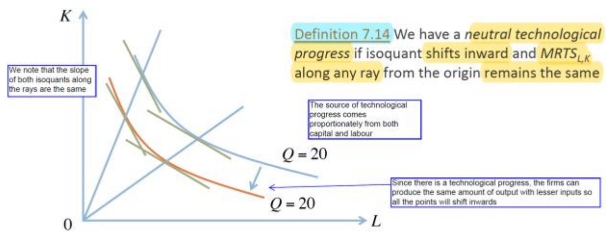
Technological Progress

When technology improves

Definition 7.13: We have technological progress if for any given combination of inputs, the firm produces higher Q. Or, to produce any Q, the firm uses less inputs

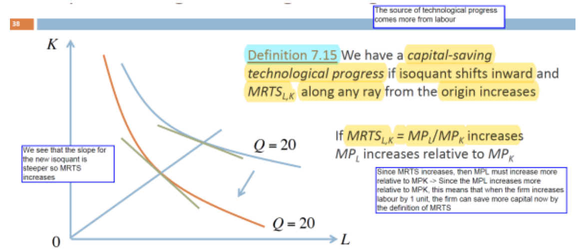
Note that we look at the ray from the origin because we want to compare at the same ratio of L and K, what is the difference in MRTS

Neutral Technological Progress



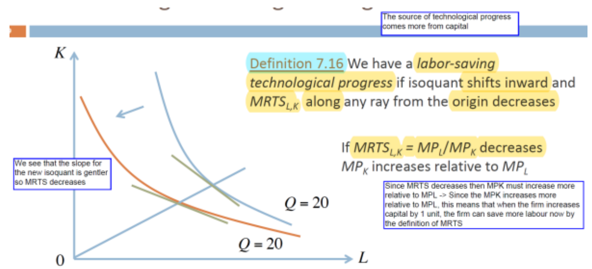
Note that $MRTS_{L,K}$ just being the same does not mean the same thing as the $MRTS_{L,K}$ along any ray

Capital-Saving Technological Progress



Note: Since MRTS increases,
→ MPL must increase more relative to MPK
→ Since the MPL increases more relative to MPK
→When the firm increases labour by 1 unit, the firm can save more capital now by the definition of MRTS

Labour-Saving Technological Progress

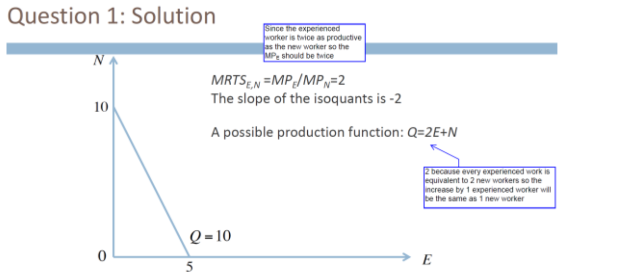


Note: Since MRTS decreases
→ MPK must increase more relative to MPL
→ Since the MPK increases more relative to MPL
→ When the firm increases capital by 1 unit, the firm can save more labour now by the definition of MRTS

Common Misunderstandings:

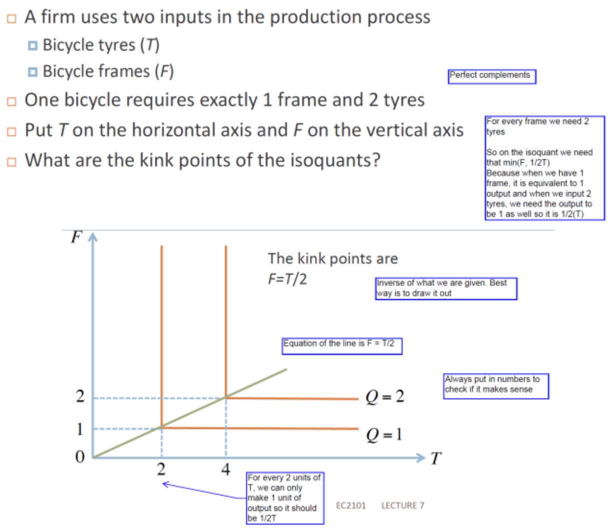
Linear Production Functions:

- A firm uses two inputs in the production process
  - Experienced worker (E)
  - New worker (N)
- Suppose 1 experienced worker is always equivalent to 2 new workers
- Put E on the horizontal axis and N on the vertical axis



Note that if we say that x of L is equivalent to y of K, it means that one of L is as productive as y/x of K note that inverse relation

Fixed Production Functions:



The best thing to do is just to put in numbers and see if it makes sense

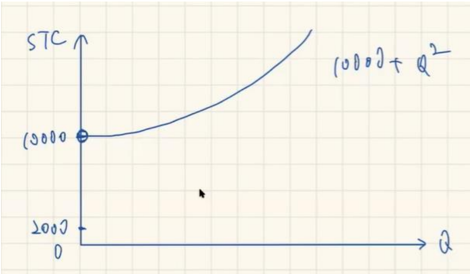
Note that since for every unit of F, we need 2 T. Therefore, we need $\min(F, 1/2T)$, *because we need to take into account that the 2 tyres produces 1 unit of output and therefore, it should be divided by 2*

Topic 8: Costs I

Common Misunderstanding

Sunk Costs Plotting:

Note that if there are some parts of the fixed cost that are sunk and some that are not, we can make use of a step function to plot it



Note that we can make use of an open circle to denote the continuity where if the value of $Q = 0$, we do not include 10000 and only include 2000

Average Variable Cost connection with SAC:

Note that the AVC has similar characteristics to the SAC since they differ by the AFC

- Since

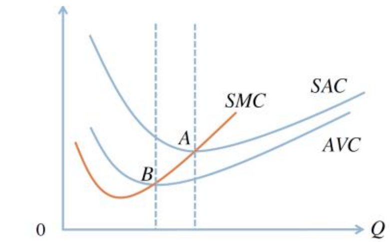
$$AVC(Q) = \frac{VC(Q)}{Q}$$

- We have

$$\frac{dAVC(Q)}{dQ} = \frac{d(\frac{VC(Q)}{Q})}{dQ} = \frac{SMC(Q)Q - VC(Q)}{Q^2} = \frac{SMC(Q) - AVC(Q)}{Q}$$

When we differentiate the AVC, we get a similar result to when we differentiate the SAC where we can get the relation that SMC > AVC when AVC is increasing and SMC < AVC when AVC is decreasing

Therefore, AVC and SAC have similar shape



Restrictions on Production Functions

For the short run:
Case 1: Since capital is fixed, when we have a normal linear function for instance where the inputs are perfect substitutes

$$Q = L + K$$

Suppose $K = 2$,
We note that since capital is fixed at 2, to produce anything less or equal to 2 units of output, the firm does not need to use any additional units of labour. Therefore, the optimal choice of labour if the firm produces less than or equal to 2 units is 0

The short run total cost when the output is less than or equal to 2 is just the total cost of capital, when output more than 2, the short run total cost is the cost of labour and the cost of capital

STC will be given by a **piecewise function**:

$$STC(Q) = \begin{cases} 4, & if\ Q \le 2 \\ Q + 2, & if\ Q > 2 \end{cases}$$

Case 2: Since capital is fixed, when we have a production function where the inputs are perfect complements:

$$Q = \min(L, K)$$

Suppose $K = 2$
We note that since capital is fixed at 2, the output can never be more than 2. For any value of L, where L>2, min(L,2)=2. Therefore, the maximum output that is possible is 2 in this case due to the fixed capital. Therefore, we can only produce where L⩽2

We note that the STC only exists for Q⩽2 since the maximum value of Q is 2 as per the above point.

STC will be bounded by the amount of quantity that can be produced by the fixed capital. Cost of labour is just the amount of quantity required

$$STC(Q) = Q + 2(2) = Q + 4, \quad if\ Q \le 2$$

Finding the Short Run Average Cost:

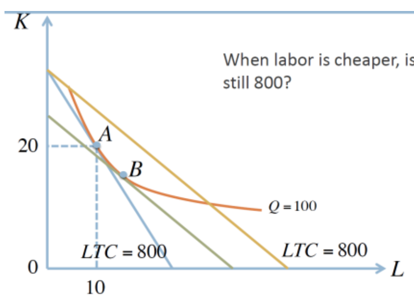
Note that since in the short run, capital is fixed, the short run average cost can just be determined from the production function

1) Choice of labour can just be determined from the production function by just substituting the value of the fixed capital and finding L in terms of Q
2) Substitute the values of L and K into the equation of STC
$$STC(Q) = wL + r\bar{K}$$
3) Short run average cost can be computed by just dividing STC by Q by definition
$$SAC(Q) = \frac{STC(Q)}{Q}$$

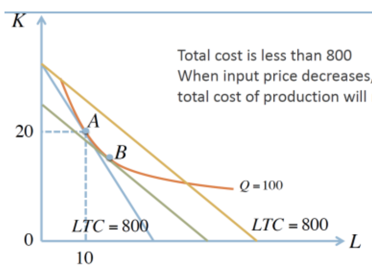
Topic 9: Costs II

Common Misunderstanding

Input Prices and LTC



Note that because the cost of labour is cheaper, the slope of the isocost will get gentler since the relative price of labour is lower now P_L/P_K will be smaller now



If we consider the same isocost of $LTC = 800$ then it should be higher than the original isoquant. However, we note that we only want to produce at $Q = 100$, therefore, we will produce at B which is tangent to the green line and should cost less than $LTC = 800$ (Since it is a lower isocost)

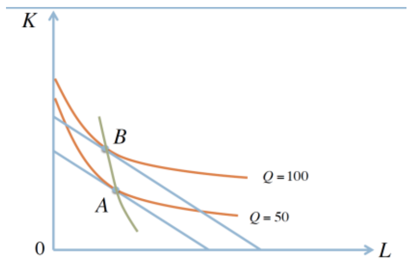
Note that in general, when input prices decrease, the long run total cost will not increase, it could still be the same or lesser

Expansion Path with Inferior Input

Definition 9.2: Inferior Input – The cost minimizing quantity of the input decreases when output increases (Holding input prices fixed)

Definition: Expansion path is the curve that goes through all the optimal choices when we keep the relative prices the same and change the level of outputs.

If labour is inferior and capital is normal



We note that the firm should use more capital since it is a normal good so the choice minimizing choice should move up but the firm should use less labour since it is an inferior input, therefore, it should move to the left.
Therefore, the resulting graph will be a downward sloping graph as we can see above

Giffen Input

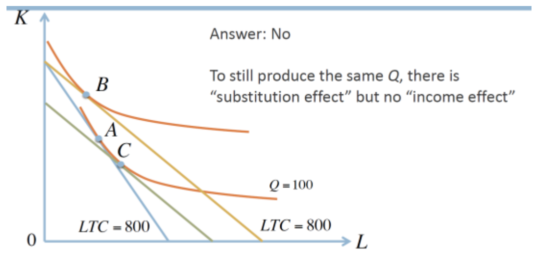
Definition 4.3: A good is a Giffen good if

- As price decreases, quantity demanded for the good drops
- A price increases, quantity demanded for the good goes up
- Holding other factors constant

Requirement
A Giffen good needs to be an inferior good and when there is a price change, Income Effect > Substitution Effect

Is it is possible for labour to be a Giffen Good?

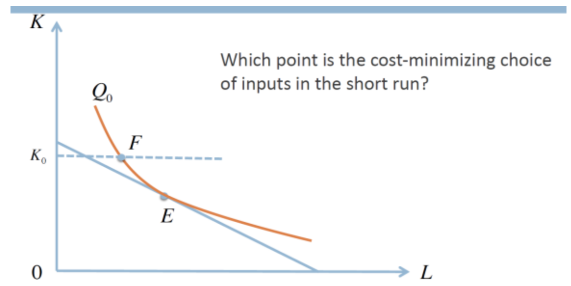
- When labour becomes cheaper, to produce the same quantity of output, the firm uses less labour?



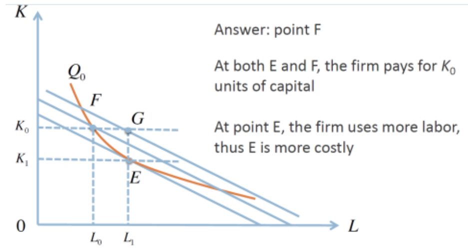
Note that the only reason why we have a Giffen Good, it is because the $IE > SE$. But we note that since we are still producing at the same level of output, there isn't the "income effect". Therefore, we will not have the Giffen Good effect
If there is the income effect, we will get to point B instead of C which is the outcome for a Giffen Good

Short Run Cost-Minimizing Choice of Inputs

From the lectures, if the fixed capital is below the optimal choice of capital, the firm will still have to pay more in the short run since the point will be on a higher indifference curve



If the fixed capital is above the optimal choice of capital, we want to determine what is the optimal choice of production for the firms



We note that the firm will still produce at point F where $K = K_0$ because if the firm were to produce at point E, the firm will still have to pay the K_0 units of capital.
We can see that even though point E lies on a lower isocost than point F, the actual cost is point G since the firm will still have to pay the same level of K_0 even at L_1, therefore, we can see that producing at E will be more costly and therefore, the cost-minimizing choice will still be the point F

Checking Normal Input or Inferior Input

Look at the demand function for the inputs. Note that we will need the actual functional form of the production function to know this

1) Find the tangency condition first
$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
2) Substitute the tangency condition into the production function to get rid of the other variable. For instance

Demand of L: we substitute K into the production function and get it only in terms of Q, w, r, L

Demand of K: we substitute L into the production function and get it only in terms of Q, w, r, K

Finding the Long Run Average Cost

Note that the LTC is the minimized cost total costs for given levels of output

1) Find the cost-minimizing points, Tangency Condition with the isocost

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

2) Substitute into the production function and solve for L and K, note that we will need to solve them in terms of Q. *Note that we need the values of r and w. Note that by doing this, we are technically finding the demand functions of L and K also since they will be in terms of Q, r, w*
3) Substitute into the equation for LTC

$$LTC(Q) = wL + rK$$

4) Divide the LTC by Q which is the definition of LAC

$$LAC(Q) = \frac{LTC(Q)}{Q}$$

Note that the LAC must always be less than or equal to SAC since in the long run, we can vary capital so if we cost minimize, then it should be that it is lower than the SAC

If the level of capital is never a cost minimizing choice, then the LAC will always be lower than the SAC (won't even be equal at any point) since there are other choices of capital that minimizes cost more

Perfect Substitutes

We can compare the per dollar marginal product to see what the firm chooses as the main input. Note that in the long run, both inputs are variable so we can choose a combination or just solely one

$$\frac{MP_L}{w} \& \frac{MP_K}{r}$$

If the per dollar marginal product for labour is higher,

$$Q = L$$

Then to find the LTC, just need to substitute into the equation

$$LTC(Q) = wL + rK$$

Topic 10: Perfect Competition in Short Run

Common Misunderstanding

Reason for $MR = MC$

First Order Condition for maximizing profit:

$$MR(Q) = MC(Q)$$

In perfectly competitive market:

$$P = MC(Q)$$

Suppose $P < MC$:

Marginal Cost: If we reduce production by 1 unit, how much we will save
Marginal Revenue: If we reduce production by 1 unit, how much we will lose out on earning

Therefore, the firm can do better by producing less when $MC > P$, since the rate of decrease of cost is higher than the rate of decrease of revenue. Therefore, if they produce less, they will cut more costs

- Suppose market price is P=12
- MR=P=12
  - If the firm decreases the production level, the total revenue decreases at a rate of 12
- Suppose at the current production level, SMC=16
  - If the firm decreases the production level, the total cost decreases at a rate of 16
- When P<SMC, total revenue decreases slower than total cost as production level decreases
  - Similarly, total revenue increases slower than total cost when Q increases
- Suppose the market price is still P=12
- MR=P=12
  - If the firm increases the production level, the total revenue increases at a rate of 12
- Suppose at the current production level, SMC=4
  - If the firm increases the production level, the total cost increases at a rate of 4
- When P>SMC, total revenue increases faster than total cost as production level increases

Therefore:

$P > SMC$: Firms should produce more
$P < SMC$: Firms should produce less

Since firms can increase profit by producing more when $P > SMC$ or producing less when $SMC < P$

Then the firms are profit maximizing when $P = SMC$ which is the profit maximizing condition.

Choice of using SMC, ANSC, SAC

Decision for Production:
- **ANSC**
- Firms will only produce when $P \geq \min(ANSC)$

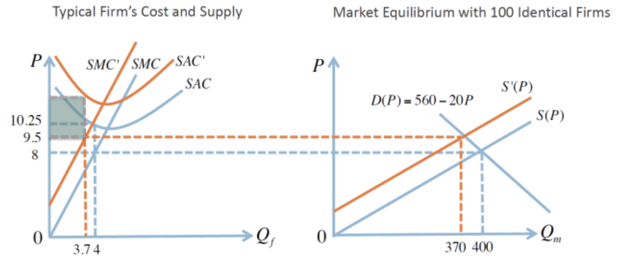
Assuming the firm produces, decision for **how much to produce**:
- **SMC**
- Firms will produce where $P = SMC$ and SMC is not downward sloping

Checking whether the firm is **making profit**:
- **SAC**
- If $P > SAC$, then the firms are making profit

Comparative Statics of Short-Run Equilibrium

Suppose a market has reached short-run equilibrium:
From lectures, we have seen what happens in the market when demand increases

When input prices change:



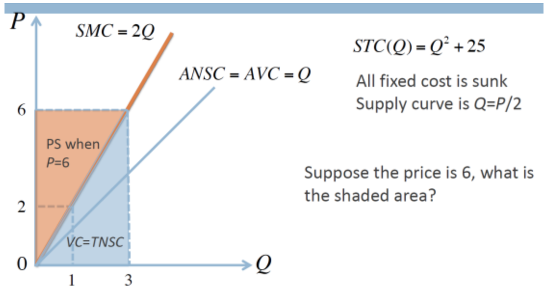
We note that when input prices change, the SMC and SAC will change as well. Since the cost of production will change.
The market supply curve shifts because the individual firms' supply curve is the SMC where the firm is producing. Since the SMC shifted, then the supply curve will also change. But demand curve does not change since it is a supply shock, it will be a movement along the demand curve

Note that the dark green area is the negative profit for the firms. It is negative profit since the SAC is above P and therefore, the average cost is higher than the revenue they get

Producer Surplus

Producer Surplus: Total Revenue – Total Non Sunk Cost
Area under Supply Curve: Total Non Sunk Cost
Note when the firm produces then the supply curve is the SMC, if the firm doesn't produce, then it is a vertical line at the y-axis

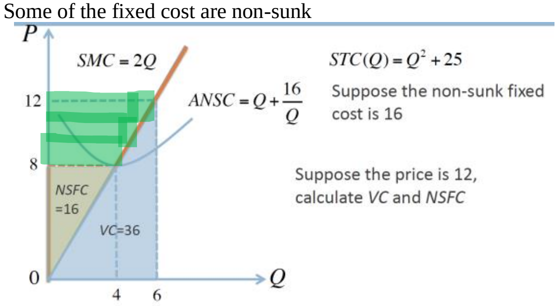
This is when we have no non-sunk fixed cost



Area under the SMC is the variable cost since the SMC tells us the additional cost we have to incur when we produce 1 extra unit of the good. So the total area of it is just the total additional cost that we have to incur where we know that the additional costs comes from the variable cost since it is the one responsible for the changes in SMC.

By the definition of producer surplus, it is the area that above the SMC and below the price or basically total revenue – TNSC which is also the whole rectangle minus the blue triangle which gives us the orange triangle

Case: Some of the fixed cost are non-sunk



We note that the firm only produces when $P > \min(ANSC)$
Area under the SMC will still be the variable cost as per the previous argument.

Producer Surplus is the green area since it is above the market supply curve (Since we only produce when $P > 8$ so anything below 8 we will not produce where those parts of the SMC will not be included and below the price which is 12 now

Area under the supply curve must be the total non sunk cost. Since we know that the blue triangle is the variable cost, then the orange triangle is the non-sunk fixed cost.

Number of Firms in the economy:

When we have the market supply and the quantity at the current market price

$$Number\ of\ firms = \frac{Market\ Supply}{Supply\ per\ firm}$$

Noting that we are assuming all firms are identical

One useful thing is that when the market is in equilibrium, market demand = market supply so we can determine the level of market supply from the market demand and solve the number of firms from there. Note that we will still need to figure out the equilibrium price as well so that we know how much each firm supplies at the equilibrium price

Finding Total Fixed Cost:

When all fixed costs are sunk:
Given the market price, *Note that the market price may not be the equilibrium price*
When the firm does not earn any economic profit,

$$Total\ Revenue = Total\ Cost$$
$$Total\ Cost = VC + FC$$

We can find the total revenue and variable cost to find the total fixed cost

Finding Supply Curve when there is non-sunk fixed cost

1) Find non-sunk fixed cost
2) Find total non sunk cost which is total variable cost and total non sunk fixed costs

$$TNSC = TVC + TNSFC$$

3) Find the average non sunk cost which is TNSC divided by Q

$$ANSC = \frac{TNSC}{Q}$$

4) Find the minimum of ANSC by either equating to SMC or find the first derivative to find the point of Q that is the minimum

When $P < \min(ANSC)$: Short-run supply curve is vertical axis
When $P \geq \min(ANSC)$: Short-run supply curve is marginal cost curve

Relation Between SMC, STC, AVC

SMC & STC:

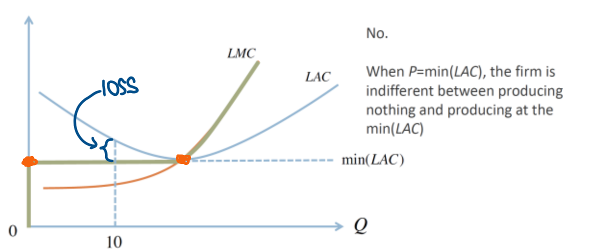
$STC \rightarrow SMC$: Differentiate with respect to Q
$SMC \rightarrow STC$: Integrate with respect to Q

STC and AVC:
$STC \rightarrow AVC$: Divide by Q
$AVC \rightarrow STC$: Multiply by Q

Topic 11: Perfect Competition in the Long Run

Common Misunderstanding

Long-Run Supply Curve

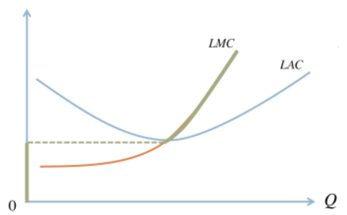


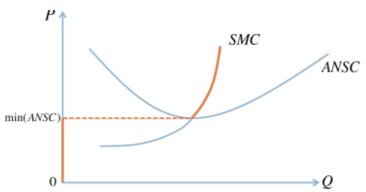
Note that the firms produce where $P > \min(LAC)$ in the long run. However, they do not produce along the horizontal part of the long-run supply curve. *Note that at this point, the firm only chooses between 2 points, where it is min(LAC) or not producing at all.*

The firm does not choose to produce between the 2 points because it will result in a negative profit as seen in the graph where LAC > P

Remember that points on the supply curve are profit maximizing given the prices

Short-Run vs. Long-Run Supply Curve





Choice of whether to produce:

Short Run: Supply curve is the SMC curve

- When price is $\geq \min(ANSC)$

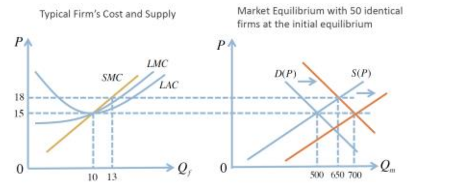
Note that for the short run because we have fixed costs, it is not the SAC but the ASNC which is the average non sunk costs

Long Run: Supply curve is the LMC curve

- When price is $\geq \min(LAC)$

Note that for the long run, we can look at the LAC since the firms can leave the market with no loss if they are making negative profits while producing. Note that everything is non sunk in the long run

Increase in Demand in a Constant-Cost Industry



Equilibrium price increases in the short run but goes back to the same level in the long run

Determining whether each firm is producing more or more firms in the market

500 to 650:

- Short run response after the demand shock
- No entry and exit yet
- Each firm produces more since there is a higher market price now

650 to 700:

- Long run response after the demand shock
- Prices will drop due to the entry of new firms
- Since it is a constant cost industry, the price will drop back to $P = \min(LAC)$ where the firms will produce 10 each
- Since each firm produces the same amount as before the demand shock, there is an increase in the number of firms

Long-Run Equilibrium with 2 Types of Firms

Long-Cost Firm vs High-Cost Firms

- Consider the example in the lecture
- Suppose every manager is paid the reservation wage of \$70000
  - The firm with the great manager has lower cost
- Suppose for a firm with an average manager (high-cost firm)
  - min(*LAC*)=20
- For the firm with the great manager (low-cost firm)
  - min(*LAC*)=16
- What happens when *P*=16?

- All potential entrants have average managers
  - There is only 1 great manager
  - At the price of 16, no one has incentive to enter
- Existing high-cost firms want to leave
  - They are making negative profit
- 16 is not the long-run equilibrium price if both types of firms are in the market

Note that this is assuming that there are only average managers left in the market. Then at the price of 16, it will be below the min(LAC) of the high cost firms and therefore, high cost firms will leave. Long run market equilibrium will go back up to 20 which is min(LAC) of high-cost since we only have one great manager

However, if all potential entrants have access to great managers (low-cost), then all the low-cost firms will drive out high cost firms and therefore, the long run equilibrium will become 16

Possible Shocks that occurred in the Economy

Useful when the question only gives us numbers and want us to explain the possible sequence of events that occurred

Demand Shocks:

Positive Demand Shocks:

- Demand curve shifts to the right
- Short-run equilibrium prices rises
- Short-run equilibrium quantity rises as well

Negative Demand Shocks:

- Demand curve shifts to the left
- Short-run equilibrium prices falls
- Short-run equilibrium quantity falls

Supply Shocks:

Positive Supply Shocks:

- Supply curve shifts to the right
- Short-run equilibrium prices fall
- Short-run equilibrium quantity rises

Negative Supply Shocks:

- Supply curve shifts to the left
- Short-run equilibrium prices rises
- Short-run equilibrium quantity falls

Constant, Increasing, Decreasing Costs Industries

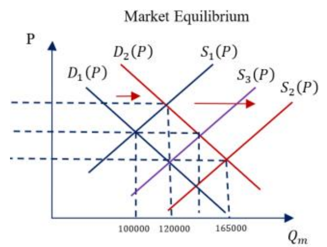
Note that this is determined through by whether the changes in industry output causes any changes in the prices of inputs to change

Constant Cost Industry:

- Long-run equilibrium prices does not change

Increasing Costs Industry:

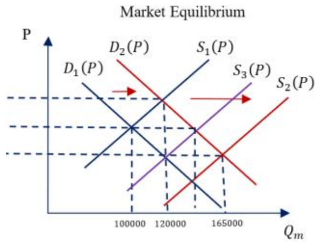
- Long-run equilibrium prices increases
- Increase output → Increase input prices → Higher cost of production → Long run equilibrium prices will be higher
- For instance for a positive demand shock, the shift in supply curve will be less than a constant cost industry, therefore, the final output will be less and the equilibrium price will be higher



For instance, the purple line will be the one for increasing cost and the red is the constant cost, it will shift less than the constant cost due to the higher $\min(LAC)$ due to increased cost of production

Decreasing Costs Industry:

- Long-run equilibrium prices decreases
- Increase output → Decrease input prices → Lower cost of production → Long run equilibrium prices will be lower
- For instance for a positive demand shock, the shift in supply curve will be more than a constant cost industry, therefore, the final output will be more and the equilibrium price will be lower



For instance, the purple line will be the one for constant cost and the red is the decreasing cost, it will shift more than the constant cost due to the lower $\min(LAC)$ due to decreased cost of production

Note:
When determining the type of industry, we normally need to see that:

$$\Delta Output \rightarrow \Delta Input\ Required\ \rightarrow \Delta Input\ Prices$$

However, if we have a negative supply shock, we will have a reverse in the process, because when we have a negative supply shock, then it must mean that cost of production changed.
This is because supply curve is determined by SMC and LAC and therefore, the costs must change first before the supply can change. This will result in the following which we cannot use to analyse the type of industry

$$\Delta Input\ Prices \rightarrow \Delta Input\ Required\ \rightarrow \Delta Output$$

Topic 12: Government Intervention

Common Misunderstanding

Demand for Inputs may not need to change with input prices

In the long run, when price of inputs becomes more expensive for instance, the demand for the input cannot increase (Since there are no Giffen Inputs)

Demand for inputs can remain the same as well, it does not have to vary with prices

If the production function is:

$$Q = \min(4L, 5K)$$

Cost minimizing point is at the kink point: (*This is the demand function for both inputs)*

$$4L = 5K = Q$$

Demand for labour/capital is independent of input prices
(Note that this is just a special case to prove that the input prices does not change the demand)

Short-run Firm Behaviour:

If a profit-maximizing firm in a perfectly competitive market currently produces at a point where its SAC is upwards sloping

Intuition:

- Firms are making positive profits
- When SAC is increasing, $SMC > SAC$
- We note that profit maximizing firms produce at $SMC = P$ if there are producing and therefore $P = SMC > SAC$
- Therefore, firms are earning positive profits

Long Run Equilibrium

- Two perfectly competitive markets have the same market demand curves. The minimum level of the *LAC* is the same for firms in market 1 and market 2. The quantity at the min(*LAC*) for firms in market 1 is higher than that for firms in market 2.
- Are the long-run equilibrium prices the same in the two markets?
- Which market has more firms in the long run equilibrium?

Equilibrium prices are the same:

- Since long-run equilibrium price is given by $\min(LAC)$, therefore, since the minimum level of LAC is the same for firms in market 1 and market 2, the equilibrium prices are the same

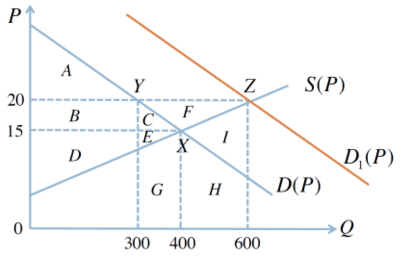
Market 1 fewer firms:

- Market demand is the same for both markets and the long-run equilibrium price is the same
- Therefore, the long-run equilibrium must be the same since the equilibrium price determines the equilibrium quantity with a demand curve
- In equilibrium, demand = supply and therefore, the supply should be the same for both markets
- Since each firm produces more at the equilibrium price in market 1 than 2, there will be fewer firms needed to produce that level of supply

Government Purchase

DWL=C+E+G+H+I

Why isn't F part of the DWL?



Solution:

- Under free market equilibrium
  - TS=A+B+C+D+E

    In the original free market, F was never part of the total surplus
- Under government intervention
  - Producers receive F, government pays F

    Therefore, it gets cancelled out and the total surplus after govt intervention does not contain F at all
  - TS=A+B+D-G-H-I
- What goes into the DWL?
  - C and E are part of the total surplus before government purchase but no longer part of the total surplus under government purchase

    Because after the government intervention, no one gets it anymore
  - G, H, and I are not part of the total surplus before government purchase but they are part of the expenditure under government purchase

    No one receives this and the government has to pay for this

EC2101 LECTURE 12

Note that when we want to compare the DWL, we will compare the total surplus before the government purchase and after. We see which are the parts that someone receives before and after (vice versa)