HW 12

1. Given the problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xy\,,\quad 0 < x < \pi\,,\quad 0 < y < \pi/2$$

$$u(0, y) = \cos y\,,\quad u(\pi, y) = -\cos y\,,\quad 0 \le y \le \pi/2\,,$$

$$u(x, 0) = \cos x\,,\quad u(x, \pi/2) = 0\,,\quad 1 \le y \le 2$$

To calculate $u(x, y)$ by using $h = k = 0.1\pi$.

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> Environment updated. Reloading
u(0.00, 0.00) = 1.00000
u(0.31, 0.00) = 0.95106
u(0.63, 0.00) = 0.80902
u(0.94, 0.00) = 0.58779
u(1.26, 0.00) = 0.30902
u(1.57, 0.00) = 0.00000
u(1.88, 0.00) = -0.30902
u(2.20, 0.00) = -0.58779
u(2.51, 0.00) = -0.80902
u(2.83, 0.00) = -0.95106
u(3.14, 0.00) = -1.00000
u(0.00, 0.31) = 0.95106
u(0.31, 0.31) = 0.79083
u(0.63, 0.31) = 0.63061
u(0.94, 0.31) = 0.44381
u(1.26, 0.31) = 0.22989
u(1.57, 0.31) = 0.00000
u(1.88, 0.31) = -0.22989
u(2.20, 0.31) = -0.44381
u(2.51, 0.31) = -0.63061
u(2.83, 0.31) = -0.79083
u(3.14, 0.31) = -0.95106
u(0.00, 0.63) = 0.80902
u(0.31, 0.63) = 0.63061
u(0.63, 0.63) = 0.47879
u(0.94, 0.63) = 0.32696
u(1.26, 0.63) = 0.16673
u(1.57, 0.63) = 0.00000
u(1.88, 0.63) = -0.16673
u(2.20, 0.63) = -0.32696
u(2.51, 0.63) = -0.47879
u(2.83, 0.63) = -0.63061
u(3.14, 0.63) = -0.80902
u(0.00, 0.94) = 0.58779
u(0.31, 0.94) = 0.44381
u(0.63, 0.94) = 0.32696
u(0.94, 0.94) = 0.21852
u(1.26, 0.94) = 0.11007
u(1.57, 0.94) = 0.00000
u(1.88, 0.94) = -0.11007
u(2.20, 0.94) = -0.21852
u(2.51, 0.94) = -0.32696
u(2.83, 0.94) = -0.44381
u(3.14, 0.94) = -0.58779
u(0.00, 1.26) = 0.30902
u(0.31, 1.26) = 0.22989
u(0.63, 1.26) = 0.16673
u(0.94, 1.26) = 0.11007
u(1.26, 1.26) = 0.05504
u(1.57, 1.26) = 0.00000
u(1.88, 1.26) = -0.05504
u(2.20, 1.26) = -0.11007
u(2.51, 1.26) = -0.16673
u(2.83, 1.26) = -0.22989
u(3.14, 1.26) = -0.30902
u(0.00, 1.57) = 0.00000
u(0.31, 1.57) = 0.00000
u(0.63, 1.57) = 0.00000
u(0.94, 1.57) = 0.00000
u(1.26, 1.57) = 0.00000
u(1.57, 1.57) = 0.00000
u(1.88, 1.57) = 0.00000
u(2.20, 1.57) = 0.00000
u(2.51, 1.57) = 0.00000
u(2.83, 1.57) = 0.00000
u(3.14, 1.57) = 0.00000
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2. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{4K}\frac{\partial T}{\partial t}\,,\quad \frac{1}{2}\le r\le 1\,,\quad 0\le t\,,$$

$$T(1,t)=100+40t\,,\quad 0\le t\le 10\,;\qquad \frac{\partial T}{\partial r}+3T=0\quad \text{at}\quad r=\frac{1}{2}$$

$$T(r,0)=200(r-0.5)\,,\quad 0.5\le r\le 1\,,$$

and use $\Delta t = 0.5$, $\Delta r = 0.1$, and $K = 0.1$ to calculate $T(r,t)$

By (a) the forward-difference method

(b) the backward-difference method

© the Crank-Nicolson algorithm.

> Environment updated. Reloading shell...

Forward Difference

time	r=0.5	r=1.0
0.00	0.000	120.000
0.50	15.385	120.000
1.00	28.205	140.000
1.50	26.585	160.000
2.00	152.091	180.000
2.50	-1314.279	200.000
3.00	18801.702	220.000
3.50	-302288.456	240.000
4.00	5472330.735	260.000
4.50	-103866990.259	280.000
5.00	1987346117.551	300.000
5.50	-37736904592.534	320.000
6.00	708241212190.765	340.000
6.50	-13141591993979.787	360.000
7.00	241476291238962.938	380.000
7.50	-4401988935641867.500	400.000
8.00	79739913600003616.000	420.000
8.50	-1437265966172255744.000	440.000
9.00	25804674882183323648.000	460.000
9.50	-461876510844660219904.000	480.000
10.00	8247195801748298006528.000	500.000

Backward Difference

time	r=0.5	r=1.0
0.00	0.000	120.000
0.50	216.533	120.000
1.00	477.853	140.000
1.50	861.374	160.000
2.00	1419.625	180.000
2.50	2222.146	200.000
3.00	3364.815	220.000
3.50	4980.816	240.000
4.00	7255.393	260.000
4.50	10446.274	280.000
5.00	14912.035	300.000
5.50	21151.576	320.000
6.00	29859.030	340.000
6.50	42000.164	360.000
7.00	58918.680	380.000
7.50	82484.110	400.000
8.00	115297.591	420.000
8.50	160978.159	440.000
9.00	224561.091	460.000
9.50	313052.114	480.000
10.00	436198.508	500.000

Crank-Nicolson

time	r=0.5	r=1.0
0.00	0.000	120.000
0.50	129.162	120.000
1.00	317.956	140.000
1.50	569.989	160.000
2.00	931.124	180.000
2.50	1430.993	200.000
3.00	2116.968	220.000
3.50	3048.998	240.000
4.00	4308.243	260.000
4.50	6001.338	280.000
5.00	8270.623	300.000
5.50	11304.419	320.000
6.00	15353.140	340.000
6.50	20748.814	360.000
7.00	27932.395	380.000
7.50	37488.967	400.000
8.00	50195.249	420.000
8.50	67082.064	440.000
9.00	89517.698	460.000
9.50	119318.198	480.000
10.00	158894.037	500.000

3. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0\,,\quad \frac{1}{2}\le r\le 1\,,\quad 0\le t\le \pi/3\,,$$

$$T(r,0)=0\,,\quad T(r,\pi/3)=0\,,\quad T(1/2,\theta)=50\,,\quad T(1,\theta)=100\,.$$

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theta = 0.00, T(r=0.5) = 0.00, T(r=1.0) = 0.00
theta = 0.10, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.21, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.31, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.42, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.52, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.63, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.73, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.84, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 0.94, T(r=0.5) = 50.00, T(r=1.0) = 100.00
theta = 1.05, T(r=0.5) = 0.00, T(r=1.0) = 0.00
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4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \le x \le 1, \quad 0 \le t$$

$$p(0,t) = 1, \quad p(1,t) = 2, \quad p(x,0) = \cos(2\pi x), \quad \frac{\partial p}{\partial t}(x,0) = 2\pi \sin(2\pi x), \quad 0 \le x \le 1$$

To calculate p by using $\Delta x = \Delta t = 0.1$.

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> Environment updated. Reloading shell...
t=0.0, p(x=0.5) = -1.000
t=1.0, p(x=0.5) = 4.000
t=2.0, p(x=0.5) = -1.000
t=3.0, p(x=0.5) = 4.000
t=4.0, p(x=0.5) = -1.000
t=5.0, p(x=0.5) = 4.000
t=6.0, p(x=0.5) = -1.000
t=7.0, p(x=0.5) = 4.000
t=8.0, p(x=0.5) = -1.000
t=9.0, p(x=0.5) = 4.000
t=10.0, p(x=0.5) = -1.000
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