HW5

1. The initial-value problem

$y' = 1 + (y/t) + (y/t)^2$, $1 \le t \le 2$, $y(1) = 0$ has the exact solution $y(t) = t\tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

```
python3 mai...
> Environment updated. Reloading shell...

Euler's Method Results:
    t      Euler y      Exact y        Error
 1.00     0.000000     0.000000     0.000000
 1.10     0.100000     0.105160     0.005160
 1.20     0.209917     0.221243     0.011325
 1.30     0.330471     0.349121     0.018651
 1.40     0.462354     0.489682     0.027328
 1.50     0.606285     0.643875     0.037590
 1.60     0.763041     0.812753     0.049711
 1.70     0.933475     0.997494     0.064019
 1.80     1.118537     1.199439     0.080902
 1.90     1.319293     1.420116     0.100823
 2.00     1.536943     1.661282     0.124338
```

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

```
Taylor's Method (Order 2) Results:
    t      Taylor y      Exact y        Error
 1.00     0.000000     0.000000     0.000000
 1.10     0.105000     0.105160     0.000160
 1.20     0.220919     0.221243     0.000324
 1.30     0.348612     0.349121     0.000509
 1.40     0.488954     0.489682     0.000728
 1.50     0.642883     0.643875     0.000993
 1.60     0.811438     0.812753     0.001315
 1.70     0.995787     0.997494     0.001707
 1.80     1.197252     1.199439     0.002187
 1.90     1.417344     1.420116     0.002772
 2.00     1.657795     1.661282     0.003487
```

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

```
=== Results for h = 0.05 ===

     t    u1 approx     u1 exact     u1 error    u2 approx     u2 exact     u2 error
  0.00     1.333333     1.333333     0.00e+00     0.666667     0.666667     1.11e-16
  0.05     1.721880     1.912059     1.90e-01    -0.499599    -0.909077     4.09e-01
  0.10     1.726915     1.793063     6.61e-02    -0.832598    -1.032002     1.99e-01
  0.15     1.617161     1.601967     1.52e-02    -0.890373    -0.961459     7.11e-02
  0.20     1.481687     1.423902     5.78e-02    -0.861042    -0.874681     1.36e-02
  0.25     1.348945     1.267646     8.13e-02    -0.807505    -0.795221     1.23e-02
  0.30     1.227063     1.131577     9.55e-02    -0.750341    -0.724999     2.53e-02
  0.35     1.117478     1.012999     1.04e-01    -0.695886    -0.663060     3.28e-02
  0.40     1.019525     0.909409     1.10e-01    -0.645732    -0.608214     3.75e-02
  0.45     0.931977     0.818630     1.13e-01    -0.599934    -0.559389     4.05e-02
  0.50     0.853541     0.738788     1.15e-01    -0.558092    -0.515658     4.24e-02
  0.55     0.783017     0.668275     1.15e-01    -0.519706    -0.476225     4.35e-02
  0.60     0.719337     0.605710     1.14e-01    -0.484290    -0.440411     4.39e-02
  0.65     0.661560     0.549909     1.12e-01    -0.451407    -0.407635     4.38e-02
  0.70     0.608868     0.499860     1.09e-01    -0.420673    -0.377404     4.33e-02
  0.75     0.560547     0.454695     1.06e-01    -0.391754    -0.349296     4.25e-02
  0.80     0.515980     0.413671     1.02e-01    -0.364365    -0.322954     4.14e-02
  0.85     0.474633     0.376158     9.85e-02    -0.338259    -0.298076     4.02e-02
  0.90     0.436043     0.341614     9.44e-02    -0.313226    -0.274409     3.88e-02
  0.95     0.399812     0.309583     9.02e-02    -0.289089    -0.251739     3.74e-02
  1.00     0.365600     0.279675     8.59e-02    -0.265698    -0.229888     3.58e-02
```

```
=== Results for h = 0.1 ===

     t     u1 approx     u1 exact     u1 error     u2 approx     u2 exact     u2 error
  0.00      1.333333     1.333333     0.00e+00      0.666667     0.666667     1.11e-16
  0.10     -3.052437     1.793063     4.85e+00      8.989305    -1.032002     1.00e+01
  0.20    -23.847795     1.423902     2.53e+01     51.192704    -0.874681     5.21e+01
  0.30   -130.165202     1.131577     1.31e+02    269.269193    -0.724999     2.70e+02
  0.40   -680.231485     0.909409     6.81e+02   1399.368584    -0.608214     1.40e+03
  0.50  -3531.299585     0.738788     3.53e+03   7258.241839    -0.515658     7.26e+03
  0.60 -18312.795052     0.605710     1.83e+04  37634.955483    -0.440411     3.76e+04
  0.70 -94951.331907     0.499860     9.50e+04 195131.871735    -0.377404     1.95e+05
  0.80 -492306.465639     0.413671     4.92e+05 1011721.872078    -0.322954     1.01e+06
  0.90 -2552513.623867     0.341614     2.55e+06 5245578.826590    -0.274409     5.25e+06
  1.00 -13234278.789168     0.279675     1.32e+07 27197287.206587    -0.229888     2.72e+07
```