Dynamics Identification: Criteria and Features

We extract multiple fingerprints from observed time series to distinguish network dynamics types. Each criterion targets specific dynamical or structural characteristics, with scores weighted by discriminatory power.

Extraction Methods and Target Features

1. State Value Range (Weight: 3)

Computation: Calculate minimum, maximum, and mean of all states across time and nodes.

Extracted features:

- **Dynamical:** Characteristic state space of different processes
- **Network:** None (topology-independent)

Decision logic:

- $[0, 1]$ range → SIS (infection probability bounded)
- $[-1, 1]$ range → Ising (spin values)
- $[0, 2\pi+]$ range → Kuramoto (phase angles)

Physical basis: Each dynamics type has intrinsic state space constraints determined by variable definitions.

2. Distribution Shape (Weight: 2)

Computation:

- Calculate skewness: $\gamma_1 = \mathbb{E}[(X-\mu)^3]/\sigma^3$
- Detect histogram peaks through smoothed derivative

Extracted features:

- **Dynamical:** Equilibrium vs. non-equilibrium signatures
- **Network:** Indirectly reflects structural influence on state distributions

Decision logic:

- Right-skewed (skewness > 0.5) → SIS (most nodes low infection, occasional outbreaks)
- Bimodal (multiple peaks) → Ising below T_c (spontaneous symmetry breaking, states cluster at ± 1)
- Near-uniform → Kuramoto (phases distributed across $[0, 2\pi]$ when desynchronized)

Physical basis: SIS is a driven dissipative system biased toward disease-free state; Ising exhibits ordered phases with discrete preferred states; Kuramoto phases are naturally periodic.

3. Fluctuation-Dissipation Theorem Test (Weight: 4)

Computation:

1. Compute correlation matrix $C_{ij} = \langle x_i x_j \rangle$
2. Apply perturbations h_j to sampled nodes, measure response $\chi_{ij} = \partial\langle x_i \rangle / \partial h_j$
3. Calculate correlation: FDT score = $\mathrm{corr}(\chi_{ij}, C_{ij})$

Extracted features:

- **Dynamical:** Equilibrium vs. non-equilibrium distinction
- **Network:** None (FDT is a fundamental thermodynamic property)

Decision logic:

- FDT score $> 0.7 \rightarrow$ Ising (thermal equilibrium, detailed balance satisfied)
- FDT score $< 0.3 \rightarrow$ SIS or Kuramoto (non-equilibrium, driven or conservative)

Physical basis: In thermal equilibrium, spontaneous fluctuations and response to perturbations are related through temperature. Non-equilibrium systems (SIS: driven by infection-recovery; Kuramoto: conservative but not thermalized) violate this relation.

4. Degree-Activity Correlation (Weight: 3)

Computation: Calculate Pearson correlation between node degree k_i and time-averaged activity $\langle x_i \rangle_t$.

Extracted features:

- **Dynamical:** Whether dynamics exhibit preferential activation of hubs
- **Network:** Degree heterogeneity (requires non-uniform degree distribution to be meaningful)

Decision logic:

- Strong positive correlation ($r > 0.7$) $\rightarrow$ SIS (high-degree nodes are superspreaders, $\langle x_i \rangle \propto k_i$ in mean-field)
- Weak correlation ($|r| < 0.3$) $\rightarrow$ Ising or Kuramoto (degree plays secondary role)

Physical basis: In epidemic spreading, infection probability scales with neighbor count. In Ising without external field, magnetization is degree-independent. In Kuramoto, hubs synchronize first but relationship is nonlinear.

5. Frequency Spectrum (Weight: 3)

Computation:

1. Apply FFT to each node's time series
2. Identify dominant frequency $\omega_{\max}$ where power spectrum peaks
3. Compute mean and standard deviation across nodes

Extracted features:

- **Dynamical:** Oscillatory vs. relaxational behavior
- **Network:** Collective mode structure (coherent vs. distributed oscillations)

Decision logic:

- Coherent oscillation (high $\bar{\omega}$, low σ_ω) $\to$ Kuramoto (synchronized oscillators share frequency)
- Broad low-frequency spectrum $\to$ SIS or Ising (relaxation-dominated, no intrinsic oscillations)

Physical basis: Kuramoto oscillators have natural frequencies; synchronization creates coherent spectral peaks. SIS and Ising relax exponentially without sustained oscillations.

6. Kuramoto Order Parameter (Weight: 3)

Computation: Calculate $r(t) = |\langle e^{i\theta_j(t)}\rangle_j|$ where θ_j interpreted as phase angles.
Extracted features:

- **Dynamical:** Synchronization degree (only meaningful for oscillatory systems)
- **Network:** Topology's ability to support synchronization (depends on spectral properties)

Decision logic:

- High $\bar{r} > 0.7 \to$ Kuramoto (strong phase coherence)
- Low $\bar{r}$ is ambiguous (could be desynchronized Kuramoto or non-oscillatory dynamics)

Physical basis: Kuramoto order parameter measures phase coherence. High values indicate collective synchronization, a hallmark of coupled oscillator systems.

7. Community Coherence (Weight: 2)

Computation:

1. Detect communities using Louvain algorithm
2. Calculate intra-community correlations: $C_{\text{intra}} = \langle C_{ij}\rangle_{i,j\in\text{same comm}}$
3. Calculate inter-community correlations: $C_{\text{inter}} = \langle C_{ij}\rangle_{i,j\in\text{different comm}}$
4. Coherence: $\eta = (C_{\text{intra}} - C_{\text{inter}})/(C_{\text{intra}} + C_{\text{inter}})$

Extracted features:

- **Dynamical:** Modular dynamics (within-module synchronization vs. between-module independence)
- **Network:** Community structure strength (modularity)

Decision logic:

- High coherence ($\eta > 0.5$) $\to$ SIS or Ising (communities act as dynamical barriers; epidemic confined within modules; Ising communities magnetize independently)
- Low coherence ($\eta < 0.2$) $\to$ Kuramoto (synchronized oscillators ignore community boundaries)

Physical basis: Epidemic spreading and spin coupling are mediated by direct connections, making community structure a strong barrier. Kuramoto synchronization can occur globally through weak inter-community links due to phase-locking mechanisms.

8. Correlation Length (Weight: 2)

Computation:

1. Group node pairs by network distance d_{ij} (shortest path length)
2. Calculate average correlation vs. distance: $C(d) = \langle |C_{ij}| \rangle_{d_{ij}=d}$
3. Fit exponential decay: $C(d) = Ae^{-d/\xi}$ to extract correlation length ξ

Extracted features:

- **Dynamical:** Spatial range of dynamical influence
- **Network:** Effective connectivity (how far influences propagate through topology)

Decision logic:

- Long ξ ($\xi/\langle d \rangle > 2$) $\rightarrow$ Ising near criticality (diverging correlation length at T_c)
- Short ξ ($\xi/\langle d \rangle < 1$) $\rightarrow$ SIS (finite spreading range)
- Moderate ξ is ambiguous

Physical basis: Near second-order phase transitions, correlation length diverges ($\xi \rightarrow \infty$ at T_c). In epidemic spreading, correlation decays exponentially with network distance. Kuramoto shows intermediate behavior depending on synchronization.

9. Response Matrix Spectrum (Weight: 2)

Computation:

1. Construct response matrix $\chi_{ij} = \partial \langle x_i \rangle / \partial h_j$ by perturbing sampled nodes
2. Compute eigenvalue decomposition of $\boldsymbol{\chi}$
3. Extract spectral gap: $\Delta\lambda = \lambda_1 - \lambda_2$, spectral radius: $\max_i |\lambda_i|$

Extracted features:

- **Dynamical:** Dominant response modes and stability
- **Network:** Spectral properties inherited from adjacency matrix (for linear dynamics: $\boldsymbol{\chi} \propto f(\mathbf{A})$)

Decision logic:

- Large spectral gap ($\Delta\lambda > 1$) $\rightarrow$ SIS or Kuramoto (clear dominant mode: epidemic principal eigenmode or synchronization manifold)
- Small gap suggests degenerate modes or critical fluctuations

Physical basis: For linear or linearized dynamics, $\boldsymbol{\chi} = \mathbf{M}^{-1}(\mathbf{A})$ where $\mathbf{M}$ is the dynamics matrix. Response eigenvalues relate to network spectral properties, revealing how topology shapes collective responses.

10. Phase Transition Detection (Weight: 2)

Computation:

1. Compute global order parameter evolution (mean state, magnetization, or sync parameter)
2. Calculate time derivative: $d\Psi/dt$
3. Smooth derivative with Gaussian filter
4. Identify maximum change rate; flag if exceeds $\bar{r} + 3\sigma_r$

Extracted features:

- **Dynamical:** Abrupt transitions vs. smooth evolution
- **Network:** Critical coupling strengths related to spectral properties

Decision logic:

- Detected transition → Ising or Kuramoto (systems with phase transitions)
- No transition → ambiguous (could be away from critical point)

Physical basis: Ising exhibits temperature-driven phase transitions; Kuramoto shows coupling-driven synchronization transitions. SIS has continuous transitions but typically reaches steady state smoothly in single simulations without parameter sweeps.

Summary Table

Criterion	**Weight**	**Network**	**Dynamics**	**Key Discriminator**
State range	3	-		Intrinsic state space
Distribution shape	2			Equilibrium symmetry
FDT test	4	-		Equilibrium vs. non-equilibrium
Degree-activity	3			Hub role in spreading
Frequency spectrum	3			Oscillatory vs. relaxational
Kuramoto order	3			Synchronization
Community coherence	2			Modular dynamics
Correlation length	2			Criticality, spatial range
Response spectrum	2			Dominant modes
Phase transition	2			Critical behavior
Total	**26**			

Table 1: Ten criteria for dynamics identification. Network column indicates if criterion extracts network structural information; Dynamics column indicates if it extracts dynamical process characteristics. Most criteria extract both, leveraging the network-dynamics coupling.

Feature Categories

Purely Dynamical Features (Topology-Independent)

1. State range: Determined by variable definitions (probability, spin, phase).

2. FDT satisfaction: Fundamental property of equilibrium systems, independent of specific network structure.

Network-Mediated Dynamical Features

These features exist only because dynamics occur *on networks*, revealing both process type and structural properties:

1. Degree-activity correlation: Reveals if dynamics preferentially activate hubs. Strong positive correlation indicates degree-dependent spreading (SIS superspreaders). Requires heterogeneous degree distribution to manifest.

2. Community coherence: Quantifies if community structure acts as dynamical barrier. High coherence indicates modules evolve semi-independently, characteristic of contact-mediated processes (SIS contagion, Ising local coupling). Low coherence suggests global coupling mechanisms (Kuramoto phase-locking).

3. Correlation length: Measures spatial extent of dynamical influence through network paths. Diverges at criticality (Ising at T_c), remains finite for epidemic spreading, varies with synchronization strength in Kuramoto.

4. Response spectrum: Eigenvalues of $\boldsymbol{\chi}$ inherit network spectral properties. For linearized SIS, $\boldsymbol{\chi}^{-1} \propto (-\gamma\mathbf{I} + \beta\mathbf{A})$, directly encoding adjacency spectrum. Large spectral gap indicates dominant collective mode.

5. Frequency spectrum coherence: Oscillation frequencies distributed across nodes reveal if network topology supports coherent collective modes (synchronized Kuramoto) or independent node dynamics.

6. Phase transitions: Critical points depend on network spectral radius: SIS threshold $\tau_c = 1/\lambda_{\max}$, Kuramoto critical coupling $K_c \propto 1/\lambda_{\max}$. Transition detection indirectly probes spectral properties.

Hybrid Features (Distribution Shape)

State distributions reflect both intrinsic dynamics and network-imposed constraints. Bimodality in Ising arises from discrete spin values and symmetry breaking. Right-skewness in SIS results from network heterogeneity creating infection inequality—hubs sustain higher infection while most nodes remain susceptible.

Weighting Rationale

Weights reflect discriminatory power and reliability:

Highest weight (4): FDT test—uniquely distinguishes equilibrium (Ising) from non-equilibrium (SIS, Kuramoto) with theoretical foundation.

High weight (3): State range, degree-activity, frequency spectrum, Kuramoto order—strong discriminators with clear physical interpretation.

Moderate weight (2): Distribution shape, community coherence, correlation length, response spectrum, phase transition—informative but context-dependent or requiring additional validation.

Complementarity and Robustness

No single criterion suffices due to potential ambiguities:

- Desynchronized Kuramoto may show low order parameter (ambiguous with non-oscillatory systems)
- SIS far from threshold may show weak degree-activity correlation
- Ising at high temperature may appear disordered

Multi-criteria voting ensures robustness: different features capture orthogonal aspects (temporal vs. spatial, equilibrium vs. network-mediated), and weighted aggregation provides confidence quantification rather than binary classification.

Computational Complexity

Criterion	**Complexity**	**Requires Simulator**
State range	$O(NT)$	No
Distribution	$O(NT)$	No
FDT test	$O(N^2T)$	Yes
Degree-activity	$O(N)$	No
Frequency spectrum	$O(NT \log T)$	No
Kuramoto order	$O(NT)$	No
Community coherence	$O(N^2 + C^2)$	No
Correlation length	$O(N^2)$	No
Response spectrum	$O(N^2T)$	Yes
Phase transition	$O(NT)$	No
Total (with simulator)	$O(N^2T)$	-
Total (without)	$O(NT \log T)$	-

Table 2: Computational complexity. N: nodes, T: time steps, C: communities. Simulator-dependent features require perturbation experiments; others use passive observation only.

Expected Fingerprint Patterns

Feature	**SIS**	**Ising**	**Kuramoto**
State range	$[0, 1]$	$[-1, 1]$	$[0, 2\pi]$
Skewness	Positive	≈ 0	Variable
FDT	Violated	Satisfied	Violated
Deg-activity corr	Strong (> 0.7)	Weak	Nonlinear
Frequency	Low, broad	Low, broad	High, narrow
Sync order r	Low	N/A	High (if sync)
Community η	High	Moderate	Low
Corr length ξ	Finite	Diverges at T_c	Moderate
Response gap	Large	Small-moderate	Large
Phase transition	Threshold	T_c	Sync transition

Table 3: Expected fingerprint patterns for three dynamics types. These patterns guide the weighted voting in identification algorithm.

Future Extensions

Higher-order dynamics: Extend fingerprints to simplicial contagion (hypergraph SIS) or higher-order Kuramoto by adding:

- Simplicial participation: fraction of dynamics explained by higher-order interactions
- Motif-specific correlation patterns

Adaptive weighting: Learn optimal criterion weights from labeled training data (supervised learning on synthetic benchmarks) to maximize identification accuracy.

Bayesian evidence: Replace heuristic scoring with rigorous Bayesian model comparison: compute $P(\text{obs}|\text{dyn}_k) = \int P(\text{obs}|A, J, \text{dyn}_k)P(A, J)\, dA\, dJ$ via Laplace approximation, incorporating network structural priors.