

EE 4715: Array Processing

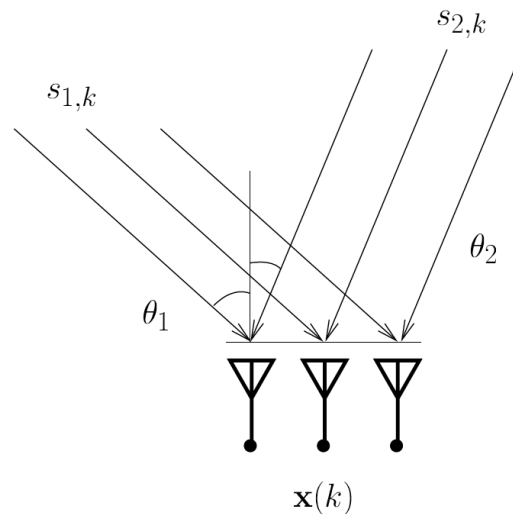
Spring 2022

EXAM ASSIGNMENT

The exam consists of two parts: (1) estimating directions and frequencies of sinusoidal signals entering a uniform linear array, and (2) equalizing a signal received over a coded channel. Make a short report containing the required Matlab files, plots, and answers, and bring it with you when you have your exam. The report can be made alone or in groups of two. The report will be discussed during an oral exam of about 30 minutes per student. During this exam I will ask you to explain your programs, results, and answers. The most important thing is that you understand WHY certain steps are taken. To further test your understanding, I may also ask you about the applicability of some of the studied algorithms to related problems. Sufficient time will be provided to express yourself, in case you have problems with the English language. If you have never taken an oral exam before and/or do not feel comfortable with taking one, please organize a trial exam with one of your fellow students.

In case something is unclear, (limited) assistance is given by Didem Dogan Baskaya, room HB17.250, e-mail: d.doganbaskaya-1@tudelft.nl. The exam is in room 17.270. Make sure you have registered on Brightspace for the oral exam.

ESTIMATION OF DIRECTIONS AND FREQUENCIES



We consider the case of d complex sinusoidal signals impinging on a uniform linear array of M antennas. We want to estimate their directions and frequencies.

Signal model

1. Make a data generator function

```
function [X,A,S] = gendata(M,N,Delta,theta,f,SNR)
```

to generate the received signal matrix $\mathbf{X} : M \times N$, where M represents the number of antennas, and N the number of samples. The antenna array is a uniform linear array with $\Delta = 1/2$ wavelengths antenna spacing. Further, $\boldsymbol{\theta} = [\theta_1, \dots, \theta_d]^T$ contains the directions of the sources in degrees, $-90 \leq \theta_i < 90$, and $\mathbf{f} = [f_1, \dots, f_d]^T$ the normalized frequencies of the sources, $0 \leq f_i < 1$. Finally, SNR denotes the signal-to-noise ratio (SNR) **per source** (this means it is the ratio of the signal power of one source over the noise power). We assume for simplicity that the noise is temporally and spatially white, complex Gaussian. The i th source is a complex sinusoid with normalized frequency f_i , given by:

$$s_{i,k} = \exp(j2\pi f_i k).$$

The noise is scaled to obtain the desired SNR per source.

2. Make plots of the singular values of \mathbf{X} , for $d = 2$ sources, $M = 5$, $N = 20$, $\boldsymbol{\theta} = [-20, 30]^T$, $\mathbf{f} = [0.1, 0.3]^T$, and $SNR = 20$ dB. What happens to the singular values if the number of samples doubles, if the number of antennas doubles, if the angles between the sources becomes small, if the frequency difference becomes small? Why?

Estimation of directions

1. Implement the ESPRIT algorithm to estimate the angles of arrival from a data matrix \mathbf{X} , assuming the number of sources is known:

```
function theta = esprit(X,d)
```

2. Check the correctness in case there is no noise (SNR very high). The angles of arrival should then be perfectly estimated (up to an order ambiguity).

Estimation of frequencies

1. For the model $\mathbf{X} = \mathbf{A}\mathbf{S}$, is there any special structure present in the \mathbf{S} matrix?
2. Modify the ESPRIT algorithm to estimate the frequencies directly from \mathbf{X} , using the structure that is present in \mathbf{S} :

```
function f = espritfreq(X,d)
```

3. Check the correctness in case there is no noise (SNR very high). The frequencies should then be perfectly estimated (up to an order ambiguity).

Joint estimation of directions and frequencies

1. Write a data model using the Khatri-Rao product structure. Use smoothing in time (not space) with a factor m to turn this problem into a joint diagonalization problem.
2. Use joint diagonalization to jointly estimate the directions and frequencies using a smoothing factor of m :

```
function [theta,f] = joint(X,d,m)
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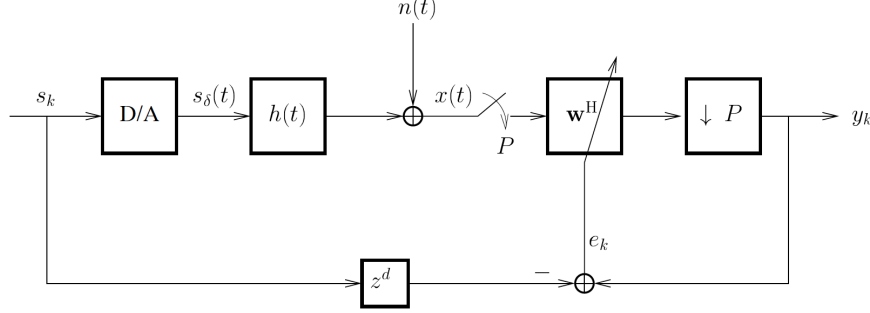
The code for performing the joint diagonalization is available in a separate file.

3. Check again the correctness in case there is no noise (SNR very high).

Comparison

1. Make a plot of the estimation performance of the **three** algorithms (mean values and standard deviations of the estimated angles and frequencies) as a function of the SNR (0, 4, 8, \dots , 20 dB), for $d = 2$ sources, $M = 3$, $N = 20$, $\boldsymbol{\theta} = [-20, 30]^T$, $\mathbf{f} = [0.1, 0.12]^T$. To compute the mean value and the standard deviation, take the statistics over 1000 test runs with independent complex Gaussian noise.
2. Compute **two** zero-forcing beamformers, one based on the direction estimates from ‘esprit’, and one based on the frequency estimates from ‘espritfreq’. Hence, you do NOT have to use any intermediate result from the ESPRIT algorithm. Test the correctness of the two beamformers in case there is no noise. In that case $\mathbf{W}^H \mathbf{X}$ should perfectly recover the transmitted sources (up to an order ambiguity).
3. Compare the two above zero-forcing beamformers by plotting their spatial responses $y(\theta) = |\mathbf{w}^H \mathbf{a}(\theta)|$ for $-90^\circ \leq \theta \leq 90^\circ$. Use the same parameters as before, and take $SNR = 10$ dB. Which one gives the best suppression of the interference? Why?

sxsubsection*CHANNEL EQUALIZATION



Here we consider the scenario where a single source is transmitting data towards a single receive antenna over a specific channel. The received signal $x(t)$ can be modeled as

$$x(t) = \sum_k h(t - k)s_k + n(t).$$

We assume the channel $h(t)$, which includes the transmit and receive filter effects, is some kind of a coded channel and looks like

$$h(t) = \begin{cases} 1 & \text{for } t \in [0, 0.25) \\ -1 & \text{for } t \in [0.25, 0.5) \\ 1 & \text{for } t \in [0.5, 0.75) \\ -1 & \text{for } t \in [0.75, 1) \end{cases}$$

Furthermore, the data symbols s_k are assumed to be QPSK data symbols, i.e., $s_k \in \{\pm 1/\sqrt{2} \pm j/\sqrt{2}\}$. The additive noise $n(t)$ is finally modeled as complex zero-mean Gaussian with standard deviation σ . The goal now is to reconstruct the transmitted data symbols by linear equalization.

Signal model

1. For a given data symbol sequence $\mathbf{s} = [s_0 \ s_1 \ \dots \ s_{N-1}]^T$, construct the sequence $\mathbf{x} = [x(0) \ x(1/P) \ \dots \ x(N-1/P)]^T$, which is obtained by sampling the received signal $x(t)$ at rate $1/P$, i.e., the sampling rate is P times larger than the data rate:

`function x = gendata_conv(s,P,N,sigma)`

2. Take $P = 4$ (i.e., we sample at 4 times the symbol rate), $N = 500$, and $\sigma = 0$ and construct the data matrix

$$\mathcal{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(N-2) \\ x(\frac{1}{P}) & x(1 + \frac{1}{P}) & \dots & x(N-2 + \frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ x(1 + \frac{P-1}{P}) & x(2 + \frac{P-1}{P}) \dots & x(N-1 + \frac{P-1}{P}) \end{bmatrix} : 2P \times (N-1) \quad (1)$$

Note that the rows of this matrix \mathcal{X} span two data symbol periods. What is the rank of \mathcal{X} and why?

3. What if we double P ? How does this change the rank of \mathcal{X} and why?

Zero-forcing and Wiener equalizer

1. Take again $P = 4$ and $N = 500$, but add some noise with standard deviation $\sigma = 0.5$. Compute the zero-forcing and Wiener receiver for the data matrix \mathcal{X} constructed in (1), assuming the channel $h(t)$ and the standard deviation of the noise σ are known. What is a good delay for these receivers, i.e., on which shift of the data symbol sequence should these receivers focus? For this delay, plot the estimated symbols of the two equalizers in the complex plane such that you observe four clusters. Is there a large difference in performance between the two receivers? Explain.
2. Take again $N = 500$ and $\sigma = 0.5$, but double P . Plot again the estimated symbols of the zero-forcing and Wiener equalizer in the complex plane. Consider the same delay as the one you selected above. Compare the plots and explain the differences.