Assignment 4 - Analyzing Functional Ultrasound Images using Tensor Decompositions

Background

Functional ultrasound (fUS) is a recently developed neuroimaging technique that measures fluctuations in local blood dynamics induced by changes in neuronal activity [1]. Our focus in this assignment will be on capturing responses given to sensory stimuli based on the fUS signal using tensor decompositions. Functional ultrasound data is typically stored as a 3D array $\underline{\mathbf{X}} \in \mathbb{R}^{N_z \times N_x \times N_t}$, wherein the first two modes represent the width and depth of power-Doppler images (PDIs), respectively, and the last mode stands for the time samples of the experiment.

The fUS dataset that we will be working on was acquired from a mouse, which was presented with a series of visual stimuli (high-contrast images) during the experimental recording. A specific brain slice was imaged along time, which contains a brain region called the lateral geniculate nucleus (LGN); this region plays a significant role in the processing of visual information. As such, we expect the response within this region to be coupled to the timings of the visual stimulus shown to the mouse subject.

In the assignment, we will first validate this hypothesis through a correlation image. To obtain the correlation image, the Pearson correlation coefficient (PCC) between the stimulus time-series and the fUS time-series of each pixel is computed. The resulting correlation image is of size N_z by N_x, where the values within the image show the PCC between the corresponding pixel and the stimulus. Correlation images provide a very important insight while analyzing neuroimaging data, as they demonstrate which regions are activated by the stimulus.

Despite their usefulness, correlation images require that the stimulus is known in advance. Alternatively, data-driven techniques such as independent component analysis and tensor decompositions can be used to reveal active brain regions by extracting components that are spatio-temporally coherent, without any a priori information of the stimulus.

In this assignment, you will use the canonical polyadic decomposition (CPD) and the block-term decomposition (BTD) to discover task-related components within the brain, i.e., the regions which were activated by the stimulus task. Two of these regions are the LGN and the visual cortex, but you will also notice some large blood vessels reacting to the stimulus in the correlation image.

To summarize, in this assignment you will first create the correlation image, which can be considered as a guide that shows the regions that we will be searching for using the tensor decompositions. Next, you are asked to compare the performance of CPD and BTD when analyzing fUS data. Note that as part of this assignment you are expected to implement BTD. The two files you are asked to fill in are `assignment_4_fUS.m` and `btd_ll1_als_3d.m`.

Assignment

Correlation Image

The stimulus is represented as a binary signal showing the on-times (value 1) and off-times (value 0) of the visual paradigm (this signal is given to you). To obtain the correlation image, you should calculate the PCC between the pixel time-series and the stimulus signal and display it on an $N_z \times N_x$ image. Note that the brain response to a given stimulus is often delayed compared to the stimulus onset. As such, the best (i.e., most informative) correlation image is obtained by delaying the stimulus.

In Fig. 1, the correlation image you should come up with is provided for you as a reference. Note that this correlation image only displays the regions that are significantly correlated with the stimulus (with a PCC value above ~ 0.3), which are then overlaid across the average PDI (average along the time mode). When you apply a tensor decomposition, you should search for the component(s) whose spatial map resembles the active areas portrayed in the correlation image. These areas include the LGN, visual cortex, and the large blood vessels on top.

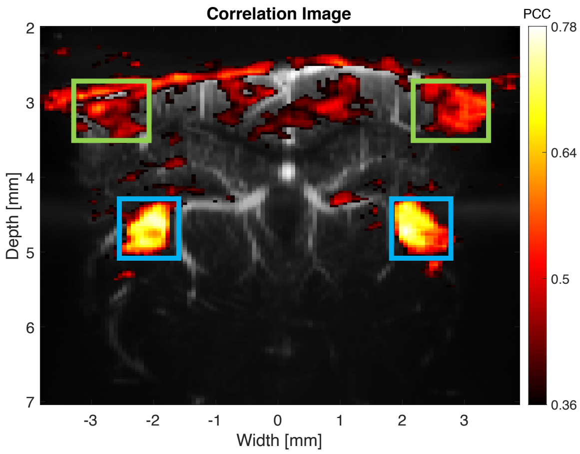

Figure 1: Correlation image. The blue and green colored windows show the LGN and visual cortex, respectively. In addition to these regions, the blood vessels on top of the image (in-between the left and right visual cortex) share a significant correlation with the stimulus.

1. To find the optimal amount of delay for calculating the correlation image, you should apply a lag on the stimulus at various values (from 0 to 10 seconds), and calculate the average absolute correlation value over the whole image for each lag. Then, you should determine the lag that maximizes this average, and display the correlation image at this lag. The code that can plot the correlation image has been provided to you (`display_brain_img.m`).

Block-Term Decomposition

The BTD can be considered as a generalization of the CPD where the extracted components can have higher ranks in each mode. In this assignment, we will be focusing on a particular case of BTD, known as $(L_r, L_r, 1)-$BTD, which decomposes a 3D input tensor into multilinear rank-$(L_r, L_r, 1)$ terms (Fig. 2). The $(L_r, L_r, 1)-$BTD achieves a more general low-rank structure compared to the CPD while preserving uniqueness under relatively mild conditions.

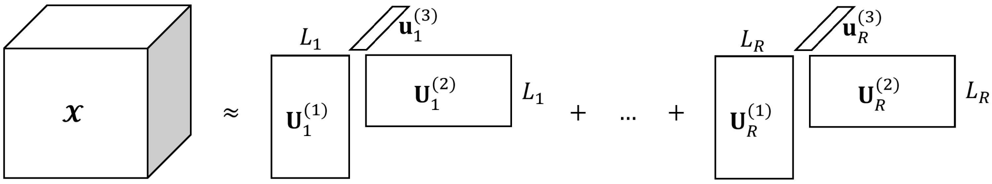

Figure 2: Decomposition into multilinear rank-$(L_r, L_r, 1)$ terms.

The $(L_r, L_r, 1)-$BTD can be expressed as:

$$\underline{\mathbf{X}} = \sum_{r=1}^{R} \left(\mathbf{U}_1^{(r)} \mathbf{U}_2^{(r)\,\mathrm{T}} \right) \circ \mathbf{u}_r^{(3)} \tag{1}$$

where $\circ$ indicates outer product and the matrices $\mathbf{U}_1^{(r)}$ and $\mathbf{U}_2^{(r)}$ have a full column rank of L_r. Note that the above equation can be viewed as a CPD by rewriting it as follows:

$$\underline{\mathbf{X}} = \sum_{r=1}^{R} \sum_{l=1}^{L_r} \mathbf{U}_1^{(r)}(:, l) \circ \mathbf{U}_2^{(r)}(:, l) \circ \mathbf{u}_r^{(3)}. \tag{2}$$

Thus, similar to CPD, the $(L_r, L_r, 1)-$BTD can be solved using alternating least squares.

1. Fill in the code that implements the $(L_r, L_r, 1)-$BTD in `btd_ll1_als_3d.m`.

   Hint: You can look in the literature for guidance in this task.

Problem formulation

1. Give a short problem definition that describes the problem we are solving and the methods we are investigating (you should not include the method math). You are encouraged to include specific research questions. Do not copy the assignment text for this.

Applying and comparing CPD and $(L_r, L_r, 1)-$BTD

Apply the CPD (you can use the assignment 2 solution `cpd_als_3d.p` provided to you) on the fUS data and answer the following questions:

1. Are you able to extract a component whose spatial map points to any of the expected areas?

   Hint 1: You should combine the factor matrices of the first two modes to construct a spatial map. Do not forget to use a colorbar!

   Hint 2: Keep in mind the sign ambiguity of CPD. In other words, you might see negative amplitude in the expected areas, but then that negativity will be reflected in one of the other modes.

2. If your answer to the previous question is yes, what information does the temporal signature of that component entail? Is the temporal signature significantly correlated (i.e., with a PCC value above 0.3) to the stimulus?

3. How did you determine the number of sources? Justify your choice.

- Are you still able to extract a meaningful component when you select it differently?
- What happens when you run the CPD with different initializations and a fixed number of sources?

Now apply the BTD and answer the following questions:

1. The first and second question as in the CPD case above.
2. While applying BTD, which mode of the fUS tensor did you select to be rank-1 and why?
3. Compared to the CPD, what kind of differences do you observe in the extracted component(s) of interest?
4. What do you think is the reason for your observations in the previous question?

   Hint: Think of the assumptions made by the CPD and the BTD.
5. How did you determine the number of sources and the factor matrices rank L?
   - Are you still able to extract a meaningful component when you select them differently?
   - What happens when you run the BTD with different initializations and a fixed number of sources?

Tips

- You can check [2] to get inspiration for answering the comment questions.
- Some functions from previous assignments that could be useful have been provided to you.

Rubric

You will be graded based on how well you follow the guidelines of the assignment, as well as your observations and critical thinking. In particular out of 20 points:

- Correlation image [1 point],
- $(L_r, L_r, 1)-$BTD implementation [5 points],
- Problem formulation [2 points]
- Applying CPD and answering the corresponding questions [6 points]
- Applying BTD and answering the corresponding questions [6 points]
- Bonus points can be earned by means of additional discussions or experiments.

References

[1] E. Macé, G. Montaldo, I. Cohen, M. Baulac, M. Fink, and M. Tanter, "Functional ultrasound imaging of the brain," *Nature Methods*, vol. 8, pp. 662–664, 2011.

[2] C. Chatzichristos, E. Kofidis, M. Morante, and S. Theodoridis, "Blind fMRI source unmixing via higher-order tensor decompositions," *Journal of Neuroscience Methods*, vol. 315, pp. 17–47, 2019.