

Streaming Factor Trajectory Learning for Temporal Tensor Decomposition

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(i) $u_{3,3}^3(t)$

(h) $u_{3,2}^3(t)$

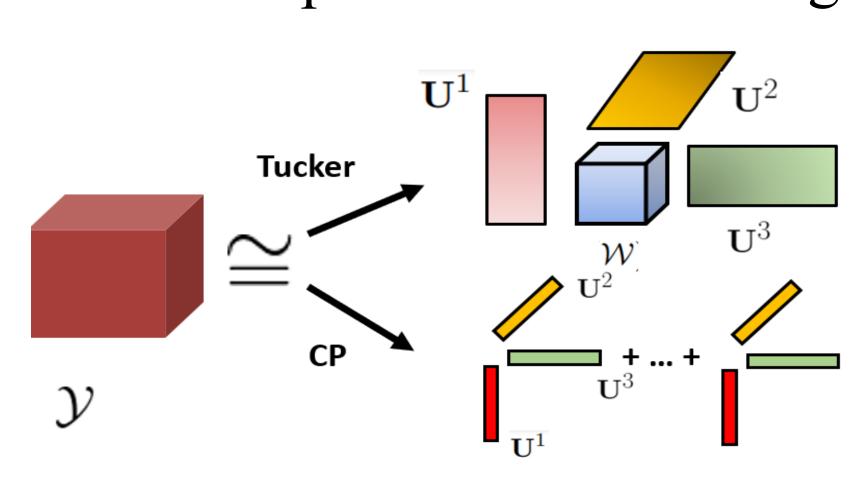
> Temporal Tensor Data

. Tensor-valued time series!



> Tensor Decomposition

. Low-rank representation learning!



> Most Existing work:

Evolving weights + Static Factors

$$y_{\mathbf{i}}(\mathbf{t}) \approx \mathbf{w}(\mathbf{t})^{\top} \left(\mathbf{u}_{i_1}^1 \circ \dots \circ \mathbf{u}_{i_K}^K \right)$$

- Over-Simplistic!
- Evolving factors dominate in many cases

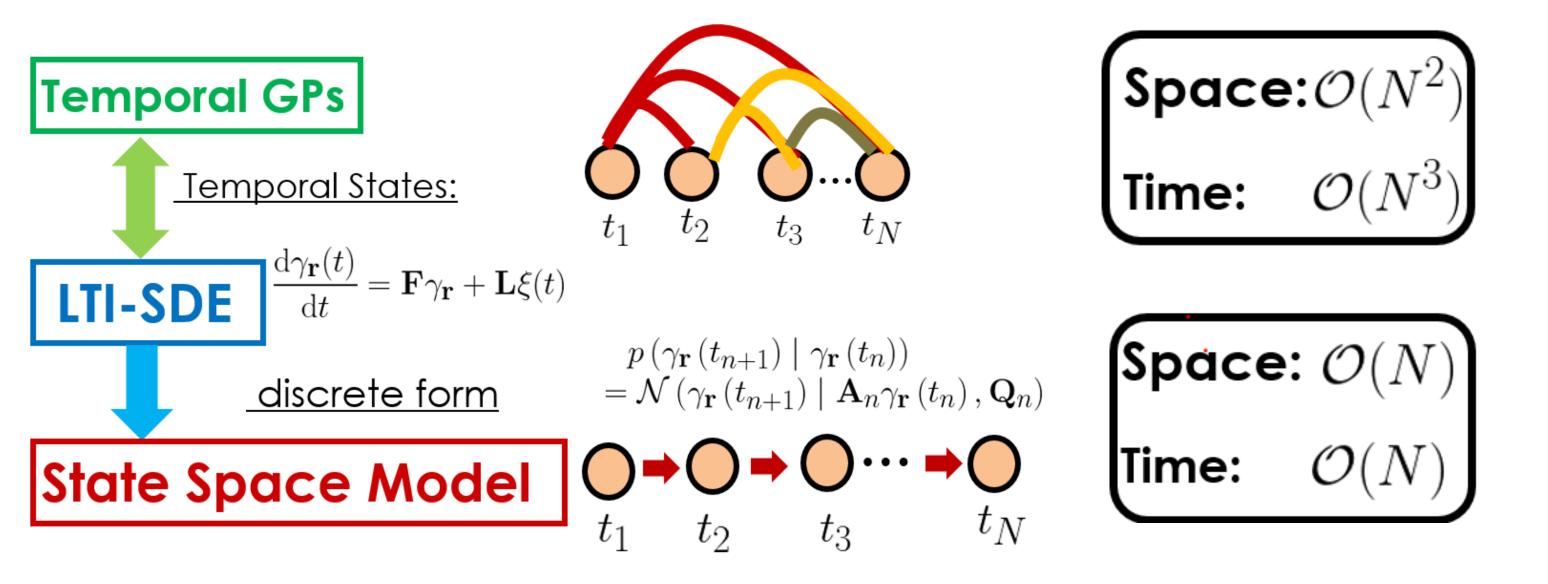
>SFTL: envolving factor trajectories!

CP: $y_{\mathbf{i}}(t) \approx \mathbf{w}^{\top} \left(\mathbf{u}_{i_{1}}^{1}(t) \circ \dots \circ \mathbf{u}_{i_{K}}^{K}(t) \right)^{\top}$ Tucker: $y_{\mathbf{i}}(t) \approx \operatorname{vec}(\mathcal{W})^{\top} \left(\mathbf{u}_{i_{1}}^{1}(t) \otimes \dots \otimes \mathbf{u}_{i_{K}}^{K}(t) \right)^{\top}$

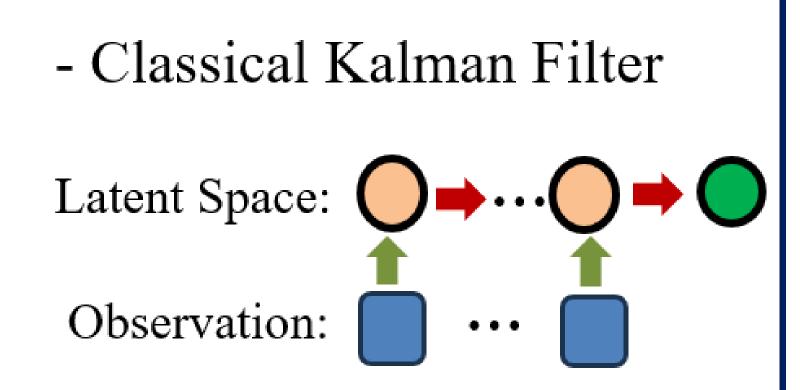
Gaussian Process(GP)

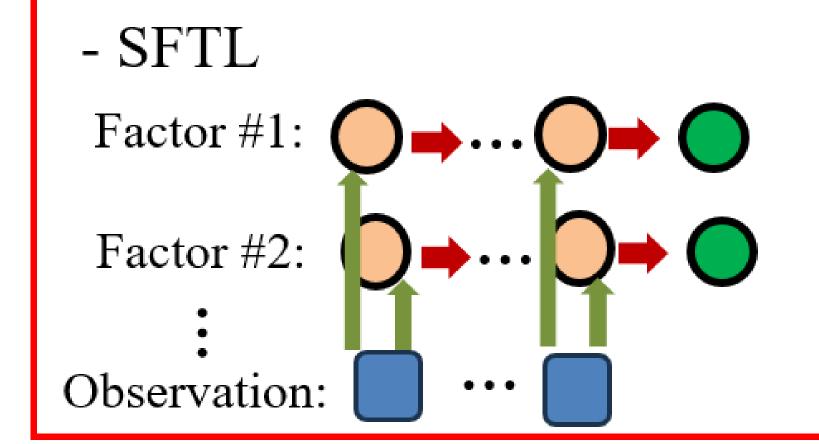
GP(0, k(t, t'))

- Non-parametric(flexiable) + Non-scalable: O(N^3)
- > State-Space GP: linear-cost + chain-structure

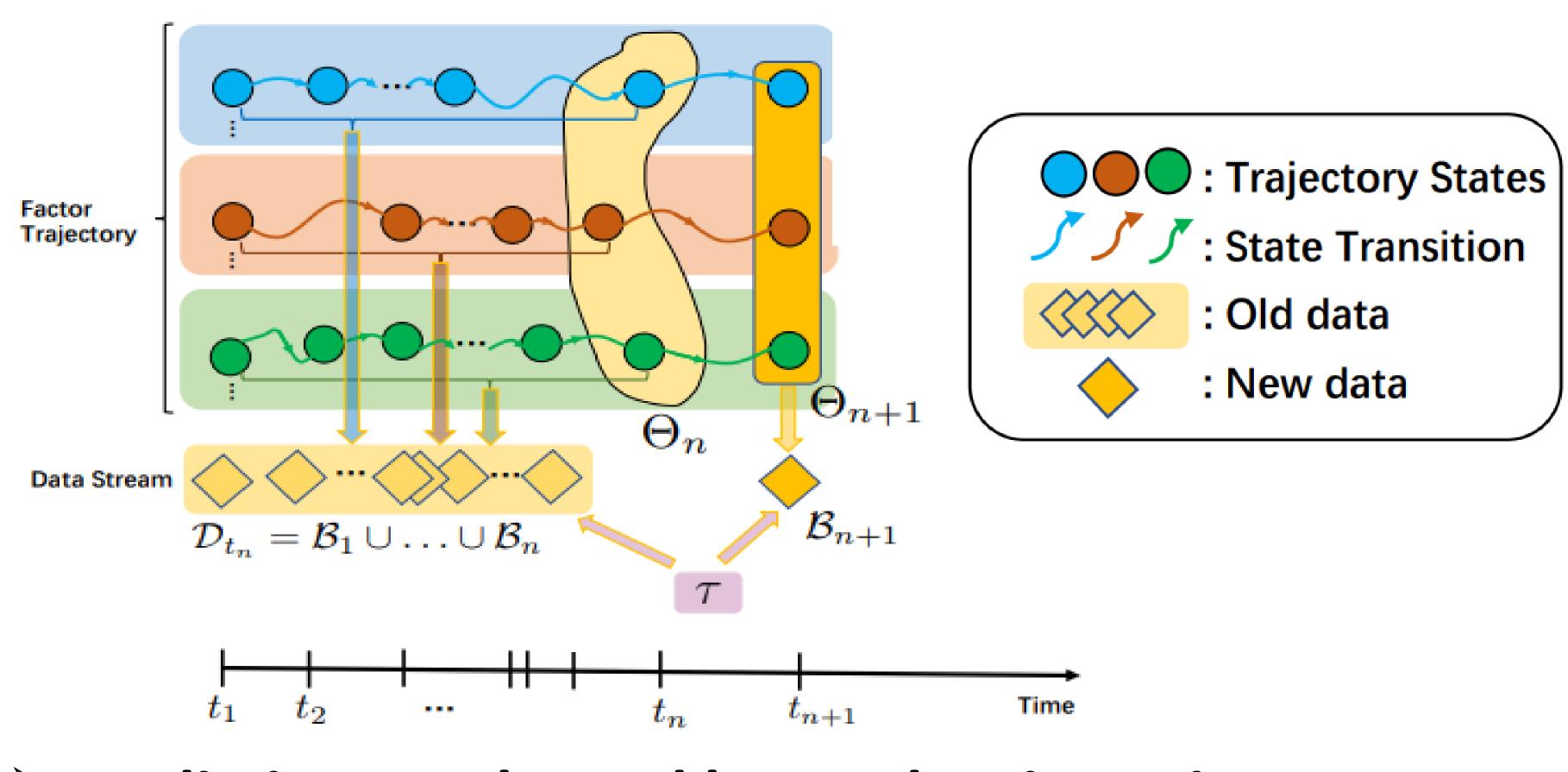


> Streaming Infer: Kalman Filter + Moment-Match





> Efficient online algorithem :



> Prediction results and learned trajectories

			0.5	0.0	0.5
	RMSE	FitRecord	0.0	-0.5	0.0
Static	PTucker	0.656 ± 0.147	0.5	-1.0	-0.5
	Tucker-ALS	0.846 ± 0.005	Time	Time	Time
	CP-ALS	0.882 ± 0.017	(a) $u_{1,1}^1(t)$	(b) $u_{1,2}^1(t)$	(c) $u_{1,3}^1(t)$
	CT-CP	0.664 ± 0.007	0.4	0.8	0.7
	CT-GP	0.604 ± 0.004		mm m	
	BCTT	0.518 ± 0.007	0.5	0.0	0.0
	NONFAT	0.503 ± 0.002		-0.5	-0.7
	THIS-ODE	0.526 ± 0.004	1.4 Time	Time	Time
Stream	POST	0.696 ± 0.019	(d) $u_{2,1}^2(t)$	(e) $u_{2,2}^2(t)$	(f) $u_{2,3}^2(t)$
	ADF-CP	0.648 ± 0.008		1	
	BASS-Tucker	0.976 ± 0.024	Madalan III.		0
	SFTL-CP	$\boldsymbol{0.424 \pm 0.014}$	- Commont	0	
	SFTL-Tucker	0.430 ± 0.010			-1
			Time	Time	Time

(g) $u_{3,1}^3(t)$