

# CMM HW5

Xuanhao Lin

December 3 2023

## 1 The Ising Model

### 1.1 Part 1

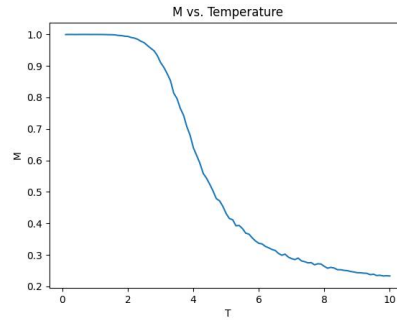


Figure 1: M vs. Temperature

This part requires to plot magnetization  $M$  vs. Temperature  $T$ . For simplification, the Boltzmann constant  $k_B$  is set to be 1. Given that  $J = 1.5$ , the analytical solution of the critical temperature of the Ising model should be  $T_c = 3.402$ . To equilibrate a system with a size of  $n \times n$ ,  $10000n^2$  Monte Carlo sweeps are needed (very heavy computation). Figure 1 is the plot generated by Metropolis-Hastings algorithm. We can find that the  $T_c$  of the system approximately equals to 3.402.

### 1.2 Part 2

Theoretically, the maximum heat capacity  $C_{max}$  will appear at  $T_c$ . Due to the heavy computation of the large system (e.g.  $n = 100$ ), I only equilibrate the system at theoretical  $T_c = 3.402$  to find  $C_{max}$ . I haven't get the answer of the system with size of 200 and 500, since these sizes broke my PC system. The result is shown as Figure 2.

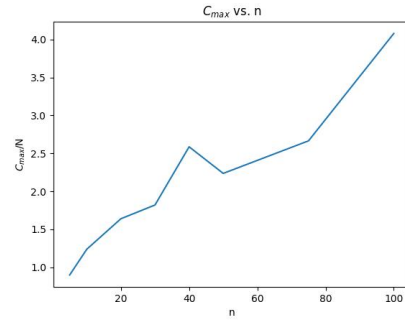


Figure 2:  $C_{max}/N$  vs.  $n$

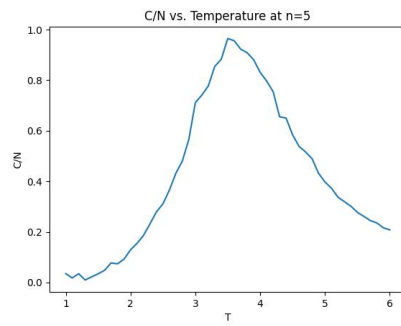


Figure 3:  $C/N$  vs. Temperature at  $n=5$

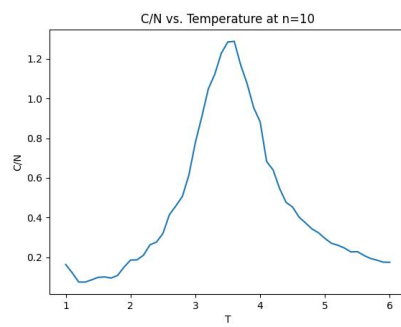


Figure 4:  $C/N$  vs. Temperature at  $n=10$

However, the results look not convincing. Additionally, there are some result of  $C/N$  vs.  $T$  for systems with small sizes (Figure 3, Figure 4).