CMM HW3

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1 Oscillatory Motion and Chaos

1.1 Part 1

When the Resonance occurs, the driving frequency Ω_D should approximately equal to the natural frequency ω_0:

$$\Omega_D = \omega_0 = \sqrt{\frac{g}{l}} = 1sec^{-1} \tag{1}$$

1.1.1 Part 2

The numerical results are shown in Figure 1 and Figure 2. By using obtained numerical results, we can use the steady sate of $\theta(t)$ to extract $\theta_0(t)$ and $\phi(t)$ (Figure 3). From these two result, we can find that the amplitude are largest and there is jump in value of phase shift when resonance occurs. The numerical result of FWMH is 0.799 which much larger than 2γ. This error comes from the lack of iteration of the driving frequency Ω_D.

1.2 Part 3

The driving frequency Ω_D is chosen to be 1 which is close to the resonance. The result is shown as Figure 4.

1.3 Part 4

The results are shown as Figure 5, Figure 6, Figure 7, and Figure 8.

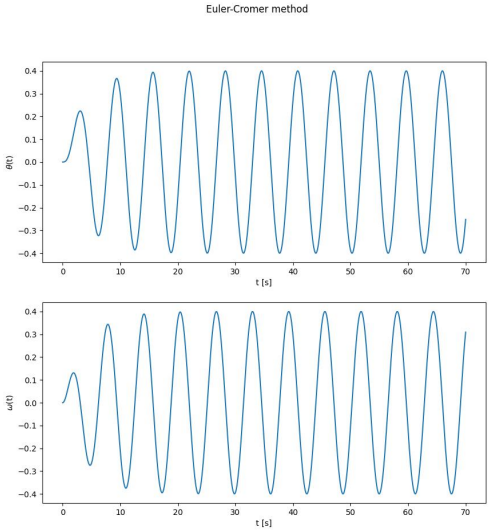

Figure 1: Euler-Cromer method

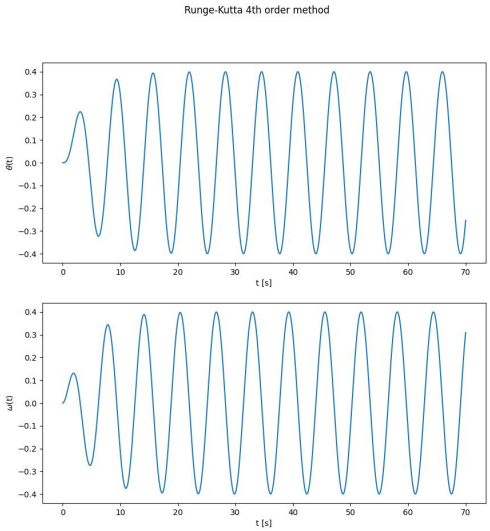

Figure 2: Runge-Kutta method

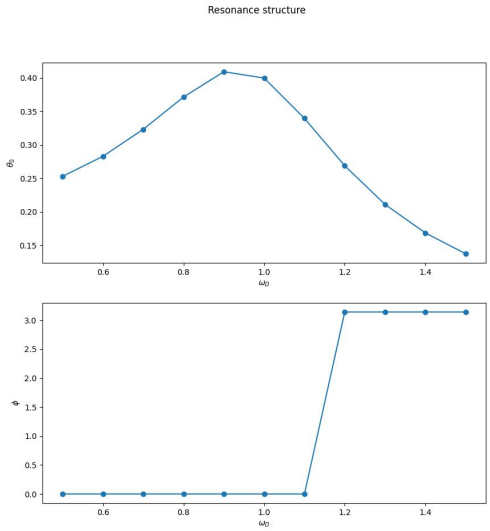

Figure 3: Resonance structure

1.4 Part 5

The analytical result is shown as Figure 9. We can see that when changing the initial conditions, the pendulum of third case becomes unpredictable. The Lyapunov exponents λ are -0.2503, -0.2499, 0.1076 corresponding to α_D equal to 0.2, 0.5, and 1.2. This makes sense: when $\lambda < 0$ the driven force effects weakly on pendulum, therefore the motion can quickly go back on its periodicity; however, when $\lambda > 0$, the motion tends to developing in a unpredictable way.

2 Poisson Equation for Dipole

2.1 Part 1

Using Jacobi relaxation, Figure 10 can be obtained. By ploting Vr in one direction, we can get Figure 11. Theoretically, V is proportional to r^2. The figure does not fit to the theory due to unknown error.

2.2 Part 2

Surprisingly, even decreasing the tolerance in a logarithmic scale. The iterationgs times are same for all tolerances (Figure 12).

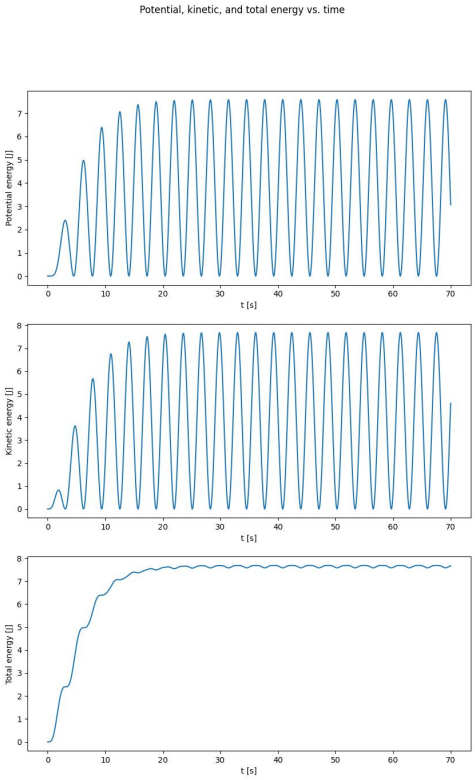

Figure 4: Potential, kinetic, and total energy vs. time

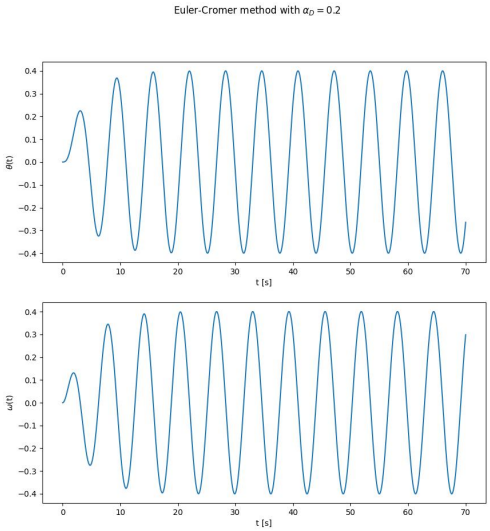

Figure 5: Euler-Cromer method with $\alpha_D = 0.2$

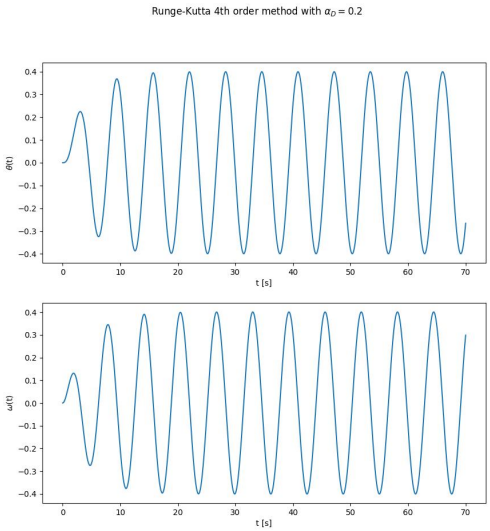

Figure 6: Runge-Kutta method with $\alpha_D = 0.2$

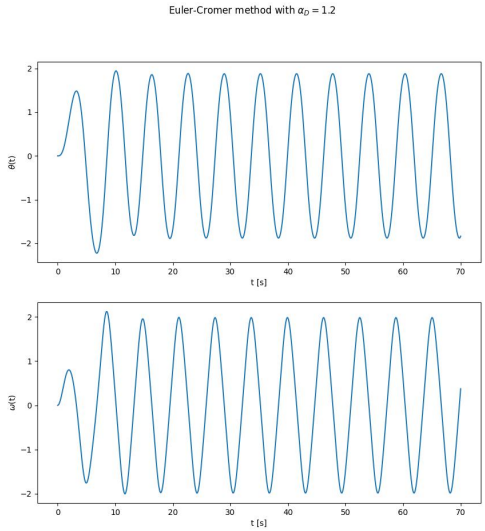

Figure 7: Euler-Cromer method with $\alpha_D = 1.2$

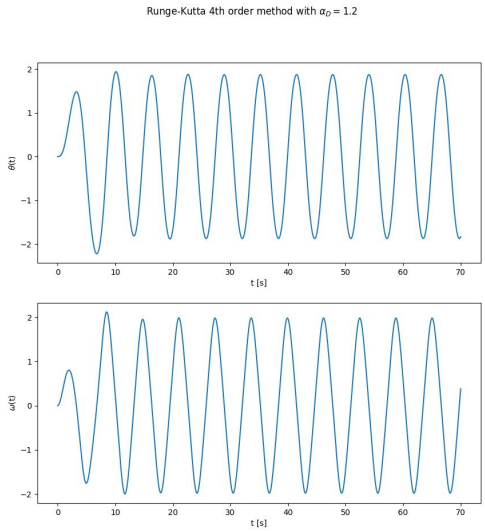

Figure 8: Runge-Kutta method with $\alpha_D = 1.2$

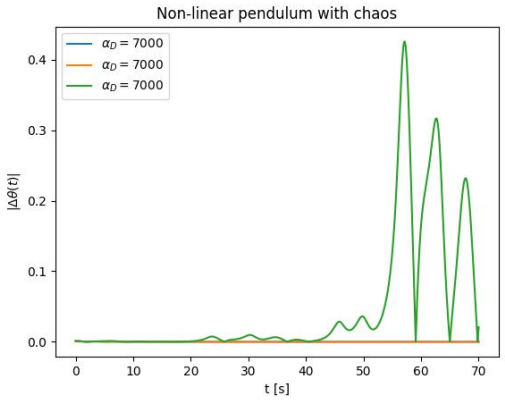

Figure 9: Non-linear pendulum with chaos

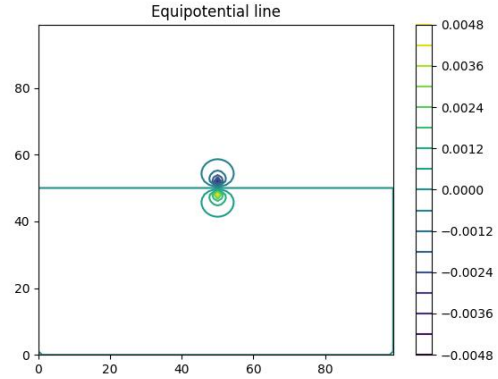

Figure 10: Enter Caption

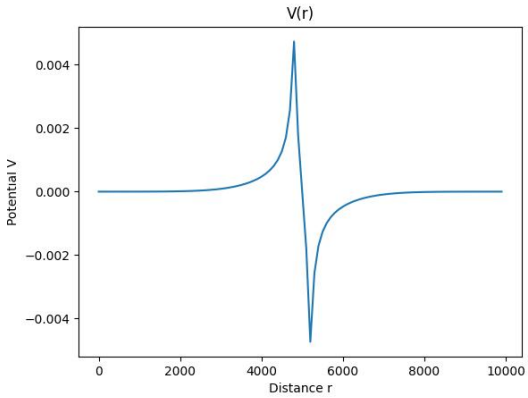

Figure 11: Enter Caption

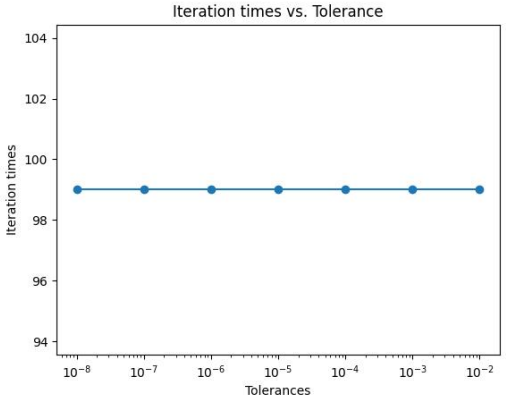

Figure 12: Enter Caption

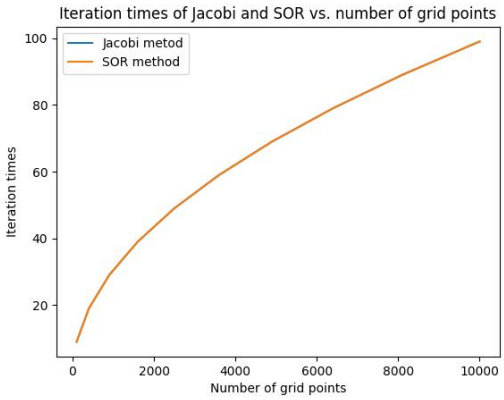

Figure 13: Enter Caption

2.3 Part 3

Unfortunately, I cannot solve this problem due to unexpectable error (Figure 13).