Math 61 Fall 2022

Homework 2

Instructor: Colleen Robichaux *Due by 10/12 at 5PM through GradeScope*

Exercises (from textbook and additional exercises (A.E.) below) Fully justify each answer:

- 3.3 - 28, 29, 31, 32, A.E.1
- 3.4 - 6, 8, 12, 13, 15, 16, 22, A.E.2
- 3.5 - 9
- 6.1 - 5, 9*, 94, 96, 97, A.E.3 (*Assume a car license plate has 3 letters then 3 numbers.)

Additional Exercise 1. Let $X = \{a, b, c, d\}$ and consider the relation

$$R = \{(a, a), (a, c), (a, d), (b, d), (c, a), (d, b)\}$$

on X. Draw the digraph associated to R.

Additional Exercise 2. For the following relations R on $\mathbb{Z}$, state whether or not each is reflexive, symmetric, transitive, and/or an equivalence relation. Fully justify each answer.

If R is an equivalence relation, also write the equivalence classes $[x]$ for arbitrary $x \in \mathbb{Z}$ as sets. *(For example, if R is the equivalence relation on $\mathbb{Z}$ such that xRy when $x \mod 3 = y \mod 3$ for $x, y \in \mathbb{Z}$, then $[x] = \{y \in \mathbb{Z} \mid x - y = 0 \mod 3\}$ for $x \in \mathbb{Z}$.)*

For the following, for $x, y \in \mathbb{Z}$, xRy if and only if:

(a) $(x + y)^2 \equiv \pm 1 \mod 3$

(b) $(x - y)^2 \equiv 0 \mod 4$

(c) $2x - y \equiv 0 \mod 7$

(d) $x^2 + 5x + 6 = y^2 + 5y + 6$

(e) $x^2 + y^2 = 1$

(f) $x^2 - y^2 = 1$

Additional Exercise 3. How many 8-bit strings are there such that contain the following? Justify your answer.

(a) exactly two 0's?

(b) at least one 0?