The gonality conjecture is true for linear ear decompositions

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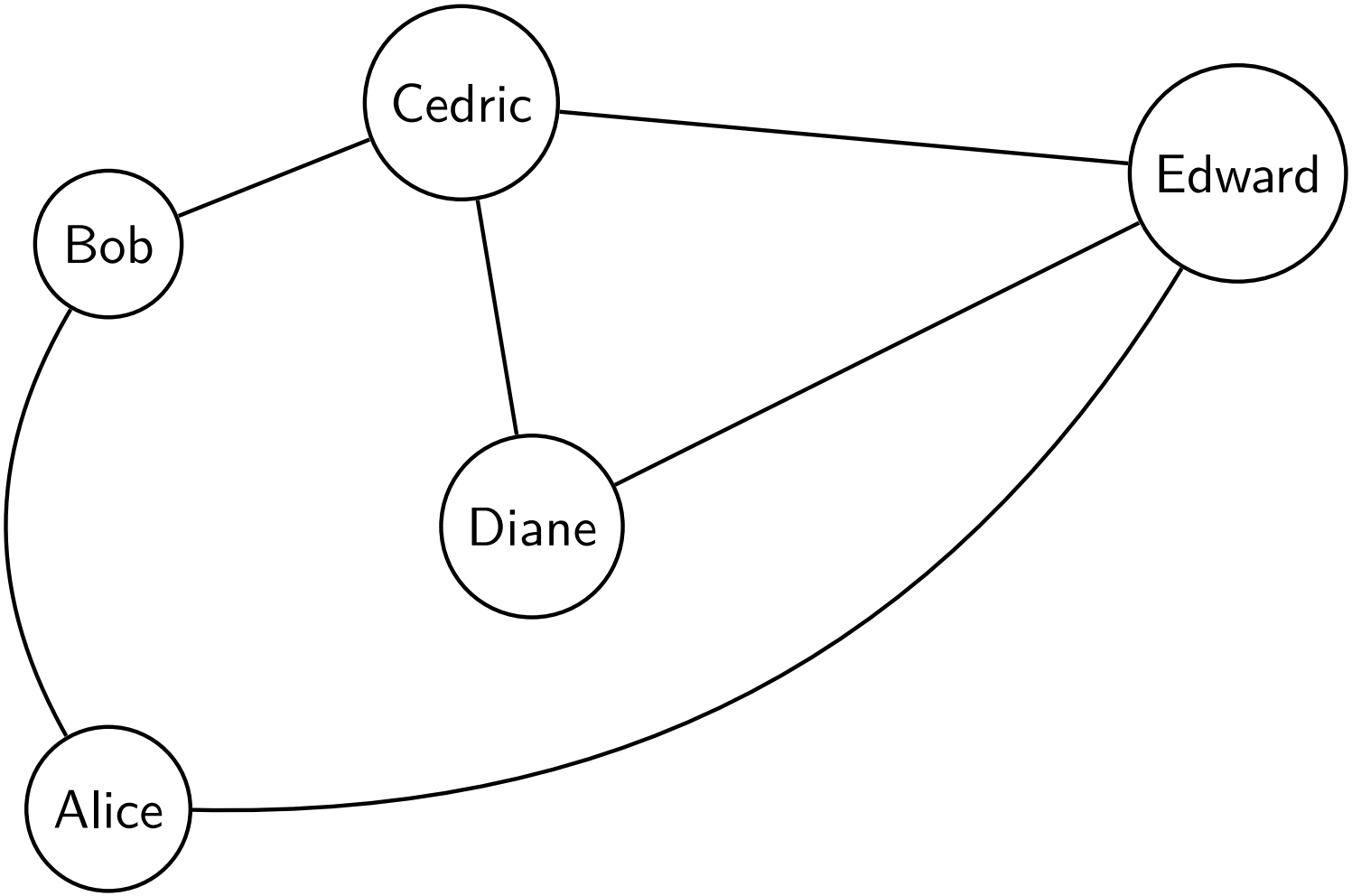
Outline

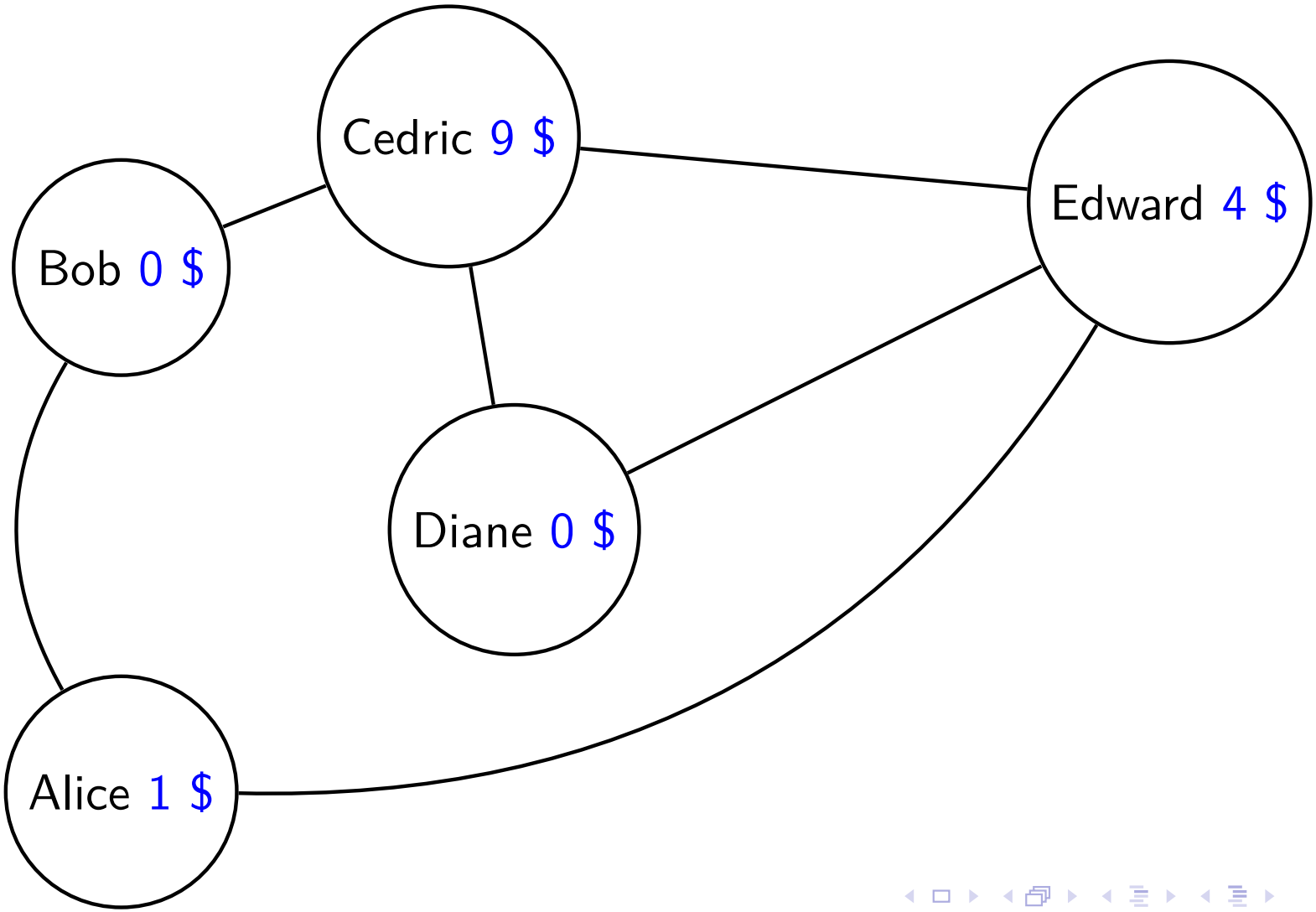
The chip firing game

A simple, unoriented graph:



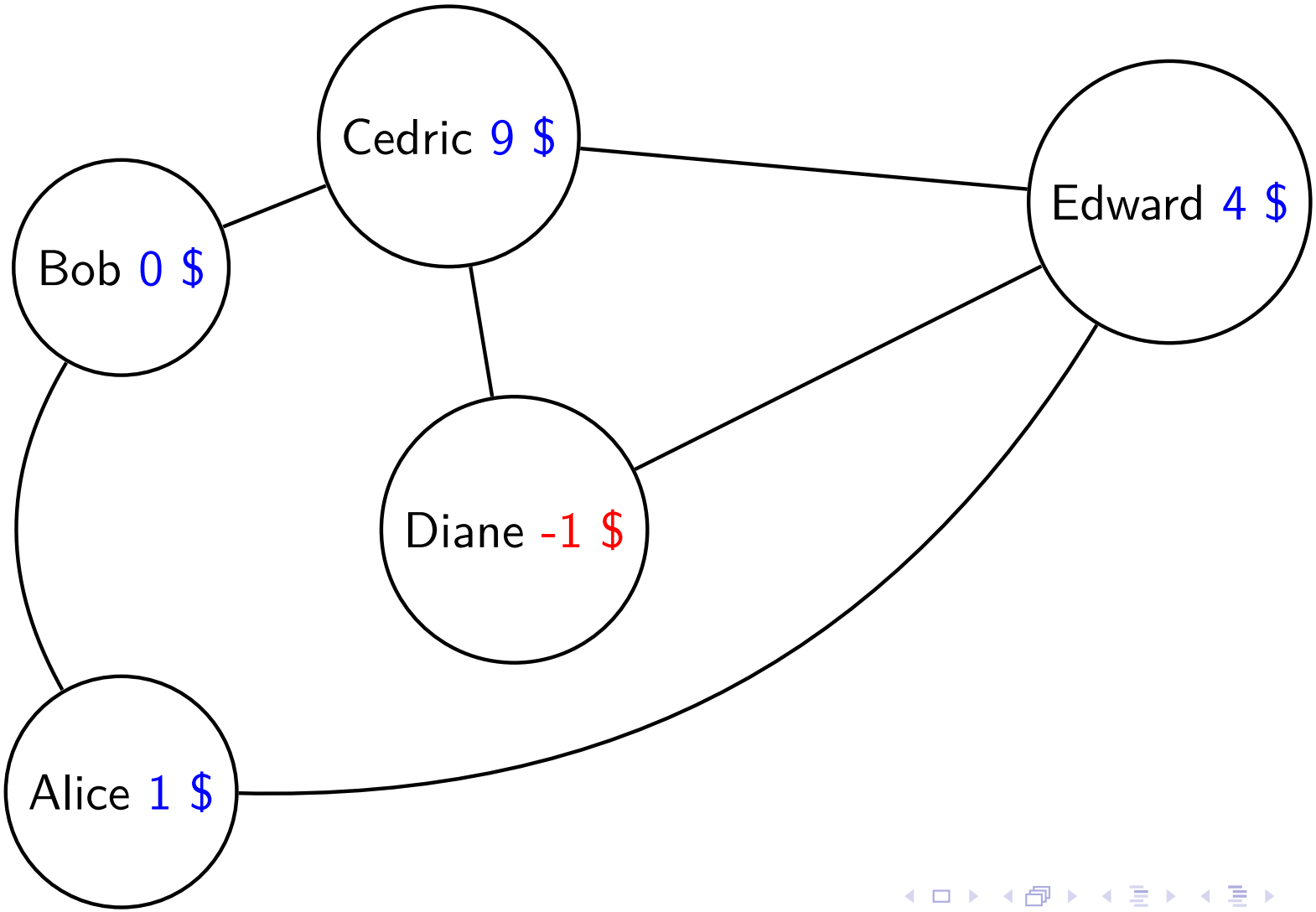
The chip firing game

The starting configuration:



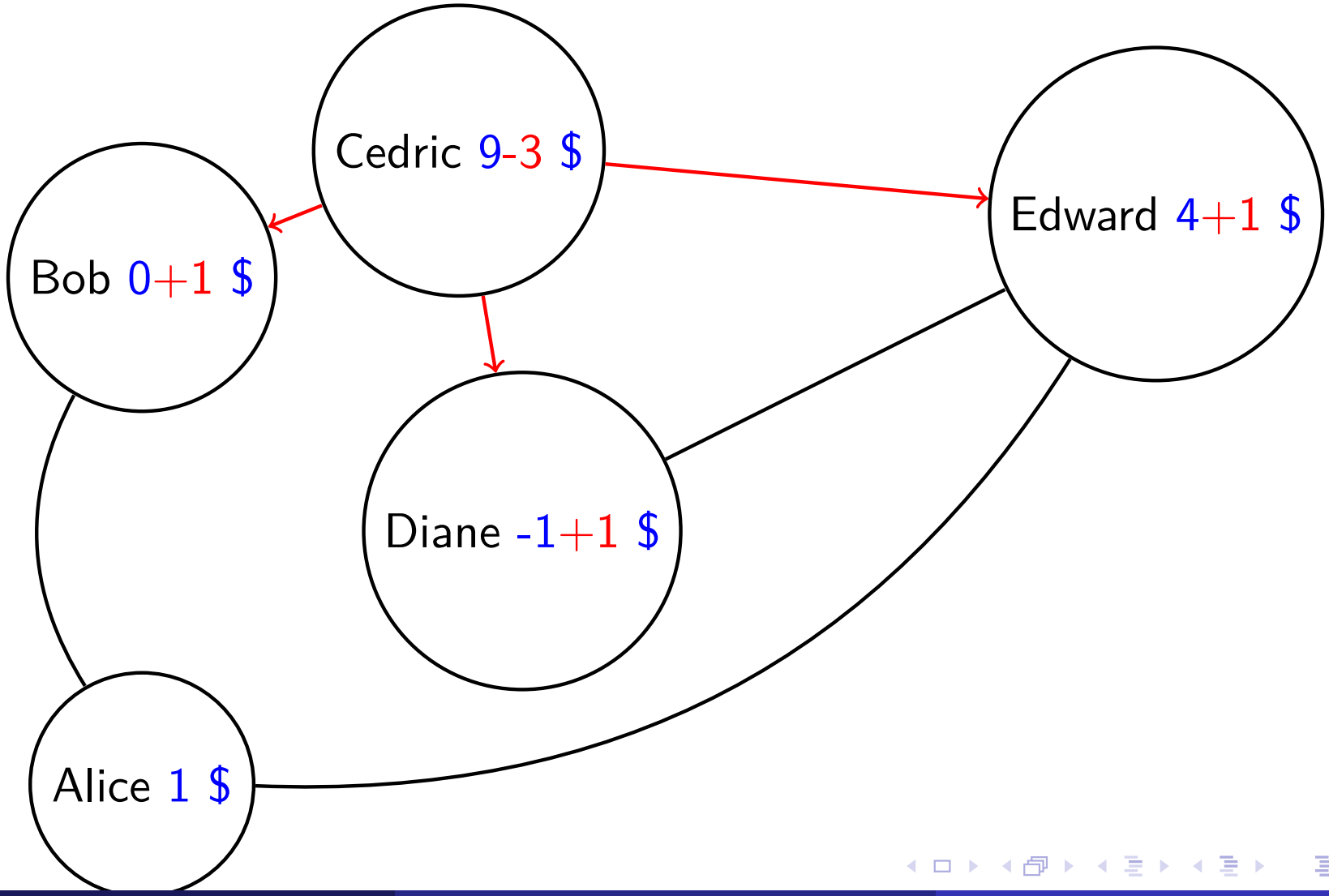
The chip firing game

A debt:



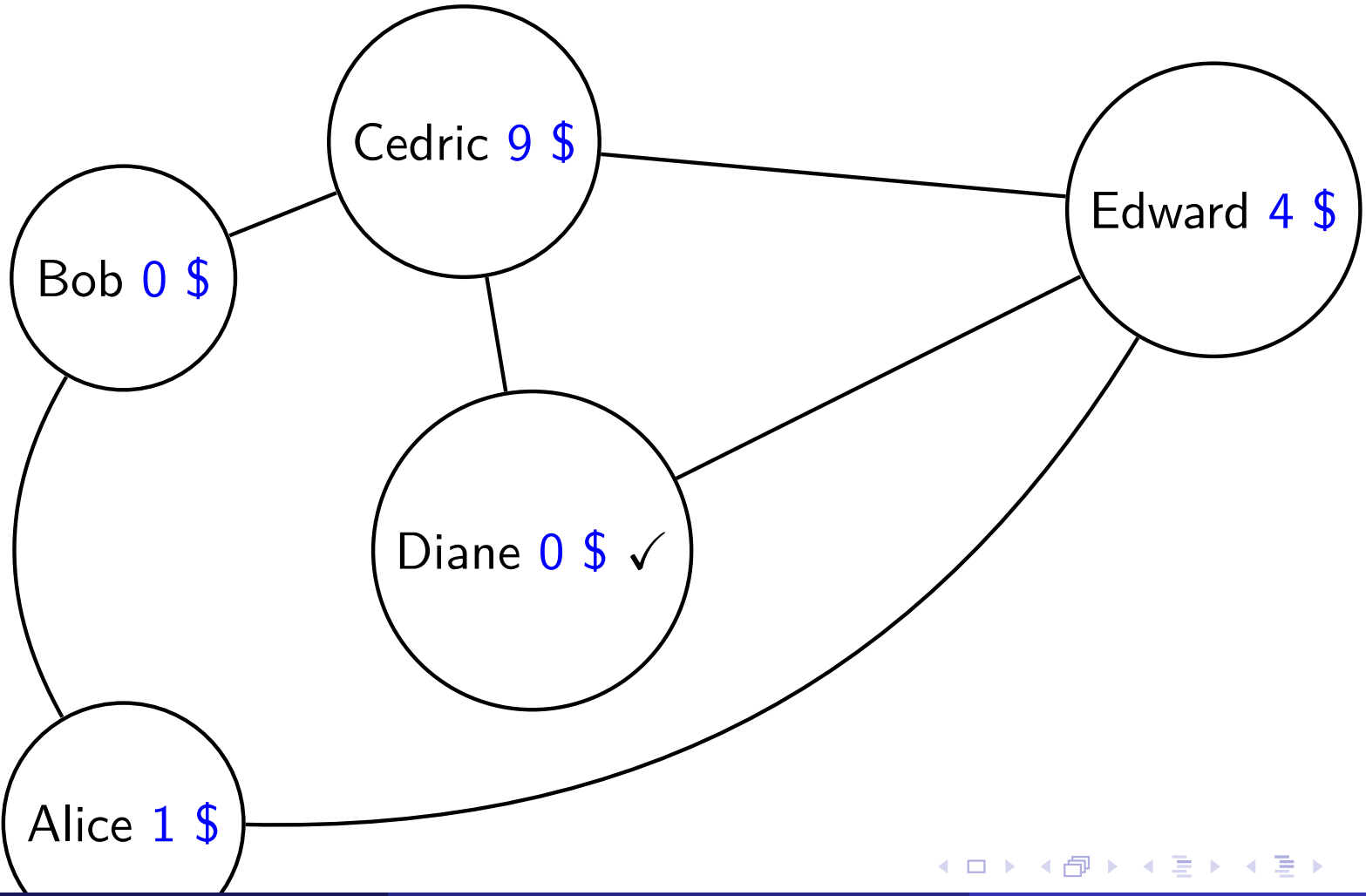
The chip firing game

A toppling:



The chip firing game

Given a configuration, if any one person loses a dollar, can we still reach a winning configuration through topplings ?



The chip firing game

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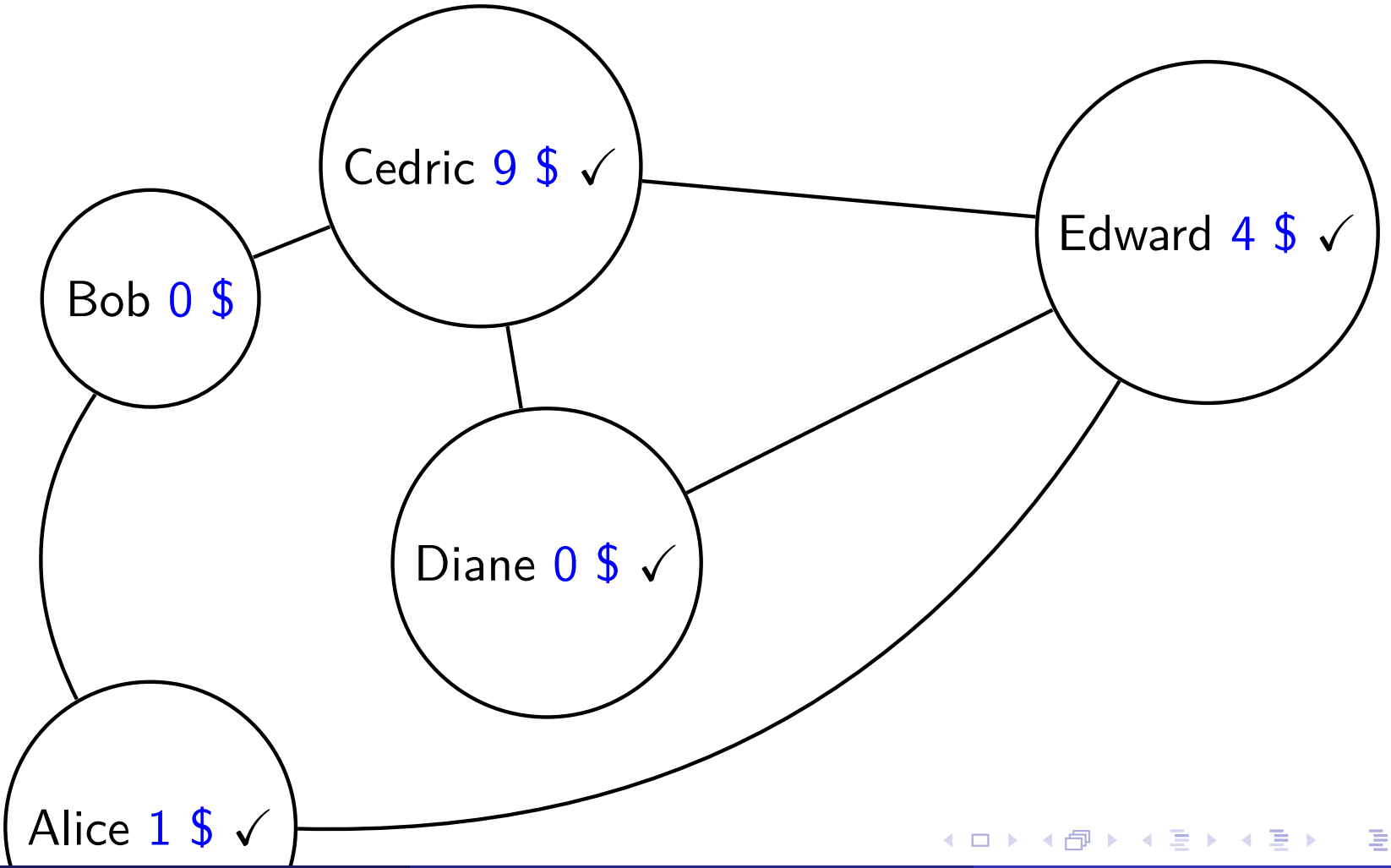
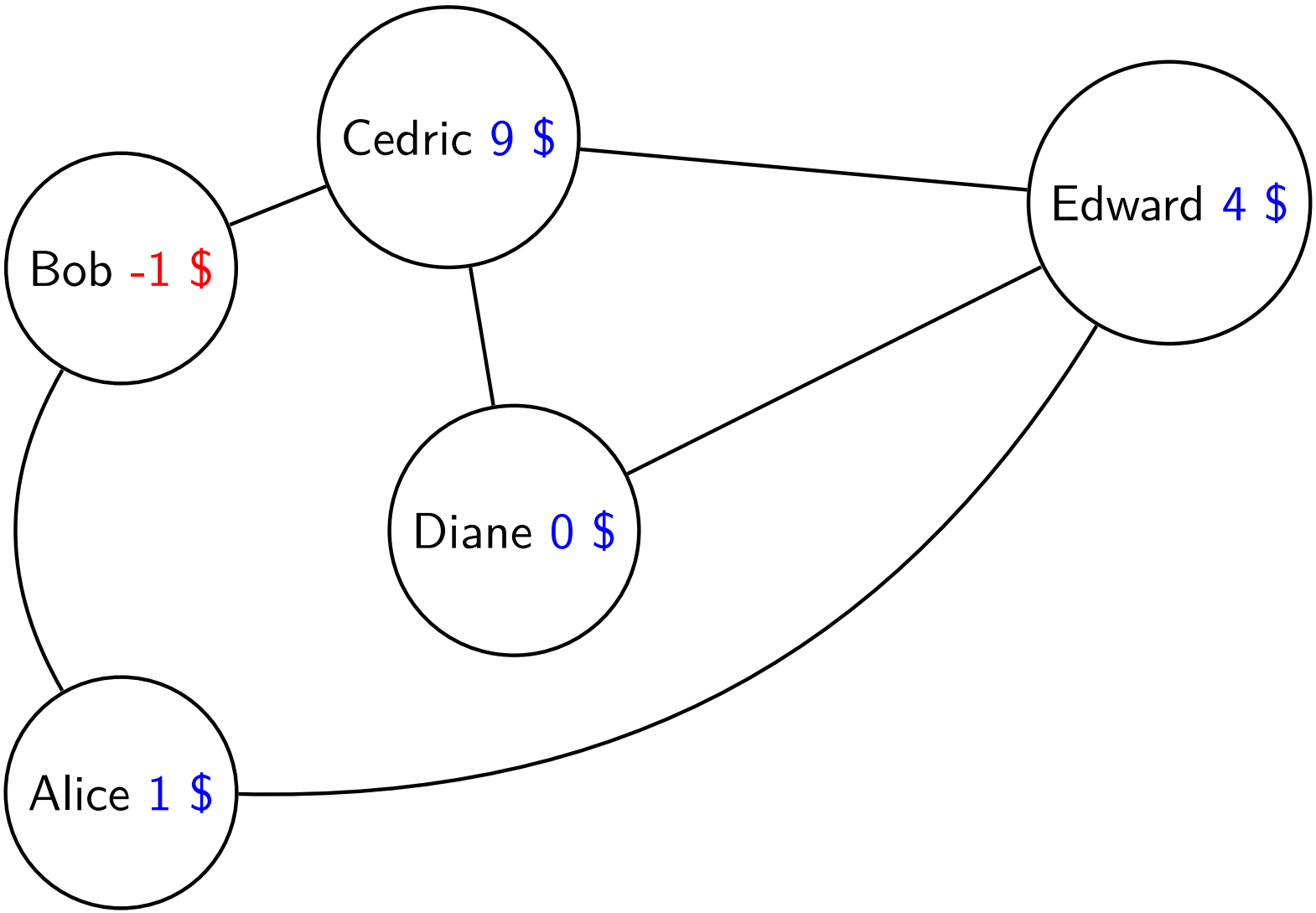
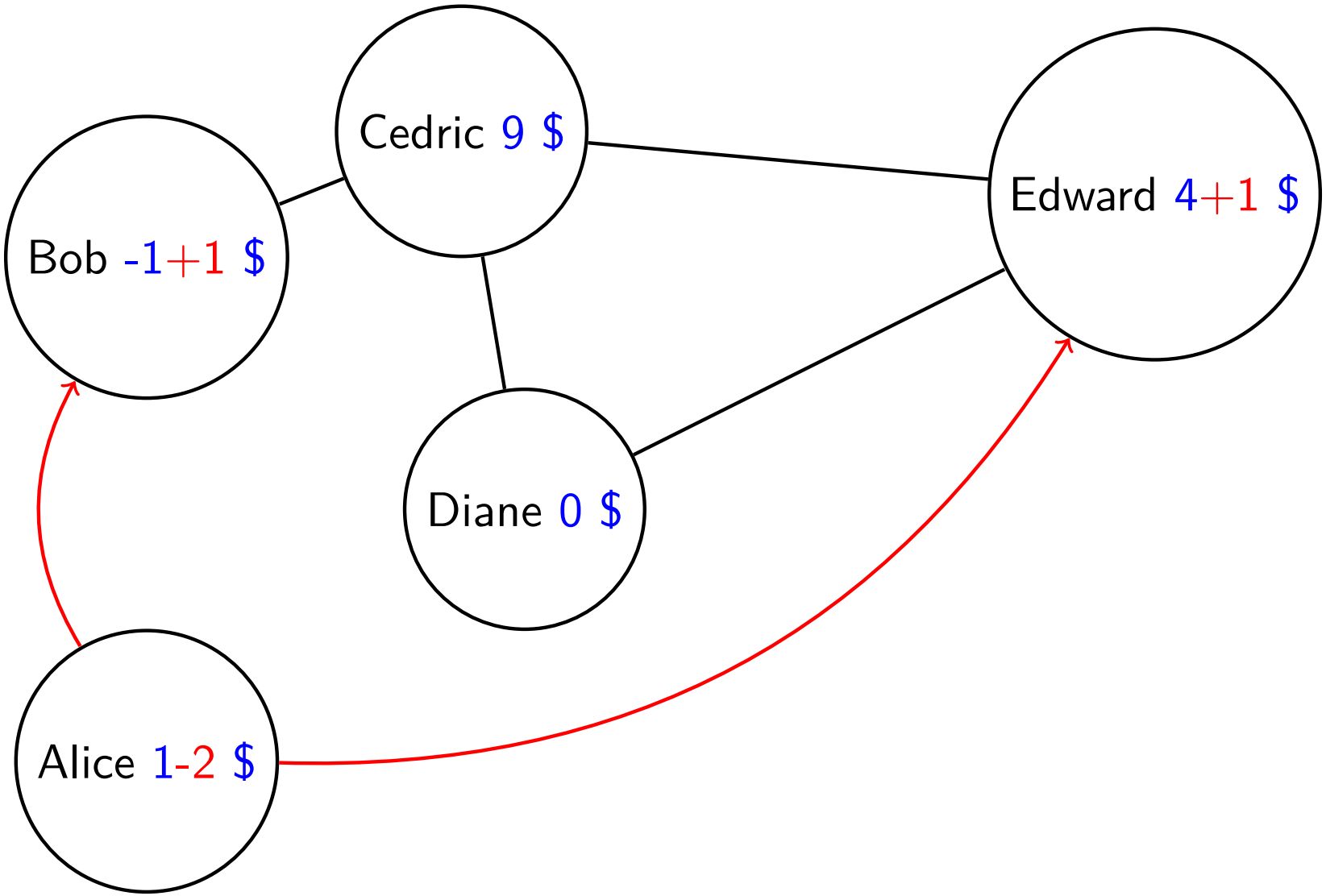


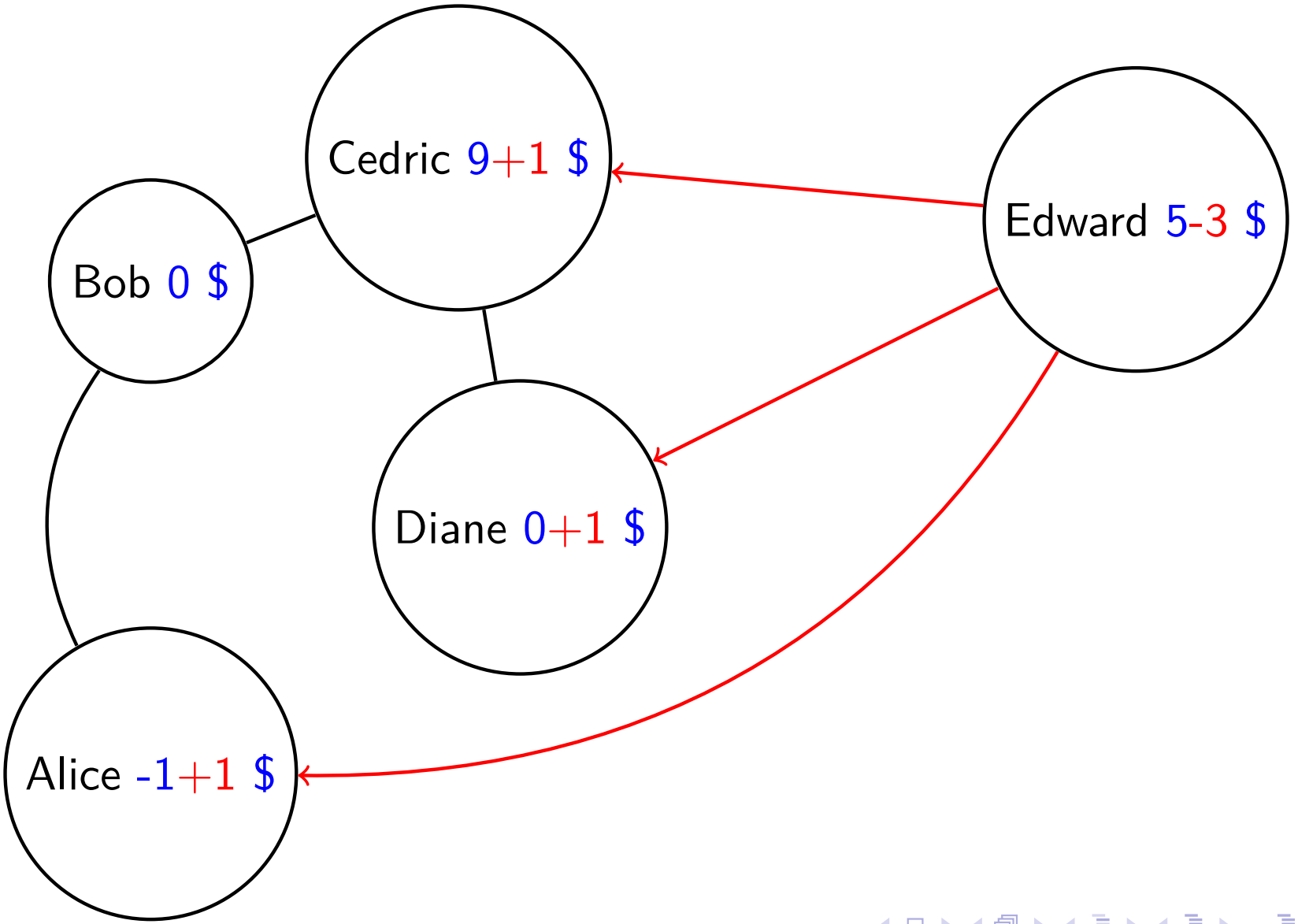
The chip firing game



The chip firing game

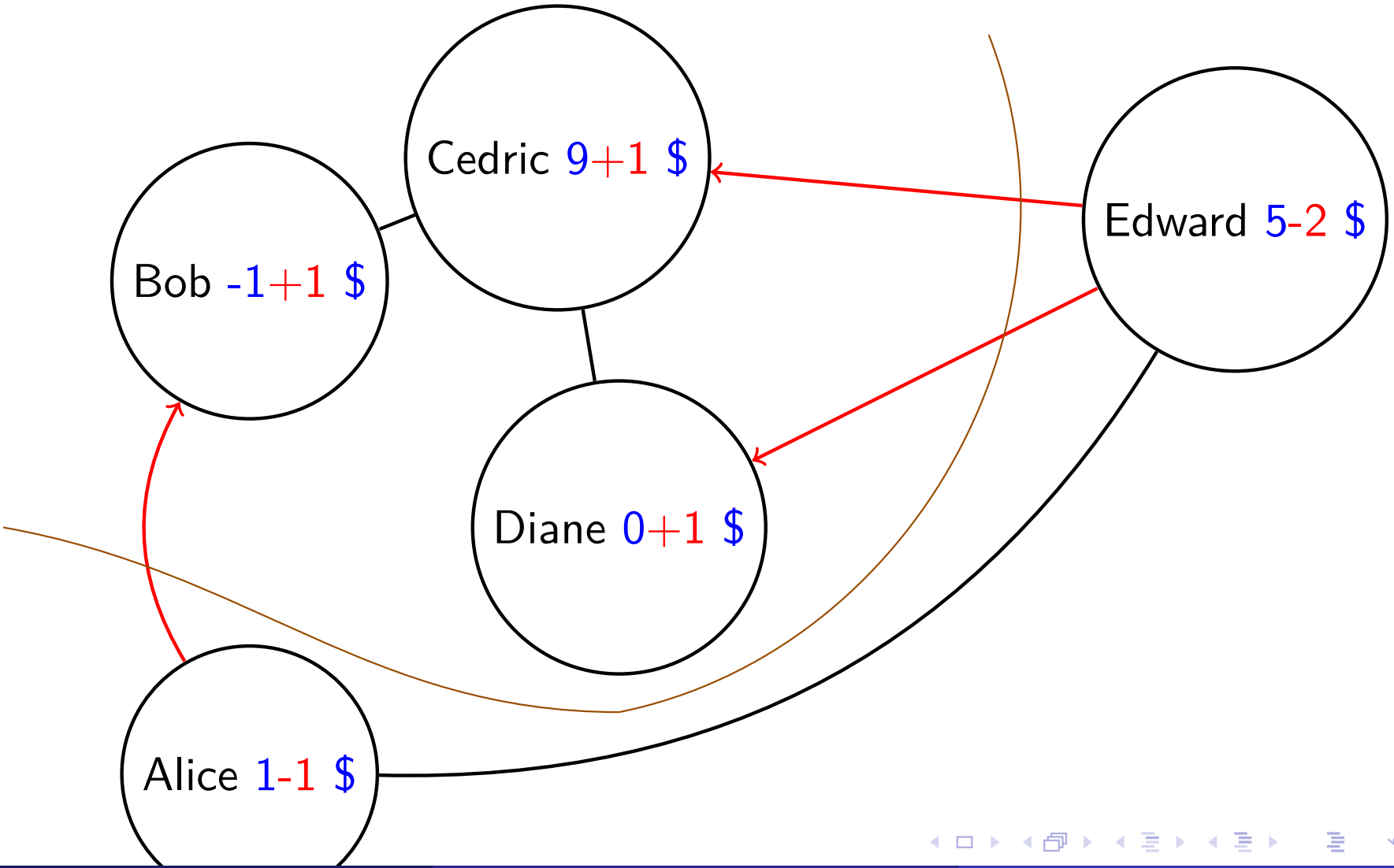


The chip firing game



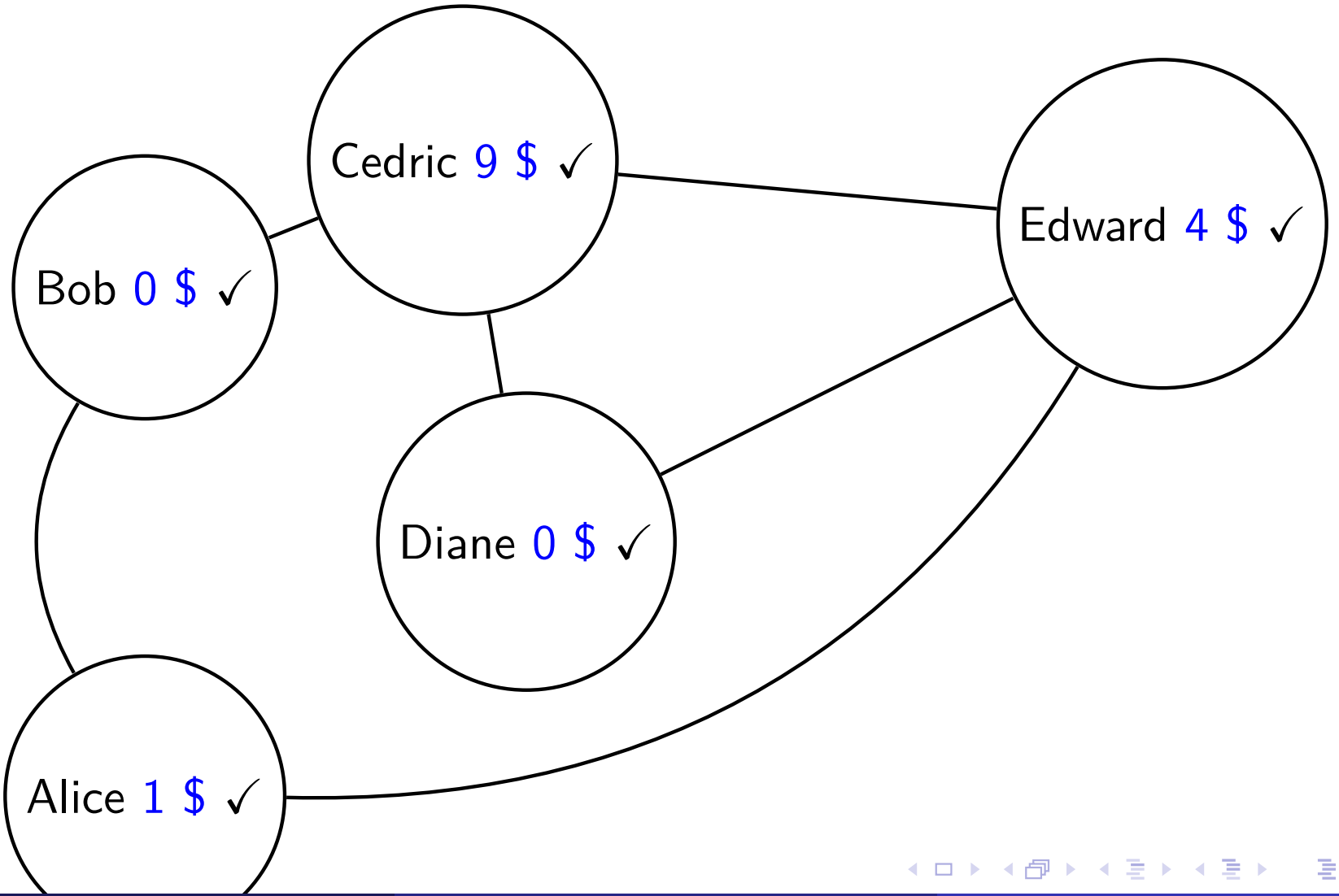
The chip firing game

Toppling a set:



The chip firing game

A winnable starting configuration:



The gonality conjecture

The game

A graph: $G = (V, E)$
A configuration: D
The question: If any one person loses a dollar, can we still win?

The gonality conjecture

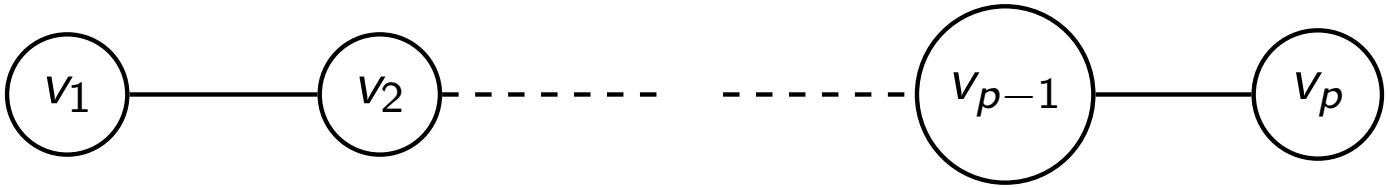
Given $\lfloor \frac{g+3}{2} \rfloor$ chips, there always exists a winnable starting configuration.

g is G's genus:

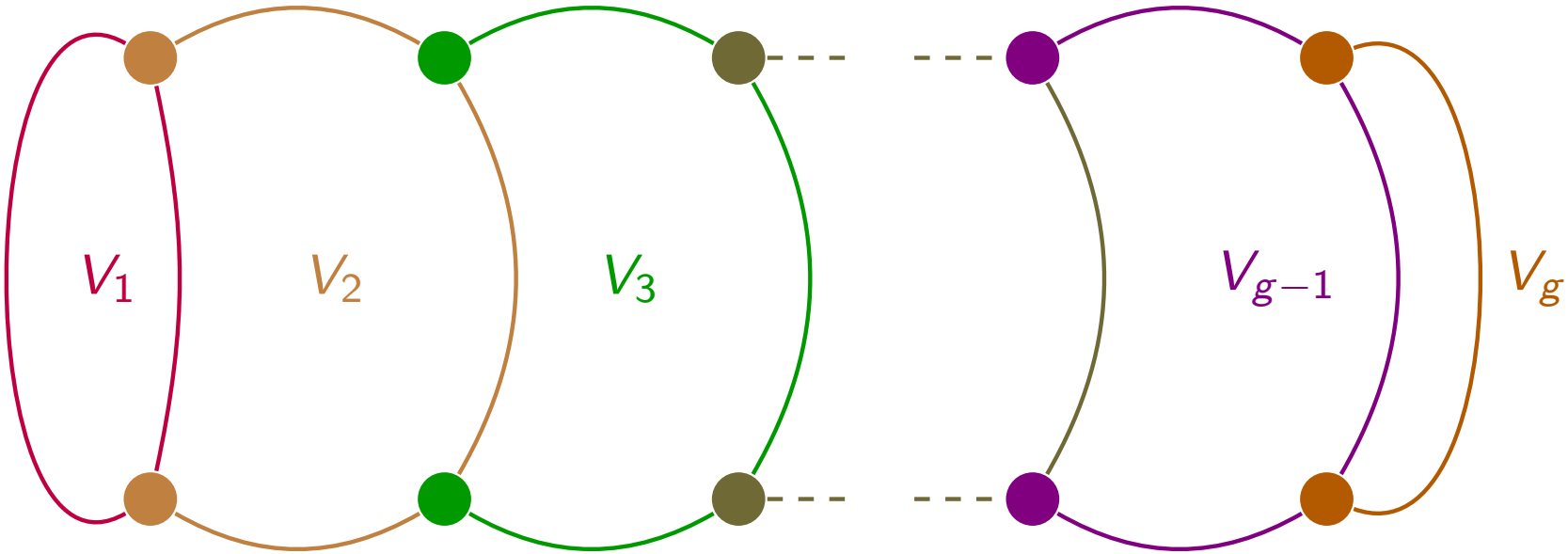
$$g = |E| - |V| + 1$$

The linear ear decomposition

An ear:

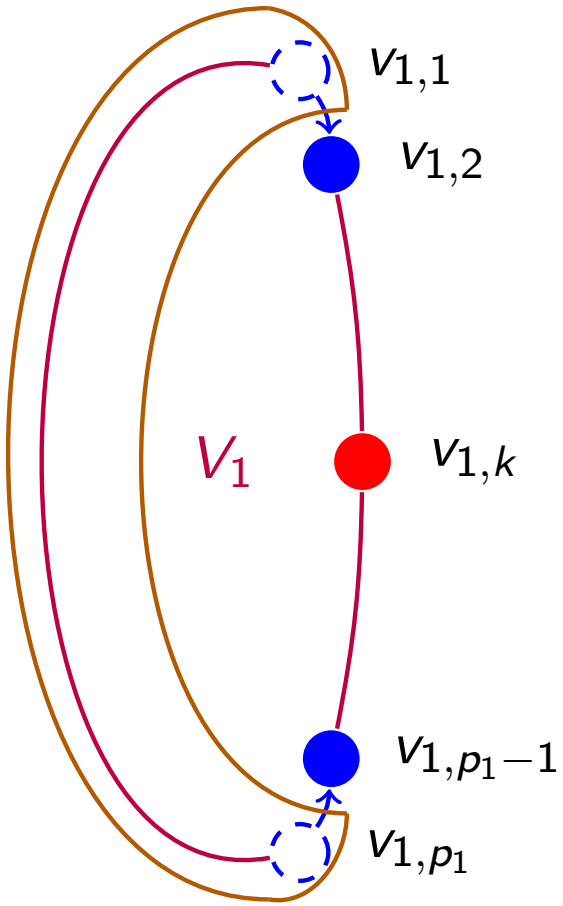


A linear ear decomposition:



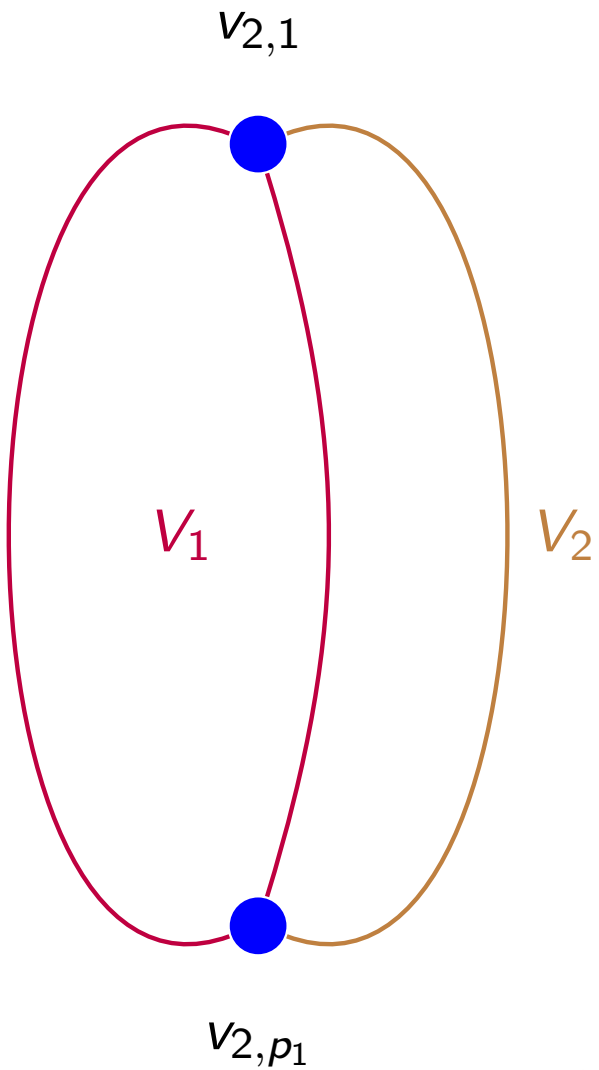
Proof

Induction on g:
$g = 1$: 2 chips



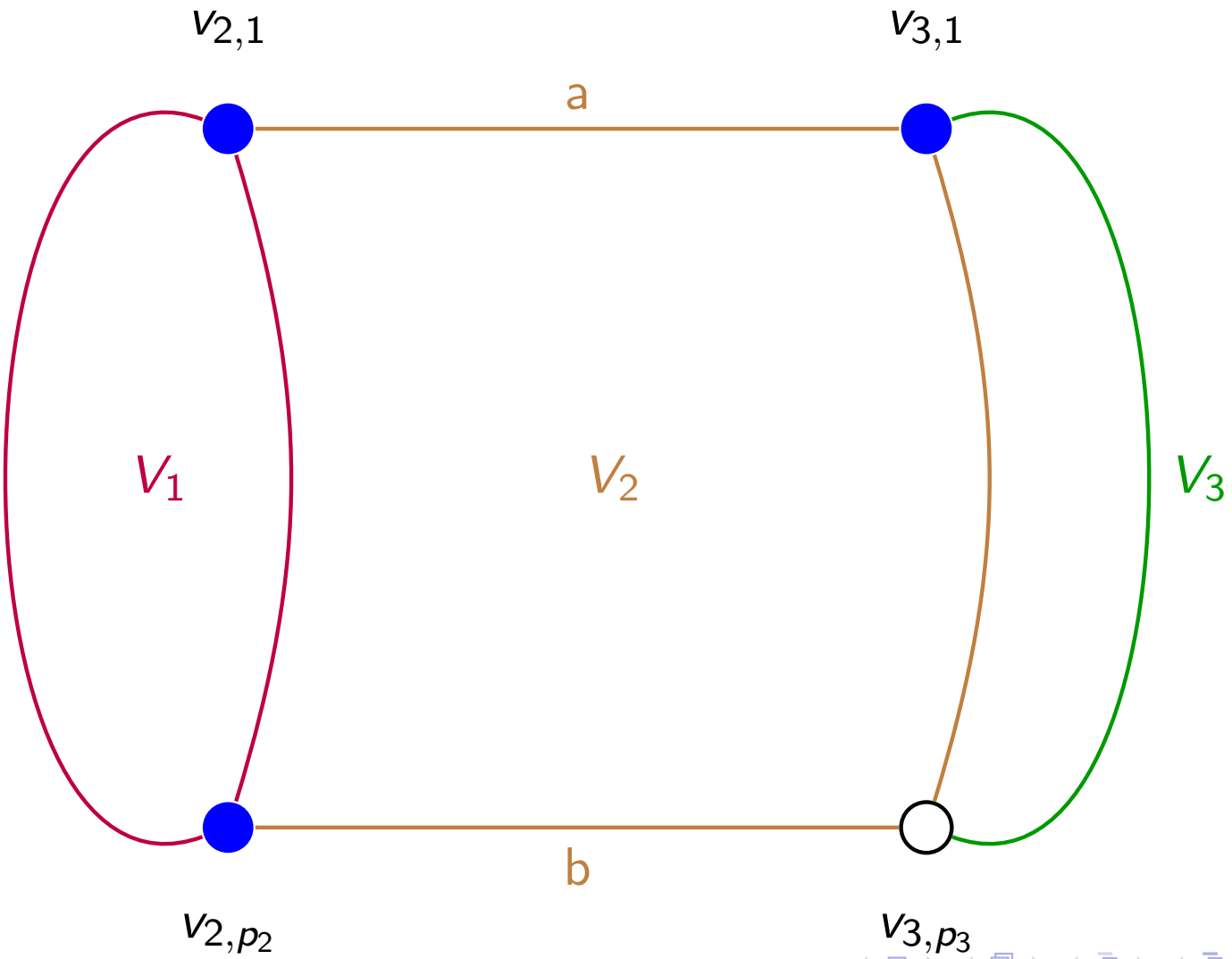
Proof

$g = 2$: 2 chips

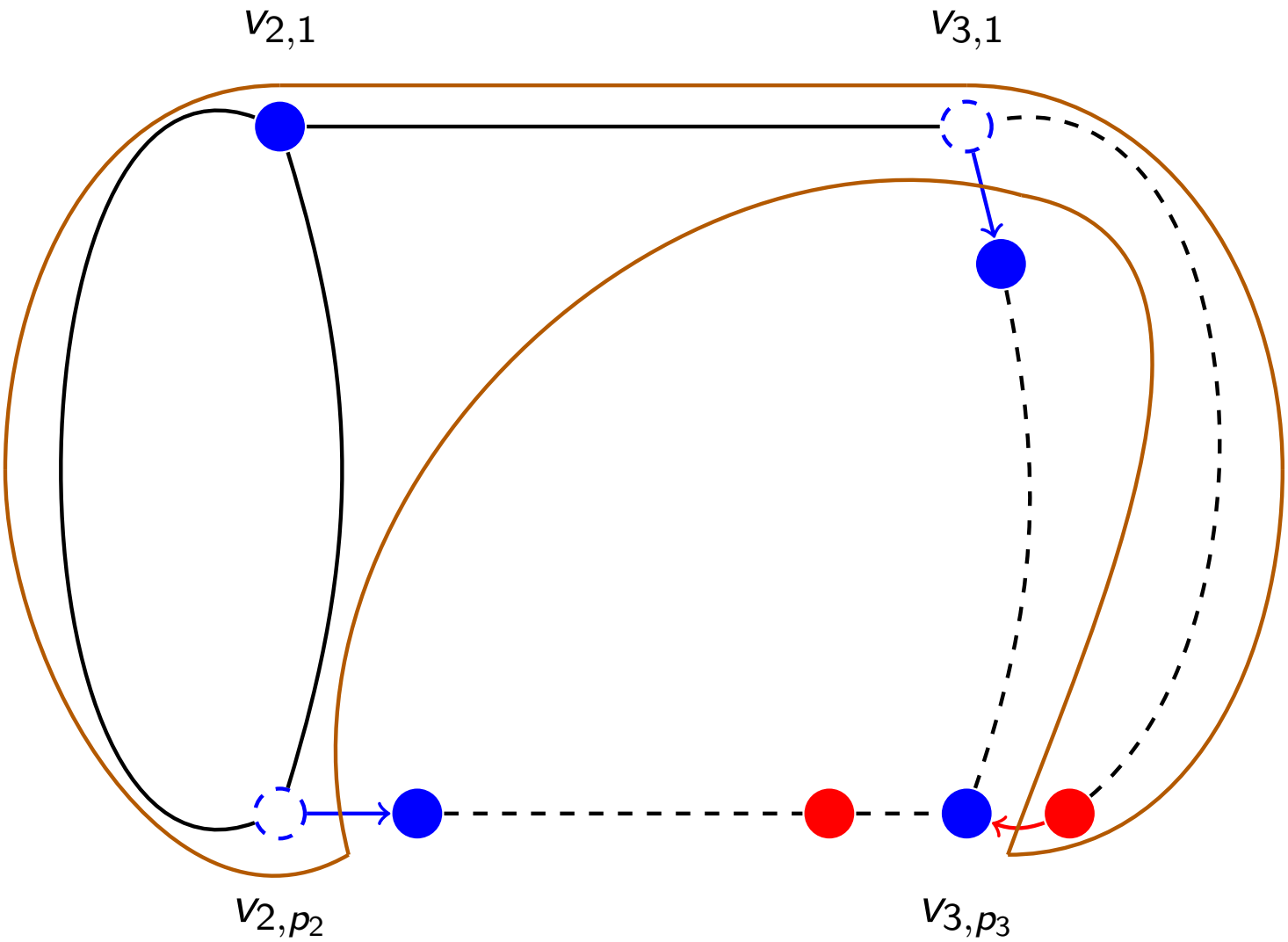


Proof

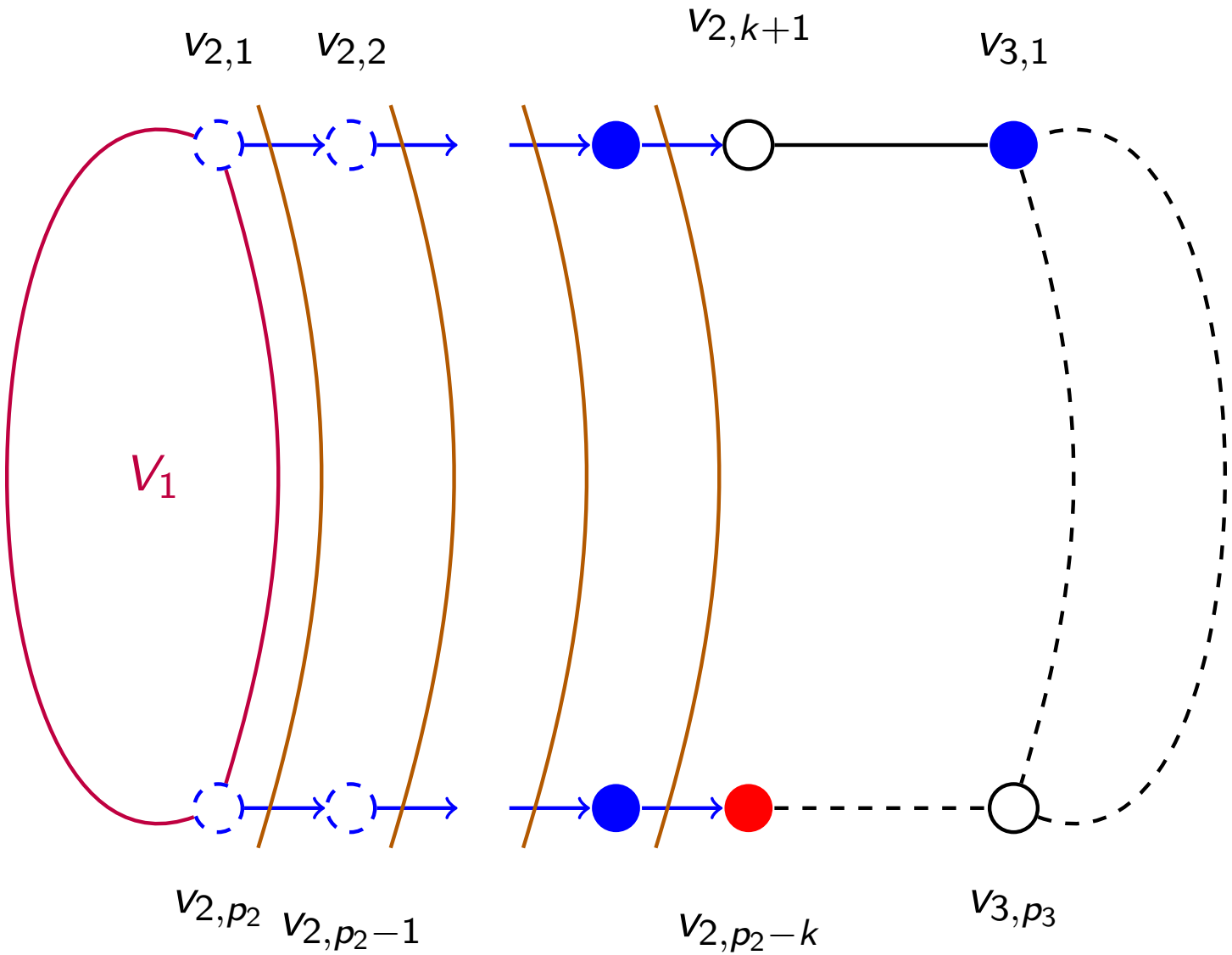
$g = 3$: $a \geq b$, 3 chips



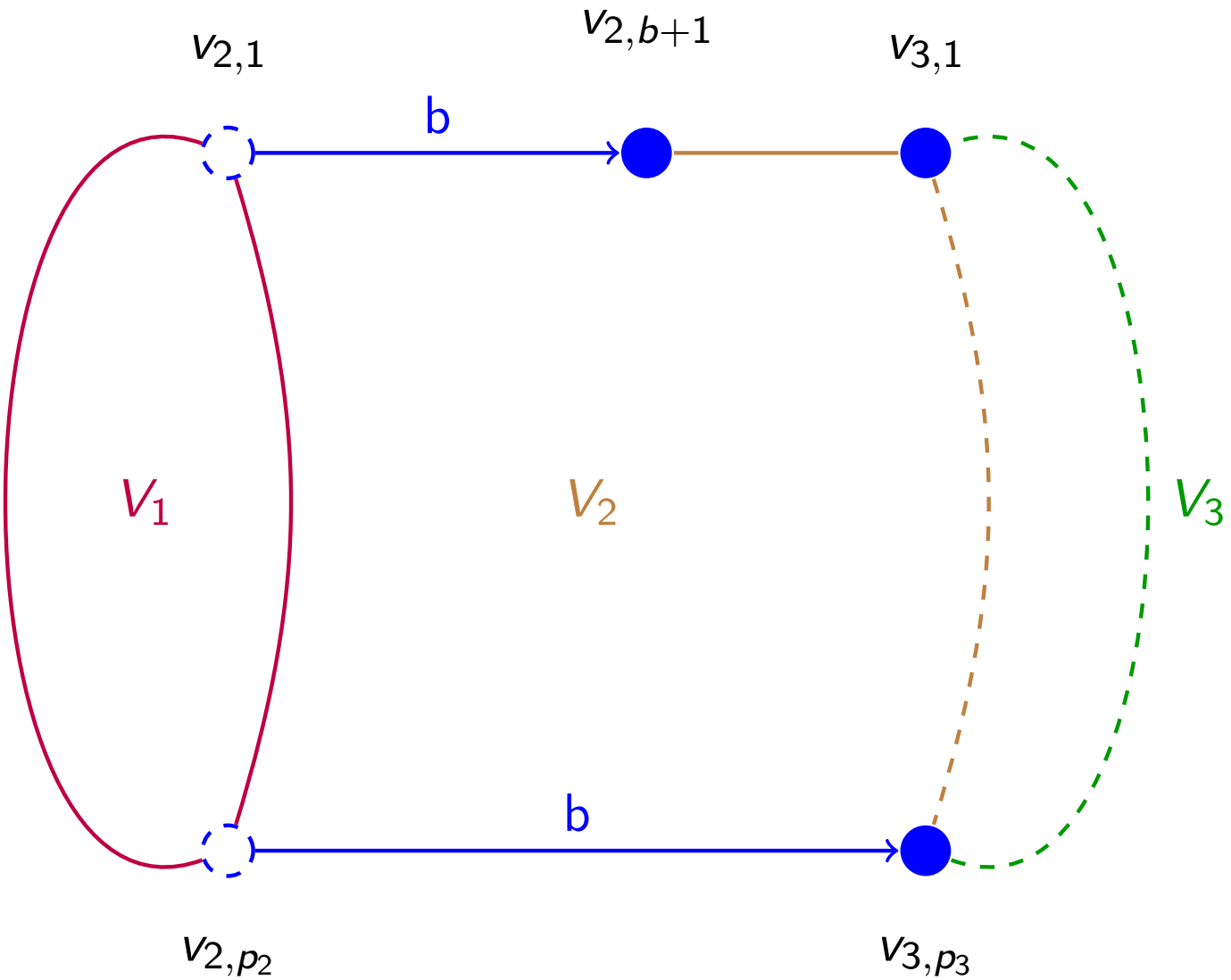
Proof



Proof

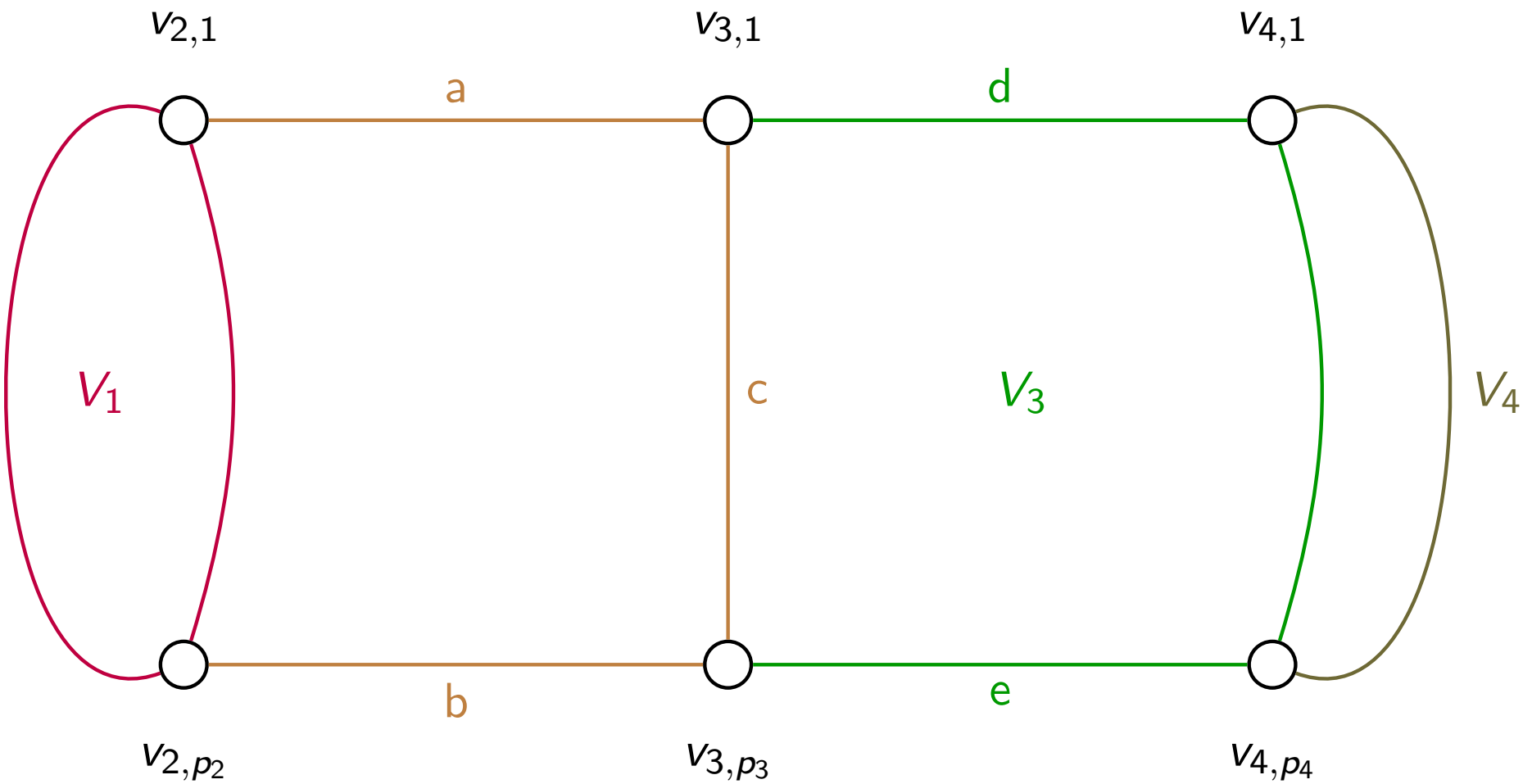


Proof



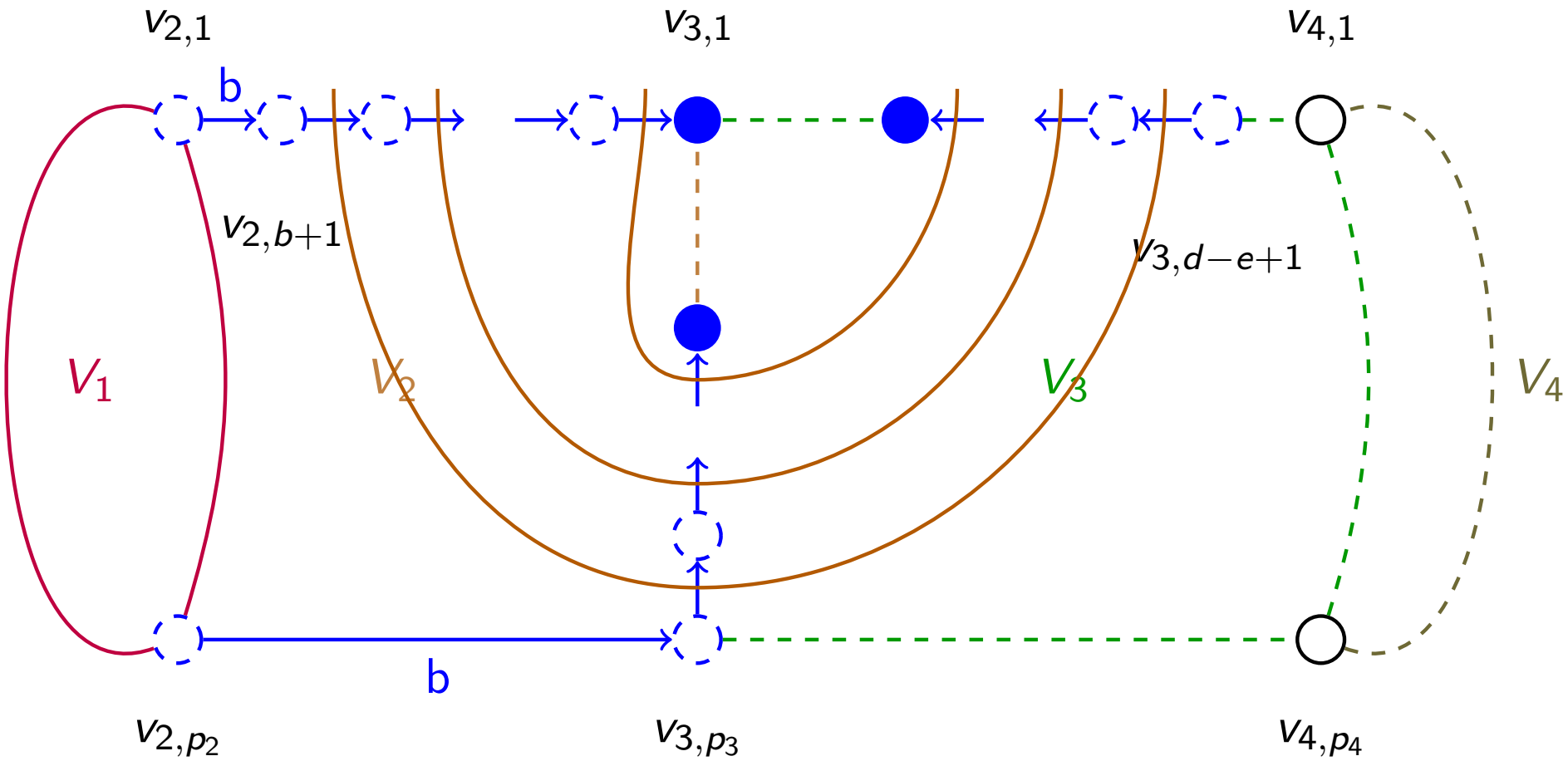
Proof

$g = 4$: 3 chips

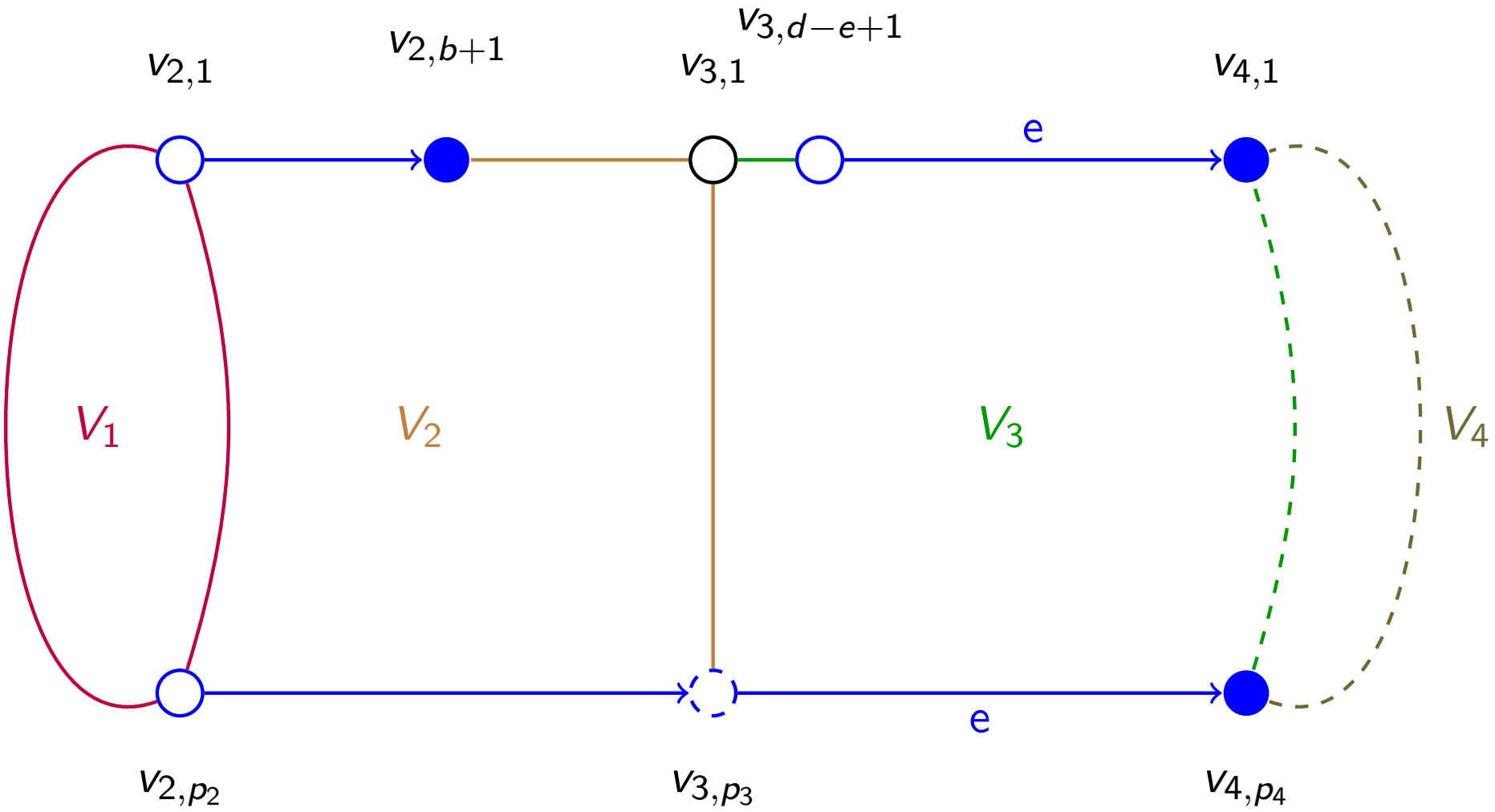


Proof

$g = 4$: $a > b, d > e, a - b \leq min(c, d - e)$

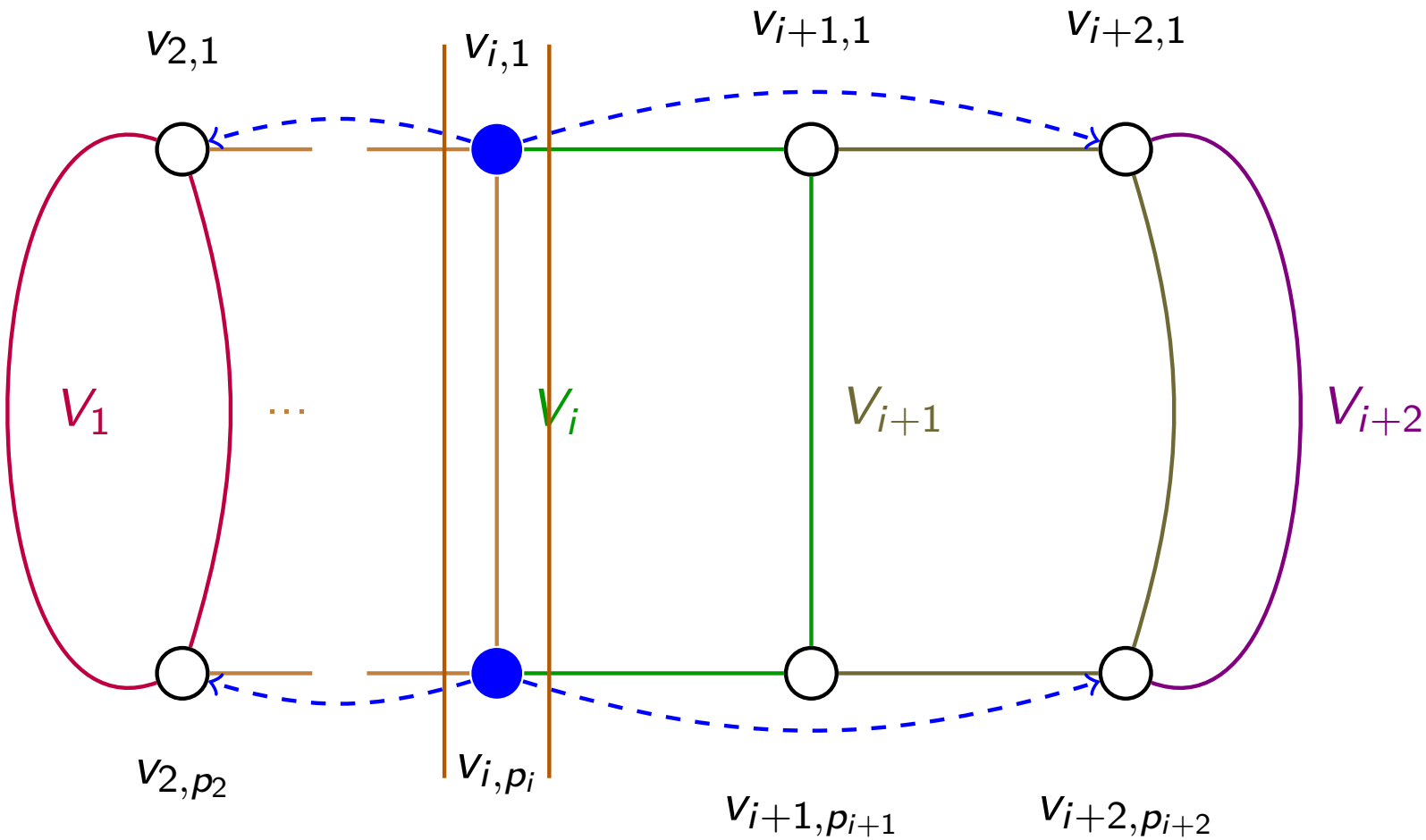


Proof



Proof

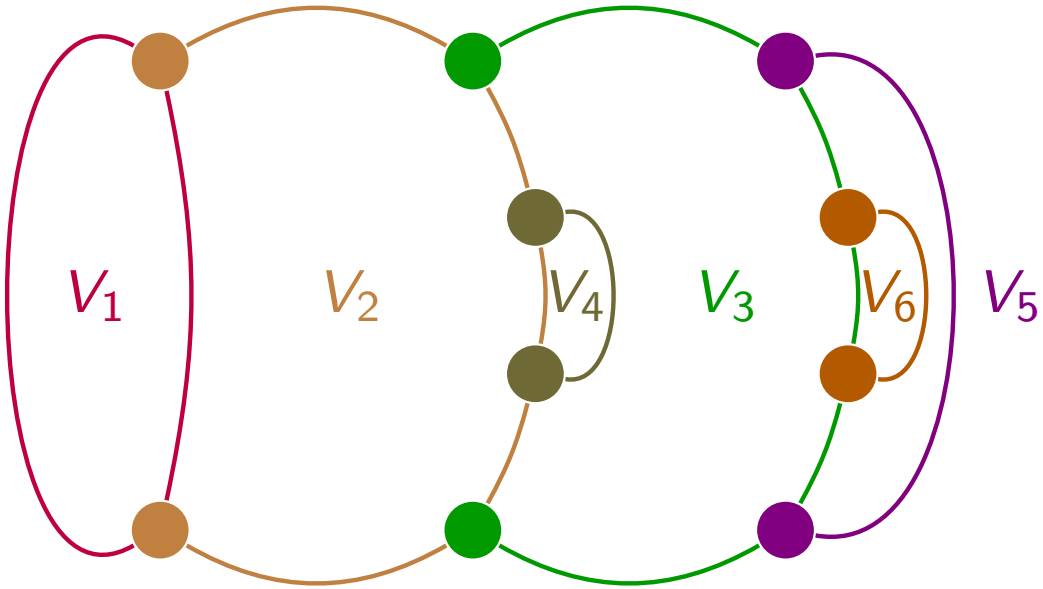
$g = i + 2$: $+1$ chip for every 2 ears



place chips similar to when $g = i$ place one chip similar to when $g = 4$

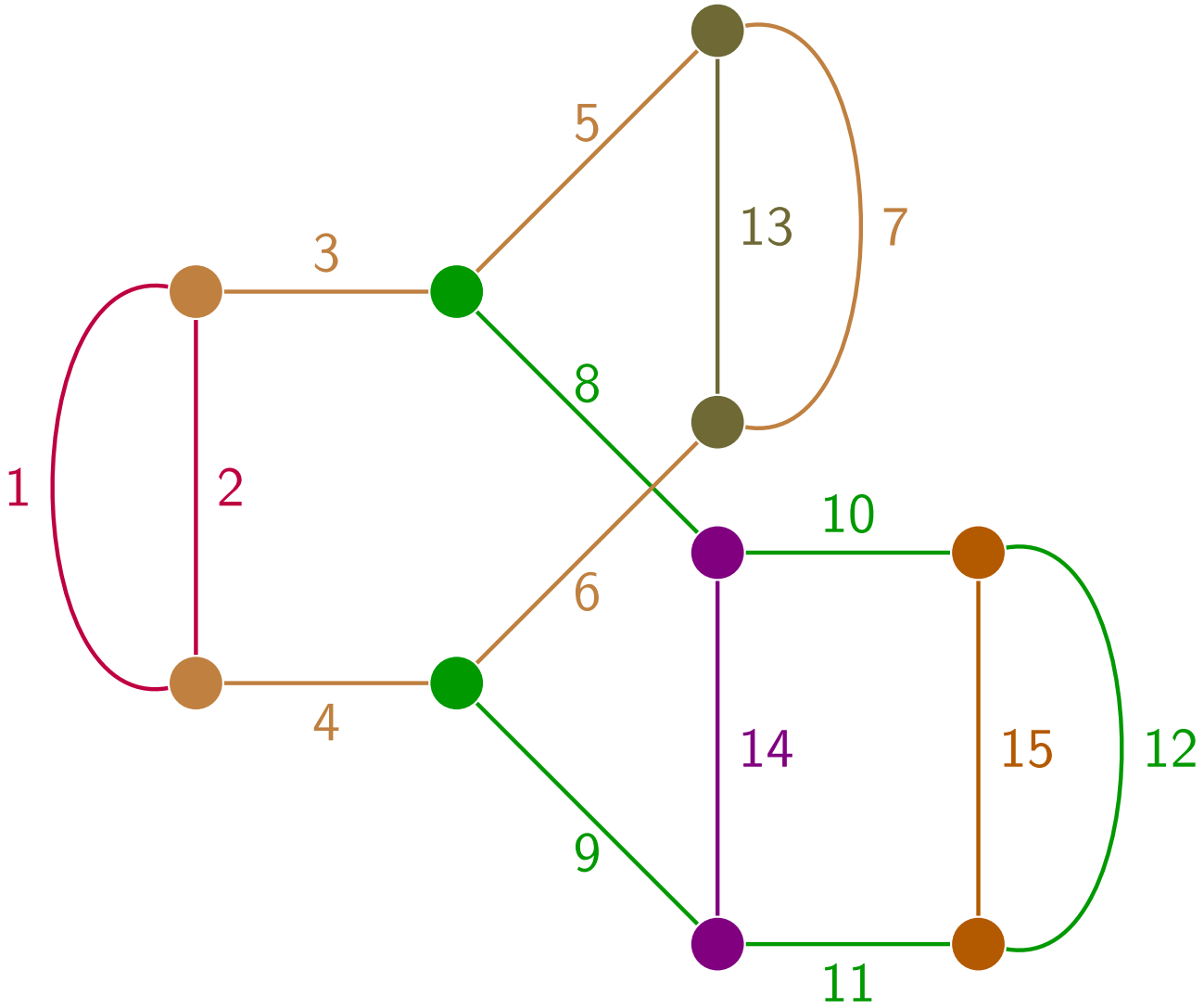
The nested ear decomposition, a work in progress...

An example of a nested ear decomposition:



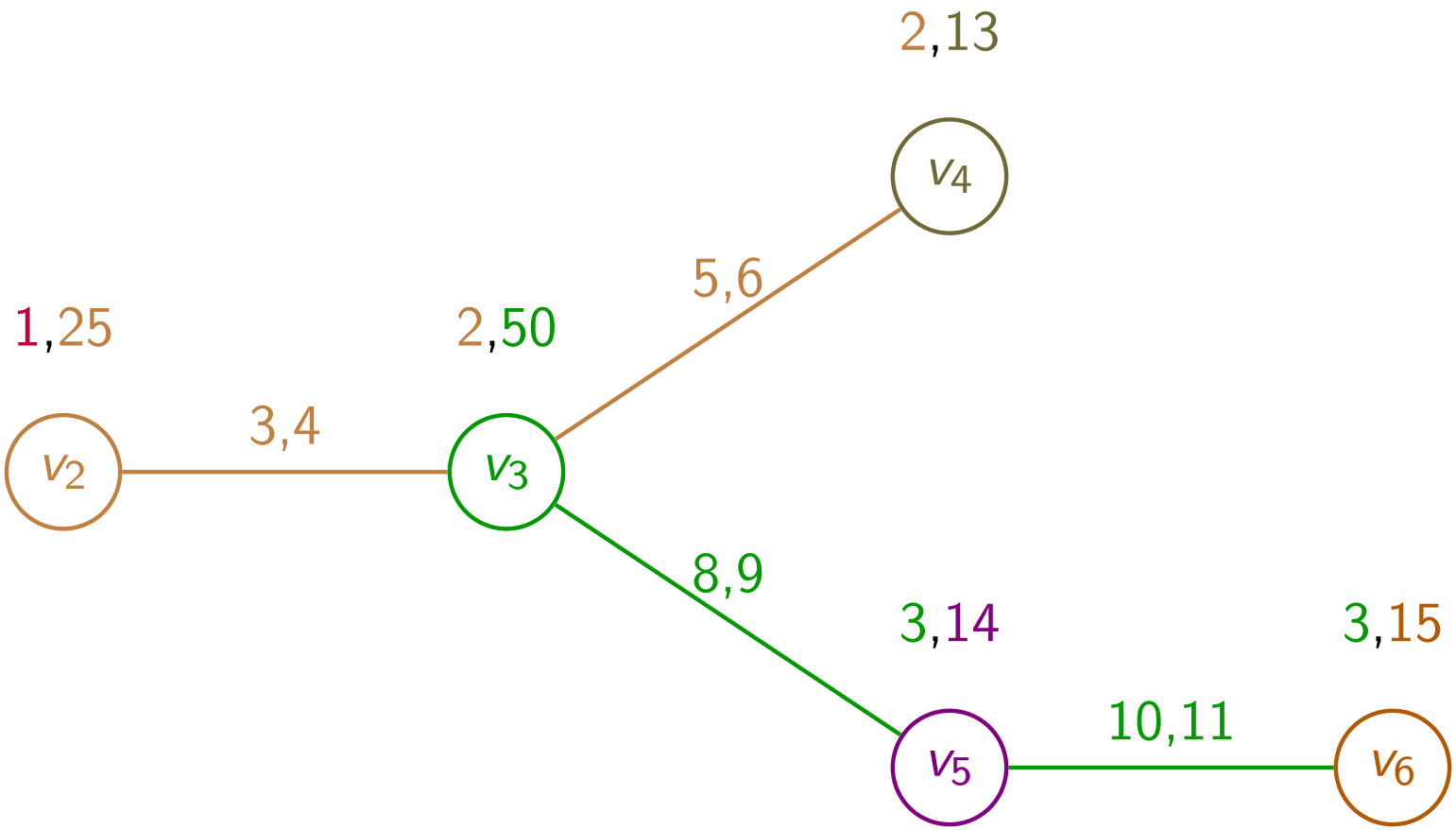
An idea

The same graph with the length of the paths added:



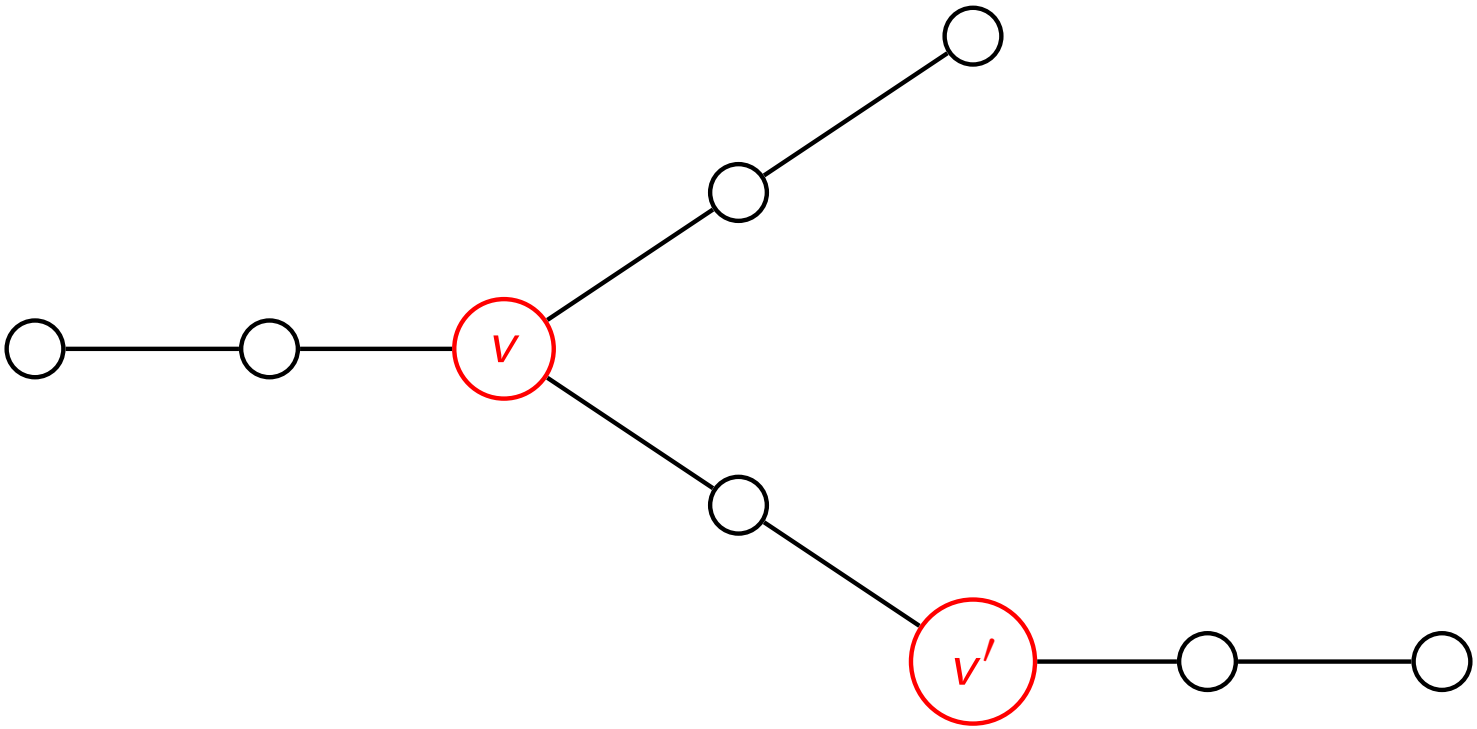
An idea

The corresponding tree:



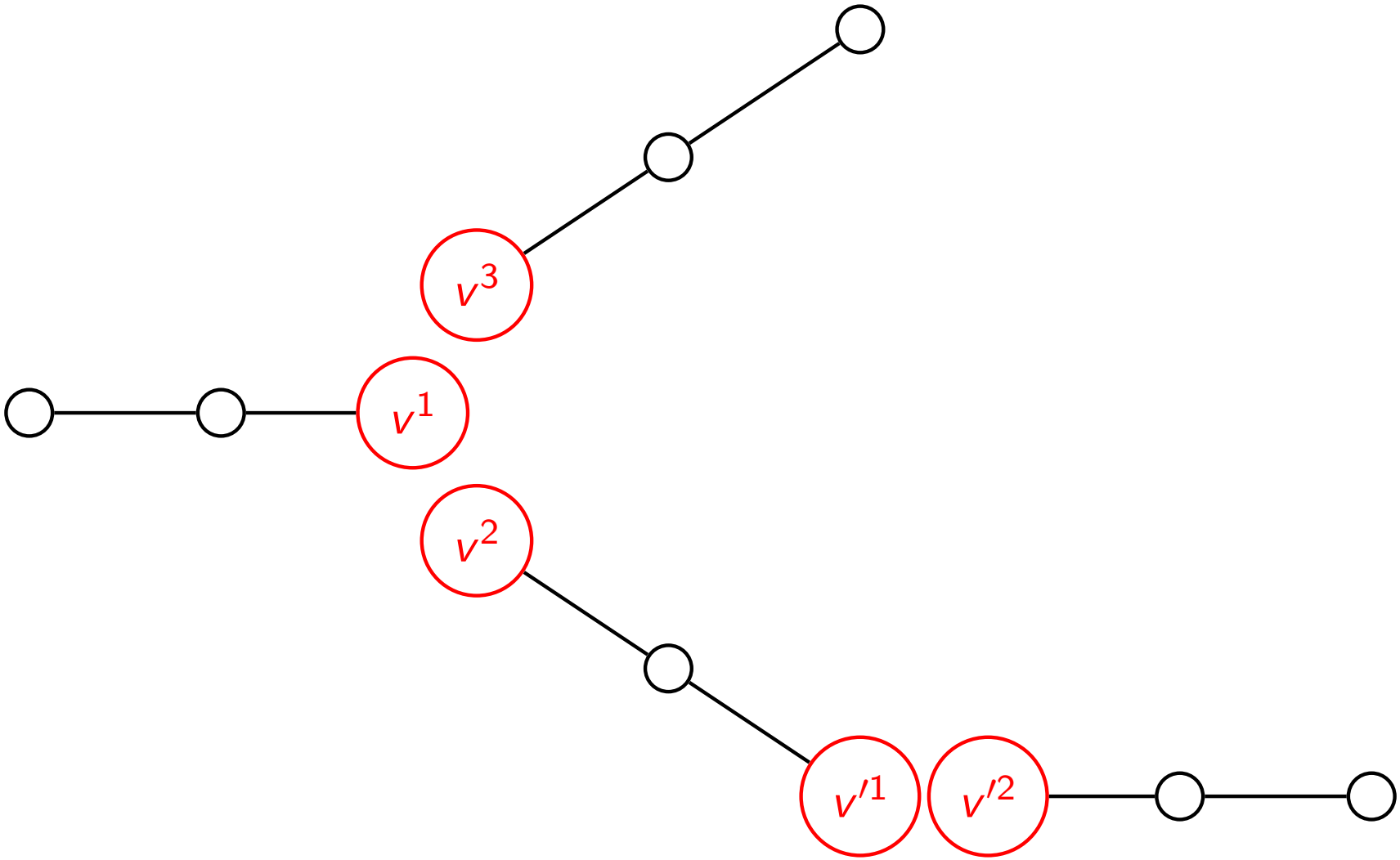
An idea

Breakers:



An idea

Components:



An idea

How to solve it on nested ear decompositions that look like full binary trees?