

# Possible $D^{(*)}\bar{D}^{(*)}$ and $B^{(*)}\bar{B}^{(*)}$ molecular states in the extended constituent quark models

You-Chang Yang · Zhi-Yun Tan · Jialun Ping · Hong-Shi Zong

arXiv:1703.09718v2 [hep-ph] 5 Apr 2017

the date of receipt and acceptance should be inserted later

**Abstract** The possible neutral  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  molecular states are studied in the framework of the constituent quark models, which is extended by including the  $s$ -channel one gluon exchange. Using different types of quark-quark potentials, we solve the four-body Schrödinger equation by means of the Gaussian expansion method. The bound states of  $D^{(*)}\bar{D}^{(*)}$  with  $J^{PC} = 1^{++}, 2^{++}$  and  $B^{(*)}\bar{B}^{(*)}$  with  $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$  are obtained. The molecular states  $D^*\bar{D}$  with  $J^{PC} = 1^{++}$  and  $B^*\bar{B}$  with  $J^{PC} = 1^{+-}$  are good candidates for the  $X(3872)$  and  $Z_b^0(10610)$ , respectively.

**PACS** 12.39.jh · 21.45.-v · 14.40.Lb · 14.40.Nd

---

youcyang@163.com

jlping@njnu.edu.cn (corresponding author)

zonghs@nju.edu.cn

---

You-Chang Yang  
Department of Physics, Nanjing University, Nanjing 210093,  
School of Physics and Electrical Science, Zunyi Normal University, Zunyi 563006, and State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing, 100190, China

Zhi-Yun Tan  
School of Physics and Electrical Science, Zunyi Normal University, Zunyi 563006, China

Jialun Ping  
Department of Physics, Nanjing Normal University, Nanjing 210023, P. R. China

Hong-Shi Zong  
Department of Physics, Nanjing University, Nanjing 210093, and State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing, 100190, China

## 1 Introduction

Since 2003, more than twenty new meson states (called  $XYZ$  particles) [1,2,3] have been observed by Belle, BaBar, BES, LHCb and other collaborations in hadronic final states that contain either a  $c\bar{c}$  or a  $b\bar{b}$  quark pair. In general, the properties of these states do not match to the expectations for any of the currently unassigned  $c\bar{c}$  charmonium or  $b\bar{b}$  bottomonium states. A well established one among these  $XYZ$  states is the  $X(3872)$ , which was first discovered in 2003 by Belle Collaboration [4] in the  $\pi^+\pi^-J/\psi$  invariant mass spectrum in  $B \rightarrow K\pi^+\pi^-J/\psi$ , and later confirmed by six other experiments [5,6,7,8,9,10]. Its quantum number have been studied by Belle, BaBar, CDF and LHCb, and determined to be  $I^GJ^{PC} = 0^+1^{++}$  [11]. The most striking feature of the  $X(3872)$  is the narrow total width about 1.2 MeV and the average mass  $3871.69 \pm 0.17$  MeV, which is extremely close to the  $D^0\bar{D}^{0*}$  mass threshold [12].

Most of  $XYZ$  states are unlikely interpreted as a conventional  $c\bar{c}$  or  $b\bar{b}$  meson for their unusual properties. During past decades, various pictures like molecular state, compact tetraquark state, hybrid state, and so on, have been proposed to explain the nature of them. For explaining the structure of  $X(3872)$ , the most popular explanation is the molecular state. Swanson [14] proposed to interpret the  $X(3872)$  as a  $D^0\bar{D}^{0*}$  molecular state with  $J^{PC} = 1^{++}$  which bound by both the pion and quark exchange. However, no  $D^0\bar{D}^{0*}$  molecular state was obtained in Ref.[14] if taking into account only of one pion exchange between  $D^0$  and  $\bar{D}^{0*}$ . Wong [13] applied a quark-based model, which is similar to add short-range quark-gluon force, to study the molecular sates composed of two heavy mesons. They found an S-wave  $D^0\bar{D}^{0*}$  molecular state with binding energy

about 7.5 MeV. Suzuki [15] believes that one pion exchange potential can not bind  $\bar{D}^0$  and  $D^{0*}$  to molecular state. Thomas and Close [16] found that the  $D^0\bar{D}^{0*}$  can be a bound state, when the pion exchange between charm and bottom mesons is considered. However, their results are very sensitive to a poorly constrained parameter. In Ref.[17], the author also obtained  $D^0\bar{D}^{0*}$  bound state when they systematically studied possible  $D\bar{D}$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  molecular states by considered the vector, pseudoscalar and scalar meson exchanges. In the framework of a potential model generated by the exchange of scalar, pseudoscalar and vector mesons, which based on the effective Lagrangian of heavy hadron chiral perturbation theory, a  $D^*\bar{D}^*$  bound state was got by Lee, Faessler *et al.*[18] as well. In Ref.[19], the authors believe the  $X(3872)$  should be understood as a molecular state of  $D\bar{D}^*$ , and extrapolates this information to make predictions of  $B\bar{B}^*$  molecules [20]. Gammie, Oset *et al.*[21] obtained a  $D\bar{D}$  bound state both by a model using a chiral Lagrangian already used to study flavor symmetry breaking in Skyrme models, and another model by take into account a SU(4) symmetric Lagrangian with heavy meson-exchanges. They also analyzed the  $e^+e^- \rightarrow J/\psi D\bar{D}, J/\psi D\bar{D}^*$  reactions of Belle, and found a hidden charm scalar meson with mass around 3700 MeV [22], which is compatible with the  $D\bar{D}$  bound state. In Ref. [23], Molina and Oset interpreted the  $Y(3940)$ ,  $Z(3940)$  as molecular states of  $D^*\bar{D}^*$  with quantum number  $J^{PC} = 0^{++}$ ,  $2^{++}$  and  $X(4160)$  as a  $D_s^*\bar{D}_s^*$  molecular state with  $J^{PC} = 2^{++}$ , respectively.

In constituent quark model, Vijande *et al.*[24] studied the four-quark system  $c\bar{c}n\bar{n}$  by means of the hyperspherical harmonic formalism. However no bound states have been found whether taking into account the exchange of scalar and pseudoscalar mesons or not. Yang and Ping [26] systematically studied the  $D\bar{D}$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  by means of the Gaussian expansion method (GEM). No neutral bound state of  $D^{(*)}\bar{D}^{(*)}$  was found as well. Liu and Zhang [25] obtained a  $D^0\bar{D}^{0*}$  bound state in a chiral quark model with including  $\pi, \sigma, \omega$  and  $\rho$  meson exchanges in it.

In nature only the colorless hadron is allowed, so there is no one-gluon annihilation interaction between quark and antiquark with the same flavor in a conventional colorless  $q\bar{q}$  meson. However, the *s*-channel one gluon exchange interaction can exist in the neutral  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  system and maybe plays a important role for binding them, since the color structure of a four-quark state is much richer than that of a  $q\bar{q}$  conventional meson. Based on the Bhaduri, Cohler and Nogami model(BCN), Wang *et al.* [27] believe that the *s*-channel one gluon exchange interaction is important

for binding a  $D^*\bar{D}^*$  molecular state, which is a good candidate for the  $X(3872)$ .

In this work, we would like to study the possible neutral molecular states  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  by two constituent quark models, which are extended by including the one-gluon annihilation interaction between  $u\bar{u}$  or  $d\bar{d}$  light quark pairs. We solve the four-body Schrödinger equation by means of GEM, which is a high accuracy method for few-body systems developed by Kamimura, Hiyama *et al.* [28] and extensively performed in studying the mass spectrum of multi-quark system [29,30,31,32,33,34].

This paper is organized as follows. After the introduction, we present the extended constituent quark models in Sec.2. The wave functions of  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  are constructed by considering the isospin, total angular momentum, color and the Gaussian expansion method and listed in Sec. 3. We summarize our numerical results and perform some analysis in Sec. 4 and draw some conclusions in Sec. 5.

## 2 The Constituent Quark Model with *s*-channel one Gluon Exchange

### 2.1 Bhaduri, Cohler and Nogami model

This quark model was proposed by Bhaduri and collaborators [35,36]. The Hamiltonian takes the form,

$$H = \sum_{i=1}^4 \left( m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_{c.m.} + \sum_{j>i=1}^4 (V_{ij}^C + V_{ij}^G) \quad (1)$$

with

$$V_{ij}^G = \alpha_s \frac{\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c}{4} \left( \frac{1}{r_{ij}} - \frac{1}{m_i m_j} \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right), \quad (2)$$

$$V_{ij}^C = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (-a_c r_{ij} - \Delta), \quad (3)$$

where  $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$  and  $T_{c.m.}$  is the kinetic energy of the center-of-mass motion.  $\boldsymbol{\sigma}, \boldsymbol{\lambda}$  are the SU(2) Pauli matrices and the SU(3) Gell-Mann matrices, respectively. The  $\boldsymbol{\lambda}$  should be replaced by  $-\boldsymbol{\lambda}^*$  for the antiquark.

### 2.2 The chiral constituent quark model(ChQM)

The chiral constituent quark model(ChQM) [37] includes Goldstone-boson exchange potential in addition to color confinement potential and *t*-channel one-gluon-exchange (OGE) potential between quarks (antiquarks). The chiral partner,  $\sigma$ -meson exchange potential, is also introduced here, although its effect is still in controversy [38].

The Hamiltonian of the ChQM used here is given as follows,

$$H = \sum_{i=1}^4 \left( m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_{c.m.} + \sum_{j>i=1}^4 (V_{ij}^G + V_{ij}^C + V_{ij}^\chi + V_{ij}^\sigma), \quad \chi = \pi, K, \eta, \quad (4)$$

The OGE potential reads

$$V_{ij}^G = \alpha_s \frac{\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c}{4} \left[ \frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \delta(\mathbf{r}_{ij}) \right], \quad (5)$$

where,  $T_{c.m.}$ ,  $\boldsymbol{\sigma}$ ,  $\boldsymbol{\lambda}$  have the same meaning as the above. In non-relativistic quark model, the function  $\delta(\mathbf{r}_{ij})$  should be regularized [39, 40]. It reads

$$\delta(\mathbf{r}_{ij}) = \frac{1}{4\pi r_{ij} r_0^2(\mu)} e^{-r_{ij}/r_0(\mu)}, \quad (6)$$

where  $r_0(\mu) = \hat{r}_0/\mu$  and  $\mu$  is the reduced mass of the interacting quark/antiquark-quark/antiquark pair,  $\hat{r}_0$  is a parameter to be determined from the experimental data. In non-relativistic quark model, the wide energy covered from light to heavy quark requires an effective scale-dependent strong coupling constant  $\alpha_s$  in Eq.(5) that cannot be obtained from the usual one-loop expression of the running coupling constant because it diverges when  $Q \rightarrow \Lambda_{QCD}$ . So one use an effective scale-dependent strong coupling constant given by

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln [(\mu^2 + \mu_0^2)/\Lambda_0^2]}, \quad (7)$$

where  $\mu_0$  and  $\Lambda_0$  are the parameters to be obtained by fitting the normal meson spectrum.

A screened potential simulating the results of unquenched lattice calculations is given by

$$V_{ij}^C = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \{-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\}, \quad (8)$$

where  $\Delta$  is a global constant to be fixed from experimental data.

Due to the spontaneous breaking of original  $SU(3)_L \otimes SU(3)_R$  chiral symmetry at some momentum scale, the Goldstone meson exchange occurs between quarks (an-

tiquarks). The potential takes the form

$$V_{ij}^\pi = C(g_{ch}, \Lambda_\pi, m_\pi) \frac{m_\pi^2}{12m_i m_j} H_1(m_\pi, \Lambda_\pi, r_{ij}) \times (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \sum_{a=1}^3 \lambda_i^a \lambda_j^a, \quad (9)$$

$$V_{ij}^\eta = C(g_{ch}, \Lambda_\eta, m_\eta) \frac{m_\eta^2}{12m_i m_j} H_1(m_\eta, \Lambda_\eta, r_{ij}) \times (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) [\cos\theta_P(\lambda_i^8 \lambda_j^8) - \sin\theta_P(\lambda_i^0 \lambda_j^0)], \quad (10)$$

$$V_{ij}^\sigma = -C(g_{ch}, \Lambda_\sigma, m_\sigma) H_2(m_\sigma, \Lambda_\sigma, r_{ij}), \quad (11)$$

$$H_1(m, \Lambda, r) = \left[ Y(mr) - \frac{\Lambda^3}{m^3} Y(\Lambda r) \right], \quad (12)$$

$$H_2(m, \Lambda, r) = \left[ Y(mr) - \frac{\Lambda}{m} Y(\Lambda r) \right], \quad (13)$$

$$C(g_{ch}, \Lambda, m) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m, \quad (14)$$

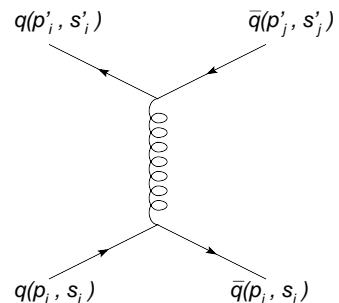
where  $Y(x)$  is the standard Yukawa function defined by  $Y(x) = e^{-x}/x$  and rest symbols have their usual meaning. The chiral coupling constant  $g_{ch}$  is determined from the  $\pi NN$  coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}, \quad (15)$$

and flavor  $SU(3)$  symmetry is assumed.

## 2.3 $s$ -channel one gluon exchange interaction

In the case of heavy-light meson and antimeson system, the contribution of  $s$ -channel annihilation interaction should be taken into account. The one-gluon annihilation of light-quark and antiquark is shown in Fig. 1. According to the Feynman rules, we can write down



**Fig. 1** The one-gluon annihilation diagrams for quark and antiquark.

the  $T$ -matrix of the process

$$T_{fi} = \frac{g_s^2}{s} \bar{u}(p'_i, s'_i) \chi_{c'_i}^\dagger \chi_{f'_i}^\dagger \frac{\lambda^a}{2} \gamma^\mu v(p'_j, s'_j) \chi_{c'_j} \chi_{f'_j} \\ \bar{v}(p_j, s_j) \chi_{c_j}^\dagger \chi_{f_j}^\dagger \frac{\lambda^a}{2} \gamma_\mu u(p_i, s_i) \chi_{c_i} \chi_{f_i}, \quad (16)$$

where  $s = (p_i + p_j)^2$  and  $p$  is four-vector momenta;  $u(p_i, s_i)$ ,  $v(p_j, s_j)$  are the free Dirac spinors of  $i$ th quark and  $j$ th antiquark;  $\chi_c$ ,  $\chi_f$  represent color and flavor wave function, respectively. After Fierz transformation [41] of SU(n) group and taking the lowest order in the non-relativistic limit, the contributions from one-gluon annihilation to the potential between quark and antiquark in momentum representation can be written as

$$V_{ij}^{Anni-G}(s) = \frac{4\pi\alpha_s}{s} \frac{1}{4} \left( \frac{16}{9} - \frac{1}{3} \lambda_i^c \cdot \lambda_j^{*c} \right) \times \left( \frac{1}{3} + \frac{1}{2} \mathbf{f}_i^a \cdot \mathbf{f}_j^{*a} \right) \left( \frac{3}{2} + \frac{\sigma_i \cdot \sigma_j}{2} \right). \quad (17)$$

In coordinate space, and under the static approximation,  $s = (m_i + m_j)^2 = 4m_q^2$  ( $q = u$  or  $d$  quark), the potential reads

$$V_{ij}^{Anni-G}(r_{ij}) = \frac{\pi\alpha_s}{4m_q^2} \left( \frac{16}{9} - \frac{1}{3} \lambda_i^c \cdot \lambda_j^{*c} \right) \times \left( \frac{1}{3} + \frac{1}{2} \mathbf{f}_i^a \cdot \mathbf{f}_j^{*a} \right) \left( \frac{3}{2} + \frac{\sigma_i \cdot \sigma_j}{2} \right) \delta(\mathbf{r}_{ij}). \quad (18)$$

Here  $\mathbf{f}^a$  is SU(3) matrix in the flavor space. The factor of first bracket represents that this interaction never occurs inside color-singlet. Obviously, the last two factors in the brackets mean that this interaction only occurs when the  $\bar{q}q$  pair is in the same flavor with spin  $S = 1$ . This interaction is always repulsive in molecular states of four-quark system, since the color matrix elements is zero and  $-\frac{14}{3}$  in  $1 \otimes 1$  and  $8 \otimes 8$ , respectively.

However, the earliest lattice simulations of gluon propagator in the Landau gauge, by Gupta *et al.* [42] were interpreted in terms of a massive particle propagator. In order to study the  $I = 0 \pi\pi$  and  $I = \frac{1}{2} K\pi$  S-wave phase shift, Barnes and Swanson [43] modified the gluon propagator by including an effective gluon mass. To analyze the mixing of the scalar glueball with scalar-isoscalar quarkonia states above 1 GeV [44], and investigate mesonic content of the nucleon and Roper resonance [45], the massive gluon propagator is also employed. So here we choose the gluon propagator [44, 46, 47]

$$D(s) = \frac{1}{s - m_g^2}, \quad (19)$$

where  $m_g$  is effective gluon mass, which should be larger than the half of the bare glueball mass deduced from lattice simulations. Typical values for the effective gluon mass are in the range 0.6 – 1.2 GeV [44].

After taking into account massive gluon propagator, the Eq.(19) turns to be,

$$V_{ij}^{Anni-G}(r_{ij}) = \frac{\pi\alpha_s}{4m_q^2 - m_g^2} \left( \frac{16}{9} - \frac{1}{3} \lambda_i^c \cdot \lambda_j^{*c} \right) \times \left( \frac{1}{3} + \frac{1}{2} \mathbf{f}_i^a \cdot \mathbf{f}_j^{*a} \right) \left( \frac{3}{2} + \frac{\sigma_i \cdot \sigma_j}{2} \right) \delta(\mathbf{r}_{ij}) \quad (20)$$

Obviously, this interaction is attractive if  $m_g > 2m_q$ .

### 3 wave function

The total wave function of four-quark system can be written as,

$$\Psi_{J,J_z}^{I,I_z} = |\xi\rangle |\eta\rangle_{II_z} \Phi_{JJ_z}, \quad (21)$$

with

$$\Phi_{JJ_z} = [|\chi\rangle_S \otimes |\Phi\rangle_{L_T}]_{JJ_z}$$

where  $|\xi\rangle$ ,  $|\eta\rangle_{II_z}$ ,  $|\chi\rangle_{SMS}$ ,  $|\Phi\rangle_{L_T M_L}$  represent color, flavor, spin and spacial wave functions with quantum numbers: color singlet, isospin  $I$ , spin  $S$  and orbital angular momentum  $L_T$ , respectively.

The molecular states  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  system can be conveniently classified in terms of total angular momentum,  $J$ , parity,  $P$  and charge conjugation,  $C$ . In this work we only consider the low-lying states, the orbital angular momentum  $L_T$  is set to 0. In this case we have the following states for the  $D^{(*)}\bar{D}^{(*)}$  system:

(i) Two states with  $J^{PC} = 0^{++}$ :  $[D\bar{D}]_0$ ,  $[D^*\bar{D}^*]_0$ , where the subscript is total angular momentum  $J$ .

(ii) One states with  $J^{PC} = 1^{++}$ :

$$\frac{1}{\sqrt{2}} ([D\bar{D}^*]_1 + [D^*\bar{D}]_1)$$

and two states with  $J^{PC} = 1^{+-}$ :

$$\frac{1}{\sqrt{2}} ([D\bar{D}^*]_1 - [D^*\bar{D}]_1) \text{ and } [D^*\bar{D}^*]_1$$

(iii) One state with  $J^{PC} = 2^{++}$ :  $[D^*\bar{D}^*]_2$ .

For the  $B^{(*)}\bar{B}^{(*)}$  system, we replace the  $D$  mesons in the above with the  $B$  mesons. The total spin function  $|\chi\rangle_{SMS}$  and flavor function  $|\eta\rangle_{II_z}$  can be easily constructed from the above expressions. For example, for  $[D^*\bar{D}^*]_1$ ,

$$\begin{aligned} |\chi\rangle_{11} &= \sqrt{\frac{1}{2}} (|11\rangle|10\rangle - |10\rangle|11\rangle) \\ &= \frac{1}{2} (\alpha\alpha\alpha\beta + \alpha\alpha\beta\alpha - \alpha\beta\alpha\alpha - \beta\alpha\alpha\alpha) \\ |\eta\rangle_{00} &= \sqrt{\frac{1}{2}} (D^0\bar{D}^0 + D^-\bar{D}^+) = \sqrt{\frac{1}{2}} (u\bar{c}c\bar{u} + d\bar{c}d\bar{d}). \end{aligned}$$

The spatial structure of molecular states are pictured in Fig. 2. We define the relative coordinate as following,

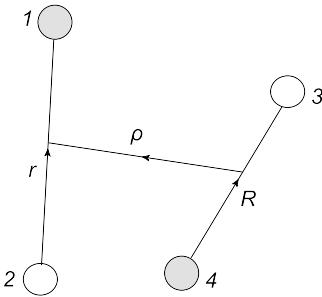
$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4, \quad (22)$$

$$\boldsymbol{\rho} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \frac{m_3 \mathbf{r}_3 + m_4 \mathbf{r}_4}{m_3 + m_4}, \quad (23)$$

and the center of mass coordinate is

$$\mathbf{R}_{cm} = \sum_{i=1}^4 m_i \mathbf{r}_i / \sum_{i=1}^4 m_i, \quad (24)$$

where  $m_i$  is the mass of the  $i$ th quark(or antiquark). Then the outer products of space and spin is



**Fig. 2** The relative coordinates for the molecular state. Darkened and open circles represent quarks and antiquarks, respectively.

$$\Phi_{JJ_z} = \left[ \left[ \left[ \phi_{lm}^G(\mathbf{r}) \chi_{s_1 m_{s_1}} \right]_{J_1 M_1} \right]_{J_2 M_2} \right]_{J_{12} M_{12}} \varphi_{\beta\gamma}^G(\boldsymbol{\rho}) \Bigg]_{JJ_z}. \quad (25)$$

Where  $\chi_{sm_s}$  is spin wave function of normal meson which is composed by quark-antiquark. The spatial wave functions  $\phi_{lm}^G(\mathbf{r})$ ,  $\psi_{LM}^G(\mathbf{R})$ ,  $\varphi_{\beta\gamma}^G(\boldsymbol{\rho})$  are written as,

$$\phi_{lm}^G(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}) \quad (26)$$

$$\psi_{LM}^G(\mathbf{R}) = \sum_{N=1}^{N_{max}} c_N N_{NL} R^L e^{-\zeta_N R^2} Y_{LM}(\hat{\mathbf{R}}) \quad (27)$$

$$\varphi_{\beta\gamma}^G(\boldsymbol{\rho}) = \sum_{\alpha=1}^{\alpha_{max}} c_\alpha N_{\alpha\beta} \rho^\beta e^{-\omega_\alpha \rho^2} Y_{\beta\gamma}(\hat{\boldsymbol{\rho}}) \quad (28)$$

Gaussian size parameters are taken as geometric progression

$$\nu_n = \frac{1}{s_n^2}, \quad s_n = s_1 a^{n-1}, \quad a = \left( \frac{s_{n_{max}}}{s_1} \right)^{\frac{1}{n_{max}-1}} \quad (29)$$

The expression of  $\zeta_N, \omega_\alpha$  in Eqs. (27)-(28) are similar to Eq. (29).

The color wave function of possible molecular states reads,

$$|\xi\rangle = \frac{1}{3} (|rr\bar{r}\bar{r}\rangle + |g\bar{g}g\bar{g}\rangle + |\bar{b}b\bar{b}\bar{b}\rangle + |r\bar{r}g\bar{g}\rangle + |r\bar{r}b\bar{b}\rangle + |g\bar{g}r\bar{r}\rangle + |g\bar{g}b\bar{b}\rangle + |\bar{b}b\bar{r}\bar{r}\rangle + |\bar{b}b\bar{g}\bar{g}\rangle), \quad (30)$$

#### 4 Numerical results and discussion

The energy of meson composed of quark-antiquark, and four-quark systems  $D^{(*)}\bar{D}^{(*)}$ ,  $B^{(*)}\bar{B}^{(*)}$  can be obtained by solving the Schrödinger equation

$$(H - E) \Psi_{J,J_z}^{I,I_z} = 0 \quad (31)$$

with Rayleigh-Ritz variational principle.

To study the spectrum of a four-quark state, one believes that whether or not the state is bound is judged by the threshold of two normal mesons, and the same parameters should used in the calculation of normal mesons and four-quark states [36, 40, 48, 49, 50].

For calculating spectrum of the normal meson, there is only one relative motion between quark and antiquark, so the Eq.(26) is employed. The model parameters of the ChQM and the BCN used in this work are shown in Table 1, which they are got from Refs.[35, 36, 37], respectively. The calculated results of normal meson spectrum listed in Table 2 are converged with  $n_{max} = 7$ ,  $s_1 = 0.1$  fm and  $s_{n_{max}} = 2$  fm, which are discussed in detail in Ref.[29]. Obviously, the meson spectrum in Table 2 calculated by GEM are agree well with the experimental data [12] and Refs.[35, 36, 37].

**Table 1** Parameters of two quark models. ChQM: the masses of  $\pi, \eta$  take the experimental values,  $m_\pi = 0.7$  fm $^{-1}$ ,  $m_\eta = 2.77$  fm $^{-1}$ ;  $m_\sigma, \Lambda_\pi, \Lambda_\eta, \theta_p$  are taken from Ref.[37], namely  $m_\sigma = 3.42$  fm $^{-1}$ ,  $\Lambda_\pi = \Lambda_\sigma = 4.2$  fm $^{-1}$ ,  $\Lambda_\eta = 5.2$  fm $^{-1}$ ,  $\theta_p = -15^\circ$ ,  $g_{ch}^2/4\pi = 0.54$ . BCN: The parameters take from Refs.[35, 36]

Quark Model	BCN	ChQM
Quark masses (MeV)	$m_{u,d}$	337
	$m_s$	600
	$m_c$	1870
	$m_b$	5259
Confinement	$a_c$ (MeV fm $^{-1}$ )	176.738
	$\Delta$ (MeV)	-171.25
	$\mu_c$ (fm $^{-1}$ )	-
OGE	$\alpha_s$	0.390209
	$\alpha_0$	-
	$\hat{r}_0$ (MeV fm)	2.118
	$r_0$ (fm)	28.17
	$\mu_0$ (MeV)	0.4545
	$\Lambda_0$ (fm $^{-1}$ )	36.976

**Table 2** Numerical results of normal meson spectrum (in MeV) for the ChQM and BCN models. The column of BCN1 and ChQM1 are taken from Refs.[35,36] and [37] respectively. The BCN2 and ChQM2 are calculated by GEM. The last column takes from the latest Particle Data Group[12]

Meson	BCN1	BCN2	ChQM1	ChQM2	Exp.
$\pi$	170	137.5	139	153.2	$139.57 \pm 0.00035$
K	537	521.4	496	484.9	$493.677 \pm 0.016$
$\rho(770)$	777	779.6	772	773.1	$775.49 \pm 0.34$
$K^*(892)$	905	907	910	907.7	$896.00 \pm 0.25$
$\omega(782)$	777	779.6	691	696.5	$782.65 \pm 0.12$
$\phi(1020)$	1018	1018.5	1020	1011.9	$1019.422 \pm 0.02$
$\eta_c(1s)$	3046	3040	2990	2999.7	$2980.3 \pm 1.2$
$J/\psi(1s)$	3102	3098	3097	3096.7	$3096.916 \pm 0.011$
$D^0$	1891	1886.7	1883	1898.4	$1864.84 \pm 0.17$
$D^*$	2021	2021.3	2010	2017.3	$2006.97 \pm 0.19$
$D_s$	2001	1997	1981	1991.8	$1968.49 \pm 0.34$
$D_s^*$	2103	2102.3	2112	2115.7	$2112.3 \pm 0.5$
$B^\pm$	5304	5302	5281	5277.9	$5279.15 \pm 0.31$
$B^0$	5304	5302	5281	5277.9	$5279.53 \pm 0.33$
$B^*$	5352	5351.5	5321	5318.8	$5325.1 \pm 0.5$
$B_s^0$	5376	5373.1	5355	5355.8	$5366.3 \pm 0.6$
$B_s^*$	5416	5414.5	5400	5400.5	$5412.8 \pm 1.3$
$\eta_b(1s)$	9431	9422.2	9454	9467.9	$9399.0 \pm 2.3$
$\Upsilon(1s)$	9448	9439.5	9505	9504.7	$9460.30 \pm 0.26$

Generally, the binding energy of the four-quark system is defined by

$$\Delta E = E_T - E_{th}. \quad (32)$$

with

$$E_{th} = E_{M_1} + E_{M_2}$$

where  $E_M$  and  $E_T$  represent the energy of  $Q\bar{q}$  ( $Q = c, b$  and  $q = u, d$ ) and  $Q\bar{q}q\bar{Q}$  systems, respectively. If  $\Delta E < 0$ , then the system is stable against the strong interaction. According to the Table 2, the thresholds of  $S$ -wave  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  of ChQM and BCN models are listed in Table 3.

**Table 3** Threshold energies (in MeV) of  $S$ -wave  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$

Configuration	$J^{PC}$	BCN	ChQM	Exp.
$DD$	$0^{++}$	3773.4	3796.8	3729.6
$D^*\bar{D}$	$1^{++}, 1^{+-}$	3908.0	3915.7	3871.7
$D^*\bar{D}^*$	$2^{++}$	4042.6	4034.6	4013.8
$BB$	$0^{++}$	10604.0	10555.8	10559.0
$B^*\bar{B}$	$1^{++}, 1^{+-}$	10653.5	10596.7	10604.6
$B^*\bar{B}^*$	$2^{++}$	10703.0	10637.6	10650.2

To calculate the spectra of the four-quark states  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$ , the Schrödinger equation Eq.(31) is solved by using the four-quark wavefunction Eq.(21). The converged results are obtained by taking the parameters of GEM as follows,  $\alpha = 12$ ,  $n = 7$ ,  $N = 7$ , and the ranges of  $s_n$  for  $\rho$  are from 0.1 to 6 fm, and 0.1

to 2 fm for  $\mathbf{R}$  and  $\mathbf{r}$ , respectively. Entem and Fernández believe the effective gluon mass  $m_g$  ranges from 0.6 GeV to 1.2 GeV [44], so we calculated spectra of  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  with  $m_g=0$  GeV, 0.9 GeV, 0.97 MeV, 1 GeV and without annihilation interaction. The results are listed in Table 4 and Table 5.

**Table 4** Energies (in MeV) of  $S$ -wave  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  with different effective gluon mass for the BCN model. 'No-anni' means without annihilation interaction

Configuration	$J^{PC}$	$E_T$	$\Delta E$	$E_T$	$\Delta E$
				No-anni	$m_g = 0$ GeV
$D\bar{D}$	$0^{++}$	3774.4	1	3774.6	1.2
$D^*\bar{D}$	$1^{++}$	3909	1	3909.2	1.2
$D^*\bar{D}$	$1^{-+}$	3909	1	3909.2	1.2
$D^*\bar{D}^*$	$2^{++}$	4043.6	1	4043.7	1.1
$B\bar{B}$	$0^{++}$	10604.3	0.3	10604.0	0.4
$B^*\bar{B}$	$1^{++}$	10653.8	0.3	10653.9	0.4
$B^*\bar{B}$	$1^{-+}$	10653.8	0.3	10653.9	0.4
$B^*\bar{B}^*$	$2^{++}$	10703.3	0.3	10703.0	0.5
				$m_g = 0.9$ GeV	$m_g = 1$ GeV
$D\bar{D}$	$0^{++}$	3765.1	-8.3	3773.1	-0.3
$D^*\bar{D}$	$1^{++}$	3891.8	-16.2	3905.4	-2.6
$D^*\bar{D}$	$1^{-+}$	3907.9	-0.1	3908.7	0.7
$D^*\bar{D}^*$	$2^{++}$	4028.5	-14.1	4040.4	-2.2
$B\bar{B}$	$0^{++}$	10659.3	-34.7	10591.4	-12.6
$B^*\bar{B}$	$1^{++}$	10608.9	-44.6	10634.5	-19.0
$B^*\bar{B}$	$1^{-+}$	10643.4	-10.1	10650.9	-2.6
$B^*\bar{B}^*$	$2^{++}$	10650.5	-42.5	10684.9	-18.1

From Table 4 and Table 5, we can find several interesting features. 1) If we don't take into account the annihilation interaction, no bound stats of  $D^{(*)}\bar{D}^{(*)}$  are found both in BCN and ChQM. However, there

**Table 5** Energies (in MeV) of  $S$ -wave  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  for the ChQM. 'No-anni' means without annihilation interaction.

Configuration	$J^{PC}$	$E_T$	$\Delta E$	$E_T$	$\Delta E$
		No-anni		$m_g = 0 \text{ GeV}$	
$D\bar{D}$	$0^{++}$	3797.8	1	3798.0	1.2
$D^*\bar{D}$	$1^{++}$	3916.6	0.9	3916.9	1.2
$D^*\bar{D}$	$1^{+-}$	3916.8	1.1	3916.9	1.2
$D^*\bar{D}^*$	$2^{++}$	4035.5	0.9	4035.8	1.2
$B\bar{B}$	$0^{++}$	10554.6	-1.2	10556.2	0.4
$B^*\bar{B}$	$1^{++}$	10592.4	-4.3	10597.2	0.5
$B^*\bar{B}$	$1^{+-}$	10597.0	0.3	10597.1	0.4
$B^*\bar{B}^*$	$2^{++}$	10633.8	-3.8	10638.1	0.5
$m_g = 0.9 \text{ GeV}$					
$D\bar{D}$	$0^{++}$	3796.9	0.1	3797.3	0.5
$D^*\bar{D}$	$1^{++}$	3913.2	-2.5	3915	-0.7
$D^*\bar{D}$	$1^{+-}$	3916.7	1.0	3916.7	1.0
$D^*\bar{D}^*$	$2^{++}$	4033.2	-1.4	4034.4	-0.2
$B\bar{B}$	$0^{++}$	10537.3	-18.5	10543	-12.8
$B^*\bar{B}$	$1^{++}$	10570.2	-26.5	10576.6	-20.1
$B^*\bar{B}$	$1^{+-}$	10595.9	-0.8	10596.6	-0.1
$B^*\bar{B}^*$	$2^{++}$	10613.4	-24.2	10619.4	-18.2

are three states of  $B^{(*)}\bar{B}^{(*)}$  with  $J^{PC} = 0^{++}, 1^{++}, 2^{++}$  having the energies lower than the corresponding thresholds in ChQM, due to the larger mass of the  $b$  quark than that of  $c$  quark, which leads to the kinetic energy of former are lower than the latter. No state appears in BCN means that the meson-exchange interaction between  $u(d, s)$  and  $\bar{u}(\bar{d}, \bar{s})$  plays an important role; 2) If we don't take into account the effective mass of the gluon in annihilation interaction, namely  $m_g = 0$  in Eq.(20), no bound state of  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  are found both in BCN and ChQM since the annihilation interaction is repulsive in this case. 3) With finite effective mass of gluon, almost all states under investigation can form molecules, having energies lower than the corresponding thresholds, and the binding energies decrease with the increasing effective mass of gluon. These results can be understood with the expression Eq.(20), where the magnitude of the denominator  $4m_q^2 - m_g^2$  increases with the increasing  $m_g$  when  $m_g > 2m_q \approx 626 \sim 674$  MeV. So the results are sensitive to the effective mass of gluon. We choose  $0.9 \sim 1.0$  GeV for the effective gluon mass, which is in accord with the effective constituent gluon mass found in the study of gluon dynamics in Ref.[51] and the glueball-quarkonia content of scalar-isoscalar mesons in Ref.[44]. For the  $D^*\bar{D}$  system, the  $D^*\bar{D}$  with  $J^{PC} = 1^{++}$  has about 2.5 MeV and 16.2 MeV binding energy for ChQM and BCN, respectively. The reason for the difference between two models is that the different masses of  $u, d$  quark in BCN and ChQM are used, and the annihilation interaction Eq.(20) is sensitive to the quark mass. If we take 1 GeV for effective gluon mass in BCN which

was chose in Ref. [27], the same binding energy as that of ChQM, 2.6 MeV is obtained. It is well known, the  $X(3872)$  was first found by Belle Collaboration in the  $J/\psi\pi^+\pi^-$  invariant mass spectrum in the decays of  $B^\pm \rightarrow K \pm J/\psi\pi^+\pi^-$ . D0, BaBar, CDF, CMS, BE-SIII, and LHCb have later confirmed the  $X(3872)$  by decays of  $B^{\pm,0}$  masons and  $pp$  collide, and affirmed the quantum number  $I^G(J^{PC}) = 0^+(1^{++})$ . The average mass of  $X(3872)$  listed in PDG is  $3871.69 \pm 0.17$  MeV, which is lower about 1 MeV than the threshold of  $D^*\bar{D}$ . Obversely, our result listed in Table 4 and 5 is agree well with the experimental data when we choose reasonable parameters, which are suggested by Giacosa, Gutsche, and Faessler in Ref.[44]. The weakly bound state of  $D^*\bar{D}^*$  with  $J^{PC} = 2^{++}$  are also obtained in our calculation. 4) For the  $B^{(*)}\bar{B}^{(*)}$  system, four bound states are obtained when we take into account the annihilation interaction. The bound state of  $B^*\bar{B}$  with  $J^{PC} = 1^{+-}$  is a good candidate for the  $Z_b^0(10610)$ , which is firstly found in the  $\Upsilon(2,3S)\pi^0$  invariant mass spectrum in the  $\Upsilon(10860) \rightarrow \Upsilon(1,2,3S)\pi^0\pi^0$  by Belle Collaboration[52].

## 5 Summary

The constituent quark model is extended by introduced the  $s$ -channel one gluon exchange interaction, which does not appear in the conventional mesons of  $q\bar{q}$ . We dynamically calculate the spectrum of  $S$ -wave  $D^{(*)}\bar{D}^{(*)}$  and  $B^{(*)}\bar{B}^{(*)}$  system in the extended quark models. The annihilation interaction is repulsive if we don't take into account the effective gluon mass in it. However, if we take massive gluon propagator and reasonable effective gluon mass in the  $s$ -channel one gluon exchange interaction, two molecular states  $D^*\bar{D}$  with  $I^G(J^{PC}) = 0^+(1^{++})$  and  $B^*\bar{B}$  with  $J^{PC} = 1^{+-}$  are obtained, which they are good candidates for the  $X(3872)$  and  $Z_b^0(10610)$ , respectively. The  $D^*\bar{D}^*$ ,  $B^*\bar{B}^*$  with  $J^{PC} = 2^{++}$ ,  $B^*\bar{B}$  with  $J^{PC} = 1^{++}$  and  $B\bar{B}$  with  $J^{PC} = 0^{++}$  are also predicted in these extended constituent quark models. Further experimental searches by LHCb, BaBar, Belle and other collaborations are needed to clarify whether these states exist or not.

In the present calculation, the one gluon annihilation interaction with effective gluon mass plays an important role. The interaction does show up in the ordinary mesons even with the same flavor because of the color structure. The effective mass of gluon is a model parameter, which does not bound much by the experimental data. Because the calculated results are sensitive to the effective mass of gluon, a better way to fix this parameter is expected.

## Acknowledgment

This work is supported partly by the National Science Foundation of China (under Contracts Nos.11265017, 11675080, 11535005, 11475085 and 11690030), and the China Postdoctoral Science Foundation (under Grant No.2015M571727), and by the Guizhou province outstanding youth science and technology talent cultivation object special funds (Grant No. QKHZ(2013)28).

## References

1. Hua-Xing Chen and Wei Chen and Xiang Liu and Shi-Lin Zhu, Phys.Rept. **639**, 1 (2016).
2. A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni, and A. D. Polosa, Int.J.Mod.Phys. **A 30** (2014); [arXiv:1411.5997].
3. A. Esposito, A. Pilloni, and A. D. Polosa, Phys. Rep. **668** 1 (2017); [arXiv:1611.07920].
4. S.-K. Choi et al. (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
5. D. Acosta et al. (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
6. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).
7. R. Aaij et al. (LHCb Collaboration), Eur. Phys. Rev. J. C **72**, 1972 (2004).
8. B. Aubert et al. (BaBar Collaboration), Phys. Rev. D **71**, 071103 (2005).
9. S. Chatrchyan et al. (CMS Collaboration), JHEP **04**, 154 (2013).
10. M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. **112**, 092001 (2014).
11. R. Aaij, et al., [LHCb Collaboration], Phys. Rev. Lett. **110**, 222001 (2013).
12. C. Patrignani et al. (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016).
13. C.-Y. Wong, Phys. Rev. C **69**, 055202 (2004).
14. E.S. Swanson, Phys. Lett. B **588**, 189 (2004).
15. M. Suzuki, Phys. Rev. D **72**, 114013 (2005)
16. C.E. Thomas, F.E. Close, Phys. Rev. D **78**, 034007 (2008).
17. X. Liu, Z.-G. Luo, Y.-R. Liu, S.-L. Zhu, Eur. Phys. J. C **61** 411C428 (2009).
18. I.W. Lee, A. Faessler, T. Gutsche, V.E. Lyubovitskij, Phys. Rev. D **80** 094005 (2009).
19. D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010)
20. A. Ozpineci, C. W. Xiao and E. Oset, Phys. Rev. D **88**, 034018 (2013)
21. D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007)
22. D. Gamermann and E. Oset, Eur. Phys. J. A **36**, 189 (2008)
23. R. Molina and E. Oset, Phys. Rev. D **80**, 114013 (2009)
24. J. Vijande, E. Weissman, N. Barnea, and A. Valcarce, Phys. Rev. D **76** 094008 (2007).
25. Yan-Rui Liu, and Zong-Ye Zhang, Phys. Rev. C **79**, 035206 (2009)
26. Youchang Yang, Jialun Ping, Chengrong Deng, Meng Wan, Int.J.Mod.Phys.Conf.Ser. **29**, 1460227(2014)
27. Bao-Kai Wang, Wei-Zhen Deng, Xiao-Lin Chen, Chin. Phys. C **34**, 105 (2010).
28. E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).
29. Youchang Yang, Chengrong Deng, Jialun Ping, and T. Goldman, Phys. Rev. D **80**, 114023 (2009).
30. Youchang Yang, and Jialun Ping, Phys. Rev. D **81**, 114025 (2010)
31. You-Chang Yang, Jialun Ping, Chengrong Deng and Hong-Shi Zong, J. Phys. G: Nucl. Part. Phys. **39**, 105001(2012)
32. Qin-Xiang Gao, You-Chang Yang and Jialun Ping, J. Phys. G: Nucl. Part. Phys. **39**, 045001 (2012)
33. Chengrong Deng, Jialun Ping, Youchang Yang, and Fan Wang, Phys. Rev. D **86**, 014008 (2012)
34. Chengrong Deng, Jialun Ping, Youchang Yang, and Fan Wang Phys. Rev. D **88**, 074007 (2013)
35. R.K. Bhaduri, L. E. Cohler, Y. Nogami, Nuovo Cimento **65A** 376 (1981).
36. D. Janc, M. Rosina, Few-Body Systems **35**, 175-196 (2004).
37. Vijande J, Fernandez F and Valcarce A, J. Phys. G **31**,481 (2005).
38. N. Kaiser, S. Grestendorfer and W. Weise, Nucl. Phys. **A637**, 395 (1998); E. Oset, H. Toki, M. Mizobe and T.T. Takahashi, Prog. Theor. Phys. **103**, 351 (2000); M.M. Kaskulov and H. Clement, Phys. Rev. **C70**, 014002 (2004); L. Z. Chen, H. R. Pang and H. X. Huang, et al., Phys. Rev. C. **76**,014001 (2007).
39. R. K. Bhaduri, L. E. Cohler and Y. Nogami, Phys. Rev. Lett. **44**, 1369 (1980).
40. John D. Weinstein, Nathan Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. D **27**, 588 (1983); **41**, 2236 (1990).
41. Hong-Shi Zong, Fan Wang, Jia-Lun Ping, Commun. Theor. Phys. **22**:479-482 (1994).
42. R. Gupta, G. Guralnik, G. Kilcup, A. Patel, S. R. Sharpe and T. Warnock, Phys. Rev D **36**, 2813 (1987).
43. Z.Li, M.Guidry, T.Barnes and E.S.Swanson,arXiv:hep-ph/9401326.
44. F. Giacosa, T. Gutsche, and Amand Faessler, Phys. Rev C **71**, 025202 (2005).
45. M.Dillig and M.Schott, Phys. Rev C **75**, 067001 (2007).
46. J. E. Mandula, Phys. Rep. 315, 273 (1999).
47. D. Dugal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, and S. P. Sorella, JHEP 0401,044 (2004).
48. A. V. Manohar, M. B. Wise, Nucl. Phys. B **399**, 17 (1993).
49. B. Silvestre-Brac and C. Semay, Z. Phys. C 57,273-282 (1993); 59,457-470(1993).
50. D. M. Brink, Fl. Stancu, Phys. Rev. D **57**,11 (1998).
51. E. Gubankova, C. R. Ji, and S. R. Cotanch, Phys. Rev. D **62**, 074001 (2000).
52. P. Krokovny, A. Bondar, I. Adachi, et al. (Belle Collaboration), ,Phys. Rev. D **88**, 052016 (2013).