

Testing the tetraquark structure for the X resonances in low-lying region

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Assuming four-quark structure for the X resonances in low-lying region, we calculate their masses using the color-spin interaction. In specific, the hyperfine masses of the color-spin interaction are calculated for the possible states in spin-0, spin-1, spin-2 channels. The two states in spin-0 channel as well as the two states in spin-1 channel are diagonalized in order to generate the physical hyperfine masses. By matching the difference in hyperfine masses with the splitting in corresponding hadron masses and using the $X(3872)$ mass as an input, we estimate the masses corresponding to the states $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$. We find that the masses of two states in 1^{+-} are close to those of $X(3823)$, $X(3900)$, and the mass of the 2^{++} state is close to that of $X(3940)$. For them, the discrepancies are about ~ 10 MeV. This may suggest that the quantum numbers of the controversial states are $X(3823) = 1^{+-}$, $X(3900) = 1^{+-}$, $X(3940) = 2^{++}$. In this work, we use the same inputs parameters, the constituent quark masses and the strength of the color-spin interaction, that have been adopted in the previous work on the D or B -meson excited states. There, it was shown that the four-quark structure can be manifested in their excited states. Thus, our results in this work provide a consistent treatment on open- and hidden-charm mesons as far as the four-quark model is concerned.

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I. INTRODUCTION

The exotic resonances, commonly referred to XYZ particles, are interesting subjects in hadron physics in recent years. This interest was triggered by the measurement of $X(3872)$ back in 2003 by the Belle collaboration [1] whose existence was further confirmed in the later experiments [2–4].

The theoretical issues on $X(3872)$ are mainly on its structure. The traditional picture would be the quarkonium where it is viewed as heavy quark-antiquark bound state, $\bar{c}c$ [5]. The other picture for $X(3872)$ is a hadronic molecule like DD^* [6, 7], where the two mesons are bounded weakly by hadronic interactions. There is also a hybrid model like the type $\bar{c}cg$ [8] containing a gluon. This type of the hybrid model is also applied [9] to the other resonance with higher mass, $Y(4260)$, whose existence is experimentally reported in Ref. [10]. Another exciting scenario for $X(3872)$ would be a long-sought tetraquark. Along this line, the tetraquark possibility was investigated under a traditional picture based on a diquark-antidiquark form [11, 12] as well as under a more extensive treatment [13, 14]. As far as we know, there is no consensus on the structure among all these possibilities. The recent review given in Ref. [15] might be useful for current status of XYZ spectroscopy and possible future direction.

Additional resonances reported from later experi-

ments, which have similar masses with $X(3872)$, escalate the interest further because of the expectation that they might share a similar structure. These include $X(3823)$ [16], $X(3900)^{\pm}$ [17, 18], $X(3900)^0$ [19], $X(3940)$ [20]. These resonances, including $X(3872)$, constitute low-lying resonances in this X spectrum. Of course, there are other resonances with higher masses, $Y(4260)$ [10], $Z(4430)$ [21] etc, which provide other exciting areas that should be investigated in future.

One practical problem, in dealing with these resonances especially in low-lying region, is that their quantum numbers are scarcely known. According to the Particle Data Group (PDG) [22], the only resonance whose quantum numbers are completely determined is $X(3872)$. The quantum numbers of others in this low-lying region are quite unknown as one can see from Table I. This precisely makes it difficult to construct a realistic model for them. In particular, for $X(3823)^0$, most of the quantum numbers are unknown except its charge conjugation being odd. One interpretation for this state is an accompanying member of $\psi(3770)$ when $\psi(3770)$ is viewed as a 3D_1 of $\bar{c}c$. Namely, $X(3823)^0$ could be a 3D_2 state but, without confirming its quantum numbers, the issue is not settled yet. The exotic possibility is still open for this resonance. Therefore, at this stage, the available data on these resonances which can be used to test a model construction are their measured masses and decay modes.

In this work, we investigate the tetraquark possibility for the X resonances in low-lying region. Especially we look for such a possibility from the electrically neutral members, $X(3823)^0$, $X(3872)^0$, $X(3900)^0$, $X(3940)^0$

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Meson	$I^G(J^{PC})$	Mass (MeV)	Γ (MeV)
$X(3823)^0$??(?) ⁻	3823.1	< 24
$X(3872)^0$	0 ⁺ (1 ⁺⁺)	3871.7	< 1.2
$X(3900)^{\pm}$?(1 ⁺)	3888.7	35
$X(3900)^0$?(?)	3904	-
$X(3940)^0$?(??)	3942	37

TABLE I. The X resonances collected from PDG [22] in low-lying region. The charge, denoted by a superscript in the name of each resonance, is assigned based on its decaying channel. Note, the measured masses in the third column are slightly different from the numbers appearing in the resonance names in the first column. The measured masses in the third column will be used in our analysis.

by assuming that they belong to the same isospin multiplet. Since $X(3872)$ is isoscalar ($I = 0$), this assumption is equivalent to the statement that the other resonances, $X(3823)^0$, $X(3900)^0$, $X(3940)^0$, are also isoscalar. Of course, according to the $SU(3)_f$ symmetry, one can expect their charged partners (isovector) in the same $SU(3)_f$ multiplet whose existence can support our isoscalar framework more. Currently there is only one candidate for them in the similar mass region, $X(3900)^{\pm}$, which is not enough to support our isoscalar tetraquark fully. As more or more resonances are updating in this scarcely explored region, one can expect that more isovector resonances might appear in future. One possible explanation for the lack of the isovector resonances in the current PDG might due to their large width, which makes them difficult to be measured. In particular, the isovector resonances can decay to $\bar{c}c + \pi$ which is however hindered by the isospin conservation for the isoscalar X resonances.

We introduce the four-quark states consisting of diquark-antidiquark, $cq\bar{c}\bar{q}$, ($q = u, d$) and test their justification by calculating the masses using the color-spin interaction. The construction of diquark-antidiquark and its application in light-quark sectors can be found in Ref. [23–25]. This type of four-quark model for the X resonances was also studied by Maiani *et al.* [11, 12]. They introduced this type of wave function and calculated the masses using the color-spin interaction. In their approach, the constituent quark masses as well as the strength of the color-spin interaction are all fitted to the low-lying baryon and meson spectra. What we want to do in this work is to upgrade this approach by implementing a few ingredients.

Firstly, we notice that the color-spin interaction alone is only useful to calculate the mass splittings through the hyperfine mass splittings. The color-spin interaction does not represent the full interaction between two quarks and, therefore, may not be enough to calculate the hadron mass itself. In specific, there are additional potentials such as the color-electric term and the constant shift as will be discussed below. What makes the color-spin interaction special is that, in the mass splittings, the color-spin interaction survives while the additional potentials

cancel. Secondly, as we will discuss below, there is a physical issue like mixing in the possible spin states. If one constructs spin states from $cq\bar{c}\bar{q}$, ($q = u, d$), there are two states in spin-0 and three states in spin-1. Some of them must mix in generating the mass eigenstates, which constitutes another ingredient in this development.

Another thing, which is more important to us, is that this type of approach is closely related to the earlier work [26]. Namely, in Ref. [26] where two authors (H.Kim and M.K.Cheoun) in the present paper are involved, we suggested that most of the D or B -meson excited states currently listed in PDG, especially their mass spectrum, can be understood if they are viewed as tetraquarks with the diquark-antidiquark form, $cq\bar{q}\bar{q}$, ($q = u, d, s$). Using the strength of the color-spin interaction fixed from the mass difference of $D_2^{*0}(2463) - D_0^{*0}(2318)$, we were able to reproduce the masses of other resonances in the excited states of D and B mesons quite successfully. Also our four-quark model provides interesting phenomenology related to decays of spin-1 mesons, which seems to fit nicely with experimental observation. Based on its phenomenological success, we made some predictions for the D and B mesons to be found in future. This approach in Ref.[26] can be straightforwardly applied to the hidden-charm case here. Through this application, one can test the parameters fixed in Ref. [26] which can help to construct a consistent four-quark picture that might be relevant for both, open- and hidden-charm mesons.

The paper is organized as follows. In Sec. II, we present four-quark wave functions that could be relevant for the X resonances. The color-spin interaction and its application to the hadron spectroscopy is discussed in Sec. III. In Sec. IV, we present our calculations of the hyperfine masses and discuss their implication in the X spectroscopy. We summarize in Sec. V.

II. WAVE FUNCTIONS FOR THE X RESONANCES

In this section, we introduce the four-quark wave functions which could be relevant for the X resonances. These wave functions, in our best recollection, was first used by Maiani *et al.* [11, 12]. Here we follow their argument closely but make a few statements which need to be implemented in this update.

The X resonances are hidden-charm mesons whose decay modes mostly entail J/ψ , $D\bar{D}$, $D\bar{D}^*$, etc, in their final states. Based on this observation, one can assume that their wave functions in the four-quark picture take the diquark-antidiquark form, $cq\bar{c}\bar{q}$, ($q = u, d$). One can show that the diquark (antidiquark) forms a bound state when it is in $\mathbf{3}_c$ ($\mathbf{3}_c$) and in the spin state $J = 0$ [23–25]. These two colorful objects, a diquark and an antidiquark, can be combined to make a four-quark state. Then following Refs. [11, 12], we take the approximate spin independence of the heavy quark interaction to assume that the $J = 1$ diquark (as well as the $J = 1$ antidiquark) also

forms a bound state.

For the other color configuration for the diquark cq , namely $\mathbf{6}_c$, we do not consider this possibility firstly because this is less compact than the $\mathbf{3}_c cq$ in the spin-0 channel. Secondly, including the $\mathbf{6}_c$ scenario through the mixing with the $\bar{\mathbf{3}}_c$ doubles the possible states, which do not seem to fit the current X phenomenology. More importantly, the similar tetraquark framework applied to the excited states of D, D_s, B, B_s mesons in Ref. [26], where the $\bar{\mathbf{3}}_c$ diquark is adopted only, provides nice phenomenological consequences, such as mass splittings. Moreover, this approach predicts two spin-1 resonances which fits very well with $D_{s1}^\pm(2460), D_{s1}^\pm(2535)$. It is our interest to test whether the similar framework works for the other hadrons like the X resonances.

Instead of this type of diquark-antidiquark picture, one can also consider a four-quark structure of the form $q\bar{q} - c\bar{c}$ because of the expectation that the color-spin interaction can be more attractive for $q\bar{q}$ than for the $\bar{\mathbf{3}}_c cq$. Indeed, the color-spin interaction is most attractive when $q\bar{q}$ is in a state with color singlet and spin zero, i.e., $(\mathbf{1}_c, J = 0)$. With this $q\bar{q}$, $q\bar{q}$ and $c\bar{c}$ are both *colorless*. This leads to a hadronic molecule of meson-meson bound state, which tends to be separate with a long-range interaction.

Now with the other possible color-spin configurations for $q\bar{q}$, $(\mathbf{8}_c, J = 0), (\mathbf{8}_c, J = 1)$, one can still construct tetraquarks of the form, $q\bar{q} - c\bar{c}$, because now $q\bar{q}$ and $c\bar{c}$ are both *colorful*. Then the question is whether the $q\bar{q}$ with $(\mathbf{8}_c, J = 0), (\mathbf{8}_c, J = 1)$ configurations are more compact than the $\bar{\mathbf{3}}_c cq$. One can check this explicitly by calculating the expectation value of the color-spin interaction with respect to the $q\bar{q}$ and the $\bar{\mathbf{3}}_c cq$ states respectively. Using the constituent quark masses adopted in our work, we find that the $\bar{\mathbf{3}}_c cq$ with $J = 0$ is most attractive among them. Thus, the diquark cq with the color $\bar{\mathbf{3}}_c$ and spin $J = 0$ is appropriate for the tetraquark study.

For the $J = 1$ diquark cq with $\bar{\mathbf{3}}_c$, its color-spin interaction is stronger than the $q\bar{q}$ with $(\mathbf{8}_c, J = 0)$ but weaker than the $q\bar{q}$ with $(\mathbf{8}_c, J = 1)$. In this sense, it is not clear whether the $J = 1$ diquark is appropriate for the tetraquark study. However, at the same time, since $c\bar{c}$ is less compact than $q\bar{q}$, it is also not clear whether the composite system, $q\bar{q} - c\bar{c}$, is easier to form than the $cq - \bar{c}\bar{q}$ when the $J = 1$ diquark is involved.

As we stated above, we take the approximate spin independence of the heavy quark limit to assume that the $J = 1$ diquark forms a bound state. Note, however, that the calculation based on the diquark-antidiquark basis, $cq\bar{c}\bar{q}$, does not rule out the $q\bar{q} - c\bar{c}$ configuration entirely. Since we calculate the color-spin interactions among all the quark pairs [see Eq.(6) below], the $q\bar{q}$ components with the color-octet as well as the color-singlet are also included in our framework.

In our work, we assume that the X resonances, $X(3823)^0, X(3872)^0, X(3900)^0, X(3940)^0$, are isoscalar. Then the flavor structure would be like $[cu\bar{c}\bar{u} + cd\bar{c}\bar{d}] / \sqrt{2}$.

Technically, this is equivalent to considering the configuration of the form, $cu\bar{c}\bar{u}$, within our framework facilitating the color-spin interaction.

For illustration purpose, we label the four quarks in the state, $cu\bar{c}\bar{u}$, with the numerical indices 1234. So [12] refers the two quarks in the diquark cu and [34] refers to the two quarks in the antidiquark $\bar{c}\bar{u}$. Using this labeling, we can explain our formalism more clearly on the one hand, and on the other hand, the general expressions involving the indices can be easily extended to other situations in future.

Assuming that all the quarks are in an S -wave state, the possible spins for the four-quark states are $J = 0, 1, 2$. If we denote these spin states in terms of the total spin J , the diquark spin J_{12} , the antidiquark spin J_{34} , *i.e.*, $|J, J_{12}, J_{34}\rangle$, then we come up with the possible spin configurations for each spin state as follow,

$$\begin{aligned} J = 0: & |000\rangle, \quad |011\rangle, \\ J = 1: & |111\rangle, \quad |110\rangle, \quad |101\rangle, \\ J = 2: & |211\rangle, \end{aligned} \quad (1)$$

where all the states have the positive parity by their construction. From these spin configurations, one can investigate how they behave under charge conjugation [11, 12]. The states in $J = 0$ and $J = 2$ are even under charge conjugation C . $|111\rangle$ is odd under C . And $|110\rangle$ and $|101\rangle$ transform among each other under C . Using these properties, one can classify the spin configurations in terms of the states based on the quantum numbers J^{PC} , which then can be identified with physical states easily.

For $J = 2$, there is only one spin configuration which can be denoted as

$$|2^{++}\rangle = |211\rangle. \quad (2)$$

For $J = 0$, we have two spin configurations and these have the same quantum numbers $J^{PC} = 0^{++}$. To distinguish the two states in $|J^{PC}\rangle$ notation, we further label them, using a and b , as

$$|0_a^{++}\rangle = |000\rangle, \quad |0_b^{++}\rangle = |011\rangle. \quad (3)$$

What we want to point out in this work is that these two are not the mass eigenstates. Indeed, as we will show in Sec. IV, the mixing element of the color-spin interaction, $\langle 0_a^{++} | V | 0_b^{++} \rangle$, is not zero. Thus, the mass eigenstates, which can be recognized as the physical states, should be linear combinations of the two in Eq. (3).

For $J = 1$, we have three spin configurations. We can assign one of them or a combination of them to $X(3872)$. $X(3872)$ has well-determined quantum numbers, *i.e.*, $J^{PC} = 1^{++}$. Only possible way to generate this state with 1^{++} from the three configurations is the following symmetric combination [11, 12],

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}[|110\rangle + |101\rangle]. \quad (4)$$

The antisymmetric combination of $|110\rangle$ and $|101\rangle$ generates the state belonging to $J^{PC} = 1^{-+}$ and the other

spin-1 state, $|111\rangle$, also belongs to $J^{PC} = 1^{+-}$. Thus, we have two states in $J^{PC} = 1^{+-}$ given by

$$|1_a^{+-}\rangle = \frac{1}{\sqrt{2}}[|110\rangle - |101\rangle], \quad |1_b^{+-}\rangle = |111\rangle, \quad (5)$$

where another labeling, a and b , has been introduced to distinguish the two. Here in $J = 1$ channel again, these two states are not the mass eigenstates as there is mixing among them through the color-spin interaction. Their linear combinations which diagonalize the hyperfine masses are the mass eigenstates and, therefore, can be identified as the physical states.

III. COLOR-SPIN INTERACTIONS

The color-spin interaction provides a simple way to calculate the *mass splittings* among hadrons with the same flavor content, once the wave functions are constructed in terms of quark fields. But, in its application to the hadron masses instead of the splittings, one may need to consider additional terms like color-electric and constant terms which are often neglected in common practices. In this section, we want to look at this aspect more carefully in order to explain why the color-spin interaction works well for the mass splittings but not for the masses themselves.

The color-spin interaction takes the following simple form [27–30]

$$V = \sum_{i < j} v_0 \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i m_j}. \quad (6)$$

Here λ_i denotes the Gell-Mann matrix for the color SU(3), J_i the spin, and m_i the constituent mass of the i -th quark. The parameter v_0 represents the strength of the color-spin interaction to be fitted from the hadron masses. It is common practice to write down the hadron masses formally by

$$M_H \sim \sum_i m_i + \langle V \rangle, \quad (7)$$

where the hyperfine mass $\langle V \rangle$ is the expectation value of the color-spin interaction evaluated with respect to an appropriate hadron wave function of concern. Phenomenologically, the successful aspect of the color-spin interaction is that the mass splitting among hadrons (with the same flavor content) can be estimated simply by the difference in the hyperfine masses [26, 31, 32],

$$\Delta M_H \sim \Delta \langle V \rangle. \quad (8)$$

This relation can be tested, for instance, in the baryon sector. In this case, one can fix the strength, v_0 in Eq.(6), for example, from the $\Delta - N$ mass difference, and using the standard values for the constituent quark masses, $m_u = 330$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV, one can calculate the mass differences among other hadrons

like $\Sigma - \Lambda$, etc, through Eq. (8). The agreement with actual mass splittings is quite good, with only ~ 10 MeV error. (See Table VI in Ref. [26].)

But, if one calculates the actual masses using the formal relation, Eq.(7), the results do not agree with the experimental masses as one can see from the third column in Table II. The disagreement is about 300 MeV for charmed baryons, but for the rest, it is about 100 MeV. So, the disagreement even has a flavor dependence also. If one sticks to the relation Eq.(7) firmly, one may attribute this disagreement to the parameters involved, v_0 and the quark masses. One can then adjust these parameters to reproduce the baryon masses and use them to calculate the masses of other hadrons [11]. But our standpoint is that there are physical reasons why Eq.(7) fails in reproducing the masses.

Physically, the interaction between two quarks inside a hadron can have two different sources, one-gluon-exchange potential V_{OGE} and instanton-induced potential V_{ins} [33, 34]. The color-spin interaction is a common ingredient in these potentials but there are additional terms, the color-electric type, $\sim \lambda_i \cdot \lambda_j$, and the constant term. Therefore, it may be more natural to think that the disagreement in the masses above may come from these types of interactions instead of adjustment of the parameters involved in Eq.(7).

Effectively, the additional potentials, being the color-electric type and the constant shift, can be parameterized as

$$V_{add} = \sum_{i < j} v_1 \frac{\lambda_i \cdot \lambda_j}{m_i m_j} + v_2, \quad (9)$$

with the additional parameters v_1 and v_2 to be fitted from some of the baryon masses. Then the baryon mass is given by

$$M_H \sim \sum_i m_i + \langle V \rangle + \langle V_{add} \rangle. \quad (10)$$

Now by fitting the additional parameters v_1 and v_2 to the masses of N and Λ , for example, we obtain other masses given in the fourth column of Table II. The results are better than the ones determined from the color-spin interaction only, though there are still disagreement in the charmed baryons more than 160 MeV or so. One can adjust the parameters at this stage to reproduce the charmed baryon masses using Eq. (10). In particular, by changing m_c from 1500 MeV to 1670 MeV, we come out with much better fits to the charmed baryon masses, namely, $M_{\Sigma_c} = 2450$ MeV, $M_{\Lambda_c} = 2294$ MeV, $M_{\Sigma_c^*} = 2508$ MeV. Although the results are nice but, at the same time, we are facing the results that are somewhat sensitive to the input parameters. Then it is questionable whether this new set of parameters can be consistently applied to other hadrons altogether.

What is interesting is that the additional potentials $\langle V_{add} \rangle$ in Eq.(10) cancel in the mass splittings among hadrons with the same flavor content. Since two quarks

Baryon	M_{expt}	M_H from Eq.(7)	M_H from Eq.(10)	M_H from Eq.(8)
N	940	844	940 (input)	940 (input)
Δ	1232	1136	1232 (input)	1232 (input)
Σ	1193	1080	1182	1182
Λ	1116	1014	1116 (input)	1116 (input)
Σ^*	1385	1273	1375	1375
Ξ	1320	1223	1330	1320 (input)
Ξ^*	1531	1415	1522	1513
Σ_c	2453	2166	2276	2438
Λ_c	2286	2014	2124	2286 (input)
Σ_c^*	2518	2230	2340	2502

TABLE II. Comparison between baryon masses from PDG and the ones calculated in the three different ways. (See also the text for detail.) All masses are given in MeV. The masses in the third column calculated using the color-spin interaction only where the coupling strength, v_0 , is fitted from the $\Delta - N$ mass difference. In obtaining the fourth column, the additional parameters v_1 and v_2 in Eq.(9) are fitted by using the N, Λ masses while v_0 is still fitted from the $\Delta - N$ mass difference. The fifth column shows the calculated the masses of $\Sigma, \Sigma^*, \Xi^*, \Sigma_c, \Sigma_c^*$, when other resonance masses are used as inputs in the context of Eq. (8).

inside a baryon are always in the color state $\bar{\mathbf{3}}_c$, the expectation value of the $\lambda_i \cdot \lambda_j$ term is the same for all the baryons considered in Table II. The constant term is also the same for all the baryons. The flavor dependence comes from the quark masses only. Hence, the relation for the mass splitting, Eq.(8), still holds regardless of the additional potentials as long as one considers hadrons with the same flavor content.

Therefore, a better estimation for the baryon masses can be made if one calculates the mass splitting, ΔM_H , via Eq. (8), and estimates one of the baryon masses involved in the splitting using other baryon mass as an input. For example, since $\Sigma - \Lambda$ mass difference is well fitted to $\Delta\langle V \rangle = \langle V \rangle_\Sigma - \langle V \rangle_\Lambda$, one can estimate the Σ mass by $M_\Sigma = M_\Lambda + \Delta\langle V \rangle$ using the experimental mass M_Λ as an input. Likewise, using $M_N, M_\Delta, M_\Xi, M_{\Lambda_c}$ as inputs, we estimate the other baryon masses and they are given in the fifth column in Table II, which fit very well with their experimental values. The reason why this method gives the best estimation among the three calculations in Table II is simply because the mass splittings fit to hyperfine mass splittings quite well without suffering from the additional potentials. To check the sensitivity to the input parameters, we also change m_c from 1500 MeV to 1670 MeV as above, and obtain $M_{\Sigma_c} = 2442$, $M_{\Sigma_c^*} = 2500$ MeV. These numbers are not so different from the ones in the fifth column in Table II, indicating that the results are not so sensitive to the input parameters. One may argue that this way of estimating the baryon masses is not so persuasive as the half of the resonances are used as inputs. The upshot that we are driving at is that the color-spin interaction can be powerful only when it is used in the context of the mass splittings.

More importantly, this way of estimating the masses, relying only on hyperfine mass splittings, can be very powerful in its application to the X resonances. As all the X resonances are assumed to have the same flavor content, once the mass splittings are calculated via Eq. (8), we can estimate all the masses using only one resonance

mass as an input. Based on the experience in the baryon sector as well as in the meson sector, this method relying on the mass splittings can determine the masses quite reliably.

IV. RESULTS AND DISCUSSION

Now we present our calculation for the hyperfine masses which can be used to estimate the mass splittings among the X resonances. The hyperfine mass is the expectation value of Eq. (6) with respect to the corresponding state that we have introduced in Sec. II. Then the color and spin parts need to be calculated separately. For the color part, since the diquark and antiquark are in $\bar{\mathbf{3}}_c, \mathbf{3}_c$ respectively, one can easily evaluate the expectation value of the terms like $\lambda_1 \cdot \lambda_2$ and $\lambda_3 \cdot \lambda_4$. For the other combinations like $\lambda_1 \cdot \lambda_3$ and $\lambda_2 \cdot \lambda_3$, etc, one can rearrange the wave functions from the diquark-antiquark ([12][34]) basis into the ([13][24]) basis or the ([14][23]) basis and evaluate the expectation values straightforwardly. One can take the similar steps to calculate the spin parts also. (See Ref. [26] for technical details.)

Table III shows our results for the hyperfine masses evaluated for the states, $|0_a^{++}\rangle, |0_b^{++}\rangle, |1^{++}\rangle, |1_a^{+-}\rangle, |1_b^{+-}\rangle, |2^{++}\rangle$, as well as the mixing terms appearing in spin-0, spin-1 channels through $\langle 0_a^{++}|V|0_b^{++}\rangle, \langle 1_a^{+-}|V|1_b^{+-}\rangle$. The origin of the mixing is that, even though the diquark ([12]) and antiquark ([34]) are in definite color and spin states, the quark pairs like [13], [14], [23], [24], are not in definite color and spin states. The color-spin interaction between two quarks in such a pair gives nonzero contribution in the matrix elements of $\langle 0_a^{++}|V|0_b^{++}\rangle, \langle 1_a^{+-}|V|1_b^{+-}\rangle$. One can explicitly see this from our formulas for the mixing terms in Table III, where only the terms related to such a pair appear. Note, the state $|1^{++}\rangle$ can not mix with $|1_a^{+-}\rangle$ or $|1_b^{+-}\rangle$ because they are in the different states in charge conjugation. Or

States, $ J^{PC}\rangle$	Hyperfine mass $\langle J^{PC} V J^{PC}\rangle_{q_1 q_2 \bar{q}_3 \bar{q}_4}$
$ 0_a^{++}\rangle$	$2v_0 \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} \right]$
$ 0_b^{++}\rangle$	$-\frac{2}{3} v_0 \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} - \frac{1}{m_1 m_3} - \frac{1}{m_1 m_4} - \frac{1}{m_2 m_3} - \frac{1}{m_2 m_4} \right]$
Mixing $[0_a^{++}\rangle, 0_b^{++}\rangle]$	$\frac{1}{\sqrt{3}} v_0 \left[\frac{1}{m_1 m_3} - \frac{1}{m_1 m_4} - \frac{1}{m_2 m_3} + \frac{1}{m_2 m_4} \right]$
$ 1^{++}\rangle$	$\frac{1}{3} v_0 \left[\frac{2}{m_1 m_2} + \frac{2}{m_3 m_4} - \frac{1}{m_1 m_3} + \frac{1}{m_1 m_4} + \frac{1}{m_2 m_3} - \frac{1}{m_2 m_4} \right]$
$ 1_a^{+-}\rangle$	$\frac{1}{3} v_0 \left[\frac{2}{m_1 m_2} + \frac{2}{m_3 m_4} + \frac{1}{m_1 m_3} - \frac{1}{m_1 m_4} - \frac{1}{m_2 m_3} + \frac{1}{m_2 m_4} \right]$
$ 1_b^{+-}\rangle$	$-\frac{2}{3} v_0 \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} - \frac{1}{2m_1 m_3} - \frac{1}{2m_1 m_4} - \frac{1}{2m_2 m_3} - \frac{1}{2m_2 m_4} \right]$
Mixing $[1_a^{+-}\rangle, 1_b^{+-}\rangle]$	$-\frac{2}{3} v_0 \left[\frac{1}{m_1 m_3} - \frac{1}{m_2 m_4} \right]$
$ 2^{++}\rangle$	$-\frac{2}{3} v_0 \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} + \frac{1}{2m_1 m_3} + \frac{1}{2m_1 m_4} + \frac{1}{2m_2 m_3} + \frac{1}{2m_2 m_4} \right]$

TABLE III. The formulas for the hyperfine masses are provided here, which are obtained from expectation values of the color-spin interaction with respect to the states indicated in the first column. The formulas presented here are for a general flavor combination, $q_1 q_2 \bar{q}_3 \bar{q}_4$. For the X resonances in this work, one needs to replace simply $m_1 = m_c, m_2 = m_u, m_3 = m_c, m_4 = m_u$.

one can explicitly check this in our approach by showing that $\langle 1^{++}|V|1_a^{+-}\rangle = \langle 1^{++}|V|1_b^{+-}\rangle = 0$.

In our numerical calculations, the input parameters are m_c , m_u , and v_0 in Eq. (6). For a consistent treatment with our earlier work on the D and B meson excited states [26], we take the same parameters, namely $m_c = 1500$ MeV, $m_u = 330$ MeV, $v_0 \sim (-193)^3$ MeV 3 . In Ref. [26], the strength v_0 was fixed by the mass difference $D_2^{*0}(2463) - D_0^{*0}(2318)$ in the four-quark formalism for the D and B meson excited states. This value is slightly different from the one fitted from the $\Delta - N$ mass difference in the baryon sector. But v_0 becomes larger when it is fitted from the $\rho - \pi$ mass difference. Thus, the extracted value for v_0 has some dependence on the number of quarks consisting the hadrons and this is a limitation of the color-spin interaction at the moment. Then the question is which v_0 to use in our work. It may be quite realistic to use the v_0 determined from the D meson spectrum in Ref. [26] because both formalisms are commonly for four-quark systems. Now, the only step left in getting actual numbers for the hyperfine masses is to identify $m_1 = m_c, m_2 = m_u, m_3 = m_c, m_4 = m_u$ in Table III.

In the spin-0 channel, the hyperfine masses for the two possible states, $|0_a^{++}\rangle, |0_b^{++}\rangle$ are obtained from the corresponding formulas in Table III. As there are the mixing terms between the two states, one has to diagonalize the matrix to calculate the physical hyperfine masses which then correspond to the physical states. We denote the physical states with the capital letters in the subscripts

as $|0_A^{++}\rangle, |0_B^{++}\rangle$.

$$\begin{array}{c|cc} & |0_a^{++}\rangle & |0_b^{++}\rangle \\ \hline |0_a^{++}\rangle & -58.0 & -23.1 \\ |0_b^{++}\rangle & -23.1 & -46.0 \end{array} \rightarrow \begin{array}{c|cc} & |0_A^{++}\rangle & |0_B^{++}\rangle \\ \hline |0_A^{++}\rangle & -28.1 & 0.00 \\ |0_B^{++}\rangle & 0.00 & -75.9 \end{array}$$

Thus, in this spin-0 channel, the hyperfine masses for physical states, $|0_A^{++}\rangle$ and $|0_B^{++}\rangle$, are the followings,

$$\begin{aligned} \langle 0_A^{++}|V|0_A^{++}\rangle &= -28.1 \text{ MeV} , \\ \langle 0_B^{++}|V|0_B^{++}\rangle &= -75.9 \text{ MeV} . \end{aligned} \quad (11)$$

Similarly, using corresponding formulas for the spin-1 channel in Table III, we can calculate the hyperfine masses for the two possible configurations, $|1_a^{+-}\rangle |1_b^{+-}\rangle$, as well as their mixing terms. Then after the diagonalization, we obtain the followings.

$$\begin{array}{c|cc} & |1_a^{+-}\rangle & |1_b^{+-}\rangle \\ \hline |1_a^{+-}\rangle & -32.7 & -41.8 \\ |1_b^{+-}\rangle & -41.8 & -13.4 \end{array} \rightarrow \begin{array}{c|cc} & |1_A^{+-}\rangle & |1_B^{+-}\rangle \\ \hline |1_A^{+-}\rangle & 19.9 & 0.00 \\ |1_B^{+-}\rangle & 0.00 & -65.9 \end{array}$$

Again, we have denoted the physical states with the capital letters in the subscripts, $|1_A^{+-}\rangle, |1_B^{+-}\rangle$. Thus, in this spin-1 channel, the hyperfine masses for physical states are the followings,

$$\begin{aligned} \langle 1_A^{+-}|V|1_A^{+-}\rangle &= 19.9 \text{ MeV} , \\ \langle 1_B^{+-}|V|1_B^{+-}\rangle &= -65.9 \text{ MeV} . \end{aligned} \quad (12)$$

There is one more state in spin-1, $|1^{++}\rangle$, and the spin-2 state, $|2^{++}\rangle$, which do not mix with the other states. Their hyperfine masses are calculated to be

$$\begin{aligned} \langle 1^{++}|V|1^{++}\rangle &= -5.96 \text{ MeV} , \\ \langle 2^{++}|V|2^{++}\rangle &= 52.0 \text{ MeV} . \end{aligned} \quad (13)$$

Now using Eq.(8), the mass splittings among these resonances can be estimated from the differences in the hyperfine masses given in Eqs. (11),(12), (13). For example, the mass difference between the states, $|2^{++}\rangle, |1^{++}\rangle$ is given by $\langle 2^{++}|V|2^{++}\rangle - \langle 1^{++}|V|1^{++}\rangle = 57.96$ MeV. As we have emphasized in Sec. III, the mass splittings calculated this way are quite insensitive to the input parameters and do not suffer from the additional potentials.

Since all these resonances have the same flavor content, we can take the mass of one resonance as an input in order to estimate the other masses. The well-known resonance $X(3872)$, which can be identified with the state $|1^{++}\rangle$ because of its quantum numbers, would be the best candidate for the reference mass. Then, combining with the mass splittings, one can estimate all the masses of the states, $|0_A^{++}\rangle, |0_B^{++}\rangle, |1_A^{+-}\rangle, |1_B^{+-}\rangle, |1^{++}\rangle, |2^{++}\rangle$. For example, the mass of the spin-2 state can be calculated through

$$\begin{aligned} M(J=2) &= M_X(3872) + \langle 2^{++}|V|2^{++}\rangle - \langle 1^{++}|V|1^{++}\rangle \\ &= 3871.7 + 52 - (-5.96) = 3929.7 \text{ MeV}. \end{aligned} \quad (14)$$

Similarly we can estimate the masses of the other states.

The states corresponding to the quantum numbers and their calculated masses are summarized as follows,

$$\begin{aligned} |0_A^{++}\rangle &: 3849.5 \text{ MeV}, \\ |0_B^{++}\rangle &: 3801.8 \text{ MeV}, \\ |1^{++}\rangle &: 3871.7 \text{ MeV}, (\text{input}), \\ |1_A^{+-}\rangle &: 3897.7 \text{ MeV}, \\ |1_B^{+-}\rangle &: 3811.7 \text{ MeV}, \\ |2^{++}\rangle &: 3929.7 \text{ MeV}. \end{aligned} \quad (15)$$

If we compare these masses with the X resonances in Table I, we find that the calculated mass of $|1_A^{+-}\rangle$ is close to $X(3900)^0$ with only 7 MeV difference, and the mass of $|1_B^{+-}\rangle$, is 11 MeV lower than $X(3823)^0$. Also the mass of $|2^{++}\rangle$ is found to be only 12 MeV lower than the mass of the resonance $X(3940)$. Thus, if our framework really works, we can identify $X(3823)^0, X(3900)^0, X(3940)$ with the quantum numbers $J^{PC} = 1^{+-}, 1^{+-}, 2^{++}$. Of course, future measurements on the quantum numbers can settle the identifying issues raised in this work.

In particular, the quantum number for $X(3823)$ is certainly not settled. It could be 3D_2 state of $\bar{c}c$ which can explain a narrow decay width observed experimentally. Our four-quark assignment with $J^{PC} = 1^{+-}$ leads to a narrow width also based on observation that its fall-apart decay to $D^*\bar{D}$ is prohibited kinematically. In addition, there is other approach based on a hadronic molecule [35] suggesting that its quantum number $J^{PC} = 1^{--}$. Eventually it will be important to confirm the quantum number of $X(3823)$ by measuring for example the angular distribution of $X(3823)$ and clarify its nature.

As we have already mentioned, in this calculation, we use the same values for the input parameters as in our previous works [26], where we develop the four-quark wave functions for the D and B meson excited states.

There, we found some of those resonances fit very nicely with our picture. To us, it is quite interesting to see that the masses of the X resonances come out very close to the experimental values without any parameter tuning at least in the spin-1 and spin-2 channels. This may provide a consistent treatment on open- and hidden-charm mesons within a four-quark framework.

Currently in PDG, there are no resonances corresponding to the states in the spin-0 channel, $|0_A^{++}\rangle$ and $|0_B^{++}\rangle$. There could be various reasons for this. One possibility may be due to their broad widths. If they decay through a fall-apart mechanism, the decay channel with low-invariant mass like $D\bar{D}$ can be open and their decay width becomes broad. If so, these resonances are hard to be measured experimentally. A similar situation can be seen also in the B meson excited states where there are no resonances observed in the spin-0 channel. (See Table III in Ref. [26].) This was explained in our four-quark model where the spin-0 mesons are expected to have broad widths due to the presence of the decay modes kinematically favorable. But this scenario does not work for the spin-1 and spin-2 channels and there are indeed some sharp resonances in PDG in these channels. It is quite likely that the similar explanation can be applied in the X resonances. Or alternatively one can look for some other QCD dynamics which might impede the formation of resonances in the spin-0 channel. Certainly more works need to be done in future to clarify this issue.

V. SUMMARY

In this work, we have calculated the masses of the X resonances in low-lying region by considering them as tetraquark states. The color-spin interaction is used to calculate the mass splittings and, using $X(3872)$ as an input, we calculate the masses corresponding to the quantum numbers $|J^{PC}\rangle = |0^{++}\rangle, |1^{+-}\rangle, |2^{++}\rangle$. We found that masses of the two states in $|1^{+-}\rangle$ and one state in $|2^{++}\rangle$ fit very nicely to $X(3823), X(3900), X(3940)$ within ~ 10 MeV. Future experiments can settle the issues of identifying the resonances by measuring the quantum numbers of the controversial states. We stressed that the color-spin interaction is powerful in generating the mass splittings but not for the mass themselves. To directly calculate the masses of hadrons of concern, the color-spin interaction may not fully represent the potential among two quarks and one needs the additional potentials which, however, cancel in the mass splittings. The input parameters that have been used in the model prediction were taken to be the same as the ones used in the D and B meson excited states in the four-quark formalism. Therefore, we believe that our results, if they are confirmed by the measurement of the X quantum numbers, can provide a consistent picture for the open- and hidden-charm mesons in this four-quark formalism.

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