Radially Excited States of η_c

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In the framework of chiral quark model, the mass spectrum of $\eta_c(ns)$ $(n = 1, \cdots, 6)$ is studied with Gaussian expansion method. With the wave functions obtained in the study of mass spectrum, the open flavor two-body strong decay widths are calculated by using ${}^3P_0$ model. The results show that the masses of $\eta_c(1S)$ and $\eta_c(2S)$ are consistent with the experimental data. The explanation of X(3940) as $\eta_c(3S)$ is disfavored for X(3940) is a narrow state, $\Gamma = 37^{+26}_{-15} \pm 8$ MeV, while the open flavor two-body strong decay width of $\eta_c(3S)$ is about 200 MeV in our calculation. Although the mass of X(4160) is about 100 MeV less than that of $\eta_c(4S)$, the assignment of X(4160) as $\eta_c(4S)$ can not be excluded because the open flavor two-body strong decay width of $\eta_c(4S)$ is consistent with the experimental value of X(4160) and the branching ratios of $\eta_c(4S)$ are compatible with that of X(4160), and the mass of $\eta_c(4S)$ can be shifted downwards by taking into account the coupling effect of the open charm channels. There are still no good candidates to $\eta_c(5S)$ and $\eta_c(6S)$.

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I. INTRODUCTION

In recent years, a lot of charmonium-like states, so called "XYZ" states [1], have been observed by Belle, BarBar, BESIII and other collaborations. Most of them cannot be accommodated in the quark models as conventional mesons because of their exotic properties. To reveal the underlying properties of these states has stimulated extensive interest in the research field of hadron physics.

In the compilation of Particle Data Group (PDG) [2], 34 states were listed under the $c\bar{c}$ section. Ten states were assigned, $\eta_c(1S)$, $\eta_c(2S)$, $J/\Psi(1S)$, $\Psi(2S)$, $\chi_{c0}(1P)$, $\chi_{c0}(2P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, $\chi_{c2}(2P)$ and $h_c(1P)$, although there are some controversy about the assignment of $\chi_{c2}(2P)$ [3–7]. Experimentally there is no sign of $X(3915) \to D\bar{D}$, which strongly contradicts the theoretical expectation of $\chi_{c2}(2P)$, the $D\bar{D}$ decay channel should dominate. In addition, the present analyses strongly favor the following assignments: $\psi(3770)$ as 1^3D_1, $\psi(4040)$ as 3^3S_1, $\psi(4160)$ as 2^3D_1 and $\psi(4415)$ as 4^3S_1 [8, 9]. The quantum numbers of $X(3872)$ are fixed recently, $I^G(J^{PC}) = 0^+(1^{++})$, so it is a good candidate of $\chi_{c1}(2P)$ [10] although there are also some arguments about this assignment[11–13]. The explanations of X(3940) as $\eta_c(3S)$ [14], X(4140) as $\chi_{c0}(3P)$ [7] are also proposed recently. However, there are about half of the $c\bar{c}$ states which remain unassigned.

To assign the state reported by experimental collaborations to the theoretical one, First the masses should be in agreement. Second the decay properties of the states should be comparable. In the present work, the $\eta_c(nS)$ $(n = 1, \cdots, 6)$ states are studied. The first two states of η_c are well established. The other states are needed to be assigned. To validate the assignment of $\eta_c(ns)$ $(n = 3, \cdots, 6)$, the more rigorous way is to calculate the decay width of these states. In the present work we study the open charm two-body strong decay widths of all the $\eta_c(nS)$ $(n = 3, \cdots, 6)$ mesons systematically in a constituent quark model. The spectrum of these $\eta_c(nS)$ $(n = 1, \cdots, 6)$ mesons are obtained by using a high-precision few-body method, Gaussian expansion method (GEM) [15], in the framework of chiral quark model [16]. In GEM all the interactions are treated equally rather than some interactions: spin-orbit and tensor terms, are treated perturbatively in other approaches. The decay amplitudes to all open charm two-body modes that are nominally accessible are derived with the ${}^3P_0$ model. In the numerical evaluation of the transition matrix elements of decay widths, the wavefunctions obtained in the study of meson spectrum, rather than the simple harmonic oscillator (SHO) ones, are used. It is expected to validate the assignment of the radially excited charmonia with spin-parity $J^{PC} = 0^{-+}$ and to provide useful information for experiment to search the still missing states.

This work is organized as follows: In section II, the chiral quark model and wavefunctions of meson are presented; the ${}^3P_0$ decay model is briefly reviewed in section III; In section IV, the numerical results of the two-body decays of $\eta_c(nS)(n = 3, \cdots, 6)$ are obtained and presented with discussions; And the last section is a short summary.

II. THE CHIRAL QUARK MODEL AND WAVE FUNCTIONS

The chiral quark model, which has given a good description of hadron spectra [16, 17], is used to obtain the masses and wavefunctions of η_c. Hamiltonian of the model for meson is taken from Ref. [16],

$$
\begin{aligned}
H_{q\bar{q}}(\mathbf{r}) &= m_1 + m_2 + \frac{p_r^2}{2\mu} + V_{12} \qquad (1) \\
V_{12} &= V_{12}^C + V_{12}^{OGE} + V_{12}^{\pi} + V_{12}^{K} + V_{12}^{\eta} + V_{12}^{\sigma},
\end{aligned}
$$

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where m_1 and m_2 are the masses of quark and antiquark, $\mathbf{p}_r$ denotes the relative momentum between quark and antiquark, and V_{12} is the interaction between quark and antiquark. In the present version of the chiral quark model, the screened color confinement potential is used

$$V_{12}^C = \lambda_1 \cdot \lambda_2 \left[-a_c(1 - e^{-\mu_c r}) + \Delta\right], \qquad (2)$$

In some sense, the channel coupling effect of $D\bar{D}$ etc. is taken into account partly according to Ref. [18]. The masses and the wavefunctions of mesons can be obtained by solving the Schrödinger equation,

$$H\Psi_{JM_J} = E\Psi_{JM_J}. \qquad (3)$$

The wavefunction Ψ_{JM_J} can be written as the direct product of orbital, color, flavor and spin wavefunctions,

$$\begin{aligned} \Psi_{JM_J} &= \left[\psi_L(\mathbf{r})\chi_S\right]_{JM_J} \phi(q\bar{q})\omega(q\bar{q}), \qquad (4) \\ \left[\psi_L(\mathbf{r})\chi_S\right]_{JM_J} &= \sum_{M_L,M_S} \langle LM_L SM_S|JM_J\rangle \psi_{LM_L}(\mathbf{r})\chi_{SM_S}, \end{aligned}$$

where $\langle LM_L SM_S|JM_J\rangle$ is the Clebsh-Gordan coefficient, χ_{SM_S}, $\phi(q\bar{q})$ and $\omega(q\bar{q})$ are spin, flavor and color wave function of meson, respectively. The Gaussian basis functions are employed to expand the orbital wavefunction $\psi_{LM_L}(\mathbf{r})$ [15]

$$\psi_{LM_L}(\mathbf{r}) = \sum_{k=1}^{k_{max}} C_{Lk}\phi^G_{LMk}(\mathbf{r}). \qquad (5)$$

$$\phi^G_{LMk}(\mathbf{r}) = N_{Lk} r^L \exp\left(-\nu_k r^2\right) \mathbf{Y}_{LM_L}(\hat{\mathbf{r}}). \qquad (6)$$

The normalization constant N_{Lk} is

$$N_{Lk} = \left[\frac{2^{L+2}(2\nu_k)^{L+\frac{3}{2}}}{\sqrt{\pi}(2L+1)!!}\right]^{\frac{1}{2}} \quad (k = 1, \cdots, k_{max}). \qquad (7)$$

The Gaussian size parameters are in geometric progression.

$$\nu_k = \frac{1}{r_k^2}, \qquad r_k = r_1 a^{k-1} \quad (k = 1, \cdots, k_{max}), \qquad (8)$$

where $r_1 = 0.001$ fm, $r_{max} = 5.000$ fm and $k_{max} = 30$ are used to arrive the convergent results. Substituting Eqs.(4-6) into Eq.(3), we obtain a general eigen-equation,

$$\mathbf{Hc} = E\mathbf{Nc}, \qquad (9)$$

where $\mathbf{H}$ and $\mathbf{N}$ are hamiltonian and overlap matrices, respectively.

III. STRONG DECAY AND QUARK-PAIR-CREATION MODEL

To calculate the open flavor two-body strong decay widths of hadrons, the quark-pair-creation model, or $^3P_0$ model, is widely used. In this model, the hadron decay occurs via a quark-antiquark pair production from the hadronic vacuum, so the quantum numbers of the created quark pair are of the hadronic vacuum, $J^{PC} = 0^{++}$. This model has given a rather good description of open flavor two-body strong decay width of hadrons [19–24], which are allowed by Okubo-Zweig-Iizuka (OZI) rule. Here the model is used to calculate the open charm two-body strong decay widths of the radially excited states of $\eta_c(ns), (n = 3, \cdots, 6)$. The transition operator used in the model [19] is

$$\begin{aligned} T = -3\,\gamma \sum_m \langle 1m1-m|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \\ \times \mathcal{Y}_1^m\left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2}\right)\chi^{34}_{1-m}\phi^{34}_0\omega^{34}_0 b_3^\dagger(\mathbf{p}_3)d_4^\dagger(\mathbf{p}_4). \quad (10) \end{aligned}$$

The created pair is characterized by a color-singlet wave function ω_0^{34}, a flavor-singlet function ϕ_0^{34}, a spin-triplet function χ^{34}_{1-m} and an orbital wave function $\mathcal{Y}_l^m(\mathbf{p}) \equiv |p|^l Y_l^m(\theta_p, \phi_p)$ which is the l-th solid spherical harmonic polynomial. $\mathbf{p}_3$ and $\mathbf{p}_4$ denote the momenta carried by the quark and anti-quark created from the vacuum. The strength of the quark pair creation γ from the vacuum is determined from the measured partial decay widths. In the present calculation, γ_n and γ_s are determined by fitting the open flavor two-body strong decay widths of the four established states $\psi(4040)$, $\psi(3770)$, $\psi(4160)$ and $\chi_{c2}(2P)$ and the decay widths are showed in Table I. Here $\gamma_n = 4.19$ for $u\bar{u}$, $d\bar{d}$ pairs and $\gamma_s = \gamma_n/\sqrt{3}$ for $s\bar{s}$ pair other than that in Ref. [4, 7].

TABLE I: The open flavor two-body strong decay widths of states used to determine γ_n and γ_s. The masses of mesons involved take the experimental values. (unit in MeV)

Meson	Γ (exp.) [2]	Γ (theo.)
$\psi(4040)$	80±10	92
$\psi(3770)$	27.2±1.0	16
$\psi(4160)$	70±10	56
$\chi_{c2}(2P)$	24±6	25

For the process $A \to B + C$, the S-matrix element is defined as

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \quad (11)$$

where the T-matrix element is

$$\langle BC|T|A\rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \quad (12)$$

$\mathbf{P}_A$, $\mathbf{P}_B$ and $\mathbf{P}_C$ are the momenta of mesons A ,B and C, respectively, and $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude for the process $A \to B + C$.

In experiments, the partial wave decay widths are often used, it can be written as

$$\Gamma = \sum_{JL} \Gamma_{JL}, \qquad \Gamma_{JL} = \pi^2 \frac{|\mathbf{P}|}{M_A^2}\left|\mathcal{M}^{JL}\right|^2. \qquad (13)$$

By the Jacob-Wick formula [25, 26], the partial wave amplitude $\mathcal{M}^{JL}$ can be further related to the helicity amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$,

$$\mathcal{M}^{JL}(A\to BC)=\frac{\sqrt{2L+1}}{2J_A+1}\sum_{M_{J_B},M_{J_C}}\langle L0JM_{J_A}|J_AM_{J_A}\rangle \times\langle J_BM_{J_B}J_CM_{J_C}|JM_{J_A}\rangle\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}),\quad(14)$$

where $\mathbf{J}=\mathbf{J}_B+\mathbf{J}_C$, $\mathbf{J}_A=\mathbf{J}_B+\mathbf{J}_C+\mathbf{L}$, $M_{J_A}=M_{J_B}+M_{J_C}$, and $\mathbf{P}=\mathbf{P}_B=-\mathbf{P}_C$) is the three momentum of the daughter mesons B and C in the center-of-mass frame of meson A.

IV. NUMERICAL CALCULATION

The masses of the involved mesons and the corresponding wavefunctions are obtained by solving the general eigen-equation Eq.(9). The running strong coupling constants are taken from Ref. [7] and the other parameters are taken from Ref. [16]. The masses of open-charm mesons and charmonium $\eta_c(ns),(n=1,\cdots,6)$ are shown in Table II. For charmonium and most open charm mesons, there is a good agreement between experimental data and theoretical results. For several open charm mesons, the theoretical masses deviate from the experimental data a few percents.

To justify the assignment, the decay width is very important. The open charm two-body strong decay modes and decay channels of $\eta_c(nS)(n=3,\cdots,6)$ allowed by the phase space and OZI law are listed in Table III. The open charm two-body decay widths of $\eta_c(nS)(n=3,\cdots,6)$ are calculated and also shown in the fourth column of Table III. In calculating the decay widths, the theoretical masses of mesons involved and the corresponding wave functions obtained in solving the Schrödinger equations are used. By this way, the calculation of the widths is more self consistent than most of the previous works, where the SHO wave functions are used. According to Ref. [7], the decay width is sensitive to the masses of mesons, especially around the threshold of the decay. For comparison, the results of using experimental masses of mesons in the calculation of the decay widths are also shown in Table III (the fifth column).

From Table II and III, one can see that the mass and the open charm two-body decay width of $\eta_c(3S)$ are 4007 MeV and 198 MeV, respectively. Around this mass and the quantum numbers of $\eta_c(3S)$ constraint, the possible candidate is X(3940). By coupling to open charm mesons, the mass of $\eta_c(3S)$ can be shifted downwards. However, the decay width is much higher than the experimental value of $X(3940)$, $\Gamma=37^{+26}_{-15}\pm8$ MeV. And if the theoretical mass of $\eta_c(3S)$ and the experimental masses of the final states are used, the decay width will reduce to 176 MeV which is still higher than that of $X(3940)$. If the mass of $\eta_c(3S)$ is shifted to 3940 MeV by coupling to $D\bar{D}^*$ channels, the decay width will rise to ~ 270 MeV. In Ref. [4], the mass of $\eta_c(3S)$ is 4043 MeV which is 40 MeV higher than our result. In this case the decay channel to $D^*\bar{D}^*$ opens and it contributes 33 MeV to the total decay width. If the mass of $\eta_c(3S)$ rises to 4043 MeV, the decay width of $\eta_c(3S)$ to $D\bar{D}^*$ will be ~ 90 MeV and that of $\eta_c(3S)$ to $D^*\bar{D}^*$ will be ~ 150 MeV, the total width is over 200 MeV. So the possibility of assigning $X(3940)$ as $\eta_c(3S)$ is not favored in the present work. In Ref. [14], the mass of $\eta_c(3S)$ was estimated from the spectrum pattern and SHO wavefunction was used in the evaluating the transition matrix element, the decay width of $\eta_c(3S)$ is around the experimental value of $X(3940)$ with appropriate SHO parameter R. So more detailed studies are needed to make the assignment of $\eta_c(3S)$.

TABLE II: The masses of the open-charm mesons and charmonium η_c (unit in MeV).

Meson	GEM	Ref. [16]	Ref. [4]	Expt. [2]
$D^0,\bar{D}^0$	1878	1883	-	1864.84±0.07
D^+,D^-	1878	1883	-	1869.61±0.10
$D^{*0},\bar{D}^{*0}$	2005	2010	-	2006.96±0.10
D^{*+},D^{*-}	2005	2010	-	2010.26±0.07
$D^{*0}(2S),\bar{D}^{*0}(2S)$	2697	-	-	-
$D^{*+}(2S),D^{*-}(2S)$	2697	-	-	-
$D_0^{*0},\bar{D}_0^{*0}$	2431	-	-	2318±29
D_0^{*+},D_0^{*-}	2431	-	-	2403±14±35
$D_1^0,\bar{D}_1^0$	2450	2492	-	2421.4±0.6
$D_1^\pm,\bar{D}_1^\pm$	2450	2492	-	2423.2±2.4
$D_1^{0\prime},\bar{D}_1^{0\prime}$	2529	-	-	-
$D_2^{*0},\bar{D}_2^{*0}$	2500	2502	-	2462.6±0.6
$D_2^{*\pm},\bar{D}_2^{*\pm}$	2500	2502	-	2464.3±1.6
D_s^+,D_s^-	1968	1981	-	1968.30±0.11
D_s^{*+},D_s^{*-}	2104	2112	-	2112.1±0.4
D_{s0}^{*+},D_{s0}^{*-}	2460	2469	-	2317.7±0.6
D_{s1}^{*+},D_{s1}^{*-}	2539	2543	-	2459.5±0.6
D_{s1}^{*+},D_{s1}^{*-}	2565	2571	-	2535.10±0.08
D_{s2}^{*+},D_{s2}^{*-}	2583	2585	-	2571.9±0.8
$J/\psi(1S)$	3096	3097	3090	3096.916±0.011
$\eta_c(1S)$	2979	2990	2982	2983.6±0.7
$\psi'(2S)$	3684	3685	3672	$3686.109^{+0.012}_{-0.014}$
$\eta_c'(2S)$	3622	3627	3630	3639.4±1.3
$\eta_c(3S)$	4007	-	4043	-
$\eta_c(4S)$	4276	-	4384	-
$\eta_c(5S)$	4470	-	-	-
$\eta_c(6S)$	4612	-	-	-

For $\eta_c(4S)$, the mass is 4276 MeV, and the decay width is around 73 MeV. Now the decay of $\eta_c(4S)$ to $D^*\bar{D}^*$ is allowed by the phase space and it is the main open flavor two-body strong decay channel. In Ref. [4], the mass of $\eta_c(4S)$ is 4384 MeV which is about 110 MeV higher than the result of this work, so that there are more decay modes allowed by phase space. But its total decay width is not far from our result. The possible candidate of $\eta_c(4S)$ is X(4160), although its mass is about 110 MeV less than the theoretical mass of $\eta_c(4S)$. Because the coupling effect of open charm channels is expected to shift the mass of $\eta_c(4S)$ a little lower. The decay width of X(4160) is $139^{+111}_{61}\pm21$ MeV with $D^*\bar{D}^*$

TABLE III: The open charm two-body strong decay modes and decay widths of the possible charmonium states of $\eta_c(nS)(n = 3, \cdot, 6)$ allowed by the OZI rule and phase space(unit in MeV). The widths in the 4th column are derived with the theoretical masses of the initial state and the final ones. The widths in the 5th column are derived with the experimental masses. And the widths in the 6th column are from Ref. [4].

State	Decay mode	Decay channel	Γ_a	Γ_b	Ref. [4]
$\eta_c(3S)$	$0^- + 1^-$	$D\bar{D}^*, D^*\bar{D}, D^{*+}D^-, D^+D^{*-}$	197.76	176.18	47
	$1^- + 1^-$	$D^*\bar{D}^*, D^{*+}\bar{D}^{*-}$	-	-	33
		Total	197.76	176.18	80
$\eta_c(4S)$	$0^- + 1^-$	$D\bar{D}^*, D^*\bar{D}, D^{*+}D^-, D^+D^{*-}$	0.97	0.32	6.3
		$D_s^+D_s^{*-}, D_s^{*+}D_s^-$	2.29	2.12	2.2
	$1^- + 1^-$	$D^*\bar{D}^*, D^{*+}\bar{D}^{*-}$	68.95	71.20	14
		$D_s^{*+}D_s^{*-}$	0.37	0.76	2.2
		Total	72.58	74.40	61
$\eta_c(5S)$	$0^- + 1^-$	$D\bar{D}^*, D^*\bar{D}, D^{*+}D^-, D^+D^{*-}$	6.35	7.96	-
		$D_s^+D_s^{*-}, D_s^{*+}D_s^-$	0.52	0.56	-
	$0^- + 0^+$	$D\bar{D}_0^{*0}, \bar{D}D_0^{*0}, D^+D_0^{*-}, D^-D_0^{*+}$	92.22	104.89	-
		$D_s^+D_{s0}^{*-}, D_s^-D_{s0}^{*+}$	0.27	2.91	-
	$0^- + 2^+$	$D\bar{D}_2^{*0}, \bar{D}D_2^{*0}, D^+\bar{D}_2^{*-}, D^-\bar{D}_2^{*+}$	44.19	2.50	-
	$1^- + 1^-$	$D^*\bar{D}^*, D^{*+}\bar{D}^{*-}$	8.69	9.99	-
		$D_s^{*+}D_s^{*-}$	0.52	0.37	-
	$1^- + 1^+$	$D^*\bar{D}_1^0, \bar{D}^*D_1^0, D^{*+}D_1^-, D^{*-}D_1^+$	38.50	75.77	-
		Total	191.26	204.95	-
$\eta_c(6S)$	$0^- + 1^-$	$D\bar{D}^*, D^*\bar{D}, D^{*+}D^-, D^+D^{*-}$	11.48	12.77	-
		$D_s^+D_s^{*-}, D_s^{*+}D_s^-$	0.03	0.04	-
		$D\bar{D}^*(2S), D^*\bar{D}(2S), D^{*+}D^-(2S), D^+D^{*-}(2S)$	1.11	5.00	-
	$0^- + 0^+$	$D\bar{D}_0^{*0}, \bar{D}D_0^{*0}, D^+D_0^{*-}, D^-D_0^{*+}$	6.99	2.39	-
		$D_s^+D_{s0}^{*-}, D_s^-D_{s0}^{*+}$	0.66	1.22	-
	$0^- + 2^+$	$D\bar{D}_2^{*0}, \bar{D}D_2^{*0}, D^+\bar{D}_2^{*-}, D^-\bar{D}_2^{*+}$	19.41	29.13	-
		$D_s^+D_{s2}^{*-}, D_s^-D_{s2}^{*+}$	0.89	0.98	-
	$1^- + 1^-$	$D^*\bar{D}^*, D^{*+}\bar{D}^{*-}$	0.03	0.12	-
		$D_s^{*+}D_s^{*-}$	0.34	0.31	-
	$1^- + 1^+$	$D^*\bar{D}_1^0, \bar{D}^*D_1^0, D^{*+}D_1^-, D^{*-}D_1^+$	2.33	3.26	-
		$D^*\bar{D}_1^{0\prime}, \bar{D}^*D_1^{0\prime}, D^{*+}D_1^{-\prime}, D^{*-}D_1^{+\prime}$	6.84	5.56	-
		$D_s^{*+}D_{s1}^{*-}, D_s^{*-}D_{s1}^{*+}$	-	0.14	-
		$D_s^{*+}D_{s1}^{*-\prime}, D_s^{*-}D_{s1}^{*+\prime}$	-	0.96	-
	$1^- + 2^+$	$D^*\bar{D}_2^{*0}, \bar{D}^*D_2^{*0}, D^{*+}D_2^{*-}, D^{*-}D_2^{*+}$	2.52	0.14	-
		Total	52.63	62.02	-

mode seen and $D\bar{D}$, $D\bar{D}^* + c.c.$ mode not seen. So the open flavor two-body strong decay width of $\eta_c(4S)$ is in the range of the experimental value of X(4160), and the branching ratios of $\eta_c(4S)$ are also compatible with that of X(4160), where the dacay $\eta_c(4S) \to D\bar{D}$ is forbidden by the angular momentum coupling and the branching ratio $\frac{\Gamma(\eta_c(4S))\to D^*\bar{D}+c.c.)}{\Gamma(\eta_c(4S)\to D^*\bar{D}^*)} = 0.014$. While $\frac{\Gamma(X(4160)\to D\bar{D})}{\Gamma(X(4160)\to D^*\bar{D}^*)} < 0.09$ and $\frac{\Gamma(X(4160)\to D^*\bar{D}+c.c.)}{\Gamma(X(4160)\to D^*\bar{D}^*)} < 0.22$ in experiment. The assignment of X(4160) as $\eta_c(4S)$ cannot be excluded. This statement is different from the results of Ref. [14], where the SHO wavefunctions are used to calculate the transition matrix elements. For the excited state, the SHO approximation is not reasonable one.

For $\eta_c(5S)$ and $\eta_c(6S)$ states, the decay to $0^- + 0^+$, $0^- + 2^+$ and $1^- + 1^+$ (even $1^- + 2^+$ for $\eta_c(6S)$) are allowed by the phase space. Moreover they are the main decay modes of $\eta_c(5S)$ and $\eta_c(6S)$ states. Comparing with experimental data, we cannot find any states with these properties. Further measurements are expected to identify these two states.

V. SUMMARY

In this work, we study the mass spectra of $\eta_c(ns)$ ($n = 1, \cdots, 6$) with Gaussian expansion method in the framework of chiral quark model and calculate the open charm two-body strong decays of $\eta_c(ns)$ ($n = 3, \cdots, 6$) with $^3P_0$ model. The results show that the masses of $\eta_c(1S)$ and $\eta_c(2S)$ are consistent with the experimental data. The explanation of X(3940) as $\eta_c(3S)$ is disfavored because X(3940) is a narrow state while the open flavor two-body strong decay width of $\eta_c(3S)$ is about 200 MeV in the present work. Although the mass of X(4160) is about 100 MeV less than that of $\eta_c(4S)$, the assignment of X(4160) as $\eta_c(4S)$ can not be excluded because the coupling effect of open charm channels may shift the mass of $\eta_c(4S)$ lower, and the open flavor two-body strong decay width of $\eta_c(4S)$ is in the range of the experimental value of X(4160) and the branching ratios of $\eta_c(4S)$ are compatible with that of X(4160).

Because of the opening of open charm decay, the spectra of $\eta_c(ns)$ $(n = 3, \cdots, 6)$ are still not clear as the bottomonium. To describe the excited spectrum of charmonium, the conventional quark model needs to be extended. To develop the quark model, the effect of quark-antiquark pair creation should be taken into account. For open flavor two-body decay model, $^3P_0$ model, the improvement is also needed. the dependence of strength γ on the momentum of the created quark has been used to improve the agreement between theoretical results and experimental data [27]. Dynamic model for the meson decay is also expected. The study of the properties of $\eta_c(ns)$ $(n = 1, \cdots, 6)$ is helpful for understanding the possible exotic, "XYZ" states.

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