

# Hidden-Charm Tetraquarks and Charged $Z_c$ States

Lu Zhao<sup>1\*</sup>, Wei-Zhen Deng<sup>1†</sup>, Shi-Lin Zhu<sup>1,2‡</sup>

<sup>1</sup> Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

<sup>2</sup> Collaborative Innovation Center of Quantum Matter, Beijing 100871, China

Experimentally several charged axial-vector hidden-charm states were reported. Within the framework of the color-magnetic interaction, we have systematically considered the mass spectrum of the hidden-charm and hidden-bottom tetraquark states. It is impossible to accommodate all the three charged states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  within the axial vector tetraquark spectrum simultaneously. Not all these three states are tetraquark candidates. Moreover, the eigenvector of the chromomagnetic interaction contains valuable information of the decay pattern of the tetraquark states. The dominant decay mode of the lowest axial vector tetraquark state is  $J/\psi\pi$  while its  $D^*\bar{D}$  and  $\bar{D}^*D^*$  modes are strongly suppressed, which is in contrast with the fact that the dominant decay mode of  $Z_c(3900)$  and  $Z_c(4025)$  is  $\bar{D}D^*$  and  $\bar{D}^*D^*$  respectively. We emphasize that all the available experimental information indicates that  $Z_c(4200)$  is a very promising candidate of the lowest axial vector hidden-charm tetraquark state.

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## I. INTRODUCTION

During the past decade, many charmonium-like states and bottomonium-like states have been reported by experimental collaborations such as Belle, *BARBAR*, CDF, D0, LHCb, BE-SIII and CLEOc.  $X(3872)$  was first observed by Belle Collaboration in the exclusive decay process  $B^\pm \rightarrow K^\pm\pi^+\pi^-J/\psi$  [1]. Its mass is very close to the  $\bar{D}^0D^{*0}$  threshold and its width is extremely narrow ( $< 1.2$  MeV). Later LHCb Collaboration determined its  $J^{PC} = 1^{++}$  [2]. Many theoretical groups interpreted  $X(3872)$  as the molecular candidate of the  $\bar{D}D^*$  system [3–6].

Besides  $X(3872)$ , a family of so called  $Y$  states were also reported.  $Y(4260)$  was observed by *BARBAR* Collaboration in the invariant mass spectrum of  $\pi^+\pi^-J/\psi$  in the initial-state radiation process  $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-J/\psi$  [7]. Later Belle Collaboration observed a peak near 4.25 GeV and a new structure around 4.05 GeV which was denoted later as  $Y(4008)$  [8, 9].  $Y(4360)$  was observed in the reaction  $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$  by *BARBAR* [10]. Almost at the same time, Belle observed two resonant structures in the  $\pi^+\pi^-\psi(2S)$  invariant mass distribution  $Y(4360)$  and  $Y(4660)$  [11], which was confirmed by *BARBAR* via the initial-state radiation process  $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$  [12].  $Y(4630)$  was reported as a near-threshold enhancement in the  $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$  process [13].

The group of charged charmonium-like and bottomonium-like states are even more exotic. The lightest charged charmonium-like state  $Z_c(3900)$  was observed in the  $J/\psi\pi^\pm$  invariant mass in the process  $Y(4260) \rightarrow J/\psi\pi^+\pi^-$  by BE-SIII Collaboration [14], by Belle Collaboration with ISR [15] and by using CLEO data [16]. Its decay mode implies that  $Z_c(3900)$  is a hidden-charm structure.  $Z_c(4025)$  was observed in the  $\pi^\mp$  recoil mass spectrum in the process  $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$  [17].  $Z_c(4020)$  was reported in the  $\pi^\pm h_c$

mass spectrum in the process  $e^+e^- \rightarrow \pi^+\pi^-h_c$  [18]. Moreover, two charged bottomonium-like resonances  $Z_b(10610)$  and  $Z_b(10650)$  were observed in the  $\pi^\pm\Upsilon(nS)$  and  $\pi^\pm h_b$  mass spectrum in the  $\Upsilon(5S)$  decays [19].  $Z_1(4050)$  and  $Z_2(4250)$  were observed in the  $\pi^+\chi_{c1}$  invariant mass distribution in the  $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$  decays [20].  $Z_c(4485)$  was observed by Belle Collaboration in the  $\pi^\pm\psi'$  invariant mass distribution in the exclusive  $B \rightarrow K\pi^\pm\psi'$  decays [21]. Later its spin and parity were determined as  $J^P = 1^+$  [22]. The charmonium-like state  $Z_c(4200)$  was observed in the  $J/\psi\pi^+$  mode with a significance of  $8.2\sigma$  when performing the amplitude analysis of  $B \rightarrow J/\psi K\pi$  [23].

These  $XYZ$  states either decay into one charmonium/bottomonium state plus light mesons or into a pair of open-charm/open-bottom heavy mesons. Many of them do not fit into the conventional  $q\bar{q}$  meson spectrum in the quark model. Some of them were interpreted as the candidates of the hybrid meson [24], molecular states [3, 4, 25–29], tetraquark states [30–35] and so on. For example,  $Z_c(3900)$  was interpreted as the isovector axial vector molecular partner of  $X(3872)$  [36–38]. Similarly  $Z_c(4025)$  was speculated to be the  $D^*D^*$  molecular candidate [39–41]. There are also some other speculations about their nature [42, 43].  $Z_b(10610)$  and  $Z_b(10650)$  are generally regarded as the candidates of the  $\bar{B}B^*$  and  $\bar{B}^*B^*$  molecular states [44–47].

However, it is not very natural to explain  $Z_c(4200)$  and  $Z_c(4485)$  as the S-wave molecular states composed of two S-wave heavy mesons. Instead,  $Z_c(4485)$  was proposed as the cousin molecular state of  $Z_c(3900)$  and  $Z_c(4025)$  composed of  $D(D^*)$  and its radial excitation [48, 49].

Another interesting possibility is that some charged  $Z_c$  states might be tetraquark candidates. The light  $q\bar{q}q\bar{q}$  tetraquark system was first studied in the MIT bag model [50, 51], where the multiquark mass spectrum mostly depend on the chromomagnetic interaction among the quarks. When considering the chromomagnetic interaction, it is convenient to adopt the  $SU(6)_{cs}$  representation which is the eigenstate of the color-magnetic (CM) interaction and can be constructed as the direct product of the  $SU(3)$  color and the  $SU(2)$  spin group. The bag model was later used to discuss the hidden-

\*Email: Luzhao@pku.edu.cn

†Email: dwz@pku.edu.cn

‡Email: zhsl@pku.edu.cn

charm/bottom tetraquark system [52, 53]. The hidden-charm tetraquarks were also studied in the constituent quark model (CQM) [33, 54].

In this work we will investigate whether some of the charged  $Z_c$  states could be the tetraquark candidates. We will discuss the mixing of the hidden-charm tetraquark states in the different color-spin representation and possible mass splitting of the hidden-charm tetraquark states in the framework of the chromomagnetic interaction. We will employ two schemes to fix the strength of the CM interaction and extract the masses and wave functions of the  $J^P = 1^+, 0^+, 2^+$  tetraquark systems. Then we compare the hidden-charm tetraquark spectrum with the current experimental data.

The paper is organized as follows. After the introduction, we present the chromomagnetic hamiltonian and the tetraquark model in Section II. In Section III, we discuss the masses of the possible tetraquark candidates. We explore the decay pattern of the tetraquark system in Section IV. The last section is the discussion and summary.

## II. HEAVY TETRAQUARK

### A. The Chromomagnetic Hamiltonian

For the tetraquark system, we consider the chromomagnetic (CM) interaction to derive the mass splitting. The Hamiltonian reads

$$H = \sum_i m_i + H_{CM} \quad (1)$$

where  $m_i$  is the mass of the  $i$ -th constituent quark.  $H_{CM}$  describes the CM interaction which is derived from one gluon exchange [50, 51, 55, 56]

$$H_{CM} = - \sum_{i>j} v_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (2)$$

where  $\vec{\lambda}_i$  is the quark color operator and  $\vec{\sigma}_i$  is the spin operator. For the anti-quark,  $\vec{\lambda}_{\bar{q}} = -\vec{\lambda}_q^*$  and  $\vec{\sigma}_{\bar{q}} = -\vec{\sigma}_q^*$ .  $v_{ij}$  represents the interaction strength between two quarks. Therefore,  $v_{ij}$  depends on the wavefunction of the multiquark system. For example,  $v_{ij}$  takes different values in the  $q\bar{q}$ ,  $qq\bar{q}$  and  $q\bar{q}q\bar{q}$  systems. In the bag model,  $v_{ij}$  depends on the bag radius and the constituent quark mass. On the other hand, the constituent quark model (CQM) is very successful in describing the meson and baryon spectrum, where the color-magnetic interaction leads to the mass splitting between the octet and decuplet baryons. We follow the CQM convention and adopt  $v_{ij} = v \frac{m_u^2}{m_i m_j}$ . The parameter  $v$  depends on the multiquark system.

### B. Hidden-charm tetraquark wavefunction

For the  $qq\bar{q}\bar{q}$  tetraquark system, the CM wavefunction can be constructed either as  $qq \otimes \bar{q}\bar{q}$  or  $q\bar{q} \otimes q\bar{q}$ . We use  $Q, \bar{Q}$

and  $\tilde{Q}$  to represent the configuration  $qq$ ,  $\bar{q}\bar{q}$  and  $q\bar{q}$  respectively. We use the notation  $|D_6, D_{3c}, S, N\rangle$  to represent the diquark configuration, where  $D_6$ ,  $D_{3c}$ ,  $S$  and  $N$  are the  $SU(6)$  color-spin coupling representations,  $SU(3)_c$  color representations, spin and number of the constituent quarks respectively. Based on the  $SU(6)_{cs} \supset SU(3)_c \otimes SU(2)_s$  group theory, there are four types of representations for the diquark  $qq$ :  $|21, \bar{3}_c, 0, 2\rangle$ ,  $|21, 6_c, 1, 2\rangle$ ,  $|15, \bar{3}_c, 1, 2\rangle$  and  $|15, \bar{6}_c, 0, 2\rangle$ . For the anti-diquark  $\bar{q}\bar{q}$ , there are also four types of representations:  $|\bar{21}, 3_c, 0, 2\rangle$ ,  $|\bar{21}, \bar{6}_c, 1, 2\rangle$ ,  $|\bar{15}, 3_c, 1, 2\rangle$  and  $|\bar{15}, 6_c, 0, 2\rangle$ . For the  $q\bar{q}$  system, there are also four types of representations:  $|1, 1_c, 0, 2\rangle$ ,  $|35, 1_c, 1, 2\rangle$ ,  $|35, 8_c, 1, 2\rangle$  and  $|35, 8_c, 0, 2\rangle$ .

For the tetraquark system  $q_1 q_2 \bar{q}_3 \bar{q}_4$  with four different flavors, the CM interaction matrix element between two  $SU(6)_{cs}$  eigenstates  $|k\rangle$  and  $|l\rangle$  is

$$\begin{aligned} V_{CM}(q_1 q_2 \bar{q}_3 \bar{q}_4) &= \langle k | H_{CM} | l \rangle = V_{12}(q_1 q_2) + V_{13}(q_1 \bar{q}_3) \\ &\quad + V_{14}(q_1 \bar{q}_4) + V_{23}(q_2 \bar{q}_3) + V_{24}(q_2 \bar{q}_4) \\ &\quad + V_{34}(\bar{q}_3 \bar{q}_4) \end{aligned} \quad (3)$$

where

$$V_{ij}(Q) = v_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j = -\frac{v_{ij}}{2} [\bar{C}(Q) - 16N] \quad (4)$$

and

$$V_{ij}(\tilde{Q}) = v_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j^* \vec{\sigma}_i \cdot \vec{\sigma}_j^* = \frac{v_{ij}}{2} [\bar{C}(\tilde{Q}) - 16N] \quad (5)$$

For the diquark system we have

$$\bar{C}(Q) = \bar{C}(\tilde{Q}) = C_6 - C_3 - \frac{8}{3} S(S+1) \quad (6)$$

where  $C_6$  and  $C_3$  are the Casimir operators of  $SU(6)_{cs}$  and  $SU(3)_c$  groups.  $S$  is the spin operator.

Based on the  $SU(6)_{cs}$  group decomposition, the color-spin wavefunction of the  $J^P = 1^+$  tetraquark  $SU(6)_{cs}$  eigenstates can be constructed in the  $Q \otimes \bar{Q}$  form

$$|35, 1_c, 1, 4\rangle = |21, 6_c, 1, 2\rangle \otimes |\bar{21}, \bar{6}_c, 1, 2\rangle \quad (7)$$

$$|35, 1_c, 1, 4\rangle = |15, \bar{3}_c, 1, 2\rangle \otimes |\bar{15}, 3_c, 1, 2\rangle \quad (8)$$

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= \sqrt{\frac{1}{3}} |21, \bar{3}_c, 0, 2\rangle \otimes |\bar{15}, 3_c, 1, 2\rangle \\ &\quad - \sqrt{\frac{2}{3}} |21, 6_c, 1, 2\rangle \otimes |\bar{15}, \bar{6}_c, 1, 2\rangle \end{aligned} \quad (9)$$

$$\begin{aligned} |280, 1_c, 1, 4\rangle &= \sqrt{\frac{2}{3}} |21, \bar{3}_c, 0, 2\rangle \otimes |\bar{15}, 3_c, 1, 2\rangle \\ &\quad + \sqrt{\frac{1}{3}} |21, 6_c, 1, 2\rangle \otimes |\bar{15}, \bar{6}_c, 1, 2\rangle \end{aligned} \quad (10)$$

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= \sqrt{\frac{1}{3}} |\bar{21}, 3_c, 0, 2\rangle \otimes |15, \bar{3}_c, 1, 2\rangle \\ &\quad - \sqrt{\frac{2}{3}} |\bar{21}, \bar{6}_c, 1, 2\rangle \otimes |15, 6_c, 1, 2\rangle \end{aligned} \quad (11)$$

$$\begin{aligned} |280, 1_c, 1, 4\rangle &= \sqrt{\frac{2}{3}}|\bar{2}1, 3_c, 0, 2\rangle \otimes |15, \bar{3}_c, 1, 2\rangle \\ &+ \sqrt{\frac{1}{3}}|\bar{2}1, \bar{6}_c, 1, 2\rangle \otimes |15, 6_c, 1, 2\rangle \quad (12) \end{aligned}$$

The CM wavefunctions of the  $J^P = 0^+$  and  $J^P = 2^+$  tetraquark states are listed in the appendix. These wavefunctions are the eigenstates of the CM interaction  $V_{ij}(Q)$  and  $V_{ij}(\bar{Q})$ . The CM interaction  $V_{CM}$  also has the form  $V_{ij}(\tilde{Q})$ . In order to get their eigenstates, we need to do the recoupling from  $Q \otimes \bar{Q}$  to  $\tilde{Q} \otimes \tilde{Q}$ . Based on Wigner and Racah coefficients of  $SU(6)_{cs} \supset SU(3)_c \otimes SU(2)_s$  [57, 58], the  $1^+ SU(6)_{cs}$  eigenstates in terms of  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  are

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= \frac{\sqrt{3}}{3}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &+ \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &+ \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &+ \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \quad (13) \end{aligned}$$

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= \frac{\sqrt{6}}{6}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &- \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &+ \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &- \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \quad (14) \end{aligned}$$

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= \frac{1}{2}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &- \frac{1}{2}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &- \frac{2}{3}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &- \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \quad (15) \end{aligned}$$

$$\begin{aligned} |280, 1_c, 1, 4\rangle &= -\frac{1}{2}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &+ \frac{1}{2}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &- \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &+ \frac{2}{3}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \quad (16) \end{aligned}$$

$$\begin{aligned} |35, 1_c, 1, 4\rangle &= -\frac{1}{2}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &+ \frac{1}{2}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &- \frac{2}{3}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &- \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \quad (17) \end{aligned}$$

$$\begin{aligned} |280, 1_c, 1, 4\rangle &= \frac{1}{2}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &- \frac{1}{2}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &- \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \\ &+ \frac{2}{3}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \quad (18) \end{aligned}$$

According to the  $SU(3)_c$  and  $SU(2)_s$  symmetry, the  $SU(6)_{cs}$  eigenstates in terms of  $q_2\bar{q}_3 \otimes q_1\bar{q}_4$  have the same form with those of  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  in Eqs. (13), (15) and (16). There appears an extra minus sign in the tetraquark states in Eqs. (14), (17) and (18) when we change the basis from  $q_2\bar{q}_3 \otimes q_1\bar{q}_4$  to  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$ . For the  $J^P = 0^+$  and  $J^P = 2^+$  tetraquark states, the  $SU(6)_{cs}$  eigenstates in the form of  $\tilde{Q} \otimes \tilde{Q}$  are listed in the appendix.

Using the above  $SU(6)_{cs}$  eigenstates in Eqs. (7)-(18), we can calculate each individual term in Eq. (3), obtain the eigenvalues of the CM interaction matrix  $V_{CM}$ , and derive the wave function and mass of the tetraquark system.

### III. POSSIBLE TETRAQUARK CANDIDATES AMONG VARIOUS $Z_c$ STATES

In order to extract the tetraquark mass, we need the values of the constituent quark mass and the parameter  $v$ . Recall that the charmonium  $J/\psi$  and  $\eta_c$  can be treated as the  $SU(6)_{cs}$  diquark  $c\bar{c}$  state  $|35, 1_c, 1, 2\rangle$  and  $|1, 1_c, 0, 2\rangle$ . Similarly, the charmed mesons  $D^*$  and  $D$  can be treated as  $SU(6)_{cs}$  diquark  $c\bar{u}$  state  $|35, 1_c, 1, 2\rangle$  and  $|1, 1_c, 0, 2\rangle$ . With Eq. (4) and the meson masses from PDG [59], we can extract the masses of the  $u, c, s$  and  $b$  constituent quarks.

$$\left\{ \begin{array}{l} M(J/\psi) = 2m_c + \frac{16}{3}v_{c\bar{c}}(\frac{m_u}{m_c})^2; \\ M(\eta_c) = 2m_c - 16v_{c\bar{c}}(\frac{m_u}{m_c})^2; \\ M(D^*) = m_u + m_c + \frac{16}{3}v_{c\bar{u}}\frac{m_u}{m_c}; \\ M(D) = m_u + m_c - 16v_{c\bar{u}}\frac{m_u}{m_c}; \\ M(D_s^*) = m_s + m_c + \frac{16}{3}v_{c\bar{s}}\frac{m_u}{m_c}\frac{m_s}{m_u}; \\ M(D_s) = m_s + m_c - 16v_{c\bar{s}}\frac{m_u}{m_c}\frac{m_s}{m_u}; \\ M(\Upsilon) = 2m_b + 16v_{b\bar{b}}(\frac{m_u}{m_b})^2 \approx 2m_b. \end{array} \right. \quad (19)$$

From the above equation, we get

$$\left\{ \begin{array}{l} m_c = 1534 \text{ MeV}; \\ m_u = 437 \text{ MeV}; \\ m_s = 542 \text{ MeV}; \\ m_b = 4730 \text{ MeV}; \end{array} \right. \quad (20)$$

According Eq. (3),  $V_{CM}(qc\bar{q}\bar{c})$  reads

$$\begin{aligned} V_{CM}(qc\bar{q}\bar{c}) &= \frac{m_u}{m_c} V_{12} + V_{13} + \frac{m_u}{m_c} V_{14} \\ &+ \frac{m_u}{m_c} V_{23} + \left(\frac{m_u}{m_c}\right)^2 V_{24} + \frac{m_u}{m_c} V_{34} \end{aligned} \quad (21)$$

After diagonalizing the mass matrix  $V_{CM}$  for the  $J^P = 1^+$   $qc\bar{q}\bar{c}$  tetraquark states, we get six eigenvalues:  $-15.9v, -4.1v, -1.5v, 1.7v, 5.6v, 5.8v$  which are listed in Table VII. Sometimes we use the eigenvalues to denote the state. In the following we discuss two schemes to fix the parameter  $v$  and extract the tetraquark spectrum.

### A. Scheme I: Using the mass of one of the $Z_c$ states as input

Assuming that  $Z_c(3900)$  is one of the six tetraquark states, the parameter  $v$  can be fixed. Similarly,  $Z_c(4025)$  and  $Z_c(4200)$  can also be used as input to extract the value of  $v$ . Throughout our discussion, we require  $v$  to be positive. Then we use the obtained  $v$  to calculate the masses of the other eigenstates, which are listed in Table I.

If  $Z_c(3900)$  is appointed as the state with the eigenvalue  $-15.9v, -4.1v$  and  $-1.5v$ , it is quite difficult to accommodate either  $Z_c(4025)$  or  $Z_c(4200)$  among the six states. If  $Z_c(4025)$  is appointed as the state with the eigenvalue  $1.7v$ , the mass of the state with the eigenvalue  $5.6v$  is 4215.4 MeV which is close to  $Z_c(4200)$ , while the mass of the state with the eigenvalue  $-1.5v$  is 3868.8 MeV which is 30 MeV lower than  $Z_c(3900)$ . Unfortunately, the lowest axial vector tetraquark state is around 3166 MeV. Such a scheme is not realistic.

If  $Z_c(4200)$  is appointed as the state with the eigenvalue  $5.6v$ , the mass of the state with the eigenvalue  $1.7v$  is 4020.3 MeV which is close to  $Z_c(4025)$ . The mass of the state with the eigenvalue  $-1.5v$  is 3872.9 MeV, which is 28 MeV lower than  $Z_c(3900)$ . In this case the lowest state is around 3210 MeV, which is also quite unrealistic. It's almost impossible to accommodate all the three charged states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  within the axial vector tetraquark spectrum simultaneously. In other words, at least one or two of these states is not a tetraquark candidate.

TABLE I: The masses of the six axial vector  $qc\bar{q}\bar{c}$  tetraquark states when the parameter  $v$  is fixed by the mass of  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$ . The eigenvalue is used to denote the state as the subscript.

$v$	$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$	$5.6v$	$5.8v$
$v_{Z_c(3900)}^{Z_c(3900)}$	2.6	3900	3931.2	3938.0	3946.5	3956.8
$v_{-4.1}^{Z_c(3900)}$	10.2	3779.1	3900	3926.6	3959.4	3999.4
$v_{-1.5}^{Z_c(3900)}$	28.0	3496.8	3827.2	3900	3989.6	4098.8
$v_{1.7}^{Z_c(4025)}$	48.8	3165.7	3741.8	3868.8	4025	4215.4
$v_{5.6}^{Z_c(4025)}$	14.8	3706.3	3881.2	3919.8	3967.2	4025
$v_{5.8}^{Z_c(4025)}$	14.3	3714.5	3883.3	3920.5	3966.3	4022.1
$v_{5.6}^{Z_c(4200)}$	46.1	3209.4	3753.1	3872.9	4020.3	4200
$v_{5.8}^{Z_c(4200)}$	44.5	3234.7	3759.6	3875.3	4017.6	4191.1

### B. Scheme II: using the mass splitting of two $Z_c$ states as input

The parameter  $v$  can be extracted from the mass splitting if we assume two of the three states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  are the  $1^+$   $qc\bar{q}\bar{c}$  tetraquark states. As pointed out in Section IV, the state with the eigenvalue  $-4.1v$  does not decay to  $J/\psi\pi$ . Thus it is not appropriate to assign it as  $Z_c(3900)$ . Therefore we only assume  $Z_c(3900)$  as the state either with the eigenvalue  $-15.9$  or  $-1.5v$ . Once the value of  $v$  is extracted, we obtain the whole spectrum. The results are listed in Table II.

If  $Z_c(3900)$  and  $Z_c(4025)$  are assigned as the state with the eigenvalue  $-1.5v$  and  $1.7v$  respectively, the resulting mass of the state with the eigenvalue  $5.6v$  is 4177.3 MeV, which is close to  $Z_c(4200)$ . Unfortunately the lowest state is around 3338 MeV, which is unrealistic. Similarly, if  $Z_c(3900)$  and  $Z_c(4200)$  are assigned as the state with the eigenvalue  $-1.5v$  and  $5.6v$  respectively, the mass of the state with the eigenvalue  $1.7v$  is 4035.2 MeV, which is close to  $Z_c(4025)$ . If  $Z_c(4025)$  and  $Z_c(4200)$  are treated as the state with the eigenvalue  $1.7v$  and  $5.6v$  respectively, the mass of the state with the eigenvalue  $-1.5v$  is 3881.4 MeV, which is close to  $Z_c(3900)$ . Now the lowest state is around 3235 MeV. Although we could accommodate all three charged states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  as the axial vector tetraquark candidates, the resulting mass of the lowest state is always too low and unrealistic. In other words, not all these three states are tetraquark candidates, which is consistent with the conclusion in the previous subsection.

TABLE II: The masses of the  $1^+$   $qc\bar{q}\bar{c}$  tetraquark states when the parameter  $v$  is fixed by the mass difference of two  $Z_c$  states.

$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$	$5.6v$	$5.8v$
$Z_c(3900) \rightarrow -15.9v, Z_c(4025) \rightarrow -4.1v, v = 10.6$					
3900	4025	4052.5	4086.4	4127.8	4129.9
$Z_c(3900) \rightarrow -15.9v, Z_c(4025) \rightarrow -1.5v, v = 8.7$					
3900	4002.4	4025	4052.8	4086.6	4088.4
$Z_c(3900) \rightarrow -15.9v, Z_c(4025) \rightarrow 1.7v, v = 7.1$					
3900	3983.8	4002.3	4025	4052.7	4054.1
$Z_c(3900) \rightarrow -15.9v, Z_c(4025) \rightarrow 5.6v, v = 5.81$					
3900	3968.6	3983.7	4002.3	4025	4026.2
$Z_c(3900) \rightarrow -15.9v, Z_c(4025) \rightarrow 5.8v, v = 5.76$					
3900	3968	3983	4001.4	4023.9	4025
$Z_c(3900) \rightarrow -1.5v, Z_c(4025) \rightarrow 1.7v, v = 39.1$					
3337.5	3798.4	3900	4025	4177.3	4185.2
$Z_c(3900) \rightarrow -1.5v, Z_c(4025) \rightarrow 5.6v, v = 17.6$					
3646.5	3854.2	3900	3956.3	4025	4028.5
$Z_c(4025) \rightarrow 1.7v, Z_c(4200) \rightarrow 5.6v, v = 44.9$					
3235.3	3764.7	3881.4	4025	4200	4209
$Z_c(4025) \rightarrow 1.7v, Z_c(4200) \rightarrow 5.8v, v = 42.7$					
3273.8	3777.4	3888.4	4025	4191.5	4200
$Z_c(3900) \rightarrow -1.5v, Z_c(4200) \rightarrow 5.6v, v = 42.3$					
3291.6	3790.1	3900	4035.2	4200	4208.5

### C. The $qc\bar{c}$ , $sc\bar{c}$ and hidden-bottom tetraquark states

We assume  $Z_c(4025)$  as the  $qc\bar{c}$  tetraquark state with the eigenvalue  $1.7v$  to fix the parameter  $v$  and collect the numerical results for the  $qc\bar{c}$  and  $sc\bar{c}$  tetraquark states in Table III.

TABLE III: The masses of the  $qc\bar{c}$ ,  $qc\bar{c}$  and  $sc\bar{c}$  tetraquark states with  $J^P = 0^+, 1^+, 2^+$ . The parameter  $v$  is fixed assuming  $Z(4025)$  as the tetraquark state with the eigenvalue  $1.7v$

		$0^+$	$1^+$	$2^+$
$qc\bar{c}$	$V_{CM}$	$-18.6v$	$-15.9v$	$2.7v$
	$M(\text{MeV})$	3033.9	3165.7	4073.8
	$V_{CM}$	$-7.4v$	$-4.1v$	$5.8v$
	$M(\text{MeV})$	3580.7	3741.8	4225.2
$qc\bar{s}c$	$V_{CM}$	$-15.6v$	$-12.8v$	$2.5v$
	$M(\text{MeV})$	3285.4	3422.1	4169.1
	$V_{CM}$	$-6.5v$	$-3.8v$	$4.7v$
	$M(\text{MeV})$	3729.7	3861.5	4276.5
$sc\bar{s}c$	$V_{CM}$	$-13.1v$	$-10.3v$	$2.3v$
	$M(\text{MeV})$	3512.4	3649.1	4264.3
	$V_{CM}$	$-5.7v$	$-3.2v$	$3.9v$
	$M(\text{MeV})$	3873.7	3995.8	4342.4

We extend the same formalism to investigate the hidden-bottom tetraquark states. The results are collected in Tables IV, V, VI.

TABLE IV: The eigenvalues of the  $qb\bar{q}\bar{b}$ ,  $qb\bar{s}\bar{b}$  and  $sb\bar{s}\bar{b}$  tetraquark states with  $J^P = 0^+, 1^+, 2^+$ .

configuration	$J^P$	$V_{CM}$			
$qb\bar{q}\bar{b}$	$0^+$	$-16.2v$	$-3.0v$	$1.8v$	$5.8v$
	$1^+$	$-16.0v$	$-1.8v$	$-0.8v$	$2.0v$
	$2^+$	$0.4v$	$5.4v$		
$qb\bar{s}\bar{b}$	$0^+$	$-13.1v$	$-2.7v$	$1.5v$	$4.7v$
	$1^+$	$-12.9v$	$-1.6v$	$-0.6v$	$1.6v$
	$2^+$	$0.5v$	$4.3v$		
$sb\bar{s}\bar{b}$	$0^+$	$-10.7v$	$-2.3v$	$1.1v$	$3.9v$
	$1^+$	$-10.4v$	$-1.3v$	$-0.5v$	$1.3v$
	$2^+$	$0.5v$	$3.5v$		

### IV. DECAY PATTERNS OF HIDDEN-CHARM TETRAQUARKS

The eigenvalues of the CM interaction matrix  $V_{CM}$  can be used to derive the mass of tetraquark system, while the eigenvectors of  $V_{CM}$  contain important information on their decay pattern. Therefore, we carefully investigate the eigenvectors of the tetraquark systems with the configuration  $qc\bar{q}\bar{c}$ ,  $qc\bar{s}\bar{c}$  and  $sc\bar{s}\bar{c}$  and  $J^P = 0^+, 1^+, 2^+$ . We first list the eigenvalues of  $V_{CM}$  for the  $qc\bar{q}\bar{c}$ ,  $qc\bar{s}\bar{c}$  and  $sc\bar{s}\bar{c}$  tetraquark configuration in Table VII.

TABLE V: The masses of the  $1^+$   $qb\bar{q}\bar{b}$  tetraquark states. The parameter  $v$  is fixed using the  $Z_c$  mass as input.

$v$	$-16.0v$	$-1.8v$	$-0.8v$	$2.0v$	$5.3v$	$5.4v$
$v_{-15.9v}^{Z_c(3900)}$	2.6	10292.4	10329.3	10331.9	10339.2	10347.8
$v_{-14.1v}^{Z_c(3900)}$	10.2	10170.8	10315.6	10325.8	10354.4	10388.1
$v_{-1.5v}^{Z_c(3900)}$	28.0	9886	10283.6	10311.6	10390	10482.4
$v_{1.7v}^{Z_c(4025)}$	48.8	9553.2	10246.2	10295	10431.6	10592.6
$v_{5.6v}^{Z_c(4025)}$	14.8	10097.2	10307.4	10322.2	10363.6	10412.4
$v_{5.8v}^{Z_c(4025)}$	14.3	10105.2	10308.3	10322.6	10362.6	10409.8
$v_{5.6v}^{Z_c(4200)}$	46.1	9596.4	10251	10297.1	10426.2	10578.3
$v_{5.8v}^{Z_c(4200)}$	44.5	9622	10253.9	10298.4	10423	10569.9

TABLE VI: The eigenvalues and masses of the  $qb\bar{q}\bar{b}$ ,  $qb\bar{s}\bar{b}$  and  $sb\bar{s}\bar{b}$  tetraquark states with  $J^P = 0^+, 1^+, 2^+$ . The parameter  $v$  is fixed assuming  $Z(4025)$  as the tetraquark state with the eigenvalue  $1.7v$ .

		$0^+$	$1^+$	$2^+$
$qb\bar{q}\bar{b}$	$V_{CM}$	$-16.2v$	$-16.0v$	$0.4v$
	$M(\text{MeV})$	9543.1	9552.8	10353.5
	$V_{CM}$	$-3.0v$	$-1.8v$	$5.4v$
	$M(\text{MeV})$	10187.5	10246.1	10597.6
$qb\bar{s}\bar{b}$	$V_{CM}$	$-13.1v$	$-12.9v$	$0.5v$
	$M(\text{MeV})$	9799.4	9809.2	10463.4
	$V_{CM}$	$-2.7v$	$-1.6v$	$4.3v$
	$M(\text{MeV})$	10307.2	10360.9	10648.9
$sb\bar{s}\bar{b}$	$V_{CM}$	$-10.7v$	$-10.4v$	$0.5v$
	$M(\text{MeV})$	10021.6	10036.2	10568.4
	$V_{CM}$	$-2.3v$	$-1.3v$	$3.5v$
	$M(\text{MeV})$	10431.7	10480.5	10714.9

For the  $J^P = 0^+, 1^+$  case, we only list the eigenvectors with the negative eigenvalues. When we present the eigenvectors using the diquark representation  $q\bar{q} \otimes q\bar{q}$ , we omit the  $N$  in the diquark representation  $|D_6, D_{3c}, S, N\rangle$  for brevity since  $N = 2$ . We present the expressions of the eigenvectors for the  $qc\bar{q}\bar{c}$ ,

TABLE VII: The eigenvalues of  $V_{CM}$  for the tetraquark configuration  $qc\bar{q}\bar{c}$ ,  $qc\bar{s}\bar{c}$  and  $sc\bar{s}\bar{c}$  with  $J^P = 0^+, 1^+, 2^+$ .

configuration	$J^P$	$V_{CM}$			
$qc\bar{q}\bar{c}$	$0^+$	$-18.6v$	$-7.4v$	$0.8v$	$8.3v$
	$1^+$	$-15.9v$	$-4.1v$	$-1.5v$	$1.7v$
	$2^+$	$2.7v$	$5.8v$		
$qc\bar{s}\bar{c}$	$0^+$	$-15.6v$	$-6.5v$	$0.5v$	$7.1v$
	$1^+$	$-12.8v$	$-3.8v$	$-1.3v$	$1.3v$
	$2^+$	$2.5v$	$4.7v$		
$sc\bar{s}\bar{c}$	$0^+$	$-13.1v$	$-5.7v$	$0.3v$	$6.1v$
	$1^+$	$-10.3v$	$-3.2v$	$-1.3v$	$1.0v$
	$2^+$	$2.3v$	$3.9v$		

$qc\bar{s}\bar{c}$ , and  $sc\bar{s}\bar{c}$  tetraquark systems in Tables VIII- X, XI-XIII, and XIV-XVI respectively.

We notice that  $J/\psi$  and  $\eta_c$  can also be expressed as the  $SU(6)_{cs} c\bar{c}$  state  $|35, 1_c, 1\rangle$  and  $|1, 1_c, 0\rangle$ . Similarly,  $D^*$  and  $D$  can be treated as the  $SU(6)_{cs} c\bar{u}$  state  $|35, 1_c, 1\rangle$  and  $|1, 1_c, 0\rangle$ . Therefore, we can identify the decay patterns of the tetraquark states from the expression of their CM interaction eigenvectors. The branching fraction of each decay mode is proportional to the square of the coefficient of the corresponding component in the eigenvectors if we ignore the phase space difference. From the very beginning, we want to emphasize the following point: so long as the phase space allows, the  $\psi'\pi$  decay mode is also allowed if  $J/\psi\pi$  is one of the allowed decay modes.

For the  $J^P = 0^+$  state, the lowest state corresponds to the eigenvalue  $-18.6v$ . From Table IX, its dominant decay mode is  $\eta_c\pi$ . The  $\bar{D}D$  mode is also important. The  $\bar{D}^*D^*$  mode is suppressed by a factor of eight if we compare the coefficients of the  $\bar{D}D$  and  $\bar{D}^*D^*$  components only. In fact, the  $\bar{D}^*D^*$  mode is further suppressed by phase space.

The  $J^P = 0^+$  state with the eigenvalue  $-7.4v$  also decays into  $\eta_c\pi$ ,  $\bar{D}D$  and  $\bar{D}^*D^*$ . However,  $\bar{D}D$  becomes its dominant decay mode. The  $\bar{D}^*D^*$  mode is also severely suppressed.

From Table IX, the  $J^P = 2^+$  state with the eigenvalue  $5.8v$  mainly decays into  $J/\psi\rho$  while its  $\bar{D}^*D^*$  mode is suppressed. In contrast, the  $J^P = 2^+$  state with the eigenvalue  $2.7v$  decays into  $\bar{D}^*D^*$  only. Its  $J/\psi\rho$  mode is forbidden.

Experimentally several charged axial vector hidden-charm states were reported in different decay channels. All the four charged axial vector states  $Z_c(3900)$ ,  $Z_c(4025)$ ,  $Z_c(4200)$  and  $Z_c(4485)$  were observed in the  $J/\psi\pi$  channel.  $Z_c(4485)$  was also observed in the  $\psi'\pi$  mode. The dominant decay mode of  $Z_c(3900)$  and  $Z_c(4025)$  is  $\bar{D}D^*$  and  $\bar{D}^*D^*$  respectively. Up to now, the dominant decay mode of  $Z_c(4200)$  and  $Z_c(4485)$  has not been established yet. Moreover,  $Z_c(4025)$  does not decay into  $\bar{D}D^*$ .

It's very interesting to investigate the decay patterns of the low lying tetraquark states and compare their typical decay modes with the available experimental data. From Table VIII, the lowest axial vector  $qc\bar{q}\bar{c}$  tetraquark state corresponds to the eigenvalue  $-15.9v$ . Its dominant decay mode is  $J/\psi\pi$ . The  $\bar{D}^*D$  mode is suppressed by a factor of sixteen if we compare the coefficients of the  $J/\psi\pi$  and  $D^*\bar{D}$  components and ignore the phase space difference. The  $\bar{D}^*D^*$  mode is further suppressed roughly by a factor of two compared with the  $D^*\bar{D}$  mode. Considering the decay phase space, the  $D^*\bar{D}$  and  $\bar{D}^*D^*$  modes are further suppressed. This state mainly decay into  $J/\psi\pi$ , which is in strong contrast with the fact that the dominant decay mode of  $Z_c(3900)$  and  $Z_c(4025)$  is  $\bar{D}D^*$  and  $\bar{D}^*D^*$  respectively. In other words, neither  $Z_c(3900)$  nor  $Z_c(4025)$  is a good candidate of this lowest lying axial vector tetraquark state. On the other hand, either  $Z_c(4200)$  or  $Z_c(4485)$  could be a candidate of this tetraquark state. In fact,  $Z_c(4200)$  is a very promising tetraquark candidate.

The second axial vector tetraquark state with the eigenvalue  $-4.1v$  decays into  $D^*\bar{D}$  only. It's quite particular that this state neither decays into  $J/\psi\pi$  nor into  $\bar{D}^*D^*$  even phase space allows. Since all the four charged  $Z_c$  states decay into the  $J/\psi\pi$

mode, none of them is the candidate of this tetraquark state.

The third  $J^P = 1^+$  state corresponds to the eigenvalue  $-1.5v$ , which decays into  $J/\psi\pi$ ,  $\eta_c\rho$ ,  $D^*\bar{D}$  and  $\bar{D}^*D^*$ . Its dominant decay mode is  $D^*\bar{D}$ . For comparison, both the  $\bar{D}^*D^*$  and  $\eta_c\rho$  modes are suppressed roughly by a factor of eight if we ignore the phase space difference. In contrast, the  $J/\psi\pi$  mode is strongly suppressed. If we ignore the phase space difference, the suppression factor is roughly 25 compared with the dominant  $D^*\bar{D}$  mode. Based on the current experimental information, all the three  $Z_c(3900)$ ,  $Z_c(4200)$  and  $Z_c(4485)$  can be assigned as this third tetrakar state with the eigenvalue  $-1.5v$ . Especially, the characteristic decay pattern of this third axial vector tetraquark state matches well with that of  $Z_c(3900)$ . With such an assignment, we would expect two more axial vector tetraquark states with the eigenvalue  $-15.9v$  and  $-4.1v$  which are very close to (or even below) the open charm threshold and lie below  $Z_c(3900)$ . Their decay patterns are listed in the previous paragraphs.

TABLE VIII: The eigenvectors of  $V_{CM}$  for the  $qc\bar{q}\bar{c}$  tetraquark states with  $J^P = 1^+$ .

$V_{CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$-15.9v$	$+0.99 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$+0.24 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$+0.14 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.44 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$+0.24 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$+0.44 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.69 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.15 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$-4.1v$	$+ 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.67 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
		$-0.24 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$-0.67 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$+0.24 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
$-1.5v$	$+0.12 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$-0.65 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$-0.56 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$	$+0.18 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
	$-0.22 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$	$-0.65 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
	$-0.79 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$	$+0.18 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.17 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.23 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

TABLE IX: The eigenvectors of  $V_{CM}$  for the  $qc\bar{q}\bar{c}$  tetraquark states with  $J^P = 0^+$ .

$V_{CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$-18.6v$	$+0.94 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$+0.45 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$+0.33 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.16 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
		$+0.35 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$-0.80 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
$-7.4v$	$+0.33 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$-0.85 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$-0.34 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$	$-0.25 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
	$+0.88 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.39 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$-0.27 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$

TABLE X: The eigenvectors of  $V_{CM}$  for the  $qc\bar{q}\bar{c}$  tetraquark states with  $J^P = 2^+$ .

$V_{CM}$	$q\bar{q} \otimes c\bar{c}$	$c\bar{q} \otimes q\bar{c}$
$5.8v$	$+ 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$	$+0.94 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$+0.33 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$2.7v$	$- 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.33 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.94 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

TABLE XI: The eigenvectors of  $V_{CM}$  for the  $qc\bar{s}\bar{c}$  tetraquark states with  $J^P = 1^+$ .

$V_{CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$-12.8v$	$+0.99 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$+0.24 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$+0.16 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$	$+0.43 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$+0.25 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$+0.43 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.69 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.14 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$-3.8v$	$-0.16 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$	$+0.5 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$-0.14 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$	$-0.2 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
	$-0.97 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.8 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
$-1.3v$	$+0.13 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$-0.78 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$-0.57 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$	$+0.24 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
	$-0.24 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$	$-0.49 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
	$-0.74 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$	$-0.09 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
	$+0.22 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.21 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.2 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

TABLE XII: The eigenvectors of  $V_{CM}$  for the  $qc\bar{s}\bar{c}$  tetraquark states with  $J^P = 0^+$ .

$V_{CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$-15.6v$	$+0.92 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$+0.5 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$-0.38 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.14 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
		$+0.33 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$-0.79 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
$-6.5v$	$+0.37 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$-0.83 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$-0.35 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$	$-0.24 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
	$+0.85 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.4 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$-0.31 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$

TABLE XIII: The eigenvectors of  $V_{CM}$  for the  $qc\bar{s}\bar{c}$  tetraquark states with  $J^P = 2^+$ .

$V_{CM}$	$q\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes q\bar{c}$
$4.7v$	$+ 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$	$+0.94 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$+0.33 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$2.5v$	$+ 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.33 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.94 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

TABLE XIV: The eigenvectors of  $V_{CM}$  for the  $sc\bar{s}\bar{c}$  tetraquark states with  $J^P = 1^+$ .

$V_{CM}$	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$-10.3v$	$+0.98 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$+0.25 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$+0.17 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$	$+0.43 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$+0.26 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$+0.43 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.69 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.13 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$-3.2v$	$+ 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.67 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
		$-0.24 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$-0.67 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$+0.24 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
$-1.3v$	$+0.14 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$	$-0.65 1, 1_c, 0\rangle \otimes  35, 1_c, 1\rangle$
	$-0.63 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$	$+0.14 35, 8_c, 0\rangle \otimes  35, 8_c, 1\rangle$
		$-0.31 35, 1_c, 1\rangle \otimes  1, 1_c, 0\rangle$
		$-0.7 35, 8_c, 1\rangle \otimes  35, 8_c, 0\rangle$
		$-0.29 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.15 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

TABLE XV: The eigenvectors of  $V_{CM}$  for the  $sc\bar{s}\bar{c}$  tetraquark states with  $J^P = 0^+$ .

$V_{CM}$	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$-13.1v$	$+0.91 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$+0.53 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$-0.41 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$-0.13 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
		$+0.31 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$-0.78 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
$-5.6v$	$+0.41 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$	$-0.81 1, 1_c, 0\rangle \otimes  1, 1_c, 0\rangle$
	$-0.36 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$	$-0.24 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
	$+0.83 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.4 35, 8_c, 0\rangle \otimes  35, 8_c, 0\rangle$
		$+0.11 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
		$-0.35 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$

## V. SUMMARY

Within the framework of the color-magnetic interaction, we have systematically considered the mass spectrum of the hidden-charm and hidden-bottom tetraquark states with the configurations  $qc\bar{q}\bar{c}$ ,  $qc\bar{s}\bar{c}$ ,  $sc\bar{s}\bar{c}$ ,  $qb\bar{q}\bar{b}$ ,  $qb\bar{s}\bar{b}$ ,  $sb\bar{s}\bar{b}$  and  $J^P = 1^+, 0^+, 2^+$ .

Experimentally several charged axial-vector hidden-charm states were reported. We have adopted two schemes to fix the parameter  $v$  and extracted the tetraquark spectrum. We first

TABLE XVI: The eigenvectors of  $V_{CM}$  for the  $sc\bar{s}\bar{c}$  tetraquark states with  $J^P = 2^+$ .

$V_{CM}$	$s\bar{s} \otimes c\bar{c}$	$c\bar{s} \otimes s\bar{c}$
$3.9v$	$+ 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$	$+0.94 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$+0.33 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$
$2.3v$	$+ 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$	$+0.33 35, 8_c, 1\rangle \otimes  35, 8_c, 1\rangle$
		$-0.94 35, 1_c, 1\rangle \otimes  35, 1_c, 1\rangle$

tried to assume one of  $Z_c$  states is a tetraquark state and use its mass as input to determine  $v$  and the masses of the other tetraquark states. We notice that it is impossible to accommodate all the three charged states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  within the axial vector tetraquark spectrum simultaneously. Then we tried to use the mass splitting between two  $Z_c$  states as input. With the second scheme we could accommodate all three charged states  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_c(4200)$  as the axial vector tetraquark candidates simultaneously. However, the resulting mass of the lowest axial vector tetraquark state is always too low and unrealistic. We have to conclude that not all these three states are tetraquark candidates. Instead of being a tetraquark candidate, at least one or two of these states is probably a molecular state or some other structure.

Moreover, the eigenvectors of the chromomagnetic interaction contains valuable information of the decay pattern of the tetraquark states. For example, the dominant decay mode of the lowest axial vector  $qc\bar{q}c$  tetraquark state is  $J/\psi\pi$ . Its  $D^*\bar{D}$  and  $\bar{D}^*D^*$  modes are strongly suppressed. Recall that the dominant decay mode of  $Z_c(3900)$  and  $Z_c(4025)$  is  $\bar{D}D^*$  and  $\bar{D}^*D^*$  respectively. We tend to conclude that neither  $Z_c(3900)$  nor  $Z_c(4025)$  is a good candidate of the lowest lying axial vector tetraquark state. In fact,  $Z_c(3900)$  and  $Z_c(4025)$  is close to the  $\bar{D}D^*$  and  $\bar{D}^*D^*$  mass threshold. They are good molecular candidates. Their mass and decay pattern agree with the naive expectation within the molecular picture.

On the other hand, the charmonium-like charged state  $Z_c(4200)$  is observed in the  $J/\psi\pi$  channel with significance  $8.2\sigma$ . Its mass is far away from the mass threshold of two S-wave heavy mesons. In fact, the axial vector hidden-charm tetraquark state was predicted to lie around 4.2 GeV several years ago [60]. As expected as a tetraquark candidate,  $Z_c(4200)$  is very broad with a width around 370 MeV. All the available experimental information indicates that  $Z_c(4200)$  is a very promising candidate of the lowest axial vector hidden-charm tetraquark state. Future experimental investigations of this state will be very desirable.

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### Appendix

The  $SU(6)_{cs}$  eigenstates of the  $0^+$  tetraquark in the  $Q \otimes \bar{Q}$  form are:

$$\begin{aligned} |1, 1_c, 0, 4\rangle &= \sqrt{\frac{6}{7}}|21, 6_c, 1, 2\rangle \otimes |\bar{2}\bar{1}, \bar{6}_c, 1, 2\rangle \\ &+ \sqrt{\frac{1}{7}}|21, \bar{3}_c, 0, 2\rangle \otimes |\bar{2}\bar{1}, 3_c, 0, 2\rangle \end{aligned} \quad (22)$$

$$\begin{aligned} |405, 1_c, 0, 4\rangle &= \sqrt{\frac{1}{7}}|21, 6_c, 1, 2\rangle \otimes |\bar{2}\bar{1}, \bar{6}_c, 1, 2\rangle \\ &- \sqrt{\frac{6}{7}}|21, \bar{3}_c, 0, 2\rangle \otimes |\bar{2}\bar{1}, 3_c, 0, 2\rangle \end{aligned} \quad (23)$$

$$\begin{aligned} |1, 1_c, 0, 4\rangle &= \sqrt{\frac{3}{5}}|15, \bar{3}_c, 1, 2\rangle \otimes |\bar{1}\bar{5}, 3_c, 1, 2\rangle \\ &+ \sqrt{\frac{2}{5}}|15, 6_c, 0, 2\rangle \otimes |\bar{1}\bar{5}, \bar{6}_c, 0, 2\rangle \end{aligned} \quad (24)$$

$$\begin{aligned} |189, 1_c, 0, 4\rangle &= \sqrt{\frac{2}{5}}|15, \bar{3}_c, 1, 2\rangle \otimes |\bar{1}\bar{5}, 3_c, 1, 2\rangle \\ &- \sqrt{\frac{3}{5}}|15, 6_c, 0, 2\rangle \otimes |\bar{1}\bar{5}, \bar{6}_c, 0, 2\rangle \end{aligned} \quad (25)$$

The  $SU(6)_{cs}$  eigenstates of the  $0^+$  tetraquark in the  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  form are:

$$\begin{aligned} |1, 1_c, 0, 4\rangle &= \frac{\sqrt{21}}{6}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &+ \frac{\sqrt{7}}{14}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &+ \frac{\sqrt{42}}{21}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &- \frac{\sqrt{14}}{7}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \end{aligned} \quad (26)$$

$$\begin{aligned} |405, 1_c, 0, 4\rangle &= \frac{2\sqrt{42}}{21}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &+ \frac{3\sqrt{7}}{14}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &+ \frac{5\sqrt{21}}{42}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \end{aligned} \quad (27)$$

$$\begin{aligned} |1, 1_c, 0, 4\rangle &= \frac{15}{6}|q_1\bar{q}_3 1, 1_c, 0, 2\rangle \otimes |q_2\bar{q}_4 1, 1_c, 0, 2\rangle \\ &+ \frac{5}{10}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &- \frac{\sqrt{30}}{15}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &+ \frac{\sqrt{10}}{5}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \end{aligned} \quad (28)$$

$$\begin{aligned} |189, 1_c, 0, 4\rangle &= -\frac{2\sqrt{30}}{15}|q_1\bar{q}_3 35, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 1_c, 1, 2\rangle \\ &- \frac{3\sqrt{5}}{10}|q_1\bar{q}_3 35, 8_c, 0, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 0, 2\rangle \\ &- \frac{\sqrt{15}}{30}|q_1\bar{q}_3 35, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_4 35, 8_c, 1, 2\rangle \end{aligned} \quad (29)$$

According to the  $SU(3)_c$  and  $SU(2)_s$  symmetry, the  $SU(6)_{cs}$  eigenstates of the  $q_2\bar{q}_3 \otimes q_1\bar{q}_4$  form are the same as those of the  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  form in the first two tetraquark states, while there appears an extra minus sign in the last two tetraquark states.

The  $SU(6)_{cs}$  eigenstates of the  $2^+$  tetraquark in the  $Q \otimes \bar{Q}$  form are:

$$|405, 1_c, 0, 4\rangle = |21, 6_c, 1, 2\rangle \otimes |\bar{2}1, \bar{6}_c, 1, 2\rangle \quad (30)$$

$$|189, 1_c, 2, 4\rangle = |15, \bar{3}_c, 1, 2\rangle \otimes |\bar{1}5, 3_c, 1, 2\rangle \quad (31)$$

The  $SU(6)_{cs}$  eigenstates of the  $2^+$  tetraquark in the  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  form are:

$$\begin{aligned} |405, 1_c, 2, 4\rangle &= \frac{1}{3}|q_1\bar{q}_335, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_435, 8_c, 1, 2\rangle \\ &+ \frac{2}{3}|q_1\bar{q}_335, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_435, 1_c, 1, 2\rangle \end{aligned} \quad (32)$$

$$\begin{aligned} |189, 1_c, 2, 4\rangle &= -\frac{\sqrt{2}}{3}|q_1\bar{q}_335, 8_c, 1, 2\rangle \otimes |q_2\bar{q}_435, 8_c, 1, 2\rangle \\ &- \frac{3\sqrt{1}}{3}|q_1\bar{q}_335, 1_c, 1, 2\rangle \otimes |q_2\bar{q}_435, 1_c, 1, 2\rangle \end{aligned}$$

According to the  $SU(3)_c$  and  $SU(2)_s$  symmetry, the  $SU(6)_{cs}$  eigenstate of the  $q_2\bar{q}_3 \otimes q_1\bar{q}_4$  form is the same as that of the  $q_1\bar{q}_3 \otimes q_2\bar{q}_4$  form in the first tetraquark state, while there appears an extra minus sign in the second tetraquark state.

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