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Search for $C = +$ charmonium and XYZ states in $e^+e^- \to \gamma + \ H$ at BESIII

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Abstract: Within the framework of nonrelativistic quantum chromodynamics, we study the production of $C = +$ charmonium states H in $e^+e^- \to \gamma + H$ at BESIII with $H = \eta_c(nS)$ (n=1, 2, 3, and 4), $\chi_{cJ}(nP)$ (n=1, 2, and 3), and ${}^1D_2(nD)$ (n=1 and 2). The radiative and relativistic corrections are calculated to next-to-leading order for S and P wave states. We then argue that the search for $C = +$ XYZ states such as $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ in $e^+e^- \to \gamma \ + \ H$ at BESIII may help clarify the nature of these states. BESIII can search XYZ states through two body process $e^+e^- \to \gamma H$, where H decay to $J/\psi\pi^+\pi^-$, $J/\psi\phi$, or $D\bar{D}$. This result may be useful in identifying the nature of $C = +$ XYZ states. For completeness, the production of $C = +$ charmonium in $e^+e^- \to \gamma + \ H$ at B factories is also discussed.

Contents

1 Introduction

During the last 10 years, many heavy quarkonium or heavy quarkonium-like XYZ states had been discovered (more details can be found in Ref.[1] and related papers). The $X(3872)$ state is the first and the most famous state among them. It was discovered by the Belle collaboration[2], and confirmed by the CDF [3], D0[4], BaBar[5], LHCb[6], and CMS[7] collaborations. One of the most conspicuous properties of $X(3872)$ is its mass, which is close to the $D^0\bar{D}^{\star 0}$ threshold within 1 MeV; hence, $X(3872)$ is suggested to be a $D^0\bar{D}^{\star 0}$ molecule [8–11]. The contribution of the charged component $D^+D^{\star -}$ is also considered in Ref.[12, 13]. The molecule model may be puzzled to explain the production cross-sections of $X(3872)$ in hadron colliders (which may be large in some phenomenological models[14]) [15]. The quantum numbers of $X(3872)$ have been determined to be $J^{PC} = 1^{++}$ by LHCb collaboration [16]. The J^{PC} of $X(3872)$ is the same as $\chi_{c1}(nP)$. On the contrary, the mass 3.872 GeV seems too low for a $\chi_{c1}(2P)$ state. The coupled-channel and screening effects may draw its mass down to 3.87 GeV [17]. However, next-to-leading order (NLO) prediction of $X(3872)$ production in hadron colliders within nonrelativistic quantum chromodynamics (NRQCD) disfavors the interpretation of $X(3872)$ as pure $\chi_{c1}(2P)$ [18]. The possibility that $X(3872)$ might be a mixture state with the $\chi_{c1}(2P)$ and the $D^0\bar{D}^{\star 0}$ components was proposed in Ref.[19]. The prompt $X(3872)$ hadroproduction is studied at NLO in α_s[20] and the result is consistent with the CMS [7] and the CDF data[3]. This idea is also favored the data of some other measurements and predictions [15, 17, 21, 22].

Besides $X(3872)$, other $C = +$ XYZ states are listed in Table 1. These states are particularly interesting and the interpretations for their nature are still inconclusive[23]. $X(3915)$

Table 1. $C = + XYZ$ states. $X(3915)$, $X(3945)$, and $Y(3940)$ is considered as $\chi_{c0}(2P)$ for compatible properties. $Z(3930)$ is considreed as $\chi_{c2}(2P)$[1, 33].

State	$m(\Gamma)$ in MeV	J^{PC}	Production (Decay)	Ref
$X(3872)$	3871.68 ± 0.17 (< 1.2)	1^{++}	$B \to K\,(\pi^+\pi^- J/\psi)$	[2]
			$p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$	[3, 34]
			$B \to K\,(\omega J/\psi)$	[35, 36]
			$B \to K\,(D^0 \bar{D}^*)$	[37, 38]
			$B \to K\,(\gamma J/\psi, \gamma\psi(2S))$	[39]
			$pp \to (\pi^+\pi^- J/\psi) + ...$	[6, 7, 16]
$X(3915)$	3917.5 ± 2.7 $(27 \pm 10\ )$	0^{++}	$B \to K\,(\omega J/\psi)$	[40, 41]
			$e^+e^- \to e^+e^-\,(\omega J/\psi)$	[36, 42]
$X(3940)$	3942^{+9}_{-8} (37^{+27}_{-17})	J^{P+}	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	[43]
$Y(4140)$	4143.0 ± 3.1 (12^{+9}_{-6})	J^{P+}	$B \to K\,(\phi J/\psi)$	[44]
$X(4160)$	4156^{+29}_{-25} (139^{+110}_{-60})	J^{P+}	$e^+e^- \to J/\psi\,(D^{*+}\bar{D}^{*-})$	[43]
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$ (32^{+22}_{-15})	J^{P+}	$B \to K\,(\phi J/\psi)$	[44]
$X(4350)$	$4350.6^{+4.6}_{-5.1}$ ($13.3^{+18.4}_{-10.0}$)	$0/2^{++}$	$e^+e^- \to e^+e^-\,(\phi J/\psi)$	[45]

($X(3945)$ or $Y(3940)$) and $Z(3930)$ are assigned as the $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$ states by the Particle Data Group[24]. However this identification may be called into question[25]. The experimental results for these $C = +$ states have induced renewed theoretical interest in understanding the nature of charmonium-like states. The double charmonium production in e^+e^- annihilation at B factories[26, 27] turned out to be a possible way to identify the $C = +$ charmonium or charmonium-like states, recoiling against the easily reconstructed 1^{--} charmonium J/ψ and $\psi(2S)$. In addition to $\eta_c, \eta_c(2S)$, χ_{c0}, $X(3940)$ (decaying into $D\bar{D}^*$), and $X(4160)$ (decaying into $D^*\bar{D}^*$) have also been observed in double charmonium production at B factories. However, χ_{c1} and χ_{c2} states are missing in production associated with J/ψ at B factories. Identifying the $C = +$ charmonium states H in the $e^+e^- \to \gamma^* \to \gamma + H$ process at B factories is also proposed[28, 29]. The quantum chromodynamics (QCD) corrections of $e^+e^- \to \gamma^* \to \gamma + H$ at B factories are calculated in Ref.[30, 31]. The relativistic correction of $e^+e^- \to \gamma^* \to \gamma + \eta_c$ is also included in Ref.[31]. Indirect measurement of quarkonium in the two-photon process is also proposed[32].

Recently, BesIII reports the cross-sections of $e^+e^- \to \gamma X(3872)$[46, 47]

$$\begin{aligned}
&\sigma[e^+e^- \to \gamma X(3872)] \times \mathrm{Br}[J/\psi\pi\pi] < 0.13\mathrm{pb}\ \ \text{at 90\% CL.} && \sqrt{s} = 4.009\mathrm{GeV} \\
&\sigma[e^+e^- \to \gamma X(3872)] \times \mathrm{Br}[J/\psi\pi\pi] = 0.32 \pm 0.15 \pm 0.02\mathrm{pb} && \sqrt{s} = 4.230\mathrm{GeV} \\
&\sigma[e^+e^- \to \gamma X(3872)] \times \mathrm{Br}[J/\psi\pi\pi] = 0.35 \pm 0.12 \pm 0.02\mathrm{pb} && \sqrt{s} = 4.260\mathrm{GeV} \\
&\sigma[e^+e^- \to \gamma X(3872)] \times \mathrm{Br}[J/\psi\pi\pi] < 0.39\mathrm{pb}\ \ \text{at 90\% CL.} && \sqrt{s} = 4.360\mathrm{GeV}
\end{aligned} \tag{1.1}$$

Where $\mathrm{Br}[J/\psi\pi\pi]$ means $\mathrm{Br}[X(3872) \to J/\psi\pi\pi]$. And the studies of $\psi(4160) \to X(3872)\gamma$

[48] and $\psi(4260) \to X(3872)\gamma$ [49] are proposed to probe the molecular content of the $X(3872)$.

Many NLO relativistic and radiative corrections for heavy quarkonium production are considered within nonrelativistic QCD (NRQCD)[50]. By introducing the color octet mechanism, one can obtain the infrared-safe calculations for the decay rates of P wave [51–53] and D wave[54–56] quarkonium states. The color octet contributions of the diphoton decay of P wave quarkonium states are calculated in Ref.[57]. $O(\alpha_s v^2)$ corrections to the decays of h_c, h_b and η_b are studied in Ref.[58, 59]. The NLO QCD corrections[60–70], relativistic corrections[71–78], and $\mathcal{O}(\alpha_s v^2)$ corrections [79, 80] largely compensate for the discrepancies between theoretical values and experimental measurements at B factories. The contributions of higher-order QCD corrections for charmonium production [18, 20, 81–88] and polarization [89–92] in hadron colliders are also significant. The relativistic corrections to J/ψ hadroproduction are significant[93–95].

We calculate the production of $C = +$ charmonium at e^+e^- annihilation at BESIII to test the nature of $C = +$ XYZ states. Our paper is organized as follows. The calculation framework is given in Sec. 2. The numerical results of the cross-sections of $C = +$ charmonium are discussed in Sec. 3. A discussion of $X(3872)$ and other $C = +$ XYZ states is given in Sec. 4. The summary is given in Sec. 5.

2 The frame of the calculation

In the NRQCD factorization framework, we can express the amplitude in the rest frame of H as[28, 30, 31]

$$\begin{aligned}
&\mathcal{A}(e^-(k_1)e^+(k_2) \to H_{c\bar{c}}(^{2S+1}L_J)(2p_1) + \gamma) \\
&\quad = \sum_{L_z S_z} \sum_{s_1 s_2} \sum_{jk} \int \mathrm{d}^3\vec{q}\,\Phi_{c\bar{c}}(\vec{q}) \langle s_1; s_2 \mid SS_z\rangle \langle 3j; \bar{3}k \mid 1\rangle \\
&\quad \times \mathcal{A}\left[e^-(k_1)e^+(k_2) \to c_j^{s_1}(p_1+q) + \bar{c}_k^{s_2}(p_1-q) + \gamma(k)\right],
\end{aligned} \tag{2.1}$$

where $\langle 3j; \bar{3}k \mid 1\rangle = \delta_{jk}/\sqrt{N_c}$, $\langle s_1; s_2 \mid SS_z\rangle$ is the color Clebsch-Gordan coefficient for $c\bar{c}$ pairs projecting out appropriate bound states, and $\langle s_1; s_2 \mid SS_z\rangle$ is the spin Clebsch-Gordan coefficient. $\mathcal{A}\left[e^-(k_1)e^+(k_2) \to c_j^{s_1}(p_1+q) + \bar{c}_k^{s_2}(p_1-q) + \gamma(k)\right]$ is the quark level scattering amplitude. In the rest frame of H, $q = (0, \vec{q})$, and $p_1 = (\sqrt{m_c^2 + \vec{q}^2}, 0, 0, 0)$. $\Phi_{c\bar{c}}^H(\vec{q})$ is the $c\bar{c}$ component wave function of hadron H in momentum space. For $v^2 = \vec{q}^2/m_c^2 \ll 1$[50], we can expand Eq.(2.1) with v^2:

$$\begin{aligned}
\mathcal{A}(q) &= \mathcal{A}(0) + \left.\frac{\partial \mathcal{A}(\vec{q})}{\partial \vec{q}^\alpha}\right|_{q=0} \vec{q}^\alpha + \left.\frac{\partial^2 \mathcal{A}(\vec{q})}{\partial \vec{q}^\alpha \partial \vec{q}^\beta}\right|_{q=0} \frac{\vec{q}^\alpha \vec{q}^\beta}{2} \\
&\quad + \left.\frac{\partial^3 \mathcal{A}(\vec{q})}{\partial \vec{q}^\alpha \partial \vec{q}^\beta \partial \vec{q}^\delta}\right|_{q=0} \frac{\vec{q}^\alpha \vec{q}^\beta \vec{q}^\delta}{3!} + \ldots.
\end{aligned} \tag{2.2}$$

Here $\mathcal{A}(q) = \mathcal{A}\left[e^-(k_1)e^+(k_2) \to c_j^{s_1}(p_1+q) + \bar{c}_k^{s_2}(p_1-q) + \gamma(k)\right]$. We consider the Fourier transform between the momentum space and position space as: [50, 94],

$$\begin{aligned}\int \mathrm{d}^3\vec{q}\ \ \Phi_{c\bar{c}}(\vec{q}) &\propto \sqrt{Z^H_{c\bar{c}}}R_{c\bar{c}}(0)\\ \int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\Phi_{c\bar{c}}(\vec{q}) &\propto \sqrt{Z^H_{c\bar{c}}}R'_{c\bar{c}}(0)\\ \int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\vec{q}^{\beta}\Phi_{c\bar{c}}(\vec{q}) &\propto \sqrt{Z^H_{c\bar{c}}}R''_{c\bar{c}}(0)\\ \int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\vec{q}^{\beta}\vec{q}^{\delta}\Phi_{c\bar{c}}(\vec{q}) &\propto \sqrt{Z^H_{c\bar{c}}}R'''_{c\bar{c}}(0).\end{aligned} \tag{2.3}$$

Here $Z^H_{c\bar{c}}$ is the possibility of $c\bar{c}$ component in hadron H. $R_{c\bar{c}}(0)$ is the radial Schrodinger wave function at the origin. $R^l_{c\bar{c}}(0)$ is the derivative of the radial Schrodinger wave function at the origin

$$R^l_{c\bar{c}}(0) = \left.\frac{\mathrm{d}^l R_{c\bar{c}}(r)}{\mathrm{d}^l r}\right|_{r=0} \tag{2.4}$$

$R_{c\bar{c}}(0)$ corresponds to the $\mathcal{O}(v^0)$ S wave matrix element, $R'_{c\bar{c}}(0)$ corresponds to the $\mathcal{O}(v^0)$ P wave matrix element, $R''_{c\bar{c}}(0)$ corresponds to the $\mathcal{O}(v^2)$ S wave matrix element or $\mathcal{O}(v^0)$ D wave matrix element, and $R'''_{c\bar{c}}(0)$ corresponds to the $\mathcal{O}(v^2)$ P wave matrix element.

$R_{c\bar{c}}(0)$ is also written as long-distance matrix elements (LDMEs) as discussed in Ref.[94]. For example,

$$\langle 0|\mathcal{O}^{\chi_{c1}}(^3P_1^{[1]})|0\rangle = \frac{27}{2\pi}|R'_{1P}(0)|^2, \tag{2.5}$$

We calculated the relativistic corrections for the S wave and P wave states and obtain two LDMEs for η_c, four LDMEs for χ_{cJ}, and one LDMEs for $^1D_2$ states. To simplify the discussion of the numerical result, we assumed that

$$< 0|\mathcal{O}^{\chi_{cJ}}(^3P_J^{[1]})|0 > = (2J+1) < 0|\mathcal{O}^{\chi_{cJ}}(^3P_0^{[1]})|0 > . \tag{2.6}$$

$$v^2 = \frac{\langle 0|\mathcal{P}^H(^{2s+1}L_J^{[c]})|0\rangle}{m_c^2\langle 0|\mathcal{O}^H(^{2s+1}L_J^{[c]})|0\rangle}. \tag{2.7}$$

Then there is only one LDME for S wave, P wave, and D wave respectively. More details can be found in Ref.[94].

The relativistic correction K factor is

$$\begin{aligned}K_{v^2}[\eta_c] &= -\frac{5v^2}{6} - \frac{rv^2}{1-r},\\ K_{v^2}[\chi_{c0}] &= -\frac{\left(55r^2-28r+13\right)v^2}{10\left(3r^2-4r+1\right)} - \frac{rv^2}{1-r},\\ K_{v^2}[\chi_{c1}] &= -\frac{\left(21r^2+30r-11\right)v^2}{10\left(r^2-1\right)} - \frac{rv^2}{1-r},\\ K_{v^2}[\chi_{c2}] &= -\frac{\left(90r^3+113r^2+4r-7\right)v^2}{10(r-1)\left(6r^2+3r+1\right)} - \frac{rv^2}{1-r},\end{aligned} \tag{2.8}$$

where $r = 4m_c^2/s$. $-\frac{rv^2}{1-r}$ is the relativistic correction of the phase space. If we select $r \to 0$, the K_{v^2} factor is consistent with the K factor at large p_T in Ref.[94].

Our leading order (LO) cross-sections of $e^+e^- \to \gamma^* \to \gamma + H$ is consistent with Ref.[28, 30, 31]. The QCD corrections of $e^+e^- \to \gamma^* \to \gamma + H$ is consistent with Ref.[30, 31]. And the relativistic corrections of $e^+e^- \to \gamma^* \to \gamma + \eta_c$ is consistent with Ref.[31, 77, 78].

We can obtain a similar amplitude for the $D\bar{D}$ component in the molecule model. We can estimate the off-resonance amplitude of $e^+e^- \to H + \gamma$ from the $D\bar{D}$ component. The parton-level amplitudes may be compared with the hadron-level amplitudes:

$$\mathcal{A}\left[e^-(k_1)e^+(k_2) \to c\bar{c}(2p_1) + \gamma\right] \sim \mathcal{A}\left[e^-(k_1)e^+(k_2) \to D\bar{D}(2p_1) + \gamma\right] \tag{2.9}$$

By contrast, the $R^l_{c\bar{c}}(0) \sim v^{2l} R^S_{c\bar{c}}(0) \gg R_{D\bar{D}}(0)$ with the S wave $l = 0$ and P wave $l = 1$ for the binding energies of $c\bar{c}$ and $D\bar{D}$ are several hundreds of MeV and several MeV, respectively. If $Z^H_{c\bar{c}} \sim Z^H_{D\bar{D}}$, we can consider the $c\bar{c}$ contributions only.

In the numerical calculation, we consider the charm quark mass as half of the hadron mass consistent with the physics phase space. With a large charm quark mass, the wave functions at the origin are identified as the Cornell potential result in Ref.[96]. The sellected parameters are as follows:

$$\begin{aligned}
&m_c = m_H/2, && \alpha_s = 0.23, && \alpha = 1/133,\\
&v^2 = 0.23, && R_{1S} = 1.454\mathrm{GeV}^3, && R_{2S} = 0.927\mathrm{GeV}^3,\\
&R_{3S} = 0.791\mathrm{GeV}^3, && R'_{1P} = 0.131\mathrm{GeV}^5, && R'_{2P} = 0.186\mathrm{GeV}^5,\\
&R''_{1D} = 0.031\mathrm{GeV}^7.
\end{aligned} \tag{2.10}$$

The wave functions at origin for higher states are estimated as

$$\begin{aligned}
R_{4S} &= 2 \times R_{3S} - R_{2S} = 0.655\mathrm{GeV}^3,\\
R'_{3P} &= (R'_{1P} + R'_{2P})/2 = 0.159\mathrm{GeV}^5,\\
R''_{2D} &= R''_{1D} = 0.031\mathrm{GeV}^7.
\end{aligned} \tag{2.11}$$

In the numerical result, "σ_{LO}" is the LO cross-section, "σ_{v^2}" is the cross-section including the LO and the relativistic correction, "σ_{α_s}" is the cross-section including the LO and the radiative correction, and "σ_{α_s,v^2}" is the cross-section including the LO, the relativistic correction, and the radiative correction. In addition, "LO" is the LO cross-section, "RC" is the relativistic correction, "QCD" is the radiative correction, and "Total" is the cross-section including the LO, the relativistic correction, and the radiative correction.

For the LO, the cross-section is $\mathcal{O}(\alpha_s^0 v^0)$. As $\alpha_s = 0.23 \pm 0.03$ and $v^2 = 0.23 \pm 0.03$ are reasonable estimates, we can estimate that the uncertainty of the numerical result from α_s and v^2 is $< 10\%$.

3 Pure $C = +$ charmonium states

We can estimate the cross-sections for pure $C = +$ charmonium states H in $e^+e^- \to \gamma + H$ at BESIII with $H = \eta_c(nS)$ (n=1, 2, 3, and 4), $\chi_{cJ}(nP)$ (n=1, 2, and 3), and ${}^1D_2(nD)$ (n=1 and 2). The mass of the lower states can be found in Ref.[24], and the mass of the higher states is selected from Ref.[17].

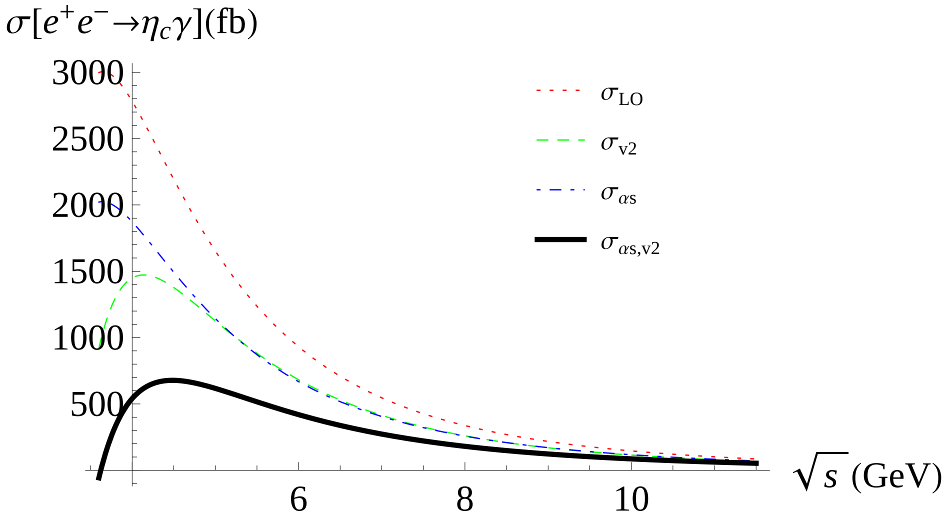

Figure 1. The cross-sections of $e^+e^- \to \eta_c + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2.

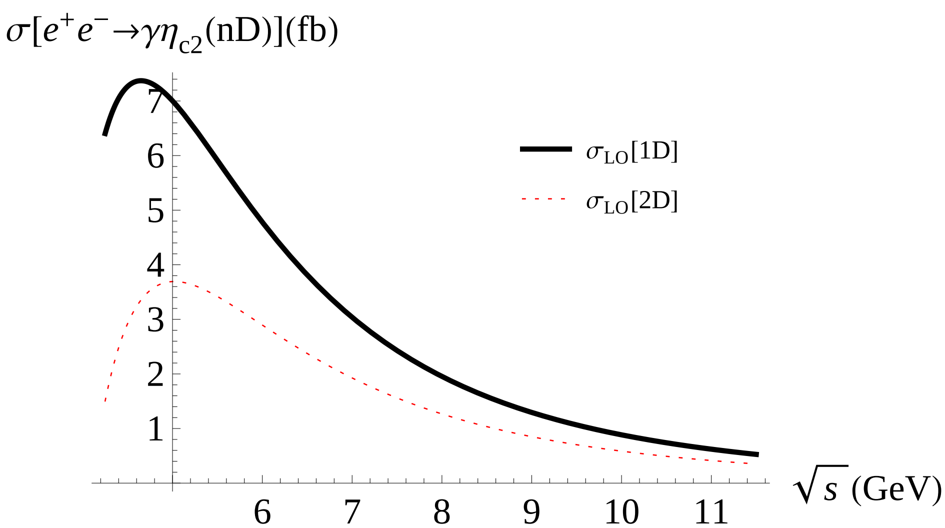

Figure 2. The cross-sections of $e^+e^- \to \eta_{c2}(1D, 2D) + \gamma$ as a function of $\sqrt{s}$ in fb.

The cross-section of $e^+e^- \to \eta_c + \gamma$ as a function of $\sqrt{s}$ is shown in Fig.1. The cross-sections of $e^+e^- \to \eta_{c2}(1D, 2D) + \gamma$ as a function of $\sqrt{s}$ are shown in Fig.2. The numerical results for nS with $n = 1, 2, 3, 4$ and nD with $n = 1, 2$ are listed in Table 2. We determined

Table 2. The cross-sections of $e^+e^- \to H+\gamma$ for $\eta_c(nS)$ with $n=1,2,3,4$ and $\eta_{c2}(nD)$ for $n=1,2$ charmonium states in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The mass of $\eta_c(3S)$, $\eta_c(4S)$, $\eta_{c2}(1D)$, and $\eta_{c2}(2D)$ are selected from Ref.[17]. The other mass can be found in Ref.[24].

$\sqrt{s}$(GeV)		4.00	4.25	4.50	4.75	5.00	10.6	11.2
$\eta_c(2981)$	LO	2781	2494	2192	1906	1652	117	95
	RC	-1332	-1033	-814	-650	-526	-25	-20
	QCD	-909	-807	-700	-598	-508	-22	-16
	Total	540	653	678	658	617	70	58
$\eta_c(2S)(3639)$	LO	563	684	706	679	629	58	48
	RC	-730	-563	-442	-352	-284	-13	-10
	QCD	-177	-221	-231	-222	-205	-13	-10
	Total	-344	-100	33	105	141	32	27
$\eta_c(3S)(3994)$	LO		233	337	374	377	44	36
	RC		-450	-352	-279	-225	-10	-8
	QCD		-72	-107	-121	-123	-10	-8
	Total		-228	-122	-27	29	24	20
$\eta_c(4S)(4250)$	LO			133	198	225	34	28
	RC			-279	-221	-178	-8	-6
	QCD			-41	-63	-73	-8	-7
	Total			-186	-86	-26	17	15
$\eta_{c2}(1D)(3796)$	LO	4.0	6.4	7.3	7.3	7.0	0.71	0.58
$\eta_{c2}(2D)(4099)$	LO		1.5	2.9	3.5	3.7	0.47	0.38

that the radiative and relativistic corrections are negative and large for $\eta_c(nS)$, respectively. The LO cross-sections for $\eta_{c2}(1D, 2D)$ is very small at BESIII; hence, the high order corrections are ignored.

The cross-sections of $e^+e^- \to \chi_{cJ}+\gamma$ as a function of $\sqrt{s}$ are shown in Fig.3, Fig.4, and Fig.5 for $J=0,1,2$, respectively. The numerical results for $\chi_{cJ}(nP)$ with $n=1,2,3$ are listed in Table 3, Table 4, and Table 5 for $J=0,1,2$, respectively. We determined that the QCD corrections are large but negative and the relativistic corrections are large and positive. Hence, many P wave states can be searched at BESIII.

The NRQCD requires that the energy of photon at the center of the mass frame of e^+e^-

$$E_\gamma = \frac{s-M_H^2}{2\sqrt{s}} \sim \sqrt{s}-M_H+\mathcal{O}\left[(1-M_H/\sqrt{s})^2\right] \tag{3.1}$$

be larger than $\Lambda_{QCD} \sim 300$ MeV $\sim m_c v^2$. Although this process is a QED process, the prediction is not reliable and only a reference value if this requirement is not satisfied. If we replace photon with gluon, the soft photon contributions correspond to the long-distance color octet contributions[31, 50].

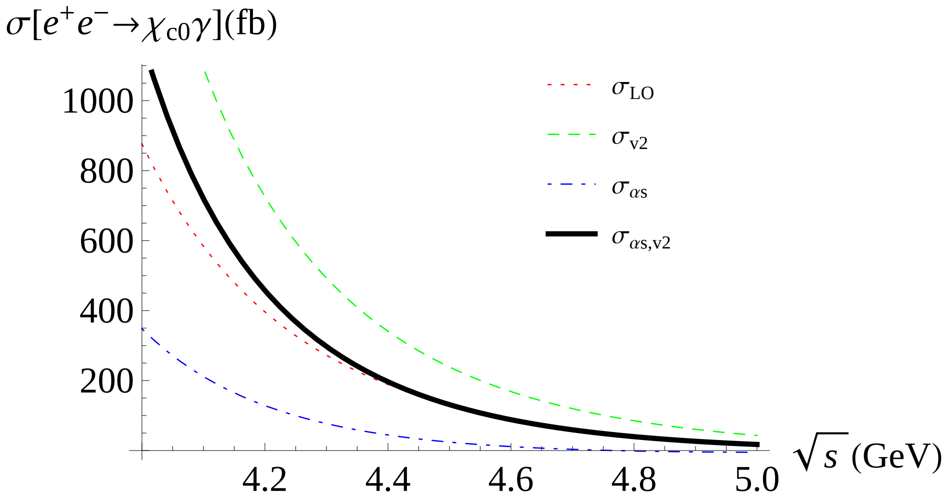

Figure 3. The cross-sections of $e^+e^- \to \chi_{c0} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2.

Table 3. The cross-sections of $e^+e^- \to \chi_{c0}(nP)+\gamma$ with $n = 1, 2, 3$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The $\chi_{c0}(2P)$ is considreed as $X(3915)(X(3945)/Y(3940))$ [1, 33]. The mass of $\chi_{c0}(3P)$ are selected from Ref.[17]. The other mass can be found in Ref.[24].

$\sqrt{s}$(GeV)		4.00	4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c0}(3415)$	LO	877	328	132	53	21	1.81	1.6
	RC	825	268	107	48	22	-0.77	-0.63
	QCD	-528	-228	-107	-52	-26	-0.38	-0.29
	Total	1173	368	131	49	17	1.42	1.22
$\chi_{c0}(2P)(3918)$	LO		1991	665	271	119	1.30	1.18
	RC		3102	680	230	96	-0.64	-0.54
	QCD		-1013	-384	-177	-89	0.39	0.30
	Total		4080	962	324	127	1.04	0.94
$\chi_{c0}(3P)(4131)$	LO			1073	384	164	0.82	0.75
	RC			1600	391	140	-0.44	-0.38
	QCD			-551	-223	-107	0.29	0.23
	Total			2121	554	198	0.67	0.61

4 $C = +$ XYZ states

$X(4160)$ and $Y(4274)$ are found in the B decay $B \to K + H \to K + \phi J/\psi$ by CDF collaboration[44]. No signal of $X(4160)$ or $Y(4274)$ is reported by B factories. Hence, the cross-sections for $X(4160)$ or $Y(4274)$ at BESIII may be too small. The cross-sections of $e^+e^- \to \gamma H$ for $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ are discussed here. The 1^{--}

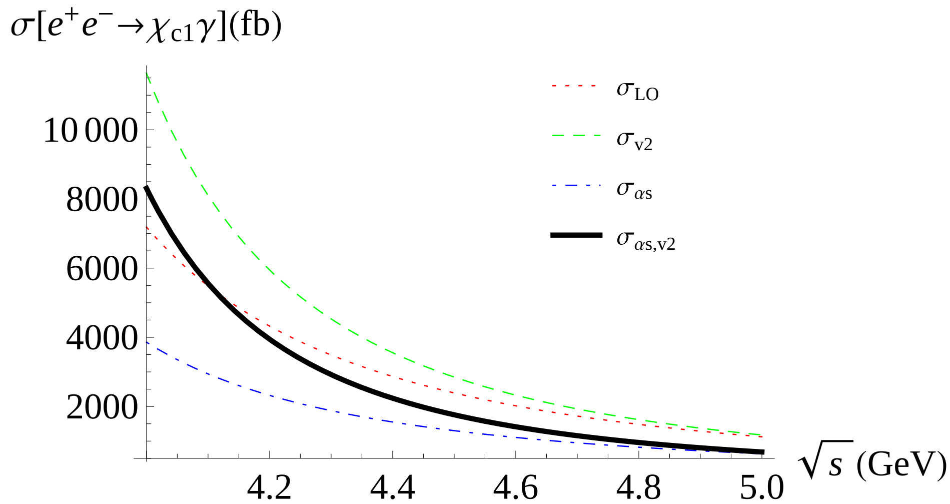

Figure 4. The cross-sections of $e^+e^- \to \chi_{c1} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2.

Table 4. The cross-sections of $e^+e^- \to \chi_{c1}(nP) + \gamma$ with $n = 1, 2, 3$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. The mass of $\chi_{c1}(2P, 3P)$ are selected from Ref.[17]. And the mass of $\chi_{c1}(1P)$ can be found in Ref.[24].

$\sqrt{s}$(GeV)		4.00	4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c1}(3511)$	LO	7186	3874	2392	1597	1124	23.5	18.5
	RC	4448	1296	459	168	52	-4.8	-3.8
	QCD	-3327	-1791	-1091	-715	-492	-6.5	-4.9
	Total	8307	3379	1760	1051	685	12.3	9.7
$\chi_{c1}(2P)(3901)$	LO		8854	4244	2495	1624	25.7	20.0
	RC		9585	2297	789	312	-4.9	-3.9
	QCD		-4041	-1967	-1152	-741	-7.7	-5.70
	Total		14397	4573	2131	1195	13.2	10.3
$\chi_{c1}(3P)(4178)$	LO			1073	384	164	0.82	0.75
	RC			1600	391	140	-0.44	-0.38
	QCD			-551	-223	-107	0.29	0.23
	Total			2121	554	198	0.67	0.61

resonance contributions are ignored here.

4.1 $X(3872)$

In the light of the mixture state of the $\chi_{c1}(2P)$ and $D^0\bar{D}^{\star 0}$ molecule, the cross-sections of $X(3872)$ at hadron collides can be expressed as[20]:

$$d\sigma[X(3872) \to J/\psi\pi^+\pi^-] = d\sigma[\chi_{c1}(2P)]\times k, \tag{4.1}$$

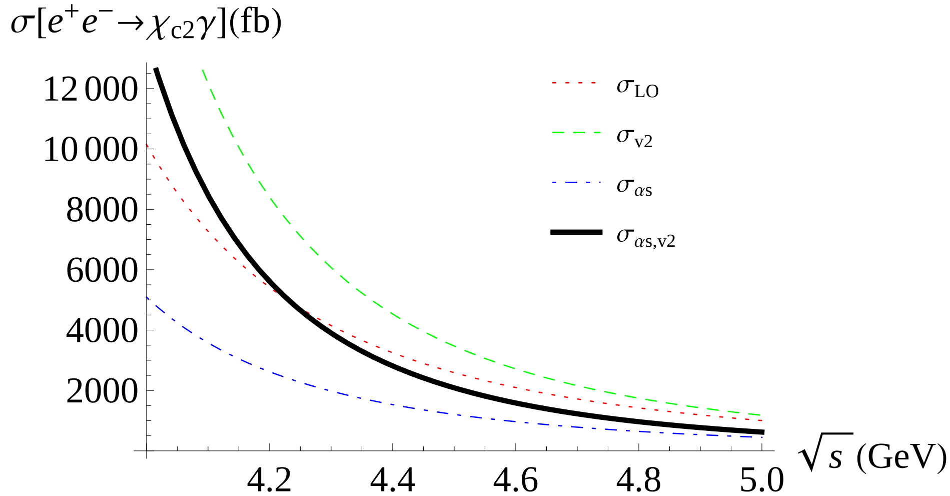

Figure 5. The cross-sections of $e^+e^- \to \chi_{c2} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2.

Table 5. The cross-sections of $e^+e^- \to \chi_{c2}(nP) + \gamma$ with $n = 1, 2, 3$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2. $\chi_{c2}(2P)$ is considreed as $Z(3930)$, [1, 33]. The mass of $\chi_{c2}(3P)$ are selected from Ref.[17]. And the mass of $\chi_{c2}(1P)$ can be found in Ref.[24].

$\sqrt{s}$(GeV)		4.00	4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c2}(3556)$	LO	10149	4724	2590	1562	1004	9.66	7.37
	RC	8587	2385	880	376	173	-1.16	-0.93
	QCD	-5056	-2455	-1384	-851	-557	-6.27	-4.82
	Total	13679	4655	2087	1086	621	2.22	1.63
$\chi_{c2}(2P)(3927)$	LO		13419	5581	2931	1927	11.29	8.53
	RC		17835	3965	1355	565	-1.22	-0.99
	QCD		-6423	-2822	-1533	-926	-7.25	-5.52
	Total		24862	6723	2754	1368	2.82	2.03
$\chi_{c2}(3P)(4208)$	LO			8938	3607	1886	8.55	6.40
	RC			14212	2949	995	-0.83	-0.68
	QCD			-4210	-1803	-977	-5.43	-4.10
	Total			18941	4753	1904	2.28	1.62

where $k = Z_{c\bar{c}}^{X(3875)} \times Br[X(3872) \to J/\psi\pi^+\pi^-]$. $Br[X(3872) \to J/\psi\pi^+\pi^-]$ is the branching fraction for $X(3872)$ decay to $J/\psi\pi^+\pi^-$. $Z_{c\bar{c}}^{X(3875)}$ is the possibility of the $\chi_{c1}(2P)$ component in $X(3872)$. And $k = 0.018 \pm 0.04$ [19, 20].

To clarify the nature of $X(3872)$, we also give the numerical calculation of $e^+e^- \to$

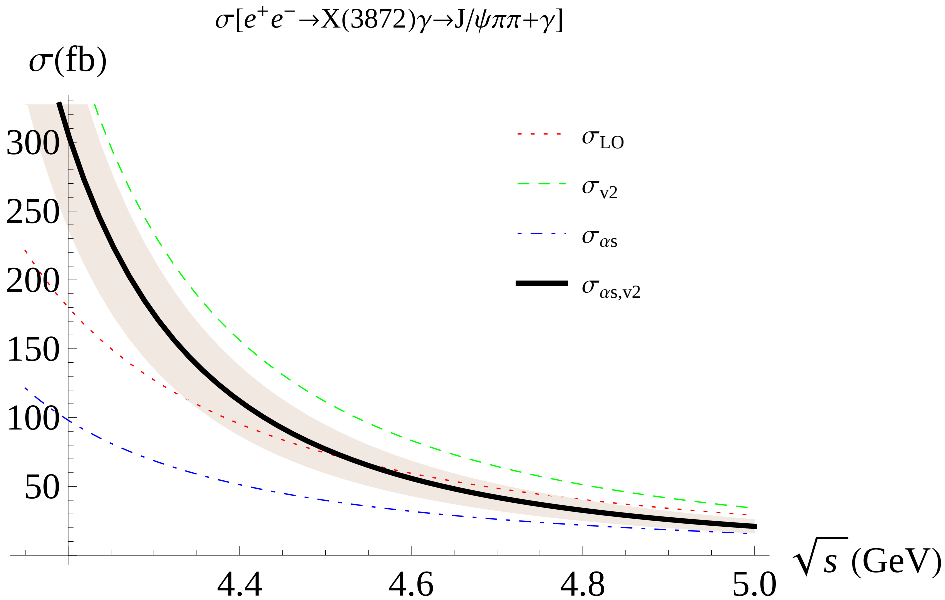

Figure 6. The cross-sections of $e^+e^- \to \chi_{c2} + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2. The uncertainty bind of σ_{α_s,v^2} is from the uncertainty of $k = 0.018 \pm 0.04$.

Table 6. The cross-sections of $e^+e^- \to X(3872) + \gamma \to J/\psi\pi\pi + \gamma$ in fb. The labels LO, RC, QCD and Total are defined near the end of Section 2.

$\sqrt{s}$(GeV)	4.15	4.2	4.25	4.3	4.35	4.45	4.55
LO	221±49	180±40	150±33	127±28	110±24	84±19	66±15
RC	310±69	208±46	146±32	106±24	80±18	47±10	30±7
QCD	-100±22	-82±18	-69±15	-59±13	-51±11	-39±9	-31±7
Total	431±96	306±68	227±51	175±39	138±31	92±20	65±14

$\sqrt{s}$(GeV)	NRQCD prediction for continue	BESIII [46, 47]
4.009		<130 at 90% CL.
4.160	401 ± 89	
4.230	255 ± 57	$320 \pm 150 \pm 20$
4.260	215 ± 48	$350 \pm 120 \pm 20$
4.360	133 ± 29	<130 at 90% CL.
4.415	105 ± 23	
4.660	47 ± 10	

$\gamma X(3872) \to J/\psi\pi^+\pi^-\gamma$ in this picture

$$\begin{aligned} &\sigma[e^+e^- \to \gamma X(3872)] \times \mathrm{Br}[X \to J/\psi\pi\pi] \\ &= \sigma[e^+e^- \to \gamma\chi_{c1}(2P)(3872)] \times (0.018 \pm 0.004) \end{aligned} \tag{4.2}$$

The cross-sections as a function of $\sqrt{s}$ is shown in Fig.6. Many 1^{--} states with $M_H < 5$ GeV are also observed. We can predict the cross-sections from continuous contributions at this point, and the result is listed in Table 6. We ignore the 1^{--} resonances contributions here. We emphasize that if we select $\sqrt{s} = 4.009$GeV, the energy of photon $E_\gamma = 134$ MeV and smaller than $\Lambda_{QCD} \sim m_c v^2 \sim 300$ MeV. Hence, NRQCD cannot accurately predict the cross-sections with a soft photon with $\sqrt{s} = 4.009$GeV[50]. If $\sqrt{s} = 4.160$GeV, the energy of photon is $E_\gamma = 270$MeV. Although this process is a QED process, the prediction is not reliable and only a reference value[31]. We determined that the NRQCD prediction of the continuous contributions can be compared with the BESIII data of the cross-sections of $e^+e^- \to \gamma X(3872)$ [46, 47] in Eq.(1.1).

When we only considered the continuum production, the resonance contributions can be estimated as that:

$$\sigma_{Res}[s] = \frac{12\pi\Gamma[Res \to e^+e^-]\Gamma[Res \to \gamma X]}{(s-M^2)^2 + (M\Gamma_{tot}[Res])^2}. \tag{4.3}$$

We take into account only one resonance here and ignore continuum and other resonances here. If we ignore the interference between one resonance and continuum and other resonances, the *gamma* energy dependence of the $\Gamma[Res \to \gamma X]$, and $D\bar{D}$ contributions of decay of $Res \to \gamma X$, we can estimate the resonance contributions. With $X(3872)$ considered as $2P$ states, the largest decay widths are $\psi(4040)$ and $\psi(4160)$, which are considered as the mixing of $\psi(3S)$ and $\psi(2D)$ [97, 98]. The $\Gamma[Res \to \gamma X]$ for other states will be less than 1 keV [98], and $\Gamma_{tot} \sim 100$ MeV, $\Gamma[Res \to e^+e^-] \sim 1$ keV. Hence, we ignore the contributions from other resonances. With the parameters for $\psi(4040)$ and $\psi(4160)$[24, 98]:

$$\begin{aligned}
&\Gamma[\psi(4040) \to e^+e^-] = 0.87 \text{ keV} \quad \Gamma[\psi(4040) \to \gamma X] = 40 \text{ keV} \quad \Gamma_{tot}[\psi(4040)] = 80 \text{ MeV} \\
&\Gamma[\psi(4160) \to e^+e^-] = 0.83 \text{ keV} \quad \Gamma[\psi(4160) \to \gamma X] = 140 \text{ keV} \quad \Gamma_{tot}[\psi(4160)] = 103 \text{ MeV}
\end{aligned}$$

Hence, we can determine the contributions of these parameters to $X(3872)\gamma \to J/\psi\pi^+\pi^-\gamma$

$$\begin{aligned}
&(\sigma_{\psi(4040)}[4.23] + \sigma_{\psi(4160)}[4.23]) \times k = (62 \pm 14) fb \\
&(\sigma_{\psi(4040)}[4.26] + \sigma_{\psi(4160)}[4.26]) \times k = (37 \pm 8) fb
\end{aligned} \tag{4.4}$$

If we considered the interference, the result would be more complex. On the other hand, we have calculated the quark-level intermediate states, which do not clearly deal with the hadron-level intermediate states.

4.2 $X(3940)$ and $X(4160)$

$X(3940)$ and $X(4160)$ are observed in $e^+e^- \to J/\psi\,(D\bar{D})$ at B factories [43]. η_c and χ_{c0} are recoiled with J/ψ, but χ_{c1} and χ_{c2} are missed[43]. The theoretical predictions are consistent with the experimental data[61, 69, 99, 100]. So there should be large $\eta_c(nS)$ and $\chi_{c0}(nP)$ component in $X(3940)$ and $X(4160)$, respectively. The mass of $\eta_c(3S)$ and $\chi_{c0}(3P)$ are predicted as 3994 MeV and 4130 MeV respectively[17]. Compared with Table 2 and Table

3, we can found that the cross-sections of $\eta_c(3S)$ is small even negative at $\sqrt{s} < 5$ GeV. But $\chi_{c0}(3P)$ is large. The cross-sections as a function of $\sqrt{s}$ is shown in Fig 7. Here $Z^X_{c\bar{c}} \leq 1$ is the possibility of $\eta_c(3S)$ and $\chi_{c0}(3P)$ component in $X(3940)$ and $X(4160)$ respectively. The BESIII collaboration can search $X(3940)$ and $X(4160)$ in the process $e^+e^- \to \gamma\ + X(D\bar{D})$. The result may be useful in identifying the nature of $X(3940)$ and $X(4160)$.

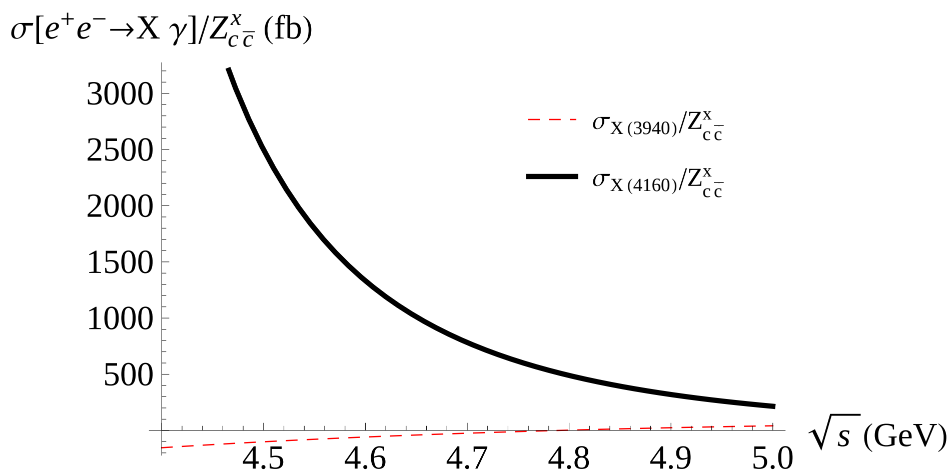

Figure 7. The cross-sections of $e^+e^- \to X(3940)(X(4160)) + \gamma$ as a function of $\sqrt{s}$ in fb.

4.3 $X(4350)$

$X(4350)$ are found in $\gamma\gamma \to H \to \phi J/\psi$ at B factories [45]. And J^{PC} is 0^{++} or 2^{++}. So there should be large $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$. In Ref.[17], The mass of $\chi_{c2}(3P)$ is 4208 MeV. Ignore more detail of the mass, we considered it as $\chi_{c0}(nS)$ or $\chi_{c2}(nP)$, the wave function at origin are estimated as

$$R' = R'_{3P} = (R'_{1P} + R'_{2P})/2 = 0.159\text{GeV}^5, \tag{4.5}$$

The cross-sections of $e^+e^- \to X(4350) + \gamma$ as a function of $\sqrt{s}$ is show in Fig.8. Here $Z^X_{c\bar{c}}$ is the possibility of $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$. The cross-section for $\chi_{c2}(nP)$ is larger than $\chi_{c0}(nP)$ by a factor of 6. The result may be useful in identifying the nature of $X(4350)$.

5 Summary and discussion

While BESIII and Belle have collected a large amount of data, some final states may be searched by the experimentalists. We can estimate the possible event number at BESIII and Belle. The possible event number is

$$N = \sigma[e^+e^- \to \gamma + c\bar{c}[n]] \times Z^H_{c\bar{c}} \times Br \times \mathcal{L} \times \epsilon, \tag{5.1}$$

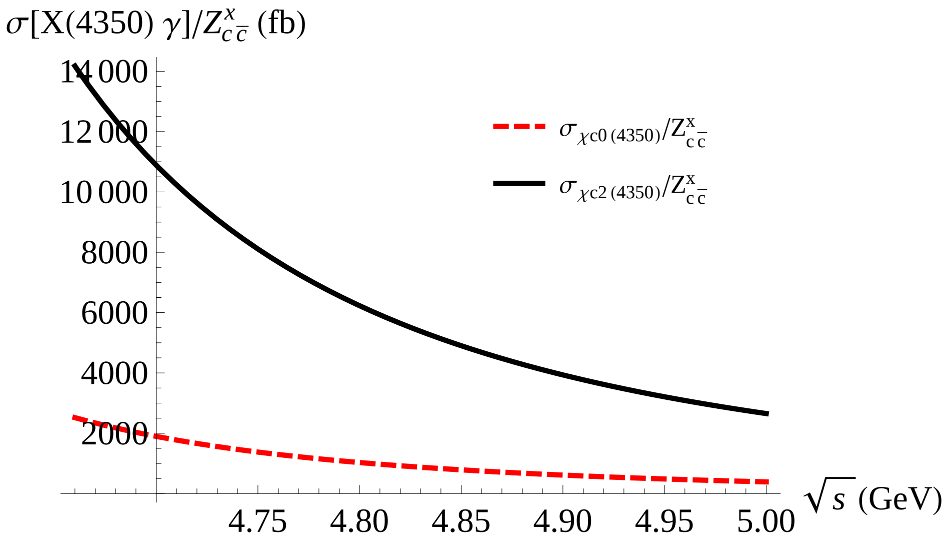

Figure 8. The cross-sections of $e^+e^- \to X(4350) + \gamma$ as a function of $\sqrt{s}$ in fb. The cross-section "σ_{LO}", "σ_{v^2}", "σ_{α_s}", and "σ_{α_s,v^2}" are defined near the end of Section 2. And $Z^X_{c\bar{c}}$ is the possibility of $\chi_{c0}(nP)$ or $\chi_{c2}(nP)$ component in $X(4350)$.

where ϵ is the efficiency of detectors selected as 20%, Br is the branch ratio of H to the decay mode, and $\mathcal{L}$ is the luminosity. The result is listed in Table 7.

Table 7. The possible event number of $C = +$ charmonium and XYZ states through $e^+e^- \to \gamma + H$ at BESIII and Belle. The efficiency of detectors are selected as 20%. The integrated luminosity is $1.0fb^{-1}$@4.23 GeV, $1.0fb^{-1}$@4.26 GeV, $0.5fb^{-1}$@4.66 GeV, and $1ab^{-1}$@10.6 GeV. The decay mode of $nKm\pi$ corresponds to $D\bar{D}$ decay, and the branch ratio is estimated as 1%.

H	Decay	Br	$Z^H_{c\bar{c}}$	4.23	4.26	4.66	10.6
η_c	$K\bar{K}\pi$	7.2%	1	9	9	5	1012
χ_{c0}	$2\pi^+2\pi^-$	2.2%	1	2	2		6
χ_{c1}	$\gamma l^+l^-(\gamma J/\psi)$	4.1%	1	29	27	5	101
χ_{c2}	$\gamma l^+l^-(\gamma J/\psi)$	2.3%	1	23	20	3	10
$\eta_{c2}(1D)$	$\gamma\gamma K\bar{K}\pi$	1.5%	1				2
$\eta_c(2S)$	$K\bar{K}\pi$	1.9%	1				123
$X(3872)(\chi_{c1}(2P))$	$\pi^+\pi^-l^+l^-(\pi^+\pi^-J/\psi)$	0.6%	0.36	6	5	1	6
$X(3915)(\chi_{c0}(2P))$	$\pi^+\pi^-\pi^0l^+l^-(\omega J/\psi)$	1%	1	9	8		2
$Z(3930)(\chi_{c2}(2P))$	$nKm\pi(D\bar{D})$	1%	1	57	46	4	6
$X(3940)(\eta_c(3S))$	$nKm\pi(D\bar{D})$	1%	1				48

As a summary, we study the production of $C = +$ charmonium states H in $e^+e^- \to \gamma + H$ at BESIII with $H = \eta_c(nS)$ (n=1, 2, 3, and 4), $\chi_{cJ}(nP)$ (n=1, 2, and 3), and ${}^1D_2(nD)$ (n=1 and 2) within the framework of NRQCD. The radiative and relativistic corrections are calculated to next-to-leading order for S and P wave states. We then argue that the search for $C = +$ XYZ states such as $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ in $e^+e^- \to \gamma + H$ at BESIII may help clarify the nature of these states. BESIII can search XYZ states through two body process $e^+e^- \to \gamma H$, where H decay to $J/\psi\pi^+\pi^-$, $J/\psi\phi$, or $D\bar{D}$. This result may be useful in identifying the nature of $C = +$ XYZ states. For completeness, the production of $C = +$ charmonium in $e^+e^- \to \gamma +$ H at B factories is also discussed.

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