η_c- and J/ψ-isoscalar meson bound states in the hadro-charmonium picture

J. Ferretti
CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

We study η_c- and J/ψ-isoscalar meson bound states in the hadro-charmonium picture. In the hadro-charmonium, the four $q\bar{q}c\bar{c}$ quarks are arranged in terms of a compact charm-anticharm pair, $c\bar{c}$, embedded in light hadronic matter, $q\bar{q}$, with $q = u$, d or s. The interaction between the charmonium core and the light matter can be written in terms of the multipole expansion in QCD, with the leading term being the $E1$ interaction with chromo-electric field $\mathbf{E}^a$. The spectrum of η_c- and J/ψ-isoscalar meson bound states is calculated and the results compared with the existing experimental data.

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I. INTRODUCTION

Recent discoveries by Belle and BESIII Collaborations of charged and neutral exotic quarkonium-like resonances, which do not fit into a traditional quark-antiquark interpretation, have driven new interest in theoretical and experimental searches for exotics. Charged states, like $Z_c(3900)$ [1, 2], $Z_c(4025)$ [3], $Z_b(10610)$ and $Z_b(10650)$ [4], have similar features and must be made up of four valence quarks because of their exotic quantum numbers. There are also several examples of neutral exotic quarkonium-like resonances, the so-called X states, whose unusual properties do not fit into a quark-antiquark classification [5].

A famous example is the $X(3872)$ [6, 7], whose quark structure is still an open puzzle. This resonance is characterized by $J^{PC} = 1^{++}$ quantum numbers, a very narrow width, and a mass $50 - 100$ MeV lower than quark model (QM) predictions [5]. The charmonium interpretation of the $X(3872)$ as a $\chi_{c1}(2^3P_1)$ state is incompatible with the present experimental data, because the difference between the calculated [8–10] and experimental [5] values of the meson mass is larger than the typical error of a QM calculation, of the order of $30 - 50$ MeV. Because of these discrepancies between theory and data, several alternative interpretations for X states have been proposed in addition to quarkonium, including: I) Meson-meson molecules [11–17]; II) The result of kinematic or threshold effects caused by virtual particles [18–24]; III) Compact tetraquark (or diquark-antidiquark) states [25–33]; IV) Hadro-quarkonia (hadro-charmonia) [34–43]; V) The rescattering effects arising by anomalous triangular singularities [44–46]. For a review, see Refs. [47–50]. Here, we focus on the hadro-charmonium picture.

The hadro-charmonium is a tetraquark configuration, where a compact $c\bar{c}$ state (ψ) is embedded in light hadronic matter ($\mathcal{X}$) [34]. The interaction between the two components, ψ and $\mathcal{X}$, takes place via a QCD analog of the van der Waals force of molecular physics. It can be written in terms of the multipole expansion in QCD [51–53], with the leading term being the $E1$ interaction with chromo-electric field $\mathbf{E}^a$.

The hadrocharmonium picture was motivated by the observation that several charmonium-like states are only found in specific charmonium-light hadron final states. Some examples include $X(4260)$, observed in the $J/\Psi\pi\pi$ channel [54], $Z_c(4430)$ discovered in $\psi(2S)\pi$ [55], $X(4360)$ and $X(4660)$, observed in $\psi(2S)\pi\pi$ [56, 57]. The recent BESIII observation of similar cross sections for $J/\Psi\pi^+\pi^-$ and $h_c\pi^+\pi^-$ at 4.26 and 4.36 GeV in e^+e^- collisions [1, 58] stimulated Li and Voloshin to extend the hadrocharmonium model by including also heavy-quark spin-symmetry breaking. As a result, $X(4260)$ and $X(4360)$ were described as a mixture of two hadrocharmonia, $|\psi_1\rangle \sim |1^{+-}\rangle_{c\bar{c}} \otimes |0^{-+}\rangle_{q\bar{q}}$ and $\psi_3 \sim |1^{--}\rangle_{c\bar{c}} \otimes |0^{++}\rangle_{q\bar{q}}$, with a large mixing angle, $\theta_{\rm mix} \simeq 40°$ [38, 40]. Recently, the hadro-charmonium model was also used to discuss the emergence of $\phi - \psi(2S)$ bound states, including the principal decay modes [43]. According to the previous study, the $\phi{-}\psi(2S)$ bound state is a good candidate for a tetraquark with hidden charm and strangeness. See also Refs. [35, 36], where the $Y(4660)$ is interpreted as a $\Psi(2S){-}f_0$ bound state, with spin partner $\eta_c(2S){-}f_0$, and Ref. [37], where a hadro-charmonium assignment for the $Z_c(3900)$ is discussed.

In the present manuscript, we calculate the spectrum of η_c- and J/ψ-isoscalar meson bound states under the hadro-charmonium hypothesis. The $q\bar{q}c\bar{c}$ masses are computed by solving the Schrödinger equation for the hadro-charmonium potential [34]. This is approximated as a finite well whose width and size can be expressed as a function of the chromo-electric polarizability, $\alpha_{\psi\psi}$, and light meson radius. The chromo-electric polarizability is estimated in the framework of the $1/N_c$ expansion [53, 59]. Finally, the hadro-charmonium masses and quantum numbers are compared with the existing experimental data. Some tentative assignments are also discussed.

The hypothesis of charmed and bottom pentaquarks as light baryon-quarkonium bound states will also be investigated [59–62].

II. A MASS FORMULA FOR THE HADRO-CHARMONIUM

The hadro-charmonium is a tetraquark configuration, where a compact $c\bar{c}$ state (ψ) is embedded in light hadronic matter ($\mathcal{X}$) [34]. The interaction between the charmonium core, ψ, and the gluonic field inside the light-meson, $\mathcal{X}$, can be written in terms of the QCD multipole expansion [51–53], considering as leading term the $E1$ interaction with chromo-electric field $\mathbf{E}^a$ [34, 62].

The effective Hamiltonian we consider is the same describing a $\psi_2 \to \psi_1$ transition in the chromo-electric field. It can can be written as [63]

$$H_{\rm eff} = -\frac{1}{2}\alpha_{ij}^{(12)} E_i^a E_j^a \ , \tag{1}$$

where

$$\alpha_{ij}^{(12)} = \frac{1}{16} \left\langle \psi_1 \right| \xi^a r_i \mathcal{G} r_j \xi^a \left| \psi_2 \right\rangle \tag{2}$$

is the chromo-electric polarizability. It is expressed in terms of the Green function $\mathcal{G}$ of the heavy-quark pair in a color octet state (having the same color quantum numbers as a gluon), the relative coordinate between the quark and the antiquark, $\mathbf{r}$, and the difference between the color generators acting on them, $\xi^a = t_1^a - t_2^a$. A schematic representation of a hidden-flavor $\psi_1 \to \psi_2 + h$ transition in the QCD multipole expansion approach is given in Fig. 1. Here, ψ_1 and ψ_2 are the initial and final charmonium states, h light hadron(s). The two vertices are those of the multipole gluon emission (MGE) and hadronization (H).

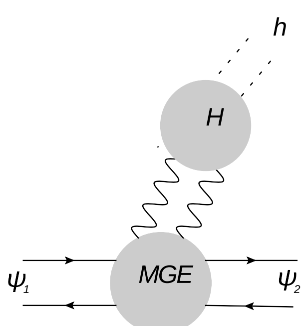

FIG. 1: Hidden-flavor amplitude $\psi_1 \to \psi_2 h$ in the QCD multipole expansion approach. Here, ψ_1 and ψ_2 are the initial and final charmonium states, h light hadron(s). The two vertices are those of the multipole gluon emission (MGE) and hadronization (H).

In order to calculate the hadrocharmonium masses, we have to compute the expectation value of Eq. (2) on the charmonium state $|\psi\rangle$, i.e. the diagonal chromo-electric polarizability $\alpha_{\psi\psi}$, and also the diagonal matrix elements $\langle \mathcal{X} | E_i^a E_i^a | \mathcal{X} \rangle$.

A. Diagonal chromo-electric polarizability

In the following, we discuss three possible prescriptions for the diagonal chromo-electric polarizabilities, $\alpha_{\psi\psi}$.

1. It is possible to provide an estimation of the off-diagonal chromo-electric polarizability, $\alpha_{\psi\psi'}$, from the decay rate $\psi(2S) \to J/\psi\pi^+\pi^-$; the resulting value is [63, 64]:

$$\alpha_{\psi\psi'} \approx 2 \text{ GeV}^{-3} \ . \tag{3}$$

After introducing final state interactions, $\alpha_{\psi\psi'}$ from Eq. (3) is reduced to about $\frac{1}{3}$ of its value [65]. Even if we expect diagonal α parameters, $\alpha_{\psi\psi}$, to be larger than off-diagonal ones, $\alpha_{\psi\psi'}$, one possibility is to take $\alpha_{\psi\psi} = \alpha_{\psi\psi'} = 2$ GeV^{-3}. Because of the smallness of (3), this prescription only gives rise to a few weakly-bounded states, like $\eta_c(2S) \otimes f_0'$ and $\psi(2S) \otimes f_0'$, with masses of 4981 and 5027 MeV, respectively. Thus, this first possibility is neglected.

2. Alternately, one can calculate the chromo-electric polarizability by considering quarkonia as pure Coulombic systems. While this is a very good approximation in the case of $b\bar{b}$ states, one may object that it is questionable in the case of charmonia.

The perturbative result in the framework of the $1/N_c$ expansion is [53, 59]

$$\alpha_{\psi\psi}(nS) = \frac{16\pi n^2 c_n a_0^3}{3 g_c^2 N_c^2} \ . \tag{4}$$

Here, n is the radial quantum number; $c_1 = \frac{7}{4}$ and $c_2 = \frac{251}{8}$; $N_c = 3$ is the number of colors; $g_c = \sqrt{4\pi\alpha_s} \simeq 2.5$, with α_s being the QCD running coupling constant; finally,

$$a_0 = \frac{2}{m_c C_F \alpha_s} \tag{5}$$

is the Bohr radius of nonrelativistic charmonium [41], with $C_F = \frac{N_c^2-1}{2N_c}$ and $m_c = 1.5$ GeV. By using Eqs. (4) and (5) and the previous values of the constants and parameters, one obtains

$$\alpha_{\psi\psi}(1S) \simeq 4.1 \text{ GeV}^{-3} \tag{6a}$$

and

$$\alpha_{\psi\psi}(2S) \simeq 296 \text{ GeV}^{-3} \ . \tag{6b}$$

As discussed in the following, the value of $\alpha_{\psi\psi}(1S)$ gives rise to hadro-charmonium states with binding energies $\mathcal{O}(10-100)$ MeV. On the contrary, the largeness of $\alpha_{\psi\psi}(2S)$ gives rise to unphysical states, characterized by negative masses. A possible explanation is the following: $2S$ are larger than $1S$ $c\bar{c}$ states; thus, the QCD multipole expansion, where one assumes the quarkonium size to be much smaller than the soft-gluon wave-length, is not applicable anymore.

In the bottomonium case, considering $\alpha_s \simeq 0.35$ and $m_b \simeq 5.0$ GeV [41], one gets: $\alpha_{\Upsilon\Upsilon}(1S) \simeq 0.47$ GeV^{-3} and $\alpha_{\Upsilon\Upsilon}(2S) \simeq 33$ GeV^{-3}. $\alpha_{\Upsilon\Upsilon}(1S)$ may be too small to generate bounded states; on the contrary, $\alpha_{\Upsilon\Upsilon}(2S)$ may give rise to hadro-bottomonia with large binding energies, $\mathcal{O}(1)$ GeV, which may be unphysical.

3. The third possibility is to calculate the expectation value of Eq. (2) on charmonia by inserting string-vibrational or continuum-octet intermediate states [41, 66–70] in the matrix element of Eq. (2).

Specifically, Eq. (2) can be re-written as [52, 71, 72]

$$\alpha_{\psi\psi} = \frac{1}{24} \langle\psi| \, r_i G_8 r_i \, |\psi\rangle \ . \tag{7}$$

Here, the condition $\langle\text{singlet}| \, \xi^a \xi^b \, |\text{singlet}\rangle = \frac{2}{3}\delta^{ab}$ is used, because the operator ξ^a turns a singlet state into an octet one, and vice-versa (only the octet states contribute), and

$$G_8 = \frac{1}{E_\psi - E_8} = \sum_{k\ell} \frac{|\nu k\ell\rangle \, \langle\nu k\ell|}{E_\psi - E^\nu_{k\ell}} \tag{8}$$

is the color-octet Green's function. Here, E_ψ and $E^\nu_{k\ell}$ are charmonium and string-vibrational state [73, 74] energies. After introducing the propagator of Eq. (8) in (7), the chromo-electric polarizability calculation essentially reduces to evaluating dipole matrix elements between quarkonium and string-vibrational states.

B. $E^a_i E^a_i$ product

The product $E^a_i E^a_i$ in Eq. (1) can be re-written using the anomaly in the trace of the energy-momentum tensor $\theta_{\mu\nu}$ in QCD [71],

$$\begin{array}{rcl} \theta^\mu_\mu & = & -\frac{9}{32\pi^2} \, G^a_{\mu\nu} G^{a\mu\nu} \\ & = & \frac{9}{16\pi^2} \left(E^a_i E^a_i - B^a_i B^a_i \right) \end{array} \ , \tag{9}$$

where B^a_i is the chromo-magnetic field. If we neglect the contribution due to the chromo-magnetic fields, which is expected to be smaller than the chromo-electric one [63], Eq. (9) can be re-written as:

$$E^a_i E^a_i \approx \frac{16\pi^2}{9} \, \theta^\mu_\mu \ . \tag{10}$$

The expectation value of the operator θ^μ_μ on a generic state $\mathcal{X}$ is given by [34]

$$\langle\mathcal{X}| \, \theta^\mu_\mu(\mathbf{q}=0) \, |\mathcal{X}\rangle = M_\mathcal{X} \ , \tag{11}$$

where a non-relativistic normalization for $\mathcal{X}$, $\langle\mathcal{X}|\mathcal{X}\rangle = 1$, is assumed.

C. An Hamiltonian for the hadro-charmonium

The effective potential $V_{\rm hc}$, describing the coupling between ψ and $\mathcal{X}$, can be approximated as a finite well [34]

$$\int_0^{R_\mathcal{X}} d^3r \ V_{hc} \approx -\frac{8\pi^2}{9} \ \alpha_{\psi\psi} M_\mathcal{X} \ , \tag{12}$$

where

$$R_\mathcal{X} = \int_0^\infty d^3r \Psi^*_\mathcal{X}(\mathbf{r}) r \Psi_\mathcal{X}(\mathbf{r}) \tag{13}$$

is the radius of the light meson $\mathcal{X}$ [9]. Thus, we have:

$$V_{\rm hc}(r) = \begin{cases} -\frac{2\pi\alpha_{\psi\psi} M_\mathcal{X}}{3R^3_\mathcal{X}} & \text{for} \ \ r < R_\mathcal{X} \\ 0 & \text{for} \ \ r > R_\mathcal{X} \end{cases} \ . \tag{14}$$

By analogy with calculations of the interaction between heavy quarkonia and the nuclear medium [62–64], we get a potential that is a constant square well inside the light meson $\mathcal{X}$ and null outside. We can estimate the order of magnitude of the strength of $V_{\rm hc}$ by introducing into Eq. (14) typical values for $R_\mathcal{X}$ and $M_\mathcal{X}$. If we take $R_\mathcal{X} = 0.5$ fm, $M_\mathcal{X} = 1$ GeV and $\alpha_{\psi\psi}$ from Eq. (3), we get a potential well with a depth of the order of 250 MeV. The Hamiltonian of the hadro-charmonium system also contains a kinetic energy term,

$$T_{\rm hc} = \frac{k^2}{2\mu} \ , \tag{15}$$

where $\mathbf{k}$ is the relative momentum (with conjugate coordinate $\mathbf{r}$) between ψ and $\mathcal{X}$, and μ the reduced mass of the $\psi\mathcal{X}$ system.

The total hadro-charmonium Hamiltonian is thus:

$$H_{\rm hc} = M_\psi + M_\mathcal{X} + V_{\rm hc}(r) + T_{\rm hc} \ . \tag{16}$$

III. RESULTS AND DISCUSSION

Below, we calculate the spectrum of η_c- and J/ψ-isoscalar meson bound states in the hadro-charmonium picture by solving the eigenvalue problem of Eq. (16). The time-independent Schrödinger equation is solved numerically by means of both Multhopp method, see [75, Sec. 2.4] and [76, Sec. II.D], and a finite differences algorithm [77, Vol. 3, Sec. 16-6] as a check. The theoretical predictions are extracted by using the prescription 2. for the chromo-electric polarizability of Sec. II A.

The calculated hadro-charmonium spectrum is shown in Table I; here, we also try some tentative assignments to experimental X states. See [49, Table I].

The hadro-charmonium quantum numbers are shown in the third column of Table I. They are obtained by combining those of the charmonium core, ψ, and light meson, $\mathcal{X}$, as

$$|\Phi_{\rm hc}\rangle = \left|(L_\psi, L_\mathcal{X})L_{\rm hc}, (S_\psi, S_\mathcal{X})S_{\rm hc}; J^{PC}_{\rm hc}\right\rangle \ , \tag{17}$$

Composition	Quark content	$J_{\rm hc}^{PC}$	Binding [MeV]	Mass [MeV]	Assignment
$\eta_c \otimes \eta'$	$c\bar{c}s\bar{s}$	0^{++}	12	3929	$X(3915)$
$\eta_c \otimes {\rm f}_0$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{-+}, 2^{-+}	28	3946	$X(3940)$
$\eta_c \otimes \phi$	$c\bar{c}s\bar{s}$	1^{+-}	20	3983	–
$J/\psi \otimes \eta'$	$c\bar{c}s\bar{s}$	1^{+-}	13	4042	–
$J/\psi \otimes {\rm f}_0$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	29	4058	–
$J/\psi \otimes \phi$	$c\bar{c}s\bar{s}$	0^{++}, 1^{+-}, 2^{++}	21	4096	–
$\eta_c \otimes {\rm h}_1$	$c\bar{c}q\bar{q}$	1^{--}	37	4116	–
$\eta_c \otimes {\rm f}_0'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{-+}, 2^{-+}	151	4191	–
$\eta_c \otimes {\rm f}_1$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{-+}, 2^{-+}	61	4204	$X(4160)$
$J/\psi \otimes {\rm h}_1$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{-+}, 2^{-+}	38	4229	–
$\eta_c \otimes {\rm f}_2$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{-+}, 2^{-+}	25	4234	–
$\eta_c \otimes {\rm h}_1'$	$c\bar{c}s\bar{s}$	1^{--}	105	4285	–
$\eta_c \otimes {\rm f}_1'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{-+}, 2^{-+}	118	4292	–
$J/\psi \otimes {\rm f}_0'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	153	4303	–
$J/\psi \otimes {\rm f}_1$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	62	4317	$Y(4260)$
$J/\psi \otimes {\rm f}_2$	$c\bar{c}q\bar{q}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	26	4346	$Y(4360)$
$J/\psi \otimes {\rm h}_1'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{-+}, 2^{-+}	107	4397	–
$J/\psi \otimes {\rm f}_1'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	120	4404	–
$\eta_c \otimes {\rm f}_2'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{-+}, 2^{-+}	85	4423	–
$J/\psi \otimes {\rm f}_2'$	$c\bar{c}s\bar{s}$	0^{-+}, 1^{--}, 1^{-+}, 2^{-+}, 2^{--}, 3^{--}	87	4535	–

TABLE I: Hadro-charmonium model predictions (fourth and fifth columns), calculated by solving the Schrödinger equation (16) with the chromo-electric polarizability of Eq. (4). The ${\rm f}_0'$ mass used in the calculations, $M_{{\rm f}_0'} = 1359$ MeV, is calculated in the relativized quark model [9].

where the hadro-charmonium P- and C-parity are given by: $P = (-1)^{L_{\rm hc}}$ and $C = (-1)^{L_{\rm hc}+S_{\rm hc}}$.

Starting from the lowest part of the spectrum, the $X(3915)$, observed by Belle and BaBar in $B \rightarrow K + (J\psi\omega)$ [78] and $e^+e^- \rightarrow e^+e^- + (J\psi\omega)$ [79], is interpreted as a $\eta_c \otimes \eta'$ hadro-charmonium state. The $X(3940)$, discovered by Belle in $e^+e^- \rightarrow J/\Psi +$ anything [80] and later observed in $e^+e^- \rightarrow J/\Psi + (D^*\bar{D})$ [81], is here interpreted as a $\eta_c \otimes {\rm f}_0$ state. The $X(4160)$, observed by Belle in $e^+e^- \rightarrow J/\Psi + (D^*\bar{D}^*)$ [81], may be interpreted as a $\eta_c \otimes {\rm f}_1$ state with 0^{-+} quantum numbers. The $X(4260)$, observed by BaBar [54, 82], CLEO [83] and Belle [2, 84] in $e^+e^- \rightarrow \gamma + (J/\Psi\pi^+\pi^-)$, and $X(4360)$, observed by BaBar [57, 85] and Belle [56, 86] in $e^+e^- \rightarrow \gamma + [\Psi(2S)\pi^+\pi^-]$, and BESIII [87] in $J/\Psi\pi^+\pi^-$ and $h_c\pi^+\pi^-$, are both characterized by 1^{--} quantum numbers. According to our results, $X(4260)$ and $X(4360)$ may be described in terms of $J/\psi \otimes {\rm f}_1$ and $J/\psi \otimes {\rm f}_2$ states, respectively. In Refs. [38–40], they are interpreted as a mixture of two hadrocharmonia, $|\psi_1\rangle \sim |1^{+-}\rangle_{c\bar{c}} \otimes |0^{-+}\rangle_{q\bar{q}}$ and $\psi_3 \sim |1^{--}\rangle_{c\bar{c}} \otimes |0^{++}\rangle_{q\bar{q}}$, with a large mixing angle, $\theta_{\rm mix} \simeq 40°$. The mixing is due to the exchange of one chromo-electric and one chromo-magnetic gluon between the hadro-charmonium $c\bar{c}$ cores.

Finally, it is worth noticing that: I) The quantum number assignments in Table I for several states are not univocal. A possible way to distinguish between them is to calculate the hadro-charmonium main decay amplitudes and compare the theoretical results with the data; II) The results strongly depend on the chromo-electric polarizability, $\alpha_{\psi\psi}$. Up to now, the value of $\alpha_{\psi\psi}$ cannot be fitted to the experimental data; it has to be estimated phenomenologically. Because of this, it represents one of the main sources of theoretical uncertainty on the results; III) In the calculation of $\langle \mathcal{X}|\, \theta_\mu^\mu(\mathbf{q}=0)\, |\mathcal{X}\rangle$ matrix elements on light mesons, $\mathcal{X}$, the contributions due to the chromo-magnetic field, $\mathbf{B}^a$, are neglected. This may represent another source of theoretical uncertainties; IV) By combining ψ and $\mathcal{X}$ quantum numbers, several $J_{\rm hc}^{PC}$ configurations are obtained. Thus, once the value of the J/ψ and η_c chromo-electric polarizability is measured (and thus the main source of theoretical uncertainties removed), it would be interesting to introduce spin-orbit and spin-spin corrections in order to split the degenerate configurations.

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