Production of the X_b in $\Upsilon(5S,6S) \to \gamma X_b$ radiative decays

Qi Wu, Gang Li*, Fenglan Shao, Qianwen Wang, Ruiqin Wang, Yawei Zhang and Ying Zheng

[1] *Department of Physics, Qufu Normal University, Qufu 273165, China*

(Dated: August 28, 2018)

In this work, we investigate the production of X_b in the process $\Upsilon(5S,6S) \to \gamma X_b$, where X_b is assumed to be the counterpart of $X(3872)$ in the bottomonium sector as a $B\bar{B}^*$ molecular state. We use the effective Lagrangian based on the heavy quark symmetry to explore the rescattering mechanism and calculate their production ratios. Our results have shown that the production ratios for the $\Upsilon(5S,6S) \to \gamma X_b$ are orders of 10^{-5} with reasonable cutoff parameter range $\alpha \simeq 2 \sim 3$. The sizeable production ratios may be accessible at the future experiments like forthcoming BelleII, which will provide important clues to the inner structures of the exotic state X_b.

PACS numbers: 14.40.Pq, 13.20.Gd, 12.39.Fe

I. INTRODUCTION

In the past decades, many so called XYZ have been observed by the Belle, BaBar, CDF, D0, CMS, LHCb, and BESIII collaborations [1]. Some of them cannot fit into the conventional heavy quarkonium in the quark model [2–5]. Up to now, many studies on the production and decay of these XYZ states have been carried out in order to understand its nature (for a recent review, see Refs. [6–8]).

In 2003, the Belle Collaboration discovered an exotic candidate $X(3872)$ in the process $B^+ \to K^+ + J/\psi\pi^+\pi^-$ [9] which was subsequently confirmed by the BaBar Collaboration [10] in the same channel. It was also discovered in proton-proton/antiproton collisions at the Tevatron [11, 12] and LHC [13, 14]. The $X(3872)$ is a particularly intriguing state because on the one hand its total width $\Gamma < 1.2$ MeV [1] is tiny compared to typical hadronic widths; on the other hand the closeness of its mass to the $D^0\overline{D}^{*0}$ threshold ($M_{X(3872)} - M_{D^0} - M_{D^{*0}} = (-0.12 \pm 0.24)$ MeV) and its prominent decays to $D^0\overline{D}^{*0}$ [1] suggest that it may be an meson-meson molecular state [15, 16].

Many theoretical works have been carried out in order to understand the nature of $X(3872)$ since the first observation of $X(3872)$. It is also natural to look for the counterpart with $J^{PC} = 1^{++}$ (denoted as X_b hereafter) in the bottom sector. These two states are related by heavy quark symmetry which should have some universal properties. The search for X_b may provide us important information on the discrimination of a compact multiquark configuration and a loosely bound

* gli@mail.qfnu.edu.cn

hadronic molecule configuration. Since the mass of X_b may be very heavy and its J^{PC} is 1^{++}, it is less likely for a direct discovery at the current electron-positron collision facilities, though the Super KEKB may provide an opportunity in $\Upsilon(5S, 6S)$ radiative decays [17]. In Ref. [18], a search for X_b in the $\omega\Upsilon(1S)$ final states has been presented and no significant signal is observed for such a state.

The production of X_b at the LHC and the Tevatron [19, 20] and other exotic states at hadron colliders [21–26] have been extensively investigated. In the bottomonium system, the isospin is almost perfectly conserved, which may explain the escape of X_b in the recent CMS search [27]. As a result, the radiative decays and isospin conserving decays will be of high priority in searching for X_b [28–30]. In Ref. [28], we have studied the radiative decays of $X_b \to \gamma\Upsilon(nS)$ $(n = 1, 2, 3)$, with X_b being a candidate for the $B\bar{B}^*$ molecular state, and found that the partial widths into γX_b are about 1 keV. In Ref. [29], we studied the rescattering mechanism of the isospin conserving decays $X_b \to \Upsilon(1S)\omega$, and our results show that the partial width for the $X_b \to \Upsilon(1S)\omega$ is about tens of keVs.

In this work, we will further investigate the X_b production in $\Upsilon(5S, 6S) \to \gamma X_b$ with X_b being a $B\bar{B}^*$ molecule candidate. To investigate this process, we calculate the intermediate meson loop (IML) contributions. As well know, IML transitions have been one of the important nonperturbative transition mechanisms been noticed for a long time [31–33]. Recently, this mechanism has been used to study the production and decays of ordinary and exotic states [34–60] and B decays [61–68], and a global agreement with experimental data were obtained. Thus this approach may be suitable for the process $\Upsilon(5S, 6S) \to \gamma X_b$.

The paper is organized as follows. In Sec. II, the effective Lagrangians for our calculation. Then in Sec. III, we present our numerical results. Finally we give the summary in Sec. IV.

II. EFFECTIVE LAGRANGIANS

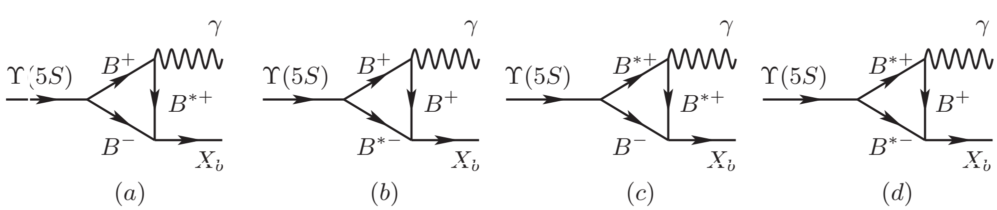

FIG. 1: Feynman diagrams for X_b production in $\Upsilon(5S) \to \gamma X_b$ under the $B\bar{B}^*$ meson loop effects.

Based on the heavy quark symmetry, we can write out the relevant effective Lagrangian for the

$\Upsilon(5S)$ [68, 69]

$$\begin{aligned}\mathcal{L}_{\Upsilon(5S)B^{(*)}B^{(*)}} &= ig_{\Upsilon BB}\Upsilon_\mu(\partial^\mu B\bar{B} - B\partial^\mu\bar{B}) - g_{\Upsilon B^*B}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu\Upsilon^\nu(\partial^\alpha B^{*\beta}\bar{B} + B\partial^\alpha\bar{B}^{*\beta}) \\ &\quad -ig_{\Upsilon B^*B^*}\big\{\Upsilon^\mu(\partial_\mu B^{*\nu}\bar{B}^*_\nu - B^{*\nu}\partial_\mu\bar{B}^*_\nu) + (\partial_\mu\Upsilon_\nu B^{*\nu} - \Upsilon_\nu\partial_\mu B^{*\nu})\bar{B}^{*\mu} \\ &\quad +B^{*\mu}(\Upsilon^\nu\partial_\mu\bar{B}^*_\nu - \partial_\mu\Upsilon^\nu\bar{B}^*_\nu)\big\},\end{aligned} \tag{1}$$

where $B^{(*)} = \big(B^{(*)+}, B^{(*)0}\big)$ and $\bar{B}^{(*)T} = \big(B^{(*)-}, \bar{B}^{(*)0}\big)$ correspond to the bottom meson isodoublets. $\epsilon_{\mu\nu\alpha\beta}$ is the anti-symmetric Levi-Civita tensor and $\epsilon_{0123} = +1$. Since $\Upsilon(5S)$ is above the threshold of $B^{(*)}\bar{B}^{(*)}$, the coupling constants between $\Upsilon(5S)$ and $B^{(*)}\bar{B}^{(*)}$ can be determined via experimental data for $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$ [1]. The experimental branching ratios and the corresponding coupling constants are listed in Table I. Since there is no experimental information on $\Upsilon(6S) \to B^{(*)}\bar{B}^{(*)}$ [1], we choose the coupling constants between $\Upsilon(6S)$ and $B^{(*)}\bar{B}^{(*)}$ the same values as that of $\Upsilon(5S)$.

TABLE I: The coupling constants of $\Upsilon(5S)$ interacting with $B^{(*)}\bar{B}^{(*)}$. Here, we list the corresponding branching ratios of $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$.

Final state	$\mathcal{B}(\%)$	Coupling	Final state	$\mathcal{B}(\%)$	Coupling	Final state	$\mathcal{B}(\%)$	Coupling
$B\bar{B}$	5.5	1.76	$B\bar{B}^* + c.c.$	13.7	0.14 GeV^{-1}	$B^*\bar{B}^*$	38.1	2.22
$B_s\bar{B}_s$	0.5	0.96	$B_s\bar{B}_s^* + c.c.$	1.35	0.10 GeV^{-1}	$B_s^*\bar{B}_s^*$	17.6	5.07

In order to calculate the process depicted in Fig. 1, we also need the photonic coupling to the bottomed mesons. The magnetic coupling of the photon to heavy bottom meson is described by the Lagrangian [72, 73]

$$\mathcal{L}_\gamma = \frac{e\beta Q_{ab}}{2}F^{\mu\nu}\mathrm{Tr}[H_b^\dagger\sigma_{\mu\nu}H_a] + \frac{eQ'}{2m_Q}F^{\mu\nu}\mathrm{Tr}[H_a^\dagger H_a\sigma_{\mu\nu}], \tag{2}$$

with

$$H = \left(\frac{1+\not{v}}{2}\right)[\mathcal{B}^{*\mu}\gamma_\mu - \mathcal{B}\gamma_5], \tag{3}$$

where β is an unknown constant, $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ is the light quark charge matrix, and Q' is the heavy quark electric charge (in units of e). $\beta \simeq 3.0$ GeV^{-1} is determined in the nonrelativistic constituent quark model and has been adopted in the study of radiative D^* decays [73]. In the b and c systems, the β value is the same due to heavy quark symmetry [73]. In Eq. (2), the first term is the magnetic moment coupling of the light quarks, while the second one is the magnetic moment coupling of the heavy quark and hence is suppressed by $1/m_Q$.

At last, assuming that X_b is an S-wave molecule with $J^{PC} = 1^{++}$ given by the superposition of $B^0\bar{B}^{*0} + c.c$ and $B^-\bar{B}^{*+} + c.c$ hadronic configurations as

$$|X_b\rangle = \frac{1}{2}[(|B^0\bar{B}^{*0}\rangle - |B^{*0}\bar{B}^0\rangle) + (|B^+B^{*-}\rangle - |B^-B^{*+}\rangle)]. \tag{4}$$

As a result, we can parameterize the coupling of X_b to the bottomed mesons in terms of the following Lagrangian

$$\mathcal{L} = \frac{1}{2} X^{\dagger}_{b\mu} [x_1 (B^{*0\mu} \bar{B}^0 - B^0 \bar{B}^{*0\mu}) + x_2 (B^{*+\mu} B^- - B^+ B^{*-\mu})] + h.c., \tag{5}$$

where x_i denotes the coupling constant. Since the X_b is slightly below the S-wave $B\bar{B}^*$ threshold, the effective coupling of this state is related to the probability of finding the $B\bar{B}^*$ component in the physical wave function of the bound states and the binding energy, $\epsilon_{X_b} = m_B + m_{B^*} - m_{X_b}$ [36, 70, 71]

$$x_i^2 \equiv 16\pi (m_B + m_{B^*})^2 c_i^2 \sqrt{\frac{2\epsilon_{X_b}}{\mu}}, \tag{6}$$

where $c_i = 1/\sqrt{2}$, $\mu = m_B m_{B^*}/(m_B + m_{B^*})$ is the reduced mass. Here, we should also notice that the coupling constant x_i in Eq. (6) is based on the assumption that X_b is a shallow bound state where the potential binding the mesons is short-ranged.

Based on the relevant Lagrangians given above, the decay amplitudes in Fig. 1 can be generally expressed as follows,

$$M_{fi} = \int \frac{d^4 q_2}{(2\pi)^4} \sum_{B^* \text{ pol.}} \frac{T_1 T_2 T_3}{D_1 D_2 D_3} \mathcal{F}(m_2, q_2^2) \tag{7}$$

where T_i and $D_i = q_i^2 - m_i^2$ $(i = 1, 2, 3)$ are the vertex functions and the denominators of the intermediate meson propagators. For example, in Fig. 1 (a), T_i $(i = 1, 2, 3)$ are the vertex functions for the initial $\Upsilon(5S)$, final X_b and photon, respectively. D_i $(i = 1, 2, 3)$ are the denominators for the intermediate B^+, B^- and B^{*+} propagators, respectively.

Since the intermediate exchanged bottom mesons in the triangle diagram Fig. 1 are off-shell, in order to compensate this off-shell effects arising from the intermediate exchanged particle and also the non-local effects of the vertex functions [74–76], we adopt the following form factors,

$$\mathcal{F}(m_2, q_2^2) \equiv \left(\frac{\Lambda^2 - m_2^2}{\Lambda^2 - q_2^2} \right)^n, \tag{8}$$

where $n = 1, 2$ corresponds monopole and dipole form factor, respectively. $\Lambda \equiv m_2 + \alpha \Lambda_{\text{QCD}}$ and the QCD energy scale $\Lambda_{\text{QCD}} = 220$ MeV. This form factor is supposed and many phenomenological studies have suggested $\alpha \simeq 2 \sim 3$. These two form factors can help us explore the dependence of our results on the form factor.

The explicit expression of transition amplitudes can be found in Appendix (A.2) in Ref. [77], where radiative decays of charmonium are studied extensively based on effective Lagrangian approach.

TABLE II: Predicted branching ratios for $\Upsilon(5S) \to \gamma X_b$. The parameter in the form factor is chosen as $\alpha = 2.0$, 2.5, and 3.0. The last column is the calculated branching ratios in NREFT approach.

Binding Energy	Monopole form factor			Dipole form factor			NREFT
	$\alpha = 2.0$	$\alpha = 2.5$	$\alpha = 3.0$	$\alpha = 2.0$	$\alpha = 2.5$	$\alpha = 3.0$	
$\epsilon_{X_b} = 5$ MeV	2.02×10^{-5}	2.06×10^{-5}	2.08×10^{-5}	1.90×10^{-5}	1.99×10^{-5}	2.04×10^{-5}	1.52×10^{-6}
$\epsilon_{X_b} = 10$ MeV	2.58×10^{-5}	2.66×10^{-5}	2.71×10^{-5}	2.32×10^{-5}	2.47×10^{-5}	2.57×10^{-5}	2.12×10^{-6}
$\epsilon_{X_b} = 25$ MeV	3.24×10^{-5}	3.42×10^{-5}	3.54×10^{-5}	2.61×10^{-5}	2.90×10^{-5}	3.09×10^{-5}	3.88×10^{-6}
$\epsilon_{X_b} = 50$ MeV	3.37×10^{-5}	3.65×10^{-5}	3.85×10^{-5}	2.37×10^{-5}	2.75×10^{-5}	3.04×10^{-5}	6.41×10^{-6}
$\epsilon_{X_b} = 100$ MeV	2.91×10^{-5}	3.27×10^{-5}	3.54×10^{-5}	1.65×10^{-5}	2.05×10^{-5}	2.38×10^{-5}	1.20×10^{-5}

TABLE III: Predicted branching ratios for $\Upsilon(6S) \to \gamma X_b$. The parameter in the form factor is chosen as $\alpha = 2.0$, 2.5, and 3.0. The last column is the calculated branching ratios in NREFT approach.

Binding Energy	Monopole form factor			Dipole form factor			NREFT
	$\alpha = 2.0$	$\alpha = 2.5$	$\alpha = 3.0$	$\alpha = 2.0$	$\alpha = 2.5$	$\alpha = 3.0$	
$\epsilon_{X_b} = 5$ MeV	9.71×10^{-6}	1.02×10^{-5}	1.05×10^{-5}	8.16×10^{-6}	9.04×10^{-6}	9.63×10^{-6}	3.38×10^{-6}
$\epsilon_{X_b} = 10$ MeV	1.25×10^{-5}	1.33×10^{-5}	1.38×10^{-5}	9.97×10^{-6}	1.13×10^{-5}	1.22×10^{-5}	4.89×10^{-6}
$\epsilon_{X_b} = 25$ MeV	1.62×10^{-5}	1.76×10^{-5}	1.85×10^{-5}	1.14×10^{-5}	1.34×10^{-5}	1.49×10^{-5}	8.27×10^{-6}
$\epsilon_{X_b} = 50$ MeV	1.76×10^{-5}	1.96×10^{-5}	2.12×10^{-5}	1.08×10^{-5}	1.32×10^{-5}	1.52×10^{-5}	1.30×10^{-5}
$\epsilon_{X_b} = 100$ MeV	1.66×10^{-5}	1.92×10^{-5}	2.12×10^{-5}	8.12×10^{-6}	1.06×10^{-5}	1.28×10^{-5}	2.24×10^{-5}

III. NUMERICAL RESULTS

Before proceeding the numerical results, we first briefly review the predictions on mass of X_b. The existence of the X_b is predicted in both the tetraquark model [78] and those involving a molecular interpretation [79–81]. In Ref. [78], the mass of the lowest-lying 1^{++} $\bar{b}\bar{q}bq$ tetraquark is predicated to be 10504 MeV , while the mass of the $B\bar{B}^*$ molecular state is predicated to be a few tens of MeV higher [79–81]. For example, in Ref. [79], the mass was predicted to be 10562 MeV, which corresponds to a binding energy to be 42 MeV, while the mass was predicted to be (10580^{+9}_{-8}) MeV, which corresponds to a binding energy (24^{+8}_{-9}) MeV in Ref. [81]. As can be seen from the theoretical predictions, it might be a good approximation and might be applicable if the binding energy is less than 50 MeV. In order to cover the range the previous molecular and tetraquark predictions on Ref. [78–81], we present our results up to a binding energy of 100 MeV, and we will choose several illustrative values: $\epsilon_{X_b} = (5, 10, 25, 50, 100)$ MeV.

In Table II, we list the predicted branching ratios by choosing the monopole and dipole form factors and three values for the cutoff parameter in the form factor. As a comparison, we also list

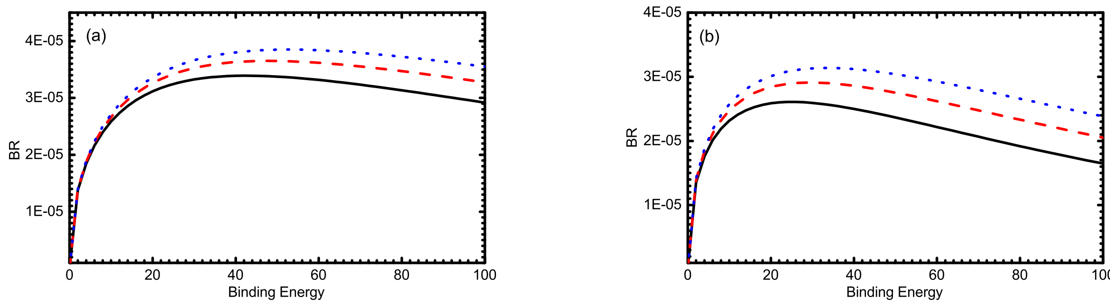

FIG. 2: (a). The dependence of the branching ratios of $\Upsilon(5S) \to \gamma X_b$ on the ϵ_{X_b} using monopole form factors with $\alpha = 2.0$ (solid lines), $\alpha = 2.5$ (dashed lines), and $\alpha = 3.0$ (dotted lines), respectively. (b). The dependence of the branching ratios of $\Upsilon(5S) \to \gamma X_b$ on the ϵ_{X_b} using dipole form factors with $\alpha = 2.0$ (solid lines), $\alpha = 2.5$ (dashed lines), and $\alpha = 3.0$ (dotted lines), respectively. The results with binding energy up to 100 MeV might make the molecular state assumption inaccurate.

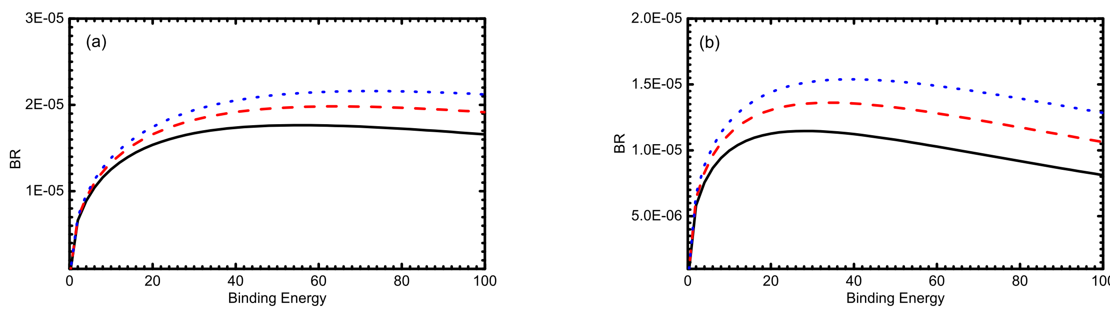

FIG. 3: (a). The dependence of the branching ratios of $\Upsilon(6S) \to \gamma X_b$ on the ϵ_{X_b} using monopole form factors with $\alpha = 2.0$ (solid lines), $\alpha = 2.5$ (dashed lines), and $\alpha = 3.0$ (dotted lines), respectively. (b). The dependence of the branching ratios of $\Upsilon(6S) \to \gamma X_b$ on the ϵ_{X_b} using dipole form factors with $\alpha = 2.0$ (solid lines), $\alpha = 2.5$ (dashed lines), and $\alpha = 3.0$ (dotted lines), respectively. The results with binding energy up to 100 MeV might make the molecular state assumption inaccurate.

the predicted branching ratios in NREFT approach. From this table, we can see that the branching ratios for $\Upsilon(5S) \to \gamma X_b$ are orders of 10^{-5}. The results are not sensitive to both the form factors and the cutoff parameter we choose.

In Fig. 2 (a), we plot the the branching ratios for $\Upsilon(5S) \to \gamma X_b$ in terms of the binding energy ϵ_{X_b} with the monopole form factors $\alpha = 2.0$ (solid line), 2.5 (dashed line), and 3.0 (dotted line), respectively. The coupling constant of X_b in Eq. (6) and the threshold effects can simultaneously influence the binding energy dependence of the branching ratios. With the increasing of the binding energy ϵ_{X_b}, the coupling strength of X_b increases, and the threshold effects decrease. Both the coupling strength of X_b and the threshold effects vary quickly in the small ϵ_{X_b} region and slowly in the large ϵ_{X_b} region. As a result, the behavior of the branching ratios is relatively sensitive at small ϵ_{X_b}, while it becomes smooth at large ϵ_{X_b}. Results with the dipole form factors $\alpha = 2.0$, 2.5, and 3.0 are shown in Fig. 2 (b) as solid, dash, and dotted curves, respectively. The behavior is similar to that of Fig. 2 (a).

We also predict the branching ratios of $\Upsilon(6S) \to \gamma X_b$ and present the relevant numerical results in Table III and Fig. 3 with the monopole and dipole form factors. At the same cutoff parameter α, the predicted rates for $\Upsilon(6S) \to \gamma X_b$ are a factor of 2-3 smaller than the corresponding rates for $\Upsilon(5S) \to \gamma X_b$. It indicates that the intermediate B-meson loop contribution to the process $\Upsilon(6S) \to \gamma X_b$ is smaller than that to $\Upsilon(5S) \to \gamma X_b$. This is understandable since the mass of $\Upsilon(6S)$ is more far away from the thresholds of $B^{(*)}B^{(*)}$ than the $\Upsilon(5S)$. But their branching ratios are also about orders of 10^{-5} with a reasonable cutoff parameter $\alpha = 2 \sim 3$.

In Ref. [51], authors introduced a nonrelativistic effective field theory method to study the meson loop effects of $\psi' \to J/\psi\pi^0$. Meanwhile they proposed a power counting scheme to estimate the contribution of the loop effects, which is used to judge the impact of the coupled-channel effects. For the diagrams in Fig. 1, the vertex involving the initial bottomonium is in P-wave. The momentum in this vertex is contracted with the final photon momentum q, and thus should be counted as q. The decay amplitude scales as follows,

$$\frac{v^5}{(v^2)^3} q^2 \sim \frac{q^2}{v}, \tag{9}$$

where v is understood as the average velocity of the intermediate bottomed mesons.

As a cross-check, we also present the branching ratios of the decays in the framework of NREFT. The relevant transition amplitudes are similar to that given in Ref. [36] with only different masses and coupling constants. The obtained numerical results for $\Upsilon(5S) \to \gamma X_b$ and $\Upsilon(6S) \to \gamma X_b$ in terms of the binding energy are listed in the last column of Table II and III, respectively. As shown in Table II, except for the largest binding energy $\epsilon_{X_b} = 100$ MeV, the NREFT predictions of $\Upsilon(5S) \to \gamma X_b$ are about 1 order of magnitude smaller than the ELA results at the commonly accepted range. For $\Upsilon(6S) \to \gamma X_b$ shown in Table III, the NREFT predictions are several times smaller than the ELA results in small binding energy range, while the predictions of these two methods are comparable at large binding energy. These difference may give some sense of the theoretical uncertainties for the predicted rates and indicates the viability of our model to some extent.

Here we should notice, for the isoscalar X_b, the pion exchanges might be nonperturbative and produce sizeable effects [81–83]. In Ref. [81], their calculations show that the relative errors of C_{0X} are about 20% for the X_b. Even if we take into account this effect, the estimated order of the magnitude for the branching ratio $\Upsilon(5S, 6S) \to \gamma X_b$ may also be sizeable, which may be measured in the forthcoming BelleII experiments.

IV. SUMMARY

In this work, we have investigated the production of X_b in the radiative decays of $\Upsilon(5S, 6S)$. Based on the $B\bar{B}^*$ molecular state picture, we considered its production through the mechanism with intermediate bottom meson loops. Our results have shown that the production ratios for the $\Upsilon(5S, 6S) \to \gamma X_b$ are about orders of 10^{-5} with a commonly accepted cutoff range $\alpha = 2 \sim 3$. As a cross-check, we also calculated the branching ratios of the decays in the framework of NREFT. Except for the large binding energy, the NREFT predictions of $\Upsilon(5S) \to \gamma X_b$ are about 1 orders of magnitude smaller than the ELA results. The NREFT predictions of $\Upsilon(6S) \to \gamma X_b$ are several times smaller than the ELA results in small binding energy range, while the predictions of these two methods are comparable at large binding energy. In Ref. [28, 29], we have studied the radiative decays and the hidden bottomonium decays of X_b. If we consider that the branching ratios of the isospin conserving process $X_b \to \omega\Upsilon(1S)$ are relatively large, a search for $\Upsilon(5S) \to \gamma X_b \to \gamma\omega\Upsilon(1S)$ may be possible for the updated BelleII experiments. These studies may help us investigate the X_b deeply. The experimental observation of X_b will provide us with further insight into the spectroscopy of exotic states and is helpful to probe the structure of the states connected by the heavy quark symmetry.

Acknowledgements

This work is supported in part by the National Natural Science Foundation of China (Grant Nos. 11275113, 11575100, 11505104) and the Natural Science Foundation of Shandong Province (Grant No. ZR2015JL001).

---

[1] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[2] N. Brambilla, S. Eidelman, B. K. Heltsley, R. Vogt, G. T. Bodwin, E. Eichten, A. D. Frawley and A. B. Meyer *et al.*, Eur. Phys. J. C **71**, 1534 (2011) [arXiv:1010.5827 [hep-ph]].

[3] S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. **58**, 51 (2008) [arXiv:0801.3867 [hep-ph]].

[4] N. Drenska, R. Faccini, F. Piccinini, A. Polosa, F. Renga and C. Sabelli, Riv. Nuovo Cim. **033**, 633 (2010) [arXiv:1006.2741 [hep-ph]].

[5] G. T. Bodwin, E. Braaten, E. Eichten, S. L. Olsen, T. K. Pedlar and J. Russ, arXiv:1307.7425.

[6] W. Chen, W. Z. Deng, J. He, N. Li, X. Liu, Z. G. Luo, Z. F. Sun and S. L. Zhu, PoS Hadron **2013**, 005 (2013) [arXiv:1311.3763 [hep-ph]].

[7] X. Liu, Chin. Sci. Bull. **59**, 3815 (2014) [arXiv:1312.7408 [hep-ph]].

[8] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, arXiv:1601.02092 [hep-ph].

[9] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003) [hep-ex/0309032].

[10] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **71**, 071103 (2005) [hep-ex/0406022].

[11] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004) [hep-ex/0405004].

[12] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **103**, 152001 (2009) [arXiv:0906.5218 [hep-ex]].

[13] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1304**, 154 (2013) [arXiv:1302.3968 [hep-ex]].

[14] RAaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **110**, no. 22, 222001 (2013) [arXiv:1302.6269 [hep-ex]].

[15] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004) [hep-ph/0402237].

[16] C. Hanhart, Y. .S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. D **76**, 034007 (2007) [arXiv:0704.0605 [hep-ph]].

[17] T. Aushev, W. Bartel, A. Bondar, J. Brodzicka, T. E. Browder, P. Chang, Y. Chao and K. F. Chen *et al.*, arXiv:1002.5012 [hep-ex].

[18] X. H. He *et al.* [Belle Collaboration], Phys. Rev. Lett. **113**, no. 14, 142001 (2014) [arXiv:1408.0504 [hep-ex]].

[19] F. K. Guo, U. G. Meißner, W. Wang and Z. Yang, Eur. Phys. J. C **74**, no. 9, 3063 (2014) [arXiv:1402.6236 [hep-ph]].

[20] F. -K. Guo, U. -G. Meißner and W. Wang, Commun. Theor. Phys. **61**, 354 (2014) [arXiv:1308.0193 [hep-ph]].

[21] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa and C. Sabelli, Phys. Rev. Lett. **103**, 162001 (2009) [arXiv:0906.0882 [hep-ph]].

[22] AEsposito, FPiccinini, APilloni and A. D. Polosa, J. Mod. Phys. **4**, 1569 (2013) [arXiv:1305.0527 [hep-ph]].

[23] P. Artoisenet and E. Braaten, Phys. Rev. D **81**, 114018 (2010) [arXiv:0911.2016 [hep-ph]].

[24] P. Artoisenet and E. Braaten, Phys. Rev. D **83**, 014019 (2011) [arXiv:1007.2868 [hep-ph]].

[25] A. Ali and W. Wang, Phys. Rev. Lett. **106**, 192001 (2011) [arXiv:1103.4587 [hep-ph]].

[26] A. Ali, C. Hambrock and W. Wang, Phys. Rev. D 88, **054026** (2013) [arXiv:1306.4470 [hep-ph]].

[27] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **727**, 57 (2013) [arXiv:1309.0250 [hep-ex]].

[28] G. Li and W. Wang, Phys. Lett. B **733**, 100 (2014) [arXiv:1402.6463 [hep-ph]].

[29] G. Li and Z. Zhou, Phys. Rev. D **91**, no. 3, 034020 (2015) [arXiv:1502.02936 [hep-ph]].

[30] M. Karliner, arXiv:1401.4058 [hep-ph].

[31] H. J. Lipkin, Nucl. Phys. B **291**, 720 (1987).

[32] H. J. Lipkin and S. F. Tuan, Phys. Lett. B **206**, 349 (1988).

[33] P. Moxhay, Phys. Rev. D **39**, 3497 (1989).

[34] Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013) [arXiv:1303.6355 [hep-ph]].

[35] X. -H. Liu and G. Li, Phys. Rev. D **88**, 014013 (2013) [arXiv:1306.1384 [hep-ph]].

[36] F. -K. Guo, C. Hanhart, U. -G. Meißner, Q. Wang and Q. Zhao, Phys. Lett. B **725**, 127 (2013)

[arXiv:1306.3096 [hep-ph]].

[37] Q. Wang, C. Hanhart and Q. Zhao, Phys. Lett. B **725**, no. 1-3, 106 (2013) [arXiv:1305.1997 [hep-ph]].

[38] M. Cleven, Q. Wang, F. -K. Guo, C. Hanhart, U. -G. Meißner and Q. Zhao, Phys. Rev. D **87**, no. 7, 074006 (2013) [arXiv:1301.6461 [hep-ph]].

[39] D. -Y. Chen and X. Liu, Phys. Rev. D **84**, 094003 (2011) [arXiv:1106.3798 [hep-ph]].

[40] G. Li, F. -l. Shao, C. -W. Zhao and Q. Zhao, Phys. Rev. D **87**, no. 3, 034020 (2013) [arXiv:1212.3784 [hep-ph]].

[41] G. Li and X. -H. Liu, Phys. Rev. D **88**, 094008 (2013) [arXiv:1307.2622 [hep-ph]].

[42] G. Li, C. S. An, P. Y. Li, D. Liu, X. Zhang and Z. Zhou, Chin. Phys. C **39**, no. 6, 063102 (2015) [arXiv:1412.3221 [hep-ph]].

[43] G. Li, X. H. Liu and Z. Zhou, Phys. Rev. D **90**, no. 5, 054006 (2014) [arXiv:1409.0754 [hep-ph]].

[44] G. Li, X. H. Liu and Q. Zhao, Eur. Phys. J. C **73**, 2576 (2013).

[45] G. Li, Eur. Phys. J. C **73**, no. 11, 2621 (2013) [arXiv:1304.4458 [hep-ph]].

[46] G. Li, X. h. Liu, Q. Wang and Q. Zhao, Phys. Rev. D **88**, no. 1, 014010 (2013) [arXiv:1302.1745 [hep-ph]].

[47] G. Li and Q. Zhao, Phys. Rev. D **84**, 074005 (2011) [arXiv:1107.2037 [hep-ph]].

[48] M. B. Voloshin, Phys. Rev. D **87**, no. 7, 074011 (2013) [arXiv:1301.5068 [hep-ph]].

[49] M. B. Voloshin, Phys. Rev. D **84**, 031502 (2011) [arXiv:1105.5829 [hep-ph]].

[50] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk and M. B. Voloshin, Phys. Rev. D **84**, 054010 (2011) [arXiv:1105.4473 [hep-ph]].

[51] F. -K. Guo, C. Hanhart and U. -G. Meißner, Phys. Rev. Lett. **103**, 082003 (2009) [Erratum-ibid. **104**, 109901 (2010)] [arXiv:0907.0521 [hep-ph]].

[52] F. -K. Guo, C. Hanhart, G. Li, U. -G. Meißner and Q. Zhao, Phys. Rev. D **82**, 034025 (2010) [arXiv:1002.2712 [hep-ph]].

[53] F. -K. Guo, C. Hanhart, G. Li, U. -G. Meißner and Q. Zhao, Phys. Rev. D **83**, 034013 (2011) [arXiv:1008.3632 [hep-ph]].

[54] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D **84**, 074032 (2011) [arXiv:1108.4458 [hep-ph]].

[55] D. -Y. Chen, X. Liu and T. Matsuki, arXiv:1208.2411 [hep-ph].

[56] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D **88**, 036008 (2013) [arXiv:1304.5845 [hep-ph]].

[57] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D **88**, 014034 (2013) [arXiv:1306.2080 [hep-ph]].

[58] Q. Wang, G. Li and Q. Zhao, Phys. Rev. D **85**, 074015 (2012) [arXiv:1201.1681 [hep-ph]].

[59] Y. J. Zhang, G. Li and Q. Zhao, Chin. Phys. C **34**, no. 9, 1181 (2010).

[60] G. Li, Y. J. Zhang, Q. Zhao and B. S. Zou, Chin. Phys. C **34**, 842 (2010).

[61] D. -S. Du, X. -Q. Li, Z. -T. Wei and B. -S. Zou, Eur. Phys. J. A **4**, 91 (1999) [hep-ph/9805260].

[62] C. -H. Chen and H. -n. Li, Phys. Rev. D **63**, 014003 (2001) [hep-ph/0006351].

[63] X. Liu and X. -Q. Li, Phys. Rev. D **77**, 096010 (2008) [arXiv:0707.0919 [hep-ph]].

[64] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B **542**, 71 (2002) [hep-ph/0207061].

[65] H. -Y. Cheng, C. -K. Chua and A. Soni, Phys. Rev. D **71**, 014030 (2005) [hep-ph/0409317].

[66] C. -D. Lu, Y. -L. Shen and W. Wang, Phys. Rev. D **73**, 034005 (2006) [hep-ph/0511255].

[67] X. Liu, Z. -T. Wei and X. -Q. Li, Eur. Phys. J. C **59**, 683 (2009) [arXiv:0805.2804 [hep-ph]].

[68] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Rev. D **69**, 054023 (2004) [hep-ph/0310084].

[69] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. **281**, 145 (1997) [hep-ph/9605342].

[70] S. Weinberg, Phys. Rev. **137**, B672 (1965).

[71] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B **586**, 53 (2004) [hep-ph/0308129].

[72] J. Hu and T. Mehen, Phys. Rev. D **73**, 054003 (2006) [hep-ph/0511321].

[73] J. F. Amundson, C. G. Boyd, E. E. Jenkins, M. E. Luke, A. V. Manohar, J. L. Rosner, M. J. Savage and M. B. Wise, Phys. Lett. B **296**, 415 (1992) [hep-ph/9209241].

[74] X. -Q. Li, D. V. Bugg and B. -S. Zou, Phys. Rev. D **55**, 1421 (1997).

[75] M. P. Locher, Y. Lu and B. S. Zou, Z. Phys. A **347**, 281 (1994) [nucl-th/9311021].

[76] X. -Q. Li and B. -S. Zou, Phys. Lett. B **399**, 297 (1997) [hep-ph/9611223].

[77] C. -W. Zhao, G. Li, X. -H. Liu and F. -L. Shao, Eur. Phys. J. C **73**, 2482 (2013).

[78] A. Ali, C. Hambrock, I. Ahmed and M. J. Aslam, Phys. Lett. B **684**, 28 (2010) [arXiv:0911.2787 [hep-ph]].

[79] N. A. Tornqvist, Z. Phys. C **61**, 525 (1994) [hep-ph/9310247].

[80] M. Karliner and S. Nussinov, JHEP **1307**, 153 (2013) [arXiv:1304.0345 [hep-ph]].

[81] F. -K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D **88**, 054007 (2013) [arXiv:1303.6608 [hep-ph]].

[82] T. Mehen and J. W. Powell, Phys. Rev. D **84**, 114013 (2011) [arXiv:1109.3479 [hep-ph]].

[83] M. P. Valderrama, Phys. Rev. D **85**, 114037 (2012) [arXiv:1204.2400 [hep-ph]].

[84] W. -S. Hou, Phys. Rev. D **74**, 017504 (2006) [hep-ph/0606016].