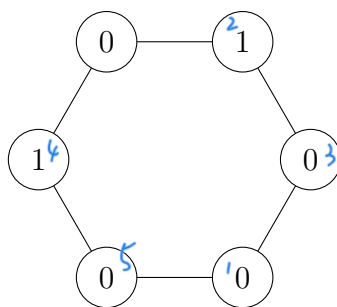


Assignment #3

This assignment is out of 30 possible marks with each question worth 5 marks. Your solutions will be graded based on their quality as well as their correctness.

1. For how many integers n with $1 \leq n \leq 2024$ is the expression $\frac{n(n+3)}{27}$ equal to an integer?
2. Ian and Mike play a game where they start with a pile of n stones where $1 \leq n \leq 100$. They take turns removing stones from the pile with Mike going first. On a turn, they are allowed to remove exactly 2, exactly 3, or exactly 4 stones from the pile. A player loses the game if they are unable to make a legal move. Determine all n for which Mike has a winning strategy.
3. How many times do the graphs of $y = \sin x$ and $y = \frac{x}{2024}$ intersect? The variable x is in degrees.
4. Sebastian flips a fair coin repeatedly until he either flips three heads in a row or two tails in a row. What is the probability that the last coin he flips is a head?
5. Emma begins by placing integers at the vertices of a regular hexagon as shown below.



She then performs a sequence of “moves” where a move consists of adding 1 to each of two adjacent vertices (“adjacent” means connected by an edge).

Is it possible for Emma to perform a sequence of moves that will cause all six integers to be equal? Justify your answer.

6. For a real number x , the *floor* of x , denoted $\lfloor x \rfloor$, is the largest integer that is less than or equal to x . For example, $\lfloor x \rfloor = x$ when x is an integer, $\lfloor -2.5 \rfloor = -3$, and $\lfloor \pi \rfloor = 3$.

Determine the number of integers n with $1 \leq n \leq 2024$ for which there exists a positive real number x such that $n = x \lfloor x \rfloor$.

Q1: 37

Q2: 1-100 except 1,6,7,12,13,18,19,24,25,29,30,34...84,85,90,91,96,97

Q3: 不会

Q4: 1/8, but not right answer

Q5: As diagram shows (btw I don't have any process)

Q6: Not started yet