The Distance between Forms and Experience in Mathematics

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Prelude

Why is mathematics so powerful in statistical analysis, physical applications, technical developments and also in many other areas? Where does its power come from? How are we, as human beings, connected to it? These three questions keep puzzling me. I have learned many techniques concerning the logic of geometrical proofs or principles of algebraic calculations, but I have never been told clearly about what its essence is, how it works, why it appears as it is in front of us and where it comes from and goes to - who is it? I wish to make these questions be my sword to break its shield. Let it be alive and less shrouded. Let me get closer to appreciate its beauty. This essay aims to develop a reasonable interpretation to respond to the above questions and to sketch a portrait of mathematics. I believe that the path which drives me comes from the view about the distance between forms and experience in mathematics itself.

Mathematics clearly is a very abstract subject, however, we can always solve complex problems by manipulations and the problems can be either practical or theoretical. Tiny daily tasks such as grocery shopping require elementary algebra. Complicated astronomical observations and predictions require advanced calculations. Look! It is everywhere in our lives, like parents who take care of her children all the time. Though it looks somber and silent; it works as a shadow, there is no way to deny its influence on us, and it certainly has a connection to us in our real living experience. On the other hand, in the historical development of quantum physics, we have seen its power in an imaginary or hypothetical world, where it is hard to verify our theories. Quantum physics is a realm under control of Mathematics and it performs as a king. In this corner, it is the stentorian leader that directs us to explore unknown secrets. In Rutherford's atomic model, mathematics is the light that warms us in the darkness of tiny but numerous particles. He helps to define the internal structure of atoms by taking account of possibilities. It, mathematics, wanders as a ghost in between the mountains and seas. It is transparent and mysterious. Who are you, Mathematics?

Part I Do you remember?

"... But how is it
That this lives in thy mind? What seest thou else
In the dark backward and abysm of time?
If thou remember'st aught ere thou cam'st here,
How thou cam'st here thou mayst.
But that I do not."

- *The Tempest*, Shakespeare[1]

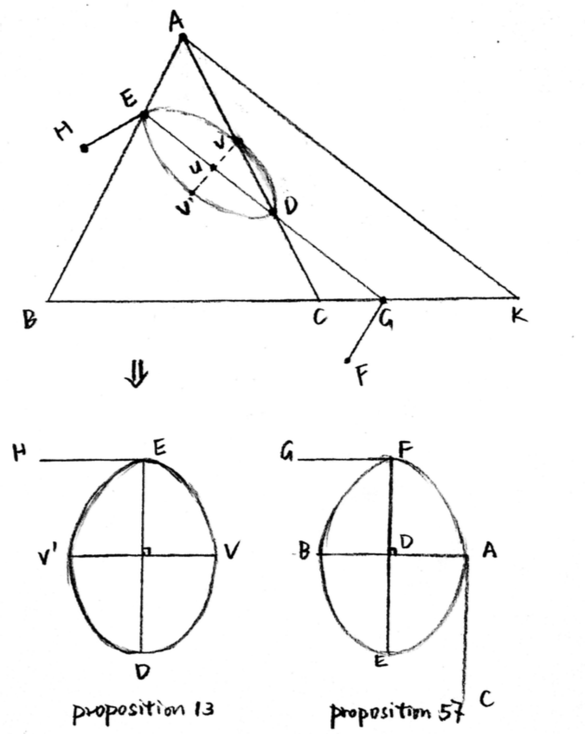
The *Elements* of Euclid has a great influence on mathematics for its introduction of fundamental rules in geometry. It starts with definitions of points and then builds a figurative world step by step, if we take the definitions, common notions and postulates as given. The book explores a perfect realm in that there is no bargaining and struggling in the succeeding proofs. Each proposition follows the previous one and brings to us a new perspective to enlarge our view. For example, the first few definitions move from a point to a line and to a surface; the propositions move from a simple triangle, a circle and a square to a polygon and a hexagon. The transition is smooth. Apollonius writes in the same style for *Conics*. Compared to Euclid, Apollonius focuses more on the features of curves and he introduces three different kinds of curves that were beyond Euclid's knowledge: parabola, hyperbola and ellipse.

[1] Shakespeare, *The Tempest,* Act I, Scene II, 48-52

Reading both Euclid and Apollonius, there is unclarity shared by them. Everyone is impressed by the contents of their books but it is unclear how they construct their entire systems. The source of their intuition is mysterious. Apollonius' *Conics* provides me a clue to understand their way of representation. Propositions 11, 12 and 13 are essential propositions and they show fundamental traits of parabola, hyperbola and ellipse. When I study them, I notice there is a condition that has been preset. For example, in Proposition 13, a condition is expressed as sq. AK : rect. BK,KC : : DE : EH (Figure below).[2] Similar ones are held in Propositions 11 and 12 too without information telling the source of this ratio nor how Apollonius discovers it. In my own investigation, I find that the conditional ratio is a necessary expansion of the later propositions, the Proposition 57 in the same book.[3] Following Proposition 57, the ratio sq. FE = rect. BA, AC, I deduced the ratio sq. AK : rect. BK,KC : : DE : EH. It confuses me about the order of propositions presented in the book. It seems like a circle to me that the conditional ratio helps to prove Propositions 11, 12, and 13, then, these three fundamental propositions are necessary preparations of the later proposition. However, the later propositions can show us how to deduce the primary ratio we have used in earlier three propositions.[4] This makes me wonder what is behind the order of propositions in this book.

	Proposition 13, Book I	Proposition 57, Book I
Transverse diameter	DE	AB
Upright side	HE	AC
Proportion in between	V'V	FE

[2] Apollonius of Perga, *Conics Books I - III*, Green Lion Press, 2010, page 25
[3] *Conics Books I - III*, page 109
[4] For Proposition 12, the ratio HF : FL : : Sq. AK : Rect. BK, BC can be deduced from Proposition 4, Book II, *Conics Books I - III*, page 22, page 121

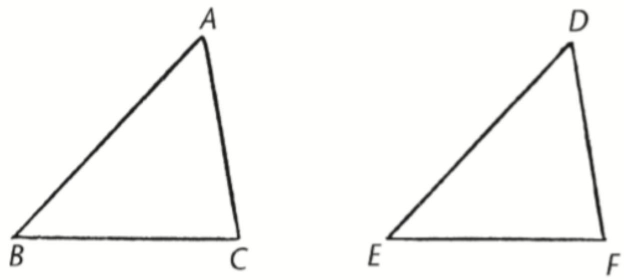


If the conditional ratio of the earlier propositions can be shown following the late propositions, then the argument follows that the order of the propositions in the writings is possible to be different from how the author discovers them. For Apollonius, it is possible that he primarily discovers the qualities of cone and curves separately, but later reorganizes his work to make it continuous and logical. If this is the case, the book certainly cannot show us through its content how the fundamental qualities behind the propositions are first reached. The propositions are fruits of thoughts which come after a complete construction. In such a process, the real approach of mathematical principles is hidden; we are in the middle of nowhere to know the source of early mathematics. The journey of finding their origin has to be halted under such conditions.

Besides the above example taken from Apollonius' propositions, it is also arguable about how to interpret the original Greek vocabulary of parabola, hyperbola and ellipse: παραβολή (beside + to throw), ὑπερβολή (above + to throw), ἔλλειψις (leaves out). In Apollonius' investigation, it is clear that he defines these three curves by different cuts. The shape of a curve depends on whether he cuts the cone parallel to the generator, above or below. The three prefixes παρα, ὑπερ and ἔλλειψ- indicate types of cuts - parallel, above and below - respectively; however, why Apollonius chooses the word βολή is unclear. Later on, from Galileo, we know that a parabola can certainly show us the trajectory of a ball which is thrown out. Are there any physical images of a thrown ball in Apollonius' mind while he analyzes these curves mathematically? No one knows but at least, the Greek words leave spaces for possibilities.

Not only Euclid and Apollonius, but also another Greek mathematician Archimedes writes propositions in a different way. From Archimedes, we can see more shadows of experience that has been mapped and reflected in the images of mathematics. In his letter to Eratosthenes, he claims that his observation of mechanics inspire him to invent a geometrical proof.[5] The proof is about the equality between a segment and the ¾ of the corresponding triangle. The center of weight plays an important role in the process and it is defined according to his physical observations and experiments, the real experience. This personal statement in the letter reveals a possibility that mechanics might inspire mathematicians to invent geometrical proofs. If this is acceptable, we have more reasons now to suspect that Euclid's and Apollonius' propositions come from their practical experience too, such as in observations.

[5] *Reading 4. Archimedes' Heuristic Method*, The Junior Mathematics Manual, 2014-2015 Edition, page 17



In fact, Euclid's own language conforms to this conjecture. One particular example can be found in Euclid's Proposition IV of Book I, *Elements*.[6] This proposition aims to prove that two triangles are equal. In the middle of the proof, we are asked to *apply* one side of the triangle ABC, AB, to another corresponding and equal side of the triangle DEF, DE, by placing one extremity of each, point A, to point D. Then, point B must *coincide* with point E. (Figure above) We can admit these two triangles are purely mathematical, but it is hard to categorize these two actions: *apply* and *coincide*. Can these two actions be well situated in a pure mathematical realm? It seems the interpretations of "applying" and "coinciding" are problematic in a pure geometrical proof. Because, to accomplish these two steps, at least an imitation of real human actions is needed. The thought experiment of imitating motions in the mind is built upon our experience. If we think the experience has got involved in a geometrical proof and it is inseparable from the proof, the mathematical judgment then is not independent from our experience. The choice of the words, *apply* and *coincide*, in this proposition reveals the secret that Euclid tries to hide from us.

Later, Descartes makes an argument in his book *Rules for the Direction of the Mind*, concerning the hidden methods of ancestors. Following Rule Four, in his

[6] *Euclid's Elements*, Green Lion Press, 2010, page 3

discussion, he says, "for the human mind has within it a sort of spark of the divine, in which the first seeds of useful ways of thinking are sown, seeds which, however neglected and stifled by studies which impede them, often bear fruit of their own accord."[7] The "neglected and stifled seeds" represent an origin of connection, and this connection is represented as a method of analysis. Following Descartes' argument, for analysis, there must be an object that can be analyzed. There must be an origin. I should ask what the object is and where we should start to analyze in this analysis. By asking a question in this way, we are already under the assumption that there is a something prior to pure forms. Aren't they our practical experience? The ancestors that Descartes speaks of, aren't they Euclid and Apollonius?

The mathematical forms are considered to be generated from experience, though I could not find a clear answer or persuadable evidence in the above-mentioned books to claim that mathematical forms and experience are closely bound together. A Chinese approach reveals a way of thinking about this issue more clearly. Different from western traditions of developing mathematics by pure forms, the Chinese version of Euclid, *Nine Chapters of the Mathematical Art*, clearly shows how experience and mathematical forms are closely connected to each other. The book is written in the style of proposing practical questions and responses to show how mathematics functions. For example, Chapter I concerns calculating the areas of farmlands. The first question asks how big the farmland is if this farmland is in a rectangle with 15 steps of width and 16 steps of length[8]. The

[7] *The Philosophical Writings of Descartes*, Volume I, Cambridge University Press, 1985, page 17 (373-374)

[8] 李继闵,*《九章算术》导读与译注*, 陕西科学技术出版社, 1998, page 229

Jimin Li, *Introductions and Commentary on Nine Chapters of the Mathematical Art*, Shanxi Science and Technology Press,1998, page 229

Translations of original context:

" Question 1: There is a farmland, the width is 15 steps and the length is 16 steps, how big is this farmland?

second question asks again about the area contained by a farmland but it changes the descriptions of width and length, which become 12 and 14 steps, respectively. This style of writing shows how simple multiplication matters in our real life and where we can apply the rules of multiplication. In this case, the mathematical form - rectangle - is made for helping farmers find a way to represent the size of their farmland. Once this rule is established, farmers and governments can make better arrangements on counting distribution, collecting taxes and assigning tasks. Though we know there is no farmland in the exact shape of a rectangle, the imaginary, absolute, perfect mathematical form of a rectangle overlaps with the real, approximate, defective farmland. These two rectangles definitely are not the same, but in the realm of our intelligence, they are.

Euclid has a similar idea in his Proposition 1 of Book II to provide a rule declaring the areas contained by rectangles.[9] Even Euclid passed over the step of calculating areas of rectangle. (Figure 4)In this proposition, Euclid is only concerned about the ideal form of rectangles and there is no connection to reality, such as farmland in the above-mentioned example. In Euclid's way of isolating geometrical proof from any relevant visual images in our life, mathematics, in the western tradition, becomes an independent subject, and this subject has the capacity to develop itself. Along with such independence, the question of origin has been buried underground and we are left with a unsolved puzzle.

Answer: 1 Chinese Acre
Question 2: There is a farmland, the width is 12 steps and the length is 14 steps, how big is this farmland?
Answer: 168 squared steps
The methods of counting area of farmland: Multiply the length and the width of the rectangle to know its squared steps." (1 Chinese Acre = 240 squared steps =15 steps * 16 steps, 1 step = 1.386 m, 1 squared step = 1.92 m^2)

[9] *Euclid's Elements*, page 37
Proposition 1, Book II: If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.

Plato proposes a possibility for the origin of forms that contrast to my hypothesis. In Plato's *Republic*, Book VI, Plato suggests that forms are beyond images.[10] Geometrical images are only representations of perfect and ideal forms, and they are unreal, because images in life are not ideal but approximate. Notwithstanding, through them, we have access to real perfection, which is reality to Plato. He believes there is an ideal, absolute and perfect world beyond our knowledge which houses forms and therefore the forms are real. This argument of reality and the realm of forms is a hypothesis or a belief that cannot be proved mathematically, nor be observed. From such an unprovable perspective of ideality and perfection, Plato claims that the forms are prior to any corresponding physical representations; however, the ideality cannot guarantee a priority. Setting the realm of form as the origin of forms is questionable.

The supporters of Plato might defend the priority of forms because the realm of forms can cross the boundary of experience. There are some parts of forms that no experience fits. In Plato's theory, the realm of forms definitely is larger than the realm of experience. While there are forms beyond experience, it does not necessary mean that the realm of forms contains the realm of experience. It also could be the case that the realm of forms and the realm of experience intersect each other. From both possibilities, there could be found the phenomenon that a form goes beyond experience. If we think that the genesis of mathematics comes from experience, we can also arrive at the same intersection where forms contain a larger view than experience. Because the genesis cannot be understood as restrictive, but potent, like Descartes' words "seeds" and "fruits" in his earlier mentioned-quotations. We can use this metaphor to explain that forms are like seeds and we find them from experience, but the growth of seeds fosters

[10] *The Republic of Plato*, Tr. Allan Bloom, Basic Books, 1968, page 191 (520-511)

independence and their fruits do not rely on experience anymore. In such a way, there are still wide spaces for forms to extend themselves in all different directions. There are possibilities that forms, as the fruits of themselves, share a different realm with experience and even create a new realm that experience contains.

Another hypothesis, which is bound together with the priority of form, is the recollection theory that Socrates shows with a slave boy in *Meno*. In explaining the slave boy's learning process of squares to Meno, Socrates argues that the forms have been buried in our minds before birth and they can be dug out again and again with the aid from recollection in our memories. This is how Plato and Socrates believe where the intuition of mathematics comes from and how mathematicians establish a connection, such as between a circle and the sun, in their minds primarily; however, such a recollection theory lacks explanations for how the first memory is stored and how the first understanding of forms are built. Plato's recollection theory is not convincing to me.

Though we cannot assert a path to mathematics now, leaving the possibility of abstracting mathematics from experience will be helpful later in arguing how mathematics applies its pure forms in our real life. Now, If I wish to develop my hypothesis of the genesis of mathematics further, I must consider the following questions: In detail, how does form develops independently in mathematics? What is the relationship between form and experience in the growth of mathematics? How does this genesis hypothesis explain the application of mathematics in reality?

Part II Self-growth and achievements

"As for his present loneliness and suffering,
this, too, no doubt is part of the God's plan
that he may not bend against Troy
the divine invincible bow
until the time shall be fulfilled, at which it is decreed,
that Troy, as they say, shall fall to that bow. "

- *Philoctete*, Sophocles[11]

In the last part, where the form is considered to be an extraction from geometrical figures in mathematics, it seems that form and mathematics are fairly equivalent to each other with treating geometry as the common ground. In fact, such an image is a temporary one and it only happens in the primary stage of the development of mathematics. Though astronomy is a different subject from mathematics because of the necessity of the knowledge about phenomena, the temporary image of equivalence holds in early astronomy in that we see astronomical theories shown through forms.

Ptolemy's form of planet motion is called eccentric circle and epicycle. With these two hypotheses, Ptolemy attempts to explain astronomical phenomena in mathematics. Here are two problems that should be noticed: why can mathematics can lead us to astronomical phenomena? how has the mathematical model been established? We will listen to mathematics and let it speak for itself and leave the question of origin on the side.

The first problem is an extension of the overlapping rectangles from our earlier discussion. From intelligence, the rules built on imaginary and perfect rectangles can be applied to real but approximate farmland. There is less difficulty to understand this

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[11] *Sophocles II: Ajax, The Women of Trachis, Electra & Philoctete*, Tr. Michael Jameson, John Moore, University of Chicago Press, 1969, page 203 (This quotation is dedicated to astronomers and mathematicians, especially for who suffered in history, such as Nicolaus Copernicus, Galileo Galilei, etc.)

overlap because both rectangles are stationary. When we now introduce mathematical models in astronomical phenomena, we start to consider motions. Because astronomy requires a joint vision of stars in its analysis to see motion, in our model, the form is still stationary but we think it can lead to motion. Besides, we need to think how circles represent planets like the rectangle and farmland, we also need to question how the composition of circles leads to the motion of planets. In other words, we are questioning how the application of mathematics leads to experience.

Before building a path from mathematics to astronomical phenomena in our intelligence, there should be a mathematical model allowing the path to be set. The construction of this mathematical model is also worth attention, because in this model, mathematics mostly develops by itself and only with limited reference to phenomena. The independent and self-growth mathematical model should be looked over carefully. Why, in the construction of this model, don't we need references from experience? Let us evaluate this problem and see the self-construction of the mathematical model first.

In Ptolemy's model of eccentric circles and epicycles, he is affected by Aristotle and he takes Aristotle's assumption that the planet motion has to be uniform and circular. Following this, Ptolemy attempts to combines multiple perfect circles to explain his hypothesis with the aid of *deferent* and *equant*, because he notices that a single simple circle cannot explain observations very well. Ptolemy also is a supporter of the geocentric model. Both of his hypotheses, eccentric circle and epicycle are established and based on this belief. Therefore, as a result, we see two complex models have been established and they are expressed by multiple circles; they also support the same belief in the geocentric model. After the primary set-up in the model that corresponds to the geocentric view of the universe and the circular motion of planets, there is no more reference to experience. The follow-up arguments of moving circles are purely mathematical because they are built solely on geometry and calculations. For this reason, I call these two models independent and self-growth mathematical models.

To understand the independent development of mathematical models, we might firstly see a general image of mathematics and experience. If there is an image that contains both of them, they should be set up like two parallel lines. Unlike Euclid's parallel line on a flat surface where two lines cannot meet, it is more like Leibniz's infinite case in that two lines are parallel but they meet at an infinitely far place where is the origin of mathematics. When we take a look at a mathematical model, besides the original cross with experience, the line that model falls on can be treated separately as parallel to the line of experience. In this line of the model, the self-contained logics push it to extend itself. In Ptolemy's case, the logic fits in between the abstract representation of the primary settings and the desired representations, which are corresponding to an astronomical phenomenon. It likes the logic that flows in Euclid's proposition, plays in between the given and the conclusion, but also is different from Euclid in that there is a clear and determined destination for logic to arrive after it departs from the primary settings. For Euclid, we can look for a certain conclusion by using reasoning, there is another possibility that we can play with the givens and run reasoning in different directions until we arrive at somewhere worth it to make an annunciation. Astronomy, the phenomenon-oriented case desires reasonable logic to finally arrive at a determined claim. The role of logic is the same for both cases in that it builds bridges for us to see unclear connections, either among geometrical qualities or apparent phenomena. In the transition, logic is the controller, on the one hand because of its self-sufficiency, and on the other hand, because we might not be able to experience or even notice these possible behind-the-scene reasonings in reality that only exists in our intelligence. Under such condition, there is no need to refer to experience in the flowing of logic. By self-sufficiency, I mean that logic does not need external confirmation to claim its validity and it always follows the prior established rules. The rules are set originally in agreements, so that it necessarily leads to a certainty while we put multiple rules together.

In Ptolemy's two independent models, we see each hypothesis develops itself while following rigid mathematical rules; however, what is more interesting is that these two models both explain the same astronomical phenomena as near as possible. In other words, Ptolemy proposes two distinct hypotheses, which are built on mathematics itself from two different logical lines, and they give the same desired ending - the astronomical phenomenon. Meanwhile, it is impossible to make a judgement on which one is better from the perspective of mathematics itself. For both hypotheses, no logical or arithmetic errors could be located, then how can we pick the correct understanding of astronomical phenomena? If the self-development of mathematics can follow its rigid logical rules but does not assure us of a corresponding true reality, just as a sentence that is correct in grammar but not saying the truth, then the validity of mathematics itself cannot help us make a judgement on the correctness of understanding a realistic phenomenon. The independent development of mathematics cannot give us a true picture of universe. What, then, is the meaning of the independence of mathematics? This analysis of astronomical phenomena also shows the difficulty of defining the path from mathematics to experience based on the self-growth of mathematics. There are connections between mathematics and experience but meanwhile mathematics has its own individuality.

For the image of parallel lines to show the connection between mathematics and experience, if I turn it to being physical and practical, the line of mathematics would be much thinner and brighter, but the line of experience, thicker and mottled. It is because the real phenomena contain more possibilities than what mathematics selects to show. In our primary settings of mathematics, the “seeds” have already experienced a selection. Not everything in experience could be generated as being our seeds for mathematics. In this selection, the frequency of appearances, the level of clarity and accessity and other factors have been considered. The extraction of seeds filters the complexity of phenomena. The independency of the mathematics models, which are built upon those selected seeds, is not necessary to lead to a certainty in phenomena. But it always helps us to understand the possibilities behind apparent phenomena. The independent development of mathematical models shows the attempt human beings made by their intelligence to get closer to reality. Therefore, when we map two rectangles, map the star and circles, we are mapping an imaginary form and the real phenomenon in our intelligence. Neither are we transiting forms into experience nor are we claiming the experience certainly is the form. Mathematics and experience occasionally are considered to be each other, only in the realm of our intelligence. From this perspective, the transition from stationary form into a movable one becomes possible in my imaginations. It is our ability of imagination to draw the connection between the stationary and movable. Our imaginations make the imperfect shape of farmland to be a perfect rectangle and our imaginations allow stationary forms to be movable.

In the parallel image, the logic of mathematics has been developed independently but also under the supervision of experience. The predetermined destination supervises the direction of logic and the flowing of logic, to different degrees, reflects experience. The mapping process is implied under the metaphor of parallel, and further, in this reflective mapping process, the reduction aims to find one form to reveal reality because there is only one nature. No matter how complex of an experience, there is one experience. Though both of Ptolemy's models can possibly explain the phenomena, we still desire a way to determine which one is more possible. His difficulty of selecting one out of two represents a deficiency in his understanding of mathematics and this already predicts the failure of his explanation. Later, Copernicus responds and makes a strong argument to explain the missing consideration in Ptolemy's arguments.

Copernicus argues from a perspective of simplicity in his book *On the Revolutions of Heavenly Spheres* and helps us to make judgements solely on the independent mathematics. In Copernicus' theory, he uses quasi-circle instead of perfect circles because he no more fully believes in Aristotle's assumption of perfect planet motion. He is distinct from Ptolemy also because his theory supports heliocentrism. Since they have very different views, Copernicus establishes a very different model of the universe from Ptolemy, though their theories are all built upon the observations of Theon and this means there is no difference for the precision of their data. Then, the core puzzle here is how Copernicus claims the advantage of his model and let readers trust him. In his book, he makes a comparison of Ptolemy's model and his. He uses two diagrams, corresponding to each theory, to explain why the same phenomenon can be interpreted both ways.[12] He claims that his model is simpler, which fits nature better. The assumption of quasi-circle will hugely reduce the work of calculation based on multiple superpositions of perfect circles. This argument seems to represent our preference, but how can this explain the preference of the universe?

Nonetheless, Copernicus is not the only mathematician who thinks this way. Newton, in *Principia*, Book III, Rule 1, argues "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances".[13] They both think that the simplicity of mathematics itself implies this preference among the different models. This is an assumption built upon a belief on the beauty of mathematics. The simpler, the more beautiful, the more real. I attempt to understand their beliefs on the simplicity of nature and I think the following argument can provide further support.

In my parallel image between mathematics and experience, there is a fact underlying the reduction that form, as a reduced product, can only reveal a possibility among many, which is contained by experience. Form, because of its own quality of absolute, perfect and imaginary, only shows us an ideal situation for things to happen. In reality, there is always a high chance for things to happen differently than our expected ideals because of the involvement of uncounted factors. If we want to have a better prediction of the phenomena, then it is necessary to leave enough space for uncounted factors to be counted in reality. The preferred model is to be as simple, general, common as it can be. Probability theory can help us understand that the more complex the model is, the least probability that the phenomena may appear. For example, let there be an

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[12] Nicolaus Copernicus, *On the Revolutions of Heavenly Spheres*, Prometheus Books, 1995, page 23
[13] Isaac Newton,*The Principia*, Prometheus Books, 1995, page 320

event A and let it be composed of many parts, such as a,b,c,d,e. In its composition process, there is a chance for each part to happen in reality, just as forms suggested, no matter how small or big this chance is, the probability of the whole event A to happen is the product under the multiplication of each factor, counted as a*b*c*d*e. The more complex the model is, either suggests the chance of each individual part will be smaller, or the whole event A should be dissected into more parts, like f, g et etc, and the probability of an event will be more folded into a complex model. If this is the case, the chance of that happening will decrease exponentially with the complexity of the model. The simplicity of the model implies a preference which corresponds to experience. Besides the above speculation, the later discovery of the principle of least action also shows that nature follows the most efficient path to express itself that a body's actual path always cost itself the least amount of action.

If the preference of simplicity is given to forms, but the experience is really complex, does the simplicity of the form and the complexity of experience contrast with one another? No. Simplicity is a reduced complexity, and it is sketched after grasping the essence of complexity. Picking variables and drawing models from complex experience depends on human intelligence of how they look at a phenomenon and what parts they think can be counted as reasons for that phenomenon. Simplicity is considered to be a product of thoughts and a representation of human intelligence, of how we understand the essence of the phenomenon. Form itself does not change, but our selections of models evolve along with our understanding of the universe. The historical development of astronomy in earlier discussions shows this tendency.

Through overviewing the early astronomical examples, though we try not to ask the origin of mathematics, nonetheless, its capacity and the connection to experience is still puzzling. Human intelligence plays a great role in the performance of mathematics.

Part III A new habitation

"...And are we in doubt to extend our manhood
Thus far with our deeds? Or do we fear to place feet
On the soil of Ausonia?"

"Then come, and at first light of sun let us search out, delighted,
What places these are, what men, and where is the city
These people defend. Let us scatter in different directions
Away from the harbor."

- *Aeneid*, Virgil[14]

Remember the terms, form and mathematics, are not the same, though they have been used closely in Greek times, when mathematics appears to be more or less pure forms. After experiencing a long tradition of equating forms and mathematics, particularly in geometry, Newton shows the possibility that mathematics stands in a different place from forms. Before him, mathematics was more inclined to be situated in the realm of forms; but with the invention of calculus, Newton shows in his book *Principia* how the role of mathematics swings in between the geometrical forms and real experience. This change in the position of mathematics leads to new discoveries and new connections between mathematics and experience. Let us take Proposition I as our example to see how he takes mathematics out of the realm of pure form and places it in between the forms and experience. [15]

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[14] Virgil, *The Aeneid*, Tr. L.R. Lind, Indiana University Press, page 125 (Book VI, 813-815); page 132 (Book VII, 140-143)
[15] *The Principia*, page 39

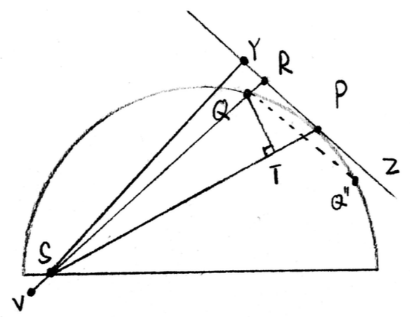
Newton's Proposition 1 follows his first law[16], which suggests that the motion in a rectilinear path is primary. The body will always move in a rectilinear motion unless there are other forces compelling it. Newton certainly is going to make his own model that does not follow Ptolemy and Copernicus. He shows that the curvilinear figures can be understood by setting up infinite numbers of rectilinear figures in their ultimate cases. The geometrical construction of the polygon leads to the curve, which we aim to understand. In the ultimate case, the polygon is the circle. It is worth attention that Newton is not the first mathematician to claim the relationship between a rectilinear figure and curvilinear figure . In Greek time, Archimedes used the method of exhaustion to show the ratio between a triangle and its corresponding parabola. The proof is a reductio that he claims neither it can be less nor more that the parabola has to be ¾ of the corresponding triangle.[17] Newton has proved a relationship between rectilinear figure (polygon) and curvilinear figure (circle) in a different way, which is a positive proof. In proposition I, he really shows us how polygon is the circle. This implies that Newton believes that the primary rectilinear motions are not just a way to explain the phenomenon, but it is what really happened.

Proposition 1 - The areas, which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described.

[16] *The Principia*, page 19

Law 1 - Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

[17] *Reading 3: Archimedes: Quadrature of the Parabola,* The Junior Mathematics Manual, Santa Fe 2014-2015 Edition, page 15



Another evidence can be found in his later Proposition 6[18]. In Proposition 6, Newton aims to explain the instantaneous motion of the body at P (Figure Above). But the next moment Q is set and the previous moment (Q") is implied in the construction of sagitta. Though we don't see rectilinear motions very clearly, QR is set to be equal to the sagitta of the arc from the previous moment to the next moment Q with setting current moment P in the middle. Newton is not using rectilinear motion to explain curvilinear appearances. He actually believes that the universe runs in such a way.

This belief situates him in a very unique place, that for him, mathematics is not only pure abstracted forms, but motions in transition. Compared to Ptolemy's model, which treats motion as grown, Newton tries to reveal motion more in details through forms. Following this logic, the self-development of mathematics now is actually corresponding to some middle and discontinuous steps that happen in nature. Is this true? There are questions underlying his proof: he assumes that the rectilinear motion has a distinct nature from curvilinear motions. By composing multiple rectilinear motions, we can approach a curvilinear motion. In other words, the accumulations of one kind of magnitude can cause the change its quality.

[18] *The Principia*, page 46

At first, I cannot be fully persuaded by Newton's proposition, though he introduces the term *ultimate equality* earlier in his Lemmas[19]. Particularly, because he situates the Proposition 1 in between the geometrical mathematics and practical physics. It is ambiguous to me whether we should be persuaded by a geometrical proof for a practical conclusion. I call the conclusion practical because the purpose of this proposition is to prove the characteristics of centripetal force, which can either be physical or mathematical.

Later I am aware that my uncomfortableness comes from an old habit, in which I used to think that mathematics is stationary and is closer to forms. Newton's proposition is new in that he tries to push mathematics to depart from forms and to get closer to practical experience. Mathematics is still based on logical reasoning, but mathematics is also new in the way that it tries to break our old impressions and resides in a new place, a place more shadowed by physics and bound to mechanics.

The characteristics of physics and mechanics can be investigated further by understanding Newton's new conception of the *body* (corpus) in his propositions. It is clear that Newton works on motion and there is always a body. This is not unfamiliar to us because Galileo discussed body and motions before him. But different from Galileo's body, which is a pure abstraction of an object, Newton adds on the practical trait that the body holds force in itself. Previously, when we overlap a perfect rectangle and the farmland in our minds, we mainly consider the analogy in between their shapes, as does Ptolemy's planet and circle. Then, in Galileo's body, we extract objects and neglect

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[19] *The Principia*, page 31
Lemma 1 - Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer the one to the other than by any given difference, become ultimately equal.

qualities of objects. Body is more like an abstract representation, our intelligence assumes and also creates their connections in order to understand experience by forms. For Newton, when the conception of the body functions in our mind, it carries the quality that closely connects to practical experience. Our intelligence is not the only bridge between forms and experience. The conception of the body opens a new access to link them. Newton clearly intends to move mathematics into a different position, which is in between form and experience. Mathematics transforms itself from simple forms, which are mostly represented by shapes, to abstract conceptions that rely on forms but expresses itself through physical phenomena. For example, the mathematical force is no more built on a single shape, and in fact, the shape of it is less important than the understanding of it. When form starts to mix with experience, mathematics becomes complex too.

Not only does Newton's proposition in *Principia* brings mathematics to a new place, it also provides a new view on two common methods of reasoning - deduction and induction. Newton's propositions are made by a mix of the two. On the one hand, he sets up laws and definitions at the beginning of all his work. On the other hand, in his propositions, he starts from phenomena and attempts to provide readers a possible explanation, which is built on his previous laws. There are dangers underlying both two methods and they reveal another trait of mathematics about possibility and certainty.

For deduction and induction, the former usually starts from laws or principles and then goes through logical steps to a destination that we go from our imaginary forms to experience; the latter does the opposite, in that it starts from an observation or perception and then goes backwards to a beginning point, which is more like a path made from phenomenon to forms. The difficulty of deduction is about the presupposed laws and principles. Logically, we can assure that each of our steps in the middle is correct, but it does not mean we will arrive at the desired outcome, because we might pick the wrong origin. For induction, we find that there is one path closely connected to our phenomena, but it could be one of many possible paths. We are unclear about whether this is the one that really happens in nature. These two methods in fact show us the limitation in the connection between experience and forms. Deduction requires a better knowledge of the primary form in order to develop, and induction requires more precise data for the phenomena in order to establish a better model. Their concentrations are obviously different: deduction focuses on the origin, the cause, and aims to an end with what the phenomenon is. Induction requires more effort to prove that the way it proposed is the right one. The danger here is that we might mix *what really happened* and *the way it possibly happened.* Thus not only mathematical proofs cannot assure true understanding of realistic phenomenon, but neither can they assure the certainty of methods, because of the possibility underlying mathematical proofs. We can always admit its mathematical validity, but we cannot be sure that this is really what happens in reality. The mathematics here creates artificial experience to help us establish connections between what we can understand and what we really see. I name artificial experience as the product of human intelligence to build up relationships in between pure forms and experience through mathematics, particularly in the direction from forms to experience. It is not necessary to be the same as real experience; instead, it represents a possibility of real experience and represents how we possibly think about the real experience.

The above argument will also help us understand the mystery behind Ptolemy and Copernicus from our earlier discussion about how each connection has been established and then relinquished. Ptolemy assumes circular motion is inherent in bodies and it is the primary motion. From his theory, we can explain the phenomena relevant to the sun well, though, for every hundred years, we notice there is a deviation in the calculation. Ptolemy's geometrical proof and data supports his idea that the earth is situated at the center of the universe. Copernicus later makes a different argument that the earth moves around the sun. Mathematics again becomes his weapon and shield to support his idea. Then, Kepler, Newton, Einstein and many other scientists, keep refreshing the image of the universe. The reason behind this cycle of refreshment is that the product of mathematics is an artificial experience, not exactly the same as real experience.

The meaning of the independence of mathematics has also been partly discussed. To add, the independence of mathematics allows and also pushes itself to create an artificial experience that is analogous to real experience. In the original imaginary realm of forms, it can show the possibility for things to happen in a particular way. Then, our intelligence can understand it and there is high chance that we even take it as truth. But it should be distinguished that artificial experience follows the independent development of mathematics and shows an understandable path to explain phenomena, but it might be the same as real experience or not.

Though I use the same term, artificial experience, to describe the achievement of mathematics in general, there are differences between early and modern mathematics. For example, Archimedes and Newton both talk about mechanics, but their propositions reveal reality in different ways. Archimedes' geometrical proof is about the relation between already existing figures, which we can also call *ready made*; Newton attempts to reveal the underlying process, establishing a relation, which is *being made*. In Newton's proposition, the ultimate case always suggests a motion underlying the geometrical diagram. Archimedes stops at stationary forms and does not move as far as Newton.

This difference also inspires me to think about how to understand truth and forms. We start from explaining things by stationary figures, like Archimedes, because they have easier access for us. The preference depends on our intelligence and it does not suggest that eternal truth lies down in forms. The big disadvantage of assuming the primary and essential object is stationary has already been revealed to us in the historical development of astronomy, that the forms are always in a changing process and we need to refresh our understanding very frequently.

Further, the frequent change of astronomical theory seems to contradict the stability of mathematics, but it does not. Though mathematics is different from other subjects, as Descartes suggested, which are made by conjecture or lack of solid ground, each step in mathematics is certain and clear by logic. When we say the stability and certainty, we are talking about the logical essence and reasoning which reside in mathematics. But the breakthrough is also found at the two extremities, either on the new abstraction for forms or the new data gained from experience. This is true not only in astronomy, but also in pure mathematics, physics or even chemistry. For example, the Non-Euclidean geometry challenges Euclid's fifth common notion of Book I in *Elements* to develop a new type of geometry. Mathematics itself is still stable. We just have a new understanding on the same theory by knowing how to place an old theory in our experience. We always look for a mathematical model that can be placed closer to experience. The essential logic does not change but our desire and knowledge grows. We should leave spaces and be patient while applying it to reality, for the certainty of mathematics only improves the chance of it to possibly explain the phenomena but it cannot cross the distance between it and experience, though mathematics is rigid about the certainty of itself. An open mind to embrace new possible explanations due to the development of mathematics is needed.

Till now, though most of my examples are taken from geometry, and algebra has been developed especially after the invention of calculus, I think geometry can represent mathematics because their essence is the same - reasonings and logics. In the early establishment of algebra, we can see a clearer connection between algebra and geometry from Descartes' writing *La Geometrie* about simple algebra moves, like multiplication and division.[20] The separation between geometry and algebra gets extended now and the connection between them becomes less evident and less visible. They contain the same essence and they should take equal weight in mathematics, just with different advantages. We have easier access through geometry because of its figurative characteristics. Algebra has an advantage on expressing relations or phenomena in a more complexed level. Algebraic notations express abstract conceptions easier. Another excellence of Newton is that he succeed to show the interaction between forms and experience in geometrical figures, and also express abstract conceptions, like the example we discussed regarding *body*. In the next section, I will use examples expressed in algebra.

[20] *The Geometry of René Descartes*, The Open Court Publishing Company, 1925, page 2

Part IV Finding a way back home

"Light of my days, Telémakhos,
You made it back! When you took ship for Pylos
I never thought to see you here again.
Come in, dear child, and let me feast my eyes;
Here you are, home from the distant places!"

- *Odyssey*, Homer[21]

After Newton finds a new habitation for mathematics in between forms and experience, mathematics is pushed further, closer to experience, especially in the study of mathematical-physics. For example, there was a beautiful encounter between Maxwell and Faraday. Maxwell's mathematical equations function as representations for what Faraday has discovered in his experiments but unable to express due to his limited knowledge of mathematics. Through Maxwell's equations, along with the corresponding Faraday experiments, we certainly see that there is a path that leads us from mathematics to experience. But can mathematics fully express experience? In the same example, Maxwell's equation, it is questionable because of the asymmetry of the third and the fourth equation, which is not well explained until Einstein's theory of relativity.[22] But after the theory of relativity, which relates all different phenomena to their conditions, people have a strong belief in the power of mathematics. It seems that experience can fully resolve itself in advanced mathematics. Is this true?

We now accept the fact that the certainty of mathematics itself does not lead to the certainty of phenomena, and we have seen from examples in astronomy that our understanding of the universe is continually refreshed by correct math. How can it be the case that mathematics expresses experience fully? If not, to what degree can mathematics

[21] Homer, *The Odyssey*, Farrar, Straus and Giroux, Inc., 1998, page 290 (Book XVI, 30-38)
[22] *The Principle of Relativity*, Dover Publications, Inc., 1923, page 48

help us build our understanding of reality? This section aims to evaluate the power and also limitations of mathematics and ask how far we should expect mathematics to reach real experience. We will firstly take Minkowski and Bergson's argument as our example. In their argument, Minkowski has as strong a belief as Einstein in the power of mathematics. He seems to believe there is no limitation to mathematics being applied in reality and our understanding of reality grows together with our knowledge of mathematics. On the other hand, Bergson thinks there are limitations to mathematics, and this limitation is primarily contained in mathematics, which should be taken into account. Because of such limitations, we also need to be careful while applying mathematics to reality.

Minkowski and Bergson's argument mainly relies on one equation: $ds^2 = dx^2 + dy^2 + dz^2 - cdt^2$, and this equation can be rewritten in the following form: $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ while let $x_1= x$, $x_2=y$, $x_3=z$ and $x_4 = ict$. It can be found in Minkowski's paper *Space and Time*.[23] Einstein also uses this equation in his book *Relativity: The Special and the General Theory*.[24] They both think there is no problem in the equation because it is simply a substitution of x to *i*ct. However, Bergson has a different opinion about the validity of this step: he agrees that this step is technically correct but mathematics now tries to achieve a reality beyond its ability. Bergson's query arises from the substitution of x for *ict*, which both Einstein and Minkowski think is simple. In fact, most scientists say that the substitution is a really smart mathematical move. They think that the form of parallel addition indicates a way of counting distance. By substitution, we find a way to take account of time in space. This is also Minkowski's purpose, that

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[23] *The Principle of Relativity*, page 88
[24] Albert Einstein, *Relativity: The Special and the General Theory,*Three Rivers Press, 1961, page 102

the substitution will allow him to unify space and time, which helps him to identify his four-dimensional absolute world. On the contrary, Bergson thinks the ability of counting distance, what this equation has gained from the rewritten action, should be approached with caution, because the conceptual time in the original equation cannot be transformed and counted as distance. Their different understanding of time leads to such a divergence. To understand this argument, it is necessary to look at Bergson's understanding of the nature of time, which is discussed in his book *Duration and Simultaneity*.

"There is no doubt but that for us time is at first identical with the continuity of our inner life."[25] For Bergson, time is necessary to be bound to our consciousness and he uses the example of melody to help us understand its continuity. "A melody to which we listen with our eyes closed, heeding it alone, comes close to coinciding with this time which is the very fluidity of our inner life."[26] From the analogy between melody and time, we take time as a continuous duration in the experience, which coincides with our inner life. Following this argument, cutting time into seconds is inappropriate in that time cannot be equally distributed in experience. Parts in time cannot be duplicated and there is no unit underlying it. Because time is a continuous entity, like a melody that only exists with a full collection of every note. We cannot take a note as a unit of melody. The division behind notes is only a distribution of time on keys, but this is not the whole meaning that notes carry. Each note is unique because it has to follow the preceding key and it predicts the upcoming key. There are rules that limit the options of notes. The distribution of each key can be measured and cut, but the consistency and the continuity inside the melody cannot be cut. An incomplete melody is not the melody. For time, we

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25 Henri Bergson, *Duration and Simultaneity*, The Bobbs-Merrill Company, Inc.,1965, page 44
26 *Duration and Simultaneity*, page 44

can assign a mathematical unit for our convenience to count it, but there is a uniqueness underlying each unit which is bound to our consciousness in ongoing experience. This uniqueness is also the essential quality of time, which cannot be quantified.

The quality of time also leads Bergson to hold his objection to Einstein's theory of relativity. Bergson calls it plurality of time. According to the theory of relativity, we can convert different experiential expressions for the same body, particularly for the stationary one and moving one. Following the transformation equations to understand the correctness of all different visions , consequently, there is an objective world that each vision has equal weight, which is also what Minkowski declares in his system. In Minkowski's construction of his absolute world, there is a t-axis that changes with x-axis, depending on the hyperbola. For each t-axis, it is just a conversion. The existence of multiple and possible t-axes corresponds to the plurality of time that appears in the theory of relativity. Bergson thinks that there is only one real vision among all these "phantasmal visions"[27] because there is only one consciousness for each person that allows them to make judgements about the time, which is solely associated to the self.

From the above discussion, we can see that time for Einstein and Minkowski is universal and mathematical, and time for Bergson is personal and experiential. In fact, the conversion and the measurement of time in mathematics implies that time itself has an absolute meaning and it is independent from us. Bergson thinks that time is inseparable from us and thus the conversion and measurement is meaningless. If there is a debate between them, Minkowski would defend himself that time flows regardless of the individual. Bergson would say time disappears with a person's consciousness. He also would say Minkowski's time is artificial because universal time excludes the uniqueness

[27] *Duration and Simultaneity*, page 127

of a personal experience, and, thus, it is not real time for an individual. Clearly, such a different understanding of time leads to the disagreement on the substitution of x for *i*ct. The consequence of this substitution is that Minkowski aims to derive a conclusion to apply to experiential time and to human beings. He wishes to define a new physical entity in reality, space-time, which as he claims at the beginning of his paper, "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."[28] Mathematical time can be unified with space, but not the experiential time. There is a quality of time that cannot be expressed mathematically. In this example, we see a boundary between the mathematical world and the experiential world.

Bergson's argument is not only challenging Minkowski and Einstein, but he also questions the tradition of mathematics to that measures time and takes account of time in physical phenomena. Since Descartes creates coordinate systems, time is usually assigned to an axis and we have units to measure time. But before Minkowski's new conception of space-time, time maintains its individuality. For example, in the physical quantity, velocity, we mark it as distance/time, such as miles/hour, in which time is independent. When we calculate velocity, we do not focus on the variables that might change along with time, such as emotions or consciousness. We focus on its quantitative characteristics and disregard all others. Because of the independence of time, the quality of experiential time is hidden but does not disappear, and mathematical time functions in the same way as experiential time. Bergson's theory does not say that Descartes and other previous mathematicians are wrong about mathematical time, but he warns to notice that the experiential time is different, though the experiential one can sometimes be represented

[28] *The Principle of Relativity*, page 75

mathematically. The difference among them become serious in Minkowski's case because Minkowski wants to deprive time's individuality to claim his new conception of space-time. The loss of individuality for time eliminates the least possibility for it to be considered experiential.

If we take Bergson's argument of experiential time, it also helps to understand the distance between forms and experience. Time definitely performs as one of the most important factors that distinguishes form from experience. Time itself only focuses on the present. For the past, it is not time but memory; for the future, it is not time but the intelligent activity of imagination. There is no distinction in itself of among past, present or future, it is only what is happening now, with connections to parts that we call the past and the future. Experience is attached to time and it takes time to complete. It is continuous, but for our purposes, we cut it at certain points to create pieces and allow us to analyze them. We can experience, but we cannot completely analyze experience. In analysis, we have to cut experience into pieces (phenomena) and only pick parts from it to start. There is no beginning or ending for real experience in nature. But artificially, in our analysis, there has to be a beginning and ending to set up boundaries. These limited ones are phenomena, which the influence from what happened earlier is automatically eliminated and we take the assumption that it is not important and can be left out. However, phenomena can never replace experience. Experience itself is vivid but we try to take out its continuity and changeability and then to stabilize and formalize it in forms. As such, they become stable and discontinuous. Then we give hope to our ability to make forms be continuous in reality, but they can only be continuous through human intelligent activities and under an assumption of artificial time. This leads to the artificial experience that I discussed earlier. Therefore, how we account for experience at its beginning predicts the difficulty with which we will meet it. In our initial selection, it is hard to judge what roles each tiny part plays in the experience. When we assume the importance of a selected variable, the importance is claimed by humans, but the importance is not up to us in reality.

What is more, experience for us only runs in one direction, but in the independent development of forms, it can go any direction, forwards and backwards, as long as the reasoning and logic performs well. Taking out of experiential time in forms also leads to a pursuit of eternity in forms. We should notice that this eternity is gained by the sacrifice of not being able to change. However, the changeability and the possibility of being varied are also crucial in experience. From this view, mathematics can never be placed exactly on the position of experience, because of the loss due to the disability of expressing experiential time in mathematical forms.

From the above example concerning time, we infer that there is a difference between a mathematical entity and an experiential entity. There can be more entities like time, which cannot be fully expressed in a quantitative way. The power of mathematics will be limited because of this difference. This consequential limitation will also lead to difficulties of *restoring* mathematics in experience. I coined the term *restoration* because the process from mathematics to experience is a way from simplified to complicated, abstract to practical, possible to certain. Certainty here means a definite happening in reality. We make buildings in experience based on the structures drawn in mathematics and the forms contained by mathematics sometimes are primarily set be further away from experience or close to experience. In building houses of experience, the blueprint can be drawn freely and we can make different versions to plan its structure: the actualization of every blueprint can be stable, but there is only one house in reality. From this perspective, forms are freer and have a larger realm than experience, the same as when Plato starts his argument on the priority of forms. We now gain the same view considering the relationship between forms and experience, but from a contrasting hypothesis.

The advantage of mathematics also implies its limitations. As Descartes has stated in Rule 2, it is clear that "(arithmetic and geometry) alone are concerned with an object so pure and simple that they make no assumptions that experience might render uncertain."[29] In the independence of establishing mathematics, the elimination of the uncertain experiential disturbances help us to avoid the external influences. From another perspective, this simplification, at the same time, also limits the function of mathematics itself. The reality has been reduced and simplified in forms. Through forms, it is easier to discuss their relations, but the relations themselves would not help to restore the quality of real objects. The quality can be approached but this is different from being restored. When we try to find a simplified way to approach the experience, we also need to suffer the consequential imperfections of restoration. Bergson's argument certainly calls our attention to make the distinction between the approach and the restoration. We can always build up a mathematical world which is infinitely closer to restoring an existing experiential world, but is still different from our experiential world.

Restoring a reality is different from creating a reality and mathematics has a stronger power in creation. The difficulty of restoration indicates the human activities of attempting to offer perfect explanations for existing experience. It does not mean that we

[29] *The Philosophical Writings of Descartes*, page 12 (365-366)

can not achieve a reality from forms. When we try to explain our observations by mathematics, we are building up artificial experience which possibly explain the real one. On the other hand, if we start from our forms and make our constructions, we can actually create new experiences and achieve them in reality. I call this progress *actualization* instead of *restoration* because there is no predetermined existing reality and we are not aiming to arrive at a destined place. The main aim here is to make forms be achieved and realized. The application of forms definitely can happen because our task is to create either physical objects or experience that strictly follows the forms. The task here no longer is to fit a form in a complex composition of the universe. It is implied in my probability theory which interprets the simplicity in Part II that the form can only possibly be a part of an existing phenomenon but it still contain a certainty to be realized.

The creative power of mathematics explains their powerful applications in our daily life. Though we cannot perfectly explains nature and the universe, in our attempt to understand them better, our intelligence is trained and improved. The experience created by mathematics has a strong influence on what reality is because those created experiences gradually integrate with reality and change our lifestyle. The achievements of modern technology is impressive and the development of computer science in recent centuries clearly demonstrates it. This reminds me of the story in Borges' *Tlön, Uqbar, Orbis Tertius.* Tlön has been written down in an encyclopedia and then many scholars admit its existence. They write research papers and they locate it on maps. Tlön is gradually made real. The application of mathematics is also a transformation under our efforts, from words and drawings to a physical entity, the product of human intelligence to a real actualization.

Recall my image of two parallel lines between forms and experience, that the line of forms is a thinner and brighter one, and the line of experience is thicker and mottled. We can revise this model after the newly-gained thinking from Bergson's argument. They are not two parallel lines on a flat surface, but two parallel helices in space. One helix still represents the line of form, but the other one should be specified as the line of existing experience, which particularly corresponds to the phenomena of nature and the universe. Their primary intersection is still infinitely far away at some point. If we view these two parallel helices from the top, there is only one perfect circle, because two helices coincide perfectly or imperfectly with each other in our temporary vision. If we look at them from a side view, we see the image which indicates their independence from one another. Mathematics can be put in between forms and existing experience, and while it is closer to existing experience, the distance between two helices from a side view is shorter. The shorter distance always indicates the growing influence from experience to forms. The interactions make them closer and bind them to each other. However, the helix of forms can never be the helix of existing experience. Though they can coincide with each other by the aid of an advanced mathematics, like the most updated and fundamental theory of relativity, the thinner and brighter line cannot make itself be the thicker and mottled one. The overlapping always belongs to a temporary vision and the improvements of mathematics always lead to further adjustments. Even though we enhance the helix of forms all the time, we cannot claim that it can be made to coincide with the helix of existing experience. In pushing the helix of forms to be closer to the helix of experience, meanwhile, we can also creates more helices in between them. We can turn the helix of forms into being a helix of new and created experience. Those helices can be intertwined with the helix of existing experience and make us to adjust our helix of forms.

Epilogue

For our mysterious Mathematics, we tried to recall the memory of its birth and we witnessed its growth, moving, and returning home. Its veil is gradually uncovered in our process of identifying it, though it is surrounded by rumors and different opinions. We now have a better window into its beauty.

Its mother is called Experience. When it is born, it has been given the nickname Form. Inside its body, there is a gene that stimulates its growth, which is logic and reasoning. It likes to play alone in the sky, especially at night, among all the shining stars. It grows up while playing in the universe, and unfortunately it forgets its birth, though it always hears a voice from its mother Experience, to call it back. In its growth, it moves a few times and it attempts valiantly to find its home. Meanwhile, it grows to look like its mother. Oh, it is HER. When she gets home after a long journey to embrace her mother Experience, we see a warming union between them. When they stands together in front of us, we finally can distinguish them from each other and also feel their inseparability.

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