Asymptotic notation and analysis (mit 6.042J)

- Leave out lower-order terms
- Leave out the coefficient in the leading term
- example 5n^3+4n+3 = O(n^3)
 ~(tilde notation) 等价无穷小量 如 f(x)~g(x)就是 $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$

Summary

- O means <=
- o means <
- Ω means >=
- ω means >
- Θ means =
- 不用在归纳证明中使用渐进符号,不然会证明一些疯狂的东西

Why use it

Because running time is too precise and depends on particular machines.

O notation

DEFINITION

Let f(n) and g(n) be functions from positive integers to positive reals. We say f = O(g) (which means that "f grows no faster than g") if there is a constant c > 0 such that $f(n) \le c \cdot g(n)$.

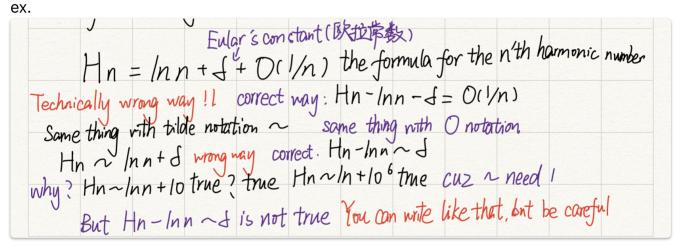
• f(n) = O(n): $f(n) \le c \bullet n$ for some constant c and large n. • i.e. $\exists c, \exists N > 0$ s.t. $\forall n > N$, we have $f(n) \le c \bullet n$.

General definition

- f(n) = O(g(n)): for some constant c, $f(n) \le c \cdot g(n)$, when n is sufficiently large.
 - i.e. $\exists c$, $\exists N$ s.t. $\forall n > N$, we have $f(n) \leq c \bullet g(n)$.

FOR CALCULATE

 $f(\mathbf{x}) = \mathsf{O}(\mathsf{g}(\mathbf{x})) \text{ if } \lim_{x \to \infty} \frac{f(x)}{g(x)} < \infty(finite)$ and it can be represent as $f(\mathbf{x}) <= \mathsf{)}(\mathsf{g}(\mathbf{x}))$, $f(\mathbf{x})$ is $\mathsf{O}(\mathsf{g}(\mathbf{x}))$, and $f(x) \in O(g(x))$ Be careful to write things on the right side, or you will write some wrong thing that technically fit the defination



Don't use f(x) >= O(g(x)), it is meaningless, we have another symbol Ω

Ω notation

General definition

- $f(n) = \Omega(g(n))$: $f(n) \ge c \cdot g(n)$ for some constant c and large n.
 - i.e. $\exists c$, $\exists N$ s.t. $\forall n > N$, we have $f(n) \geq c \bullet g(n)$.

For calculate

$$\begin{split} &\mathsf{f}(\mathsf{x}) = \Omega(\mathsf{g}(\mathsf{x})) \; \mathsf{if} \; \lim_{x \to \infty} |\frac{f(x)}{g(x)}| > 0 \\ &\mathsf{f}(\mathsf{x}) = \mathsf{O}(\mathsf{g}(\mathsf{x})) \; \mathsf{is} \; \mathsf{equal} \; \mathsf{to} \; \mathsf{g}(\mathsf{x}) = \Omega(\mathsf{f}(\mathsf{x})) \end{split}$$

⊕ notation

General definition

• $f(n)=\Theta(g(n))$: f(n)=O(g(n)) and $f(n)=\Omega(g(n))$ • i.e. $c_1\cdot g(n)\leq f(n)\leq c_2\cdot g(n)$ for two constants c_1 and c_2 and large n.

For calculate

f(x) =
$$\Theta(g(x))$$
 if $\lim_{x \to \infty} |rac{f(x)}{g(x)}| > 0$ and $< \infty$, Ω and O both true