Asymptotic notation and analysis(mit 6.042J)

- Leave out **lower-order terms**
- Leave out **the coefficient in *the leading term***
- example 5n^3+4n+3 = O(n^3)
  ~(tilde notation) 等价无穷小量 如 f(x)~g(x)就是 $\lim_{x\to\infty}\frac{f(x)}{g(x)} = 1$

Summary

- O means <=
- o means <
- Ω means >=
- ω means >
- Θ means =
- 不用在归纳证明中使用渐进符号，不然会证明一些疯狂的东西
  ex.

Thm (not!) Let $f(n) = \sum_{i=1}^{n} i$ please proof $f(n) = O(n)$

it is n^2 actually

Flase pf by induction on n, fix value of n here

$\boxed{\text{Induction hypothesis: } P(n): f(n) = O(n)}$ really bad

f(n) is not a function

big O notation only apply in function

Base case $f(1) = 1 = O(1)$

Ind step. Assume P(n) is true to prove P(n+1)

$P(n) \Rightarrow f(n) = O(n)$

$f(n+1) = f(n) + n + 1 = O(n) + O(n) = O(n)$

Never use asymptotic notation with inductive proofs

不要将渐近符号和归纳证明一起使用

Why use it

Because running time is too precise and depends on particular machines.

O notation

DEFINITION

Let $f(n)$ and $g(n)$ be functions from positive integers to positive reals. We say $f = O(g)$ (which means that "f grows no faster than g") if *there is a constant $c > 0$ such that $f(n) \le c \cdot g(n)$*.

Extended definitions

- $f(n) = O(n)$: $f(n) \le c \bullet n$ for some constant c and large n.
  - i.e. $\exists c$, $\exists N > 0$ s.t. $\forall n > N$, we have $f(n) \le c \bullet n$.

General definition

- $f(n) = O(g(n))$: for some constant c, $f(n) \le c \bullet g(n)$, when n is sufficiently large.
  - i.e. $\exists c$, $\exists N$ s.t. $\forall n > N$, we have $f(n) \le c \bullet g(n)$.

FOR CALCULATE

f(x) = O(g(x)) if $\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty (finite)$

and it can be represent as f(x) <=)(g(x)), f(x) is O(g(x)), and $f(x) \in O(g(x))$

`Be careful to write things on the right side, or you will write some wrong thing that technically fit the defination`

ex.

Eular's constant (欧拉常数)

$H_n = \ln n + \delta + O(1/n)$ the formula for the n'th harmonic number

Technically wrong way !! correct way: $H_n - \ln n - \delta = O(1/n)$

Same thing with tilde notation $\sim$ same thing with O notation

$H_n \sim \ln n + \delta$ wrong way correct. $H_n - \ln n \sim \delta$

why? $H_n \sim \ln n + 10$ true? true $H_n \sim \ln n + 10^6$ true cuz $\sim$ need !

But $H_n - \ln n \sim \delta$ is not true You can write like that, but be careful

`Don't use f(x) >= O(g(x)), it is meaningless, we have another symbol Ω`

Ω notation

General definition

- $f(n) = \Omega(g(n))$: $f(n) \ge c \bullet g(n)$ for some constant c and large n.
  - i.e. $\exists c$, $\exists N$ s.t. $\forall n > N$, we have $f(n) \ge c \bullet g(n)$.

For calculate

f(x) = Ω(g(x)) if $\lim_{x\to\infty}|\frac{f(x)}{g(x)}| > 0$

`f(x) = O(g(x)) is equal to g(x) = Ω(f(x))`

Θ notation

General definition

- $f(n) = \Theta(g(n))$: $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$
  - i.e. $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for two constants c_1 and c_2 and large n.

For calculate

f(x) = Θ(g(x)) if $\lim_{x\to\infty}|\frac{f(x)}{g(x)}| > 0 \ and \ < \infty$,Ω and O both true