Graphics' Dependencies

- Basic mathematics
  - Linear algebra, calculus, statistics
- Basic physics
  - Optics, Mechanics
- Misc
  - Signal processing
  - Numerical analysis
- A bit of Aesthetics(大噓) 美感也是图形学的重要组成部分

注意：本课程中的坐标系都为右手系，即z和x叉乘y的同向，在有些API中(OpenGL)中是左手系，其各有优势。
图形学中默认向量以一列的矩阵表示（nx1）,但是向量转换成矩阵在后面的推导中有用到

Vector

Magnitude (length) of a vector written as $||\vec{a}||$(注意是双竖线，和以前的不一样)

Vector Addition

Parallelogram law & Triangle law

Vector Multiplication

DOT(SCALAR) PRODUCT

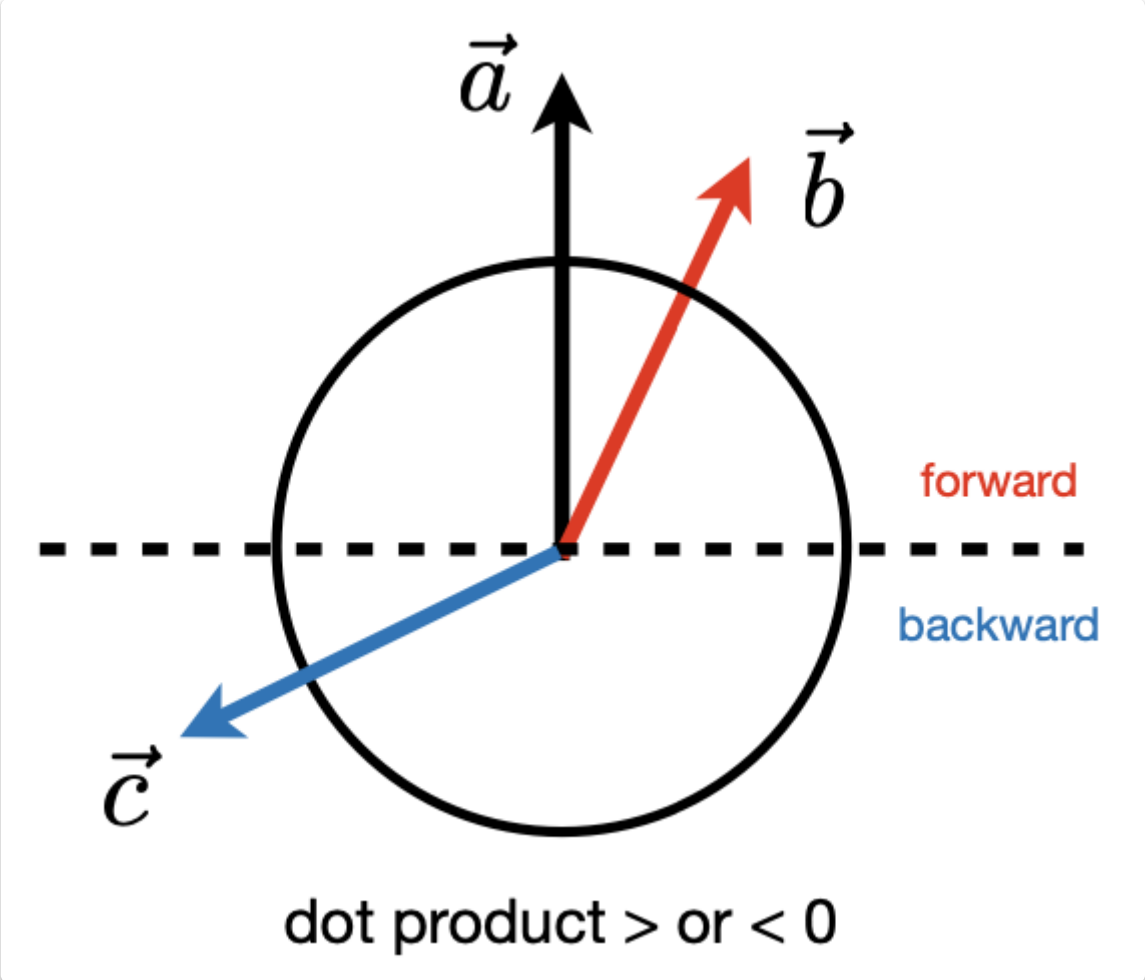
- Dot Product in Cartesian Coordinates

$$\vec{a} * \vec{b} = \begin{pmatrix} x_a \\ y_a \end{pmatrix} * \begin{pmatrix} x_b \\ y_b \end{pmatrix} = x_a x_b + y_a y_b$$

- in matrix form

$$\vec{a} * \vec{b} = \vec{a}^T \vec{b} = x_a x_b + y_a y_b + z_a z_b$$

- Use In graphics
  Find angle between two vectors(e.g. cosine of angle between light source and surface)
  Measure how close two directions are
  Finding projection of one vector on another
  Determine forward / backward（看点乘正负）



Decompose a vector

CROSS PRODUCT

Right-hand rule
x,y,z叉乘的时候注意方向
$\vec{a} \times \vec{a} = \vec{0}$

- Cross Product: Cartesian Formula

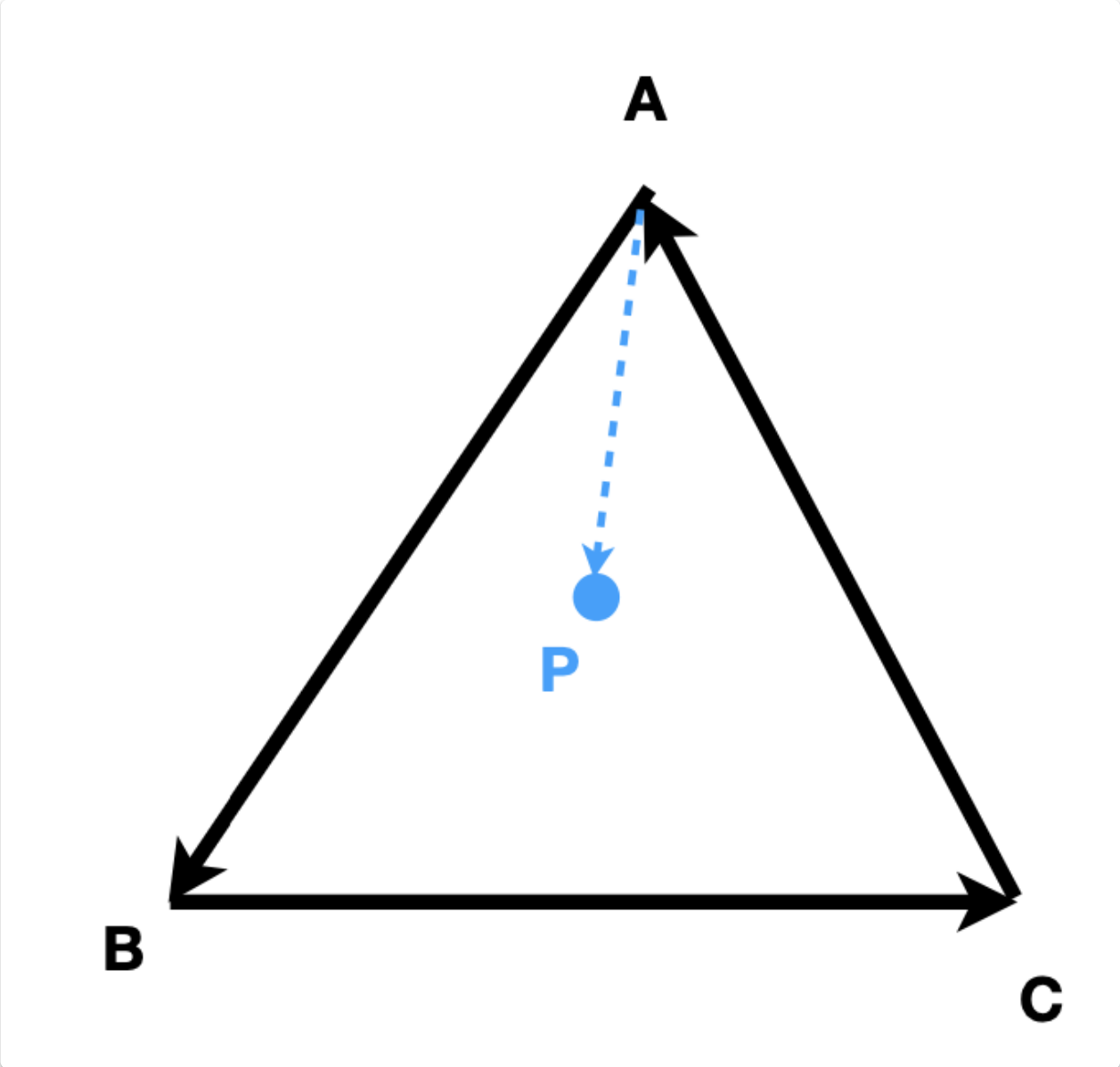
$$\vec{a} \times \vec{b} = \begin{pmatrix} y_a z_b - y_b z_a \\ z_a x_b - x_a z_b \\ x_a y_b - y_a x_b \end{pmatrix}$$

- In matrix form

$$\vec{a} \times \vec{b} = A^* b = \begin{pmatrix} 0 & -z_a & y_a \\ z_a & 0 & -x_a \\ -y_a & x_a & 0 \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

- Use in Graphics
  Determine left / right(顺时针/逆时针)
  看叉乘结果的正负
  Determine inside/outside

看点在所有边的左边还是右边



1. 在一般的常识或者教科书中规定叉乘只有3d才拥有，其实2d也可以拓展出来一个叉乘形式，而且非常有用。

拓展方式：假设有两个2d向量a,b，我们直接把他们视为3d向量，z轴补0，那么这个时候的a，b向量的叉乘结果c,c.x=0,c.y=0,c.z=a.x*b.y-b.x*a.y,

这个时候可以吧2d的叉乘值定义为得到一个值，而不是得到一个向量，那么这个值k， k = c.z=a.x*b.y-b.x*a.y，我们可以通过这个k值得到很多有用的性质

1.a，b向量构成的平行四边形的面积。

2.如果k>0时，那么a正旋转到b的角度为<180°，如果k<0，那么a正旋转到b的角度为>180°,如果k=0 那么a，b向量平行。

ORTHONORMAL BASES / COORDINATE FRAMES

Critical issue : transforming between these systems/ bases

- Important for representing points, positions, locations(无坐标系没坐标)

Matrices

In Graphics, pervasively used to represent transformations

Multiplication

- (M x N) (N x P) = (M x P)
- Element (i,j) in the product is the dot product of row i from A and column j from b

PROPERTIES

Non-commutative 不满足交换律

Transpose of a Matrix

Switch rows and columns (ij -> ji)

PROPERTY

$(AB)^T = B^T A^T$
注意是B在前A在后

Identity Matrix and Inverses

- Identity Matrix

$$I_{3x3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Inverses matrix's property

$$AA^{-1} = A^{-1}A = I$$

$$(AB)^{-1} = B^{-1}A^{-1}$$