(a) $x_a(n) = x_a(n \% N)$. $\qquad X(k) = DFT[x(n)] = \sum_{n=0}^{N-1} x(n) W_N^{nk}$

$X_a(k) = DFT[x_a(n)] = \sum_{n=0}^{MN-1} x_a(n) W_{MN}^{nk} = \sum_{q=0}^{M-1} \sum_{n=0}^{N-1} x_a(qN+n) W_{MN}^{(qN+n)k}$

$\qquad = \sum_{q=0}^{M-1} \sum_{n=0}^{N-1} x_a(n) W_{MN}^{nk} W_M^{qk} = \left(\sum_{q=0}^{M-1} W_M^{qk} \right) \left(\sum_{n=0}^{N-1} x(n) W_N^{nk} \right)$

$\qquad = \begin{cases} 0, & \frac{k}{m} \notin \mathbb{Z} \\ M X(\frac{k}{m}), & \frac{k}{m} \in \mathbb{Z} \end{cases}$

(b) $x_b = Y(k) = DFT[y(n)] = \sum_{n=0}^{MN-1} y(n) W_{MN}^{nk} = \sum_{n=0}^{MN-1} [n \% M = 0] y(n) W_{MN}^{nk}$

$\qquad = \sum_{n=0}^{N-1} y(n) W_{MN}^{nMk} = \sum_{n=0}^{N-1} y(n) W_N^{nk} \qquad \therefore Y(k) = X(k \% N)$.

(c) $Y(k) = \sum_{n=0}^{MN-1} y(n) W_{MN}^{nk} = \sum_{n=0}^{N-1} y(n) W_{MN}^{nk} = \sum_{n=0}^{N-1} y(n) W_N^{n \frac{k}{M}}$

$\qquad \therefore$ 当 $\frac{k}{m} \in \mathbb{Z}$ 时, $Y(k) = X(\frac{k}{m})$.

$\qquad$ 当 $\frac{k}{m} \notin \mathbb{Z}$ 时, $Y(k)$ 未知.