

1. 设 $z = \cos x + i \sin x$.

$$\frac{dz}{dx} = -\sin x + i \cos x = i^2 \sin x + i \cos x = iz.$$

$$\therefore \frac{dz}{z} = i dx. \quad \int \frac{dz}{z} = \int i dx \Rightarrow \ln z = ix + C.$$

取 $z=1$ 时 $x=0$, 则 $z=1$, $C = \ln z - ix = 0$.

$$\therefore \ln z = ix. \quad \text{即 } z = e^{ix}$$

$$\therefore e^{ix} = \cos x + i \sin x.$$

2. ① $e^{jn_1 \omega_0 t}$ 与 $e^{jn_2 \omega_0 t}$ 在 $[-\frac{\pi}{\omega_0}, \frac{\pi}{\omega_0}]$ 上正交. ($n_1 \neq n_2, n_1, n_2 \in \mathbb{Z}$).

$$\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{jn_1 \omega_0 t} \cdot e^{-jn_2 \omega_0 t} dt$$

$$= \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{j(n_1 - n_2) \omega_0 t} dt = \frac{1}{j(n_1 - n_2) \omega_0} e^{j(n_1 - n_2) \omega_0 t} \Big|_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}}$$

$$= \frac{1}{j(n_1 - n_2) \omega_0} (e^{j(n_1 - n_2) \pi} - e^{-j(n_1 - n_2) \pi}) = \frac{1}{j(n_1 - n_2) \omega_0} ((-1)^{n_1 - n_2} - (-1)^{n_2 - n_1}).$$

$$\therefore \forall x \in \mathbb{Z}, (-1)^x = (-1)^{-x}. \quad \therefore \text{上式} = 0$$

② $e^{jn_1 \omega_0 t}$ 与 $e^{jn_2 \omega_0 t}$ 在 $[-\frac{\pi}{\omega_0}, \frac{\pi}{\omega_0}]$ 上正交. ($n_1 = n_2$).

$$\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{jn_1 \omega_0 t} \cdot e^{-jn_1 \omega_0 t} dt.$$

$$= \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{j(n_1 - n_1) \omega_0 t} dt = \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} 1 dt = \frac{2\pi}{\omega_0}.$$

$\therefore e^{jn \omega_0 t}$ ($n \in \mathbb{Z}$) 在区间 $[-\frac{\pi}{\omega_0}, \frac{\pi}{\omega_0}]$ ($\omega_0 \in \mathbb{R}$) 上是正交函数集.

全额退款有保障

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咨询过很多家留学机构, 感觉还是太傻的服务最好, 咨询老师不仅专业还很亲切, 那个不满意全额退款的服务也给了我多一层保障, 更安心, 所以最后还是决定在新的申请季来临前找太傻帮忙。