$e^{-(a+j\omega)t}dt.$

No.

Date

1. $f(t)=e^{-at}.$

$\cancel{\because \mathcal{F}[e^{j\omega_0 t}]=2\pi\delta(\omega-\omega_0).}$

$\cancel{\therefore \mathcal{F}[e^{-at}]=\mathcal{F}[e^{j(ja)t}]}$

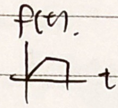
$\cancel{=2\pi\delta(\omega-ja).}$

$f(t)\Leftrightarrow F(\omega).$ $\mathcal{F}[e^{-at}]=\int_0^{+\infty}e^{-at}e^{-j\omega t}dt.$

$=-\frac{1}{a+j\omega}e^{-(a+j\omega)t}\Big|_0^{+\infty}$

$=-\frac{1}{a+j\omega}[0-1]=\frac{1}{a+j\omega}.$

2. $f(t)=\begin{cases} t, & 0\le t<\tau \\ \tau, & \tau\le t<2\tau \\ 0, & \text{else}. \end{cases}$




$f(t)\Leftrightarrow F(\omega).$

$\mathcal{F}[f(t)]=\int_{-\infty}^{+\infty}f(t)e^{-j\omega t}dt$

$=\int_0^{2\tau}f(t)e^{-j\omega t}dt.$

$=\int_0^{\tau}te^{-j\omega t}dt+\tau\int_{\tau}^{2\tau}e^{-j\omega t}dt.$

$=e^{-j\omega t}\left(\frac{1}{\omega^2}+\frac{jt}{\omega}\right)\Big|_0^{\tau}-\frac{\tau}{j\omega}e^{-j\omega t}\Big|_{\tau}^{2\tau}$

$=\frac{1}{\omega^2}[j\omega\tau e^{-j\omega 2\tau}+e^{-j\omega\tau}-1].$

3. a. $f(t)=e^{-\frac{t^2}{20}}.$

$f(t)\Leftrightarrow F(\omega).$

$\mathcal{F}[f(t)]=\int_{-\infty}^{+\infty}e^{-\frac{t^2}{20}}e^{-j\omega t}dt$

$=\int_{-\infty}^{+\infty}e^{-\frac{1}{20}(t+10j\omega)^2-5\omega^2}dt$

$=\sqrt{20}e^{-5\omega^2}\int_{-\infty}^{+\infty}e^{-\left(\frac{t}{\sqrt{20}}+\frac{10j\omega}{\sqrt{20}}\right)^2}d\left(\frac{t}{\sqrt{20}}\right).$

$=2\sqrt{5\pi}e^{-5\omega^2}.$

b. $\cancel{f(\omega)}.$ $f_w(t,2)=e^{-\frac{11}{20}t^2}.$

$\mathcal{F}[e^{-\frac{11}{20}t^2}]=\int_{-\infty}^{+\infty}e^{-\frac{11}{20}\left(x+\frac{10j\omega}{11}\right)^2-\frac{5\omega^2}{11}}dt.$

$=\frac{2\sqrt{5\pi}}{\sqrt{11}}e^{-\frac{5}{11}\omega^2}.$

c.