No.

Date

卷积：$(f*g)(t)=f(t)*g(t)=\int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$.

① 证明 $\frac{d}{dt}[f_1(t)*f_2(t)]=f_1(t)*[\frac{d}{dt}f_2(t)]=[\frac{df_1(t)}{dt}]*f_2(t)$.

$$\frac{d}{dt}[f_1(t)*f_2(t)]=\frac{d}{dt}\int_{-\infty}^{+\infty} f_1(t-\tau)f_2(\tau)d\tau=\int_{-\infty}^{+\infty}\frac{d}{dt}[f_1(t-\tau)f_2(\tau)]d\tau$$

$$=\int_{-\infty}^{+\infty}\left[\frac{d}{dt}f_1(t-\tau)\right]f_2(\tau)d\tau=\left[\frac{df_1(t)}{dt}\right]*f_2(t).$$

∵ $\frac{d}{dt}[f_1(t)*f_2(t)]=[\frac{df_1(t)}{dt}]*f_2(t)$. 且 $f_1*f_2=f_2*f_1$.

∴ 上式可变为 $\frac{d}{dt}[f_2(t)*f_1(t)]=f_2(t)*[\frac{df_1(t)}{dt}]$. 完全等价于 $\frac{d}{dt}[f_1(t)*f_2(t)]=f_1(t)*[\frac{d}{dt}f_2(t)]$ □

② 证明 $\int_{-\infty}^{t}(f_1*f_2)(\lambda)d\lambda=f_1(t)*\int_{-\infty}^{t}f_2(\lambda)d\lambda=(\int_{-\infty}^{t}f_1(\lambda)d\lambda)*f_2(t)$.

原式 $=\int_{-\infty}^{t}\int_{-\infty}^{+\infty}f_1(\lambda-\tau)f_2(\tau)d\tau d\lambda \overset{\alpha=\lambda-\tau}{=\!=} \int_{-\infty}^{t-\tau}\int_{-\infty}^{+\infty}f_1(\alpha)f_2(\tau)d\tau d\alpha \overset{\lambda=\alpha}{=} \int_{-\infty}^{+\infty}(\int_{-\infty}^{t-\tau}f_1(\lambda)d\lambda)f_2(\tau)d\tau$

$=(\int_{-\infty}^{t}f_1(\lambda)d\lambda)*f_2(t)$.

∵ $\int_{-\infty}^{t}(f_1*f_2)(\lambda)d\lambda=(\int_{-\infty}^{t}f_1(\lambda)d\lambda)*f_2(t)$. 且 $f_1*f_2=f_2*f_1$.

∴ 原式可变为 $\int_{-\infty}^{t}(f_2*f_1)(\lambda)d\lambda=f_2(t)*(\int_{-\infty}^{t}f_1(\lambda)d\lambda)$ 即第一个等号成立.

③ 冲激函数抽样特性：$\int_{-\infty}^{+\infty}f(t)\delta(t-t_0)dt=f(t_0)$.

∵ $t\neq t_0$ 时. $\delta(t-t_0)=0$.

∴ $\int_{-\infty}^{+\infty}f(t)\delta(t-t_0)dt=\int_{-\infty}^{t_0^-}f(t)\delta(t-t_0)dt+\int_{t_0^+}^{+\infty}f(t)\delta(t-t_0)dt+\int_{t_0^-}^{t_0^+}f(t)\delta(t-t_0)dt$

$=\int_{t_0^-}^{t_0^+}f(t)\delta(t-t_0)dt = f(t_0)\int_{t_0^-}^{t_0^+}\delta(t-t_0)dt=f(t_0)$.

实际上只有$t=t_0$时有非零被积值