Problem 1
Calculate and compare the expected value and standard deviation of price at time t $\left(P_t\right)$, given each of the 3 types of price returns, assuming $r_t \sim N\left(0, \sigma^2 = 0.01\right)$ and the price at t-1 $\left(P_{t-1}\right)$.
Simulate each return equation using $r_t \sim N\left(0, \sigma^2\right)$ and show the mean and standard deviation match your expectations.

For simplicity, I use first 5 days of SPY price and simulate their next days price, the mean and std meet my expectation overall. Classic price's std is small because it's formula is count as drift term directly as price difference, and the rt simulation is small so its std is small. By looking at actual price, it's actual price is more fluctuated than simulation, if we want to simulate the Pt, we should increase variance of simulation.

Output of code:

for day1 SPY price, expected price is (443.04376997495604, 443.39794583318314, 445.6724504270646), std is (0.10126840145807187, 44.866253222550824, 45.09769910775857)

for day2 SPY price, expected price is (440.0629573968157, 439.33924014928994, 441.589979762478), std is (0.10112891442988567, 44.50325585476032, 44.71544832475863)

for day3 SPY price, expected price is (438.7153777436695, 439.5368640445973, 441.76630831244216), std is (0.10069446085640182, 44.17601945125952, 44.439911670363685)

for day4 SPY price, expected price is (439.37407227080183, 439.28845456604506, 441.6246082024265), std is (0.10312420604802766, 45.31012250192765, 45.49365152488549)

for day5 SPY price, expected price is (442.26440143994415, 442.4920679662477, 444.6592738501966), std is (0.09859262943665519, 43.60395937612404, 44.288698178415075)

Problem 2
Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

You own 1 share of META. Remove the mean from the series so that the mean(META)=0

Calculate VaR
1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historic Simulation.

Compare the 5 values.

Here the value of investment is last day price of Meta in 'DailyPrice',around 521$
for historic simulation, I didn't use remove mean list because it will lose meaning. VaR of those five method overall meet my expectation.

Output of code:

Delta-Normal VaR: $19.94
EWMA VaR: $15.46
MLE t-distribution VaR: $16.42
AR(1) VaR: $19.83
Historic Simulation VaR: $13.74

Problem 3
Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.97, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

Discuss your methods and your results.

Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

code output:
Var based on EWMA for portfolio A is 20247.041091704225
Var based on EWMA for portfolio B is 12035.820474831105
Var based on EWMA for portfolio C is 27839.30753640576
Var based on EWMA for total portfolio is 55682.33134303159

discussion:
VaR represent the most potential loss in our 95% confidence interval, the total holding and amount for each portfolio is high so VaR is also high.
EWMA's sigma is more influenced by sigma that are going to approach the final time, so if we use this to calculate VaR, not whole
Var based on AR1 for portfolio A is 16636.443227135213
Var based on AR1 for portfolio B is 9992.58188108131
Var based on AR1 for portfolio C is 22847.49261616034
Var based on AR1 for total portfolio is 45201.76874702878