Algorithm for 2D irregular-shaped nesting problem based on the NFP algorithm and lowest-gravity-center principle[*]

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Abstract: The nesting problem involves arranging pieces on a plate to maximize use of material. A new scheme for 2D irregular-shaped nesting problem is proposed. The new scheme is based on the NFP (No Fit Polygon) algorithm and a new placement principle for pieces. The novel placement principle is to place a piece to the position with lowest gravity center based on NFP. In addition, genetic algorithm (GA) is adopted to find an efficient nesting sequence. The proposed scheme can deal with pieces with arbitrary rotation and containing region with holes, and achieves competitive results in experiment on benchmark datasets.

INTRODUCTION

2D irregular-shaped nesting problem is the problem of finding an efficient arrangement for pieces in a containing region without overlapping (Fig.1), and is aimed at maximizing use of material. Nesting problem is of great interest to garment, paper, ship building, and sheet metal industries since small improvement of layout can lead to large savings in material. Many effective solutions have been proposed for the case when pieces and containing region are both rectangular. There are also many cases where the pieces or the containing region have irregular shape. Due to the geometrical complexity introduced by irregular shapes, irregular nesting problems are not studied as extensively as rectangular nesting problem.

Nesting problem is NP-complete even for rectangular shapes; for irregular shapes, additional geometric complexity increases the computational burden, so it is difficult to apply a deterministic nesting algorithm for the irregular-shaped nesting problem. Hence to get an approximate solution, some heuristic and meta-heuristic nesting algorithms were proposed. Hopper and Turton (2001) and Dowsland and Dowsland (1995) provided a comprehensive review of recent work in the field of 2D irregular nesting. Jakobs (1996) used an order-based genetic algorithm for nesting and extended the work on nesting of rectangles to polygons, and realized 2D irregular nesting by introducing a shrinking algorithm to the rectangles nesting layout. Fischer and Dagli (2004) and Tay *et al.*(2002) proposed some approaches based on genetic algorithm, although the performance of these algorithms needs enhancement. It is still difficult to outperform an experienced worker with a purely automatic algorithm, so that irregular nesting problem has become an attractive research topic.

IRREGULAR-SHAPED NESTING PROCEDURE

The procedure for irregular-shaped nesting mainly includes two steps: First, place the pieces into
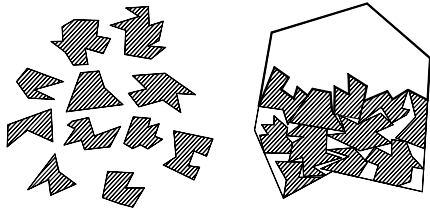

Fig.1 2D irregular-shaped nesting problem

the containing region without overlapping; Then re-generate the containing region by subtracting the region occupied by nested pieces from it. Fig.2 shows the nesting procedure.
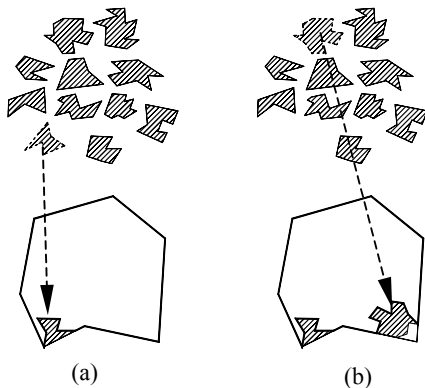

Fig.2 2D irregular-shaped nesting procedure. (a) Nest one piece and form a new boundary; (b) Nest next piece in the new boundary

NFP AND ITS FUNCTION IN NESTING

With computational geometry tools such as No Fit Polygon (NFP) and piece placement principles, pieces can be precisely placed inside the container's boundary. The No Fit Polygon (NFP) (Art, 1966; Adamowicz and Albano, 1976), has practically become a prerequisite for solving irregular packing problems; it gives the set of non-overlapping placements for polygons. NFP includes outer-fit NFP and inner-fit NFP defined as follows:

Definition 1   Given a container and one piece with fixed orientation, where container is defined as the fixed polygon and piece is the sliding polygon. The NFP is a closed path formed by tracing the locus of a reference point on the piece while the piece slides around the inside boundary of the container, as shown in Fig.3 below.
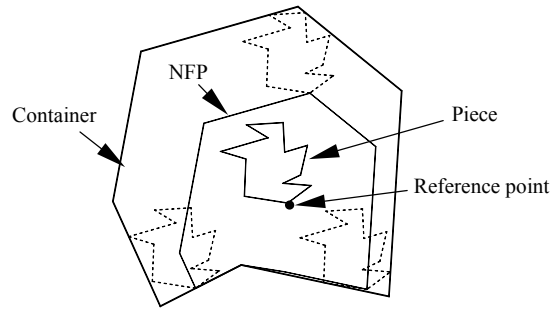

Fig.3 Inner-fit NFP is the locus of the reference point on the piece while the piece slides around the inside boundary of the container

The relevant property of NFP with respect to the interaction between the container and the piece is as follows: if the piece is positioned with its reference point on the boundary of NFP then the container and the piece will touch; if the piece is positioned with its reference point inside the boundary of NFP then the piece will be inside the container without overlapping. Thus, the interior of NFP represents all feasible placement positions for the piece, and the boundary of NFP represents all touching positions. In the nesting procedure, pieces are arranged touching each other for saving material, therefore, based on the NFP, the problem of nesting one piece can be simplified to selecting the optimal position on NFP.

Approaches for generating the NFP have focused on three core methodologies: Minkowski sums, largely based on Ghosh's boundary addition theorem (Bennell *et al*., 2001); the collision approach or orbiting approach, designed to simulate the sliding motion of the polygons; and decomposition, where polygons are decomposed into component polygons that can be more easily handled.

PLACEMENT PRINCIPLE FOR IRREGULAR PIECES

The searching space of the irregular nesting problem is infinite because any movement or rotation of the pieces may lead to a new nesting pattern. In order to reduce the searching space, bottom-left (BL) principle has been widely adopted as the placement principle. BL-principle is to place the piece as close as possible to the bottom and left; it works for rectangles, but not for irregular polygons because it does not

consider the irregularity on shape and the arbitrary rotation on pieces, hence other placement principles must be developed. Because NFP gives all feasible placement positions within the containing region, placement principle can be developed based on NFP. Given NFP, the placement principle for one piece can be simplified to finding the optimal placement position on NFP, as shown in Fig.3.

Gravity center NFP

To search for a suitable placement position on NFP, some researchers choose the point on NFP where the piece and the previous nested pieces form a minimal area. Grinde and Cavalier (1995) proposed an algorithm for calculating the minimal enclosure convex-area of two polygons based on NFP. However, the minimal-area method is aimed at finding a local optimal position and does not consider the global optimization, while finding a minimal-area position is rather complicated and adds computational burden to the whole nesting algorithm.

To get a fast and effective placement principle for pieces, this paper proposed a new principle named lowest-gravity-center principle to rapidly find the placement position on NFP. In this placement principle, the placement position on NFP where the piece has the lowest gravity center is chosen. This principle is aimed to "push" the piece to the nested pieces as closely as possible, and make a relatively flat nesting boundary for the remaining pieces to be nested in. The method based on principle makes a better boundary for the remaining pieces, and is much faster than the algorithm for computing the minimal-area polygon.

To find the lowest gravity center, we first calculated the gravity center of the piece, and then the gravity-center NFP can be found by making the gravity center as the reference point, as illustrated in Fig.4.

Note that different rotation angles of the piece will lead to different gravity-center NFPs, the lowest gravity center is the global lowest point among all NFPs. Thus if the NFP with the global lowest point is first determined, the corresponding rotation angle of the piece can be determined accordingly. Therefore just place the piece in the position where the gravity-center is the lowest one and rotate it to the corresponding rotation angle, as shown in dashed lines in Fig.4.
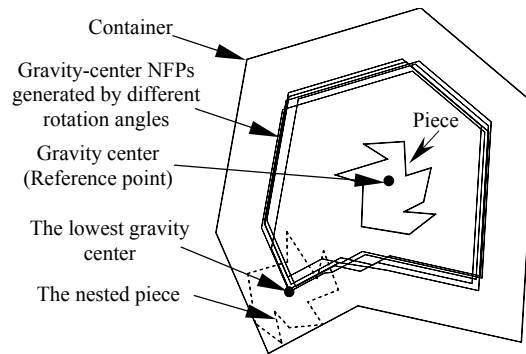

Fig.4 Finding the lowest gravity center on gravity center NFPs generated by different rotation angles

To find the rotation angle leading to the lowest gravity center, a two-stage searching method was adopted. First, divide the whole rotation range (360 degrees) averagely into a number of rotation angles, and test each of the angles to find the one with the lowest gravity center. Then, repeat similar division and testing within the angle found in the former stage and its two neighboring angles so as to achieve a more precise rotation angle.

Calculating the gravity center

We adopted a division-into-strips method to calculate the gravity center of a polygon. Project and divide the polygon into several strips (Fig.5), then calculate every strip's gravity center and area, finally calculate the whole gravity center by the following formula:

$$x = \sum A_i x_i \Big/ \sum A_i \, ; \; y = \sum A_i y_i \Big/ \sum A_i \, ,$$

here A_i is the area of strip i, x_i, y_i the x and y coordinates of a strip's gravity center.

Fig.5 shows that the polygon's edges are projected on x axis by lines parallel to y axis, with each edge corresponding to a strip such as A_0 and A_2. If the x coordinate of the polygon edge's end-point is greater than that of the start-point, then the area is positive, otherwise it is negative. For example, A_0 is a negative area and A_2 is a positive area in Fig.5. After calculating all strips' area and gravity center separately, the whole gravity center of the polygon can be calculated according to the above gravity center formula.
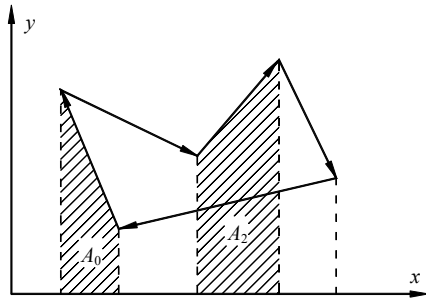
Fig.5   Calculate the gravity center by strip division

Fig.6   Hole-recursive-nesting to reduce holes

ALGORITHMS FOR DETERMINING THE NESTING SEQUENCE

The lowest-gravity-center (LGC) principle determines the nesting position and rotation angle for one piece, and in the same way, all pieces can be nested one by one in a given nesting sequence. In other words, the key point of solving 2D irregular nesting problem will shift to finding an efficient nesting sequence. According to (Hopper and Turton, 2001), meta-heuristic search algorithms including GA (genetic algorithm), SA (simulated annealing), TS (tabu search) and ANN (artificial neural networks), etc. are widely used in nesting problems to find the nesting sequence. For example, Jakobs (1996) adopted an order-based genetic algorithm in bin-nesting algorithm to find the nesting sequence. This paper proposes a recursive algorithm and a GA to search for an effective nesting sequence to meet different requirements on solution quality and computing time.

Hole-recursive nesting approach

If the pieces were sorted by area, there would be a number of holes between big pieces if they were nested together. Actually these holes can be filled by smaller pieces. From this point of view, we proposed the hole-recursive-nesting (HRN) approach to get an effective permutation.

As shown in Fig.6, pieces were first sorted by area in descending order, and then were nested in this sequence. Each time after a piece is nested, there may be formed some holes which are big enough to accommodate one or more of the remaining smaller pieces. In order to maximize the usage of such holes, smaller piece(s) that can be put into such holes should be nested earlier.
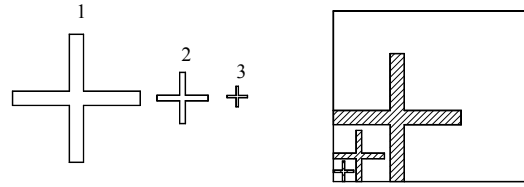
For example, in Fig.6, the original sorted-by-area nesting sequence is 1→2→3, piece No. 1 is first nested, then the biggest hole formed between piece No. 1 and the boundary of the containing region is tested, if this hole is big enough to accommodate piece No. 2, piece No. 2 should be nested earlier than piece No. 1, so that the original nesting sequence is altered into 2→1→3. In the same principle, piece No. 3 should be nested before piece No. 2, so that the nesting sequence has become 3→2→1. Continue to alter the nesting sequence until the newly-formed hole is too small to hold any other remaining pieces.

Hole-recursive nesting approach not only has minimal time requirement, but also achieves moderate solution quality in practice. It can be adopted when high nesting speed is a priority. Another important point is that it can be used to generate initiative solutions (individuals) for genetic nesting algorithm.

Order-based GA algorithm

To achieve high quality nesting sequence, GA is adopted in this paper. Some of its important features are listed below.

1. Gene encoding of the solution

Because the proposed lowest-gravity-center placement principle can determine the gravity center and rotation angle of a piece (Fig.4), there is no need to encode rotation angle into the chromosome, the only thing we need to encode is the nesting sequence (permutation order).

2. Initialization

The first generation of individuals has great influence on the final GA result. To get a set of well-defined initial individuals, sorted-by-area nesting permutation is used as the first individual, and then other individuals are created by mutations of the first individual with all these individuals forming the first generation.

3. Fitness function and fitness scaling

The height of the remaining containing region is

taken as the fitness, a fitness linear scaling function is adopted to scale the fitness value to a reasonable range:

$$f'=\alpha f+\beta+\sigma,$$

here f is the original fitness and f' is the output scaled fitness with

$$\alpha=\frac{f_{avg}}{f_{avg}-f_{min}}, \ \beta=\frac{-f_{min}f_{avg}}{f_{avg}-f_{min}}, \ \sigma=\frac{f_{avg}}{K},$$

where K is a constant.

Because proportional selection method is adopted in this GA, some individuals with very low fitness will be rejected too early, which may lead to premature convergence, so σ is added to the fitness function to avoid this kind of rejection.

4. Crossover

OX (order-crossover (Davis, 1985)) crossover is adopted in this paper. Suppose one parent is (5, 2, 3, 7, 6, 1, 4) and another parent is (4, 6, 2, 1, 3, 5, 7), the OX crossover procedure is illustrated in Fig.7.
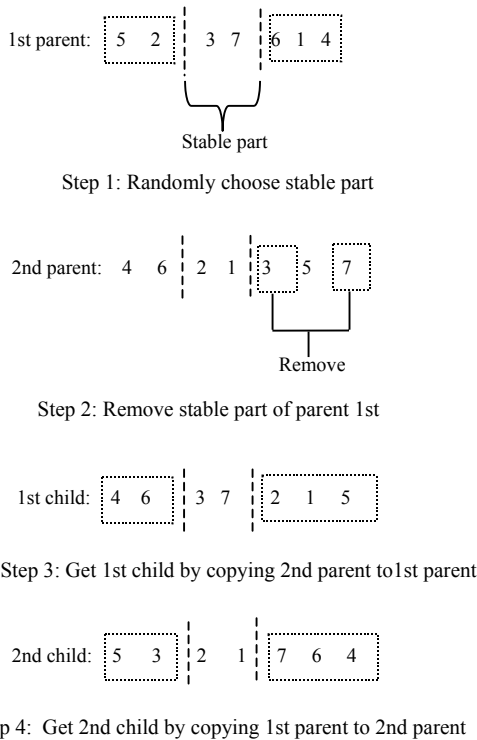

Fig.7 Get two child-individuals by OX crossover

5. Mutation

Mutation occurs after crossover and is applied to newly generated child-individuals. Here mutation is used to exchange two numbers in the chromosome, the mutation probability is 0.1~0.2. Sometimes two segments of the chromosome can also be exchanged, in such case the mutation probability should be decreased since the chromosome is greatly changed.

RESULTS

The proposed 2D irregular nesting algorithm is designed to deal with the nesting problem involving pieces with arbitrary rotation, it can also deal with containing region with holes inside it. Below are the testing benchmark datasets:

1. Dataset 1 from Jakobs (1996)

The first dataset came from Jakobs (1996) who used GA to pack rectangular pieces and extended the algorithm to polygons. He first simplified the polygons into rectangles with Embedding Shrinking algorithm, then after a GA bin-nesting procedure, he adopted a shrinking algorithm to "push" the polygons as close as possible to each other based on the idea of BL-principle. The results are shown in Table 1 (all tests are conducted on a PC with 1.5 GHz Pentium IV CPU and 640 MB memory).

2. Dataset 2 (swimming suit)

The swimming suit dataset came from Gomes and Oliveira (2002). Here we remove the rotation restriction, as shown in Fig.8.
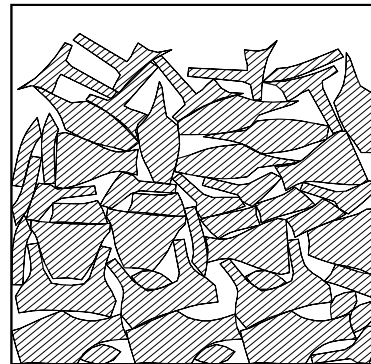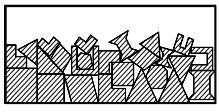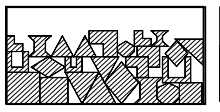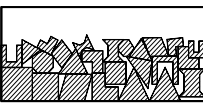

Fig.8 Swimming suit nesting result by algorithm based on lowest-gravity-center NFP and hole-recursive-nesting approach; with arbitrary rotation; height=6320.7; area utilization percentage=79.1%; runtime=156.5 s

Table 1 Results comparison between our approach and Jakobs (1996)'

	Results comparison			
	Jakobs' approach	Hole-recursive approach of this paper	Order-based GA approach of this paper	
Feasible rotations (°)	0	Arbitrary	0	Arbitrary
Result layout				
Layout height	16.0	14.0	14.0	13.2
Material utilization percentage (%)	77.4	83.6	81.0	84.9
Runtime (s)	Not provided	11	1472	2005

3. Dataset 3 (rectangular nesting)

This dataset is also from Jakobs (1996). As a special kind of polygon, rectangular nesting can be realized by adding orthogonal rotation restriction on pieces. Our result is shown in Fig.9.
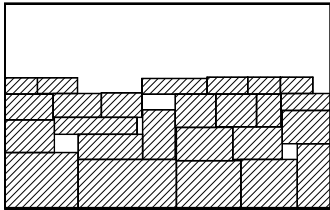

Fig.9 Rectangular nesting result by algorithm based on lowest-gravity-center NFP and GA (33 generations, population size=15, objective is height); with orthogonal rotation; height=16.07; area utilization percentage= 96.6%; runtime=257.7 s

This nesting algorithm provides an integrated feasible application both for irregular and regular pieces. We can control every piece's rotation angle range, some piece can be arbitrarily rotated, others can be restricted in a specific rotation range.

4. Dataset 4 (dataset polygons from ESICUP)

This dataset is from the website of ESICUP (EURO Special Interest Group on Cutting and Packing), it includes both convex and concave polygons, as illustrated in Fig.10.

5. Dataset 5 (container with holes)

Holes not only affect the calculation of NFP, but also affect the calculation of nesting boundary. To test the dataset with holes, we created some holes in the containing region in Dataset 4. Fig.11 illustrates the calculation of NFP with holes.

6. Dataset 6 (comparison on dataset shapes from Gomes and Oliveira (2002))
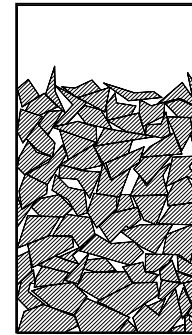

Fig.10 Artificially created polygons (75 pieces) nesting result by algorithm based on lowest-gravity-center NFP and hole-recursive-nesting approach; with arbitrary rotation; height=60.12; area utilization percentage= 83.3%; runtime=37.6 s
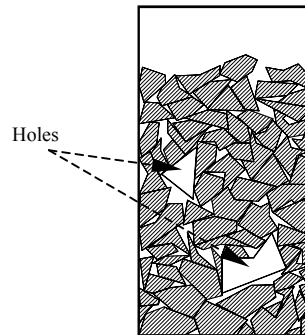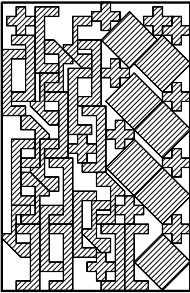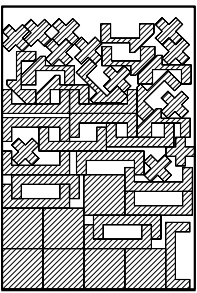

Fig.11 Nesting with holes by algorithm based on lowest-gravity-center NFP and hole-recursive-nesting approach; with arbitrary rotation; height=63.183; area utilization percentage=82.0%; runtime=79.8 s

Existing irregular nesting algorithms are usually restricted on the rotations of pieces, to compare our approach with existing best known nesting algorithms, we took the dataset shapes from Gomes and Oliveira (2002), Table 2 listed the results.

With rotation angle restriction on this dataset,

Table 2 Results compare with existing best known nesting algorithms on dataset shapes from Gomes and Oliveira (2002)

	Results comparison		
	Best known result in the literature[a]	Order-based GA approach of this paper	
Feasible rotations (°)	0 and 180	0 and 180	Arbitrary
Result layout			
Layout height	59.0	61.0	56.9
Material utilization percentage (%)	69.5	70.5	73.1
Runtime (s)	2019	1191	4082

[a] 2-exchange algorithm in (Gomes and Oliveira, 2002)

the quality of the result of our approach is close to the best known result. Without rotation restriction, the nesting layout height and the material utilization percentage are notably improved by our approach.

CONCLUSION

A new 2D irregular-shaped nesting scheme and a new placement principle are proposed in this paper, the new placement principle named lowest-gravity-center principle. With this lowest-gravity-center principle, pieces could be nested closely to each other and result in a flat nesting boundary. At the same time, the pieces' placement position and rotation angle can be determined by this placement principle. A recursive algorithm and a GA are proposed to search for an effective nesting sequence. Based on these proposed algorithms, the new nesting scheme achieved competitive results in nesting pattern height, material utilization percentage and computation time. In addition, it works well for pieces with arbitrary rotation and containing region with holes, this is useful for many practical nesting applications in which arbitrary rotation and holes are usually presented.

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