doi:10.3772/j.issn.1006-6748.2012.03.014

A statistical end-to-end performance model for networks with complex topologies①

Chen Yanping (陈艳平)②*, Wang Huiqiang*, Gao Yulong**
(* College of Computer Science and Technology, Harbin Engineering University, Harbin 150001, P. R. China)
(** Communication Research Center, Harbin Institute of Technology, Harbin 150001, P. R. China)

Abstract

Network calculus provides new tools for performance analysis of networks, but analyzing networks with complex topologies is a challenging research issue using statistical network calculus. A service model is proposed to characterize a service process of network with complex topologies. To obtain closed-form expression of statistical end-to-end performance bounds for a wide range of traffic source models, the traffic model and service model are expanded according to error function. Based on the proposed models, the explicit end-to-end delay bound of Fractional Brownian Motion (FBM) traffic is derived, the factors that affect the delay bound are analyzed, and a comparison between theoretical and simulation results is performed. The results illustrate that the proposed models not only fit the network behaviors well, but also facilitate the network performance analysis.

Key words: statistical network calculus, arrival curve, service curve, end-to-end delay bound

0 Introduction

Performance analysis for high network utilization and statistical service guarantees is a major challenge to network information theory[1]. Statistical network calculus has been recognized by researchers as a crucial step towards it[2]. In recent years, many attempts have been made for developing a statistical (or stochastic) network calculus[3]. Nevertheless, it has not yet developed maturely and still faces many difficulties.

One challenge is arrival modeling. Obviously, one goal of arrival modeling is to capture all characteristics of traffic as much as possible, now the results of statistical network calculus for the direction include models of EBB (exponentially bounded burstiness)[4], SBB (stochastically bounded burstiness)[5] and gSBB (generalized stochastically bounded burstiness) model[6]. The EBB model covers important classes such as multiplexed regulated traffic, Markov-modulated on-off traffic, and the Poisson process. With the continual emerging applications, network traffic exhibits long-range correlations characteristic which isn't depicted by the EBB model. Thus SBB is presented. The SBB model includes some classes but does not belong to EBB, such as fractional Brownian motion traffic. In fact, besides the long-range correlations and self-similitude, the aggregate traffic shows the characteristics of non-Gaussian. For the traffic with the characteristic of non-Gaussian, David Starobinski presented the gSBB model to capture the traffic behaviors. However, one aim of traffic modeling is to measure the performance for QoS guarantees, the feasibility of network performance analysis should be taken into account for the traffic modeling, which isn't considered in the current traffic modeling.

Another challenge is service modeling, especially the end-to-end service, which is a crucial step towards the end-to-end performance analysis. Based on the (min, +) convolution operation, the authors of Refs[3,7,8] formulated statistical end-to-end service curve, but failed to account for sample paths which are considered in Refs[6,9]. Although some achievements are obtained for statistical end-to-end performance model, the following issues need to be improved. Min-plus or max-plus algebra under statistical network calculus can resolve complex network modeling under some ideal assumption, such as restricting the network topologies to be concatenation, but it's a challenge if the assumptions don't exist. Also the algebra makes statistical network calculus rather eccentric to the general research community, especially to derive closed-form

① Supported by the National Natural Science Foundation Major Research Plan of China (No. 90718003), the National Natural Science Foundation of China (No. 60973027), and the National High Technology Research and Development Program of China (No. 2007AA01Z401).

② To whom correspondence should be addressed. E-mail: yanping1009@163.com

Received on Aug. 10, 2010

performance bounds for some traffic models.

In this paper, by extending the traffic and service model, we analyze and reason about the closed-form expressions of statistical end-to-end performance for a wide range of traffic source models in a network with complicated topology. The remainder of the paper is organized as follows. In Section 1, we introduce the basic theory of statistical network calculus which is the foundation of the following sections. In Section 2, the extended arrival and service curves are defined based on the theory in Section 1, and the service model of network with complex topologies is presented. In Section 3, we discuss an application of our main results to FBM traffic. Finally, brief conclusions are given in Section 4.

1 Statistical network calculus preliminaries

In this section, we review the notations and main results of statistical network calculus. Let $A(t)$ and $D(t)$ denote cumulation of the arrival process and departure process by time t respectively, and they are non-negative, non-decreasing, left-continuous functions with $A(t) = D(t)$ and $A(0) = D(0) = 0$.

Definition 1 (**statistical arrival curve**): A flow is said to have a stochastic arrival curve $\alpha \in F$ with error function $f \in \bar{F}$, if for all $0 \leqslant s \leqslant t$ and $x \geqslant 0$

$$P\{A(s, t) - \alpha(t - s) > x\} \leqslant f(x)$$

Definition 2 (**statistical service curve**): A server S is said to provide a stochastic service curve $\beta \in F$ with error function $f \in \bar{F}$, if for all $t \geqslant 0$ and $x \geqslant 0$

$$P(D(t) < A \otimes [\beta - x]_+ (t + \tau_0)) \leqslant f(x)$$

where $f(x)$ satisfies $\int_0^\infty f(u)\mathrm{d}u < \infty$. Let F represent the set of all functions $f(x)$, for all $0 \leqslant x \leqslant y$, $f(x) \geqslant 0$, there holds $F = \{f(\cdot): f(x) \leqslant f(y)\}$. Similarly, $\bar{F}$ is defined as $\bar{F} = \{f(\cdot): f(y) \leqslant f(x)\}$.

In the context of statistical network calculus, backlog and delay can be deduced from the arrival curve and service curve, which use two formulas: convolution and deconvolution. Here, we only introduce the convolution definition. For the deconvolution and other statistical network calculus theories, please see Refs[10-12].

Definition 3 (**convolution**): For all $f, g \in F$, $t \geqslant 0$, the (min, +) convolution of functions f and g is $(f \otimes g)(t) = \inf_{0 \leqslant s \leqslant t} \{f(s) + g(t - s)\}$

Definition 4 (**delay bound**): For all $t \geqslant 0$, the statistical delay W is given by

$$P\{W(t) > d(x)\} \leqslant \varepsilon(x) \quad (1)$$

Here, the delay bound d is defined by

$$d(x) = \inf\{d: S(s + d) \geqslant \alpha(s) + x\},\ s \geqslant 0 \quad (2)$$

here S, s represent the service curve and traffic arrival time respectively, and x represents delay bound parameter.

2 A statistical service model for networks with complex topologies

Arrival curve and service curve are the foundation of network performance modeling. In the following, the extended definitions of 1 and 2 are presented, and then the statistical service model for networks with complex topologies is explored.

2.1 The improved arrival curve and service curve

To unfold the improvement, we analyze the EBB, SBB and gSBB models, and their definitions given as follows:

$$P(Q(t) > x) \leqslant Me^{-\theta x} \quad (3)$$

$$P(Q(t) > x) \leqslant f(x) \quad (4)$$

$$P(\widehat{A}(t, \rho) > x) \leqslant g(x) \quad (5)$$

where $f(x)$ is a nonnegative, nonincreasing function and satisfies $(\int_x^\infty \mathrm{d}u)^n f(u) < \infty$. $g(x)$ is defined on $[0, \infty)$, and $1 - g(x)$ is a distribution function. These definitions imply that the expanding of traffic models is driven mainly by the error function change. To learn its influence on performance analysis, we analyze main methods to obtain statistical end-to-end performance: integration[9] and convolution[3]. At present, the integration method is only applied to EBB model. For the SBB and gSBB models, there exist integral difficulties. For example, for the SBB model with the error function $\varepsilon(x) = \beta e^{-\alpha x^r}$, $0 < r < 1$, the integral $\int_0^{+\infty} e^{-\alpha x^r}\mathrm{d}x$ can't be calculated. And the closed-form performance bounds using the convolution method are not seen yet. To realize the compatibility of covering more types of flows and facilitate end-to-end performance analysis for traffic models, we analyze the relationships of the three specific traffic models in advance.

The EBB model: $P(Q(t) > x) \leqslant Me^{-\theta x}$ (6)

FBM model (which belongs to the SBB model):

$$P(Q(t) > x) \leqslant Me^{-\theta x^r} \quad (7)$$

Long-tailed traffic model (which belongs to gSBB model):

$$P(Q(t) > x) \leqslant Kx^{-\alpha} \quad 0 \leqslant \alpha \leqslant 2 \quad (8)$$

Based on the trend of error function with x, inequations (4) and (5) are the generalization of inequations (7) and (8) respectively. According to the evolution of the traffic models, it can be seen that they can be formalized by composite functions, that is the error function is expressed with respect to $f(x)$. The advantage is to help the end-to-end statistical performance analysis, which is fully reflected in Section 4. According to the above ideas, the following definitions are obtained.

Definition 5 (improved arrival curve): A non-decreasing function $\alpha \in F$ is said to be a stochastic arrival curve with error function $\varepsilon \in \tilde{F}$ for an arrival process, refer to it as $A \sim_A \langle \varepsilon(y), \alpha \rangle$, $y = f(x)$, if for all $0 \leqslant s \leqslant t$ and all $x \geqslant 0$

$$P\left(\sup_{0<s<t}\left[A(s,t) - \alpha(t-s)\right] > x\right) \leqslant \varepsilon(y)$$

Definition 6 (improved service curve): A function $S(t)$ is a statistical service curve with error function ε for an arrival process A, denoted by $S \sim_S \langle \varepsilon(y), \beta \rangle$, if for all $t \geqslant 0$

$$P\left(\sup_{0\leqslant s\leqslant t}\{A \otimes [\beta - x]_+(s+\tau_0) - D(s)\} > 0\right) \leqslant \varepsilon(y)$$

here $\varepsilon(y) \in \tilde{F}$ and $y = f(x)$.

Remark: if $\varepsilon(y)$ satisfies $\int_0^\infty \varepsilon(u)\,du < \infty$ in definitions 5 and 6, the service curve can be applied to performance analysis of EBB and SBB traffic, and if the limitation is free, it can also be applied to the gSBB model.

2.2 Modeling end-to-end service for networks with complex topologies

End-to-end performance modeling relates to the network topology and routing. Current performance analysis is mainly done for the network with line topologies as illustrated in Fig. 1. However, the topology is complex and the routing is dynamic in practice network, such as the topology described in Fig. 2, modeling such network is a challenge in the context of stochastic network calculus. In this section, a preliminary solution is given.

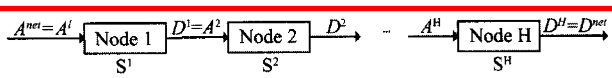

Fig. 1 Tandem network topology

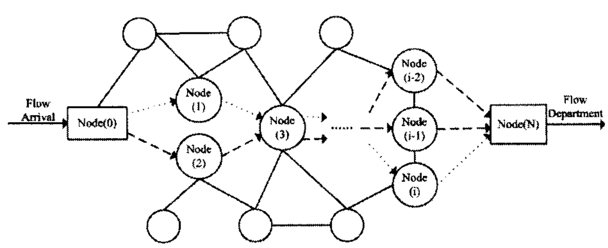

Fig. 2 Non-tandem network topology

Simple line topology is the foundation of a complex one. Using inductive method, the complex topology can be deformed and transformed into the simple line one. The method is to divide the nodes on the possible paths traversed by a flow into tandem virtual nodes, as shown in Fig. 3. One virtual node contains one or more nodes, and at any time, at least one node provides service to the flow. For simplicity, it is assumed that only one node in a virtual node provides service to the flow at any time.

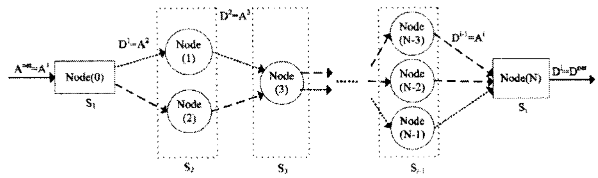

Fig. 3 Transformation of network structure

Theorem 1: In Fig. 3, let A^h, D^h, S^h, $h = 1, 2, \ldots, i$ denote the arrival process, departure process and service process of the h th virtual node respectively, with $D^h(t) = A^{h+1}(t)$. $A^{net} = A^1$, $D^{net} = D^H$ represent the network arrival process, departure process of through flow, and A^h, S^h satisfy the definition of 5 and 6 respectively, that is $S^h \sim_S \langle \varepsilon^h(y), \beta^h \rangle$, $A^h \sim_A \langle \varepsilon^h(y), \alpha^h \rangle$, here $\beta^h = \min_{j\in N}\{\beta^{hj}\}$, N denotes the number of nodes in the h th virtual node, then stochastic service curve provided by network for the flow satisfies ${}_S\langle \varepsilon^{net}(y), S^{net} \rangle$, it is expressed by

$$S^{net} = S^1 \otimes S^2_{-\delta} \otimes \ldots \otimes S^i_{-(i-1)\delta} \tag{9}$$

$$\varepsilon^{net}(y) = \inf_{y^1+y^2+\ldots+y^i=y}\left\{\varepsilon^i(y^i) + \sum_{h=1}^{i-1}\frac{1}{\delta\tau_0}\int_{y^h}^{\infty}\varepsilon^h(u)\,du\right\} \tag{10}$$

where $y = f(x)$.

The presented model extends the conclusion in Ref. [9] from the following two aspects. (1) Enlarge classes of applicable traffic models. In Ref. [9], $\int_\sigma^\infty \varepsilon(x)\,dx$ must be integral and bounded, which limit the application of theorem to EBB model. While theorem 1 presented in this paper can also be applied to SBB model. (2) Extend line structure of topologies for end-to-end performance modeling, which is a step toward analyzing the performance of arbitrary complex network topologies.

3 Statistical end-to-end delay bound analysis using the improved models

To illustrate the feasibility of the method presented in this paper, the end-to-end delay bound of FBM traffic under non line topology is studied, and then comparison between the simulation result and theoretical result is performed to validate the fitting between modified model and practical network. The topology and parameters of the network are given in Fig. 4, and real-traffic traces BC-pAug89. TL, which were collected at the Bellcore, are used as the traffic source. Fractional Gaussian Noise (FGN) is used as the traffic model, because of its good fitting to BC-pAug89. TL traces, it is shown in Fig. 5. The statistical multiplexing FGN traffic conforms to the FBM model which belongs to the SBB model.

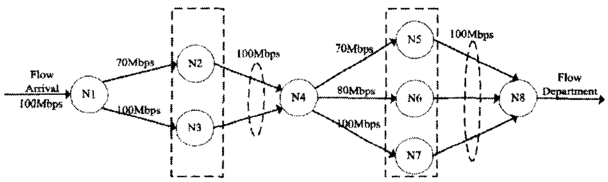

Fig. 4 Network topology for simulation

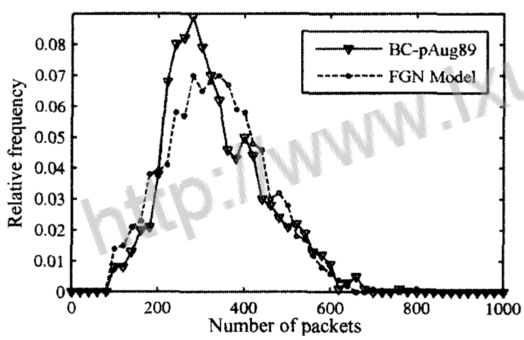

Fig. 5 The Fitting of the FGN model to dataset (BC-pAug89) with self-similar property

The statistical arrival process A of the FBM traffic is defined as

$$P(A(t) - A(s) > \rho(t-s) + x) \leqslant Me^{-\theta x^r} \tag{11}$$

here ρ is the mean rate of arrival traffic, $0 < r < 1$ is the self-similar parameter, θ is the decay rate of the error function, $M \geqslant 1$ is a constant. Let $\varepsilon(y) = Me^{-\theta f(x)}$ and $y = f(x) = x^r$, the arrival process A of the FBM traffic satisfies $A \sim_A \langle \rho, \varepsilon(y) \rangle$. By definition 8 and Theorem 3 in Ref. [9], the service curve at the hth node for the through flows satisfies

$$S^h(t) = (C - \rho_c - \delta)t \tag{12}$$

with error function given by $\varepsilon^{s,h}(y) = \frac{1}{\delta\tau_0}\int_y^{\infty} \varepsilon(u)\,\mathrm{d}u = \frac{1}{\delta\tau_0}\int_u^{\infty} Me^{-\eta u}\mathrm{d}u = \frac{M}{\eta\delta\tau_0}e^{-\eta y}$, and $y = f(x) = (x - C\tau_0)^r$. According to Theorem 1, the network service curve for the FBM traffic is denoted by

$$S^{net}(t) = (C - \rho_c - H\delta)t \tag{13}$$

The error function is given by

$$\begin{aligned}\varepsilon^{s,net}(y) &= \inf_{y^1+y^2+\dots+y^H=y}\left\{\varepsilon^H(y^H) + \sum_{h=1}^{H-1}\frac{1}{\delta\tau_0}\int_y^{\infty}\varepsilon^{s,h}(u)\,\mathrm{d}u\right\} \\ &= \inf_{y^1+y^2+\dots+y^H=y}\left\{\frac{M}{\eta\delta\tau_0}e^{-\eta y^H} + \sum_{h=1}^{H-1}\frac{M}{(\eta\delta\tau_0)^2}e^{-\eta y^h}\right\}\end{aligned}$$

The optimal result based on Ref. [9] is obtained:

$$\varepsilon^{s,net}(y) = M\frac{H}{(\theta\delta\tau_0)^{\frac{2H-1}{H}}}e^{-\frac{\theta}{H}y} \tag{14}$$

here $y = (x - C\tau_0)^r$.

Given arrival process $A \sim_A \langle Me^{-\theta x^r}, \rho t \rangle$ and service curve $S \sim_S \langle \varepsilon^{net}, S^{net} \rangle$, by the definition 4, the delay is upper bounded as

$$P\{W^{net}(t) > d\} \leqslant M^{net}e^{-\frac{\theta}{H+1}x^r} \tag{15}$$

where $M^{net} = M(1 + H)(1/\theta\delta\tau_0)^{\frac{2H}{1+H}}$. According to Eq. (2), arrival curve A and service curve S, there holds

$$d(x) = \frac{x}{C - \rho_c - H\delta} \tag{16}$$

Substituting x in inequation (15) by $(C - \rho_c - H\delta)d$, inequation (17) is obtained

$$P\{W^{net}(t) > d\} \leqslant M^{net}e^{-\frac{\theta}{H+1}(C-\rho_c-H\delta)^r d^r} \tag{17}$$

Set the right side of inequation (17) equal to ε, Eq. (18) is yielded

$$d = \frac{1}{C - \rho_c - H\delta}\frac{(H+1)^{\frac{1}{r}}}{\theta^{\frac{1}{r}}}[\log(M^{net}/\varepsilon)]^{\frac{1}{r}} \tag{18}$$

Together with Eq. (18), the factors that affect delay bound is analyzed. Fig. 6 depicts the delay bound as a function of the number of nodes traversed by the through flows. It is obvious that the delay bound increases with load and number of nodes along the network path. Traffic has long-range and short-range correlation under self-similar parameters $r > 0.5$ and $r < 0.5$ respectively, and its two scenarios are illustrated in Fig. 7 and Fig. 8. As shown in Fig. 7(a) – (c), the tiny changes of self-similar parameter r result in large fluctuation of delay bound when traffic has short - range

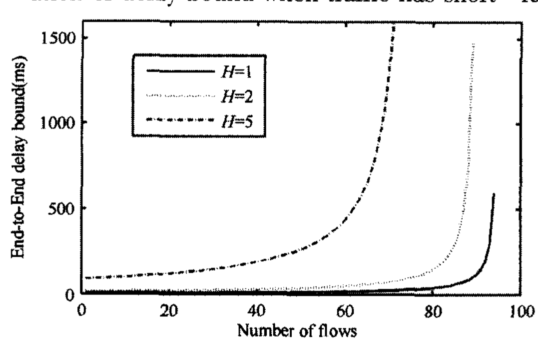

Fig. 6 Delay bound in network with node number ($r = 0.5$)

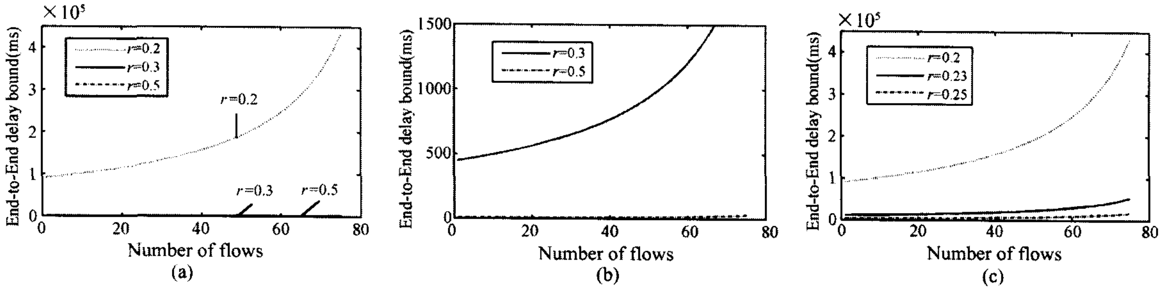

Fig. 7 Delay bound trend ($r \leqslant 0.5$, $H = 1$)

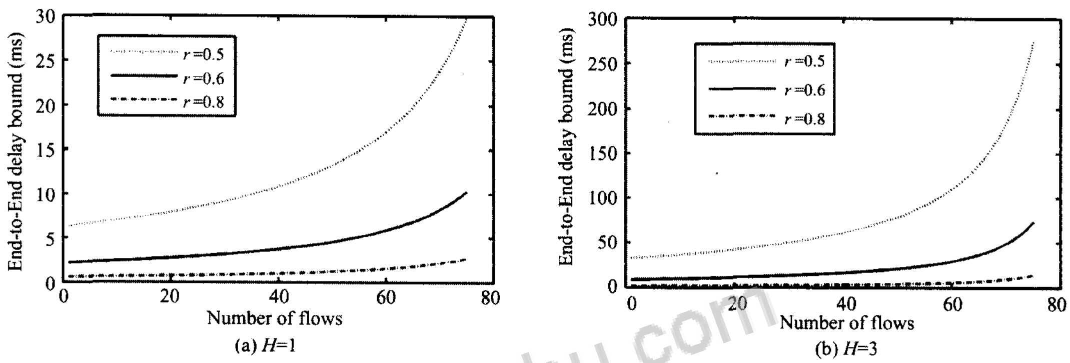

Fig. 8 Delay bound trend ($r \geqslant 0.5$)

correlation. Fig. 8 gives the situation under $r > 0.5$. Compared with $r < 0.5$, self-similar parameters affect delay bounds softly. In addition, the delay bound increases with the number of nodes traversed by the FBM traffic for different self-similar parameters.

To analyze the influence of network load on delay bound and illustrate the fitting between the theoretical analysis and situation in practical network, the simulation is performed for network topology shown in Fig. 4 ($H = 5$). The traffic data are obtained from the file BC-pOct89. TL(r = 0. 83[13]). Fig. 9 shows that delay bounds have almost the same trend for theoretical analysis and simulation, it is proved that the theoretical result fit with the simulation. But, the theoretical result is more conservative. The reason is based on the following facts: (1) There are discrepancies between the

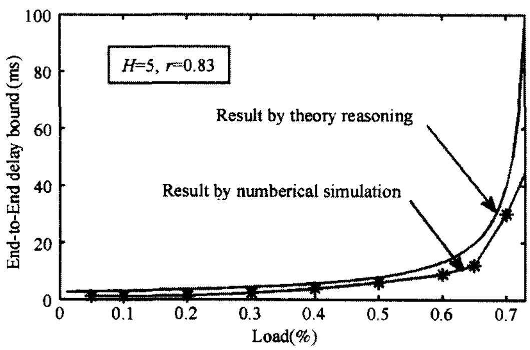

Fig. 9 Theoretical and simulation results on end-to-end delay bound

traffic model and the real traffic traces, which is shown in Fig. 7; (2) the service model is conservative, especially for dynamic rooting. It is assumed that only one service node in one virtual node provides minimal service, which leads to the conservative performance in this paper.

The above analysis shows that Eq. (18) fully describes the relationship between the delay bound and network factors including traffic characteristic, number of hops (or network topology) and the network load.

4 Conclusions

In this paper, the service model characterized by a service process of network and the traffic model characterized by an arrival process of traffic are extended, such that closed-form statistical end-to-end performance bounds can be obtained for more types of business under network with complex topologies. As an application, the closed-form expression of delay bound of FBM traffic is derived and the factors affecting the delay bound are studied, which states the models presented expand the traffic types for closed-form performance bounds; Comparison of theoretical reason and simulation results illustrates the theoretical analysis fits practical situation well.

References

[1] Burchard A, Liebeherr J, Patek S. A min-plus calculus for end-to-end statistical service guarantees. *IEEE Trans-*

actions on Communications, 2006, 52(9):4105-4114

[2] Liu Y, Chen-Khong T, Jiang Y M. A calculus for stochastic QoS analysis. *Performance Evaluation*, 2007, 64(6):547-572

[3] Jiang Y M. A basic stochastic network calculus. In: Proceedings of ACM SIGCOMM Conference on Network Architectures and Protocols, New York, USA, 2006. 123-134

[4] Yaron O, Sidi M. Performance and stability of communication networks via robust exponential bounds. *IEEE/ACM Transactions on Networking*, 1993, 1(3):372-385

[5] Starobinski D, Sidi M. Stochastically bounded burstiness for communication networks. *IEEE Transactions on Information Theory*, 2000, 46(1):206-212

[6] Yin Q H, Jiang Y M, Jiang S M, et al. Analysis on generalized stochastically bounded bursty traffic for communication networks. In: Proceedings of the 27th Annual IEEE Conference on Local Computer Networks, USA 2002. 141-149

[7] Fan B H, Zhang H Y, Dou W H. A max-plus network calculus. In: Proceedings of 8th IEEE/ACIS International Conference on Computer and Information Science, Shanghai, China, 2009. 149-154

[8] Liu Y, Tham C, Jiang Y. A stochastic network calculus: [Technical Report]. Department of Electrical and Computer Engineering, National University of Singapore, 2003

[9] Ciucu F, Burchard A, Liebeherr J. Scaling properties of statistical end-to-end bounds in the network calculus. *IEEE Transactions on Information Theory*, 2006, 52(6): 2300-2312

[10] Jiang Y, Liu Y. Stochastic Network Calculus. London: Springer, 2008. 1-17

[11] Markus F. A survey of deterministic and stochastic service curve models in the network calculus. *IEEE Communications Surveys and Tutorials*, 2010, 12(1): 59-86

[12] Kobayashi K, Takahashi Y, Takada H. A stochastic network calculus for many flows. In: Proceedings of the 21st International Teletraffic Congress: Traffic and Performance Issues in Networks of the Future, Paris, France, 2009. 1-8

[13] Ramirez Pacheco J C, Torres Roman D. Local and cumulative analysis of self-similar traffic traces. In: Proceedings of the 16th International Conference on Electronics, Communications and Computers, Waxhington DC, USA, 2006. 27-33

Chen Yanping, born in 1981. She received her B. S. and M. S. degrees from Harbin University of Commerce and Harbin Institute of Technology in 2005 and 2007 respectively. Her research interests include the theory and modeling of cognitive network, network calculus and so on.