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Performance Analysis of a Cognitive Radio Network with Imperfect Spectrum Sensing

Yuehong Gao†‡ and Yuming Jiang‡
†Wireless Theories and Technologies (WT&T) Lab
Beijing University of Posts and Telecommunications (BUPT), 100876 Beijing, China
‡Centre for Quantifiable Quality of Service in Communication Systems (Q2S)
Norwegian University of Science and Technology (NTNU), 7491 Trondheim, Norway

***Abstract*—In this paper, we use stochastic network calculus to analyze a cognitive radio network, where influences of imperfect spectrum sensing and different retransmission schemes are considered. In particular, stochastic arrival curves for spectrum sensing error processes are derived firstly, based on which stochastic service curve for each class of users is obtained under different retransmission schemes, including without retransmission, retransmission until success and maximum-N-times retransmission. Then backlog and delay bounds for primary and secondary users are derived. Finally, numerical results are shown for different types of traffic, where the influence of different retransmission schemes is further discussed.**

I. Introduction

Cognitive radio is a newly proposed wireless communication technology [3], which can improve spectrum utilization and thus increasing communication demands can be better fulfilled. In this paper, we adopt the opportunistic secondary spectrum access view of cognitive radio. In such a cognitive radio network, there are two types of users, namely primary users (PU) and secondary users (SU). Secondary users have the ability to sense and use available spectrum holes when primary users do not transmit data on the assigned spectrum. However, spectrum sensing errors may happen due to uncertainty of wireless channels and unpredictable interference, and imperfect spectrum sensing can influence system performance. It is hence important to conduct performance analysis for both primary and secondary users taking into consideration the impact of imperfect sensing.

Among existing analysis tools, queueing theory has been proved to be a useful method to deal with queueing problems in communication networks. It has also been employed in performance analysis of cognitive radio networks [2, 7, 9], where some results have been obtained, such as packet waiting time in queue and delay. However, the focus has mainly been on average values in steady states. In addition, the current analysis mostly assumes $M/G/1$ model with Poisson arrival, and the obtained results cannot be easily extended to other types of arrival processes. Moreover, the influence of imperfect spectrum sensing is not well considered either; although some results based on Monte Carlo simulation are reported in [7], no analytical research is known. Besides, the impact of retransmission is not found.

The objective of this paper is to conduct performance analysis of a cognitive radio network by considering both spectrum sensing and retransmission. Specifically, stochastic network calculus [1, 4–6] is employed to analyze performance distribution bounds. First, we obtain stochastic arrival curve for the sensing error process, followed by the derivation of stochastic service curve for both primary users and secondary users under different retransmission schemes. Then performance analysis is conducted based on stochastic network calculus, where expressions for backlog and delay bounds are shown. Lastly, numerical results under various configurations are presented for both Poisson traffic and (σ, ρ)-constraint traffic, where the impact of retransmission is also discussed.

The paper is organized as following. The cognitive radio network is modeled as a preemptive queueing system in Section II, where basic assumptions, traffic and server models as well as retransmission schemes are described. In Section III, we first derive stochastic arrival curves and bounding functions for the sensing error process based on different retransmission mechanisms. We then analyze stochastic service guarantees provided to the primary and secondary users, followed by general expressions of performance bounds. Numerical results are shown in Section IV, from which more insights are discussed. The summary is given in the last section.

II. System Model

A. System Model

In this paper, we consider a cognitive radio network with two input flows as illustrated in Fig.1, where fl^P and fl^S represent the aggregated flows from primary users and secondary users, respectively. For ease of expression and with the focus on the impact of sensing error and retransmission, the wireless channel is assumed to be error free with a constant service rate C. The analysis can be easily extended to consider stochastical

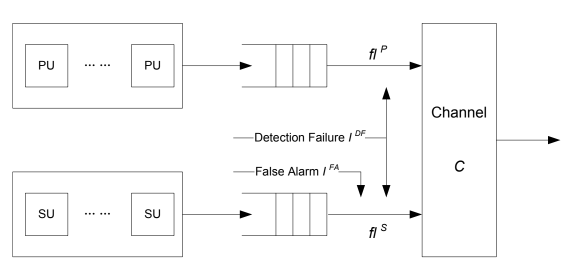


Fig. 1. System Model

channel that can be expressed with a stochastic service curve. The primary users' flow has preemptive priority over the secondary users' flow. If one packet arrives into the system and cannot be transmitted immediately, it will be stored in the corresponding buffer in a First-In-First-Out (FIFO) manner, where the buffer is assumed to be large enough and therefore no packet will be dropped.

The system is supposed to be synchronized and the time is divided into slots with length T and indexed by $[0, 1, ..., s, ..., t, ...]$. At the beginning of each slot, secondary users will try to sense the spectrum to decide whether it is idle or busy. In this paper, it is assumed that the time period used for spectrum sensing is small and its effect is not considered. It is also assumed that PUs and SUs can negotiate respectively among themselves before transmitting so no collision will happen between PUs or between SUs. However, spectrum sensing errors may occur and they can have significant influence on the system performance.

Typically, spectrum sensing errors can be classified into two types [8], i.e., mis-detection (MD) and false alarm (FA). Mis-detection means that the spectrum is occupied by PUs but the spectrum sensing result says it is available for SUs, which will result in transmission collision and influence both PUs' and SUs' current transmission. However, false alarm occurs in the opposite way, when SUs believe that the spectrum is being used by PUs but actually the spectrum is idle, which will waste transmission opportunities for SUs. Let p_e denote the average probability that a sensing error (either MD or FA) happens in one time slot. Let ϕ be the probability that this error is a mis-detection. Then the average probability in one time slot for MD and FA can be respectively expressed as $p_e^{MD} = p_e \cdot \phi$ and $p_e^{FA} = p_e \cdot (1 - \phi)$.

Generally, packet arrivals and sensing errors are stochastic processes, which will only lead to stochastic service guarantees. While the system model described above has already been overly simplified, to the best of our knowledge, it is difficult (if not impossible) to obtain explicit results from the traditional queueing theory, particularly when the involving stochastic processes are not Poisson or with exponentially distributed rates. To address this problem, we resort to stochastic network calculus.

B. Stochastic Network Calculus Basics

Stochastic network calculus is a newly developed queueing theory for service guarantee analysis (e.g., [4–6] and references therein) , which contains two fundamental concepts, stochastic arrival curve and stochastic service curve.

In stochastic network calculus, a stochastic arrival curve (SAC) is used to describe the stochastic characteristics of a flow. There are several definitions for SAC, and in this paper the following definition is used, which explores the *virtual backlog property* of deterministic arrival curve [5].

Definition 1. ***(Stochastic Arrival Curve).*** *A flow $A(t)$ is said to have a virtual-backlog-centric (v.b.c) stochastic arrival curve $\alpha \in F^1$ with bounding function $f \in \bar{F}^2$, denoted by $A(t) \sim_{sac} \langle f, \alpha \rangle$, if for all $t \geq 0$ and all $x \geq 0$ there holds:*

$$P\left\{\sup_{0 \leq s \leq t} \{A(s,t) - \alpha(t-s)\} > x\right\} \leq f(x).$$

In Definition 1, $A(s,t)$ denotes the cumulative amount of traffic of the flow during period $(s, t]$, $A(t) = A(0, t)$, and $\alpha(t)$ is a non-decreasing function.

While the stochastic arrival curve describes the traffic, the stochastic service curve shows the service guarantee provided by a server. Similarly with SAC, stochastic service curve (SSC) can also be defined in different ways. In this paper, we use the following one [5].

Definition 2. ***(Stochastic Service Curve).*** *A system S is said to provide a stochastic service curve $\beta \in F$ with bounding function $g \in \bar{F}$, denoted by $S \sim_{ssc} \langle g, \beta \rangle$, if for all $t \geq 0$ and all $x \geq 0$ there holds:*

$$P\{A \otimes \beta(t) - A^*(t) > x\} \leq g(x).$$

Here, $A \otimes \beta(t) \equiv \inf_{0 \leq s \leq t} \{A(s) + \beta(t-s)\}$, and $A^*(t)$ denote the cumulative output traffic amount up to time t.

Given SAC and SSC, the following bounds have been derived under stochastic network calculus [5]:

Theorem 1. *Consider a system S with input A. Suppose the input has a v.b.c stochastic arrival curve as $A \sim_{sac} \langle f, \alpha \rangle$; and server S provides the input with a stochastic service guarantee as $S \sim_{ssc} \langle g, \beta \rangle$. Then for any $t \geq 0$ and $x \geq 0$, the backlog $B(t)$ and delay $D(t)$ is bounded by:*

$$\begin{aligned} P\{B(t) > x\} &\leq [f \otimes g(x - \alpha \oslash \beta(0))]_1 \\ P\{D(t) > h(\alpha + x, \beta)\} &\leq [f \otimes g(x)]_1 \end{aligned}$$

where $\alpha \oslash \beta(0) = sup_{u \geq 0}\{\alpha(u) - \beta(u)\}$, $h(\alpha + x, \beta) = sup_{s \geq 0}\{inf\{\tau \geq 0 : \alpha(s) + x \leq \beta(s + \tau)\}\}$ and $[\cdot]_1$ denotes $max(min(\cdot, 1), 0)$.

In order to apply stochastic network calculus results to performance analysis of the considered cognitive radio network, a critical challenge is to find stochastic service curve for both PUs and SUs. Major contribution of this paper is in finding out stochastic service curve provided to both PUs and SUs, considering sensing errors and retransmission schemes, which are presented below.

C. Retransmission Schemes

As discussed above, transmission collisions may happen due to sensing errors, which may significantly influence the system performance. Retransmission technology is a commonly used method to deal with transmission errors, and different schemes can result in different outcomes. In this paper, the following three retransmission schemes will be discussed, where the first two are extreme cases and the third one is a tradeoff.

[1] F: the set of non-negative wide-sensing increasing functions
[2] $\bar{F}$: the set of non-negative wide-sensing decreasing functions

1) WithOut-ReTransmission (WO-RT): In this scheme, it is assumed that there is no retransmission. In other words, when one packet is transmitted through the wireless channel, it will be removed from the waiting queue no matter whether it will be received correctly or not. Therefore, sensing error process will not influence backlog and delay, but will affect transmission error.

2) ReTransmission until Success (RT-S): Compared with WO-RT, RT-S goes to the other extreme. One packet will be removed from the waiting queue only if it has been received by the receiver successfully. Otherwise, it will be backlogged in buffer as long as needed. Therefore, no transmission error will occur. However, spectrum sensing impairments will lead to larger backlogged queue and longer waiting time.

3) Max-N-times ReTransmission (Max-N-RT): This scheme is a tradeoff between WO-RT and RT-S, in which one packet can be retransmitted at most N times. After that, the packet will be removed from the queue no matter it has been received correctly or not. It can be expected from Max-N-RT that the transmission error can be reduced to some extend as compared with WO-RT, while the backlog and delay can be better guaranteed as compared with RT-S.

III. Performance Analysis

In this section, performance analysis of the considered cognitive radio network is conducted. The focus is on finding probabilistic bounds on delay and backlog of both primary users (PUs) and secondary users (SUs), and the theoretical tool we rely on is stochastic network calculus. As highlighted in Sec.II.B, in order to apply Theorem 1, it is essential to find stochastic service curves for both PUs and SUs which take into consideration sensing errors and consider different retransmission schemes. To achieve this, we present in the following an analytical approach. First, we study the impact of the sensing error process under different retransmission schemes. Particularly, if a collision happens due to sensing error *and* the collided packets need retransmission, we say the corresponding amount of service (i.e., the corresponding time slot) has been *wasted*, and we shall characterize the *wasted* service with stochastic arrival curve. Then, we treat the *wasted* service process as an interference process, and establish the relationship between the interference process and the stochastic service curve. Based these, we conclude stochastic service curves for both PUs and SUs, where for SUs, we further treat the arrivals from PUs as an interference process to SUs. Finally, we present delay and backlog bounds for both PUs and SUs based on stochastic network calculus results. Throughout the analysis, the three different retransmission schemes are studied.

A. Impact of Sensing Error

In this subsection, we study the impact of sensing error. Particularly, we focus on the sensing error impact on the amount of service that will otherwise be delivered to the users successfully. We shall say such service is *wasted* to the sender in the sense that it has not helped in reducing the number of packets in the sending queue. We shall characterize the *wasted* service process using stochastic arrival curve.

For WO-RT, interestingly, there is no *wasted* service to the sending queue. This is due to that even though the packet under transmission is collided, the sending queue (no matter whether it belongs to PUs or SUs) does not care and the packet is still removed from the corresponding buffer. As a result, from the sending queue viewpoint, it works just as if there had been no error, and hence the *wasted* service to the sending queue is zero. However, if retransmission takes place due to sensing error, some service will be *wasted* as seen by the sending queue, since no packet is moved out of the queue in a *wasted* service slot.

Assume the sensing error probability is the same on each time slot, denoted by p_e. The average number of errors during any time period $(s,t]$ will be $p_e(t-s)$. Let $I_n(s,t)$ denote the number of sensing errors during $(s,t]$, and $\gamma_n(s,t) = K \cdot p_e(t-s)$, where $K > 0$ is a constant parameter facilitating later analysis in obtaining performance bounds.

For RT-S, the equivalent amount of *wasted* service in $(s,t]$ can be expressed as $I(s,t) = I_n(s,t) * CT$. Let $M_X(\theta)$ denote the moment generating function of random variable X, i.e., $M_X(\theta) = E[e^{\theta X}]$ for any $\theta > 0$. Then, we have:

Lemma 1. *The* wasted *service process I under RT-S has a stochastic arrival curve $\langle f^I, \gamma^I \rangle$, where*

$$\begin{aligned} \gamma^I(s,t) &= K \cdot p_e(t-s) \cdot CT \\ f^I(x) &= e^{-\frac{\theta x}{CT}} \frac{e^{-\theta K p_e}(p_e e^{\theta} + 1 - p_e)}{1 - e^{-\theta K p_e}(p_e e^{\theta} + 1 - p_e)} \end{aligned}$$

for any $\theta > 0$.

Proof: By using definition of stochastic arrival curve, Boole's inequation and Chernoff bound, we have:

$$\begin{aligned} & P\{\sup_{0 \le s \le t} \{I(s,t) - \gamma^I(s,t)\} > x\} \\ = \quad & P\{\sup_{0 \le s \le t} \sum_{k=s}^{k=t-1} [I_n(k,k+1) - \gamma_n(k,k+1)] > \frac{x}{CT}\} \\ \le \quad & e^{-\frac{\theta x}{CT}} \sum_{s=0}^{s=t-1} E[e^{\theta \sum_{k=s}^{k=t-1} [I_n(k,k+1) - \gamma_n(k,k+1)]}] \\ \le \quad & e^{-\frac{\theta x}{CT}} \sum_{i=1}^{i=+\infty} E[e^{\theta [I_n(1) - \gamma_n(1)]}]^i \\ = \quad & e^{-\frac{\theta x}{CT}} \frac{M_{I_n(1)-\gamma_n(1)}(\theta)}{1 - M_{I_n(1)-\gamma_n(1)}(\theta)} \equiv f^I(x). \end{aligned}$$

Since $\gamma_n(1) = K p_e$ and $I_n(1)$ is a Bernoulli distributed random variable, its moment generating function can be obtained as $E[e^{\theta[I_n(1)-\gamma_n(1)]}] = E[e^{\theta I_n(1)}] E[e^{-\theta K p_e}] = e^{-\theta K p_e}(p_e e^{\theta} + 1 - p_e)$. Then, the process I has a stochastic arrival curve as $\langle f^I, \gamma^I \rangle$. ■

Similarly, the stochastic arrival curve characterization of the corresponding *wasted* service processes due to mis-detection I^{MD} and false alarm I^{FA} can be found in the same way by

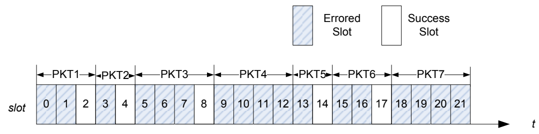


Fig. 2. An Example of Max-N-Times Transmission

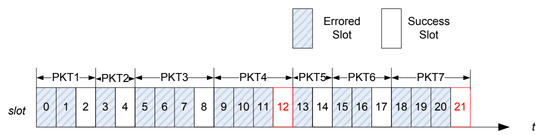


Fig. 3. Equivalent Model of Max-N-Times Transmission

replacing p_e in Lemma 1 respectively with ϕp_e and $(1-\phi)p_e$:

$$I^{MD} \sim_{sac} \langle f^{MD}, \gamma^{MD} \rangle, I^{FA} \sim_{sac} \langle f^{FA}, \gamma^{FA} \rangle$$

For Max-N-RT, before presenting the result, let us consider an example as shown in Fig.2 to see how it works.

Fig.2 shows an example of this scheme with $N = 3$. It is shown that Packet 4 and Packet 7 are not transmitted successfully after 4 times transmission, but they will be removed from the buffer and the next packet will be served. In other words, the result of the 3rd retransmission will not influence the operation of the sending queue. Therefore, we can use Fig.3 as an equivalent model, in which only the shadowed slots will influence the delay and backlog of the sending queue and are considered as *wasted*, the 3rd retransmission slots (such as slot 12 and 21) can be treated as a succeeded slot like slot 2 and 4.

Let us call all of the shadowed slots in the equivalent model as "wasted slots", and let $I'_n(s,t)$ denote the number of such slots in an equivalent system during time period $(s,t]$. It is easy to find that there are at least $\frac{1}{N+1}(t-s)$ succeeded slots during $(s,t]$. Therefore, we can know that $I'_n(s,t) \leq \frac{N}{N+1} I_n(s,t)$. In addition, the average number of failed slots will become $\frac{N}{N+1} p_e(t-s)$, and γ'_n can be expressed as $\frac{N}{N+1}\gamma_n$. Let $\eta = \frac{N}{N+1}$ denote the retransmission efficiency. Then, we have

$$\begin{aligned} & P\left\{ \sup_{0\leq s\leq t} \{I'(s,t) - \gamma^{I'}(s,t)\} > x \right\} \\ \leq \; & P\left\{ \sup_{0\leq s\leq t} \{\eta I_n(s,t) - \eta\gamma_n^I(s,t)\} > \frac{x}{CT} \right\} \\ \leq \; & e^{-\frac{\theta x}{CT}} \frac{M_{\eta I_n(1)-\eta\gamma_n^I(1)}(\theta)}{1 - M_{\eta I_n(1)-\eta\gamma_n^I(1)}(\theta)} \equiv f^{I'}(x) \end{aligned}$$

where $\langle f^{I'}, \gamma^{I'} \rangle$ is the stochastic arrival curve characterization of the *wasted* service process due to sensing error. Formally, we have proved the following result:

Lemma 2. *The* wasted *service process I under Max-N-RT has a stochastic arrival curve $\langle f^{I'}, \gamma^{I'} \rangle$, where*

$$\begin{aligned} \gamma^{I'}(s,t) &= \eta \cdot K \cdot p_e(t-s) \cdot CT \\ f^{I'}(x) &= e^{-\frac{\theta x}{CT}} \frac{e^{-\eta\theta K p_e}(p_e e^{\eta\theta} + 1 - p_e)}{1 - e^{-\eta\theta K p_e}(p_e e^{\eta\theta} + 1 - p_e)} \end{aligned}$$

for any $\theta > 0$.

In the same way, the stochastic arrival curve characterization of the corresponding *wasted* service processes due to misdetection $I^{MD'}$ and false alarm $I^{FA'}$ under the Max-N-RT scheme can be found by replacing p_e in Lemma 2 respectively with ϕp_e and $(1-\phi)p_e$:

$$I^{MD'} \sim_{sac} \langle f^{MD'}, \gamma^{MD'} \rangle, I^{FA'} \sim_{sac} \langle f^{FA'}, \gamma^{FA'} \rangle.$$

B. Stochastic Service Curves of Users

For ease of expression, we have assumed that the channel is error-free. Under this assumption, the channel provides a constant strict service guarantee, i.e., $\hat{\beta} = Ct$ (for all $t \geq 0$), which can be considered as a special form of stochastic service curve with bounding function $\hat{g}(x) = 0$ for any $x \geq 0$. In the following, we apply a concept called *interference process* [4, 5] to facilitate finding stochastic service curves for PUs and SUs. Particularly, in a system with interference, the interfered service will be treated as *wasted* or cannot be used by the sender. For the considered cognitive radio network, the performance of both PUs and SUs is influenced by some interference processes. Particularly, for PUs, the *wasted* service process due to sensing error is an interference process. For SUs, in addition to the *wasted* service process due to sensing error, the packet arrival process of PUs can also be treated as an interference process.

The following result establishes the link between the interference process and the stochastic service guarantee.

Theorem 2. *For the considered cognitive radio network, if the interference process I to an input flow F (either fl^p or fl^s) has a stochastic arrival curve $\langle g^I, \beta^I(t) \rangle$, then the network provides to the flow a stochastic service curve $\langle g^I, Ct - \beta^I(t) \rangle$.*

Proof: Let $R(t)$ and $R^*(t)$ denote the sum of inputs and outputs from flow F and interference process I, respectively, i.e., $R(t) = F(t) + I(t)$ and $R^*(t) = F^*(t) + I^*(t)$. Since the output traffic will not be larger than the input traffic, we have $F^*(t) \leq F(t)$, $I^*(t) \leq I(t)$ and $R^*(t) \leq R(t)$. It is easy to find that, for any $s \geq 0$,

$$\begin{aligned} & F(s) \otimes \left(\hat{\beta}(s) - \beta^I(s)\right) - F^*(s) \\ = \; & (R(s) - I(s)) \otimes \left(\hat{\beta}(s) - \beta^I(s)\right) - (R^*(s) - I^*(s)) \\ = \; & \inf_{0\leq u\leq s} \left[R(u) + \hat{\beta}(s-u) - \beta^I(s-u) - I(u)\right] \\ & - (R^*(s) - I^*(s)) \\ \leq \; & \inf_{0\leq u\leq s} \left[R(u) + \hat{\beta}(s-u)\right] - R^*(s) \\ & + I(s) - \inf_{0\leq u\leq s} \left[\beta^I(s-u) + I(u)\right] \\ \leq \; & \sup_{0\leq u\leq s} \left[I(u,s) - \beta^I(s-u)\right] \end{aligned}$$

where the last step is due to that for a constant rate server with rate C, it has been shown in the literature (e.g., see [5]) that $R \otimes \hat{\beta}(t) \leq R^*(t)$ for $\hat{\beta}(t) = C \cdot t$ which is the case here.

Since the interference process I has stochastic arrival curve

TABLE I
STOCHASTIC SERVICE GUARANTEE

RT Scheme	PUs Flow	SUs Flow
WO-RT	$\langle 0, \hat{\beta}\rangle$	$\langle f^P, \hat{\beta}-\alpha^P\rangle$
RT-S	$\langle f^{MD}, \hat{\beta}-\gamma^{MD}\rangle$	$\langle f^P \otimes f^I, \hat{\beta}-\alpha^P-\gamma^I\rangle$
MAX-N-RT	$\langle f^{MD'}, \hat{\beta}-\gamma^{MD'}\rangle$	$\langle f^P \otimes f^{I'}, \hat{\beta}-\alpha^P-\gamma^{I'}\rangle$

$\langle g^I, \beta^I(t)\rangle$, then based on the definition of SAC, we have:

$$\begin{aligned} & P\left\{F \otimes (\hat{\beta}-\beta^I) - F^*(s) > x\right\} \\ \leq \quad & P\left\{\sup_{0\leq u\leq s}[I(u,s)-\beta^I(s-u)] > x\right\} \leq g^I(x) \end{aligned}$$

which ends the proof. ∎

Based on this theorem and different retransmission schemes, we can get the service guarantees provided to users in each scenario as summarized in Table I.

C. Performance Bounds

With the stochastic service curves obtained above, performance bounds for each type of users can be immediately obtained from Theorem 1 and are presented below.

Theorem 3. *For the considered network, suppose respectively flow fl^P and flow fl^S have stochastic arrival curves as $A^P(t) \sim_{sac} \langle f^P, \alpha^P\rangle$ and $A^S(t) \sim_{sac} \langle f^S, \alpha^S\rangle$. For the stochastic service curves received by them, we denote by $S^P_{RT}(t) \sim ssc\langle g^P_{RT}, \beta^P_{RT}\rangle$ and $S^S_{RT}(t) \sim ssc\langle g^S_{RT}, \beta^S_{RT}\rangle$, where $RT \in \{WO-RT, RT-S, MAX-N-RT\}$, and for each combination, the corresponding bounding function and service curve are found in Table I. Then the backlog $B(t)$ and delay $D(t)$ distribution bounds can be expressed as:*

$$P\{B^U(t) > x\} \leq [f^U \otimes g^U_{RT}(x - \alpha^U \oslash \beta^U_{RT}(0))]_1$$
$$P\{D^U(t) > h(\alpha^U + x, \beta^U_{RT})\} \leq [f^U \otimes g^U_{RT}(x)]_1$$

where $U \in \{P, S\}$.

IV. NUMERICAL RESULTS

In Section III, the stochastic service curve for each flow is obtained. Given the stochastic arrival curve, performance bounds can be derived by using Theorem 3. In this section, we consider two types of input traffic: Poisson traffic and $(\sigma(\epsilon), \rho(\epsilon))$-constraint traffic. The $(\sigma(\epsilon), \rho(\epsilon))$-constraint traffic model is a general traffic model and many types traffic can be represented by it [1], which include exponential on-off, Markov modulated process and effective bandwidth. Numerical results under different configurations are shown and discussed.

A. Numerical Results for Poisson Traffic

(Poisson Traffic.) Suppose all packets of a flow have the same size L and they arrive according to a Poisson process with mean arrival rate λ. Then the flow has a stochastic arrival curve $A(t) \sim_{sac} \langle f_{Pois}, rt\rangle$ for any $r > \lambda L$ with bounding function [6]:

$$f_{Pois}(x) = 1 - (1-a)\sum_{i=0}^{k}\left[\frac{[a(i-k)]^i}{i!}e^{-a(i-k)}\right]$$

where $a = \frac{\lambda L}{r}$ and $k = \lceil \frac{x}{L} \rceil$.

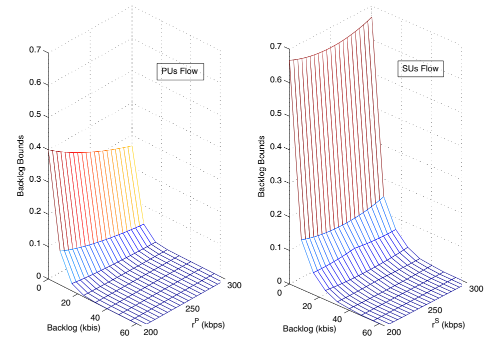
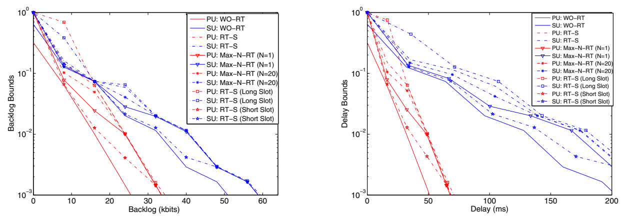


Fig. 4. Backlog Bounds for Poisson Arrivals with Different r^P

First, the influence of average rate r in Poisson arrival is studied, as Fig.4 shows, where system capacity C is set as $500kbps$, packet length for both PUs and SUs flows is set to be 8kbits, and the arrival rate is 10 packets per second. Spectrum sensing error probability in each slot, i.e., p_e, is 1%, and the probability that this is a mis-detection is 50%. Parameter K is set as 2.1 and θ is 1.3 for both sensing error process and mis-detection process. Retransmission is not considered. When applying Theorem 3 in this case, it is required that the average rate for PUs and SUs flow should fulfill $r^P \leq C$ and $r^P + r^S \leq C$, otherwise no guarantee will be provided. Therefore, r^P is set to be within $[200, C]$ and r^S set to $C - r^P$.

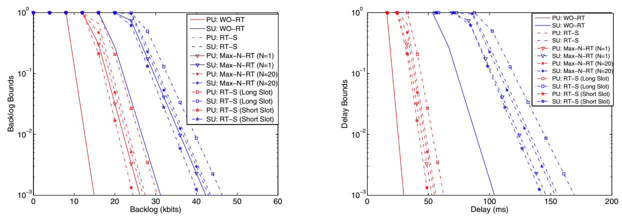
It is obvious that the results for the PUs flow locate lower than SUs flow, which means the PUs flow is provided with better QoS guarantee, since it is assigned with high priority. In addition, it is found that smaller bounds will be provided to the PUs flow when increasing r^P, which is straightforward since r^P denotes average service rate. On the other hand, bounds for SUs flows become smaller first and then larger when increasing r^S. This is due to the fact that the PUs' bounding function also has effect on SUs' bounds. Higher r^S means smaller r^P, more backlog in PUs buffer, and more time slots will be occupied by PUs, which will lead to more backlog in the SUs flow. Therefore, optimal bounds for the SUs flow are tradeoff between PUs' and SUs' service guarantees. Under the current configuration, the balance point locates in the middle where $r^P = r^S = \frac{C}{2}$. As for delay, similar results can be obtained which are not shown here due to limited space.

Fig.5 shows backlog bounds and delay bounds for PUs and SUs considering sensing errors and retransmission at the same time. When we consider the results for one flow, it is found that the distribution bounds of WO-RT scheme are smaller than RT-S scheme, and results of Max-N-RT locate between them, which is consistent with what we can expect from the description of each scheme. However, WO-RT



(a) Backlog (b) Delay

Fig. 5. Numerical Results for Poisson Traffic



(a) Backlog (b) Delay

Fig. 6. Numerical Results for $(\sigma(\theta), \rho(\theta))$-Constraint Traffic

scheme and Max-N-RT scheme may introduce transmission errors or retransmissions in higher layers. In addition, it is straightforward that Max-N-RT scheme converges to RT-S when the maximum retransmission time N is set to infinity; and here results for Max-N-RT scheme with $N = 20$ are also plotted, which lie closer to the RT-S scheme's results than Max-N-RT scheme's results when $N = 1$.

Furthermore, slot length also has significant influences on the system performance. If slot is longer, collision caused by mis-detection will lead to larger backlog and delay because one collision wastes more time; on the other hand, shorter slot results in better performance guarantee. The results when $T = 2.5ms$ and $T = 1.5ms$ are shown, where results with $T = 1.5ms$ give smaller distribution bounds.

B. Numerical Results for $(\sigma(\varepsilon), \rho(\varepsilon))$-Constraint Traffic

($(\sigma(\varepsilon), \rho(\varepsilon))$-Constraint Traffic.) If a flow is $(\sigma(\varepsilon), \rho(\varepsilon))$ upper constraint, then it has a stochastic arrival curve $\alpha(t) = \rho(\varepsilon) \cdot t + \sigma(\varepsilon)$ with bounding function $f(x) = e^{-\varepsilon x}$ [5].

This model can be used to model Markov modulated processes but cannot be easily analyzed by using traditional queueing theory. However, it is possible to use stochastic network calculus to obtain distribution bounds [1, 5].

In the following, average arrival rate $\rho(\varepsilon)$ and maximum burst $\sigma(\varepsilon)$ for both PUs and SUs flow are set to fixed values as 200kbps and 8kbits, respectively; while ε in the bounding function is set to 1. In addition, the channel capacity is 500kbps and the slot length is 2ms. Numerical results are shown in Fig.6, where similar trends as Poisson traffic can be found. In short, WO-RT scheme provides better backlog and delay guarantee. Smaller slot length and less retransmission time in RT-S and MAX-N-RT schemes can improve the bounds.

V. Conclusion

In this paper, performance analysis for a cognitive radio network has been conducted. The network is modeled as a preemptive priority queueing system, with imperfect spectrum sensing and different retransmission schemes. The spectrum sensing error process is modeled as a combination of two processes, mis-detection and false alarm, for which stochastic arrival curves of the corresponding "wasted" service process are found. Three retransmission schemes are discussed, including without retransmission, retransmission until success and maximum-N-times retransmission. Stochastic service curves provided to both PUs and SUs are proved under different retransmission schemes together with performance bounds on backlog and delay, which can be applicable to many types of traffic. Numerical results are shown for two types of input traffic, Poisson traffic and (σ, ρ)-constraint traffic, where further discussions are made. We believe that these results will shed light on deeper understanding of cognitive radio networks, and on the design of optimal retransmission schemes in such networks.

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