A Network Calculus Approach to Throughput Analysis of Stochastic Multi-Channel Networks

Zhidu Li, Yuehong Gao, Pengxiang Li, Lin Sang, Dacheng Yang
Wireless Theories and Technologies Lab
Beijing University of Posts and Telecommunications
Beijing, P.R. China, 100876
Email: prclzd@126.com

***Abstract*—In this paper, two types of throughput, which are transient throughput and delay-constraint throughput, are investigated in a Gilbert-Elliott multi-channel network by using the stochastic network calculus. We propose a multi-channel model and derive an equivalent stochastic service curve guaranteed by the whole network. With the stochastic service curve, we obtain the lower bound of the network transient throughput. The throughput is non-asymptotic in that it holds for any number of channels and also fully accounts for transient regime. After, we derive the probabilistic delay bound of the multi-channel network, with which we further ascertain the delay-constraint throughput. Finally, numerical results are presented to show the impacts of the channel memory and number of channel on the transient throughput and that of the delay constraint on the maximum arrival rate sustained by the network.**

***Keywords*—Multi-channel network; throughput and delay; stochastic service curve; stochastic network calculus**

I. Introduction

Multi-channel network, referring to the network with several parallel channels working simultaneously, are widely used in wireless communications. Example includes LTE-OFDM (orthogonal frequency division multiplexing) networks, where the available bandwidth is divided into hundreds of orthogonal channels (subcarriers) [1].

Network throughput and network delay are two key metrics of interest in multi-channel networks. As an analytical tool resembling the traditional queueing theory, the stochastic network calculus has been proved to be powerful in performance analysis [2–4]. It uses arrival curve and service curve to yield performance bounds (e.g., delay and buffer bounds) which are guaranteed by the communication systems. Unlike traditional queueing theory, the results of the stochastic network calculus are usually presented by probabilistic bounds instead of closed-form average values, which in turn simplifies the complex nature of non-linear problems albeit at the expense of some degree of accuracy. Therefore, it is applicable to the scenarios where some violations of performance criteria could be tolerated. Moreover, the stochastic network calculus has been widely employed in many fields, such as network performance analysis in [5–9] and energy consumption analysis in [10, 11] just to mention a few.

The work in this paper is supported by the Beijing Higher Education Young Elite Teacher Project under Grant No.YETP0430

In this paper, we use the theory of the stochastic network calculus to study the throughput and delay in a Gilbert-Elliott multi-channel network with unsaturated arrivals (i.e., the arrival is finite and the network is stable). We mainly focus on how to capture the characteristics of the stochastic service in the considered multi-channel network. Particularly, we are concerned with the probabilistic lower bound of the network (transient) throughput and the maximum arrival rate sustained by the network under a given probabilistic delay constraint.

In the literature, though attempts have been devoted to the throughput and delay study, to the best of our knowledge, the results are mostly restricted to average values, constant service rate or singe-channel scenario. In [12–14], researchers studied the throughput on the assumption that the arrival and service processes were both Poisson distributed. However, the corresponding results therein were limited to be long-term average values, which could not capture the transient characteristics. In [5, 6], although Florin performed logical throughput and delay analysis in a multi-hop model by considering transient cases, his work was only carried out in a single channel scenario. Throughput and delay distribution analysis in multi-channel scenarios can be found in [15–17]. However, these researches were based on the constant channel transmission rate, which failed in capturing the stochastic characterizations of wireless transmissions. In addition, similar work to conduct throughput analysis based on the network calculus in multi-channel network is [7], nevertheless, the channel transmission rates were also simply assumed to be constant. Moreover, the service curve was derived in backlog periods (i.e. saturated arrivals) with worse case, which made the throughput bound loose.

The contribution of this paper consists in the derivation of a generally equivalent stochastic service curve and its violation probability function for a multi-channel scenario. It is significant that this stochastic service curve can be easily applied to yield closed-form performance bounds with the exact conclusions of the stochastic network calculus, such that extending the application scenario of the stochastic network calculus from single-channel to multi-channel. With the stochastic service curve of the whole network, we derive the lower bound distribution of the network throughput and the distribution of the network delay. Our throughput results are then obtained with the given violation probability. We highlight that the lower bound of network throughput achieved in this paper is non-asymptotic, i.e., they hold for any finite time scale and any number of parallel channels. Moreover, our results are general since the equivalent stochastic service curve is applicable to the multi-channel scenarios no matter whether the parallel channels are independent of each others or not.

The remainder of this paper is organized as follows. Section II introduces the system model, where some definition and assumptions are given. Section III demonstrates the derivations for the stochastic service curve guaranteed by the whole network and the network throughput under delay constraints. Thereafter, numerical results of the transient network throughput and delay-constraint throughput are presented and discussed in section IV. Finally, section V concludes the paper.

II. System Model

A. Network Structure

Throughout this paper a continuous time model is employed and data units is referred to packets. Consider a multi-channel network as depicted in Fig. 1 comprising of an infinite length buffer and N parallel channels. The arrival data units are arranged into the buffer and served in the first-in-first-out (FIFO) order. A data unit can be served by only one channel and a channel can be allocated to only one data unit at the same time. Each channel can either be independent or correlated, and the service rate of each channel is time varying. Each data unit is eager for being served by a channel in the best serving condition. Therefore, how to schedule these data units and channels to improve the network performance is an interesting topic. However, our key concern is the network throughput and delay. Therefore, we suppose that the network always makes an optimal schedule to maximize the network performance.

B. Arrival, Service and Departure Processes

The cumulative arrivals and departures of the whole network within the time interval $[s,t]$ are denoted by functions $A(s,t) = A(t) - A(s)$ and $D(s,t) = D(t) - D(s)$ with $A(0) = D(0) = 0$ and $A(t) \geq D(t)$. We similarly use $S(t)$ to denote the totally cumulative service guaranteed by the considered network, where $S(t) = \sum_{i=1}^{N} S_i(t)$ and $S_i(t)$ denotes the cumulative service guaranteed by the channel indexed i.

For the network we are interested in two performance metrics, which are the throughput and delay of $A(t)$. The probabilistic lower bound of A' throughput (which is denoted by ξ) is defined as follows: for any time, the probability that the actual network throughput $D(t)$ is larger than the corresponding lower bound ξ is guaranteed beyond $1 - \epsilon_\xi$, i.e.,

$$Pr\{D(t) \geq \xi t\} \geq 1 - \epsilon_\xi, \tag{1}$$

where ϵ_ξ is also a tolerable violation probability.

The delay of $A(t)$, denoted by $W(t)$, represents the delay of A' last data unit arriving at time t. And $W(t) = \inf\{d : A(t) \leq D(t+d)\}$. The probabilistic delay bound is similarly defined as: the delay $W(t)$ exceeding a threshold d_0 is controlled within a probability ϵ_d, i.e.,

$$Pr\{W(t) \geq d_0\} \leq \epsilon_d, \tag{2}$$

where ϵ_d is also called a violation probability.

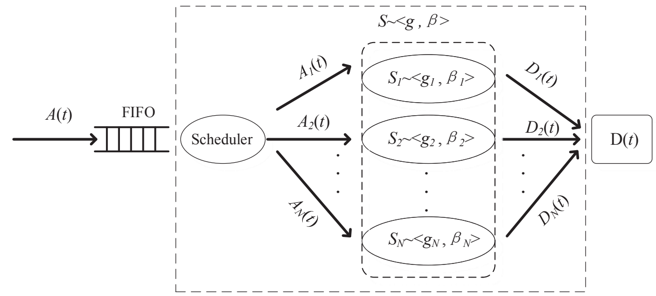

Fig. 1. Multi-channel network model

The delay-constraint throughput (denoted by ψ) in this paper is defined as the maximum arrival rate which can be sustained by the multi-channel network under a given delay constraint, i.e.,

$$\psi = \sup\{\frac{D(t)}{t} : Pr\{W(t) \geq d_0\} \leq \epsilon_d\}.$$

In this paper, the arrival traffic is assumed to have deterministic envelope without burstiness [3]. Specifically, the traffic arrival rate is upper bounded by M data units/s. We use the deterministic arrival curve to characterize the arrival process. The deterministic arrival curve, denoted by $r(t)$, is defined as $A(s,t) \leq r(s,t)$ for all $0 \leq s \leq t$, and there holds [2]

$$r(t) = Mt.$$

The channels in this paper are considered as Gilbert-Elliott (GE) channel with two Markov states ON and OFF. This model is widely used in analytical analysis of wireless networks (e.g. see [18, 19]). We assume that each channel provide service rate as K/N data units/s in ON state and no service in OFF state, which means the multi-channel network guarantees a peak service rate K data units/s. We denote the state transition rate from OFF to ON by λ and from ON to OFF by μ respectively. Due to the reason that the service rate of each channel is time varying, obtaining an exact expression of $S(t)$ is difficult or even impossible. Hence, we use the stochastic service curve to characterize the service processes of the N channels.

A non-decreasing and non-negative function $\beta(t)$ is defined as stochastic service curve of $S(t)$ if for all $t \geq 0$ and all $x \geq 0$, there holds

$$Pr\{A \otimes \beta(t) - D(t) \geq x\} \leq g(x), \tag{3}$$

where '$\otimes$' is the (min, +) convolution operator defined as $A \otimes \beta(t) = \inf_{0 \leq s \leq t}\{A(s) + \beta(s,t)\}$. Here, g is called the violation probability function satisfying $g(x) \to 0$ as $x \to \infty$. Furthermore, the stochastic service curve of a GE channel can be derived by using the theory of effective bandwidth, which holds as following [20]:

$$\begin{aligned} &\beta(t) \leq -\frac{1}{\theta} \ln \mathrm{E}[e^{-\theta S(t)}] \\ &g(x) = e^{-\theta x} \end{aligned} \quad , \tag{4}$$

where θ is a non-negative free parameter. Here, $\beta(t)$ can be constructed by any type of expression as long as which satisfies the first term of (4). We adopt a lower bound of $-\frac{1}{\theta}\ln \mathrm{E}[e^{-\theta S(t)}]$ as given in [20], i.e.,

$$\beta(t) = \frac{t}{2\theta}(\frac{K}{N}\theta + \lambda + \mu - \sqrt{(\frac{K}{N}\theta - \lambda + \mu)^2 + 4\lambda\mu}).$$

Note that the definition of stochastic service curve also implies the distribution of the lower bound of the departures. Thus the key challenge is to find an equivalent stochastic service curve of the whole network by using the stochastic service curve of each channel.

Moreover, a network is stable if all the data units have finite delay. In this paper a sufficient stability condition is adopted as

$$\lim_{t\to\infty}\frac{r(t)}{t} \le \lim_{t\to\infty}\frac{\beta(t)}{t}, \tag{5}$$

where the left-hand side term represents the long-term average arrival rate and the right-hand side term can be view as the available service rate. The sufficient stability condition is also used in [9], [10], [13] and [17].

III. Throughput and Delay Analysis

A. Probabilistic Lower Bound of the Network Throughput

In this subsection, we firstly derive a generally equivalent stochastic service curve of the whole multi-channel network with the corresponding violation probability function.

Theorem 1. *(***Equivalent Stochastic Service Curve***) Consider a multi-channel network as shown in Fig. 1. For $\forall 1 \leqslant i \leqslant N$, the stochastic service curve with violation probability function of channel S_i is given as $S_i \sim< g_i, \beta_i >$. Suppose an arrival process A is characterized by a deterministic arrival curve $r(t) = Mt$ as defined in Section II. If the system is stable, the whole network can guarantees an equivalent stochastic service curve $< g, \beta >$ with*

$$\begin{aligned} &\beta(t) = \sum_{i=1}^{N}\beta_i(t) \\ &g(x) = [g_1 \otimes g_2 \otimes \cdots \otimes g_N(x)]_1 \end{aligned},$$

where function $[\cdot]_1$ means $\min\{\cdot, 1\}$.

Proof: According to the description of the system model in Section II, the cumulative arrivals $A(t)$ can be viewed as the sum of N divided cumulative arrivals denoted by $\{A_i(t) \mid 1 \le i \le N\}$, where $A(t) = \sum_{i=1}^{N} A_i(t)$ denotes the arrivals arranged to be transmitted by channel S_i. For each of $A_i(t)$, we can find a deterministic arrival curve as $r_i(t) = M_i t$, satisfying $r(t) = \sum_{i=1}^{N} r_i(t)$. Additionally, the corresponding departures from the N channels are similarly denoted by $\{D_i(t)|1 \le i \le N\}$, and $D(t) = \sum_{i=1}^{N} D_i(t)$.

At first, we prove that

$$(A \otimes \sum_{i=1}^{N}\beta_i)(t) \le \sum_{i=1}^{N} A_i \otimes \beta_i(t).$$

$$\begin{aligned} &(A \otimes \sum_{i=1}^{N}\beta_i)(t) - \sum_{i=1}^{N} A_i \otimes \beta_i(t) \\ &= \inf_{0\le s\le t}\{A(s) + \sum_{i=1}^{N}\beta_i(s,t)\} - \sum_{i=1}^{N}\inf_{0\le s\le t}\{A_i(s) + \beta_i(s,t)\} \\ &\le \sum_{i=1}^{N}(A_i(t) + \beta_i(t,t)) - \sum_{i=1}^{N}\inf_{0\le s\le t}\{A_i(s) + \beta_i(s,t)\} \\ &\le \sum_{i=1}^{N}\sup_{t\ge 0}\{r_i(t) - \beta_i(t)\} \end{aligned}.$$

Since the whole network is stable, all the channels are stable. According to (5), we then have

$$\sum_{i=1}^{N}\sup_{t\ge 0}\{r_i(t) - \beta_i(t)\} \le \sum_{i=1}^{N}\sup_{t\ge 0}\{r_i(0) - \beta_i(0)\} = 0.$$

Therefore, the violation function is proved as following:

$$\begin{aligned} &Pr\{A \otimes \beta(t) - D(t) \ge x\} \\ &\le Pr\{\sum_{i=1}^{N} A_i \otimes \beta_i(t) - \sum_{i=1}^{N} D_i(t) \ge x\} \\ &\le \inf_{0\le x_1,\ldots,x_N,\sum_{i=1}^{N} x_i = x}\{\sum_{i=1}^{N} Pr\{A_i \otimes \beta_i(t) - D_i(t) \ge x_i\}\} \\ &\le [g_1 \otimes g_2 \otimes \ldots \otimes g_N(x)]_1 \end{aligned},$$

where the last step is in terms of the Lemma 1.5 in [4]. Using the definition of stochastic service curve ends the proof.

The equivalent stochastic service curve of the considered multi-channel network is directly obtained by using Theorem 1.

$$\begin{aligned} &\beta(t) = \frac{Nt}{2\theta}(\frac{K}{N}\theta + \lambda + \mu - \sqrt{(\frac{K}{N}\theta - \lambda + \mu)^2 + 4\lambda\mu}) \\ &g(x) = [Ne^{-\theta\frac{x}{N}}]_1 \end{aligned}.$$

Looking back the stability condition in (5), there holds

$$Mt \le \beta(t) = \frac{Nt}{2\theta}(\frac{K}{N}\theta + \lambda + \mu - \sqrt{(\frac{K}{N}\theta - \lambda + \mu)^2 + 4\lambda\mu}).$$

As a result, the regime of θ is ascertained by solving the above inequality.

$$0 \le \theta \le \frac{N(K\lambda - M(\lambda + \mu))}{M(K - M)}.$$

Further, rewriting the definition of the stochastic service curve (3) into the same form of (1) as

$$Pr\{D(t) \ge A \otimes \beta(t) - x\} \ge 1 - g(x) = 1 - \varepsilon_\xi,$$

we finally achieve the lower bound of the multi-channel network throughput as follows

$$\begin{aligned} \xi &= \sup_{0\le\theta\le\frac{K\lambda - M(\lambda+\mu)}{M(K-M)}}\{\frac{A \otimes \beta(t) - g^{-1}(\varepsilon_\xi)}{t}\} \\ &= M + \frac{M(K - M)}{(K\lambda - M(\lambda + \mu))t}\ln\frac{\varepsilon_\xi}{N} \end{aligned}, \tag{6}$$

where $g^{-1}(x)$ is the inverse function of $g(x)$. Note that ξ is non-asymptotic in which they hold for any number of channels and also fully account for transient regimes. Furthermore, ξ decays in t, $\lim_{t\to\infty}\xi = M$.

B. Delay-Constraint Throughput

In this subsection, the delay-constraint throughput are discussed and derived under the given probabilistic delay bounds. The probabilistic delay bound is firstly proposed as the follow theorem.

Theorem 2. *(**Probabilistic Delay Bound**) Consider a stable N-channel network S as described in Section II. Suppose the equivalent stochastic service curve and the violation probability function of the network are obtained in Theorem 1 as $S \sim< g, \beta >$. If an arrival process A with the deterministic arrival curve $r(t) = Mt$ requests to be served, then for any $d \geq 0$, the probabilistic delay bound $W(t)$ holds as:*

$$Pr\{W(t) > d\} \leq [g(\beta(d))]_1.$$

Proof:

$$\begin{aligned}
&Pr\{W(t) > d\} \\
\leq &Pr\{A(t) - D(t+d) > 0\} \\
\leq &Pr\{\sup_{0\leq s\leq t+d} \{A(s,t) - \beta(s,t+d)\} \\
&\quad + A \otimes \beta(t+d) - D(t+d) > 0\} \\
\leq &Pr\{\sup_{0\leq s\leq t+d} \{r(s,t) - \beta(s,t+d)\} \\
&\quad + A \otimes \beta(t+d) - D(t+d) > 0\} \\
= &Pr\{A \otimes \beta(t+d) - D(t+d) > \beta(d)\} \\
\leq &[g(\beta(d)]_1
\end{aligned},$$

where the third step is according to the definition of arrival curve, and the last step is in light of Theorem 1.

If the probabilistic delay bound is constrained by $Pr\{W(t) > d_0\} \leq \varepsilon_d$, according to the stability condition in (5), the delay-constraint throughput ψ then satisfies

$$\begin{aligned}
&\psi = \frac{N}{2\theta}\left(\frac{K}{N}\theta + \lambda + \mu - \sqrt{(\frac{K}{N}\theta - \lambda + \mu)^2 + 4\lambda\mu}\right) \\
&Ne^{-\theta\frac{Nd_0}{2N\theta}(\frac{K}{N}\theta+\lambda+\mu-\sqrt{(\frac{K}{N}\theta-\lambda+\mu)^2+4\lambda\mu})} = \varepsilon_d
\end{aligned}.$$

Solving the equation set, we have

$$\begin{aligned}
\theta &= -\frac{\ln\frac{\varepsilon_d}{N}}{d_0}\frac{(\lambda d_0 + \mu d_0 + \ln\frac{\varepsilon_d}{N})}{(\lambda d_0 + \ln\frac{\varepsilon_d}{N})\frac{K}{N}} \\
\psi &= \frac{(\lambda d_0 + \ln\frac{\varepsilon_d}{N})K}{\lambda d_0 + \mu d_0 + \ln\frac{\varepsilon_d}{N}}
\end{aligned}. \tag{7}$$

According to the description of the Gilbert-Elliott multi-channel, the long-term average service rate of the whole network is $\frac{K\lambda}{\lambda+\mu}$. If the delay constraint is not concerned (i.e., the delay requirement $d_0 \to \infty$ or the violation probability $\epsilon_d \to 1$), ψ will tend to equate to $\frac{K\lambda}{\lambda+\mu}$.

IV. Numerical Results

In this section, numerical results under different configurations are presented and discussed.

Fig. 2 depicts the probabilistic lower bound of the transient network throughput. The traffic arrival rate is set to 3 data units/s and the violation probability ε_ξ is fixed to 0.001. The number of channels is set to 5 and service rate of each channel is set to 1 data unit/s when the channel state is ON. Note that

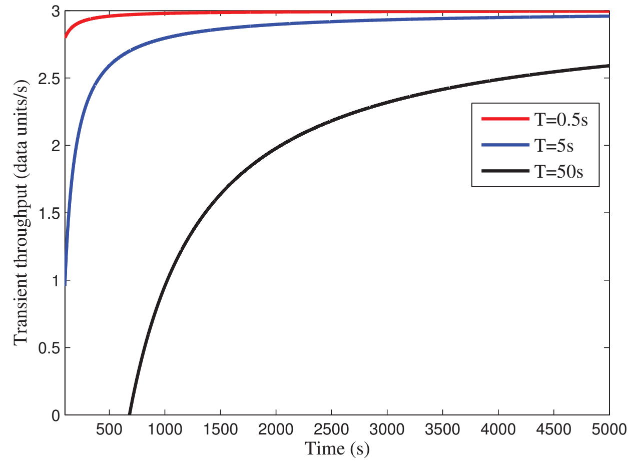

Fig. 2. Lower bound of transient network throughput with different channel memories

$T = \frac{1}{\lambda} + \frac{1}{\mu}$ represents an average cycle time of state transition (i.e. the channel memory), we set $T = 0.5$s, 5s, 50s as three cases and $\frac{\lambda}{\mu+\lambda} = 0.8$. It is found that lower bound of network throughput increases with time. It would converge to the arrival rate while time tends to infinity, which indicates that the multi-network is stable. Moreover, the increasing rate of the transient throughput varies with different channel memories. Shorter channel memory impels the network throughput rises faster. In the memoryless channel ($T \to 0$) network, the transient throughput would be asymptotic in terms of (6).

Fig. 3 illustrates the probabilistic lower bound of the transient network throughput. The peak service rate of the whole network is fixed as $K = 5$ data units/s and the number of channels $N = 5,\ 25,\ 125$ for three cases, namely, when the channel state is ON, the service rates of each channel are different for these three cases. The traffic arrival rate and the violation probability ε_ξ are set to 3 data units/s and 0.001 respectively. The channel memory T is 50s and $\frac{\lambda}{\mu+\lambda} = 0.8$. We found that in a multi-channel network with a fixed peak service rate, the throughput converging rate decays as the number of the channels increases. This phenomenon can be summarized as more channels leads to a more stochastic service guaranteed by a multi-channel network with fixed peak service rate.

Fig. 4 depicts the relationship between the delay constraint and the corresponding maximum sustained throughput. The peak service rate of the whole network is fixed as $K = 5$ data units/s. The number of channels is $N = 5,\ 10$ and the channel memory is 0.05s with $\frac{\lambda}{\mu+\lambda} = 0.8$. The violation probability ε_d is set to 0.001 and 0.01 for two cases. The phenomenon that more channels leads to a more stochastic service guaranteed by a multi-channel network with fixed peak service rate can also be verified through the delay-constraint throughput in Fig. 4. On the other hand, it is intuitive that higher traffic arrival rate can be sustained by the multi-channel network with looser delay constraint (i.e., larger delay requirement or larger tolerable violation probability). Comparing with rising the violation probability, rising the delay requirement is a more reasonable way to slack the delay constraint. Furthermore, the curves in Fig. 4 provides useful guideline on the tradeoff

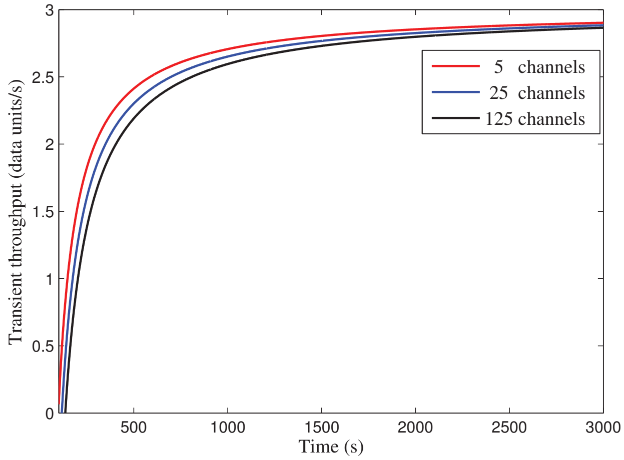

Fig. 3. Lower bound of transient network throughput with different channel numbers under a fixed peak service rate

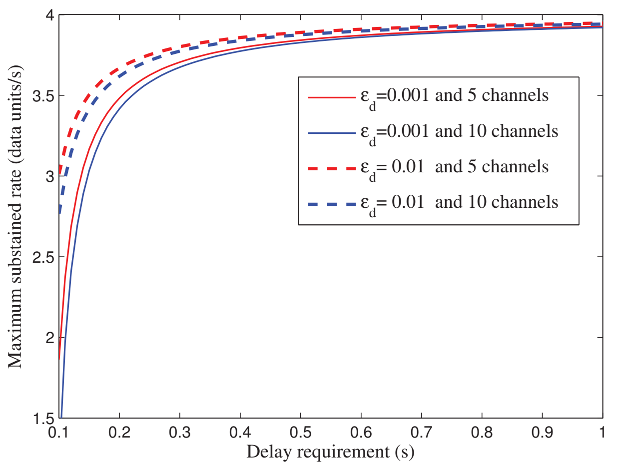

Fig. 4. Delay-constraint throughput

between the traffic throughput and network delay constraint.

V. Conclusions

In this paper, we perform the throughput and delay analysis of a Gilbert-Elliott multi-channel network by using the theory of the stochastic network calculus. We derived a general equivalent stochastic service curve of the whole network with which the lower bound of the network transient throughput was obtained. The transient throughput holds for any number of parallel channels and fully accounts for transient regime. Furthermore, the delay-constraint throughput was ascertained while the probabilistic delay constraint is taken into account. The numerical results indicate that i) the transient throughput converges faster in the transmission channel with shorter channel memory; ii) more parallel channels leads to a more stochastic service guaranteed by a multi-channel network with fixed peak service rate; iii) rising the delay requirement improves the delay-constraint throughput more obviously than rising the violation probability. Although we use the Gilbert-Elliott channel model to perform our analysis in this paper, we highlight that our approach to get the stochastic service curve of the whole multi-channel network is also applicable to other channel models as long as the stochastic service curve of each channel can be obtained.

References

[1] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser ofdm with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct 1999.

[2] J.-Y. Le Boudec and P. Thiran, *Network calculus: a theory of deterministic queuing systems for the internet*. Springer, 2001, vol. 2050.

[3] C.-S. Chang, *Performance guarantees in communication networks*. Springer, 2000.

[4] Y. Jiang and Y. Liu, *Stochastic network calculus*. Springer, 2008.

[5] F. Ciucu, O. Hohlfeld, and P. Hui, "Non-asymptotic throughput and delay distributions in multi-hop wireless networks," in *Proc. Annu. Allerton Conf. Commun., Control, and Computing (Allerton)*, Sept 2010, pp. 662–669.

[6] F. Ciucu, "On the scaling of non-asymptotic capacity in multi-access networks with bursty traffic," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, July 2011, pp. 2547–2551.

[7] Y. Gao, W. Jiang, and Y. Jiang, "Guaranteed service and delay-constrained capacity of a multi-channel cognitive secondary network," in *ICST Int. Conf. Cognitive Radio Oriented Wireless Netw. and Commun. (CROWNCOM)*, June 2012, pp. 83–88.

[8] K. Zheng, F. Liu, L. Lei, C. Lin, and Y. Jiang, "Stochastic performance analysis of a wireless finite-state markov channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 782–793, February 2013.

[9] A. Burchard, J. Liebeherr, and F. Ciucu, "On superlinear scaling of network delays," *IEEE/ACM Trans. Netw.*, vol. 19, no. 4, pp. 1043–1056, Aug 2011.

[10] K. Wu, Y. Jiang, and D. Marinakis, "A stochastic calculus for network systems with renewable energy sources," in *IEEE INFOCOM Workshop*, March 2012, pp. 109–114.

[11] Z. Li, Y. Gao, L. Sang, and D. Yang, "Analysis on the energy consumption in stochastic wireless networks," in *IEEE ICC Workshops*, June 2014, pp. 866–870.

[12] S. Fowler, C. Ha?ll, D. Yuan, G. Baravdish, and A. Mellouk, "Analysis of vehicular wireless channel communication via queueing theory model," in *IEEE Int. Conf. Commun. (ICC)*, June 2014, pp. 1736–1741.

[13] M. H. Lee, A. Dudin, and V. Klimenok, "Mathematical analysis of the multi-server queueing model for dynamic channel reservation in wireless networks," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 855–857, December 2006.

[14] G. Bianchi, "Performance analysis of the ieee 802.11 distributed coordination function," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 3, pp. 535–547, March 2000.

[15] S. Kittipiyakul and T. Javidi, "Delay-optimal server allocation in multiqueue multiserver systems with time-varying connectivities," *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2319–2333, May 2009.

[16] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, "Scheduling for small delay in multi-rate multi-channel wireless networks," in *Proc. IEEE INFOCOM*, April 2011, pp. 1251–1259.

[17] ——, "Scheduling in multi-channel wireless networks: Rate function optimality in the small-buffer regime," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 1101–1125, Feb 2014.

[18] J. G. Kim and M. Krunz, "Bandwidth allocation in wireless networks with guaranteed packet-loss performance," *IEEE/ACM Trans. Netw.*, vol. 8, no. 3, pp. 337–349, Jun 2000.

[19] L. Liu, P. Parag, J. Tang, W.-Y. Chen, and J.-F. Chamberland, "Resource allocation and quality of service evaluation for wireless communication systems using fluid models," *IEEE Trans. Inf. Theory*, vol. 53, no. 5, pp. 1767–1777, May 2007.

[20] Y. Jiang, "A note on applying stochastic network calculus. http://q2s.ntnu.no/ jiang/publications.html," 2010.