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Modelling Wireless Sensor Networks with Energy Harvesting: A Stochastic Calculus Approach

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***Abstract*—Wireless Sensor Networks (WSN) have been increasingly applied to industrial monitoring and control, where soft Quality of Service guarantees under worst-case scenarios are often required. Many performance analysis frameworks including Network Calculus are proposed to derive those guarantees. Those frameworks were originally designed for wired network and typically do not consider energy constraints, which play a crucial role in the operation of wireless sensor networks. In this paper, a novel model is proposed to integrate energy harvesting into the network calculus framework to stochastically bound the worst-case performance. In our framework, energy is converted into tokens to stochastically bound the service provided to the traffic. Meanwhile, behaviors of nodes during the situations of energy overflow and underflow are accurately characterized by two virtual queues. Various performance metrics are analyzed and bounded for a single node and for a tree-based data collection wireless sensor network. Additionally, procedures are presented to show the model instantiation for a specific wireless sensor network based on hardware parameters and empirically obtained energy harvesting traces.**

I. Introduction

One of the key components underlying the explosion of interest in *Internet of Things* applications is the rapid deployment of Low Power and Lossy or Wireless Sensor Networks (WSNs) [1]. Due to cost and design limitations, WSN systems are highly resource-constrained and require extremely careful power management techniques. To simplify the WSN management, prolong lifetime, and provide ecologically friendly solutions, there are many proposals for utilizing energy harvesting. In an energy harvesting system, energy is obtained from the ambient environment and stored in rechargeable devices such as supercapacitors for later use [2].

As energy harvesting WSNs (EH-WSNs) scale up to support larger applications, multi-hop and tree-based systems will become increasingly common. It is therefore essential to develop analytical techniques capable of quantifying system performance while considering the coupled impact of node resouce capabilities, workload model and energy harvesting profiles. The difficulty of performance modeling in such an environment is compounded by the probabilistic nature of both workload modeling and energy harvesting activities. Our work addresses this issue by describing modeling techniques for probabilistic, multi-hop energy-harvesting WSNs based on stochastic network calculus.

Network calculus is a theoretical framework that offers worse-case analysis for multi-hop networks. It use convolutional techniques that provide the derivations of performance measures for end-to-end delay and workload backlog. Using convolution greatly simplifies the extension from a single node system to a multi-hop system. Stochastic network calculus (SNC) extends earlier work in its deterministic counterpart [3] to more accurately capture the impact of probabilistic events. SNC employs statistical multiplexing analysis of arrival and service activities using the theory of effective bandwidths [4].

End-to-end analysis of EH-WSNs is known to be an important and pressing problem [5]. Accurate analysis will greatly simplify the problem of establishing workload ranges and dimensioning relatively expensive components such as storage and harvesting capabilities, but to date there has been relatively little work in applying SNC techniques to these issues. One reason is the apparent challenge in taking energy harvesting restrictions into account.

Other work in using network calculus includes Schmitt et al. [6], who first applied deterministic network calculus to dimensioning wireless sensor networks. They derived the bounds for various performance metrics including delay and backlog for both a single node and a typical wireless sensor network with features such as multiple sinks and in-network processing. Koubaa et al. [7] extended the sensor network calculus for dimensioning in the analysis of cluster-tree topologies. These two sets of approaches do not take into account energy constraints. A real-time calculus-based model was proposed by Guan et al. [8]. Similarly, in their proposed model, a node is assumed to be able to buffer the arrived traffic even if there was not enough energy available in its battery. This is not the case for EH-WSNs, where, in the situation of energy shortage, sensor nodes will be forced to sleep and any incoming traffic will be dropped instead of buffered.

In terms of SNC, Wu et al. [9] applied stochastic network calculus theories to network systems with renewable energy sources. In their proposed model, the energy arrival is used to constrain the service curve by utilizing a power-rate function inspired by [10]. The backlog and delay bound with energy constraints and the bound for remaining energy are derived. However, the cases of energy overflow and underflow – the battery becomes full when charging and the battery runs out – is not directly considered in their model. These cases,

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especially energy underflow, will have a critical effect on the operations of the EH-WSNs.

In our work, we take into account factors such as energy overflow and energy depletion using SNC. Our method is to use a token-bucket-like model with two virtual queues, where energy is viewed as the available amount of service that can be provided for traffic service. Tokens are used within the SNC technique as the method to unify the analysis of all energy harvesting, consumption, traffic arrival and service activities. Traffic arriving at a node is buffered in a service queue only if there are enough tokens available; in other words, the node has power reserves left. When there are not enough tokens, instead of buffering, the traffic arrived will be discarded immediately by flowing into the *discarded* virtual queue. Extra tokens are stored in the bucket with finite capacity determined by the internal battery or capacitor of the sensor node. When the bucket becomes full, any excess tokens will be discarded by flowing into the *wasted* virtual queue. Based on our proposed model, we have derived the stochastic bounds for delay, backlog, energy wasted and traffic discarded for individual nodes as well as multi-hop tree-topology data collection networks. In additional to providing end-to-end stochastic performance bounds for specific parameters, we are able to providing dimensioning information that enables a designer to achieve required performance levels.

To summarize, the contribution of our work is three-fold:

- Using our unified token construct, energy constraints including finite energy storage capacity and energy depletion are directly integrated into an SNC framework.
- SNC bounds on the backlog, delay, wasted energy and lost packets are derived for both single nodes and multi-hop tree-based network, directly providing performance analysis and dimensioning information.
- Using an explicit hardware model we demonstrate how to incorporate our techniques into a real system. We also numerically show how to provide a stochastic bounds on a real energy harvesting trace.

II. Background

In this section, the stochastic network calculus is briefly introduced, including its basic concepts, notations and results. Additionally, the architectural background for our targeted wireless sensor network environment is presented.

A. Stochastic Network Calculus

Network calculus is a theoretical framework for analyzing worst-case performance guarantees in computer networks [3]. It uses alternate algebras (min-plus and max-plus algebra, in contrast to the ordinary addition and multiplication) and various envelop processes for analysis such that worst-case performance can be easily derived and bounded. The bounds derived from network calculus can be either deterministic or stochastic. Due to the dynamic nature of the wireless communication between sensor nodes, the stochastic version of network calculus is adopted to derive tighter bounds.

We start with the definition of the key operations in network calculus: convolution and deconvolution. Let f and g be wide-sense increasing functions and $f(0) = g(0) = 0$. We have the convolution and deconvolution operations under both min-plus and max-plus algebra defined in [3] as follows.

Definition 1. *Min-Plus Convolution / Deconvolution*

$$f \otimes g(t) = \inf_{0\leq s\leq t} \{f(s)+g(t-s)\}$$
$$f \oslash g(t) = \sup_{0\leq s} \{f(t+s)-g(s)\}$$

Definition 2. *Max-Plus Convolution / Deconvolution*

$$f \bar{\otimes} g(t) = \sup_{0\leq s\leq t} \{f(s)+g(t-s)\}$$
$$f \bar{\oslash} g(t) = \inf_{0\leq s} \{f(t+s)-g(s)\}$$

Now, by means of the convolution and deconvolution under the alternate algebras, the arrival and service curves can be defined. We adopt the following two definitions for the arrival and service curves from [11] and extend the arrival curve with an lower bound and the service curve with upper bound to facilitate the analysis of our proposed model. For a cumulative random process $A(t)$, the convention for denoting the increment during time period $[s, t]$ as $A(s, t) = A(t)-A(s)$ is used for notation simplicity in this paper.

Definition 3. *Virtual-Backlog-Centric Arrival Curve*. Let α_u and α_l be non-negative, wide-sense increasing functions and f_u and f_l be non-negative, wide-sense decreasing functions. the arrival process $A(t)$ is said to follow a *virtual-backlog-centric* or *v.b.c* arrival curve if $\forall t, x \geq 0$, the follwing holds,

$$Prob\Big\{ \sup_{0\leq s\leq t} \{A(s,t)-\alpha_u(t-s)\} > x \Big\} \leq f_u(x)$$
$$Prob\Big\{ \sup_{0\leq s\leq t} \{\alpha_l(t-s)-A(s,t)\} > x \Big\} \leq f_l(x)$$

We call the curves α_u and α_l the upper and lower arrival curve for A with the probabilistic bounding functions f_u and f_l, respectively, denoted as the follows,

$$A \sim_{v.b.c} \langle f_u, f_l, \alpha_u, \alpha_l \rangle$$

Definition 4. *Stochastic Service Curve*. Let β_u and β_l be non-negative, wide-sense increasing functions and g_u and g_l be non-negative, wide-sense decreasing functions. A server S is said to provide a *stochastic service curve* or *s.c* curve if $\forall t \geq 0$ and $\forall x \geq 0$, the arrival process $A(t)$ and the departure process $A^*(t)$ satisfy the following,

$$Prob\Big\{ \sup_{0\leq s\leq t} \{A \otimes \beta_l(t)-A^*(t)\} > x \Big\} \leq g_l(x)$$
$$Prob\Big\{ \inf_{0\leq s\leq t} \{A^*(t)-A \bar{\otimes} \beta_u(t)\} > x \Big\} \leq g_u(x)$$

We call the curves α_u and α_l the upper and lower service curve for S with the probabilistic bounding functions g_u and g_l, respectively, denoted as the follows,

$$S \sim_{s.c} \langle g_u, g_l, \beta_u, \beta_l \rangle$$

With the definition of the arrival and service curve, we are able to obtain the upper bound for backlog and delay.

Assuming the arrival traffic satisfies $A \sim_{v.b.c} \langle f_u, f_l, \alpha_u, \alpha_l \rangle$ and the server satisfies $S \sim_{s.c} \langle g_u, g_l, \beta_u, \beta_l \rangle$, we have the following result from [11], which provides upper bounds for $BKL(t)$ – the backlog in the buffer at time t, and $DLY(t)$ – the delay at time t due to the backlog, respectively.

$$Prob\{BKL(t) > v_{max}(\alpha_u, \beta_l)+x\} \leq f_u \otimes g_l(x) \quad (1)$$

$$Prob\{DLY(t) > h_{max}(\alpha_u+y, \beta_l)\} \leq f_u \otimes g_l(y) \quad (2)$$

where v_{max} and h_{max} are the horizontal and vertical deviations between functions, respectively, defined as follows,

$$v_{max}(\alpha, \beta) = \sup_{0 \leq s}\{\alpha(s)-\beta(s)\} = \alpha \oslash \beta(0)$$

$$h_{max}(\alpha, \beta) = \sup_{0 \leq s}\Big\{ inf\{\tau > 0 : \alpha(s) \leq \beta(s+\tau)\}\Big\}$$

The following lemma [11] provides the probabilistic bounds for the sum of two random variables, which will be used to prove several useful results in later sections.

Lemma 1. Assume that the complementary cumulative distribution functions of random variables X, Y are F_X, F_Y, respectively. Assume that $f_1(x) \leq F_X(x) \leq f_2(x)$ and $g_1(x) \leq F_Y(x) \leq g_2(x)$ and $Z = X+Y$. There holds $\forall x \geq 0$,

$$f_1 \bar{\otimes} g_1(x) - 1 \leq F_Z(x) \leq f_2 \otimes g_2(x)$$

B. Wireless Sensor Network Architecture

Our WSN model assumes that each node has a CPU, a radio, an application-specific sensor suite, along with an environmental-specific energy harvesting head and an energy storage unit, such as a rechargeable battery or a supercapacitor. To account for a broad range of realistic environments, we make the common assumption that the amount of harvested power is uncontrollable but reasonably predictable, based on the source and the environmental history [12]. Prediction methods include techniques such as history-based round estimation [13]. Sample environment sources, including solar, wind and vibration, exhibit behavior that changes over time. Harvested energy is modeled as a series of finite length time varying *epochs*. The amount of energy harvested within each epoch can be stochastically characterized [14].

We target the common class of networked WSN systems that exhibit a many-to-one communication pattern [15]. The nodes form a collection tree rooted at a base station. The primary activity of the system is for each node to sense and process its data and then forward it, along with any packets received from its children on the tree, up to its parent until it is eventually received by the base station.

III. SINGLE NODE MODEL

Our goal of this paper is to obtain the probabilistic bounds for network-wide performance metrics including buffer usage, end-to-end latency, wasted energy, and dropped packets so that proper value of network parameters, including buffer and battery capacity, network topology, can be determined to stochastically quantify the network performance and dimension wireless sensor networks.

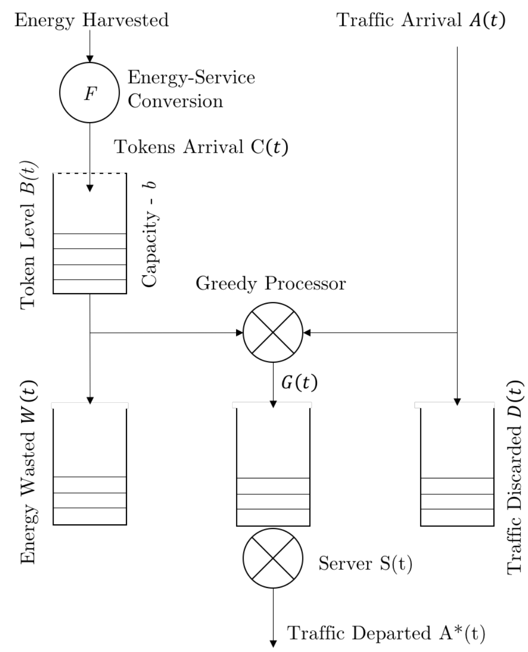

Fig. 1. Single Node Performance Model

To this end, in this section, we will start by describing the proposed model for a single node, followed by the calculus result where the bounds for traffic discarded and energy wasted at a single node will first be derived as Theorem 1 and 2, respectively. The energy-constrained traffic arrival flow will then be obtained in Theorem 3, and finally the delay and backlog bounds can be readily obtained from Equation 1 and 2.

A. Model Description

To integrate the energy constraints into network calculus, we have augmented the single queue-server model with three more queues and an additional greedy processor. Our model is depicted in Figure 1. In energy harvesting wireless sensor networks, the service available to the traffic at the sensor nodes depends closely on the amount of energy harvested and/or in the battery, which makes it difficult to accurately define a conventional service curve in a closed form. To address this issue, we have decoupled the energy constraints and the service process into two separated stages, as shown in Figure 1, where the harvested energy is viewed as tokens to constrain the arrived traffic first. The traffic passed through is then served with an energy-oblivious service curve based on the computation and communication capabilities of the sensor hardware. The details of the model and behaviors of its each component are described below.

1) Energy as Tokens: The flow of harvested energy is first converted to the flow of tokens according to an *Energy-Service Conversion* function F (which will be discussed in Section V). Without loss of generality, we assume that each token warrants one unit of incoming traffic being processed later by the server. After the conversion, the tokens are then accumulated in a bucket with a finite capacity. The bucket capacity is computed

TABLE I
LIST OF NOTATIONS

Symbol	Description
$A(t)$	Traffic Arrival
$A^*(t)$	Traffic Departed
$B(t)$	Token Bucket Level
$C(t)$	Token Arrival
$D(t)$	Traffic Discarded
$G(t)$	Greedy Processor Output
$S(t)$	Service Process
$W(t)$	Energy Wasted
$F(\cdot)$	Energy-Service Conversion Function
b	Token Bucket Capacity

using the same F function based on the capacity of the sensor's energy storage devices (battery or super-capacitor). When the bucket becomes full, overflowed tokens will be directed into a virtual queue with infinite capacity named *Energy Wasted*, where those excess tokens cannot be reclaimed later.

2) Greedy Processor: In the token-bucket-like stage, the incoming traffic flow is handled by a greedy processor which consumes tokens at the same time. The processor is greedy or work-conserving in the sense that when there is traffic to process, it will process the traffic immediately whenever there are enough tokens left in the bucket to do so. In other words, it will not happen that the bucket is non-empty while the arrived traffic is discarded. Other non-work-conserving energy usage scheduling polices may also be applied by replacing the greedy processor with other types of processors. If the token bucket is empty when traffic arrives, that portion of the incoming traffic will be directed to another virtual queue with infinite capacity named *Traffic Discarded*, where the traffic is discarded and will not be reprocessed when tokens become available later.

3) Actual Service: In the second stage, the traffic departed from the greedy processor will be processed through a single queue-server, where the traffic receives the actual service (i.e. processed, aggregated and transmitted) and finally leaves the sensor node. The server has an energy-oblivious service curve based on the hardware capability. The traffic backlog and the delay at a node are incurred only in this stage.

B. Notations and Assumptions

Suppose we have the following bounds on the flows,

- Traffic arrival is bounded by a *v.b.c* upper and lower arrival curve α_u and α_l with their corresponding probabilistic bounding functions f_u and f_l,
$$A \sim_{v.b.c} \langle f_u, f_l, \alpha_u, \alpha_l \rangle$$
- Token arrival is bounded by a *v.b.c* upper and lower arrival curve σ_u and σ_l with their corresponding probabilistic bounding functions h_u and h_l,
$$C \sim_{v.b.c} \langle h_u, h_l, \sigma_u, \sigma_l \rangle$$
- Service from the server S is bounded by a *s.c.* upper and lower service curve β_u and β_l with their corresponding probabilistic bounding functions g_u and g_l,
$$S \sim_{s.c} \langle g_u, g_l, \beta_u, \beta_l \rangle$$

Additionally, we use the discrete time domain and a generic unit for the traffic and harvested energy, which may represent real units as needed, including bits, bytes or packets.

Energy harvesting from common ambient energy sources such as solar and wind is often a time-variant cyclostationary process. These energy sources are uncontrollable but predictable. In order to model the harvested energy or tokens using the same *v.b.c* model with the traffic arrival, an epoch-based approach is assumed so that in each epoch, the energy or token arrival can be viewed as a stationary process and bounded by the *v.b.c* model.

The descriptions for the notations used in the model are provided in Table I.

C. Calculus for A Single Node

In this subsection, the calculus will be focused on a single sensor node with energy harvesting. From Figure 1, the token level at time t can be expressed recursively as follows,

$$B(t) = min\left\{b, [B(t-1)+C(t-1,t)-A(t-1,t)]^+\right\}$$

The following lemma provides a non-recursive expression of $B(t)$, which can be readily proved with result in [16].

Lemma 2. *Token Level.*

$$\begin{aligned}B(t) &= \underset{0\le s\le t}{inf}\left\{\underset{s\le u\le t}{sup}\{C(u,t)-A(u,t), C(s,t)-A(s,t)+b\}\right\}\\ &= \underset{0\le s\le t}{sup}\left\{\underset{s\le u\le t}{inf}\{C(u,t)-A(u,t), C(s,t)-A(s,t)+b\}\right\}\end{aligned}$$

From Figure 1, the discarded traffic flow and wasted energy flow at time t can be expressed recursively as follows,

$$\begin{aligned}D(t) &= [A(t-1,t)-C(t-1,t)-B(t-1)]^+\\ W(t) &= [B(t-1)+C(t-1,t)-A(t-1,t)-b]^+\end{aligned}$$

With Lemma 2, the non-recursive representations of the discarded traffic and wasted energy can be derived by substituting $B(t-1)$, as shown in the following two corollaries.

Corollary 1. *Traffic Discarded.*

$$D(t) = \Big[\underset{0\le s\le t-1}{sup}\Big\{\underset{s\le u\le t-1}{inf}\{A(u,t)-C(u,t), A(s,t)-C(s,t)-b\}\Big\}\Big]^+$$

Corollary 2. *Energy Wasted.*

$$W(t) = \Big[\underset{0\le s\le t-1}{inf}\Big\{\underset{s\le u\le t-1}{sup}\{[C(s,t)-A(s,t), C(u,t)-A(u,t)-b\}\Big\}\Big]^+$$

With Corollary 1 and the bounds satisfied by $A(t)$ and $C(t)$ as described in subsection III-B, we can stochastically bound $D(t)$ as shown in the following Theorem 1.

Theorem 1. *Traffic Discarded Bound.* For $\forall x \ge 0$,

$$Prob\{D(t) > x+v_{max}(\alpha_u, \sigma_l)-b\} \le f_u \otimes h_l(x)$$

Proof. The following holds for $\forall x \geq 0$,

$$Prob\{D(t) > x\} = Prob\{D'(t) > x\} \tag{3}$$

where $D(t) = [D'(t)]^+$. $D'(t)$ can be bounded as follows,

$$\begin{aligned}
D'(t) = & \sup_{0 \leq s \leq t-1} \{ \inf_{s \leq u \leq t-1} \{A(u,t) - C(u,t), A(s,t) \\
& \qquad\qquad\qquad\qquad - C(s,t) - b\}\} \\
\leq & \sup_{0 \leq s \leq t} \{A(s,t) - C(s,t) - b\} \\
\leq & \sup_{0 \leq s \leq t} \{A(s,t) - \alpha_u(t-s)\} \\
& + \sup_{0 \leq s \leq t} \{\sigma_l(t-s) - C(s,t)\} \\
& + \sup_{0 \leq s \leq t} \{\alpha_u(t-s) - \sigma_l(t-s)\} - b \\
\leq & \sup_{0 \leq s \leq t} \{A(s,t) - \alpha_u(t-s)\} \\
& + \sup_{0 \leq s \leq t} \{\sigma_l(t-s) - C(s,t)\} + v_{max}(\alpha_u, \sigma_l) - b
\end{aligned}$$

By Lemma 1 and Equation 3, we have the following,

$$Prob\{D(t) > x + v_{max}(\alpha_u, \sigma_l) - b\} \leq f_u \otimes h_l(x)$$

which completes the proof of Theorem 1. □

Similarly, with Corollary 2 we can stochastically bound $W(t)$ as Theorem 2 with its proof provided in Appendix A.

Theorem 2. *Energy Wasted Bound.* For $\forall x \geq 0$,

$$Prob\{W(t) > x + v_{max}(\alpha_u, \sigma_l) - b\} \leq f_u \bar{\oslash} h_l(x) + 1$$

We now proceed to the second stage of the proposed model. In the following, Theorem 3 characterizes the output from the greedy processor as a *v.b.c* arrival curve, with which the backlog and delay of the node can be readily obtained.

Theorem 3. *Greedy Processor Output.* $G(t)$ is constrained by a *v.b.c* arrival curve as follows,

$$\begin{aligned}
G \sim_{v.b.c} \langle & f_u \otimes f_u \otimes h_l, f_l \oslash f_u \otimes h_l, \\
& \alpha_u - b + v_{max}(\alpha_u, \sigma_l), \alpha_l - b + v_{max}(\alpha_u, \sigma_l) \rangle
\end{aligned}$$

Proof. Consider $G(s,t), \forall 0 \leq s \leq t$,

$$\begin{aligned}
G(s,t) &= A(s,t) - D(s,t) \\
&\leq A(s,t) - \alpha_u(t-s) + \alpha_u(t-s) + D(s) \\
&= A(s,t) - \alpha_u(t-s) + \alpha_u(t-s) + D(s) \\
&\quad + b - v_{max}(\alpha_u, \sigma_l) - b + v_{max}(\alpha_u, \sigma_l)
\end{aligned}$$

Hence,

$$\begin{aligned}
\sup_{0 \leq s \leq t} \{G(s,t) - \alpha_u(t-s)\} \leq & \sup_{0 \leq s \leq t} \{A(s,t) - \alpha_u(t-s)\} \\
& + D(t) + b - v_{max}(\alpha_u, \sigma_l) \\
& - b + v_{max}(\alpha_u, \sigma_l)
\end{aligned}$$

By Lemma 1, we have the *v.b.c* upper bound,

$$\begin{aligned}
& Prob\{\sup_{0 \leq s \leq t} \{G(s,t) - \alpha_u(t-s) + b - v_{max}(\alpha_u, \sigma_l)\} > x\} \\
& \leq f_u \otimes f_u \otimes h_l(x)
\end{aligned}$$

Similarly, we may obtain the *v.b.c* lower bound as follows,

$$\begin{aligned}
& Prob\{\sup_{0 \leq s \leq t} \{\alpha_l(t-s) - G(s,t) - b + v_{max}(\alpha_u, \sigma_l)\} > x\} \\
& \geq f_l \oslash f_u \otimes h_l(x)
\end{aligned}$$

which completes the proof of Theorem 3. □

By Theorem 3 and Equation 1 and 2, we are able to immediately obtain the bounds for the backlog in the traffic buffer and the delay as the following two corollaries.

Corollary 3. *Backlog Bound.*

$$\begin{aligned}
& Prob\{BKL(t) > v_{max}(\alpha_u, \beta_l) - b + v_{max}(\alpha_u, \sigma_l) + x\} \\
& \leq f_u \otimes f_u \otimes h_l \otimes g_l(x)
\end{aligned}$$

Corollary 4. *Delay Bound.*

$$\begin{aligned}
& Prob\{DLY(t) > h_{max}(\alpha_u - b + v_{max}(\alpha_u, \sigma_l) + y, \beta_l)\} \\
& \leq f_u \otimes f_u \otimes h_l \otimes g_l(y)
\end{aligned}$$

IV. Multi-hop Network Model

In this section, the calculus for a multi-hop network will be studied. First, the output characteristic of a sensor node with energy constraints will be provided to facilitate the analysis to a network. In other words, the output traffic flow will be characterized in the same form as the *v.b.c* arrival curve so that it can be fed directly into its next hop, enabling the iterative node-by-node analysis. Based on the output characteristic, we will then provide the superposition property of the aggregated flows, along with result of network-wide performance metrics in a tree-based data collection wireless sensor network.

Assuming the traffic and the server satisfy the same aforementioned *v.b.c* arrival curve and the *s.c.* service curve given in subsection III-B, respectively, we have the following Theorem 4 that bounds the output flow of a node with the same type of curve as the traffic arrival flow.

Theorem 4. *Output Characteristic.* The output flow of a node $A^*(t)$ satisfies a *v.b.c* arrival curve shown as follows,

$$A^* \sim_{v.b.c} \langle f_u \otimes g_l, f_l \bar{\otimes} g_u - 1, \alpha_u \oslash \beta_l, \beta_u \bar{\oslash} \alpha_l \rangle$$

Proof. The upper bound of the curve has been proved in [11]. We now provide the proof for the lower bound. Consider the output $A^*(s,t)$, for $\forall t \geq s \geq 0$, the following holds,

$$\begin{aligned}
A^*(s,t) \geq & A^*(t) - A(s) \\
= & A \bar{\otimes} \alpha_l(s) - A(s) + A^*(t) - A \bar{\otimes} \alpha_l(s) \\
\geq & A \bar{\otimes} \alpha_l(s) - A(s) + A^*(t) \\
& - \sup_{0 \leq u \leq s} \{A(u) + \beta_u(t-u)\} \\
& - \sup_{0 \leq u \leq s} \{\alpha_l(s-u) - \beta_u(t-u)\} \\
\geq & A \bar{\otimes} \alpha_l(s) - A(s) + A^*(t) - A \bar{\otimes} \beta_l(t) \\
& - \sup_{0 \leq v \leq s} \{\alpha_l(v) - \beta_u(t-s+v)\} \\
\geq & A \bar{\otimes} \alpha_l(s) - A(s) + A^*(t) - A \bar{\otimes} \beta_l(t) + \beta_u \bar{\oslash} \alpha_l(t-s)
\end{aligned}$$

Hence,

$$A^*(s,t) - \beta_u \bar{\oslash} \alpha_l(t-s) \geq A \bar{\otimes} \alpha_l(s) - A(s) + A^*(t) - A \bar{\otimes} \beta_l(t)$$

By Lemma 1, we have,

$$Prob\{A^*(s,t) - \beta_u \bar{\oslash} \alpha_l(t-s) > x\} \geq f_l \bar{\otimes} g_u(x) - 1$$

which completes the proof of Theorem 4. □

With Theorem 3 and 4, we immediately have the following corollary which bounds the output with energy harvesting.

Corollary 5. *Output Characteristic with Energy Harvesting.*

$$\begin{aligned} A^* \sim_{v.b.c} \langle & f_u \otimes f_u \otimes h_u \otimes g_l, f_l \oslash f_u \otimes h_l \bar{\otimes} g_u - 1, \\ & \alpha_u \oslash \beta_l - b + v_{max}(\alpha_u, \sigma_l), \beta_u \bar{\oslash} \alpha_l - b + v_{max}(\alpha_u, \sigma_l) \rangle \\ \sim_{v.b.c} \langle & f_u^*, f_l^*, \alpha_u^*, \alpha_l^* \rangle \end{aligned}$$

Theorem 5 describes the aggregation of multiple traffic arrival flows to facilitate the analysis of the many-to-one collection tree traffic pattern.

Theorem 5. *Superposition Property*. Consider N flows arrive at the same node with their arrival processes $A^v(t)$ satisfying $v.b.c$ arrival curve as follows,

$$A^v \sim_{v.b.c} \langle f_u^v, f_l^v, \alpha_u^v, \alpha_l^v \rangle, v = 1, 2, ..., N.$$

That node will experience an aggregated traffic flow $A(t)$ with its arrival curve defined as follows,

$$\begin{aligned} A \sim_{v.b.c} \langle & f_u^1 \otimes f_u^2 \otimes \cdots \otimes f_u^N, \\ & f_l^1 \bar{\otimes} f_l^2 \bar{\otimes} \cdots \bar{\otimes} f_l^N, \sum_{i=1}^N \alpha_u^i, \sum_{i=1}^N \alpha_l^i \rangle \end{aligned}$$

The upper bound of the aggregated flow has been proved in [11]. With similar reasoning, the lower bound can be readily proved and therefore is omitted here.

Now, given the tree topology of the network and Theorem 5, we are able to derive several network-wide metrics using a node-by-node analysis. For the aforementioned data collection wireless sensor network, the traffic arrival for a sensor node is the sum of all the traffic departed from its children plus the measurements generated by itself [6]. By Theorem 5, the traffic arrival $A^v(t)$ for node v is the following,

$$\begin{aligned} A^v \sim_{v.b.c} & \Big\langle f_u^{\prime v} \otimes \bigotimes_{i \in child(v)} f_u^{*i},\ f_l^{\prime v} \bar{\otimes} \bar{\bigotimes_{i \in child(v)}} f_l^{*i}, \\ & \alpha_u^{\prime i} + \sum_{i \in child(v)} \alpha_u^{*i},\ \alpha_l^{\prime i} + \sum_{i \in child(v)} \alpha_l^{*i} \Big\rangle \\ \sim_{v.b.c} & \langle f_u^v, f_l^v, \alpha_u^v, \alpha_l^v \rangle \end{aligned}$$

where for the notation purpose, we use $\bigotimes$ and $\bar{\bigotimes}$ to denote the multiple consecutive min-plus and max-plus convolutions, respectively. From the equation above, the arrival curve of the output flow can be readily derived by Corollary 5, so as the bound for packet loss (by Theorem 1), the bound for backlog (by Corollary 3) and the bound for delay (by Corollary 4). With these metrics available for every node, the network-wide performance metrics can be obtained given the network topology. Here we present one example regarding the data collection latency in energy harvesting wireless sensor networks. Intuitively, the data collection latency in a tree-based network is defined as follows,

$$DLY_{network} = \max_{v \in V}\{DLY_{v \to S}\} = \max_{v \in V}\Big\{\sum_{i \in path(v)} DLY_i\Big\}$$

where V is the set of all the sensor nodes in the network and $DLY_{v \to S}$ is the latency along the route from the node v to the sink S, DLY_i is the delay incurred at node i, and $path(v)$ is the set of all the nodes on the path from v to S according to the topology of the collection tree. From Equation IV and Corollary 4, DLY_i can be probabilistically bounded. Then by applying Lemma 1 recursively along the path, we are able to obtain the probabilistic bounding function for $DLY_{v \to S}$ for every node v, denoted as $f_{v \to S} \geq Prob\{DLY_{v \to S} > x\}$. Finally, we can probabilistically bound $DLY_{network}$ as follows,

$$\begin{aligned} & Prob\{DLY_{network} > x\} \\ & \quad = 1 - Prob\{\max_{v \in V}\{DLY_{v \to S}\} \leq x\} \\ & \quad = 1 - \prod_{v \in V} Prob\{DLY_{v \to S} \leq x\} \\ & \quad = 1 - \prod_{v \in V} (1 - Prob\{DLY_{v \to S} > x\}) \\ & \quad \leq 1 - \prod_{v \in V} (1 - f_{v \to S}(x)) \end{aligned}$$

We now briefly discuss the complexity to compute the probabilistic delay bound showed above. Suppose the data collection tree is bounded by its degree D and its height H. The total number of nodes V is therefore $\mathcal{O}(D^H)$. Assume the cost for each convolution operation is constant with regard to the number of nodes. First the delay incurred at each node can be computed in a bottom-up fashion from the leaf nodes to the sink. At each node, $\mathcal{O}(D)$ amount of min-plus convolutions and additions are needed. Therefore, in total we have $\mathcal{O}(D \times V)$ min-plus convolutions and additions. We are then able compute the delay along each route in a top-down fashion from the sink to every leaf node. The amount of work at each node is constant, resulting in a total complexity of $\mathcal{O}(V)$. Hence, add the two results and we finally have the complexity to compute the multi-hop network delay bound $\mathcal{O}(D \times V)$, which essentially scales linearly with the size of the network and therefore is efficient to compute for performance evaluation and network dimensioning.

V. Model Instantiation

In this section, we will present how to instantiate the proposed abstract model based on the parameters of real WSN nodes and energy harvesting traces. For the traffic arrival and service curves, various models have been proposed in [11], [17] to characterize the traffic source, processors and communications, such as periodic traffic source, constant rate servers, and Bernoulli process-impaired communication channels. Here we will focus on procedures to obtain 1) the energy-service function based on the sensor hardware parameters; and 2) the energy arrival curve from the experimental measurements of solar energy harvesting.

A. Energy-service Function

In order to convert the energy to tokens, we study the amount of energy consumed by processing one unit of the traffic. Here we adopt a common model for energy consumption of CPU and radio communication as described in [18].

The power of the CPU P^{CPU} is composed of two components: The static power P^{CPU}_{static} and the dynamic power consumption. The dynamic power consumption includes the frequency-independent P^{CPU}_{ind} and frequency-dependent components $C_{ef}{\cdot}s^{\alpha}$, where C_{ef} is the effective switching capacitance, f is the CPU frequency and α is the constant.For simplicity, we assume the CPU is running at a constant frequency f_c. Also, one unit of the traffic requires c CPU cycles to process. Hence, the total energy consumed by CPU to process n unit of traffic can be derived as follows,

$$E^{CPU}(n) = \frac{nc}{f_c}(P^{CPU}_{static}{+}P^{CPU}_{ind}{+}C_{eff}{\cdot}f^{\alpha}_c) \tag{4}$$

For the radio, similarly, we assume the transceiver is running at a constant modulation level m_c. Also, one unit of the traffic requires to transmit b bits of data. We can then derive the total energy consumed to transmit n unit of traffic as follows,

$$E^{TX}(n) = \frac{nb}{R_s{\cdot}m_c}(P^{TX}_{static}{+}C_s{\cdot}\phi(m_c){\cdot}R_s) \tag{5}$$

where C_s is determined by radio hardware and the environment and can be approximated as a constant assuming a time invariant channel, P^{TX}_{static} is the static power consumption of the radio, R_s is the symbol rate, and $\phi(b)$ is a convex function of the modulation level and its specific form depends upon the modulation scheme.

By adding Equation 4 and Equation 5 together and inverting the result with regard to n, we can obtain the energy-service function, where floor is used to get discretized token quantity.

$$F(E) = \Big\lfloor E \cdot \Big(\frac{c}{f_c}(P^{CPU}_{static} + P^{CPU}_{ind} + C_{eff} \cdot f^{\alpha}_c) + \frac{b}{R_s \cdot m_c}(P^{TX}_{static} + C_s \cdot \phi(m_c) \cdot R_s) \Big)^{-1} \Big\rfloor$$

B. Energy Curve

Sensor nodes may harvest various ambient energy including solar, thermal and vibration energy. As aforementioned, solar energy is used in this example and epoch-based analysis is utilized to approximate a stationary process. Assuming the distribution of the harvested energy process is obtained, we may obtain the *v.b.c* arrival curves of the harvested energy using the Moment-Generating Functions (MGFs) and Chernoff bound [19] as follows, $\forall\theta > 0$,

$$Prob\Big\{ \sup_{0\le s\le t} \{C(s,t){-}\sigma_u(t{-}s)\} > x \Big\} \le e^{-\theta x} M_X(\theta)$$

$$Prob\Big\{ \sup_{0\le s\le t} \{\sigma_l(t{-}s){-}C(s,t)\} > x \Big\} \le e^{-\theta x} M_{X'}(\theta)$$

given that the MGF of X and X' exists, where $X \equiv \sup_{0\le s\le t} \{C(s,t){-}\sigma_u(t{-}s)\}, X' \equiv \sup_{0\le s\le t} \{\sigma_l(t{-}s){-}C(s,t)\}$. Hence, we have the *v.b.c* energy arrival curve as follows,

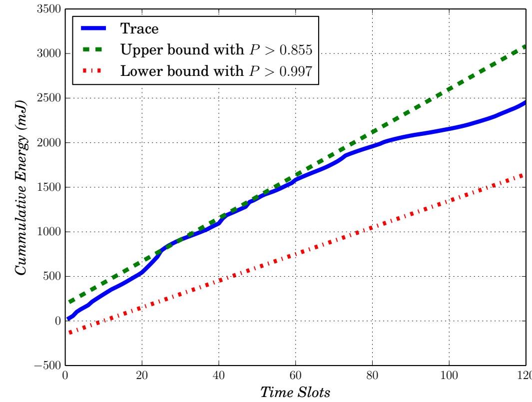

Fig. 2. Energy arrival upper and lower curves with their probabilities

$$C \sim_{v.b.c} \langle e^{-\theta x} M_X(\theta), e^{-\theta x} M_{X'}(\theta), \sigma_u, \sigma_l \rangle$$

A real solar energy harvesting trace [20] will now be used as an example to instantiate the energy arrival curve as follows. The measurement was obtained by sampling the current (in mA) generated by the solar cells ($2.3V$) at intervals of 30 seconds during a two-month period in Hamburg, Germany. Since the harvested solar energy is a time-variant or cyclostationary process, the epoch-based approach is used to obtain approximately time-invariant periods. We choose an one-hour period from the trace and use the affine arrival model $\sigma = rt{+}b$ for the upper and lower energy arrival curves.

Figure 2 shows the resulting upper and lower curves from the procedures described above. The blue solid line is the trace from the solar panel. The green dashed line ($\sigma_u(t) = 24.15t{+}186.3$) is the upper curve with a probability greater than 0.855 that the actual measurement of the energy is below that curve. The red dash-dot line ($\sigma_l(t) = 14.95t{-}147.2$) is the lower curve with a probability greater than 0.997 that the actual measurement is above that curve. In summary, we have the *v.b.c* energy arrival process $E(t)$ bounded as follows,

$$E \sim_{v.b.c} \langle 0.145e^{-x}, 0.003e^{-x}, 24.15t{+}186.3, 14.95t{-}147.2 \rangle$$

From the figure, we can see that we have chosen a tighter upper bound, i.e. closer to the real process in the graph, and a relatively looser lower bound. However, the probability when the measurement is below the upper bound (85.5%) is significantly lower than that when the measurement is above the lower bound (99.7%). In practice, those two curves can be adjusted based on the expected probabilities, which is a trade-off between the tightness and the confidence. Here we assume a general distribution and MGF of the energy arrival without any prior knowledge of the underlying governing process. More accurate upper and lower bound can be obtained using estimation techniques based on history, e.g. [13], and more explicit models for the energy arrival process from exponentially and stochastically bounded burstiness [21] to models that studies the underlying physics such as [22].

Though here we study energy instead of token arrival, with the energy-service function provided in the previous subsection, the curves for token arrival can be readily obtained. Together with the traffic curves, we are able to compute the result of theorems presented previously and dimension the network based on the expected performance level.

VI. Conclusion

In this paper, we proposed a novel model to integrate the energy into the network calculus framework for worst-case performance analysis. In the proposed model, the energy is converted into tokens to constrain the service to the traffic. Meanwhile, behaviors of the sensor during the situations of energy overflow and underflow are accurately characterized by two virtual queues. Various performance metrics are analyzed including delay, backlog, packet loss, energy waste, are stochastically bounded for both a single node and for a tree-based data collection wireless sensor network. Additionally, detailed model instantiation procedures are provided for a specific wireless sensor network based on the measurements and its hardware parameters and network organization.

Appendix A
Proof of Theorem 2

Proof. The following holds for $\forall x \geq 0$,

$$Prob\{W(t) > x\} = Prob\{W'(t) > x\} \tag{6}$$

where $W(t) = [W'(t)]^+$. $W'(t)$ can be bounded as follows,

$$\begin{aligned}
W'(t) =& \sup_{0 \leq s \leq t-1} \{ \inf_{s \leq u \leq t-1} \{C(u,t) - A(u,t) - b, C(s,t) \\
&\qquad - A(s,t)\}\} \\
\leq& \sup_{0 \leq s \leq t} \{C(s,t) - A(s,t) - b\} \\
=& \sup_{0 \leq s \leq t} \{C(s,t) - \sigma_u(t-s) + \sigma_u(t-s) - \alpha_l(t-s) \\
&\qquad + \alpha_l(t-s) - A(s,t)\} - b \\
\leq& \sup_{0 \leq s \leq t} \{C(s,t) - \sigma_u(t-s)\} \\
&+ \sup_{0 \leq s \leq t} \{\alpha_l(t-s) - A(s,t)\} \\
&+ \sup_{0 \leq s \leq t} \{\sigma_u(t-s) - \alpha_l(t-s)\} - b \\
\leq& \sup_{0 \leq s \leq t} \{C(s,t) - \sigma_u(t-s)\} \\
&+ \sup_{0 \leq s \leq t} \{\alpha_l(t-s) - A(s,t)\} + v_{max}(\sigma_u, \alpha_l) - b
\end{aligned}$$

By Lemma 1 and Equation 6, we have the following,

$$Prob\{W(t) > x + v_{max}(\sigma_u, \alpha_l) - b\} \leq h_u \otimes f_l(x)$$

which completes the proof of Theorem 2. □

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