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Throughput Analysis of Cognitive Radio Networks Via Stochastic Network Calculus

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Abstract—**Cognitive radio is a promising technique in improving the utilization of wireless spectrum by spectrum sensing. However, conceivable spectrum sensing errors may deteriorate the quality of service provided for users. In this paper, we propose a framework to analyze the per-user throughput of both primary users and secondary users. Based on this framework, we further estimate how sensing errors influence the throughput in cognitive radio networks. To this end, stochastic network calculus is employed as a basic tool, which is a powerful theory for quality of service analysis of computer networks. Impairment processes of service are introduced to construct the service models for both primary users and secondary users. Then per-user throughput bounds of both primary users and secondary users are obtained after considering arrival process of primary users. Numerical result shows that sensing errors play a decisive role on the upper bound of primary users' throughput, while have less impact on the lower bound of secondary users' throughput.**

I. Introduction

The scarce spectrum resources have been allocated to meet various wireless communication demands. However, the spectrum has been observed underutilized most of time [1]. To tackle this problem, cognitive radio is proposed as a promising technique [2]. In cognitive radio networks, unauthorized secondary users (SUs) can access to the spectrum licensed to primary users (PUs) when PUs have no data to transmit. To identify the spectrum state, spectrum sensing is employed. Based on the sensing result, SUs transmit over the channel without interfering with PUs. However, sensing errors always happen which have impact on the transmission of both PUs and SUs. Furthermore, due to the inherent hierarchy and the opportunistic scheduling, PUs and SUs may have impact on each other. Therefore, performance metrics, such as delay and capacity, of both PUs and SUs is urgently needed.

In the literature, attempts have been devoted to the performance analysis for cognitive radio networks. Queueing theory is an effective method to obtain the waiting time in the queue, the length of the queue and delay of users by using finite state Markov chain and different queueing models [3][4]. In some other works, traditional stochastic process is employed. A stochastic differential equation is adopted to conduct a queueing delay analysis [5]. To obtain the capacity metrics, the probability distribution function or cumulative distribution function of SNR (Signal to Noise Ratio) or SINR (Signal to Interference-and-Noise Ratio) is usually derived based on traditional stochastic theory [6][7].

However, these delay and capacity results, which illustrating the steady state, are always in the sense of average. Yet, cognitive radio networks have a stochastic nature, especially the spectrum-sensing-based opportunistic scheduling. In fact, the spectrum sensing results always have errors at a certain probability, these errors are classified into two categories of mis-detection and false alarm. Mis-detection results in a retransmission for PUs, while false alarm results in a waste of opportunities of transmission for SUs. These sensing errors make the performance analysis more complicated. To tackle this problem, stochastic network calculus can be employed as a more effective method. Stochastic network calculus constructs arrival curve [8][11] and service curve [9][11] for the arrival process and service process of the network. Based on these curves, performance bounds, typically delay and backlog bounds, are derived [10][11]. Stochastic network calculus evolves to throughput analysis based on Forin's work [12][13]. As to the performance analysis of cognitive radio networks, Gao *et al.* employ stochastic network calculus to derive delay bounds for both PU and SU, by considering imperfect spectrum sensing and retransmission [14]. And further, they conduct a delay-constrained capacity analysis in [15].

In this paper, we propose a framework to analyze the per-user throughput for cognitive radio networks with single PU and multiple SUs. Based on this framework, we estimate how the sensing error and contention-based (CB) access scheme will influence the throughput. To deal with these detriments, impairment processes of both PU and SUs are introduced, then service processes are derived for PU and SUs respectively. With the manipulation of min-plus convolution, throughput bounds are deduced based on arrival process and service precess of PU and SUs. For PU, it has the preemptive priority to the channel, however, the sensing error of mis-detection makes it unable to fully utilize the channel service capacity. Therefore, PU can be only provided with an upper bound of the throughput. For SUs, even though they have lower priority to the channel, SUs can still get a lower bound of the per-user throughput, considering the influence of both kinds of sensing errors and CB access scheme. Numerical result is presented for the per-user throughput bounds of PU and SUs under different configurations. Accordingly, the impact of sensing error and CB access scheme is discussed.

The rest of this paper is organized as follows. Section II describes the network model and constructs service process for PU and SUs. The per-user throughput bounds are then derived in Section III. To validate the performance bound, numerical results are presented and discussed in Section IV. Conclusions

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TABLE I. NOTATIONS

Symbol	Definition
$A^{PU}(t), A^{SU}(t)$	Arrival process of PU, SU
α^{PU}	PU's average arrival rate
$S^{PU}(t), S^{SU}(t)$	Service process of PU, SU
$D^{PU}(t), D^{SU}(t)$	Service process of PU, SU
$I^{PU}(t), I^{SU}(t)$	Impairment process of PU, SU
$I_c^{PU}(t)$	Impairment process of a tagged SU in competition
$\lambda_{U,t}^{PU}, \lambda_U^{PU}$	Transient and asymptotic upper bound of PU's throughput
$\lambda_{L,t}^{SU}, \lambda_L^{SU}$	Transient and asymptotic lower bound of SU's throughput
p_{MD}, p_{FA}	Probability of mis-detection, false alarm
p_a^{PU}	Probability that at leat one packet from PU arrivals
p_c^{SU}	Probability that a tagged SU succeed in the competition when channel is idle
p_l	Probability that channel service is ready for SU
p_I	Probability a tagged SU can transmit successfully

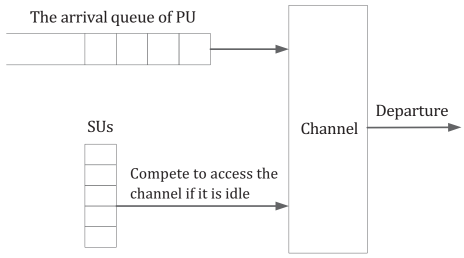


Fig. 1. PU preempts the channel service if its arrival queue is not empty. A spectrum sensing error (refers to mis-detection) will cause collision and results in PU's retransmission. When the channel is sensed to be idle, SUs compete to access the channel.

are draw in the final section.

II. Network Model

In this section we first present the basic tools used for analyzing the performance bounds, which stem from stochastic networks calculus. Based on these tools, we then describe the model for cognitive radio networks with single PU and multiple SUs, of which the arrival process and service process of PU and SUs are derived respectively. However, since the service provided by channel may be impaired by system errors, like sensing error, resulting in a waste of channel resources. To estimate the waste quantificationally, impairment process is introduced.

A. Basics tools

Stochastic network calculus [11] is a newly developed theory. Different from traditional queueing theory, stochastic network calculus concerns the arrival and service process of a queuing system, and yields the performance bounds of service expediently, such as delay bound and backlog bound. Consider a queuing system, we denote cumulative arrival process, cumulative service process and cumulative departure process during period of $[s,t]$ as $A(s,t) = A(t) - A(s)$, $S(s,t) = S(t) - S(s)$ and $D(s,t) = D(t) - D(s)$, respectively. Moreover, let $A(0) = S(0) = D(0) = 0$. According to [11], we have

$$D(t) = A \otimes S(t) = \inf_{0 \le s \le t} \{A(s) + S(s,t)\}. \qquad (1)$$

where $\otimes$ is called as min-plus convolution operator. Begin with Eq. (1), metrics of QoS, like delay and backlog, can be depicted precisely.

However, in this paper, per-user throughput is of great interest. The definition of non-asymptotic lower bound of throughput [12] is the maximal rate λ_t satisfying

$$Pr(D(t) \le \lambda_{L,t} t) \le \varepsilon, \qquad \text{for all } t \ge 0. \qquad (2)$$

which is a probabilistic definition by bounding the cumulative departure process $D(t)$ at time t, and ε is a violation probability close to zero like 10^{-3}. Analogously, a non-asymptotic upper bound of throughput is the minimum rate λ_t defined as

$$Pr(D(t) \ge \lambda_{U,t} t) \le \varepsilon, \qquad \text{for all } t \ge 0. \qquad (3)$$

Concretely, consider a cognitive radio network consisting of one PU and N SUs, they are served by a wireless channel with a service process $S(t)$, see Fig. 1. The arrival process $A^{PU}(t)$ of PU follows a Poisson process with average arrival rate α^{PU}, and its moment generation function (MGF) is

$$M_{A^{PU}(t)}(\theta) = E\left[e^{\theta A^{PU}(t)}\right] = e^{\alpha^{PU} t (e^{\theta} - 1)}.$$

The following inequality, known as the Chernoff bound and will be used frequently later, gives an upper bound on the complementary cumulative distribution function of a random variable X:

$$Pr(X \ge x) \le e^{-\theta x} E\left[e^{\theta X}\right]$$

where $\theta \ge 0$ is a free parameter.

Moreover, a discrete time model is introduced with increment Δt representing the transmission time for one data unit (i.e. packet). During time slot $[N\Delta t, (N+1)\Delta t]$, the channel serves PU exclusively if the queue of PU is not empty, otherwise the channel serves a SU who succeeds in the competition with other SUs.

B. Service Process of PU

In a cognitive radio network, SUs sense the spectrum state and compete with each other to access the channel if the channel is sensed to be idle. Naturally, sensing errors are inevitable and can be classified into two types [16], mis-detection (MD) and false alarm (FA). Mis-detection means that the spectrum is occupied by PU but the spectrum sensing result says it is available for SUs. As a result, a collision occurs and PU's packet need to be retransmitted, meanwhile SUs have to wait till next idle period. Whereas, false alarm occurs in the opposite way, when the spectrum is idle but SUs believe the spectrum is being used by PU, which will waste transmission opportunities for SUs. Let p_{MD} and p_{FA} denotes the probability of mis-detection and false alarm occurs in a time slot accordingly, let $C_{state} = 1$ and $C_{state} = 0$ denotes the event that channel state is occupied by PU or idle accordingly, let $S_{result} = 1$ and $S_{result} = 0$ denotes the event that spectrum is sensed to be busy and idle accordingly, then

$$\begin{cases} p_{MD} = Pr(S_{result} = 0 | C_{state} = 1), \\ p_{FA} = Pr(S_{result} = 1 | C_{state} = 0). \end{cases}$$

Let p_e denotes the probability that spectrum state is incorrectly sensed, then $p_e = p_{MD} + p_{FA}$. Assume a is a constant in $[0,1]$, let $p_{FA} = a p_e$ and $p_{MD} = (1-a) p_e$. As

$C_{state} = 1$ denotes the event that channel state is occupied by PU, i.e., there is at least one packet from PU arrives within this time slot, its probability is denoted by p_a^{PU}, then $Pr(C_{state} = 1) = p_a^{PU} = 1 - e^{-\alpha^{PU}}$ holds since PU's arrival is a Poisson process, meanwhile $Pr(C_{state} = 0) = e^{-\alpha^{PU}}$.

Assuming that channel's service rate is C, and it is normalized as $C = 1$ for simplicity. As a consequence, the average arrival rate of PU is restricted by $0 \le \alpha^{PU} \le 1$ for the stability concern of the system. Next, we introduce a definition which is a key analytical tool in this paper, i.e., the impairment process of channel's service.

Definition 1: The impairment process $I^{PU}(t)$ of PU is defined by a Bernoulli increment process $I^{PU}(t, t+\Delta t) := I^{PU}(t+\Delta t) - I^{PU}(t)$ as

$$I^{PU}(t, t+\Delta t) = \begin{cases} 0, & \text{if a packet from PU is successfully transmitted during } (t, t+\Delta t]. \\ \Delta t, & \text{otherwise.} \end{cases}$$

The probabilities are for $q = Pr(p_{state} = 1)p_{MD} = p_a^{PU} p_{MD}$

$$\begin{cases} Pr\left(I^{PU}(t, t+\Delta t) = 0\right) = 1 - q, \\ Pr\left(I^{PU}(t, t+\Delta t) = \Delta t\right) = q. \end{cases}$$

Naturally, the cumulative impairment process during $[0, t]$ is $\sum_{N=0}^{t/\Delta t} I^{PU}(N\Delta t, (N+1)\Delta t)$ (assume $t/\Delta t$ is an integer), it represents the cumulative time during which mis-detection happens when packets from PU is served. More generally, consider a arbitrary time period $[s, t]$, note that the service rate is normalized, then the cumulative service provided by the channel during $[s, t]$ is $t - s$. Account for the impairment process defined above, PU'S service process is

$$S^{PU}(s, t) = t - s - I^{PU}(s, t), \tag{4}$$

which is a leftover service process since the impairment process is always of highest priority. The MGF of PU's service process is given below

$$\begin{aligned} M_{S^{PU}(s,t)}(\theta) &= E\left[e^{\theta S^{PU}(s,t)}\right] = E\left[e^{\theta \sum_{i=s}^{t-1} S^{PU}(i,i+1)}\right] \\ &= \prod_{i=s}^{t-1} E\left[e^{\theta S^{PU}(i,i+1)}\right]. \end{aligned}$$

In turn, service process $S^{PU}(t, t+\Delta t)$ is actually a complement process of $I^{PU}(t, t+\Delta t)$ in time slot Δt, so $Pr(S^{PU}(t, t+\Delta t) = 0) = Pr(I^{PU}(t, t+\Delta t) = \Delta t) = q$, and vice versa. Then

$$\begin{aligned} M_{S^{PU}(s,t)}(\theta) &= \prod_{i=s}^{t-1} \left[q e^0 + (1-q)e^\theta\right] \\ &= \left[p_a^{PU} p_{MD} + (1 - p_a^{PU} p_{MD})e^\theta\right]^{t-s}, \end{aligned} \tag{5}$$

note that the service processes in any two time slots are independent, the above derivation follows intuitively.

C. Service Process of SUs

Based on the working mechanism of cognitive radio networks, the service process for SUs is actually a leftover service of PU and impairment, i.e., SUs are of lowest priority. But not only that, to a tagged SU, the competition with other SUs is also a impairment process. A specific service process for a certain SU is derived below.

Assume the arrival queue of PU during $[t, t+\Delta t]$ is empty, and the spectrum state is correctly sensed (No false alarm), thus SUs get their chance to compete with each other for the access to the channel. Similar to impairment process of PU's service process, the impairment from other SUs (i.e., during the competition) to a tagged SU's service is defined as

$$I_c^{SU}(t, t+\Delta t) = \begin{cases} 0, & \text{if the tagged SU succeeds in the competition during } (t, t+\Delta t]. \\ \Delta t, & \text{otherwise.} \end{cases} \tag{6}$$

The probability is

$$\begin{cases} Pr\left(I_c^{SU}(t, t+\Delta t) = 0\right) = 1 - q_c^{SU}, \\ Pr\left(I_c^{SU}(t, t+\Delta t) = \Delta t\right) = q_c^{SU}. \end{cases}$$

where q_c^{SU} denotes the probability that a tagged SU succeeds in a competition. Assume each SU succeeds with chance $\frac{1}{N}$ (N is the number of SUs), a tagged SU succeeds with probability $q_c^{SU} = \frac{1}{N}(1 - \frac{1}{N})^{N-1}$ since the competition is a Bernoulli process. After the above definition, a tagged SU's service process can be written as

$$S^{SU}(s, t) = t - s - A^{PU}(s, t) - I^{PU}(s, t) - I_c^{SU}(s, t). \tag{7}$$

One can find that, after a careful observation of the right-hand side of Eq. (7), a certain time slot may have four different potentials, 1)if packet from PU is under transmission and the spectrum is correctly sensed (No mis-detection), then this time slot is used for transmitting PU's packet, 2)if packet from PU is under transmission and meanwhile a mis-detection error occurs, then SUs attempt to access the channel and result in collision, 3)if the arrival queue of PU is empty that no PU's packet is under transmission, but a false alarm error occurs, this slot is wasted, 4)if no packet flow from PU is under transmission and the spectrum is correctly sensed, one SU can get a chance to transmit its packet after competition.

Now service process of both PU and SU are derived, the performance bounds of services for PU and SU are described in next section and the impact of different parameters on performance bounds will be illustrated more clearly.

III. PERFORMANCE BOUND

In Section II, the service process, for PU and SUs, are obtained based on the impairment process. In this section, the per-user throughput bounds are derived with the manipulation of min-plus convolution, from which, PU gets an upper bound and SU is provided with a lower bound.

A. Upper Bound of PU's Throughput

Similar to Eq. (3), upper bound of PU's throughput $\lambda_{U,t}^{PU}$ is defined as

$$Pr(D^{PU}(t) \ge \lambda_{U,t}^{PU} t) \le \varepsilon, \qquad \text{for all } t \ge 0. \tag{8}$$

Theorem 1: In a cognitive radio network consisting of one PU and N SUs, assume PU's average arrival rate is α^{PU}, a transient upper bound of PU's throughput is

$$\lambda_{U,t}^{PU} = -\frac{1}{\theta t}\log\varepsilon + \frac{1}{\theta}\log\left[p_{MD}p_a^{PU} + (1-p_{MD}p_a^{PU})e^{\theta}\right], \tag{9}$$

where θ is a free parameter from MGF.

Proof: This theorem is proved in a mathematic manner. Recall the relation between $D(t)$ and $A(t), S(t)$ from Eq. (1), the following inequalities are intuitional

$$\begin{aligned} & Pr(D^{PU}(t) \geq \lambda_{U,t}^{PU}t) \\ & = Pr\left(\inf_{0\leq s\leq t}\{A^{PU}(s) + S^{PU}(s,t)\} \geq \lambda_{U,t}^{PU}t\right) \end{aligned}$$

for all $t \geq 0$. Since

$$A^{PU}(0) + S^{PU}(0,t) \geq \inf_{0\leq s\leq t}\{A^{PU}(s) + S^{PU}(s,t)\}$$

and $A^{PU}(0) = 0$, the above equation can be rewritten as

$$\begin{aligned} Pr(D^{PU}(t) \geq \lambda_{U,t}^{PU}t) & \leq Pr\left(S^{PU}(0,t) \geq \lambda_{U,t}^{PU}t\right) \\ & \leq e^{-\theta\lambda_{U,t}^{PU}t}E\left[e^{\theta S^{PU}(0,t)}\right], \end{aligned}$$

Chernoff bound is used in the last step of the above derivation, and $\theta \geq 0$ is the free parameter in MGF. Note that the arrival process of PU and the service process of channel is mutually independent in each time slot, after substituting Eq. (5) the above formula goes as

$$\begin{aligned} & Pr(D^{PU}(t) \geq \lambda_{U,t}^{PU}t) \\ & \leq e^{-\theta\lambda_{U,t}^{PU}t}\left[p_{MD}p_a^{PU} + (1-p_{MD}p_a^{PU})e^{\theta}\right]^t. \end{aligned} \tag{10}$$

Then Eq. (9) holds directly after letting Eq. (10) equals a violation probability $\varepsilon > 0$. ■

A non-asymptotic upper bound of PU's throughput can be directly derived by letting $t \to \infty$ in Eq. (9), i.e.,

$$\begin{aligned} \lambda_U^{PU} & = \lim_{t\to\infty}\lambda_{U,t}^{PU} \\ & = \frac{1}{\theta}\log\left[p_{MD}p_a^{PU} + (1-p_{MD}p_a^{PU})e^{\theta}\right]. \end{aligned} \tag{11}$$

Furthermore, since $\theta \geq 0$, a potential optimization can be conducted for a smaller thus tighter upper bound, that is

$$\lambda_{U,opt}^{PU} = \inf_{\theta\in[0,\infty]}\{\lambda_U^{PU}\}. \tag{12}$$

B. Lower Bound of SU's Throughput

The fact that SUs are only substitutes for the channel make it nontrivial to address the lower bound of the their throughput, from which the guaranteed throughput can be inferred. Recall that the impairment process of the leftover service of PU in Eq. (6), the probability distribution of leftover service need to be figured out. SUs can get their chances to compete with each other to access the channel only when $C_{state} = 0$ occurs and the spectrum state is correctly sensed, denoted this probability by p_l, then

$$\begin{aligned} p_l & = Pr(C_{state} = 0)Pr(S_{result} = 0|C_{state} = 0) \\ & = (1-p_a^{PU})(1 - Pr(S_{result} = 1|C_{state} = 0)) \\ & = e^{-\alpha^{PU}}(1-p_{FA}). \end{aligned} \tag{13}$$

Similar to Eq. (2), let $\lambda_{L,t}^{SU}$ be the lower bound of a tagged SU's throughput, we have

$$Pr(D^{SU}(t) \leq \lambda_{L,t}^{SU}t) \leq \varepsilon, \qquad \text{for all } t \geq 0.$$

Theorem 2: In a cognitive radio network consisting of one PU and N SUs, assume PU's average arrival rate is α^{PU}, a transient lower bound of SU's throughput is

$$\lambda_{L,t}^{SU} = 1 + \frac{1}{\theta t}\log\varepsilon - \frac{1}{\theta}\log\left[(1-p_I) + p_I e^{\theta}\right], \tag{14}$$

where θ is a free parameter from MGF, and

$$p_I = p_l q_c^{SU} = e^{-\alpha^{PU}}(1-p_{FA})\frac{1}{N}(1-\frac{1}{N})^{N-1} \tag{15}$$

denote the thoroughly probability that a tagged SU transmits its packet within time slot Δt.

Proof: Following the route of proof of Theorem 1, this theorem is mathematically performed.

$$\begin{aligned} & Pr(D^{SU}(t) \leq \lambda_{L,t}^{SU}t) \\ & = Pr\left(\inf_{0\leq s\leq t}\{A^{SU}(s) + S^{SU}(s,t)\} \leq \lambda_{L,t}^{SU}t\right), \end{aligned}$$

Let $A^{SU}(0) = 0$ and $A^{SU}(t) = \infty$ for $t > 0$ since SUs' arrivals are always saturated. So

$$\begin{aligned} & Pr(D^{SU}(t) \leq \lambda_{L,t}^{SU}t) = Pr\left(S^{SU}(0,t) \leq \lambda_{L,t}^{SU}t\right) \\ & = Pr\left(t - I^{SU}(t) \leq \lambda_{L,t}^{SU}t\right) \\ & \leq e^{-\theta(1-\lambda_{L,t}^{SU})t}E\left[e^{\theta I^{SU}(t)}\right] \end{aligned}$$

where $I^{SU}(t)$ is short for $I^{SU}(0,t)$ and

$$I^{SU}(s,t) = A^{PU}(s,t) + I^{PU}(s,t) + I_c^{SU}(s,t)$$

denotes the thorough impairment process of a tagged SU. The righthand equation of $I^{PU}(s,t)$ consists of three elements: the arrival process of PU, impairment process caused by mis-detection when PU is transmitting and impairment process of a tagged SU caused by other SUs during competition. It follows that a tagged SU has the lowest priority when accessing the channel. Let $p_I = Pr(I^{SU}(t, t+\Delta t) = 0)$, we have

$$p_I = p_l q_c^{SU} = e^{-\alpha^{PU}}(1-p_{FA})\frac{1}{N}(1-\frac{1}{N})^{N-1}.$$

Then

$$\begin{aligned} & Pr(D^{SU}(t) \leq \lambda_{L,t}^{SU}t) \\ & = e^{-\theta(1-\lambda_{L,t}^{SU})t}\left[(1-p_I) + p_I e^{\theta}\right]^t, \end{aligned} \tag{16}$$

and Eq. (14) holds by letting Eq. (16) equals a violation probability $\varepsilon > 0$. ■

A non-asymptotic lower bound of SU's throughput can be directly derived by letting $t \to \infty$ in Eq. (14), i.e.,

$$\lambda_L^{SU} = \lim_{t\to\infty}\lambda_{L,t}^{SU} = 1 - \frac{1}{\theta}\log\left[(1-p_I) + p_I e^{\theta}\right]. \tag{17}$$

Similarly, an optimization of λ_L^{SU} can be conducted for $\theta \geq 0$.

IV. Numerical Results

In Section III, the per-user throughput bounds have been derived, respectively, for PU and SUs based on the arrival and service process. For the service process, an important impairment process is introduced which consists of the sensing error and CB access scheme. The sensing error has impact on the throughput for both PU and SUs, while the CB access scheme has influence only on the SUs. In this section, numerical results under different configurations are shown and discussed.

Since the service capacity C is normalized and PU is assumed to be unsaturated, the average arrival rate of PU α^{PU} is set to be $\alpha^{PU} \in [0,1]$. For SUs, due to the assumption of saturation, the queue of every SU always has data to be transmitted, therefore, the arrival rate of SUs can be ignored. To investigate the influence of the sensing error on the throughput, p_e is set to be in the range of $[0,1]$, though, it could be very small, for example less than 10%, in fact. Pay attention that, the following figures are drawn according to the optimized throughput bound based on the free parameter of θ.

A. Upper Bound of PU's Throughput

Due to the unsaturated condition, the arrival rate of PU and throughput bound are tightly coupled. As for the sensing error, only the mis-detection can influence the throughput. Therefore, we focus on the arrival rate α^{PU} and the probability of mis-detection p_{MD}. Refer to Fig. 2 for details.

The curves show that for a fixed arrival rate α^{PU}, the throughput bound λ_U^{PU} decreases with the probability of p_{MD}. This is evident since a higher p_{MD} implies a higher probability of collision caused by mis-detection when PU transmits in a slot. The collision will make PU's transmission failed, then retransmission is carried out which results in the decrease of the throughput.

Furthermore, we can find that for a fixed probability p_{MD}, the throughput decrease with the increase of arrival rate α^{PU}. This can be explained as follows. Referring to the probability $q = p_a^{PU} p_{MD}$ of impairment in Definition 1, a larger arrival rate α^{PU} implies a higher probability p_a^{PU} that the channel is occupied by PU, then the probability of the event of mis-detection becomes larger, which accounts for a higher probability of collision caused by SUs. As a result, the throughput bound decreases. From this perspective, it is demonstrated that an effective spectrum sensing technique is vital for the improvement of throughput in cognitive radio networks.

B. Lower Bound of SU's Throughput

Due to the condition of saturation, SUs are ready to transmit the data in the queue as soon as they contend the channel successfully. Accordingly, the arrival rate of SUs has no impact on the throughput. Therefore, we focus on the number of SUs N, probability of sensing error p_e and arrival rate of PU α^{PU}.

For parameter N, the influence is apparent to the per-user throughput bound of SUs. Comparing figures (a), (b) and (c) of Fig. 3, it is illustrated that the throughput bound decreases with the increase of N. This is obvious since the larger N is, the smaller probability $1/N$ of successful contention is, which results in less opportunities to access to the channel for each SU. A special case occurs when $\alpha^{PU} = 0$ and $p_e = 0$, which implies that there is no PU arrival nor sensing error, and the per-user throughput should be C/N ($C = 1$). However, the throughput is less than C/N. Taking $N = 2$ for example, the two SUs fairly share the spectrum due to the contention-based access scheme. Therefore, each SU has a probability of $1/2$ to transmit over the channel. In fact, the SU may fail to access to the channel for several consecutive slots, which decreases the throughput to less than $C/2$.

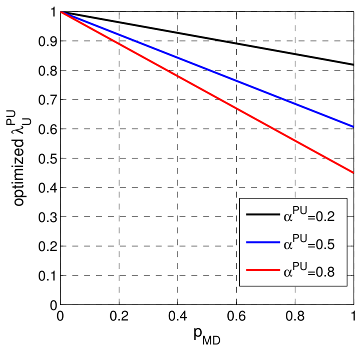
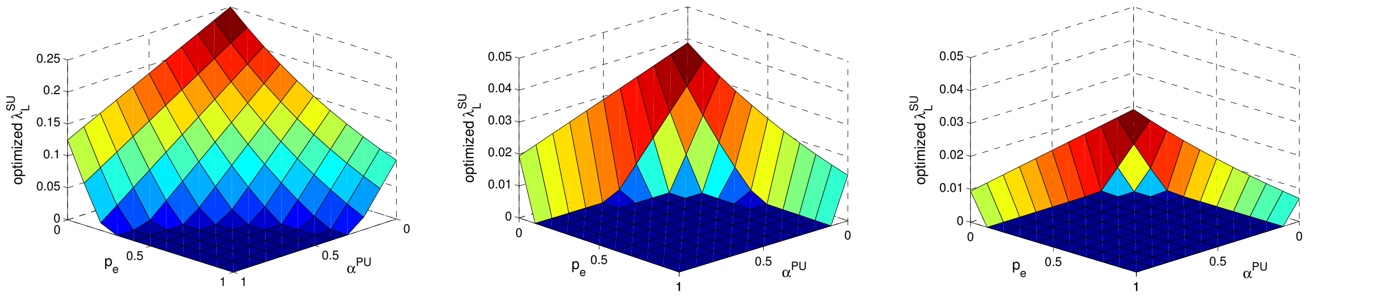


Fig. 2. These curves demonstrate the correlation of probability p_{MD} with the optimized throughput bound λ_U^{PU} when PU's average arrival rate α^{PU} varies. λ_U^{PU} decays when p_{MD} increases. And for a fixed p_{MD}, the increase of α^{PU} leads to decrease of λ_U^{PU}.

For parameter p_e, the throughput decreases with the increase of p_e when N and α^{PU} are fixed. This is due to the fact that the larger p_e is, the more collided slots (corresponding to mis-detection) are, which implies PU needs more slots to retransmit a packet. Therefore, the resource left to SUs decreases. On the other hand, the increasing p_e also leads to more slots wasted (corresponding to false alarm). Then, it takes a longer time to transmit the data for SUs. As a result, the throughput becomes smaller.

For parameter α^{PU}, with the increase of it, more data need to be transmitted for PU, therefore, more slots are occupied by PU. Referring to $q = p_a^{PU} p_{MD}$, even for a fixed p_e, the probability of the event of mis-detection increases with α^{PU}, which results in an increase of slot collision. Hence, the throughput decreases.

Among these three parameters, the number of SUs N has dominant impact on the per-user throughput bound, while the probability of sensing error p_e and arrival rate α^{PU} have less impact on the throughput. Since the throughput dramatically decreased with the increase of N, resulting in a large ratio of throughput of zero. This can be explained according to Eq. (15), the probability of successful contention, based on N, influences the value of this equation prominently. However, in this paper, a lower bound of the per-user throughput is obtained, which implies that the actual throughput is larger than this bound. Besides, the probability of sensing error p_e is always very small in realistic networks, therefore, the throughput is larger than zero.



(a) N=2 (b) N=10 (c) N=20

Fig. 3. These figures show the influence of different parameters (namely the number of SUs N, probability of sensing error p_e and average arrival rate of PU α^{PU}) on the optimized per-user lower bound of SUs' throughput λ_L^{SU}. The increase of these three parameters will result in the decrease of throughput λ_L^{SU}. Among the three, number N has a dominant impact on λ_L^{SU} over the other two. It implies that the contention-based access scheme has a dominant influence on the thoughput λ_L^{SU}, while the sensing error has less impact on it.

V. Conclusion

In this paper, per-user throughput bounds of PU and SUs are analyzed for cognitive radio networks based on stochastic network calculus. We focus on the correlation of throughput with the arrival rate of PU α^{PU}, the probability of sensing error p_e and the number of SUs N. To investigate these influence, impairment process is introduced to represent the service which cannot be utilized by PU nor SUs. Then, service process of PU and SUs is obtained. Combining with arrival process of PU and SUs, the upper bound of PU's per-user throughput and lower bound of SUs' per-user bound are derived with the manipulation of min-plus convolution. Based on numerical result, a detailed discussion reveals that sensing error plays a decisive role on the upper bound of PU's throughput, while has less impact on the lower bound of SUs' throughput. For SUs, the contention-based access scheme has dominant influence on the throughput. These results shed light on understanding and designing optimal cognitive radio networks. For cognitive radio networks, multi-hop and multi-channel sceneries are of great interest, and we will further focus on the performance analysis of these networks in the future.

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