

# Mandyoc: A finite element code to simulate thermochemical convection in parallel

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## Software

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## Summary

Mandyoc is a finite element code written in C dedicated to simulate thermochemical convection in the interior of terrestrial planets. Different linear and non-linear rheologies can be adopted, appropriately simulating the strain and stress pattern in the Earth's crust and mantle, both in extensional or collisional tectonics. Additionally, the code allows variations of boundary condition for the velocity field in space and time, simulating different pulses of tectonism in the same numerical scenario.

## Statement of need

Although Mandyoc is the acronym for MANtle DYnamics simulatOr Code, it is designed to simulate Stokes flow type thermochemical convection taking different compositional layers into account, and it is also appropriate to simulate Earth's lithospheric dynamics in the geological timescale.

One advantage of Mandyoc is the possibility to create scenarios with velocity boundary conditions variable in space and time, allowing the user to simulate different tectonic pulses in the same model run. Additionally, the current version incorporates surface processes, imposing rates of erosion and sedimentation on the top of the free surface.

Previous versions of the code were used to study the evolution of continental margins, showing the interaction of the continental lithosphere with the asthenospheric mantle ([Sacek, 2017](#); [Salazar-Mora & Sacek, 2021](#)).

## Mathematics

Mandyoc solves the equations for conservation of mass, momentum and energy using the Finite Element Method assuming the extended Boussinesq approximation, respectively:

$$u_{i,i} = 0$$

$$\sigma_{ij,j} + g_i \rho = 0$$

$$\frac{\partial T}{\partial t} + u_i T_{,i} = \kappa T_{,ii} + \frac{H}{c_p \rho} + \frac{u_i g_i \alpha T}{c_p}$$

30 where

$$\sigma_{ij} = -P\delta_{ij} + \eta(u_{i,j} + u_{j,i})$$

$$\rho = \rho_0 (1 - \alpha(T - T_0))$$

31  $u_i$  is the component  $i$  of the velocity field,  $T$  is temperature,  $t$  is time,  $\kappa$  is the thermal  
32 diffusivity,  $H$  is the volumetric heat production,  $c_p$  is the specific heat capacity,  $g$  is gravity,  
33  $\rho$  is the effective rock density dependent on temperature and composition,  $\rho_0$  is the reference  
34 rock density at temperature  $T_0$ ,  $\alpha$  is the coefficient of thermal expansion,  $P$  is the dynamic  
35 pressure,  $\eta$  is the effective viscosity, and  $\delta_{ij}$  is the Kronecker delta.

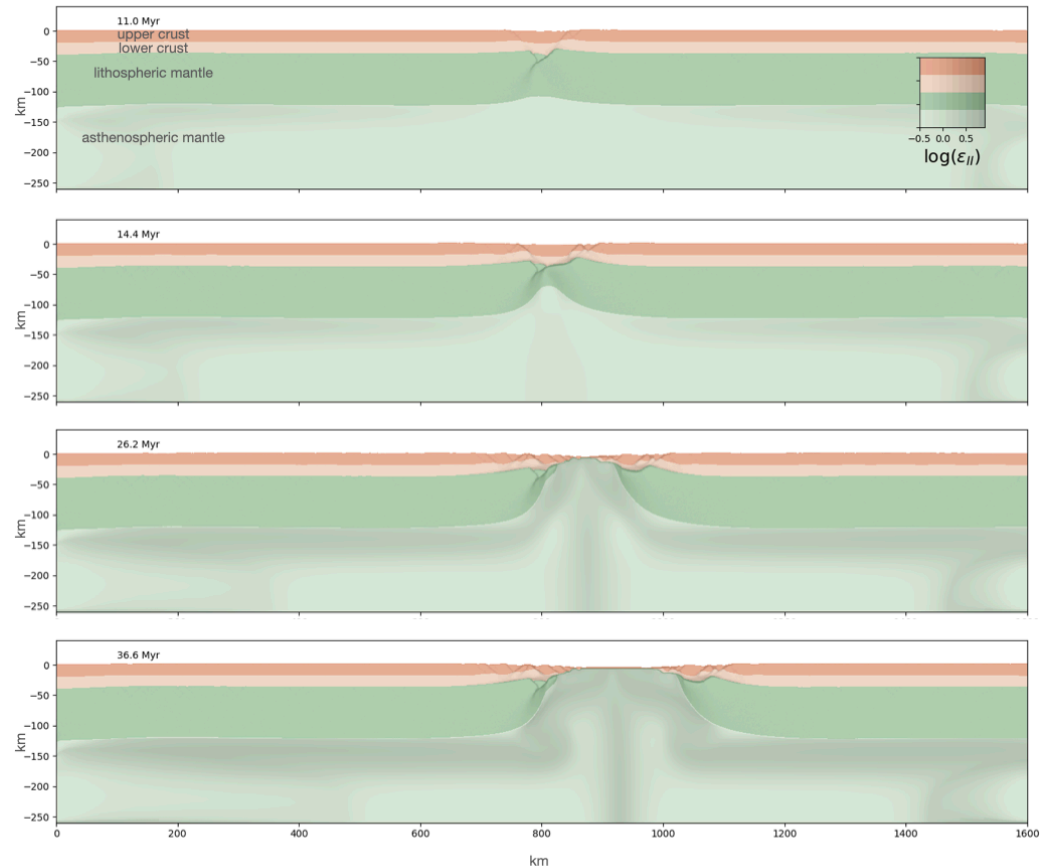
36 The code is fully parallelized using the Portable, Extensible Toolkit for Scientific Computation  
37 (PETSc) (Balay et al., 1997, 2021a, 2021b). The present version of the code can simulate  
38 thermochemical convection using different rheological formulations: Newtonian flow, non-  
39 linear viscous flow or visco-plastic deformation. For example, the lithosphere can be simulated  
40 as a combination of different visco-plastic layers in which the effective viscosity depends on  
41 a nonlinear power law viscous rheology and a plastic yield criterion, like the Drucker-Prager  
42 criterion. Additionally, strain softening is implemented to facilitate the localization of strain  
43 in the plastic regime during, for example, lithospheric stretching.

44 The composition and strain history is tracked by particles present in the interior of the fi-  
45 nite element. The exchange of particles among the subdomains of the model is efficiently  
46 parallelized in PETSc using DMSwarm (May & Knepley, 2017).

47 The free surface of the Earth can be simulated and is numerically stabilized using the Free  
48 Surface Stabilization Algorithm (Kaus et al., 2010). Surface processes of erosion and sed-  
49 imentation can also be incorporated in the thermo-mechanical model. Complex boundary  
50 conditions for the velocity field, variable both in space and time, can be adopted by the user  
51 to simulate different episodes of tectonism. Different benchmarks are available in the reposi-  
52 tory and can be reproduced by the user (e.g. thermochemical convection – Van Keken et al.  
53 (1997); plume-lithosphere interaction – Crameri et al. (2012)).

54 As an example of application of Mandyoc, Figure 1 presents snapshots of one numerical  
55 scenario of lithospheric stretching imposing a divergent flow direction, resulting in rifting and  
56 break-up. In this example, the upper crust, lower crust, lithospheric mantle and asthenosphere  
57 present different rheology and density, resulting in faulting mainly in the upper crust and part  
58 of the lithospheric mantle. Additionally, deformations in the lower crust and at the base of the  
59 lithospheric mantle are accommodated by ductile creep flow. This example can be reproduced  
60 from the repository.

## Figures



**Figure 1:** Mandyoc example of application of the thermo-mechanical model to simulate the stretching of the lithosphere, assuming different rheologies. The scales of gray represent cumulative strain in the different materials. Details can be found in the repository.

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