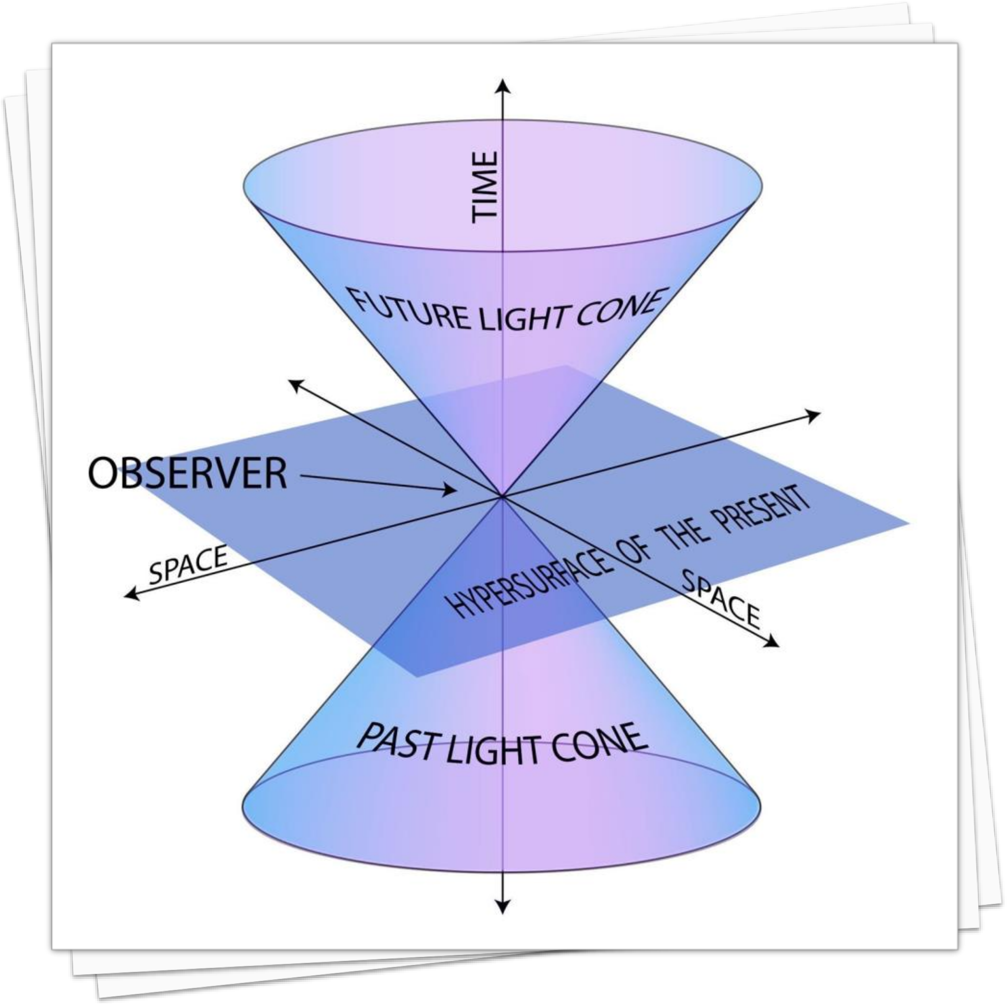

相对论量子力学*

Klein方程

Dirac方程

Schrödinger方程（1926）

平面波

$$\psi(\vec{r},t) = \psi_0 e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-Et)}$$

$$E \to i\hbar\frac{\partial}{\partial t}, \qquad \vec{p} \to -i\hbar\nabla$$

$$E = \frac{\vec{p}^2}{2m}$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi$$

几率流的连续性方程

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0$$

$$\rho \equiv \psi^*\psi, \qquad \psi(\vec{r},t) \equiv \sqrt{\rho}e^{\frac{i}{\hbar}S}$$

$$\vec{j} \stackrel{\text{def}}{=} -i\frac{\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*) = \frac{\rho}{m}\nabla S$$

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Klein-Gordon方程（1927）

平面波$\psi(\vec{r},t) = \psi_0 e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-Et)}$

$$E \to i\hbar\frac{\partial}{\partial t}, \qquad \vec{p} \to -i\hbar\nabla$$

$$E^2 = \vec{p}^2c^2 + m^2c^4$$

$$-\hbar^2\frac{\partial^2}{\partial t^2}\psi = (-\hbar^2c^2\nabla^2 + m^2c^4)\psi$$

$$\rho = \frac{i\hbar}{2mc^2}(\psi^*\dot{\psi} - \psi\dot{\psi}^*)$$

$$\vec{j} = -\frac{i\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

$$\partial_\mu j^\mu = 0, \qquad \frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0$$

Oskar Klein
1894-1977
瑞典物理学家

负能解

$$\frac{1}{(2\pi\hbar)^{3/2}}e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-(-E)t)}$$

负几率密度

Pauli, Weisskopf, 1934年：量子场，描述自旋0粒子

Dirac方程（1928年）

为了去掉负能解，对质能关系开方

$$E = \sqrt{\vec{p}^2c^2 + m^2c^4}$$

需采用非线性的算子表达式

对算子开方，设为线性：

$$\hat{H} = c\vec{\alpha}\cdot\hat{\vec{p}} + \beta mc^2$$

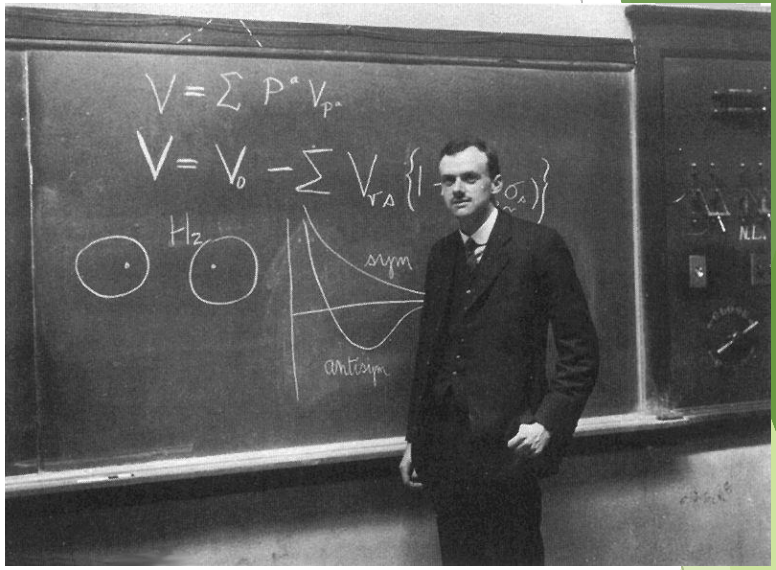
其中参数$\vec{\alpha},\beta$待定

方程

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi = (-i\hbar c\vec{\alpha}\cdot\nabla + \beta mc^2)\psi$$

算子作用两次

$$-\hbar^2\frac{\partial^2}{\partial t^2}\psi = (-i\hbar c\vec{\alpha}\cdot\nabla + \beta mc^2)^2\psi$$



P.A.M. Dirac 1902-1984
英国物理学家 **1933**年诺贝尔奖

必须满足质能关系

$$E^2 = \vec{p}^2c^2 + m^2c^4$$

$$-\hbar^2\frac{\partial^2}{\partial t^2}\psi = (-\hbar^2c^2\nabla^2 + m^2c^4)\psi$$

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Dirac方程

$$(-i\hbar c\vec{\alpha}\cdot\nabla+\beta mc^2)^2=-\hbar^2\nabla^2+m^2c^4$$

$$\boxed{\begin{gathered}\alpha_j\alpha_k+\alpha_k\alpha_j=\delta_{jk}\\ \beta^2=1\\ \alpha_j\beta+\beta\alpha_j=0\end{gathered}}$$

α_j,β最简形式为4×4的矩阵

$$\beta=\begin{pmatrix}\mathbf{1}_{2\times2} & \mathbf{0}_{2\times2}\\ \mathbf{0}_{2\times2} & -\mathbf{1}_{2\times2}\end{pmatrix},\qquad \alpha_j=\begin{pmatrix}\mathbf{0}_{2\times2} & \sigma_j\\ \sigma_j & \mathbf{0}_{2\times2}\end{pmatrix}$$

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi=(-i\hbar c\vec{\alpha}\cdot\nabla+\beta mc^2)\psi}$$

$$\rho=\psi^+\psi,\qquad \vec{j}=c\psi^+\vec{\alpha}\psi,\qquad \partial_\mu j^\mu=0,\qquad \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$$

没有负几率问题 能够推出自旋g因子为2

有负能解：Dirac海

正反粒子，自旋1/2的量子场

相对论+量子论→相对论量子场论

电磁场中的Dirac粒子

规范不变性要求

$$\partial_\mu \to \partial_\mu + iqA_\mu$$

即

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + i\frac{q}{\hbar}\phi, \qquad \nabla \to \nabla - i\frac{q}{\hbar c}\vec{A}$$

$$\boxed{\left(i\hbar\frac{\partial}{\partial t} - q\phi\right)\psi = \left(-i\hbar c\vec{\alpha}\cdot\nabla - q\vec{\alpha}\cdot\vec{A} + \beta mc^2\right)\psi}$$