Capacity and Power Allocation for Different Fading MIMO Channels with Channel Estimation Error

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***Abstract*—In wireless communication, it is generally assumed that the channel information can be accurately estimated at the receiver for the convenience of signal processing. However, in practical applications, the channel estimation obtained at the receiver always has an estimation error due to various factors. The influence of channel estimation error on system performance has been paid more and more attention. In view of this, this paper investigates the effects of channel estimation errors on Multiple Input Multiple Output(MIMO) channel capacity in Rayleigh, Rician and Nakagami fading channels. The distance between the upper and lower bounds of the fading MIMO channel mutual information with channel estimation error is calculated, indicating that the difference between these two bounds is usually small. The optimal transmission power allocation strategy for ergodic capacity is obtained. The simulation results show that the channel capacity of MIMO system is sensitive to channel estimation error. The channel capacity of MIMO system tends to be constant with the increase of SNR.**

Keywords-MIMO; channel estimation error; mutual information; power allocation

I. Introduction

MIMO system significantly improves capacity gain by increasing spatial dimension. However, this requires the system to obtain channel state information(CSI). In practical systems, especially frequency division duplex (FDD) systems, it is difficult for the system to obtain perfect CSI. Therefore, it is necessary to study the influence of imperfect CSI on the performance of MIMO system. Transmitter CSI is obtained through feedback channel or through channel reciprocity. The CSI of the receiver is usually obtained by sending a training sequences. Whether pilot-based or blind channel estimation is used, there are usually some estimation errors, which lead to the decrease of channel capacity of the system[1,2]. In [3], the author discussed the boundary of mutual information of the system when the channel estimation error exists at the receiver. In [4], the results are extended to stationary fading channels and a lower bound of system capacity is given. The authors of [5] have discussed the effect of channel estimation error on system performance. In [6], the effect of channel estimation error at the receiver on the channel capacity in Rayleigh fading channels is discussed. Optimal transmitter strategy includes power allocation in the spatial (antenna) domain and temporal (fading) domain. The [1] shows that the difference between spatial power allocation and temporal power allocation can be neglected. which means that the capacity gain given by temporal power adaptive is very small. Therefore, this paper studies the spatial power allocation. In this paper, we generalize the above literature and analyze the influence of the estimation error of the receiver on the ergodic capacity of the system in other channels. The distance between the upper and lower bounds of mutual information of fading MIMO channels with channel estimation error at the receiver is calculated, which shows that the two bounds are close. The optimal transmitter power allocation strategy suitable for ergodic capacity is obtained. The effects of channel estimation error on MIMO channel capacity under Rayleigh, Rician and Nakagami fading channels with different numbers of transmit and receive antennas and different power allocation strategies are simulated.

A. *Rayleigh Fading*

Most of the channels are modeled as complete scattering, i.e., Rayleigh fading channels. In Rayleigh fading channels, the receiver antenna receives a large number of scattered or reflected signals, usually used to simulate the environment where there is no direct path between transmitter and receiver[7].

B. *Rician Fading*

Rayleigh fading model is suitable for various fading environments in practical communication systems. In other cases, however, there are strong deterministic LOS components between the transmitter and the receiver, resulting in Rician fading model. The difference between Rician fading and Rayleigh fading is that the Rician fading channel has a strong line of sight (LOS) component and reflected waves. The receiving antenna receives interference signals from different paths. Rayleigh fading is a special case of Rician fading, and Rician fading is an extension of Rayleigh fading[8].

C. *Nakagami Fading*

Nakagami-m fading can be seen as a generalized fading model of Rayleigh and Rician. It has a fading parameter m ranging from 0.5 to ∞ channel conditions. When $m=1$, it is a generalized Rayleigh fading channel, and when $m=\infty$, it will be extended to the fading-free AWGN channel. By changing the value of m, Nakagami fading can also be transformed into a variety of fading models. The Nakagami-m distribution does not assume the existence of direct LOS components, but uses the density function of the gamma distribution to fit the experimental data to obtain an approximate distribution, which is more general. In communication theory, the Nakagami distribution is used to simulate the scattered signal arriving at the receiver through multiple paths[8-10].

II. SYSTEM MODEL

For a MIMO system with M antennas at the transmitter and N antennas at the receiver, the system can be expressed as $\mathbf{Y} = \mathbf{HX} + \mathbf{Z}$, $\mathbf{Y}$ is the channel output of $\mathrm{N} \times 1$. $\mathbf{H}$ is the channel transmission matrix of $\mathrm{N} \times \mathrm{M}$, $\mathbf{X}$ is the channel input of $\mathrm{M} \times 1$, $\mathbf{Z}$ is the Additive White Gaussian Noise vector (AWGN) of $\mathrm{N} \times 1$. In this paper, we assumes that there is no correlation between channel fading process and noise process. In such a system, if the fading process is perfectly known to the receiver, the mutual information between the channel input and output can be given expressed as [4,5]

$$I\left(\mathbf{X};\mathbf{Y}\right) = E\left\{\log_2\left|\mathbf{I} + \mathbf{H}^*\mathbf{HQ}\right|\right\} \tag{1}$$

where $\mathbf{Q}$ is the input covariance matrix $\mathbf{Q} = E(\mathbf{XX}^*)$. and $E\{\bullet\}$ is the expectation operator. Receiver performs MMSE channel estimation on channel. Let $\mathbf{H} = \Delta\mathbf{H} + \mathbf{E}$, where $\Delta\mathbf{H}$ represents the channel estima-tion information obtained by the receiver, and $\mathbf{E}$ is the estimation error[12]. Furthermore, $\Delta\mathbf{H}$ and $\mathbf{E}$ are irrelevant, and each term in $\mathbf{E}$ is a zero-mean cyclic symmetric complex Gaussian (ZMCSCG), the variance is $\sigma_{\mathbf{E}}^2 = E\left(\mathbf{H}_{ij}^2\right) - E\left(\Delta\mathbf{H}_{ij}^2\right)$, $\sigma_{\mathbf{E}}^2$ indicates the quality of the channel estimation and assumes that both the transmitter and the receiver are known.

When the receiver cannot obtain the accurate information of the channel and only obtains the estimated information of the channel, the bound of the mutual information of the MIMO system is given by[4,5,12]

$$I_{lower}\left(\mathbf{X};\mathbf{Y}\mid\Delta\mathbf{H}\right) = E\left[\log_2\left|\mathbf{I} + \frac{1}{1+\sigma_{\mathbf{E}}^2 P}\Delta\mathbf{H}^*\Delta\mathbf{HQ}\right|\right] \tag{2}$$

$$I_{upper}\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) = I_{lower}\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) + \mathrm{N}E\left[\log_2\frac{\sigma_{\mathbf{E}}^2 P + 1}{\sigma_{\mathbf{E}}^2\left\|\mathbf{X}\right\|^2 + 1}\right] \tag{3}$$

P represents the total transmitted signal power, i.e., $tr\left(\boldsymbol{Q}\right) \le P$. $tr(\bullet)$ stands for trace. [4,5] shows that under the limitation of high SNR and large number of antennas, the second term in Gaussian input (3) is close to $\left(\mathrm{N/M}\right)\log_2\sqrt{\mathrm{e}} \approx 0.72\left(\mathrm{N/M}\right)$. Therefore, the relationship between the upper two bounds when the high SNR and the number of antennas are large is given by

$$I_{upper}\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) \approx I_{lower}\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) + 0.72\left(\mathrm{N/M}\right) \tag{4}$$

The above formula shows that for MIMO systems, unless $\mathrm{N} \gg \mathrm{M}$, the difference between the upper and lower bounds of mutual information is usually very small. Therefore, this paper discusses the lower bound of mutual information as the performance criterion.

The ergodic capacity of the known estimated channel $\Delta\mathbf{H}$ in the transmitter and receiver is given by

$$C = \max I\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) \tag{5}$$

Let the singular value decomposition of the estimated channel matrix be $\Delta\mathbf{H} = \mathbf{UDV}^*$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{D}$ are diagonal matrices[11], $\Delta\mathbf{Q} = \mathbf{V}^*\mathbf{QV}$ and $\mathbf{\Lambda} = \mathbf{D}^*\mathbf{D}$. Then

$$\begin{aligned} I_{lower}\left(\mathbf{X};\mathbf{Y}\left|\Delta\mathbf{H}\right.\right) &= \log_2\left|\mathbf{I} + \frac{1}{1+\sigma_{\mathbf{E}}^2 P}\Delta\mathbf{H}^*\Delta\mathbf{HQ}\right| \\ &= \log_2\left|\mathbf{I} + \frac{1}{1+\sigma_{\mathbf{E}}^2 P}\mathbf{VD}^*\mathbf{U}^*\mathbf{UDV}^*\mathbf{Q}\right| \\ &= \log_2\left|\mathbf{I} + \frac{1}{1+\sigma_{\mathbf{E}}^2 P}\mathbf{\Lambda}\Delta\mathbf{Q}\right| \end{aligned} \tag{6}$$

(6) is maximized with $\Delta\mathbf{Q}$ a diagonal matrix, $\Delta\mathbf{Q} = \mathrm{diag}\left(p_1, \cdots, p_M\right)$, has the optimal power distribution $\left\{p_i\right\}$, so that $\sum_{i=1}^{M} p_i = P$. Therefore, the lower bound of the system capacity can be expressed as

$$\begin{aligned} C_{lower} &= \max_{\{p_i\}} E\left(\sum_{i=1}^{M}\log_2\left(1 + \frac{p_i\lambda_i}{1+\sigma_{\mathbf{E}}^2 P}\right)\right) \\ \text{subject to} \quad & E\left(P\right) = E\left(\sum_{i=1}^{M} p_i\right) \end{aligned} \tag{7}$$

where λ_i is the $(i,i)^{th}$ element of $\mathbf{\Lambda}$ and thus the i^{th} eigenvalue of $\Delta\mathbf{H}^*\Delta\mathbf{H}$.

III. OPTIMAL POWAL ALLOCATION

This section takes the lower bound as the performance criterion to discuss the optimal signal transmission form at the transmitter of the system and the channel capacity characteristics when there is channel estimation error.

A. Transmitter Unknown CSI

When the channel information is unknown at the transmitter, the maximum channel capacity can be achieved by uniformly distributing the transmission power in each direction[4,5,12]. Thus, the covariance matrix of the transmitted signal can be expressed as $\mathbf{Q} = \left(P/M\right)\mathbf{I}$, and the lower bound of the channel capacity of the system is given by

$$C_{lower} = \sum_{i=1}^{M} E\left[\log_2\left(1+\frac{P/M}{1+\sigma_{\mathbf{E}}^2 P}\lambda_i\right)\right] \tag{8}$$

B. *Transmitter Known CSI*

When the transmitter obtains the channel information of the system through feedback, it can optimize the power allocation in each transmitting direction through the principle of waterfilling, according to formula (7), the power allocation on each antenna can be expressed as[4,5,12,13]

$$p_i = \left(\mu - \frac{1+\sigma_{\mathbf{E}}^2 P}{\lambda_i}\right)^+ \tag{9}$$

the lower bound of the system capacity becomes

$$C_{lower} = \sum_{i=1}^{M}\left[\log_2\left(\frac{\mu\lambda_i}{1+\sigma_{\mathbf{E}}^2 P}\right)\right]^+ \tag{10}$$

$(\bullet)^+ = \max\{0,\bullet\}$. where the determination of μ should satisfy

$$\sum_{i=1}^{M} p_i = P \tag{11}$$

the covariance matrix of the transmitted signal can be expressed as $\boldsymbol{Q} = \boldsymbol{V}diag(p_1,\cdots,p_M)\boldsymbol{V}^*$, where $\mathbf{V}$ is the eigenvector of the channel estimation matrix $\Delta\mathbf{H}$.

IV. Numerical Results

In this part, numerical results are presented based on Monte Carlo simulations. The effects of channel estimation error on MIMO channel capacity under Rayleigh, Rician and Nakagami fading channels with different numbers of transmit and receive antennas and different power allocation strategies are studied.

Fig.1, Fig.2 and Fig.3 show the influence of the number of transmit and receive antennas on the system capacity with 10% channel estimation error under Rayleigh, Rician and Nakagami fading conditions, respectively. It can be seen from the figure that the channel capacity of the MIMO system tends to be constant with the increase of the SNR due to the influence of channel estimation error. As the number of transmit antennas and the number of receive antennas increase, the channel capacity of the system increases. By comparing the fading channels, Rician fading channels have the largest capacity and Nakagami the smallest. In the MIMO system with a 30dB SNR, Rician can achieve a capacity of approximately 26bits/s/Hz, while Rayleigh and Nakagami can only achieve capacities of approximately 19bits/s/Hz and 9bits/s/Hz, respectively. This is because the Rician fading has a strong LOS component, and the signal attenuation is small.

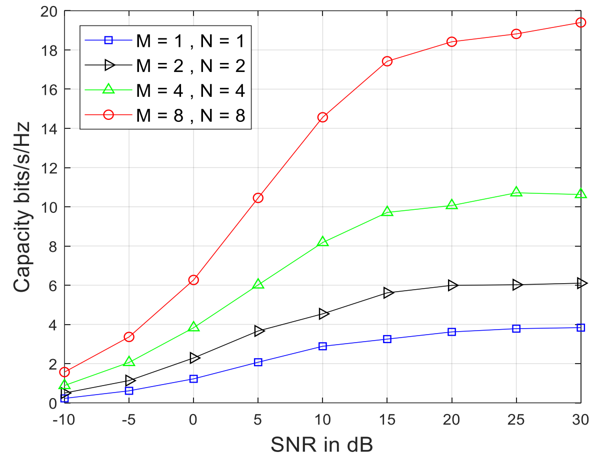

Figure 1. Rayleigh fading channel capacity with 10% estimation error

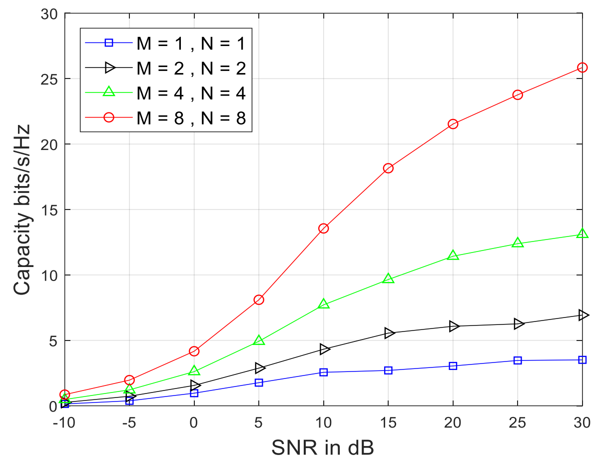

Figure 2. Rician fading channel capacity with 10% estimation error

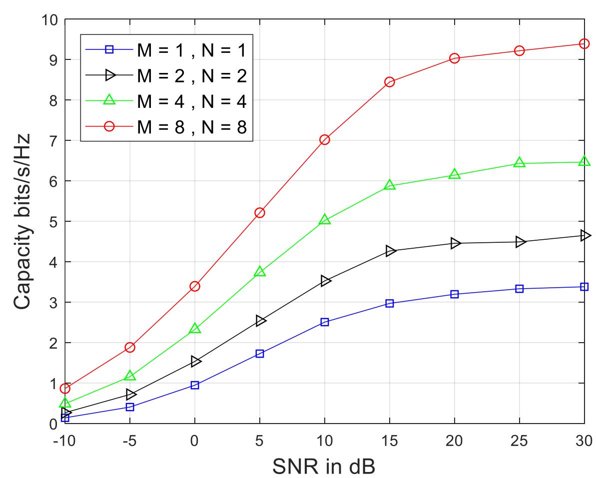

Figure 3. Nakagami fading channel capacity with 10% estimation error

Fig.4, Fig.5 and Fig.6 show the effect of channel estimation error on system channel capacity under different power allocation strategies. It can be seen that the channel capacity of the MIMO system is sensitive to channel estimation errors. When there is no estimation error in the channel, the channel capacity of the system increases proportionately with the SNR, and with the increase of the channel estimation error, the channel capacity decreases gradually. When the SNR is small, the channel estimation error has no significant effect on the average channel capacity of the system. When the channel estimation error is constant, the channel capacity reaches saturation when the SNR increases to a certain extent. Increasing SNR, the channel capacity remains basically the same, but tends to a constant value.

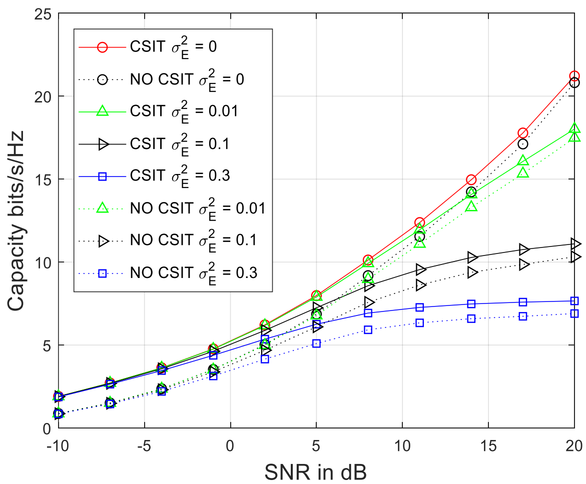

Figure 4. Ergodic capacity for MIMO channel with different power allocation strategies for 1% 10%, and 30% estimation error in Rayleigh fading.

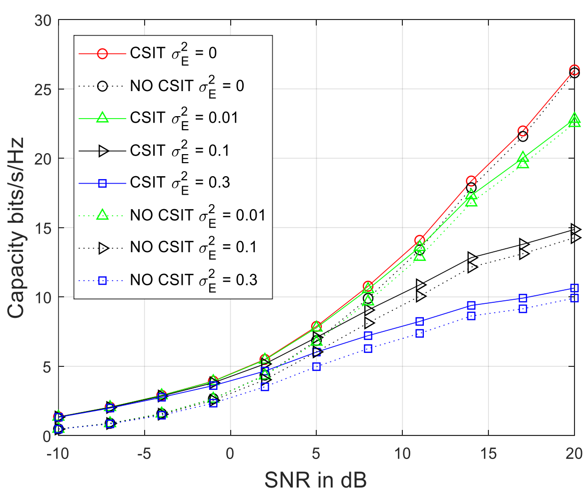

Figure 5. Ergodic capacity for MIMO channel with different power allocation strategies for 1% 10%, and 30% estimation error in Rician fading.

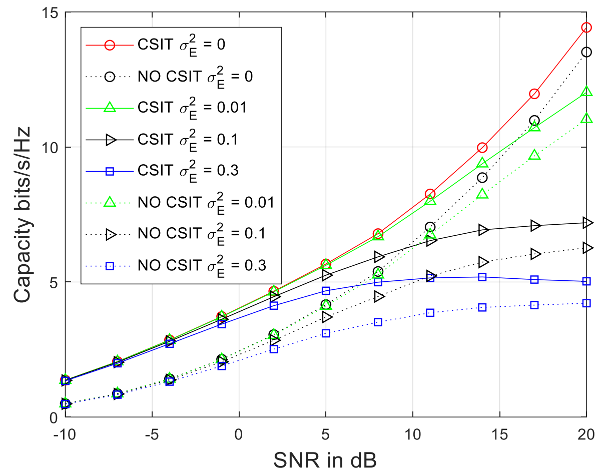

Figure 6. Ergodic capacity for MIMO channel with different power allocation strategies for 1% 10%, and 30% estimation error in Nakagami fading.

V. Conclusion

This paper investigates the influence of channel estimation errors on MIMO channel capacity in Rayleigh, Rician and Nakagami fading channels. The upper and lower bounds of the system mutual information with channel estimation and perfect feedback of MMSE channel estimation are studied. Based on the lower bound, the optimal transmitter power allocation strategy for maximizing ergodic capacity is derived. The simulation results show that the channel capacity of MIMO system is sensitive to channel estimation error. Under the fading conditions of Rayleigh, Rician and Nakagami, the channel capacity of the MIMO system tends to be constant with the increase of the SNR due to the influence of channel estimation error.

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