Inf2C - Computer Systems
Lecture 2
Data Representation

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Last lecture

- Course overview
  - Piazza: up & running. Use it!
  - Tutorials: start in week 3
  - Labs: drop-in. Start in week 3
- Moore's law
- Types of computer systems
- Computer components
- Computer system stack

Lecture 2: Data Representation

- The way in which data is represented in computer hardware affects
  - complexity of circuits
  - cost
  - speed
  - reliability
- Must consider how to design hardware for
  - Storing data: memory
  - Manipulating data: processing (e.g., adders, multipliers)

Lecture outline

- The bit – atomic unit of data
- Representing numbers
  – Integers
  – Floating point
- Representing text

The bit

- Information represented as sequences of symbols
  - Humans use letters, numerals, punctuation, whitespace
  - Computers use just 0s and 1s, ***bits***
- ***Bit*** – an acronym for Binary digiT
- Advantages: easy to do computation, very reliable, simple & reusable circuits
- Disadvantages: little information per bit → must use many bits. 512 ≡ 1 0000 0000, 'A' ≡ 0100 0001

Natural numbers representation

- Non-negative (unsigned) integers are very simple to represent in binary

Most significant bit → MSB

Least significant bit → LSB

	MSB				LSB
Bit position	n-1	n-2		1	0
Binary:	☐	☐	••••	☐	☐

Decimal: $\ast 2^{n-1} + \ast 2^{n-2} + \cdots \ast 2^{1} + \ast 2^{0}$

Basic operations

- Addition, subtraction with unsigned binary numbers is easy:

13

11

$$\begin{array}{r} {\color{red}1\,1\,1\,1} \\ 01101 \\ +01011 \\ \hline 11000 \end{array} \qquad \begin{array}{r} {\color{red}0\,0\,1\,0} \\ 01101 \\ -01011 \\ \hline 00010 \end{array}$$

24

2

Fixed bit-length arithmetic

- Hardware cannot handle infinitely long bit sequences
- We end up with a few fixed-size data types
  - Byte: always 8 bits
  - Word: the typical unit of data on which a processor operates (32 or 64 bits most common today)
- Overflow happens when a result does not fit
  - Numbers wrap-around when they become too large
  - Arithmetic is modulo 2^N, where N=number of bits

What about negative numbers?

- Sign-magnitude representation:
  - Use 1st bit (MSB) as the sign
  - 1 → negative, 0 → positive

    0010 ≡ 2   1010 ≡ -2

- Complicates addition and subtraction
  - The actual operation depends on the sign
- Has positive and negative zero
  - 0000 ≡ 0   1000 ≡ -0

Better way: 2's complement representation

Two's complement: the intuition

- Want: X + (-X) = 0
- Insight: don't need the full sum to be 0
  - Only the bits that can be represented within a computer's fixed width need to be 0
- Approach:
  - Represent the negation of X as 2^N-X
  - Then: X + (-X) = X + (2^N-X) = 2^N
    - Recall: largest number represented with N bits: 2^N-1
    - Note that N lowest bits of the sum are all 0!

Two's complement: example

Given:

- 3-bit fixed width (N=3)
- X = 2 (decimal) → 0 1 0 (binary)

$2^N = 8$ (dec) → 1 0 0 0 (bin)

$-X = 2^N - X = 8 - 2 = 6$ (dec) → 1 1 0 (bin)

Check:

$X + (-X) = 0\,1\,0 + 1\,1\,0 = 1\,\boxed{0\,0\,0}$ ✔

3-bit fixed width

Efficiently computing 2's complement

EASY!

"Flip the bits and add 1"

Example:

X = 0 1 0 (bin) → 2 (dec)

Flip the bits: 1 0 1

Add 1: 1 1 0 (bin) → -X

The roots of the idea

John von Neumann (died: 1957)

- Co-inventor of the stored program concept
- Proposed 2's complement idea in a 1945 paper
- Also came up with cellular automata, numerical weather forecasting, concept of global warming
- Outside of computing: linear programming, quantum logic, policy of mutually assured destruction, and more!

Patron saint of this class

2's complement details

- The MSB is the sign
- A – B = A + (2's complement of B)
- Arithmetic operations do not depend on the operands' signs
- Range is asymmetric: -2^{n-1} to $2^{n-1}-1$
- There are two kinds of overflows:
  – Positive overflow produces a negative number
  – Negative underflow produces a positive number

Converting between data types

- Converting a 2's complement number from a smaller to a larger representation is done by **sign extension**

Example: from byte to short (16 bits):

2 = 0 0 0 0 0 0 1 0 ⇒ ? ? ? ? ? ? ? ? 0 0 0 0 0 0 1 0

-2 = 1 1 1 1 1 1 1 0 ⇒ ? ? ? ? ? ? ? ? 1 1 1 1 1 1 1 0

2 = 00000010 ⇒ 0000000000000010

-2 = 11111110 ⇒ 1111111111111110

(byte) **(short)** **(byte)** **(short)**

Shifting

- Shifting: move the bits of a data type left or right
  - Data bits falling off the edge are lost
- For left shifts, 0s fill in the empty bit places
- For right shifts, two options:
  - Fill with 0 (logical shift): for non-numerical data
  - Fill with MSB (arithmetic shift): for 2's complement numbers
- Shift left by *n* is equivalent to multiplying by 2^n
- Shift right by *n* is equivalent to dividing by 2^n and rounding towards $-\infty$
- Example

$$6 = \boxed{0\ 0\ 0\ 0\ 0\ 1}\ 1\ 0 >> 2 \rightarrow 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1 = 1$$

$$-6 = 1\ 1\ 1\ 1\ 1\ 0\ 1\ 0 >> 2 \rightarrow 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0 = -2$$

Hexadecimal notation

- Binary numbers (and other binary-encoded information) are too long and tedious for us to use
- Solution: use a more compact encoding
  - Hexadecimal (base 16) is most common
- Hex digits: 0-9 and A-F
  – A=10_{dec}, B=11, …, F=15
- Conversion to/from binary is very easy:

  Every 4 bits correspond to 1 hex digit:

$$\underbrace{1\ 1\ 1\ 1}_{F(15)}\ \underbrace{1\ 0\ 0\ 0}_{8} \quad = 0xF8$$

Hex is just a convenience for humans
Computers use the binary form

Real numbers - floating point

- Java's `float` **(32 bits)**
  `double` **(64 bits)**

- Binary representation:
  - example 0.75 in base 10 ⇒ 0.11 in base 2

    $(2^{-1} + 2^{-2} = 0.5 + 0.25 = 0.75)$

Real numbers - floating point

- Java's `float` **(32 bits)**
  `double` **(64 bits)**

- Binary representation:
  – example 0.75 in base 10 $\Rightarrow$ 0.11 in base 2

  $(2^{-1} + 2^{-2} = 0.5 + 0.25 = 0.75)$

- Normalization:

  Mantissa (aka significand)

  exponent

  $0.11 \Rightarrow 1.1\text{x}2^{-1}$

  implicit (always 1)

Why normalize?

Three reasons:

1. Simplifies machine representation
(don't need to represent the fraction separator)

2. Simplifies comparisons

– Which one is bigger: 0.0000101 or 0.000001 ?

– $1.01\text{x}2^{-5}$ vs $1.0\text{x}2^{-6}$

3. Is more compact for very small/large numbers

– E.g., 0.0000000000000001 = 1.0×2^{-16}

or can be made more compact (by rounding fraction)

Floating point conversion example #1

Convert the number 25 to floating point with normalization

1)25 in base 10 ⇒ 11001 in base 2

2)11001 to normalized floating point ⇒ 1.1001×2^4

Understand that:

- The number is normalized
- 1.1001 is mantissa (aka significand)
- 4 is exponent
- sign is "+" (implicit here)

IEEE 754 Floating Point standard

- Need a standard to represent and compute with fixed-width floats
- 32 bit representation:

31	30 … 23	22 … 0
sign (s)	**exponent (exp)**	**mantissa (m)**

$$(-1)^s \text{ x } (1.m) \text{ x } 2^{exp-127}$$

Bias

e.g.,

$(0.75)_{10} \rightarrow (0.11)_2 \rightarrow (1.1\text{x}2^{-1})_2$

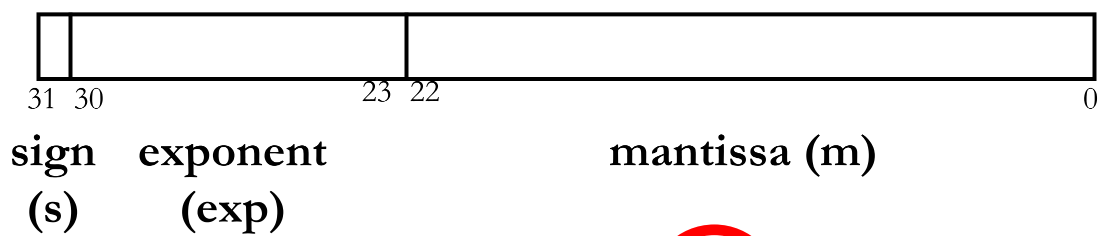
$\rightarrow$ s = 0, m = 1, exp = 126

$\rightarrow$ 0 01111110 10000000000000000000000

Note: representation does NOT use 2's complement

IEEE 754 Floating Point standard

- Need a standard to represent and compute with fixed-width floats
- 32 bit representation:



$$(-1)^s \text{ x } (1.m) \text{ x } 2^{exp-127}$$ **Bias**

e.g.,

$(0.75)_{10} \rightarrow (0.11)_2 \rightarrow (1.1 \text{x} 2^{-1})_2$

$\rightarrow$ s = 0, m = 1, exp = 126

$\rightarrow$ 0 01111110 10000000000000000000000

- 64 bit representation:
  – exponent = 11 bits; mantissa= 52 bits

IEEE 754 Floating Point standard

- Why bias?
  - Avoids the complexity of +/- exponents
  - Simplifies relative ordering of FP numbers

- Note: processors usually have specialized floating point units to perform FP arithmetic

IEEE 754 floating point conversion #2

Example: Convert 23.5 (decimal) to IEEE 754 floating point

Start: 23 in base 10 $\Rightarrow$ 10111 in base 2

IEEE 754 floating point conversion #2

Example: Convert 23.5 (decimal) to IEEE 754 floating point

Start: 23 in base 10 ⇒ 10111 in base 2

1) 23.5 in base 10 ⇒ 10111.1 in base 2

2) 10111.1 to normalized floating point ⇒ 1.01111×2^4

3) S = 0
   M = 01111 is mantissa (remember: 1. is implicit)
   Exp = 4+127 = 131 in base 10 ⇒ 1000 0011 in base 2

0	1 0 0 0 0 0 1 1	01111000000000000000000
31	30 … 23	22 … 0
sign (s)	**exponent (exp)**	**mantissa (m)**

Pad with 0s

IEEE 754 Floating Point notation

Exponent	Mantissa	Meaning
0	0	0
1-254	Anything	Floating point number
255	0	Infinity (signed)
255	Non-zero	Not-a-number (NaN)

32-bit representation

Representing characters

- Characters need to be encoded in binary too
- Operations on characters have simpler requirements than on numbers, so the encoding choice is not crucial
- Most common representation is ASCII
  - Each character is held in a byte
  - E.g. '0' is 0x30, 'A' is 0x41, 'a' is 0x61
- Java uses Unicode which can encode characters from many (all?) languages
  - 16 bits per character required

Representing strings

- Words, sentences, etc. are just strings of characters
- How is the end of a string identified?
  - No common standard exists. Different programming languages use different encodings
  - In C: a special character, encoded as 0x00
    - Also called NULL character
  - In Java: string length is kept with the string itself
    - string is an object and length is one of its member variables

Summary

- Computers use binary representation
- Signed numbers: sign-magnitude vs 2's complement
- Floating point
- Characters and strings