Best Shot Game Experiment Report

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1 BSG Background

Best Shot Game models the free rider problem. In BSG, agents play binary actions from the set $\{0, 1\}$. For the convenience of running Gradient Best Response, this experiment considers the continuous version of BSG. The expected utility of agent i is given by:

$$\mathbb{E}_{x_i \sim p_i}[u_i] := 1 + (p_i - 1) \prod_{j \in N_i} (1 - p_j) - c_i p_i$$

where p_i is the probability that i selects $x_i = 1$. **Note that p_i will become either 0 or 1 when the expected utility attains the maximum, which implies the PSNEs of BSG are action profiles consist only 0s and 1s.** With this continuous utility function, the experiment computes the gradient of the expected utility with respect to p_i.

2 Experiment Setup

Observed from the utility function for an agent in BSG, c_i, the parameter that measures the cost for choosing 1, can be used for characterizing the similarity of utility functions among each agents. Therefore, the experiment considers three different setups for sampling c: gaussian, homophily, and fully-homophily.[1]

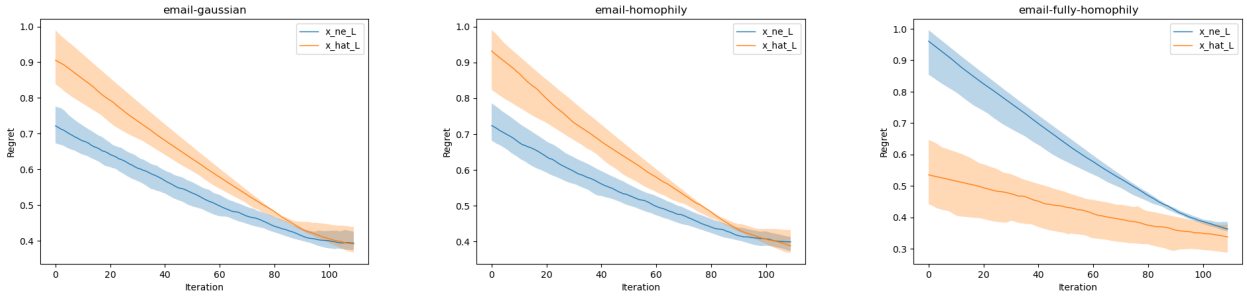
The experiment runs on the data of an email network(Leskovec, kleinberg, and Faloutsos 2007). It compares the convergence speed with random initial action profile and group PSNE action profile. The results shows MS-GBR converges faster than the randomly initialized GBR when the communities exhibit high homophily.

The convergence of GBR algorithm is measured by the regrets in the network. Different from the LQG, the PSNEs of BSG can be determined before the absolute convergence[2] when p_i clearly shows the tendency of becoming 1 or 0 for all agents. We call this point: action profile convergence point. The experiment approximates this point as being around 110 iterations based on the specific learning rate (0.005) being used.

[1] The specification of these setups can be found in the draft paper on Overleaf

[2] when the action profile only contain 0 and 1s and regret is zero

3 Experiment Result



(a) Gaussian (b) Homophily (c) Fully-Homophily

Figure 1

The Gaussian and homophily does not show much differences, but we can see that in the homophily case, x hat slightly outperforms random x near the action profile convergence point. With more similarities within a community, the fully-homophily shows a significantly faster converge speed when using MS-GBR.