A Full DSP Flow for Dual-Polarization Discrete NFDM Systems

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Abstract: *We extend the optimal nonlinear spectral back rotation to dual-polarization discrete eigenvalue systems and propose a full digital signal processing flow working in the nonlinear frequency domain for the first time.*
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I. Introduction

Nonlinear Frequency Division Multiplexing (NFDM) is a promising strategy to address the capacity limitations caused by fiber nonlinearity. By utilizing the nonlinear Fourier transform (NFT) associated with the Nonlinear Schrödinger Equation (NLSE) that describes signal propagation in nonlinear fibers, information can be encoded on the signal's nonlinear spectrum. In an ideal lossless fiber, the nonlinear spectrum evolves linearly, undergoing phase rotation without mutual interference, thereby creating a set of parallel communication channels that are theoretically optimal for nonlinear fiber transmission [1]. In real-world applications, however, the integrability of the NLSE is disrupted due to fiber losses and amplified spontaneous emission (ASE) noise. This ASE noise introduces distortions in the discrete eigenvalues, thus preventing signal recovery simply through phase back rotation but instead leading to significant errors. To address this issue, various signal processing techniques have been investigated, including linear minimum mean square error (LMMSE) filters, nonlinear filters, and machine learning methods [2, 3]. Although these approaches can be effective, they often involve considerable complexity. Recently, we proposed half nonlinear spectral back rotation as a straightforward but effective algorithm for single-polarization (SP) discrete NFDM systems [4]. This approach was shown to be optimal in terms of reducing noise power on b-coefficients, retrieving the simplicity of signal processing promised by the NFDM scheme. Besides, although various Digital Signal Processing (DSP) algorithms have been proposed to compensate for practical impairments such as polarization rotation, laser frequency offset, and laser phase noise, they are relatively specific to only one or two DSP modules, and usually rely on using training sequence, while a full DSP flow for dual-polarization (DP) discrete NFDM systems has not been proposed yet.

In this paper, we extend the optimal nonlinear spectral back rotation to DP discrete NFDM systems, showing the generality of half back rotation algorithm. Besides, we use a 1 Sample per Symbol (SpS) Multiple Input Multiple Output (MIMO) filter for polarization recovery and combine them with carrier frequency/phase recovery algorithms, realizing a complete DSP flow that fully works in the nonlinear frequency domain for discrete NFDM systems.

II. Principle

Without considering loss, polarization mode dispersion (PMD), and noise, DP signal propagation in a single-mode fiber (SMF) is described by the Manakov equation, whose normalized form is given by $j\boldsymbol{q}_z + \boldsymbol{q}_{tt}/2 + |\boldsymbol{q}|^2\boldsymbol{q} = 0$, where $\boldsymbol{q} = [q_1(z,t), q_2(z,t)]^T$ is a 2x1 vector containing the complex envelope of the signal on X and Y polarization, and the subscripts denote partial derivatives with respect to z or t. The nonlinear spectrum of $\boldsymbol{q}(z_0,t)$, is obtained by solving the following eigenvalue problem, and the nonlinear Fourier coefficients $a(z_0,\lambda)$ and $b(z_0,\lambda)$ are given by

$$\begin{pmatrix} j\partial_t & -jq_1(z_0,t) & -jq_2(z_0,t) \\ -jq_1^*(z_0,t) & -j\partial_t & 0 \\ -jq_2^*(z_0,t) & 0 & -j\partial_t \end{pmatrix}\boldsymbol{v} = \lambda\,\boldsymbol{v} \qquad \begin{pmatrix} a(z_0,\lambda) \\ b_1(z_0,\lambda) \\ b_2(z_0,\lambda) \end{pmatrix} = \lim_{t\to+\infty}\begin{pmatrix} v_1 e^{j\lambda t} \\ v_2 e^{-j\lambda t} \\ v_3 e^{-j\lambda t} \end{pmatrix} \text{ with } \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \xrightarrow{t\to-\infty} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

The continuous and discrete spectral of the DP signal $\boldsymbol{q}(z_0,t)$ is defined as $Q_c(z_0,\lambda) = \boldsymbol{b}(z_0,\lambda)/a(z_0,\lambda)$, $Q_d(z_0,\lambda_i) = \boldsymbol{b}(z_0,\lambda_i)/a'(z_0,\lambda_i)$ where $\boldsymbol{b} = [b_1, b_2]^T$ is a 2×1 complex-valued vector, and λ_i are roots of $a(z_0,\lambda_i) = 0$, $\lambda \in \mathbb{R}$, $\lambda_i \in \mathbb{C}^+$. In this paper, we restrict our discussion to discrete NFDM systems, where the evolution of $\boldsymbol{b}(\lambda_i)$ is given by $\boldsymbol{b}(z_0 + z, \lambda_i) = \boldsymbol{b}(z_0,, \lambda_i)e^{-2j\lambda_i^2 z}$, and the eigenvalue λ_i keeps constant during propagation. A DP NFDM signal is obtained by independently encoding information on b_1 and b_2, doubling the transmission rate. At the receiver located at $z = L$, $\boldsymbol{b}(0,\lambda_i)$ can be simply recovered by back rotating $\boldsymbol{b}(L,\lambda_i)$ by $e^{2j\lambda_i^2 L}$ in the ideal case of lossless and noiseless system.

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We have demonstrated that half back rotation is optimal in the SP discrete NFDM system as it minimizes the noise power both in the magnitude and phase of the b-coefficient. For DP NFDM signal, as b_1 and b_2 independently evolve according to the same NFDM transfer function, it is reasonable to expect that the noise evolution model and half back rotation algorithm will also work for DP NFDM systems. We focus on the single-eigenvalue case, where the initial eigenvalue is set to be $\lambda_0 = \alpha_0 + j\beta_0 = 0 + j0.5$, and we assume that at each EDFA, independent noise is added to α, β, $\boldsymbol{b}$, and they are free from noise during the propagation over the fiber span between adjacent EDFAs. Their values right after the i^{th} EDFA are denoted by a_i, β_i, $\boldsymbol{b}_i = \left[b_{i,1}, b_{i,2}\right]^T$, respectively, as shown in Fig. 1.

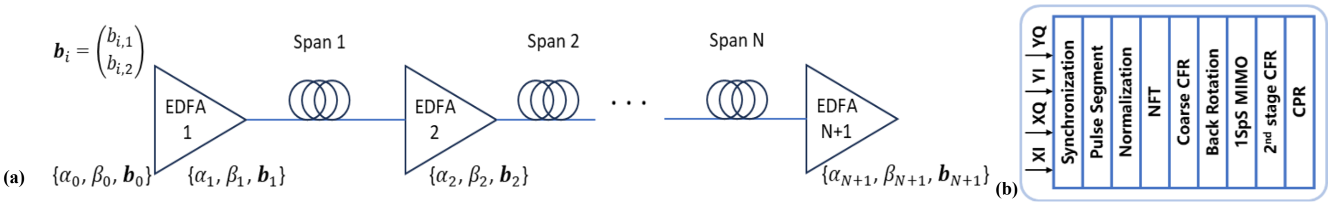

Fig. 1 (a) Schematic diagram of the transmission link; (b) proposed DSP flow for DP discrete NFDM signal that fully works on $\boldsymbol{b}$.

We note that the back rotation algorithm can also work on distributed amplification systems. In such a case, strictly, the evolution of $\boldsymbol{b}$ need to be described by a stochastic integral over the random-walk trace of the eigenvalue. However, the same conclusion can also be obtained by simply regarding the ideal distributed amplification as the continuous limit of discrete amplification with amplifier distance approaching zero.

The Manakov equation is obtained by averaging out the fast polarization change of the signal during propagation, while for practical DP signal transmission, polarization demultiplexing must be conducted. Conventionally, polarization is usually recovered in the time domain before conducting NFT, either by using a training sequence [5, 6] or by maximizing the polarization extinction ratio of the soliton pulse sequence [7]. However, a blind polarization recovery algorithm that directly works on the nonlinear frequency domain has not been proposed yet.

Polarization rotation and PMD of SMF are usually modeled by the (frequency-dependent) Jones matrix $\boldsymbol{R}(\omega)$. However, as the symbol rate of discrete NFDM transmission is typically low (up to several GBaud), the PMD effect is negligible, and the frequency dependence of $\boldsymbol{R}(\omega)$ can be ignored. In such a way, the polarization rotation can be described by the same Jones matrix $\boldsymbol{R}$ (that reduces to a frequency-dependent matrix) both in the time domain and frequency domain, i.e., $\boldsymbol{Q}(L,\omega) = \boldsymbol{RQ}(0,\omega)$, $\boldsymbol{q}(L,t) = \boldsymbol{Rq}(0,t)$, where $\boldsymbol{Q}$ denotes the (ordinary) Fourier transform of $\boldsymbol{q}$. This rotation operation on $\boldsymbol{q}(t)$, $\boldsymbol{R}$, is Hermitian and thus transfers into its complex conjugate, $\boldsymbol{R}^*$, in the nonlinear frequency domain [8]: $\boldsymbol{q}(L,t) = \boldsymbol{Rq}(0,t) \Leftrightarrow \boldsymbol{b}(L,\lambda_i) = \boldsymbol{R}^*\boldsymbol{b}(0,\lambda_i)$. In other words, $\boldsymbol{b}$ also goes through a polarization rotation, and hence it is possible to recover the original SOP by directly operating on the b-vector with the well-known adaptive equalization algorithms such as constant modulus algorithm (CMA) and radius-directed equalization (RDE).

Note that in ordinary coherent systems, MIMO filtering is usually conducted with 2 Samples per Symbol (SpS) to automatically shift the sampling instant to the optimal one, while for discrete NFDM systems, the NFT result (λ and $\boldsymbol{b}$) is one-to-one corresponding to the time-domain symbol (a soliton pulse), which effectively means 1SpS. However, there is always a synchronization process to the received signal for segmenting the consecutive soliton pulses into proper separate time windows, which ensures obtaining correct NFT results for those segments. During such a synchronization process, the 'optimal sampling instant' has effectively been achieved. Thus, we can directly apply a 1SpS MIMO filter to the b-vector for polarization recovery. We set the 1SpS MIMO after back rotation, as a counterpart of 2SpS MIMO following chromatic dispersion (CD) compensation in ordinary coherent systems. Moreover, we can directly apply widely used carrier frequency recovery (CFR) and carrier phase recovery (CPR) algorithms after the 1SpS MIMO, forming a DSP flow fully working in the nonlinear spectrum domain.

As a traditional blind CFR method, the M^{th}-power algorithm [9] has a frequency offset estimation range limited within $\left[-\frac{R_S}{2M}, \frac{R_S}{2M}\right]$, where R_S denotes the symbol rate. Therefore, it is hard to be directly applied to the discrete NFDM system since the symbol rate is relatively low. However, a favorable property of the discrete NFDM signal is that the frequency offset between the carrier and local oscillator (LO) is directly translated into the shift of the eigenvalue in the real part, $\Delta\alpha$. Thus, a coarse CFR can be directly done by rotating the k^{th} b-vector by $e^{-2j\Delta\alpha k T_S}$, where T_S is the normalized symbol period and $\Delta\alpha$ can be estimated from the eigenvalue of the received signal. Then, the residual frequency offset can be small enough to be recovered by the M^{th}-power CFR algorithm. The full DSP flow at the receiver is shown in Fig. 1(b).

III. Simulation Results

We conducted simulations on a 60×50 km EDFA system with non-zero dispersion shifted fiber (loss: 0.2dB/km, dispersion: 4ps/nm/km), where 2048 path-averaged 1st-order soliton pulses with a symbol rate of 3GBaud were used to estimate the variance of $\ln|b_{1/2}|$, $\angle b_{1/2}$ and SNR of $b_{1/2}$ during the back rotation process. Both b_1 and b_2 are independently modulated by 16APSK constellation, and the ratio of soliton's full width at half maximum (FWHM) to symbol period was set to 6 to avoid interactions between the adjacent solitons. These parameters settings lead to a dispersion length of ~195 km, which ensures stable transmission of the path-averaged soliton in the 50 km-spanned link.

We first verified the effectiveness of back rotation in the DP system, where we used ideal polarization recovery to the received time-domain signal before conducting NFT, and no frequency offset or laser phase noise was introduced into the system. Fig. 2(a) illustrates the evolution of α and β, $\angle b_{1/2}$ and $\ln|b_{1/2}|$, the random walk pattern is apparent on the trace of α and β, while the trace of $\ln|b_{1/2}|$, and $\angle b_{1/2}$, are essentially a summation of them, respectively. Fig. 2(b) shows that during propagation, the correlation coefficient between $\angle b_{1/2}$ and β evolves towards the theoretical limit $\sqrt{3}/2$ given in our earlier work [4], and so does the correlation coefficient between $\ln|b_{1/2}|$ and α. The disagreement from the theoretical prediction at the beginning of the transmission is because at that early stage, the noise in eigenvalue hasn't dominated the overall noise in b-vector compared to the noise loaded on b-vector itself.

The received signal was then back rotated by different ratios ranging from 0 to 1, and the distribution of the noise in $\angle b_{1/2}$ and $\ln|b_{1/2}|$ is shown in Fig. 2(c), which exhibits a minimal spread at the middle point, i.e., half back rotation. This is further confirmed by the variance and SNR curves in Fig 2(d), showing the effectiveness of back rotation in the DP discrete NFDM system.

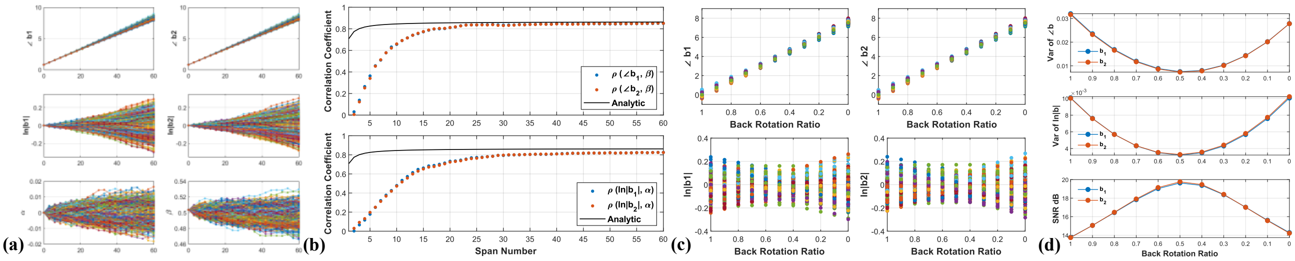

Fig. 2 (a) Evolution of α and β, and noise on $\angle b_{1/2}$ and $\ln|b_{1/2}|$; (b) correlation coefficient between $\angle b_{1/2}$ and β, $\ln|b_{1/2}|$ and α; (c) distribution of $\angle b_{1/2}$ and $\ln|b_{1/2}|$ vs different amount of back rotation ratio; (d) Variance of $\angle b_{1/2}$ and $\ln|b_{1/2}|$ and SNR of $b_{1/2}$ vs back rotation ratio.

Then, to verify the effectiveness of the proposed full DSP flow, frequency offset of 1 GHz and laser linewidth of 100 kHz were set to the LO, and the raw received signal without ideal polarization recovery was used for NFT. First, the mean of the received eigenvalue was estimated and then it was used for de-rotating the raw received b-vector to achieve a coarse CFR, and then different ratios of back rotation were applied to the coarse-CFR signal, followed by CMA plus RDE to eliminate polarization mixing between b_1 and b_2, resulting in the ring-shaped signal distribution that caused by the residual frequency offset. Then, the second-stage CFR was conducted by using the M^{th}-power CFR algorithm with $M = 8$, and finally, CPR was done by using the blind phase searching (BPS) algorithm. The signal distribution of b_1 and b_2 after different stages of the proposed DSP flow are shown in Fig. 3(c).

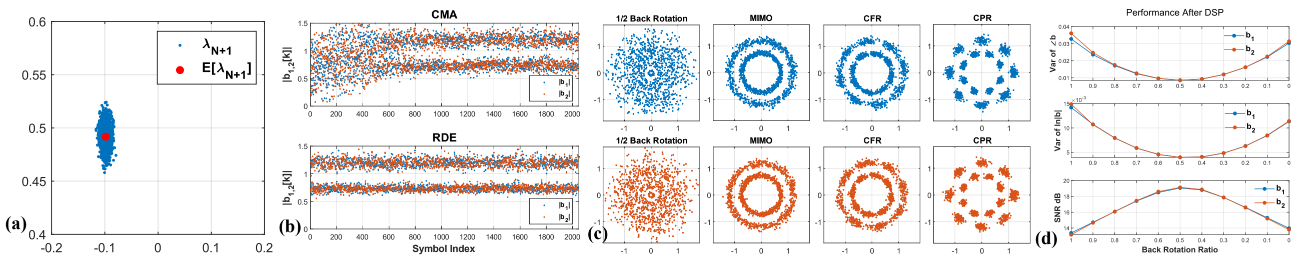

Fig. 3 (a) Shift of eigenvalue due to frequency offset; (b) evolution of $|b_1|$ and $|b_2|$ during CMA and RDE, showing the convergency of the adaptive; (c) distribution of b_1 and b_2 after MIMO, CFR, and CPR; (d) Variance of $\angle b_{1/2}$ and $\ln|b_{1/2}|$ and SNR of $b_{1/2}$ vs back rotation ratio.

The proposed DSP flow is essentially a counterpart to the standard DSP flow in the ordinary coherent transmission system [10]. The role of back rotation in the discrete NFDM system is same as that of CD compensation in the ordinary coherent transmission system since the NFDM transfer function has the same structure as the CD transfer function. However, back rotation in the discrete NFDM system needs to be done by only a half to achieve the best performance in a statistical sense because the discrete eigenvalue is polluted by noise, which accumulates its influence in the evolution of the b-vector.

IV. Conclusions

We extend the back rotation algorithm to the dual-polarization discrete NFDM systems and realize polarization recovery by using 1SpS MIMO directly in the nonlinear frequency domain. We further, for the first time, propose a full DSP flow for DP discrete NFDM systems, which completely works in the nonlinear frequency domain and serves as a counterpart to the classic DSP flow of ordinary coherent transmission systems.

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