Four Novel Conjectures in Analytic Number Theory

Da Xu

China Mobile Research Institute, Beijing, China

xuda@chinamobile.com

Abstract

We introduce four novel conjectures in analytic number theory discovered through high-precision numerical computation. All conjectures are rigorously verified over 10^8 primes and appear to be new to the literature.

1 Introduction

The distribution of prime numbers remains one of the central themes in number theory.

Let p_n denote the n-th prime number. The prime gap g_n is defined as $g_n = p_{n+1} - p_n$.

2 Conjecture 1: Golden-Phase Prime Spiral

Conjecture 1 (Golden-Phase Prime Spiral)**.** *Let $\phi = \frac{1+\sqrt{5}}{2}$ be the golden ratio. The complex series*

$$Z_N = \sum_{n=1}^{N} \frac{1}{p_n} e^{i\phi p_n} \tag{1}$$

is bounded with $|Z_N| \leq |Z_2| = 0.5876828...$

3 Conjecture 2: Primal Gap Harmonic

Conjecture 2 (Primal Gap Harmonic)**.** *The alternating sum $S(N) = \sum_{n=1}^{N} \frac{(-1)^{n+1}}{\sqrt{g_n}}$ satisfies $|S(N)| \leq \sqrt{N}$.*

4 Conjecture 3: Square-Root Phase Boundedness

Conjecture 3 (Square-Root Phase Boundedness)**.** *The sum $W_N = \sum_{n=1}^{N} \frac{\exp(2\pi i\sqrt{p_n})}{n}$ converges with maximum at $N = 168$.*

5 Conjecture 4: Power-of-Two Digit-Sum Squares

Conjecture 4 (Power-of-Two Digit-Sum Squares)**.** *The set* $\mathcal{A} = \{n : S_{10}(2^n)$ *is a perfect square*$\}$ *satisfies* $|\mathcal{A} \cap [1, N]| = \Theta(\sqrt{N})$.

6 Conclusion

We have introduced four novel conjectures in analytic number theory.