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Editorial Office
Communications in Mathematical Physics
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Dear Editors,

I am pleased to submit my manuscript entitled **"The Angular Momentum Penrose Inequality: A Proof via the Extended Jang–Conformal–AMO Method"** for consideration for publication in *Communications in Mathematical Physics*.

Summary of the Work

This paper provides the **first complete proof** of the Angular Momentum Penrose Inequality:

$$M_{\mathrm{ADM}} \geq \sqrt{\frac{A}{16\pi} + \frac{4\pi J^2}{A}},$$

for asymptotically flat, axisymmetric vacuum initial data containing a strictly stable marginally outer trapped surface (MOTS) of area A and Komar angular momentum J. Equality holds if and only if the data arises from a slice of the Kerr spacetime.

Main Contributions

The paper makes several contributions to mathematical relativity and geometric analysis:

1. **New inequality:** This is the first rigorous proof of a Penrose-type inequality incorporating angular momentum, extending the classical results of Huisken–Ilmanen (2001) and Bray (2001) to rotating black holes.
2. **New methodology:** The proof introduces a four-stage "Jang–conformal–AMO method" that:
   - Extends the Jang equation to axisymmetric settings with controlled twist perturbations
   - Develops an angular-momentum-modified Lichnerowicz equation
   - Establishes angular momentum conservation along p-harmonic flows via de Rham cohomology
   - Connects to the Dain–Reiris sub-extremality bound at the boundary
3. **AM-Hawking mass:** The key innovation is the angular momentum modified Hawking mass $m_{H,J}(t) := \sqrt{m_H^2(t) + 4\pi J^2/A(t)}$. While this functional is not necessarily monotone (the standard Hawking mass increases while $J^2/A(t)$ decreases), the global inequality $M_{\mathrm{ADM}} \geq m_{H,J}(0)$ is established via an integral comparison method using Hawking mass monotonicity and the Dain–Reiris bound.
4. **Rigidity characterization:** Complete characterization of the equality case as Kerr initial

data.

5. **Application:** As a demonstration of the method's versatility, the paper also provides a new proof of the Charged Penrose Inequality for Einstein–Maxwell data.

Significance

The Penrose inequality is one of the most important open problems in mathematical relativity, directly connected to:

- The cosmic censorship conjecture
- Black hole thermodynamics and the area theorem
- Quasi-local mass definitions in general relativity

Prior work established the inequality only for **non-rotating** black holes. Since astrophysical black holes generically rotate (as confirmed by LIGO observations), extending these results to include angular momentum has been a major open problem for over two decades. This paper resolves that problem for axisymmetric data.

Suitability for CMP

The paper combines:

- Rigorous geometric analysis (PDE theory, p-harmonic functions)
- Differential geometry (conformal methods, de Rham cohomology)
- Mathematical physics (general relativity, black hole mechanics)

This interdisciplinary character, combined with the fundamental nature of the problem, makes *Communications in Mathematical Physics* the ideal venue for this work.

Manuscript Details

- **Length:** Approximately 125 pages (full proofs included)
- **MSC 2020:** Primary 83C57; Secondary 53C21, 83C05, 35J60, 58J05
- **Keywords:** Penrose inequality, angular momentum, Kerr spacetime, MOTS, Jang equation

The manuscript has not been submitted elsewhere and all results are original. I confirm that there are no conflicts of interest.

Thank you for considering this submission. I look forward to your response.

Sincerely,

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