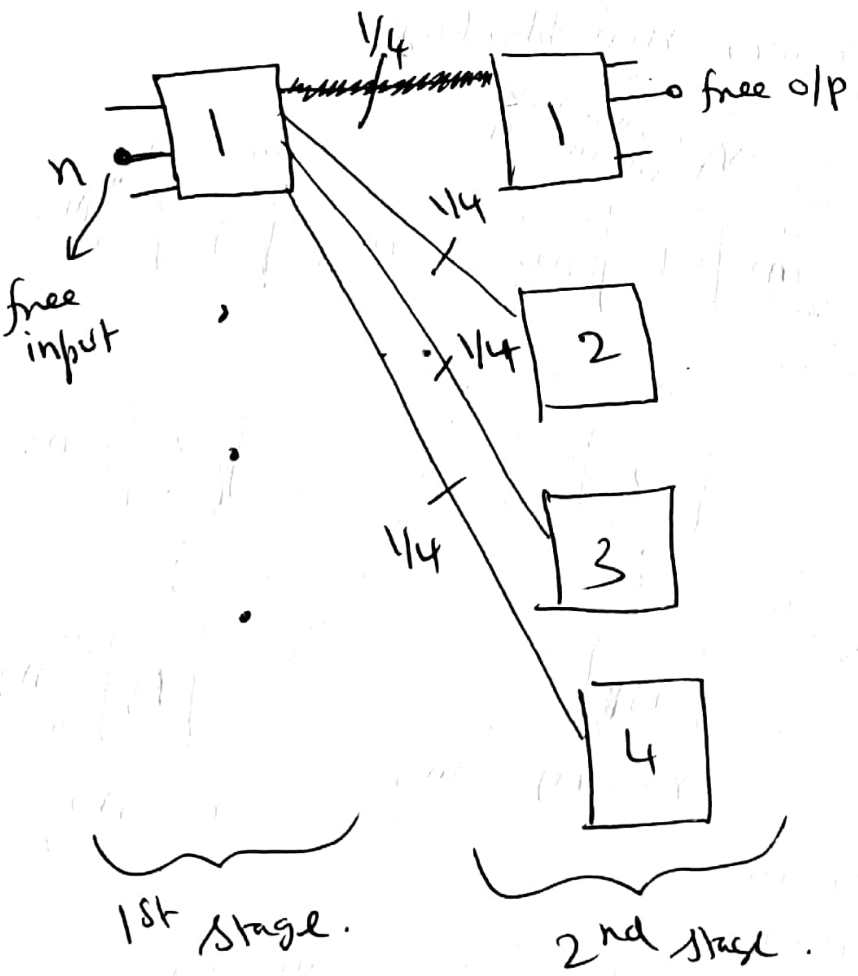
CLOS Network

Our objective – To design a strictly non-blocking interconnection switch.

Recall: Two-stage was unable to meet this strictly non-blocking requirement; In addition, switch complexity would grow quadratically.

- Bell Labs paper – by Charles Clos

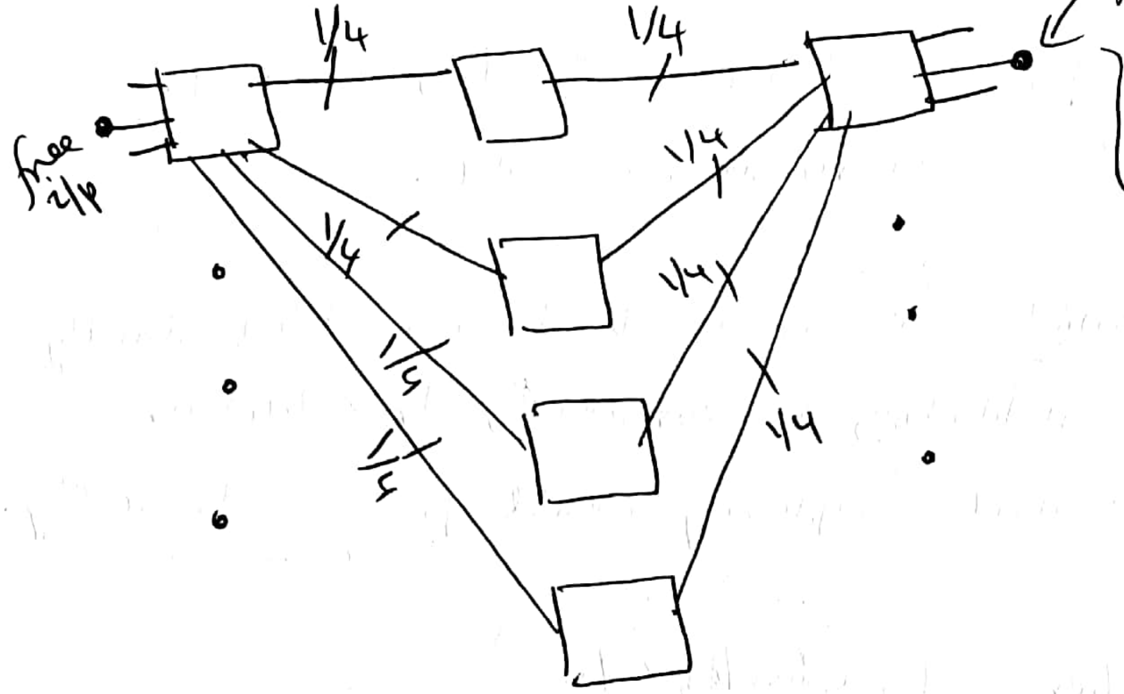
Idea Extending from 2-stage networks.



Issue is when there is one free i/p on SW_1 (1st stage) & 1 free o/p on SW_1 (2nd stage), there is no way a connection could be established as the 1/4th link are currently occupied.

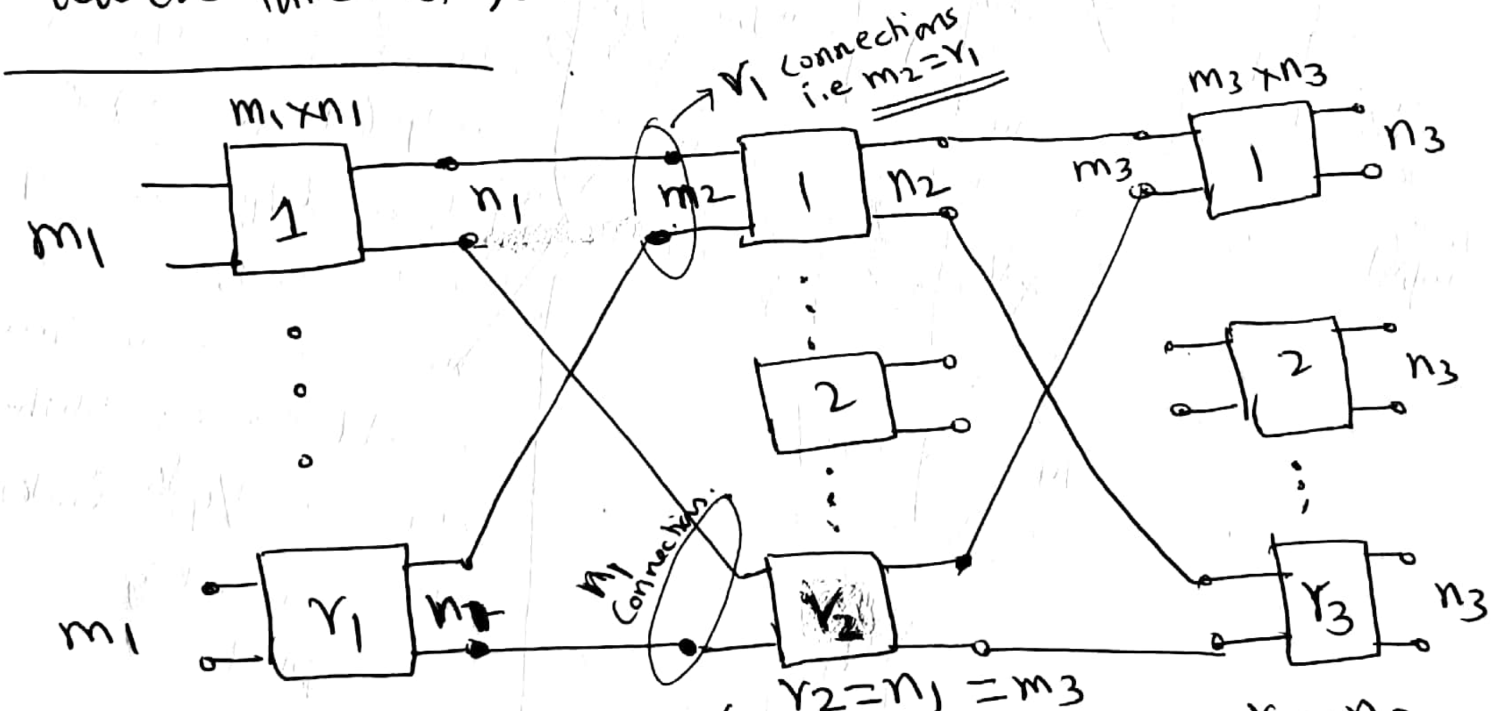
This leads to **blocking**.

If we extend to 3-stages;



Then this solution may provide a path between any two free i/p - o/p lines.

Actually, we can increase the # of switches in the middle stage! Clos came up with a condition under which three-stage becomes non-blocking.



$r_2 = n_1 = m_3$

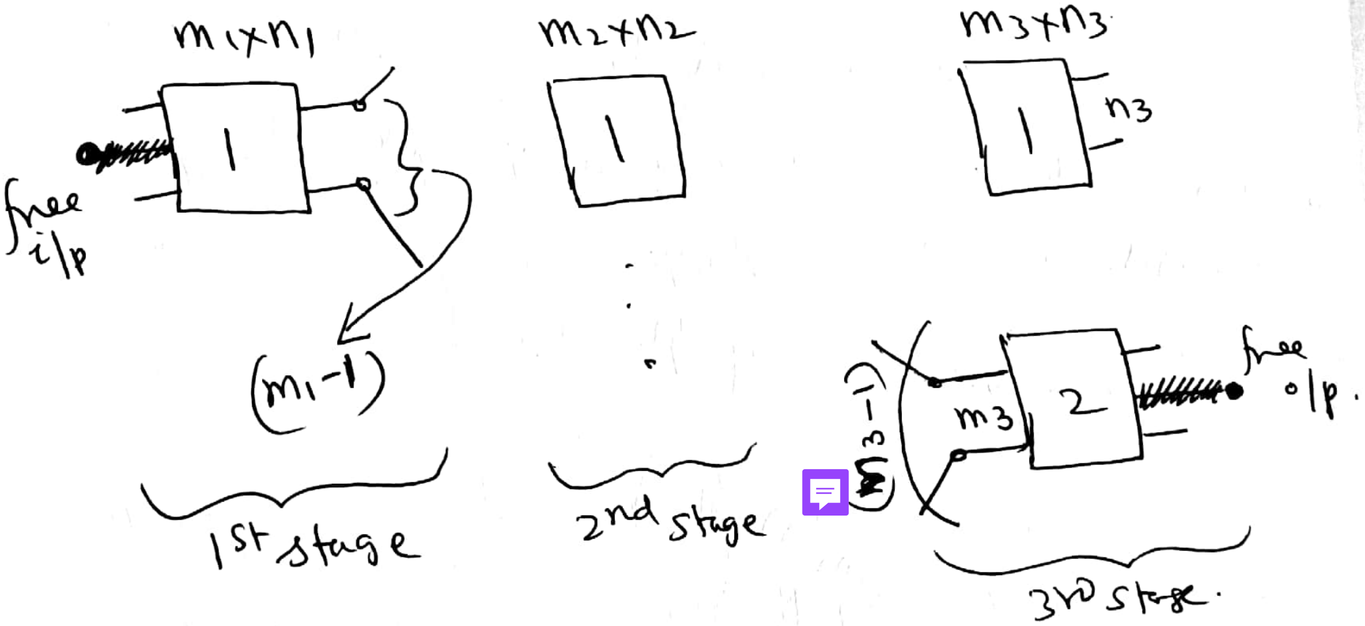
$r_3 = n_2$

exactly one connection between any pair of i/p-o/p.

$\boxed{\text{CLOS Network.}}$

note: $m_1 < n_1$ usually.

Consider two free i/p - o/p pairs & let us see if they can be connected.



<u>3rd stage</u>: Consider SW_2. Out of m_3 only n_3 can be connected (max → (m_3 could be greater than n_3)

Since one o/p is free only $(n_3 - 1)$ can be occupied or emanating from the input side.

<u>1st stage</u>: SW_1 — From SW_1 only $(m_1 - 1)$ can be going out occupied (as one is free); That is out of m_1 input lines, $n_1 - (m_1 - 1)$ will be free.

So, if $(m_1 - 1)$ rears are occupied / coming out of stage 1, this means there are $(m_1 - 1)$ switches in the middle stage that are used for all other rea from SW_1 in stage 1.

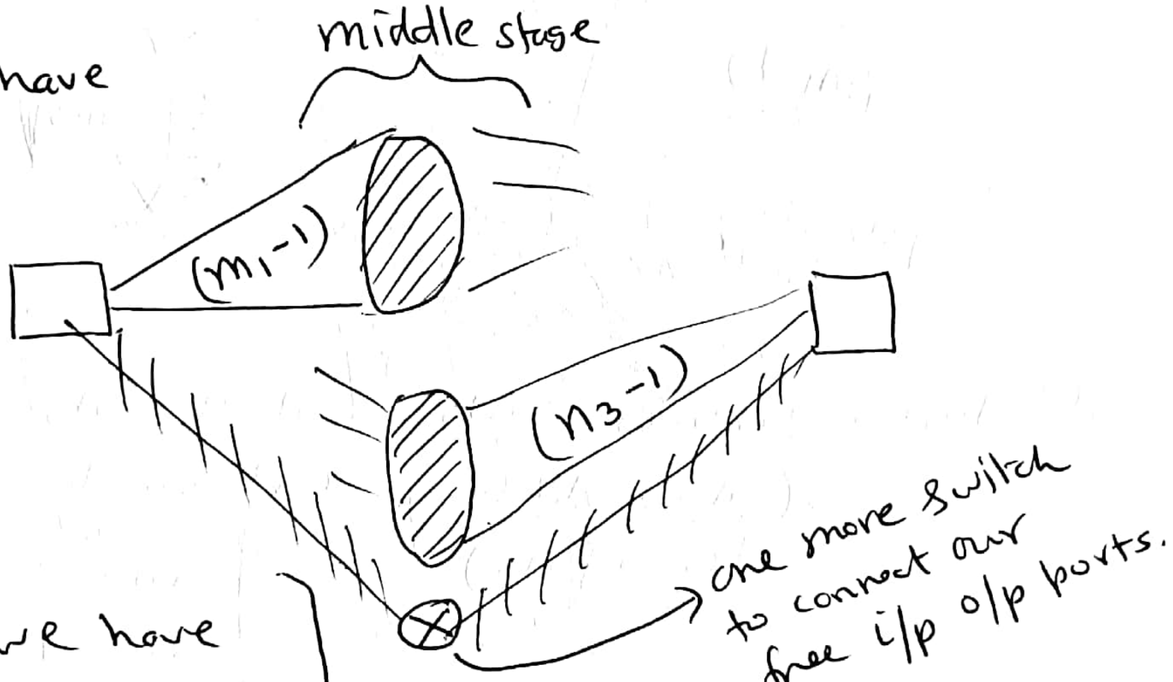
(4)

Similarly, see SW_2 of 3rd stage.

$(n_3 - 1)$ middle stage switches are used to connect the requests to the 3rd stage.

- Note that we have our free link at switch 2 at stage 3.

So, we have



So, if we have one more switch in the middle stage then we will be able to connect our free i/p & free o/p link.

Therefore,

$$r_2 > (m_1 - 1) + (n_3 - 1)$$

$$\Rightarrow \boxed{r_2 > m_1 + n_3 - 2}$$

$\to$ strictly non-blocking

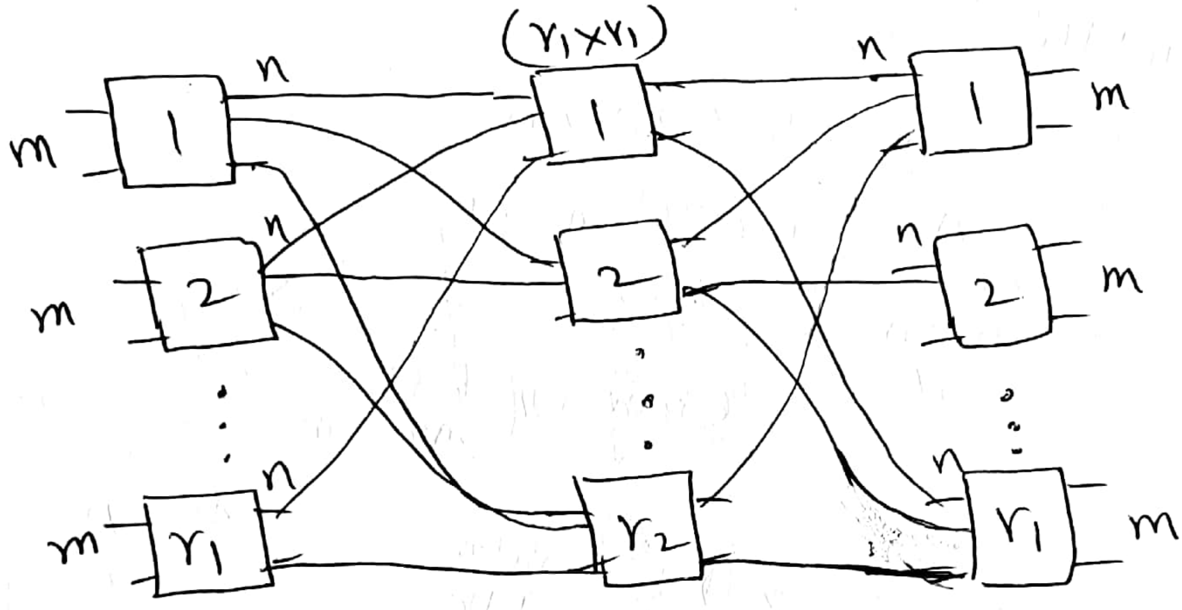
$$\boxed{\text{note: } r_2 \geq m_1 + n_3 - 1}$$

Also, note that,

$$m_3 = n_1 = r_2 \geq m_1 + n_3 - 1$$

Q: How many cross points required for this switch configuration?

Let us consider a symmetric configuration



looks the same from either →ion.

• for symmetric config. $m_1 = n_3$
$\therefore r_2 = n$

from our condition: $r_2 \geq m + m - 1$ i.e., $r_2 \geq 2m-1$

So, $\boxed{n = r_2 \geq 2m-1}$

#of input points: $N = (m \times r_1) \Rightarrow r_1 = (N/m)$

#of cross points needed: $(2m-1)$ switches in the middle stage;

So, each switch in the first stage will be having $m(2m-1)$ cross points.

There are r_1 switches in the first stage.

⑥ Therefore, # of crosspoints in the first stage is: $r_1 \cdot m \cdot (2m-1)$, i.e., $\underbrace{\left(\frac{N}{m}\right)}_{r_1} \cdot m(2m-1)$

For middle stage:

each switch : $n \times n$

& $n = r_2 = (2m-1)$

$\therefore \; n \times n =$

$$\underbrace{(2m-1)}_{\text{\# of switches in the middle stage.}}\left(\frac{N}{m}\right)\left(\frac{N}{m}\right)$$

Third stage: Same as first stage.

Therefore, # of cross point Q

$$Q = \underbrace{2m(2m-1)\frac{N}{m}}_{\text{1st \& 3rd stages}} + \underbrace{(2m-1)\frac{N}{m}\cdot\frac{N}{m}}_{\text{middle stage}}.$$

We can further simplify. If we assume if we have one more middle switch 2m-1 becomes just 2m.

$$\boxed{\therefore \; Q = 4mN + 2\frac{N^2}{m}}$$

It is better to express complexity in terms of N & hence we will find an optimum m^*.

$$Q = 4mN + \frac{2N^2}{m} \Big\}$$ # of cross-points in the system.

$$\Rightarrow \left(\frac{dQ}{dm}\right) = 0 = 4N - \frac{2N^2}{m^2}$$

$$\Rightarrow \quad 2N\underbrace{\left(2 - N/m^2\right)} = 0 \quad , N > 0$$

$$\Rightarrow \quad \boxed{m^* = \sqrt{N/2}}$$

$$\therefore Q = 4 \cdot \sqrt{N/2} \cdot N + \frac{2N^2}{\sqrt{N/2}}$$

$$Q = 2N^{3/2} + 2\sqrt{2} \cdot N^{3/2} = \underline{\underline{4\sqrt{2}\, N^{3/2}}}$$

$$\therefore \text{complexity} \simeq O(N^{3/2})$$

So, # of cross points are not growing as N^2! Are minimized to $O(N^{3/2})$.