Scaling Laws: Basic Information, Challenges and Application

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Scaling laws have made many contributions when detecting relationships between urban indicators and cities' population size.

Basics of Scaling Laws

Scaling laws are powerful summaries of the variations of urban attributes with city size.(Arcaute and Hatna, 2020) It was also used in finding the relationship between metabolic rates and body size(West, Brown and Enquist, 1997). Its mathematical form is shown below:

$$Y_i = Y_0 * N_i^{\beta}$$

Where Y_i refers to the quantity of one indicator in city i, for example the number of manufacture jobs, cars and so on. Y_0 is a constant. N_i is total population of city i. β is scaling exponent.

We are interest in discussing the number of β. Superlinear relationships ($\beta > 1$) means that population size could have positive effect on urban indicator; whereas sublinearity ($\beta < 1$) is associated with lower growth compared with population size. Linear scaling ($\beta \approx 1$) means that the speed of growth of indicator is constant across city size.(Cottineau *et al.*, 2017)

Challenges when identifying urban scaling

There are many challenges when we use scaling laws. Here we focus on two parts: How we define a city; the contradictory results of exponents in different scales.

Define a city

Cities could be measured or defined by some characteristics, such as their boundaries (natural or man-made), population density and other urban indicators. Applying different criteria may lead to different results. For example, Arcaute(2015) used two indicators to define cities: the minimum density of residents and minimum share of commuters. This method is more flexible when defining the boundaries, compared with the administrator boundary which is fixed in a period. But does it make sense to define cities by these two criteria? If there is a settlement, its population density and ratio of commuters are all below the criteria we set, but the living style of residents here is like the place we defined by population density and ratio of commuters. Although we could define other criteria to include these places, the exponents (β) may change. The contradictory exponents may have some problems, which will be explained in next part.

Exponent (β): contradictory results could emerge

Different scales of cities need to be considered when discussing the fitness of β. For example, according to the 6th national census dataset (2010), the city with the largest urban population in China is Shanghai, which contains almost 23.01 million people, compared with Ngari Prefecture in Tibet where there is only about 95 thousand residents. If we put them together or set population cutoff, the number of β could be different before and after cutoff, and the generic pattern of cities could not be easy to find.

What is more, contradictory results may emerge when we apply the scaling laws into two different level of settlements, especially one region contains the other one. In paper (Arcaute et al., 2019), without population cutoff, the author got two different β ($\beta_1>1$, at urban cores level; $\beta_2<1$, at metropolitan level) when illustrating the case for the number of individuals employed in manufacturing and in education. The example reveals that exponent for this indicator is not universal. Arcaute(2020) argues that this may because the number of people employed in a specific sector depends on the maturity of the sector itself.

Urban Scaling: what does it tell and hide?

Scaling laws consider the relationships between population size and economic indicators. This is done by looking at the value of β. The total wages (M) and population(N), for example(Bettencourt *et al.*, 2007), could be interpreted by scaling law ($M_i = M_0 * N^{\beta}_i$, $\beta=1.12$). It means that economic indicator (total wages) is increased with the growth of population size. And when we look deep into the velocity of total wage, we could find that the exponent of the new equation ($V_{M_i} = C_0 * N_i^{\beta-1}$,β-1=0.12) would be above zero but smaller than 1. It means that the velocity of total wage also increases as the growth of population size. From this example, scaling relations could be used to predict many of the characteristics when a city gains or loses population. (Bettencourt *et al.*, 2007)

However, scaling laws cannot reveal the interactions and contributions at the local level(Arcaute and Hatna, 2020). Also, some other factors, not just population size, could have impact on economic indicators. An example of this, is that the investment from central government may play an important role to help local authorities maintain or increase the size of economy.

Conclusion

Urban Scaling could be useful to detect whether the aggregation of population could speed up the growth of economic indicators. However, the flaw of urban scaling, Arcaute (2020) has summarized, is that the objects we study are independent and it can lead to inconsistent results. What is more, the way we define cities and exponent we use to interpret are challenges when applying scaling laws.

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