

# Modeling, Learning and Reasoning about Preference Trees over Combinatorial Domains

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# Preferences Are Ubiquitous

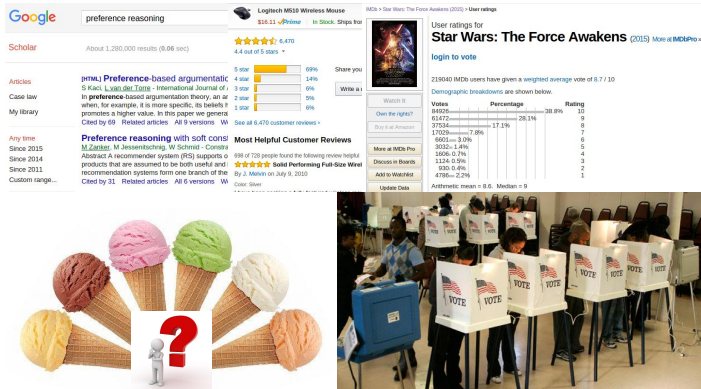


Figure : Preferences of different forms

# Describing Preferences

Car1



<mvan, 7m, gray, big, honda, med, med>

Car2



<sedan, 5, blue, med, vw, med, med>



Figure : How to express preferences?

## ① How will I rate cars?

- For BodyType, I will assign 7 points to minivans, 5 to sedans, ...
- For Color, I will assign 8 points to blue, 4 to gray, ...

## ② What are the desired properties I see in cars?

- I prefer minivans to sedans, ...
- If minivan, I prefer gray to blue; if sedan, I prefer blue to gray; ...

# Describing Preferences

Car1



<mvan, 7m, gray, big, honda, med, med>

Car2



<sedan, 5, blue, med, vw, med, med>



Figure : How to express preferences?

## ① How will I rate cars? (**Quantitative**)

- For BodyType, I will assign 7 points to minivans, 5 to sedans, ...
- For Color, I will assign 8 points to blue, 4 to gray, ...

## ② What are the desired properties I see in cars? (**Qualitative**)

- I prefer minivans to sedans, ...
- If minivan, I prefer gray to blue; if sedan, I prefer blue to gray; ...

## Combinatorial Domains

Let  $\mathcal{I}$  be a finite set of attributes  $\{X_1, \dots, X_p\}$ , associated with a set of finite domains  $\{Dom(X_1), \dots, Dom(X_p)\}$  for each attribute  $X_i$ . A *combinatorial domain*  $CD(\mathcal{I})$  is a set of *objects* described by combinations of values from  $Dom(X_i)$ :

$$CD(\mathcal{I}) = \prod_{X_i \in \mathcal{I}} Dom(X_i).$$

# Combinatorial Domains: Example

Domain of cars over set  $\mathcal{I}$  of  $p$  binary attributes:

① **BodyType**: {mvan, sedan}.

② **Capacity**: {5, 7m}.

③ **Color**: {blue, gray}.

⋮

$$CD(\mathcal{I}) = \underbrace{\{\langle \text{sedan}, 5, \text{blue}, \dots \rangle, \langle \text{mvan}, 7\text{m}, \text{gray}, \dots \rangle, \dots\}}_{2^p \text{ objects, too many!}}.$$

# Combinatorial Domains: Example

Domain of cars:

- ① **BodyType**: {mvan, sedan, sport, suv}.
- ② **Capacity**: {2, 5, 7m}.
- ③ **Color**: {black, blue, gray, red, white}.
- ④ **LuggageSize**: {big, med, small}.
- ⑤ **Make**: {bmw, ford, honda, vw}.
- ⑥ **Price**: {low, med, high, vhigh}.
- ⑦ **Safety**: {low, med, high}.

# Single Agent

Car1



<mvan, 7m, gray, big, honda, med, med>

Car2



<sedan, 5, blue, med, vw, med, med>



Figure : Dominance and Optimization



# Multi-Agent



Figure : Social Choice and Welfare

# Research Problems of Interest

- ① Preference representation formalisms to compactly model qualitative preferences over combinatorial domains.
- ② Preference elicitation and learning methods to cast preferences of agents as a theory in a preference formalism.
- ③ Preference reasoning tasks:
  - Dominance and optimization
  - Preference aggregation

Q: How do we compactly represent qualitative preferences over combinatorial domains?

- ① Preference Trees (P-trees)<sup>1,11</sup>
- ② Partial Lexicographic Preference Trees (PLP-trees)<sup>8</sup>
- ③ Lexicographic Preference Trees (LP-trees)<sup>4,13</sup>

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<sup>1</sup>Niall M Fraser. "Ordinal preference representations". In: Theory and Decision (1994)

<sup>2</sup>Xudong Liu and Mirosław Trzuszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

<sup>3</sup>Xudong Liu and Mirosław Trzuszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

<sup>4</sup>Richard Booth et al. "Learning conditionally lexicographic preference relations". In: ECAI. 2010

<sup>5</sup>Xudong Liu and Mirosław Trzuszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

Q: How do we learn predictive qualitative preference models over combinatorial domains?

- ① Partial Lexicographic Preference Trees (PLP-trees)<sup>6,7,8</sup>
  - Active and passive learning
  - Compute a (possibly small) PLP-tree consistent with all the data
  - Compute a PLP-tree that agrees with the data as much as possible
- ② Empirical Learning of PLP-trees and PLP-forests<sup>9</sup>

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<sup>6</sup>Michael Schmitt and Laura Martignon. "On the complexity of learning lexicographic strategies". In: The Journal of Machine Learning Research (2006)

<sup>7</sup>József Dombi, Csanád Imreh, and Nándor Vincze. "Learning lexicographic orders". In: European Journal of Operational Research (2007)

<sup>8</sup>Xudong Liu and Mirosław Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

<sup>9</sup>Xudong Liu and Mirosław Truszczynski. "Learning Partial Lexicographic Preference Trees and Forests over Multi-Valued Attributes". In: Review by ECAI-16 Program Committee

Q: How do we reason about preferences over combinatorial domains?

① Preference Optimization<sup>10,11,12</sup>:

- Dominance testing:  $o_1 \succ_P o_2$ ?
- Optimality testing:  $o_1 \succ_P o_2$  for all  $o_2 \neq o_1$ ?
- Optimality computing: what is the optimal outcome wrt  $P$ ?

② Preference Aggregation<sup>13</sup>:

- Winner determination: which candidate wins the election?
- “Strong” candidate: a candidate with score more than a threshold?

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<sup>10</sup>Jérôme Lang, Jérôme Mengin, and Lirong Xia. “Aggregating Conditionally Lexicographic Preferences on Multi-issue Domains”. In: CP. 2012

<sup>11</sup>Xudong Liu and Mirosław Trzuszczynski. “Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences”. In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

<sup>12</sup>Xudong Liu and Mirosław Trzuszczynski. “Reasoning with Preference Trees over Combinatorial Domains”. In: Proceedings of the 4th International Conference on Algorithmic Decision Theory (ADT). 2015

<sup>13</sup>Xudong Liu and Mirosław Trzuszczynski. “Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers”. In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

- ① Modeling qualitative preferences:
  - Preference trees (P-trees)
  - Partial lexicographic preference trees (PLP-trees)
- ② Learning PLP-trees and PLP-forests
- ③ Aggregating LP-trees
- ④ Future research directions

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# Preference Trees

- 1 Let  $\mathcal{I} = \{X_1, \dots, X_p\}$  be a set of attributes, and  $D(\mathcal{I}) = \{Dom(X_1), \dots, Dom(X_p)\}$  a set of finite domains for  $\mathcal{I}$ .
- 2 A *literal* is an assignment to an attribute. We denote by  $X_i := x_{i,j}$  the literal that assigns value  $x_{i,j} \in Dom(X_i)$  to  $X_i$ . When no confusion, we write  $x_{i,j}$ , instead of  $X_i := x_{i,j}$ , as a literal. We then denote by  $\mathcal{L} = \{x_{i,j} \in Dom(X_i) : X_i \in \mathcal{I}\}$  the set of literals given  $\mathcal{I}$  and  $D(\mathcal{I})$ .
- 3 The combinatorial domain  $CD(\mathcal{I})$  is defined as earlier.



- 4 A **P-tree**  $T$  over  $CD(\mathcal{I})$  is a binary tree, where non-leaf nodes are labeled with propositional formulas over  $\mathcal{L}$ .
- 5 Given an outcome  $o \in CD(\mathcal{I})$ , the **leaf**  $l_T(o)$  is the leaf reached by traversing the tree  $T$  according to  $o$ . When at a node  $N$  labeled with  $\varphi$ , if  $o \models \varphi$ , we descend to the left child of  $N$ ; otherwise, to the right.
- 6 For  $o_1, o_2 \in CD(\mathcal{I})$ , we have  $o_1 \succ_T o_2$  if  $l_T(o_1) \succ_T l_T(o_2)$ , and  $o_1 \approx_T o_2$  if  $l_T(o_1) = l_T(o_2)$ . Outcome  $o_1$  is **optimal** if there exists no  $o_2$  such that  $o_2 \succ_T o_1$ .

# Preference Trees (P-Trees)

Let  $\varphi$ ,  $\psi$ , and  $\pi$  be propositional formulas over the set  $\mathcal{L}$  of literals that are values from  $\bigcup_{X_i \in V} \text{Dom}(X_i)$ .

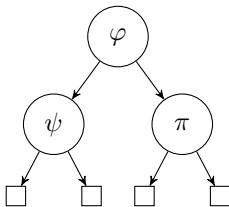


Figure : A P-tree

$$\varphi \wedge \psi \succ \varphi \wedge \neg\psi \succ \neg\varphi \wedge \pi \succ \neg\varphi \wedge \neg\pi.$$

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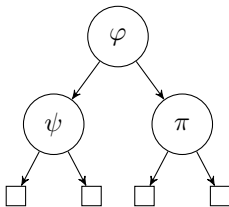


Figure : A P-tree

$$\varphi \wedge \psi \succ \varphi \wedge \neg\psi \succ \neg\varphi \wedge \pi \succ \neg\varphi \wedge \neg\pi.$$

Total preorder

## Example: The Cars Domain

- ① **BodyType**( $X_1$ ):  $\{\text{mvan}(x_{1,1}), \text{sedan}(x_{1,2}), \text{sport}(x_{1,3}), \text{suv}(x_{1,4})\}$ .
- ② **Capacity**( $X_2$ ):  $\{2, 5, 7\text{m}\}$ .
- ③ **Color**( $X_3$ ):  $\{\text{black}, \text{blue}, \text{gray}, \text{red}, \text{white}\}$ .
- ④ **LuggageSize**( $X_4$ ):  $\{\text{big}, \text{med}, \text{small}\}$ .
- ⑤ **Make**( $X_5$ ):  $\{\text{bmw}, \text{ford}, \text{honda}, \text{vw}\}$ .
- ⑥ **Price**( $X_6$ ):  $\{\text{low}, \text{med}, \text{high}, \text{vhigh}\}$ .
- ⑦ **Safety**( $X_7$ ):  $\{\text{low}, \text{med}, \text{high}\}$ .

# Example: Preference Trees over Cars

**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

**Color**( $X_3$ ): {black, blue, gray, red, white}.

**Price**( $X_6$ ): {low, med, high, vhigh}.

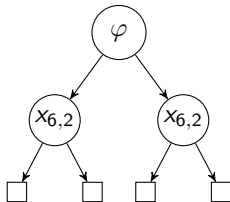


Figure : A P-tree over cars<sup>14</sup>

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<sup>14</sup> $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$ .

# Example: Preference Trees over Cars

**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

**Color**( $X_3$ ): {black, blue, gray, red, white}.

**Price**( $X_6$ ): {low, med, high, vhigh}.

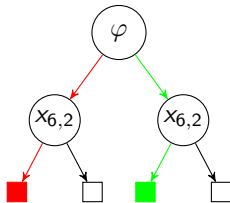


Figure : A P-tree over cars<sup>14</sup>

*Car2*  $\succ$  *Car1*

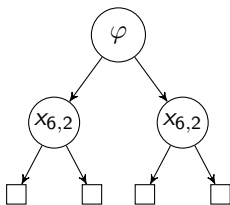
<sup>14</sup> $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$ .

# Compact Representation of P-trees

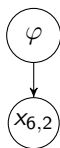
**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

**Color**( $X_3$ ): {black, blue, gray, red, white}.

**Price**( $X_6$ ): {low, med, high, vhigh}.



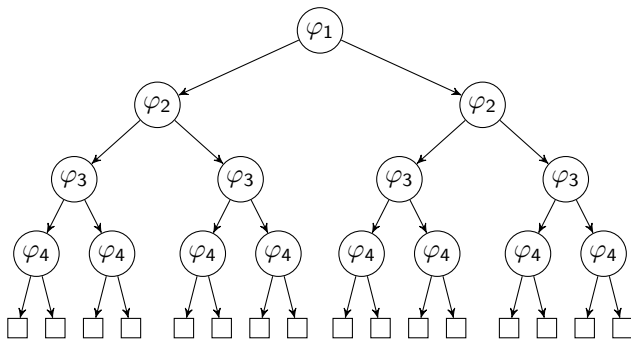
(a) Full



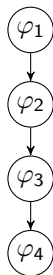
(b) Compact

Figure : Compact P-trees

# Compact Representation of P-trees



(a) Full



(b) Compact

Figure : Compact P-trees



# Compact Representation of P-trees

A *compact P-tree* over  $CD(\mathcal{I})$  is a binary tree where

- 1 every node is labeled with a Boolean formula over  $\mathcal{I}$ , and
- 2 every non-leaf node  $t$  labeled with  $\varphi$  has either two outgoing edges (Fig. (a)), or one outgoing edge pointing straight-down (Fig. (b)), left (Fig. (c)), or right (Fig. (d)).

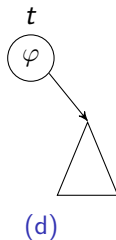
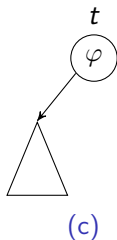
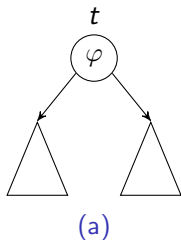
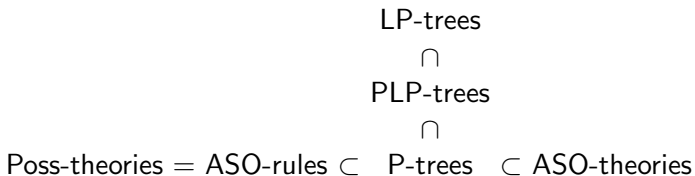


Figure : Compact P-trees

# Relative Expressivity of Preference Languages



# Computational Complexity Results

Dominance-testing (DOMTEST):  $o_1 \succ_T o_2$ ?

Optimality-testing (OPTTEST):  $o$  optimal w.r.t  $T$ ?

Optimality-with-property (OPTPROP): is there optimal  $o$  with property  $\alpha$ ?

- ① DOMTEST  $\in P$
- ② OPTTEST  $\in coNP$ -complete:
  - The complement problem is reduced from the SAT problem.
- ③ OPTPROP  $\in \Delta_2^P$ -complete:
  - The problem is reduced from the Maximum Satisfying Assignment (MSA) problem.

- ① Modeling qualitative preferences:
  - Preference trees (P-trees)
  - Partial lexicographic preference trees (PLP-trees)
- Learning PLP-trees and PLP-forests
- Aggregating LP-trees
- Future research directions

# The Cars Domain

- ① **BodyType(B)**: {mvan, sedan, sport, suv}.
- ② **Capacity(C)**: {2, 5, 7m}.
- ③ **Color(O)**: {black, blue, gray, red, white}.
- ④ **LuggageSize(L)**: {big, med, small}.
- ⑤ **Make(M)**: {bmw, ford, honda, vw}.
- ⑥ **Price(P)**: {low, med, high, vhigh}.
- ⑦ **Safety(S)**: {low, med, high}.

# Partial Lexicographic Preference Trees (PLP-Trees)

A *PLP-tree* over  $CD(\mathcal{I})$  is a tree, where

- 1 every non-leaf node  $t$  is labeled with an attribute  $Attr(t)$  in  $\mathcal{I}$ ,
- 2 every non-leaf node  $t$  has  $|Dom(Attr(t))|$  outgoing edges labeled with a value of  $Attr(t)$ , and
- 3 every attribute appears *at most* once on every branch.

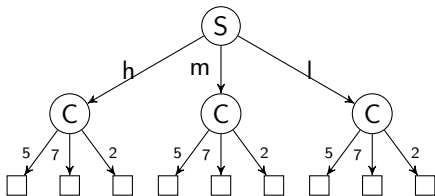


Figure : A PLP-tree over cars

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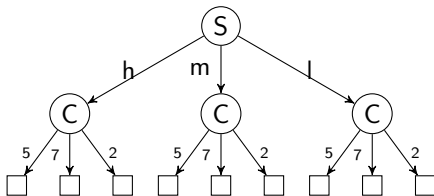


Figure : A PLP-tree over cars

Total preorder

# Partial Lexicographic Preference Trees (PLP-Trees)

Car1



<mvan, 7m, gray, big, honda, med, med>

Car2



<sedan, 5, blue, med, vw, med, med>

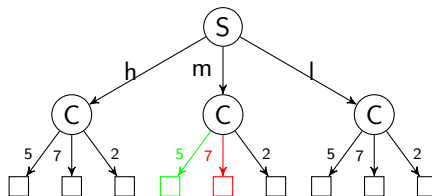
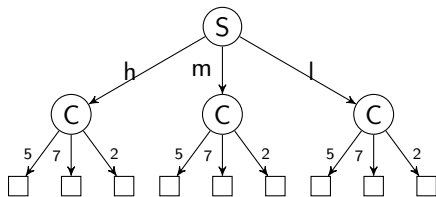


Figure : A PLP-tree over cars

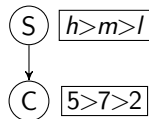
Car2  $\succ$  Car1



# Compact Representations of PLP-Tree



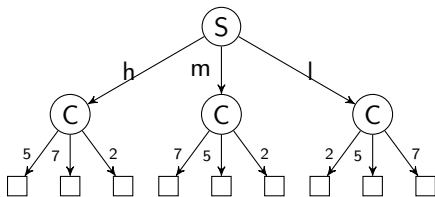
(a) Full



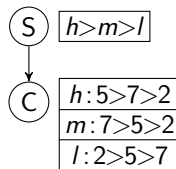
(b) Compact

Figure : Unconditional Importance & Unconditional Preference (UIUP)

# Compact Representations of PLP-Tree



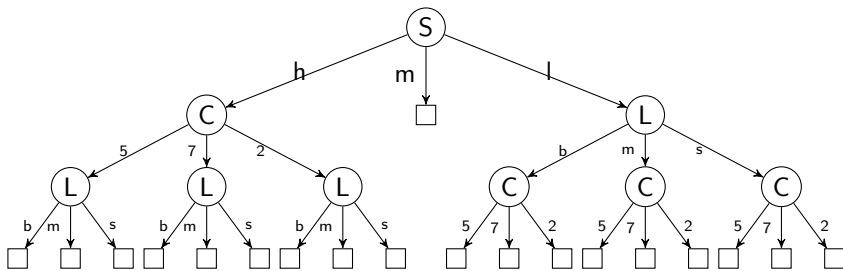
(a) Full



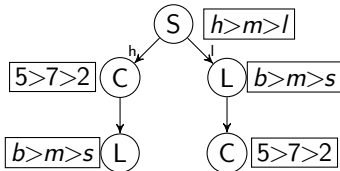
(b) Compact

Figure : Unconditional Importance & Conditional Preference (UICP)

# Compact Representations of PLP-Tree



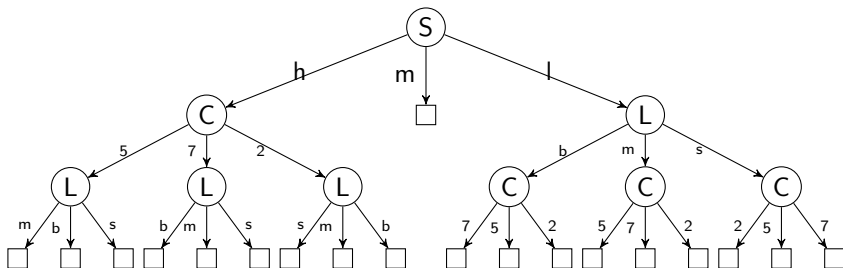
(a) Full



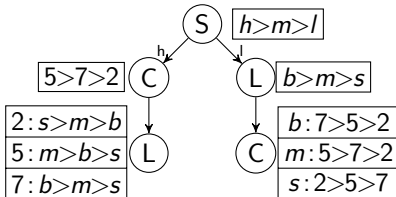
(b) Compact

Figure : Conditional Importance & Unconditional Preference (CIUP)

# Compact Representations of PLP-Tree



(a) Full



(b) Compact

Figure : Conditional Importance & Conditional Preference (CICP)

# Lexicographic Preference Trees (LP-Trees)

- ① An *LP-tree*  $\mathcal{L}$  over  $CD(\mathcal{I})$  is a PLP-tree, where
- each attribute appears **exactly once** on every path from the root to a leaf.
  - Unlike PLP-trees, an LP-tree induces a total order.

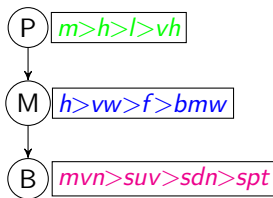
- Modeling qualitative preferences:
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  - Partial lexicographic preference trees (PLP-trees)
- ② Learning PLP-trees and PLP-forests
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- Future research directions

# Learning PLP-trees

## Consistent Learning (CONSLearn)

Given an example set  $\mathcal{E}$ , decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$ .

( $\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle$ )  
( $\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle$ )  
( $\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle$ )



UIUP tree

# Learning PLP-trees

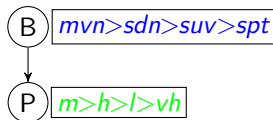
## Small Learning (SMALLLEARN)

Given an example set  $\mathcal{E}$  and a positive integer  $l$  ( $l \leq |\mathcal{E}|$ ), decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$  and  $|T| \leq l$ .

( $\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle$ )

( $\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle$ )

( $\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle$ )



UIUP tree



# Learning PLP-trees

## Maximal Learning (MAXLEARN)

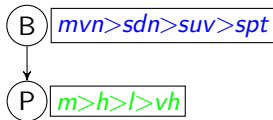
Given an example set  $\mathcal{E}$  and a positive integer  $k$  ( $k \leq m$ ), decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  satisfies at least  $k$  examples in  $\mathcal{E}$ .

( $\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle$ )

( $\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle$ )

( $\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle$ )

( $\langle \text{suv}, 7m, \text{gry}, b, \text{vw}, \text{vh}, m \rangle, \langle \text{suv}, 7m, \text{gry}, b, \text{vw}, h, m \rangle$ )



UIUP tree

# Learning PLP-trees

## Consistent Learning (CONSLearn)

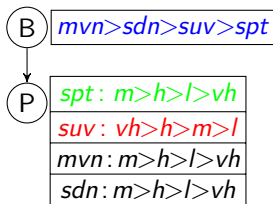
Given an example set  $\mathcal{E}$ , decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$ .

( $\langle \text{sdn}, 5, \text{blk}, \text{m}, \text{h}, \text{m}, \text{m} \rangle, \langle \text{suv}, 7\text{m}, \text{wht}, \text{b}, \text{f}, \text{m}, \text{m} \rangle$ )

( $\langle \text{spt}, 2, \text{wht}, \text{s}, \text{bmw}, \text{h}, \text{h} \rangle, \langle \text{spt}, 2, \text{wht}, \text{s}, \text{bmw}, \text{vh}, \text{h} \rangle$ )

( $\langle \text{mvn}, 7\text{m}, \text{gry}, \text{b}, \text{f}, \text{m}, \text{m} \rangle, \langle \text{sdn}, 5, \text{bl}, \text{m}, \text{f}, \text{m}, \text{m} \rangle$ )

( $\langle \text{suv}, 7\text{m}, \text{gry}, \text{b}, \text{vw}, \text{vh}, \text{m} \rangle, \langle \text{suv}, 7\text{m}, \text{gry}, \text{b}, \text{vw}, \text{h}, \text{m} \rangle$ )



UICP tree

# Computational Complexity

- ①  $P$ ,  $NP$ ,  $coNP$ : We typically believe that  $P \subset NP$  and  $P \subset coNP$ .
- ②  $\Delta_2^P$ :  $P^{NP}$ ,  $\Sigma_2^P$ :  $NP^{NP}$ , and  $\Pi_2^P$ :  $coNP^{NP}$ .
- ③  $C$ -complete: hardest decision problems in class  $C$ .

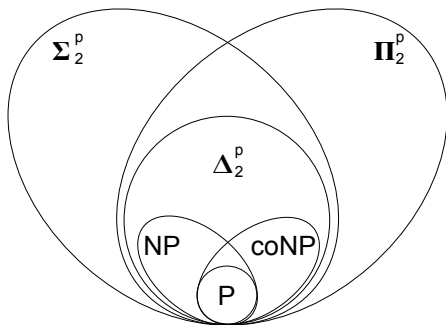


Figure : Computational complexity diagram

# Complexity Results on PLP-trees

	UP	CP
UI	P	P
CI	NPC <sup>15</sup>	P

(a) CONSLearn

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) SMALLLearn

	UP	CP
UI	NPC <sup>16</sup>	NPC
CI	NPC	NPC

(c) MAXLearn

Figure : Complexity results for learning PLP-trees

<sup>15</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

<sup>16</sup>Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006.

# Experimentation

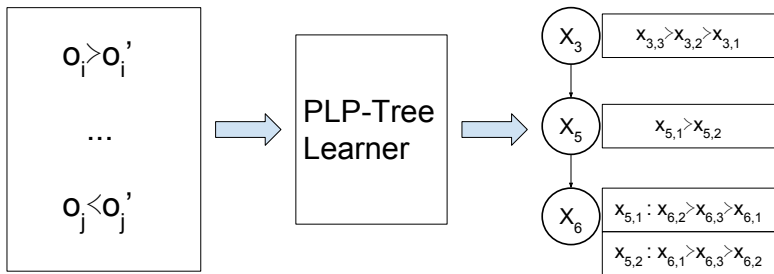


Figure : PLP-tree learning system

# Datasets

Dataset	$p$	$ \mathcal{X} $	$ \mathcal{E}^> $	$ \mathcal{E}^{\approx} $
BreastCancerWisconsin	9	270	9,009	27,306
CarEvaluation	6	1,728	682,721	809,407
CreditApproval	10	520	66,079	68,861
GermanCredit	10	914	172,368	244,873
Ionosphere	10	118	3,472	3,431
MammographicMass	5	62	792	1,099
Mushroom	10	184	8,448	8,388
Nursery	8	1,266	548,064	252,681
SPECTHeart	10	115	3,196	3,359
TicTacToe	9	958	207,832	250,571
Vehicle	10	455	76,713	26,572
Wine	10	177	10,322	5,254

Figure : Preference Learning Library<sup>17</sup>

<sup>17</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>

# PLP-Trees To Learn

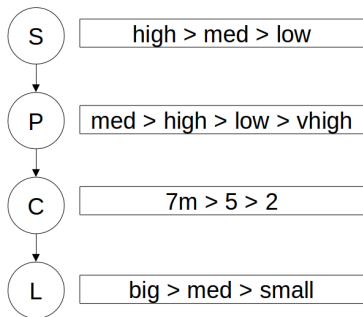


Figure : Unconditional Importance & Unconditional Preference (UIUP)

# PLP-Trees To Learn

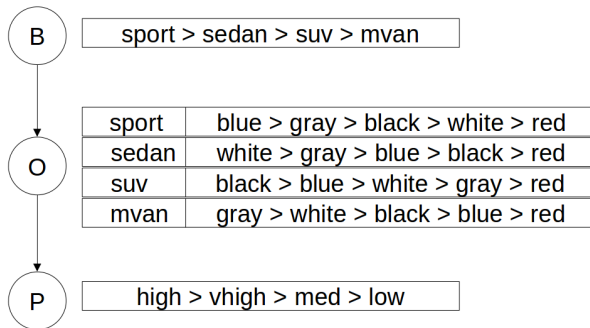


Figure : UICP with at most 1 parent (UICP-1)



# PLP-Trees To Learn

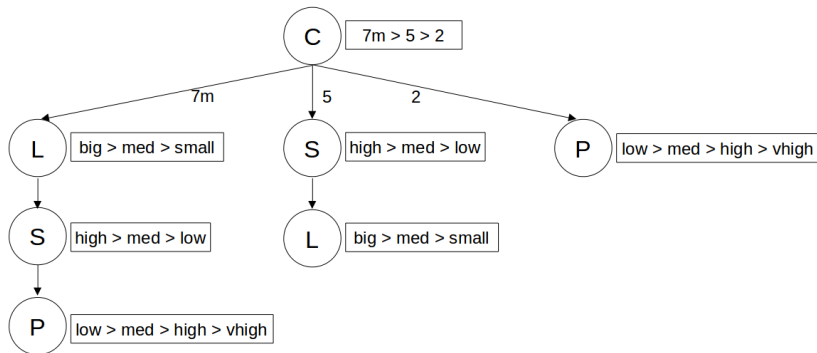


Figure : CIUP with 1 split at the root (CIUP-1)

# PLP-Trees To Learn

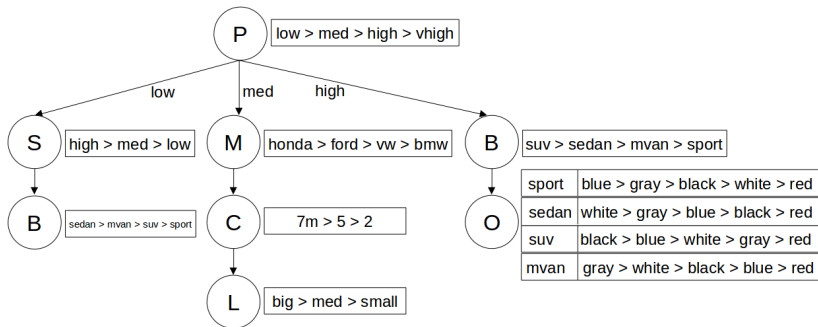
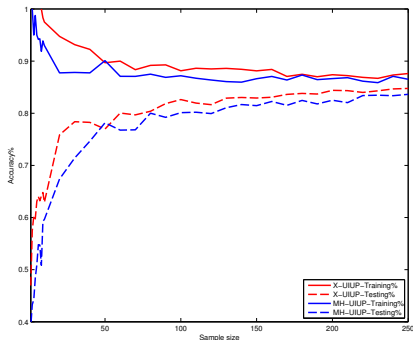


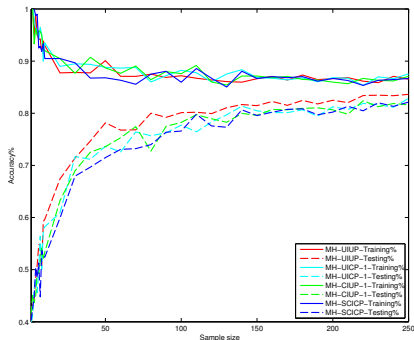
Figure : Simple CICP (SCICP)

# Experimental Results: CarEvaluation<sup>18</sup>

#attributes:6, #objects:1728, #examples:682721



(a) Compare exact & greedy heuristic



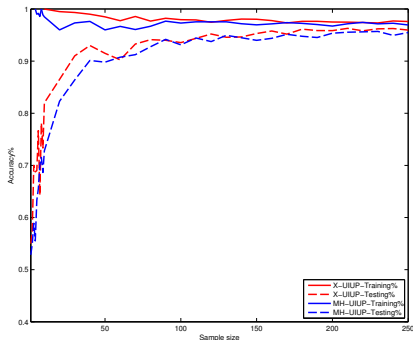
(b) Greedy heuristic

Figure : Learning curves solving MAXLEARN

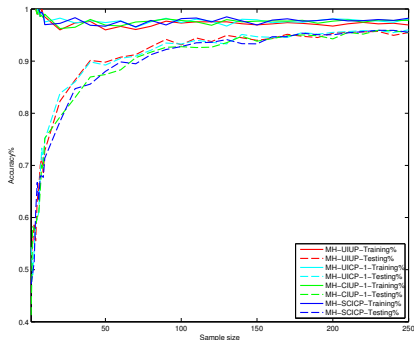
<sup>18</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>

# Experimental Results: Wine<sup>19</sup>

#attributes:10, #objects:177, #examples:10322



(a) Compare exact & greedy heuristic



(b) Greedy heuristic

Figure : Learning curves solving MAXLEARN

<sup>19</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>

- Modeling qualitative preferences:
  - Preference trees (P-trees)
  - Partial lexicographic preference trees (PLP-trees)
- Learning PLP-trees and PLP-forests
- ③ Aggregating LP-trees
- Future research directions

# Positional Scoring Rules

- $k$ -approval:  $(1, \dots, 1, 0, \dots, 0)$  with  $k$  being the number of 1's.
- $(k, l)$ -approval:  $(c, \dots, c, d, \dots, d, 0, \dots, 0)$ , where  $c$  and  $d$  are constants ( $c > d$ ), and the numbers of  $c$ 's and  $d$ 's equal to  $k$  and  $l$ .
- $b$ -Borda:  $(b, b-1, \dots, b-m+1)$ , where  $b$  is a constant and  $m$  is the number of candidates.

# The Evaluation and Winner Problems

## The Evaluation Problem

Let  $r$  be a positional scoring rule with a scoring vector  $w$ ,  $\mathcal{C}$  a class of LP-trees. Given a  $\mathcal{C}$ -profile  $P$  of  $n$  LP-trees over  $p$  attributes and a positive integer  $R$ , the *evaluation* problem is to decide whether there exists an alternative  $o \in \mathcal{X}$  such that  $s_w(o, P) \geq R$ .

## The Winner Problem

Let  $r$  be a positional scoring rule with a scoring vector  $w$ ,  $\mathcal{C}$  a class of LP-trees. Given a  $\mathcal{C}$ -profile  $P$  of  $n$  LP-trees over  $p$  attributes, the *winner* problem is to compute an alternative  $o \in \mathcal{X}$  with the maximum score  $s_w(o, P)$ .

# Complexity of the Evaluation Problem: $k$ -Approval

	UP	CP
UI	P	P
CI	P	P

(a)  $k = 2^{p-1} \pm f(p)$ ,  $f(p)$  is a poly

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $k = 2^{p-c}$ ,  $c > 1$  is a const

Figure :  $k$ -Approval



# Complexity of the Evaluation Problem: $(k, l)$ -Approval

	UP	CP
UI	P	P
CI	P	P

(a)  $k = l = 2^{p-1}$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $k = l = 2^{p-c}$ ,  $c > 1$  is a const

Figure :  $(k, l)$ -Approval

# Complexity of the Evaluation Problem: $b$ -Borda

	UP	CP
UI	P	NPC
CI	NPC	NPC

(a)  $b = 2^p - 1$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $b = 2^{p-c} - 1$ ,  $c \geq 1$  is a const

Figure :  $b$ -Borda

# Modeling the Problems in ASP

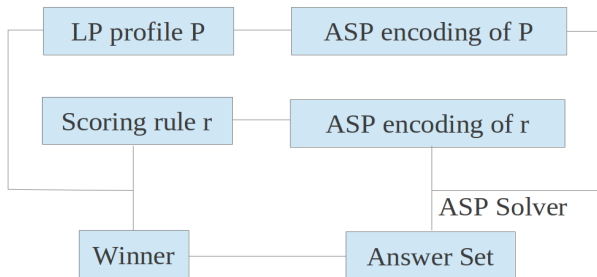


Figure : The winner problem

- Solvers: *clingo*<sup>20</sup>, *clingcon*<sup>21</sup>

<sup>20</sup>M. Gebser et al. "Potassco: The Potsdam Answer Set Solving Collection". In: AI Communications (2011)

<sup>21</sup>Max Ostrowski and Torsten Schaub. "ASP modulo CSP: The clingcon system". In: TPLP (2012)

# Modeling the Problems in W-MAXSAT

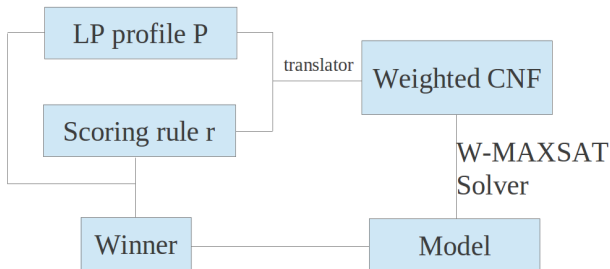


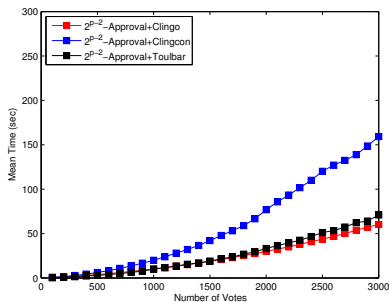
Figure : The winner problem

- Solver: *toulbar*<sup>22</sup>

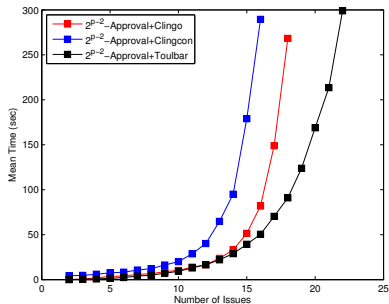
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<sup>22</sup>M Sanchez et al. "Max-CSP competition 2008: toulbar2 solver description". In: the Third International CSP Solver Competition (2008)

# Varying $p$ and $n$ : $2^{p-2}$ -approval



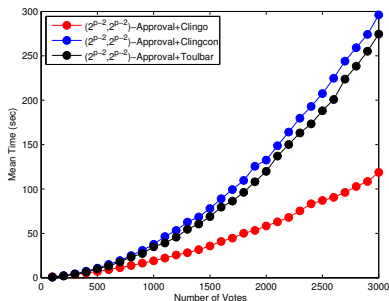
(a) Fixed #attributes (10)



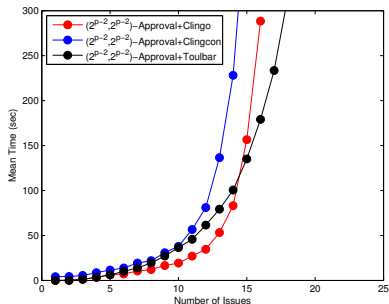
(b) Fixed #votes (1000)

Figure : Solving the winner problem

# Varying $p$ and $n$ : $(2^{p-2}, 2^{p-2})$ -approval<sup>23</sup>



(a) Fixed #attributes (10)

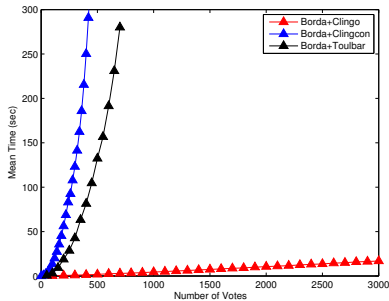


(b) Fixed #votes (1000)

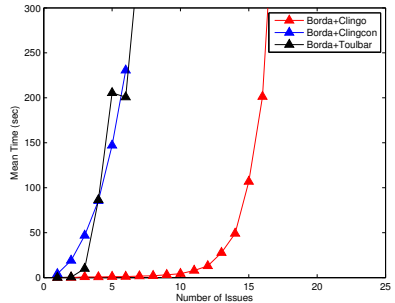
Figure : Solving the winner problem

<sup>23</sup> scoring vector:  $(2, \dots, 2, 1, \dots, 1, 0, \dots, 0)$  with the numbers of 2's and 1's equal to  $2^{p-2}$

# Varying $p$ and $n$ : Borda



(a) Fixed #attributes (10)



(b) Fixed #votes (1000)

Figure : Solving the winner problem

- Modeling qualitative preferences:
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- Learning PLP-trees and PLP-forests
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- Future research directions



## Data-Driven Preference Learning:

### ① Recommender Systems<sup>24</sup>:

- Collaborative
- Content-based
- Hybrid

### ② Machine Learning (fitting function):

- Supervised learning (e.g., decision trees, random forests)
- Label ranking<sup>25</sup>

### ③ Model-based Learning (learning interpretable decision models):

- Preference Elicitation (Human-in-the-Loop)
- Conditional Preference Networks, Preference Trees
- Stochastic Models (e.g., Choquet integral<sup>26</sup>, TOPSIS-like models<sup>27</sup>)

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<sup>24</sup>Gediminas Adomavicius and Alexander Tuzhilin. "Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions". In: Knowledge and Data Engineering, IEEE Transactions on (2005)

<sup>25</sup>Eyke Hüllermeier et al. "Label ranking by learning pairwise preferences". In: Artificial Intelligence (2008)

<sup>26</sup>Ali Fallah Tehrani, Weiwei Cheng, and Eyke Hüllermeier. "Choquistic Regression: Generalizing Logistic Regression using the Choquet Integral." In: EUSFLAT. 2011

<sup>27</sup>Manish Agarwal, Ali Fallah Tehrani, and Eyke Hüllermeier. "Preference-based Learning of Ideal Solutions in TOPSIS-like Decision Models". In: Journal of Multi-Criteria Decision Analysis (2014)

## Preference Reasoning and Applications:

- ① Social Choice and Welfare<sup>28,29</sup>:
  - Voting
  - Fair division
  - Strategyproof Social Choice
- ② Automated Planning and Scheduling<sup>30,31,32</sup>:
  - Travel scheduling
  - Manufacturing
  - Traffic control

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<sup>28</sup>Kenneth J Arrow, Amartya Sen, and Kotaro Suzumura. Handbook of Social Choice and Welfare. Vol. 1 & 2. 2010

<sup>29</sup>Felix Brandt, Vincent Conitzer, and Ulle Endriss. "Computational social choice". In: Multiagent systems (2012)

<sup>30</sup>Tran Cao Son and Enrico Pontelli. "Planning with preferences using logic programming". In: Theory and Practice of Logic Programming (2006)

<sup>31</sup>Meghyn Bienvenu, Christian Fritz, and Sheila A McIlraith. "Specifying and computing preferred plans". In: Artificial Intelligence (2011)

<sup>32</sup>Hannah Bast et al. "Route planning in transportation networks". In: arXiv preprint (2015)

- ① Xudong Liu. “Modeling, Learning and Reasoning with Qualitative Preferences”. Algorithmic Decision Theory, 2015.
- ② Xudong Liu and Mirosław Truszczyński. “Reasoning with Preference Trees over Combinatorial Domains”. Algorithmic Decision Theory, 2015.
- ③ Xudong Liu and Mirosław Truszczyński. “Learning Partial Lexicographic Preference Trees over Combinatorial Domains”. AAAI Conference on Artificial Intelligence, 2015.
- ④ Xudong Liu and Mirosław Truszczyński. “Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences”. Multidisciplinary Workshop on Advances in Preference Handling, 2014.

- ⑤ Matthew Spradling, Judy Goldsmith, Xudong Liu, Chandrima Dadi, and Zhiyu Li. “Roles and Teams Hedonic Game”. Algorithmic Decision Theory, 2013.
- ⑥ Xudong Liu and Mirosław Trzuszczynski. “Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers”. Algorithmic Decision Theory, 2013.
- ⑦ Xudong Liu. “Aggregating Lexicographic Preference Trees Using Answer Set Programming: Extended Abstract”. International Joint Conference on Artificial Intelligence Doctoral Consortium, 2013.
- ⑧ Xudong Liu and Mirosław Trzuszczynski. “Learning Partial Lexicographic Preference Trees and Forests over Multi-Valued Attributes”. (In Review by ECAI-16 Program Committee).

## ① Quantitative:

- Utility/Cost Functions<sup>33</sup>
- Possibilistic Logic<sup>34</sup>
- Fuzzy Preference Relations<sup>35</sup>
- Penalty Logic<sup>36</sup>

## ② Qualitative:

- Answer-Set Optimization Theories<sup>37</sup>
- Ceteris Paribus Networks (e.g., CP-nets<sup>38</sup>, TCP-nets<sup>39</sup>, CI-nets<sup>40</sup>)
- Conditional Preference Theories<sup>41</sup>

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<sup>33</sup>Souhila Kaci. Working with Preferences: Less Is More: Less Is More. Springer Science & Business Media, 2011

<sup>34</sup>Didier Dubois, Jérôme Lang, and Henri Prade. "A Brief Overview of Possibilistic Logic". In: ECSQARU. 1991

<sup>35</sup>SA Orlovsky. "Decision-making with a fuzzy preference relation". In: Fuzzy sets and systems (1978)

<sup>36</sup>Gadi Pinkas. Propositional non-monotonic reasoning and inconsistency in symmetric neural networks. 1991

<sup>37</sup>Gerhard Brewka, Ilkka Niemelä, and Mirosław Truszczyński. "Answer Set Optimization". In: IJCAI. 2003

<sup>38</sup>C. Boutilier et al. "CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements". In: Journal of Artificial Intelligence Research (2004)

<sup>39</sup>Ronen I. Brafman and Carmel Domshlak. "Introducing Variable Importance Tradeoffs into CP-Nets". In: UAI. 2002

<sup>40</sup>Sylvain Bouveret, Ulle Endriss, and Jérôme Lang. "Conditional importance networks: A graphical language for representing ordinal, monotonic preferences over sets of goods". In: (2009)

<sup>41</sup>Nic Wilson. "Extending CP-Nets with Stronger Conditional Preference Statements". In: AAAI-04. 2004

# Questions?

Thank you!