

# Modeling, Learning and Reasoning with Qualitative Preferences

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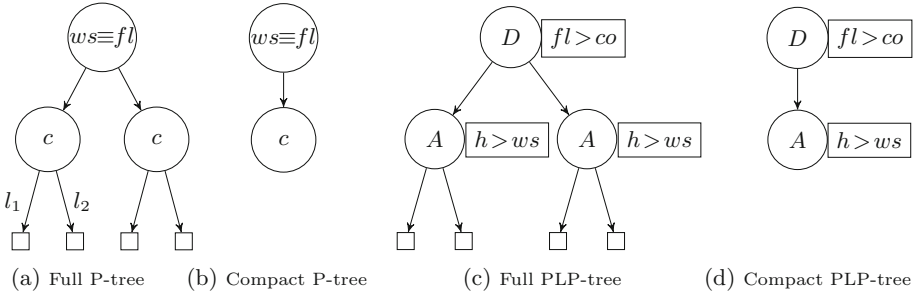
## 1 Research Overview

My research is focused on knowledge representation and reasoning, especially, preference modeling, learning and reasoning, and computational social choice. Preference modeling, learning and reasoning is a major research area in artificial intelligence (AI) and decision theory, and is closely related to the social choice theory considered by economists and political scientists. In my research I explore emerging connections between preferences in AI and social choice theory. My main focus is on qualitative preference representation languages extending and combining formalisms such as lexicographic preference trees (LP-trees) [1], answer-set optimization theories (ASO-theories) [3], possibilistic logic [4]; and conditional preference networks (CP-nets) [2], on learning problems that aim at discovering qualitative preference models and predictive preference information from empirical data; and on qualitative preference reasoning problems centered around preference optimization and strategy-proofness of preference aggregation methods. Applications of my research include recommendation systems, decision support tools, multi-agent systems, and Internet trading and marketing platforms.

## 2 Preliminaries

My research focuses on problems involving *qualitative preferences*, that is, simple and intuitive qualitative statements about *preferred properties* of alternatives. These alternatives are described in terms of *attributes* or *issues*, each assuming values from some finite domain. For instance, vacations can be described in terms of issues such as *activity* ( $A$ ), *destination* ( $D$ ), *time* ( $T$ ), and *transportation* ( $R$ ), where *activity* has values *water-sports* ( $ws$ ) and *hiking* ( $h$ ), *destination* has values *Florida* ( $fl$ ) and *Colorado* ( $co$ ), *time* has values *summer* ( $s$ ) and *winter* ( $w$ ), and *transportation* has values *car* ( $c$ ) and *plane* ( $p$ ). Thus, a sequence of attribute values, for example,  $\langle ws, fl, s, c \rangle$  describes a specific *summer* vacation involving *water-sports* in *Florida* to which we travel by *car*. Spaces of alternatives of this type are referred to as *combinatorial domains*.

The exponential size of the combinatorial domain leads to the infeasibility of correctly putting precise numbers on the utility of specific choices; thus, we turn



**Fig. 1.** P-trees and PLP-trees on vacations

to languages specifying preferences qualitatively. The sheer number of alternatives in the combinatorial domain also makes it impossible to enumerate from the most preferred to the least. Consequently, my research focuses on designing *concise* formalisms in which qualitative preferences over such domains could be expressed compactly and intuitively, and solving problems in preference learning and reasoning in the context of these formalisms.

One of such preference systems, *preference trees* (P-trees), was introduced by Fraser [5,6], and further discussed in my work [9]. Let us illustrate the formalism with preferences over the vacation domain. The most important property for our agent involves activity and destination. She prefers vacations with water sports in Florida or hiking in Colorado over the other options. This preference is described as an equivalence formula  $ws \equiv fl$ . Within each of the two groups of vacations (satisfying the formula and not satisfying the formula), driving ( $c$ ) is the preferred transportation mode. These preference statements are described in Fig. 1a. Clearly, the P-tree partitions the vacations into four clusters, denoted by the leaves, with the leftmost representing the set of most preferred vacations satisfying the formulas  $ws \equiv fl$  and  $c$ . Thus, the alternative  $\langle h, co, s, c \rangle$  is better than vacation  $\langle ws, fl, s, p \rangle$ , because the former descends to leaf  $l_1$  and the latter  $l_2$ , and  $l_1$  precedes  $l_2$  in the order of leaves. Since the subtrees of the root are the same and leaves can be omitted in Fig. 1a, we can collapse the full tree to its *compact* version in Fig. 1b. Compactness of preference models is crucial in studying problems in preference learning and reasoning.

I introduced the preference formalism of *partial lexicographic preference trees*, or PLP-trees [10], where nodes in the tree are labeled not by a formula but by an attribute and a total ordering of the values of the attribute. To illustrate PLP-trees, let us consider again preferences over vacations. As shown in Fig. 1c, our agent puts *destination* as the most important attribute on which she prefers *Florida*. Similarly as before, she next considers *activity* and prefers *hiking* for both *Florida* and *Colorado* vacations. Like the above P-tree, this full PLP-tree induces a total preorder of four clusters of equivalent alternatives as the box-labeled leaves, and it is collapsed to a much more compact one in Fig. 1d, where preferences on all attributes are *unconditional*. I have shown that

PLP-trees are special cases of P-trees but more general than the restrictive LP-trees. Studying learning and reasoning problems for PLP-trees will contribute to the most general setting of P-trees.

### 3 Research Experience

Working on my research I collaborated with professors and colleagues in our department. My work on preferences has led to publications on preference modeling [9], learning [10] and reasoning [7, 8, 12].

#### 3.1 Preference Modeling

In joint work with my Ph.D. advisor Dr. Mirosław Truszczyński, we focused on the language of P-trees, studied the relationship between P-trees and other existing preference languages, and showed that P-trees extend possibilistic logic, LP-trees and ASO-rules [9]. Moreover, we established computational complexity results of commonly considered decision problems in the setting of P-trees, such as *dominance testing* (asking if an alternative is preferred to another given the preferences), *optimality testing* (deciding if an alternative is optimal given the preferences), and *optimality testing w.r.t a property* (determining if there exists an optimal alternative satisfying a given property).

#### 3.2 Preference Learning

Another joint work with my Ph.D. advisor introduced the formalism of PLP-trees, a novel formalism for lexicographic preference models, also a subclass of P-trees, over combinatorial domains of alternatives [10]. For PLP-trees we investigated the problem of *passive learning*, that is, the problem of learning preference models given a set of pairwise preferences between alternatives, called *training examples*, provided by the user upfront. Specifically, for several classes of PLP-trees, we studied how to learn (i) a PLP-tree, preferably of a small size, consistent with a dataset of examples, and (ii) a PLP-tree correctly ordering as many of the examples as possible in case of inconsistency. We established complexity results of these problems and, in each case where the problem is in the class P, proposed a polynomial time algorithm.

#### 3.3 Preference Reasoning

In the work with Dr. Truszczyński [8, 11], we investigated two preference-aggregation problems, the *winner* problem, computing the winning alternative in an election, and the *evaluation* problem, computing an alternative scoring at least above some threshold in an election, based on *positional scoring rules* (such as *k*-approval and Borda) when preferences are represented as LP-trees. We obtained new computational complexity results of these two problems and provided computational methods to model and solve the problems in two programming formalisms, answer set programming (ASP) and weighted partial maximum

satisfiability (WPM). To support experimentation, we designed methods to generate LP-tree votes randomly and presented experimental results with ASP and WPM solvers. In a joint work [12] with Dr. Judy Goldsmith and other fellow graduate students, we introduced a new variant of hedonic coalition formation games in which agents have two levels of preference on their own coalitions: preference on the set of “roles” that make up the coalition, and preference on their own role within the coalition. We defined and studied several stability notions and optimization problems for this model.

## 4 Current and Future Research

Moving forward I plan to continue working on problems on preferences in AI and social choice theory, particularly when the preferences concern alternatives ranging over combinatorial domains.

### 4.1 Preference Learning and Approximation

I will generalize my results on learning PLP-trees to the case of P-trees. I will also design and implement algorithms to learn, from both synthetic and real-world datasets, preferences described in formalisms of LP-trees, PLP-trees and P-trees for both passive learning and active learning, where, unlike passive learning, training examples are elicited from the user interactively. To support evaluation of these learning algorithms, I will design and implement algorithms to randomly generate instances of LP-trees, PLP-trees and P-trees. To facilitate the preference learning process, I will develop datasets of examples from existing learning datasets, and apply machine learning methods to obtain preferences from these developed datasets.

Some models of preference orders do not support effective reasoning. For instance, if a preference order is represented by a CP-net, the *dominance testing* problem is known to be NP-hard even for the simple case where the dependency among the nodes is acyclic, and it is PSPACE-complete in general. Learning can provide a way to circumvent the difficulty. Compared to the formalism of CP-nets, P-trees are more practical, more intuitive and more transparent for representing preferences over combinatorial domains. Since reasoning with P-trees is easier (e.g., dominance is straightforward), approximating (or exactly representing) CP-nets using P-trees learned from examples consistent with the CP-net might open a way to more effective approximate, or even exact, reasoning with CP-nets. I plan to design algorithms to find a *small* set of P-trees that can best approximate the given CP-net.

### 4.2 Preference Aggregation

Provided that we have obtained preferences from the agents as P-trees, I will apply two approaches to aggregate P-trees to compute the collective decision: the Pareto method and voting rules. Using the Pareto method is similar to a previous

work on ASO. As for the voting rules that could be applied, I will investigate positional scoring rules (e.g., Plurality,  $k$ -Approval and Borda), comparison-based rules (e.g., the Copeland, Simpsons and Maximin rules), and distance-based rules (e.g., the Kemeny and Dodgson rules). To compare the two approaches of aggregating P-trees, I will perform experiments on both randomly generated and learned P-trees using the two methods separately, and analyze the winning alternatives computed by both of them.

### 4.3 Misrepresentation of Preferences

I will study problems relating to vulnerability of collective decisions under misrepresentation of preferences specified over combinatorial domains. Take the *coalitional manipulation problem* as an example. This problem asks to decide if the small coalition set of manipulative voters can make some candidate a winner. I have already obtained preliminary complexity results for LP-trees when the voting rules are Plurality and half-Approval where each voter approves her top half candidates. I will examine other positional scoring rules, as well as some comparison-based and distance-based voting systems, for LP-trees, and extend these results to elections over complicated domains when votes are specified as P-trees.

## 5 Conclusion

My research concerns problems in the fields pertaining to preferences and social choice, and exploits emerging connections between preferences in AI and social choice theory. My main focus is on qualitative preference representation languages extending and combining existing formalisms such as lexicographic preference trees (LP-trees) and answer-set optimization theories (ASO-theories), on learning problems that aim at discovering predictive preference models from empirical data, and on qualitative preference reasoning problems centered around preference optimization and strategy-proofness of preference aggregation methods.

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