

# Learning Partial Lexicographic Preference Trees over Combinatorial Domains

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Preferences over options are ubiquitous in real life.

- 1 People have preferences over dinners offered in a restaurant.
- 2 In political elections voters cast ballots over candidates.
- 3 Customers rate their purchased items on websites such as Bestbuy.com and Amazon.com.



$\succ$



$\succ$



$\succ \dots$

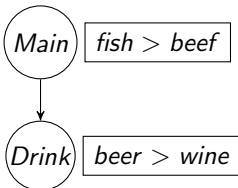
Figure: What do I like for dinner?



- 1 Preferences are **qualitative**.
- 2 Domains are **combinatorial**.
- 3 Can we model such preferences **compactly**?
- 4 If so, can we learn a compact model **efficiently**?

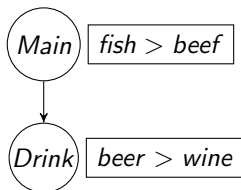
- ① What is the problem?
  - Qualitative preference modeling and learning over combinatorial domains.
- ② Why is it important?
  - Predict preferences between unseen options.
  - Support preference reasoning tasks, such as dominance testing, optimization and aggregation.

- 1 Propose *partial lexicographic preference trees* (PLP-trees):
  - lexicographic, compact, extending LP-trees<sup>1</sup>



<sup>1</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

- 1 Propose *partial lexicographic preference trees* (PLP-trees):
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UI-UP PLP-tree



<sup>1</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

- ① Propose *partial lexicographic preference trees* (PLP-trees):
  - lexicographic, compact, extending LP-trees
- ② Show results on *passive learning problems*, where all training examples ( $o_i \succ o'_i$  or  $o_k \approx o'_k$ ) are provided upfront, in the setting of PLP-trees:
  - CONSLearn, SMALLLearn, MAXLearn

- ① Let  $\mathcal{I} = \{X_1, \dots, X_p\}$  be a set of binary issues, and  $CD(\mathcal{I})$  be the combinatorial domain  $\prod_{1 \leq i \leq p} \mathcal{D}_i$ , where  $\mathcal{D}_i = \{0_i, 1_i\}$ . Given an example set  $\mathcal{E} = \{e_1, \dots, e_m\}$ , where  $e_i$  is of form either  $\alpha_i \succ \beta_i$  or  $\alpha_i \approx \beta_i$  for  $\alpha_i, \beta_i \in CD(\mathcal{I})$ , the CONSLearn problem is to learn a PLP-tree  $T$  (of a particular type) consistent with  $\mathcal{E}$ .
- ② Denote by  $\mathcal{E}^\succ = \{e_i \in \mathcal{E} : \alpha_i \succ \beta_i\}$  the set of *strict examples*, and  $\mathcal{E}^\approx = \{e_i \in \mathcal{E} : \alpha_i \approx \beta_i\}$  the set of *equivalent examples*.
- ③ Denote by  $(X_i, x_i)$  a node label in a UI-UP PLP-tree, where  $X_i$  is an issue from  $\mathcal{I}$  and  $x_i$  is the preferred value, either  $0_i$  or  $1_i$ , in  $\mathcal{D}_i$ .



- 4  $NEQ(\mathcal{E}, \mathcal{I})$ : set of issues in  $\mathcal{I}$  that incorrectly handle at least one equivalent example in  $\mathcal{E}$ , i.e.,

$$NEQ(\mathcal{E}, \mathcal{I}) = \{X_i \in \mathcal{I} : \exists \alpha \approx \beta \in \mathcal{E}, \alpha(X_i) \neq \beta(X_i)\}.$$

- 5  $EQ(\mathcal{E}, \mathcal{I})$ : set of issues in  $\mathcal{I}$  that do not order any of the strict examples in  $\mathcal{E}$ , i.e.,

$$EQ(\mathcal{E}, \mathcal{I}) = \{X_i \in \mathcal{I} : \forall \alpha \succ \beta \in \mathcal{E}, \alpha(X_i) = \beta(X_i)\}.$$

- 6  $AVI(\mathcal{E}, S)$ : set of issues in  $S$  ( $S \subseteq \mathcal{I} \setminus (NEQ(\mathcal{E}, \mathcal{I}) \cup EQ(\mathcal{E}, \mathcal{I}))$ ) available for selection as the next important issue, i.e.,

$$AVI(\mathcal{E}, S) = \{X_i \in S : \forall \alpha \succ \beta \in \mathcal{E}, \alpha(X_i) \geq \beta(X_i) \vee \\ \forall \alpha \succ \beta \in \mathcal{E}, \alpha(X_i) \leq \beta(X_i)\}.$$

1.  $1_1 1_2 1_3 \mathbf{0}_4 0_5 \approx 1_1 1_2 1_3 \mathbf{1}_4 0_5$
2.  $0_1 0_2 1_3 \mathbf{1}_4 1_5 \approx 0_1 0_2 1_3 \mathbf{0}_4 1_5$
3.  $1_1 0_2 0_3 0_4 \mathbf{1}_5 \succ 0_1 1_2 0_3 1_4 \mathbf{1}_5$
4.  $1_1 1_2 0_3 1_4 \mathbf{0}_5 \succ 1_1 1_2 1_3 0_4 \mathbf{0}_5$
5.  $0_1 1_2 0_3 0_4 \mathbf{1}_5 \succ 0_1 0_2 1_3 0_4 \mathbf{1}_5$
6.  $0_1 1_2 1_3 1_4 \mathbf{0}_5 \succ 0_1 0_2 1_3 1_4 \mathbf{0}_5$

$$NEQ = \{\mathbf{X}_4\}, EQ = \{\mathbf{X}_5\},$$

$$S = \{X_1, X_2, X_3\}$$

**Input:** A set  $\mathcal{E}$  of examples over  $\mathcal{I}$   
**Output:** A sequence  $T$  of pairs  $(X_\ell, x_\ell)$ , or FAILURE if a UI-UP tree does not exist

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     $X_\ell \leftarrow$  an element from  $AVI(\mathcal{E}, S)$ ;
    Construct  $(X_\ell, x_\ell)$ ;
     $T \leftarrow T, (X_\ell, x_\ell)$ ;
     $\mathcal{E} \leftarrow \mathcal{E} \setminus \{e \in \mathcal{E}^\succ : e \text{ is decided on } X_\ell\}$ ;
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2.  ~~$0_1 0_2 1_3 1_4 1_5 \approx 0_1 0_2 1_3 0_4 1_5$~~
3.  $1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$
4.  $1_1 1_2 0_3 1_4 0_5 \succ 1_1 1_2 1_3 0_4 0_5$
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1st:  $S = \{X_1, X_2, X_3\}$ ,  
 $AVI = \{X_1, X_3\}$ ,  $X_\ell = X_3$

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**end**  
**return**  $T$ ;

1.  $\cancel{1_1 1_2 1_3 0_4 0_5} \approx \cancel{1_1 1_2 1_3 1_4 0_5}$
2.  $\cancel{0_1 0_2 1_3 1_4 1_5} \approx \cancel{0_1 0_2 1_3 0_4 1_5}$
3.  $1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$
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5.  $0_1 1_2 0_3 0_4 1_5 \succ 0_1 0_2 1_3 0_4 1_5$
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3.  $1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$
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1st:  $S = \{X_1, X_2, X_3\}$ ,  
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$(X_3, 0_3)$

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3.  $1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$
4.  ~~$1_1 1_2 0_3 1_4 0_5 \succ 1_1 1_2 1_3 0_4 0_5$~~
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```

$S = \{X_1, X_2\}$

1.  ~~$1_1 1_2 1_3 0_4 0_5 \approx 1_1 1_2 1_3 1_4 0_5$~~
2.  ~~$0_1 0_2 1_3 1_4 1_5 \approx 0_1 0_2 1_3 0_4 1_5$~~
3.  $1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$
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5.  ~~$0_1 1_2 0_3 0_4 1_5 \succ 0_1 0_2 1_3 0_4 1_5$~~
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2nd:  $S = \{X_1, X_2\}$ ,  
 $AVI = \{X_1\}$ ,  $X_\ell = X_1$

$(X_3, 0_3)$



$(X_1, 1_1)$

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6.  $0_1 1_2 1_3 1_4 0_5 \succ 0_1 0_2 1_3 1_4 0_5$

2nd:  $S = \{X_1, X_2\}$ ,  
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$(X_3, 0_3)$



$(X_1, 1_1)$

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**end**  
**return**  $T$ ;

$S = \{X_2\}$



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2.  $0_1 0_2 1_3 1_4 1_5 \approx 0_1 0_2 1_3 0_4 1_5$
3.  ~~$1_1 0_2 0_3 0_4 1_5 \succ 0_1 1_2 0_3 1_4 1_5$~~
4.  ~~$1_1 1_2 0_3 1_4 0_5 \succ 1_1 1_2 1_3 0_4 0_5$~~
5.  ~~$0_1 1_2 0_3 0_4 1_5 \succ 0_1 0_2 1_3 0_4 1_5$~~
6.  $0_1 1_2 1_3 1_4 0_5 \succ 0_1 0_2 1_3 1_4 0_5$

3rd:  $S = \{X_2\}$ ,  
 $AVI = \{X_2\}$ ,  $X_\ell = X_2$

$(X_3, 0_3)$



$(X_1, 1_1)$



$(X_2, 1_2)$

**Input:** A set  $\mathcal{E}$  of examples over  $\mathcal{I}$   
**Output:** A sequence  $T$  of pairs  $(X_\ell, x_\ell)$ , or FAILURE if a UI-UP tree does not exist

$S = \mathcal{I} \setminus (NEQ(\mathcal{E}, \mathcal{I}) \cup EQ(\mathcal{E}, \mathcal{I}));$   
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**end**  
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3rd:  $S = \{X_2\}$ ,  
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$(X_3, 0_3)$



$(X_1, 1_1)$



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$S = \emptyset$

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6.  ~~$0_1 1_2 1_3 1_4 0_5 \succ 0_1 0_2 1_3 1_4 0_5$~~

$\mathcal{E}^\succ = \emptyset$ ,

Done!

$(X_3, 0_3)$



$(X_1, 1_1)$



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**end**  
**return**  $T$ ;

	FP	UP	CP
UI	P	P	<i>NP</i>
CI	P	NPC <sup>2</sup>	P

(a) CONSLearn

	FP	UP	CP
UI	NPC	NPC	NPC
CI	NPC	NPC	NPC

(b) SMALLLearn

	FP	UP	CP
UI	NPC <sup>3</sup>	NPC	NPC
CI	NPC	NPC	NPC

(c) MAXLearn

Figure: Complexity results for passive learning problems

<sup>2</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

<sup>3</sup>Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006. ↻ 🔍

- 1 Settle the complexity for the `CONSLearn` problem for UI-CP.
- 2 Implement algorithms handling issues of multi-valued domains.
- 3 Compare PLP-tree empirically<sup>4</sup> with other models.

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<sup>4</sup>Available datasets: UCI machine learning repository, [preference-learning.org](http://preference-learning.org), and [PrefLib.org](http://PrefLib.org)

Thank you!

- ① Graphical models: ceteris paribus<sup>5</sup> models (CP-nets<sup>6</sup>, TCP-nets<sup>7</sup>), lexicographic models (lexicographic strategies<sup>8</sup>, LP-trees<sup>9</sup>, CLP-trees<sup>10</sup>).
- ② Non-graphical models: ASO-theories<sup>11</sup>, Possibilistic logic<sup>12</sup>, CP-theories<sup>13</sup>.

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<sup>5</sup>Latin, it means “everything else being equal.”

<sup>6</sup>Boutilier et al., *CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements*, 2004.

<sup>7</sup>Brafman and Domshlak, *Introducing Variable Importance Tradeoffs into CP-nets*, 2002.

<sup>8</sup>Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006.

<sup>9</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

<sup>10</sup>Bräuning and Eyke, *Learning Conditional Lexicographic Preference Trees*, 2012

<sup>11</sup>Brewka, Niemelä and Truszczyński, *Answer Set Optimization*, 2003.

<sup>12</sup>Dubois, Lang and Prade, *A brief overview of possibilistic logic*, 1991.

<sup>13</sup>Wilson, *Extending CP-Nets with Stronger Conditional Preference Statements*, 2004. 