

# Preference Trees over Combinatorial Domains

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Sunday, 1/17/2016

# Preferences Are Ubiquitous

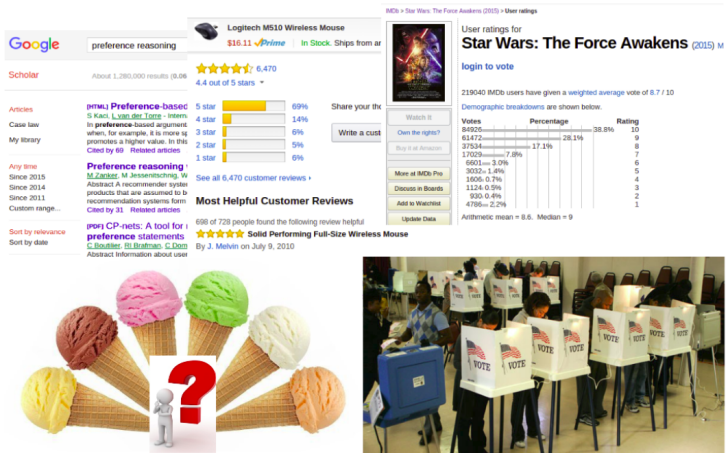


Figure : Preferences of different forms

# Describing Preferences

Car1



Car2



Figure : How to express preferences?

- ① On scale of 0 to 99, how will I rate these two cars?
  - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- ② What are the desired properties I see in cars?
  - I prefer minivans to sedans.
  - If minivan, I prefer gray to blue; if sedan, I prefer blue to gray; ...

# Describing Preferences

Car1



Car2



Figure : How to express preferences?

- ① On scale of 0 to 99, how will I rate these two cars? (**Quantitative**)
  - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- ② What are the desired properties I see in cars? (**Qualitative**)
  - I prefer minivans to sedans.
  - If minivan, I prefer gray to blue; if sedan, I prefer blue to gray; ...

## Binary Relations

Let  $O$  be a set of elements. A *binary relation*  $R$  over  $O$  is a collection of ordered pairs of elements in  $O$ ; that is,

$$R \subseteq O \times O.$$

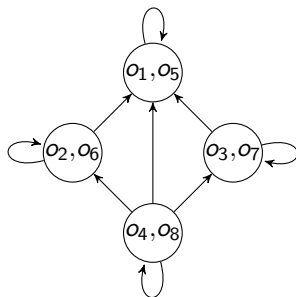
Properties of binary relations related to preferences:

- ➊ Reflexivity:  $\forall o \in O, (o, o) \in R$ .
- ➋ Irreflexivity:  $\forall o \in O, (o, o) \notin R$ .
- ➌ Totality:  $\forall o_1, o_2, (o_1, o_2) \in R$  or  $(o_2, o_1) \in R$ .
- ➍ Transitivity:  $\forall o_1, o_2, o_3$ , if  $(o_1, o_2) \in R$  and  $(o_2, o_3) \in R$ , then  $(o_1, o_3) \in R$ .
- ➎ Symmetry:  $\forall o_1, o_2$ , if  $(o_1, o_2) \in R$ , then  $(o_2, o_1) \in R$ .
- ➏ Antisymmetry:  $\forall o_1, o_2$ , if  $(o_1, o_2) \in R$  and  $(o_2, o_1) \in R$ , then  $o_1 = o_2$ .

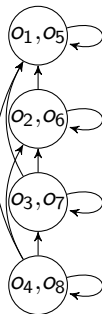
# Relations and Orderings

## Orderings

$\succeq$  is a *partial preorder* if it is reflexive and transitive, a *total preorder* if it is a partial preorder and total, a *partial order* if it is a partial preorder and antisymmetric, and a *total order* if it is a partial order and total.



(a) partial preorder

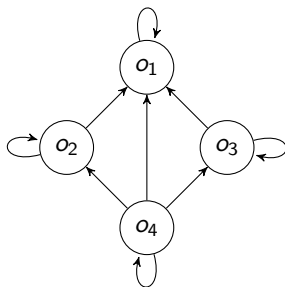


(b) total preorder

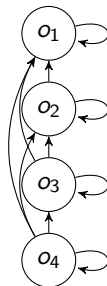
# Relations and Orderings

## Orderings

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(a) partial order

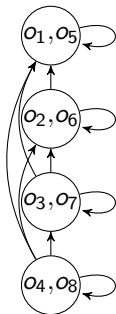


(b) total order

# Relations and Orderings

## Preference Relations

Let  $\succeq$  be a preference relation that is a total preorder over  $O$ . We say that  $o_1$  is *weakly preferred* to  $o_2$  if  $o_1 \succeq o_2$ , that  $o_1$  is *strictly preferred* ( $\succ$ ) to  $o_2$  if  $o_1 \succeq o_2$  and  $o_2 \not\succeq o_1$ , and that  $o_1$  is *indifferent* ( $\approx$ ) from  $o_2$  if  $o_1 \succeq o_2$  and  $o_2 \succeq o_1$ .



(a) total preorder

$$o_1 \succ o_5,$$

$$o_2 \succ o_4,$$

$$o_4 \approx o_8,$$

(b) preferences



## Combinatorial Domains

Let  $V$  be a finite set of variables  $\{X_1, \dots, X_p\}$ , associated with a set of finite domains  $\{Dom(X_1), \dots, Dom(X_p)\}$  for each variable  $X_i$ . A *combinatorial domain*  $CD(V)$  is a set of *outcomes* described by combinations of values from  $Dom(X_i)$ :

$$CD(V) = \prod_{X_i \in V} Dom(X_i).$$

# Combinatorial Domains: Example

Domain of cars over set  $V$  of  $p$  binary variables:

① **BodyType**: {mvan, sedan}.

② **Capacity**: {5, 7m}.

③ **Color**: {blue, gray}.

⋮

$$CD(V) = \underbrace{\{\langle \text{sedan}, 5, \text{blue}, \dots \rangle, \langle \text{mvan}, 7\text{m}, \text{gray}, \dots \rangle, \dots\}}_{2^p \text{ outcomes, too many!}}.$$

# Combinatorial Domains: Example

Domain of cars:

- ① **BodyType**: {mvan, sedan, sport, suv}.
- ② **Capacity**: {2, 5, 7m}.
- ③ **Color**: {black, blue, gray, red, white}.
- ④ **LuggageSize**: {big, med, small}.
- ⑤ **Make**: {bmw, ford, honda, vw}.
- ⑥ **Price**: {low, med, high, vhigh}.
- ⑦ **Safety**: {low, med, high}.

# Single Agent

Car1



<mvan, 7m, gray, big, honda, med, med>

Car2



<sedan, 5, blue, med, vw, med, med>



Figure : Dominance and Optimization



Figure : Social Choice and Welfare

# Research Problems of Interest

- ① Preference representation formalisms to compactly model qualitative preferences over combinatorial domains.
- ② Preference elicitation and learning methods to cast preferences of agents as a theory in a preference formalism.
- ③ Preference reasoning tasks:
  - Dominance and optimization
  - Manipulation: better off by misreporting preferences.

Q: How do we compactly represent qualitative preferences over combinatorial domains?

- ❶ Preference Trees (P-trees)<sup>1,13</sup>
- ❷ Partial Lexicographic Preference Trees (PLP-trees)<sup>8</sup>
- ❸ Lexicographic Preference Trees (LP-trees)<sup>4,12</sup>

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<sup>1</sup>Niall M Fraser. "Ordinal preference representations". In: Theory and Decision (1994)

<sup>2</sup>Xudong Liu and Mirosław Trzuszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

<sup>3</sup>Xudong Liu and Mirosław Trzuszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

<sup>4</sup>Richard Booth et al. "Learning conditionally lexicographic preference relations". In: ECAI. 2010

<sup>5</sup>Xudong Liu and Mirosław Trzuszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

# Preference Learning

Q: How do we learn predictive qualitative preference models over combinatorial domains?

- ① Partial Lexicographic Preference Trees (PLP-trees)<sup>6,7,8</sup>
  - Active and passive learning
  - Compute a (possibly small) PLP-tree consistent with all the data
  - Compute a PLP-tree that agrees with the data as much as possible
- ② Preference Forests<sup>9</sup>
- ③ Preference Approximation<sup>10</sup>

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<sup>6</sup>Michael Schmitt and Laura Martignon. "On the complexity of learning lexicographic strategies". In: The Journal of Machine Learning Research (2006)

<sup>7</sup>József Dombi, Csanád Imreh, and Nándor Vincze. "Learning lexicographic orders". In: European Journal of Operational Research (2007)

<sup>8</sup>Xudong Liu and Mirosław Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

<sup>9</sup>Xudong Liu and Mirosław Truszczynski. "Learning Preference Trees and Forests". In: Preparation

<sup>10</sup>Xudong Liu and Mirosław Truszczynski. "Approximating Conditional Preference Networks Using Lexicographic Preference Trees". In: Preparation



Q: How do we reason about preferences over combinatorial domains?

① Preference Optimization<sup>11,12,13,14</sup>:

- Dominance testing:  $o_1 \succeq_P o_2$ ?
- Optimality testing:  $o_1 \succeq_P o_2$  for all  $o_2 \neq o_1$ ?
- Optimality computing: what is the optimal outcome wrt  $P$ ?
- Preference aggregation: which candidate wins the election?

② Preference Misrepresentation<sup>15,16</sup>:

- Manipulation

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<sup>11</sup>Jérôme Lang, Jérôme Mengin, and Lirong Xia. "Aggregating Conditionally Lexicographic Preferences on Multi-issue Domains". In: CP. 2012

<sup>12</sup>Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

<sup>13</sup>Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

<sup>14</sup>Xudong Liu and Miroslaw Truszczynski. "Reasoning with Preference Trees over Combinatorial Domains". In: Proceedings of the 4th International Conference on Algorithmic Decision Theory (ADT). 2015

<sup>15</sup>Felix Brandt, Vincent Conitzer, and Ulle Endriss. "Computational social choice". In: Multiagent systems (2012)

<sup>16</sup>Xudong Liu and Miroslaw Truszczynski. "Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees". In: Preparation

Q: What fields can we apply preferences to?

- ① Role-playing Games:
  - Hedonic games<sup>17</sup>
- ② Automated Planning and Scheduling:
  - Trip planning<sup>18</sup>
- ③ Data-Driven Decision Making:
  - Predictive models<sup>19</sup>

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<sup>17</sup>Matthew Spradling et al. "Roles and Teams Hedonic Game". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

<sup>18</sup>Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

<sup>19</sup>Xudong Liu and Mirosław Trzuszczynski. "Learning Preference Trees and Forests". In: Preparation

- ① The languages of P-trees, PLP-trees, and LP-trees
- ② Learning preference models in case of PLP-trees
- ③ Reasoning with preferences:
  - Preference optimization in case of P-trees
  - Computing winners and “strong” outcomes when votes are LP-trees
  - Application in trip planning
- ④ Future research directions

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- Future research directions

# Preference Trees (P-Trees)

Let  $\varphi$ ,  $\psi$ , and  $\pi$  be propositional formulas over the set  $\mathcal{L}$  of literals that are values from  $\bigcup_{X_i \in V} \text{Dom}(X_i)$ .

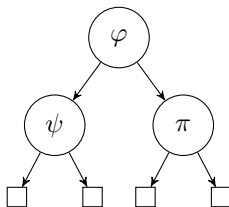


Figure : A P-tree

$$\varphi \wedge \psi \succ \varphi \wedge \neg\psi \succ \neg\varphi \wedge \pi \succ \neg\varphi \wedge \neg\pi.$$

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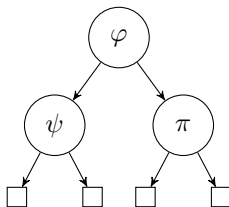


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Total preorder

# Example: The Cars Domain

- ① **BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.
- ② **Capacity**( $X_2$ ): {2, 5, 7m}.
- ③ **Color**( $X_3$ ): {black, blue, gray, red, white}.
- ④ **LuggageSize**( $X_4$ ): {big, med, small}.
- ⑤ **Make**( $X_5$ ): {bmw, ford, honda, vw}.
- ⑥ **Price**( $X_6$ ): {low, med, high, vhigh}.
- ⑦ **Safety**( $X_7$ ): {low, med, high}.

# Example: Preference Trees over Cars

**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

**Color**( $X_3$ ): {black, blue, gray, red, white}.

**Price**( $X_6$ ): {low, med, high, vhigh}.

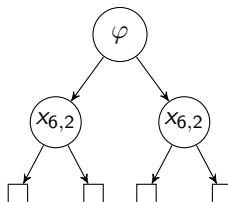


Figure : A P-tree over cars<sup>20</sup>

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<sup>20</sup>  $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$ .



# Example: Preference Trees over Cars

**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

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**Price**( $X_6$ ): {low, med, high, vhigh}.

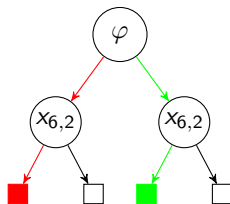


Figure : A P-tree over cars<sup>20</sup>

*Car2*  $\succ$  *Car1*

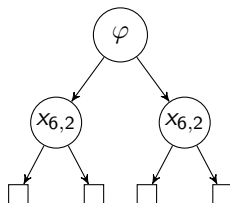
<sup>20</sup>  $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$ .

# Compact Representation of P-trees

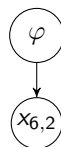
**BodyType**( $X_1$ ): {mvan( $x_{1,1}$ ), sedan( $x_{1,2}$ ), sport( $x_{1,3}$ ), suv( $x_{1,4}$ )}.

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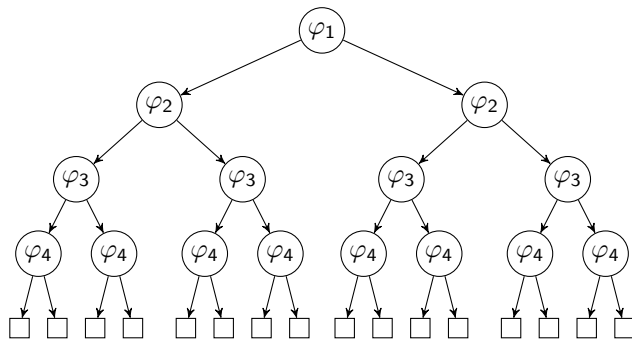
(a) Full



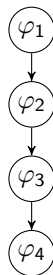
(b) Compact

Figure : Compact P-trees

# Compact Representation of P-trees



(a) Full



(b) Compact

Figure : Compact P-trees

# Compact Representation of P-trees

A *compact P-tree* over  $CD(\mathcal{I})$  is a binary tree where

- 1 every node is labeled with a Boolean formula over  $\mathcal{I}$ , and
- 2 every non-leaf node  $t$  labeled with  $\varphi$  has either two outgoing edges (Fig. (a)), or one outgoing edge pointing straight-down (Fig. (b)), left (Fig. (c)), or right (Fig. (d)).

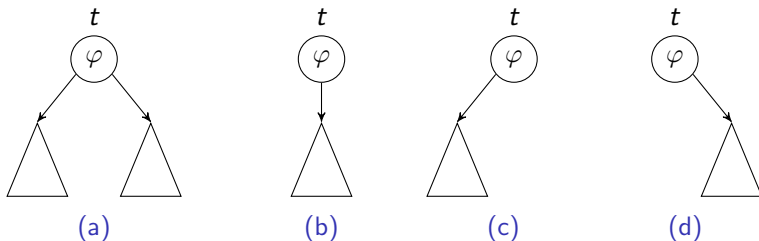
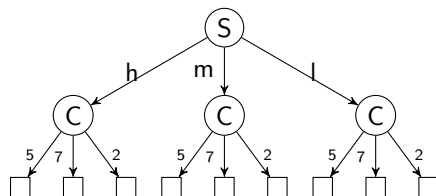
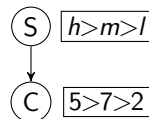


Figure : Compact P-trees

# Partial Lexicographic Preference Trees (PLP-Tree)



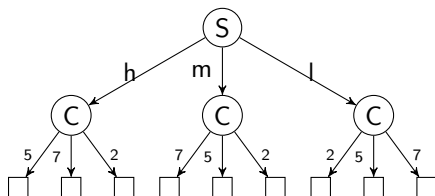
(a) Full



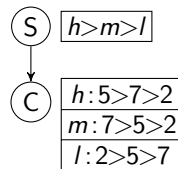
(b) Compact

Figure : Unconditional Importance & Unconditional Preference (UIUP)

# Partial Lexicographic Preference Trees (PLP-Tree)



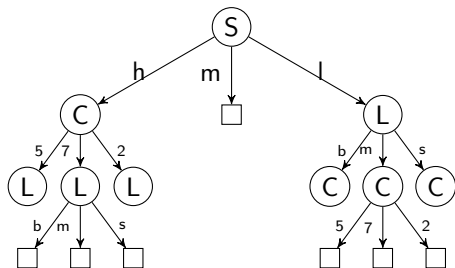
(a) Full



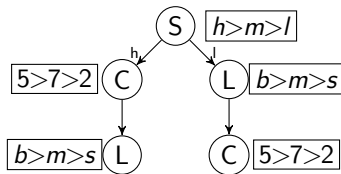
(b) Compact

Figure : Unconditional Importance & Conditional Preference (UICP)

# Partial Lexicographic Preference Trees (PLP-Tree)



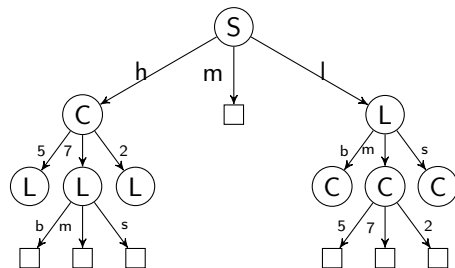
(a) Full



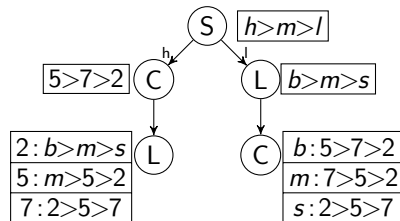
(b) Compact

Figure : Conditional Importance & Unconditional Preference (CIUP)

# Partial Lexicographic Preference Trees (PLP-Tree)



(a) Full



(b) Compact

Figure : Conditional Importance & Conditional Preference (CICP)



# Lexicographic Preference Trees (LP-Trees)

- ① An *LP-tree*  $\mathcal{L}$  over  $CD(\mathcal{I})$  is a PLP-tree, where
- each attribute appears **exactly once** on every path from the root to a leaf.
  - Unlike P-trees and PLP-trees, an LP-tree induces a total order.

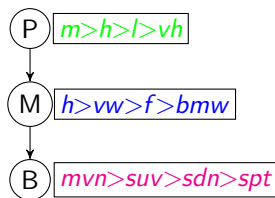
- The languages of P-trees, PLP-trees, and LP-trees
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- Future research directions

# Learning PLP-trees

## Consistent Learning (CONSLearn)

Given an example set  $\mathcal{E}$ , decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$ .

$(\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle)$   
 $(\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle)$   
 $(\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle)$



UIUP tree

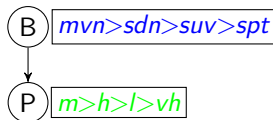
## Small Learning (SMALLLEARN)

Given an example set  $\mathcal{E}$  and a positive integer  $l$  ( $l \leq |\mathcal{E}|$ ), decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$  and  $|T| \leq l$ .

( $\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle$ )

( $\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle$ )

( $\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle$ )



UIUP tree

# Learning PLP-trees

## Maixmal Learning (MAXLEARN)

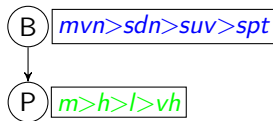
Given an example set  $\mathcal{E}$  and a positive integer  $k$  ( $k \leq m$ ), decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  satisfies at least  $k$  examples in  $\mathcal{E}$ .

( $\langle \text{sdn}, 5, \text{blk}, m, h, m, m \rangle, \langle \text{suv}, 7m, \text{wht}, b, f, m, m \rangle$ )

( $\langle \text{spt}, 2, \text{wht}, s, \text{bmw}, h, h \rangle, \langle \text{spt}, 2, \text{wht}, s, \text{bmw}, \text{vh}, h \rangle$ )

( $\langle \text{mvn}, 7m, \text{gry}, b, f, m, m \rangle, \langle \text{sdn}, 5, \text{bl}, m, f, m, m \rangle$ )

( $\langle \text{suv}, 7m, \text{gry}, b, \text{vw}, \text{vh}, m \rangle, \langle \text{suv}, 7m, \text{gry}, b, \text{vw}, h, m \rangle$ )

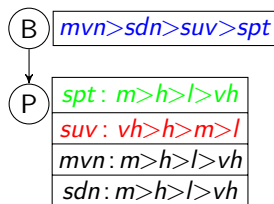


UIUP tree

## Consistent Learning (CONSLearn)

Given an example set  $\mathcal{E}$ , decide whether there exists a PLP-tree  $T$  (of a particular type) such that  $T$  is consistent with  $\mathcal{E}$ .

( $\langle \text{sdn}, 5, \text{blk}, \text{m}, \text{h}, \text{m}, \text{m} \rangle, \langle \text{suv}, 7\text{m}, \text{wht}, \text{b}, \text{f}, \text{m}, \text{m} \rangle$ )  
( $\langle \text{spt}, 2, \text{wht}, \text{s}, \text{bmw}, \text{h}, \text{h} \rangle, \langle \text{spt}, 2, \text{wht}, \text{s}, \text{bmw}, \text{vh}, \text{h} \rangle$ )  
( $\langle \text{mvn}, 7\text{m}, \text{gry}, \text{b}, \text{f}, \text{m}, \text{m} \rangle, \langle \text{sdn}, 5, \text{bl}, \text{m}, \text{f}, \text{m}, \text{m} \rangle$ )  
( $\langle \text{suv}, 7\text{m}, \text{gry}, \text{b}, \text{vw}, \text{vh}, \text{m} \rangle, \langle \text{suv}, 7\text{m}, \text{gry}, \text{b}, \text{vw}, \text{h}, \text{m} \rangle$ )



UICP tree

# Computational Complexity

- ①  $P$ ,  $NP$ ,  $coNP$ : We typically believe that  $P \subset NP$  and  $P \subset coNP$ .
- ②  $\Delta_2^P$ :  $P^{NP}$ ,  $\Sigma_2^P$ :  $NP^{NP}$ , and  $\Pi_2^P$ :  $coNP^{NP}$ .
- ③  $C$ -complete: hardest decision problems in class  $C$ .

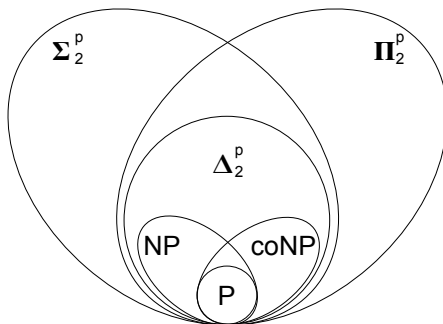


Figure : Computational complexity diagram

# Complexity Results on PLP-trees

	UP	CP
UI	P	P
CI	NPC <sup>21</sup>	P

(a) CONSLearn

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) SMALLLearn

	UP	CP
UI	NPC <sup>22</sup>	NPC
CI	NPC	NPC

(c) MAXLearn

Figure : Complexity results for learning PLP-trees

<sup>21</sup>Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

<sup>22</sup>Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006.



# Experimentation

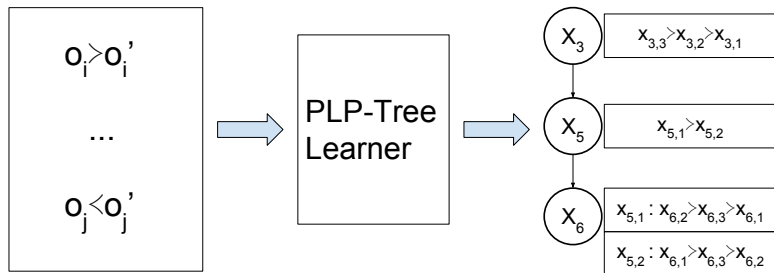


Figure : PLP-tree learning system

# Datasets

Dataset	#Attributes	#Outcomes	#Examples
BreastCancerWisconsin	9	270	9009
CarEvaluation	6	1728	682721
CreditApproval	10	520	66079
GermanCredit	10	914	172368
Ionosphere	10	118	3472
MammographicMass	5	62	792
Mushroom	10	184	8448
Nursery	8	1266	548064
SPECTHeart	10	115	3196
TicTacToe	9	958	207832
Vehicle	10	455	76713
Wine	10	177	10322

Figure : Preference Learning Library<sup>23</sup>

<sup>23</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>

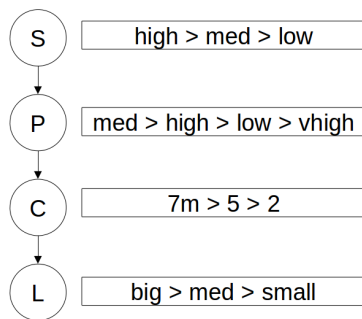


Figure : Unconditional Importance & Unconditional Preference (UIUP)

# PLP-Trees To Learn

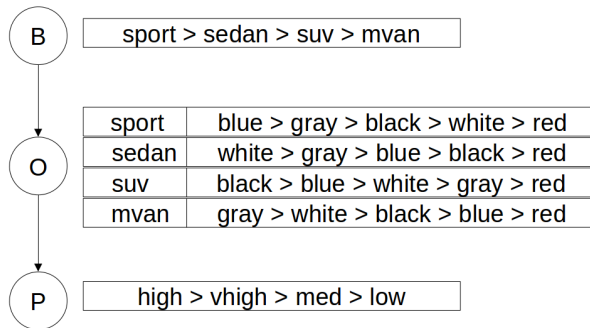


Figure : UICP with at most 1 parent (UICP-1)

# PLP-Trees To Learn

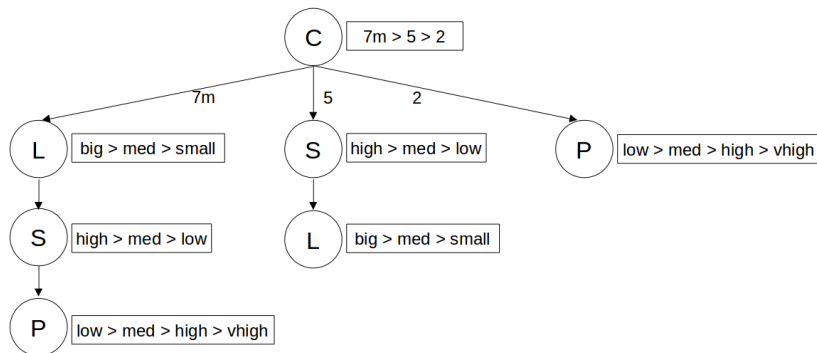


Figure : CIUP with 1 split at the root (CIUP-1)

# PLP-Trees To Learn

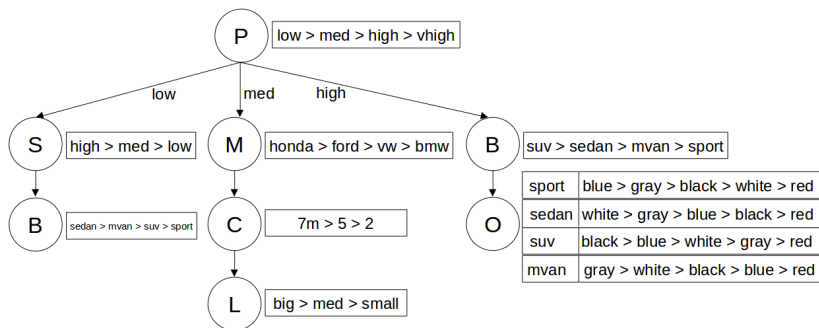
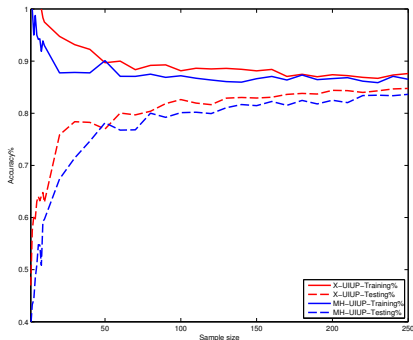


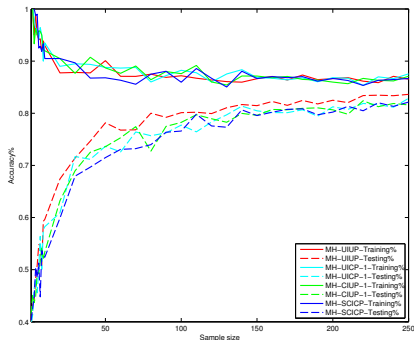
Figure : Simple CICP (SCICP)

# Experimental Results: CarEvaluation<sup>24</sup>

#attributes:6, #outcomes:1728, #examples:682721



(a) Compare exact & greedy heuristic



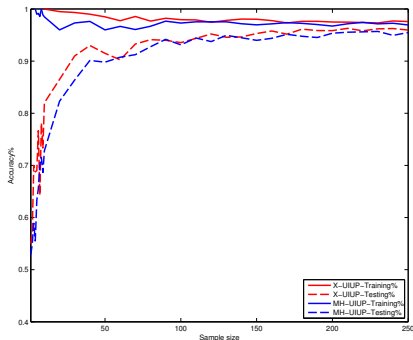
(b) Greedy heuristic

Figure : Learning curves solving MAXLEARN

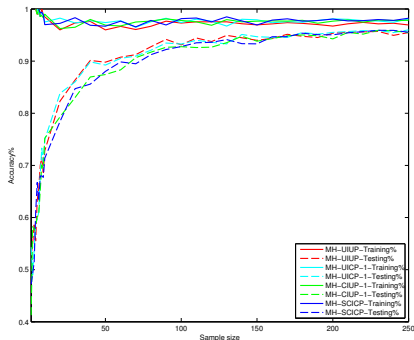
<sup>24</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>

# Experimental Results: Wine<sup>25</sup>

#attributes:10, #outcomes:177, #examples:10322



(a) Compare exact & greedy heuristic



(b) Greedy heuristic

Figure : Learning curves solving MAXLEARN

<sup>25</sup><http://www.cs.uky.edu/~liu/preflearnlib.php>



- The languages of P-trees, PLP-trees, and LP-trees
- Learning preference models in case of PLP-trees
- ③ Reasoning with preferences:
  - Preference optimization in case of P-trees
  - Computing winners and “strong” outcomes when votes are LP-trees
  - Application in trip planning
- Future research directions

# Computational Complexity Results for P-trees

Dominance-testing (DOMTEST):  $o_1 \succ_T o_2$ ?

Optimality-testing (OPTTEST):  $o$  optimal w.r.t  $T$ ?

Optimality-with-property (OPTPROP): is there optimal  $o$  with property  $\alpha$ ?

- ① DOMTEST  $\in P$
- ② OPTTEST  $\in coNP$ -complete:
  - The complement problem is reduced from the SAT problem.
- ③ OPTPROP  $\in \Delta_2^P$ -complete:
  - The problem is reduced from the Maximum Satisfying Assignment (MSA) problem.

- The languages of P-trees, PLP-trees, and LP-trees
- Learning preference models in case of PLP-trees
- ③ Reasoning with preferences:
  - Preference optimization in case of P-tree
  - Computing winners and “strong” outcomes when votes are LP-trees
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- Future research directions

# Positional Scoring Rules

- $k$ -approval:  $(1, \dots, 1, 0, \dots, 0)$  with  $k$  being the number of 1's and  $m - k$  the number of 0's where  $m = 2^p$ .
- $(k, l)$ -approval:  $(a, \dots, a, b, \dots, b, 0, \dots, 0)$ , where  $a$  and  $b$  are constants ( $a > b$ ) and the numbers of  $a$ 's and  $b$ 's equal to  $k$  and  $l$ , respectively.
- $b$ -Borda:  $(b, b - 1, \dots, 0)$ , where if  $b > m - 1$ ,  $b$ -Borda is reduced to the regular Borda rule with  $(m - 1, m - 2, \dots, 1, 0)$ .

# The Evaluation and Winner Problems

## The Evaluation Problem

Let  $r$  be a positional scoring rule with a scoring vector  $w$ ,  $\mathcal{C}$  a class of LP-trees. Given a  $\mathcal{C}$ -profile  $P$  of  $n$  LP-trees over  $p$  attributes and a positive integer  $R$ , the *evaluation* problem is to decide whether there exists an alternative  $o \in \mathcal{X}$  such that  $s_w(o, P) \geq R$ .

## The Winner Problem

Let  $r$  be a positional scoring rule with a scoring vector  $w$ ,  $\mathcal{C}$  a class of LP-trees. Given a  $\mathcal{C}$ -profile  $P$  of  $n$  LP-trees over  $p$  attributes, the *winner* problem is to compute an alternative  $o \in \mathcal{X}$  with the maximum score  $s_w(o, P)$ .

# Complexity of the Evaluation Problem: $k$ -Approval

	UP	CP
UI	P	P
CI	P	P

(a)  $k = 2^{p-1} \pm f(p)$ ,  $f(p)$  is a poly

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $k = 2^{p-c}$ ,  $c > 1$  is a const

Figure :  $k$ -Approval

# Complexity of the Evaluation Problem: $(k, l)$ -Approval

	UP	CP
UI	P	P
CI	P	P

(a)  $k = l = 2^{p-1}$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $k = l = 2^{p-c}$ ,  $c > 1$  is a const

Figure :  $(k, l)$ -Approval

# Complexity of the Evaluation Problem: $b$ -Borda

	UP	CP
UI	P	NPC
CI	NPC	NPC

(a)  $b = 2^p - 1$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)  $b = 2^{p-c} - 1$ ,  $c \geq 1$  is a const

Figure :  $b$ -Borda



# Modeling the Problems in ASP

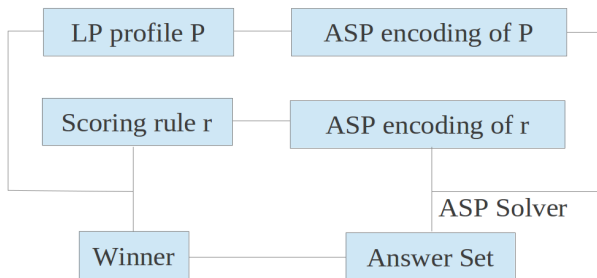


Figure : The winner problem

- Solvers: *clingo*<sup>26</sup>, *clingcon*<sup>27</sup>

<sup>26</sup>M. Gebser et al. "Potassco: The Potsdam Answer Set Solving Collection". In: AI Communications (2011)

<sup>27</sup>Max Ostrowski and Torsten Schaub. "ASP modulo CSP: The clingcon system". In: TPLP (2012)

# Modeling the Problems in W-MAXSAT

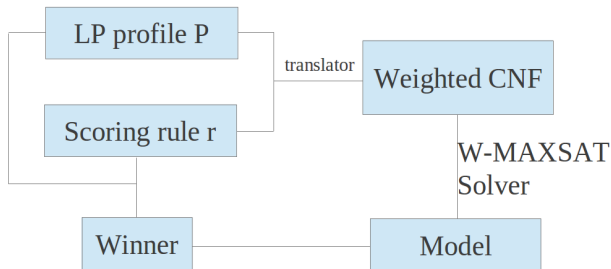


Figure : The winner problem

- Solver: *toulbar*<sup>28</sup>

<sup>28</sup>M Sanchez et al. "Max-CSP competition 2008: toulbar2 solver description". In: the Third International CSP Solver Competition (2008)

# Random LP Profiles

- To experiment with LP profiles, we developed methods to randomly generate *encodings* of a special type of CI-CP LP-tree of size linear in the number of attributes

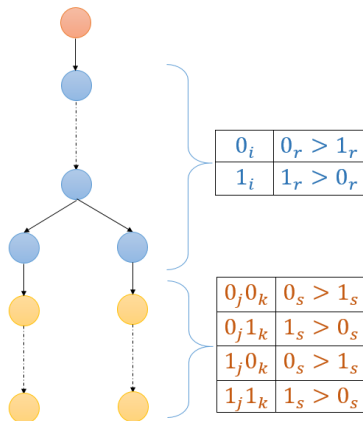
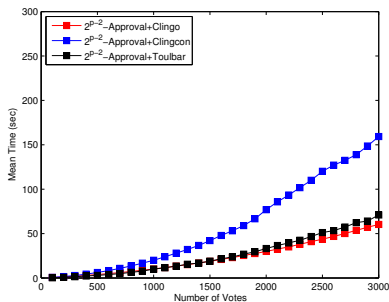
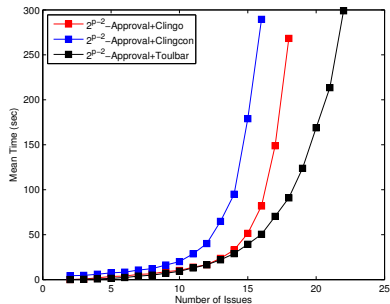


Figure : Random LP-tree

# Varying $p$ and $n$ : $2^{p-2}$ -approval



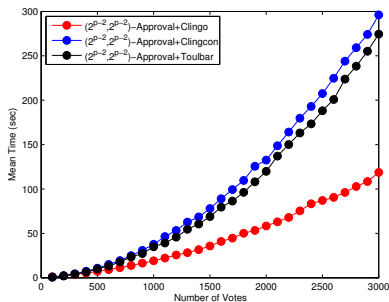
(a) Fixed #attributes (10)



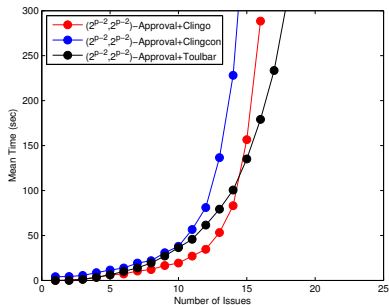
(b) Fixed #votes (1000)

Figure : Solving the winner problem

# Varying $p$ and $n$ : $(2^{p-2}, 2^{p-2})$ -approval<sup>29</sup>



(a) Fixed #attributes (10)

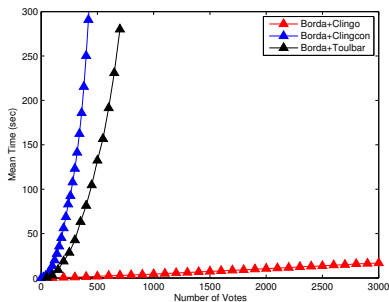


(b) Fixed #votes (1000)

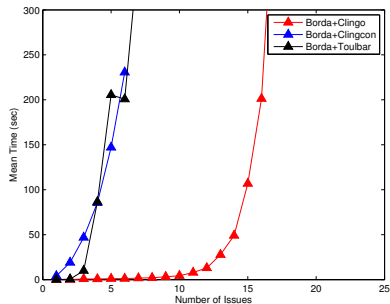
Figure : Solving the winner problem

<sup>29</sup> scoring vector:  $(2, \dots, 2, 1, \dots, 1, 0, \dots, 0)$  with the numbers of 2's and 1's equal to  $2^{p-2}$

# Varying $p$ and $n$ : Borda



(a) Fixed #attributes (10)



(b) Fixed #votes (1000)

Figure : Solving the winner problem

- The languages of P-trees, PLP-trees, and LP-trees
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- Future research directions

# Personalization in Trip Planning

- 1 Important to incorporate user constraints and preferences into trip planning systems.
- 2 Collaboration with experts (in AI, planning, optimization, multi-agent systems) at PARC.
- 3 Developed a hipergraph-based trip planner that accommodates constraints specified as *linear temporal logic* and preferences expressed as *preferential cost function* to compute optimal routes using  $A^*$ <sup>30</sup>.
- 4 Available later for trip planning in the Bay Area, LA, and Denver.

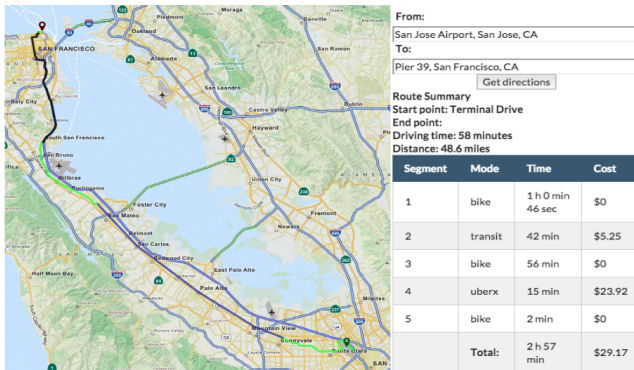
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<sup>30</sup>Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015



# Personalization in Trip Planning

- 1 From SJC, to Pier 39, Monday, 9am.
- 2 Constraints: never drive a car, and bike for 1 to 2 hours.
- 3 Preferences: bike = public (0.25) > wait(2) > walk(3), and 30\$/hr.



- The languages of P-trees, PLP-trees, and LP-trees
- Learning preference models in case of PLP-trees
- Reasoning with preferences:
  - Computing winners and “strong” outcomes when votes are LP-trees
  - Application in trip planning
- ④ Future research directions

# Data-Driven Preference Engineering

## 1 Recommender Systems<sup>31</sup>:

- 1 Collaborative
- 2 Content-based
- 3 Hybrid

## 2 Machine Learning:

- 1 Supervised learning (e.g., decision trees, random forests)
- 2 Label ranking<sup>32</sup>

## 3 Preference Elicitation (Human-in-the-Loop):

- 1 Context-based

## 4 Preference Learning:

- 1 Conditional Preference Networks, Preference Trees
- 2 Stochastic Models (e.g., Choquet integral<sup>33</sup>, TOPSIS-like models<sup>34</sup>)

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<sup>31</sup>Gediminas Adomavicius and Alexander Tuzhilin. "Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions". In: Knowledge and Data Engineering, IEEE Transactions on (2005)

<sup>32</sup>Eyke Hüllermeier et al. "Label ranking by learning pairwise preferences". In: Artificial Intelligence (2008)

<sup>33</sup>Agnes Leroy, Vincent Mousseau, and Marc Pirlot. "Learning the parameters of a multiple criteria sorting method". In: Algorithmic decision theory. 2011

<sup>34</sup>Manish Agarwal, Ali Fallah Tehrani, and Eyke Hüllermeier. "Preference-based Learning of Ideal Solutions in TOPSIS-like Decision Models". In: Journal of Multi-Criteria Decision Analysis (2014)

# Preference Reasoning and Applications

- ① Social Choice and Welfare<sup>35</sup>:
  - ① Voting
  - ② Fair division
  - ③ Strategyproof Social Choice
- ② Automated Planning and Scheduling:
  - ① Travel scheduling
  - ② Manufacturing
  - ③ Traffic control
- ③ Computer Vision and Image Processing:
  - ① Image retrieval
  - ② Image and video understanding

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<sup>35</sup>Kenneth J Arrow, Amartya Sen, and Kotaro Suzumura. Handbook of Social Choice and Welfare. Vol. 1 & 2. 2010

- ① Xudong Liu. “Modeling, Learning and Reasoning with Qualitative Preferences”. Algorithmic Decision Theory, 2015.
- ② Xudong Liu and Mirosław Truszczyński. “Reasoning with Preference Trees over Combinatorial Domains”. Algorithmic Decision Theory, 2015.
- ③ Xudong Liu and Mirosław Truszczyński. “Learning Partial Lexicographic Preference Trees over Combinatorial Domains”. AAAI Conference on Artificial Intelligence, 2015.
- ④ Xudong Liu and Mirosław Truszczyński. “Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences”. Multidisciplinary Workshop on Advances in Preference Handling, 2014.

- 5 Matthew Spradling, Judy Goldsmith, Xudong Liu, Chandrima Dadi, and Zhiyu Li. “Roles and Teams Hedonic Game”. Algorithmic Decision Theory, 2013.
- 6 Xudong Liu and Mirosław Truszczynski. “Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers”. Algorithmic Decision Theory, 2013.
- 7 Xudong Liu. “Aggregating Lexicographic Preference Trees Using Answer Set Programming: Extended Abstract”. International Joint Conference on Artificial Intelligence Doctoral Consortium, 2013.
- 8 Xudong Liu and Mirosław Truszczynski. “Learning Preference Trees and Forests”. (In Preparation).

- 9 Xudong Liu and Mirosław Truszczyński. “Approximating Conditional Preference Networks Using Lexicographic Preference Trees”. (In Preparation).
- 10 Xudong Liu and Mirosław Truszczyński. “Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees”. (In Preparation).
- 11 Xudong Liu and Mirosław Truszczyński. “Reasoning About Lexicographic Preferences Over Combinatorial Domains”. (In Preparation).
- 12 Xudong Liu and Christian Fritz. “On Personalizability and Extensibility of Multi-Modal Trip Planning”. (In Preparation).

## ① Quantitative:

- Utility/Cost Functions
- Possibilistic Logic<sup>36</sup>
- Fuzzy Preference Relations<sup>37</sup>
- Penalty Logic<sup>38</sup>

## ② Qualitative:

- Answer-Set Optimization Theories<sup>39</sup>
- Ceteris Paribus Networks (e.g., CP-nets<sup>40</sup>, TCP-nets<sup>41</sup>, CI-nets<sup>42</sup>)
- Conditional Preference Theories<sup>43</sup>

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<sup>36</sup>Didier Dubois, Jérôme Lang, and Henri Prade. "A Brief Overview of Possibilistic Logic". In: ECSQARU. 1991

<sup>37</sup>SA Orlovsky. "Decision-making with a fuzzy preference relation". In: Fuzzy sets and systems (1978)

<sup>38</sup>Gadi Pinkas. Propositional non-monotonic reasoning and inconsistency in symmetric neural networks. 1991

<sup>39</sup>Gerhard Brewka, Ilkka Niemelä, and Mirosław Truszczyński. "Answer Set Optimization". In: IJCAI. 2003

<sup>40</sup>C. Boutilier et al. "CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements". In: Journal of Artificial Intelligence Research (2004)

<sup>41</sup>Ronen I. Brafman and Carmel Domshlak. "Introducing Variable Importance Tradeoffs into CP-Nets". In: UAI. 2002

<sup>42</sup>Sylvain Bouveret, Ulle Endriss, and Jérôme Lang. "Conditional importance networks: A graphical language for representing ordinal, monotonic preferences over sets of goods". In: (2009)

<sup>43</sup>Nic Wilson. "Extending CP-Nets with Stronger Conditional Preference Statements". In: AAAI-04. 2004



# Questions?

Thank you!