Preference Trees over Combinatorial Domains

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Preferences Are Ubiquitous



Figure: Preferences of different forms

Describing Preferences



Figure: How to express preferences?

- On scale of 0 to 99, how will I rate these two cars?
 - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- Which one to me is better than the other?
 - I prefer Car1 to Car2. (Strict preference)
 - I like Car1 and Car2 equally. (Indifference/Equivalence)
 - I cannot decide. (Incomparability)

Describing Preferences



Figure: How to express preferences?

- On scale of 0 to 99, how will I rate these two cars? (Quantitative)
 - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- Which one to me is better than the other? (Qualitative)
 - I prefer Car1 to Car2. (Strict preference)
 - I like Car1 and Car2 equally. (Indifference/Equivalence)
 - I cannot decide. (Incomparability)

Binary Relations

Let O be a set of elements. A binary relation R over O is a collection of ordered pairs of elements in O; that is,

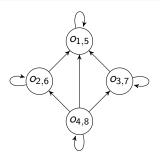
$$R \subseteq O \times O$$
.

Properties of binary relations related to preferences:

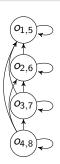
- **1** Reflexivity: $\forall o \in O, (o, o) \in R$.
- 2 Irreflexivity: $\forall o \in O$, $(o, o) \notin R$.
- **3** Totality: $\forall o_1, o_2, (o_1, o_2) \in R \text{ or } (o_2, o_1) \in R.$
- **1** Transitivity: $\forall o_1, o_2, o_3$, if $(o_1, o_2) \in R$ and $(o_2, o_3) \in R$, then $(o_1, o_3) \in R$.
- **3** Symmetry: $\forall o_1, o_2$, if $(o_1, o_2) \in R$, then $(o_2, o_1) \in R$.
- **1** Antisymmetry: $\forall o_1, o_2$, if $(o_1, o_2) \in R$ and $(o_2, o_1) \in R$, then $o_1 = o_2$.

Orderings

 \succeq is a partial preorder if it is reflexive and transitive, a total preorder if it is a partial preorder and total, a partial order if it is a partial preorder and antisymmetric, and a total order if it is a partial order and total.



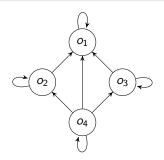
(a) partial preorder



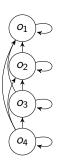
(b) total preorder

Orderings

≥ is a partial preorder if it is reflexive and transitive, a total preorder if it is a partial preorder and total, a partial order if it is a partial preorder and antisymmetric, and a total order if it is a partial order and total.



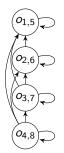
(a) partial order



(b) total order

Preference Relations

Let \succeq be a preference relation that is a total preorder over O. We say that o_1 is weakly preferred to o_2 if $o_1 \succeq o_2$, that o_1 is strictly preferred (\succ) to o_2 if $o_1 \succeq o_2$ and $o_2 \not\succeq o_1$, and that o_1 is indifferent (\approx) from o_2 if $o_1 \succeq o_2$ and $o_2 \succeq o_1$.



(a) total preorder

- $o_1 \succeq o_5$,
- $o_4 \succ o_2$,
- $o_4 \approx o_8$,
- (b) preferences

Combinatorial Domains

Combinatorial Domains

Let V be a finite set of variables $\{X_1, \ldots, X_p\}$, D a set of finite domains $\{Dom(X_1), \ldots, Dom(X_p)\}$ for each variable X_i . A combinatorial domain CD(V) is a set of outcomes described by combinations of values from $Dom(X_i)$:

$$CD(V) = \prod_{X_i \in V} Dom(X_i).$$

Combinatorial Domains: Example

Domain of cars over set V of p binary variables:

```
• BodyType: {mvan, sedan}.
```

i i

$$CD(V) = \{ \langle \text{sedan, 5, blue, } \ldots \rangle, \langle \text{mvan, 7m, gray, } \ldots \rangle, \ldots \}.$$

$$2^p \text{ outcomes, too many!}$$

Combinatorial Domains: Example

Domain of cars (cf. the Car Evaluation Dataset¹)

- **1 BodyType**: {mvan, sedan, sport, suv}.
- **2** Capacity: {2, 5, 7m}.
- Color: {black, blue, gray, red, white}.
- LuggageSize: {big, med, small}.
- **Make**: {bmw, ford, honda, vw}.
- Price: {low, med, high, vhigh}.
- Safety: {low, med, high}.

 $^{^1}$ http://www.cs.uky.edu/~liu/preflearnlib.php, slightly adapted in the talk.

Qualitative Preferences

Individual:



Figure: Dominance and Optimization

Qualitative Preferences

Collective:



Figure: Social Choice and Welfare

Research Problems of Interest

- Preference representation formalisms to compactly model qualitative preferences over combinatorial domains.
- Preference elicitation and learning methods to cast preferences of agents in a formalism.
- Preference reasoning tasks:
 - Dominance and optimization
 - Manipulation: better off by misreporting preferences untruthfully.

Preference Modeling

Q: How do we compactly represent qualitative preferences over combinatorial domains?

- Preference Trees (P-trees)^{2,14}
- Partial Lexicographic Preference Trees (PLP-trees)⁹
- **1** Lexicographic Preference Trees (LP-trees)^{5,13}

Preference Trees Research Overview University of North Florida

²Niall M Fraser. "Ordinal preference representations". In: <u>Theory and Decision</u> (1994)

³Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

⁴Xudong Liu and Miroslaw Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

⁵Richard Booth et al. "Learning conditionally lexicographic preference relations". In: <u>ECAI</u>. 2010

⁶Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

Preference Learning

Q: How do we learn predictive qualitative preference models over combinatorial domains?

- Partial Lexicographic Preference Trees (PLP-trees)^{7,8,9}
 - Active and passive learning
 - Compute a (possibly small) PLP-tree consistent with all the data
 - Compute a PLP-tree that agrees with the data as much as possible
- Preference Forests¹⁰
- Preference Approximation¹¹

⁷Michael Schmitt and Laura Martignon. "On the complexity of learning lexicographic strategies". In: The Journal of Machine Learning Research (2006)

 $^{^8}$ József Dombi, Csanád Imreh, and Nándor Vincze. "Learning lexicographic orders". In: European Journal of Operational Research (2007)

 $^{^9}$ Xudong Liu and Miroslaw Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: <u>Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI)</u>. 2015

¹⁰Xudong Liu and Miroslaw Truszczynski. "Learning Preference Trees and Forests". In: <u>IJCAI-16 (In Preparation)</u>

¹¹Xudong Liu and Miroslaw Truszczynski. "Approximating Conditional Preference Networks Using Lexicographic Preference Trees". In: AAMAS-17 (In Preparation)

Preference Reasoning

- Q: How do we reason about preferences over combinatorial domains?
 - Preference Optimization 12,13,14,15:
 - Dominance testing: $o_1 \succ_P o_2$?
 - Optimality testing: $o_1 \succ_P o_2$ for all $o_2 \neq o_1$?
 - Optimality computing: what is the optimal outcome wrt *P*?
 - Preference aggregation: which candidate wins the election?
 - 2 Preference Misrepresentation 16,17:
 - Manipulation

 $^{^{12}}$ Jérôme Lang, Jérôme Mengin, and Lirong Xia. "Aggregating Conditionally Lexicographic Preferences on Multi-issue Domains". In: $\underline{\mathsf{CP}}.\ 2012$

¹³Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

¹⁴Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

¹⁵Xudong Liu and Miroslaw Truszczynski. "Reasoning with Preference Trees over Combinatorial Domains". In: Proceedings of the 4th International Conference on Algorithmic Decision Theory (ADT). 2015

 $^{^{16}}$ Felix Brandt, Vincent Conitzer, and Ulle Endriss. "Computational social choice". In: $\underline{\text{Multiagent systems}}$ (2012)

¹⁷Xudong Liu and Miroslaw Truszczynski. "Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees". In: AAMAS-17 (In Preparation)

Preference Applications

Q: What fields can we apply preferences to?

- Game Theory:
 - Hedonic games¹⁸
- Automated Planning and Scheduling:
 - Trip planning¹⁹
- Oata-Driven Decision Making:
 - Predictive models²⁰

¹⁸Matthew Spradling et al. "Roles and Teams Hedonic Game". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

¹⁹Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

²⁰Xudong Liu and Miroslaw Truszczynski. "Learning Preference Trees and Forests". In: <u>IJCAI-16 (In Preparation)</u>

Outline

- The languages of P-trees, PLP-trees, and LP-trees
- Learning preference models in case of PLP-trees
- Reasoning with preferences:
 - Preference optimization in case of P-trees
 - Computing winners and "strong" outcomes when votes are LP-trees
 - Application in trip planning
- Future research directions

Outline

- The languages of P-trees, PLP-trees, and LP-trees
- Learning of preference models (PLP-trees and P-forests)
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Preference Trees (P-Trees)

Let φ , ψ , and π be propositional formulas over the set \mathcal{L} of literals that are values from $\bigcup_{X_i \in V} Dom(X_i)$.

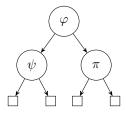


Figure: A P-tree

$$\varphi \wedge \psi \succ \varphi \wedge \neg \psi \succ \neg \varphi \wedge \pi \succ \neg \varphi \wedge \neg \pi.$$

Preference Trees (P-Trees)

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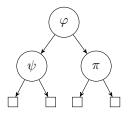


Figure: A P-tree

$$\varphi \wedge \psi \succ \varphi \wedge \neg \psi \succ \neg \varphi \wedge \pi \succ \neg \varphi \wedge \neg \pi$$
.

Total preorder

- **9 BodyType**(X_1): {mvan($x_{1,1}$), sedan($x_{1,2}$), sport($x_{1,3}$), suv($x_{1,4}$)}.
- **2** Capacity(X_2): {2, 5, 7m}.
- **3** Color(X_3): {black, blue, gray, red, white}.
- **1** LuggageSize(X_4): {big, med, small}.
- **Make**(X_5): {bmw, ford, honda, vw}.
- **o Price**(X_6): {low, med, high, vhigh}.
- **Safety**(X_7): {low, med, high}.

Example: Preference Trees over Cars

```
BodyType(X_1): {mvan(x_{1,1}), sedan(x_{1,2}), sport(x_{1,3}), suv(x_{1,4})}. Color(X_3): {black, blue, gray, red, white}. Price(X_6): {low, med, high, vhigh}.
```

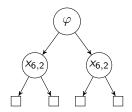


Figure : A P-tree over cars²¹

 $^{^{21}\}varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2}).$

Example: Preferences over Cars

BodyType(X_1): {mvan($x_{1,1}$), sedan($x_{1,2}$), sport($x_{1,3}$), suv($x_{1,4}$)}. **Color**(X_3): {black, blue, gray, red, white}. **Price**(X_6): {low, med, high, vhigh}.

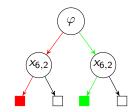


Figure : A P-tree over cars²¹ $Car2 \succ Car1$

 $^{^{21}\}varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2}).$

Compact Representation of P-trees

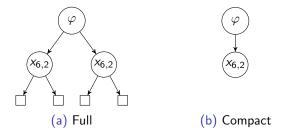


Figure: Compact P-trees

Compact Representation of P-trees

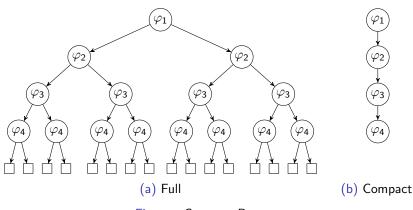


Figure : Compact P-trees

Compact Representation of P-trees

A compact P-tree over $CD(\mathcal{I})$ is a binary tree where

- lacktriangle every node is labeled with a Boolean formula over \mathcal{I} , and
- ② every non-leaf node t labeled with φ has either two outgoing edges (Figure 15a), or one outgoing edge pointing straight-down (Figure 15d), left (Figure 15c), or right (Figure 15b).

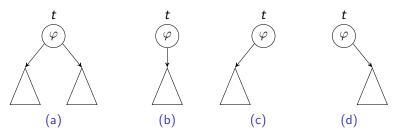


Figure: Compact P-trees

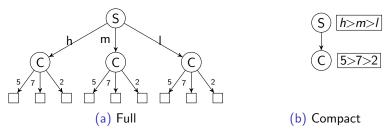


Figure: Unconditional Importance & Unconditional Preference (UIUP)

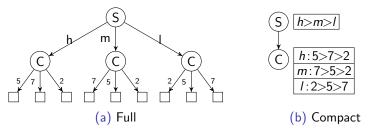


Figure: Unconditional Importance & Conditional Preference (UICP)

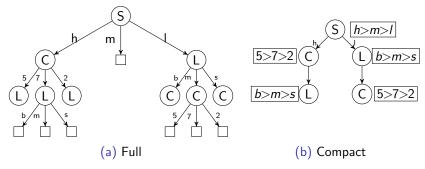


Figure: Conditional Importance & Unconditional Preference (CIUP)

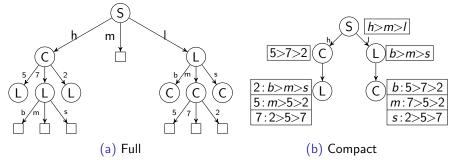


Figure: Conditional Importance & Conditional Preference (CICP)

Lexicographic Preference Trees (LP-Trees)

- **1** An LP-tree \mathcal{L} over $CD(\mathcal{I})$ is a PLP-tree, where
 - each attribute appears exactly once on every path from the root to a leaf.
 - Unlike P-trees and PLP-trees, an LP-tree induces a total order.

Outline

- The languages of P-trees, PLP-trees, and LP-trees
- Learning of preference models (PLP-trees and P-forests)
- Reasoning with preferences:
 - Preference optimization in case of P-trees
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Learning Problems on PLP-trees

Consistent Learning (CONSLEARN)

Given an example set \mathcal{E} , decide whether there exists a PLP-tree T (of a particular type) such that T is consistent with \mathcal{E} .

Small Learning (SMALLLEARN)

Given an example set \mathcal{E} and a positive integer I ($I \leq |\mathcal{E}|$), decide whether there exists a PLP-tree T (of a particular type) such that T is consistent with \mathcal{E} and $|T| \leq I$.

Maixmal Learning (MAXLEARN)

Given an example set $\mathcal E$ and a positive integer k ($k \le m$), decide whether there exists a PLP-tree $\mathcal T$ (of a particular type) such that $\mathcal T$ satisfies at least k examples in $\mathcal E$.

Computational Complexity

- **1** P, NP, coNP: We typically believe that $P \subset NP$ and $P \subset coNP$.
- ② Δ_2^P : P^{NP} , Σ_2^P : NP^{NP} , and Π_2^P : $coNP^{NP}$.
- 3 C-complete: hardest decision problems in class C.

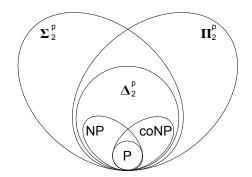


Figure: Computational complexity diagram

Complexity Results on PLP-trees

	UP	CP
UI	Р	Р
CI	NPC ²²	Р

	UP	СР
UI	NPC	NPC
CI	NPC	NPC

(a) Conslearn

(b) SMALLLEARN

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	UP	CP
UI	NPC ²³	NPC
CI	NPC	NPC

(c) MaxLearn

Figure : Complexity results for learning PLP-trees

²²Booth et al., Learning Conditionally Lexicographic Preference Relations, 2010.

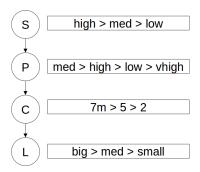


Figure: Unconditional Importance & Unconditional Preference (UIUP)

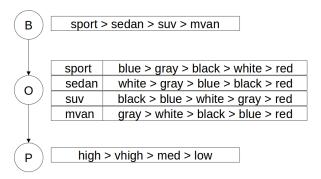


Figure: UICP with at most 1 parent (UICP-1)

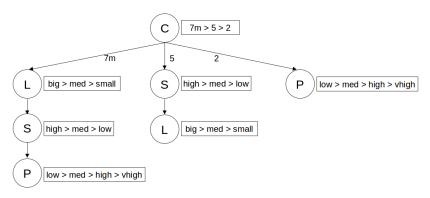


Figure: CIUP with 1 split at the root (CIUP-1)

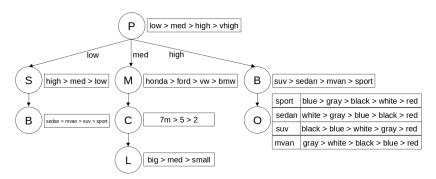


Figure: Simple CICP (SCICP)

Experimental Results on PLP-trees

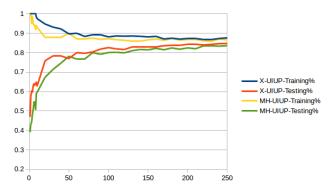


Figure: Learning curve for UIUP using ASP and greedy heuristic

Experimental Results on PLP-trees

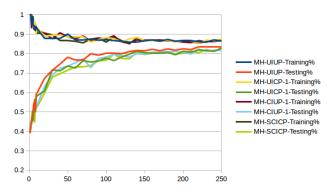


Figure: Learning curve for all four classes using greedy heuristic

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Computational Complexity Results for P-trees

```
Dominance-testing (DomTest): o_1 \succ_T o_2?
Optimality-testing (OPTTest): o optimal w.r.t T?
Optimality-with-property (OPTPROP): is there optimal o with property \alpha?
```

- **1** DomTest $\in P$
- ② OPTTEST \in *coNP*-complete:
 - The complement problem is reduced from the SAT problem.
- **3** OptProp $\in \Delta_2^P$ -complete:
 - The problem is reduced from the Maximum Satisfying Assignment (MSA) problem.

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Positional Scoring Rules

- k-approval: (1, ..., 1, 0, ..., 0) with k being the number of 1's and m k the number of 0's where $m = 2^p$.
- (k, l)-approval: $(a, \ldots, a, b, \ldots, b, 0 \ldots, 0)$, where a and b are constants (a > b) and the numbers of a's and b's equal to k and l, respectively.
- b-Borda: $(b, b-1, \ldots, 0)$, where if b>m-1, b-Borda is reduced to the regular Borda rule with $(m-1, m-2, \ldots, 1, 0)$.

The Evaluation and Winner Problems

The Evaluation Problem

Let r be a positional scoring rule with a scoring vector w, \mathcal{C} a class of LP-trees. Given a \mathcal{C} -profile P of n LP-trees over p attributes and a positive integer R, the *evaluation* problem is to decide whether there exists an alternative $o \in \mathcal{X}$ such that $s_w(o, P) \geq R$.

The Winner Problem

Let r be a positional scoring rule with a scoring vector w, \mathcal{C} a class of LP-trees. Given a \mathcal{C} -profile P of n LP-trees over p attributes, the winner problem is to compute an alternative $o \in \mathcal{X}$ with the maximum score $s_w(o, P)$.

Complexity of the Evaluation Problem: k-Approval

	UP	CP
UI	Р	Р
CI	Р	Р

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(a)
$$k = 2^{p-1} \pm f(p)$$
, $f(p)$ is a poly

(b)
$$k = 2^{p-c}$$
, $c > 1$ is a const

Figure : k-Approval

Complexity of the Evaluation Problem: (k, l)-Approval

	UP	CP
UI	Р	Р
CI	Р	Р

(a)
$$k = I = 2^{p-1}$$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b)
$$k = l = 2^{p-c}$$
, $c > 1$ is a const

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Figure : (k, l)-Approval ²⁴

²⁴ Liu and Truszczynski, Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming, ADT, 2013.

Complexity of the Evaluation Problem: b-Borda

	UP	CP
UI	Р	NPC
CI	NPC	NPC

(a)
$$b = 2^p - 1$$

	UP	СР
UI	NPC	NPC
CI	NPC	NPC

(b)
$$b = 2^{p-c} - 1$$
, $c \ge 1$ is a const

Figure : b-Borda

Modeling the Problems in ASP

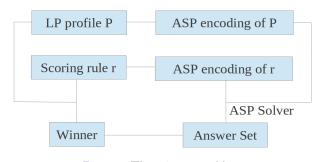


Figure : The winner problem

• Solvers: clingo²⁵, clingcon²⁶

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 $^{^{25}}$ M. Gebser et al. "Potassco: The Potsdam Answer Set Solving Collection". In: $\underline{\text{Al Communications}}$ (2011)

 $^{^{26}}$ Max Ostrowski and Torsten Schaub. "ASP modulo CSP: The clingcon system". In: $\underline{\text{TPLP}}$ (2012)

Modeling the Problems in W-MAXSAT

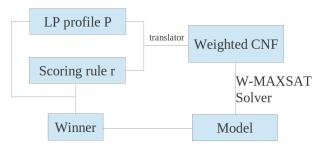


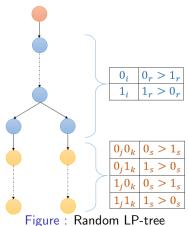
Figure: The winner problem

Solver: toulbar²⁷

 $^{^{27}\}mbox{M}$ Sanchez et al. "Max-CSP competition 2008: toulbar2 solver description" . In: the Third International CSP Solver Competition (2008)

Random LP Profiles

 To experiment with LP profiles, we developed methods to randomly generate encodings of a special type of CI-CP LP-tree of size linear in the number of attributes



Varying p and n: 2^{p-2} -approval

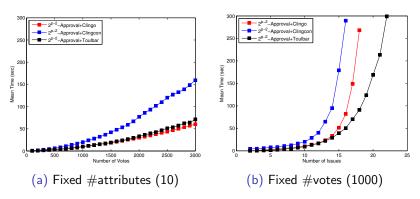


Figure: Solving the winner problem

Varying p and n: $(2^{p-2}, 2^{p-2})$ -approval ²⁸

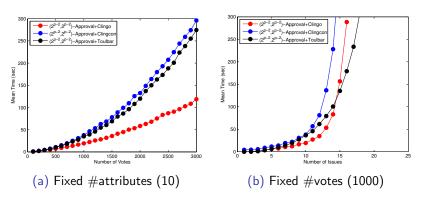


Figure: Solving the winner problem

 $^{^{28}}$ scoring vector: $(2,\ldots,2,1,\ldots,1,0,\ldots,0)$ with the numbers of 2's and 1's equal to 2^{p-2}

Varying p and n: Borda

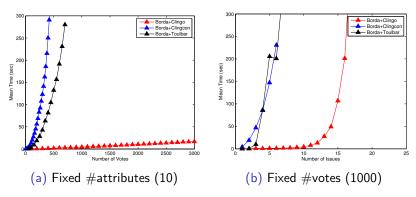


Figure: Solving the winner problem

Outline

- The languages of P-trees, PLP-trees, and LP-trees
- Learning of preference models (PLP-trees and P-forests
- Reasoning with preferences:
 - Preference optimization in case of P-tree
 - Computing winners and "strong" outcomes when votes are LP-trees
 - Application in trip planning
- Future research directions

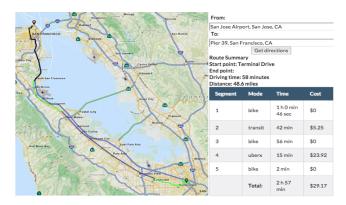
Personalization in Trip Planning

- Important to incorporate user constraints and preferences into trip planning systems.
- Ocliaboration with experts (in AI, planning, optimization, multi-agent systems) at PARC.
- Developed a hipergraph-based trip planner that accommodates constraints specified as linear temporal logic and preferences expressed as preferential cost function to compute optimal routes using A*29.
- Available later for trip planning in the Bay Area, LA, and Denver.

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Personalization in Trip Planning

- From SJC, to Pier 39, Monday, 9am.
- Constraints: never drive a car, and bike for 1 to 2 hours.
- **③** Preferences: bike = public (0.25) > wait(2) > walk(3), and 30\$/hr.



Outline

- The languages of P-trees, PLP-trees, and LP-trees
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Data-Driven Preference Engineering

- Recommender Systems³⁰:
 - Collaborative
 - Ontent-based
 - Hybrid
- Machine Learning:
 - Supervised learning (e.g., decision trees, random forests)
 - 2 Label ranking³¹
- Preference Elicitation (Human-in-the-Loop):
 - Context-based
- Preference Learning:
 - Onditional Preference Networks, Preference Trees
 - ② Stochastic Models (e.g., Choquet integral³², TOPSIS-like models³³)

 $^{^{30}}$ Gediminas Adomavicius and Alexander Tuzhilin. "Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions". In: Knowledge and Data Engineering, IEEE Transactions on (2005)

³¹Eyke Hüllermeier et al. "Label ranking by learning pairwise preferences". In: <u>Artificial Intelligence</u> (2008)

 $^{^{32}}$ Agnes Leroy, Vincent Mousseau, and Marc Pirlot. "Learning the parameters of a multiple criteria sorting method". In: Algorithmic decision theory. 2011

³³Manish Agarwal, Ali Fallah Tehrani, and Eyke Hüllermeier. "Preference-based Learning of Ideal Solutions in TOPSIS-like Decision Models". In: <u>Journal of Multi-Criteria Decision Analysis</u> (2014)

Preference Reasoning and Applications

- Social Choice and Welfare³⁴:
 - Voting
 - Pair devision
 - Strategyproof Social Choice
- Automated Planning and Scheduling:
 - Travel scheduling
 - Manufacturing
 - Traffic control
- Computer Vision and Image Processing:
 - Image retrieval
 - 2 Image and video understanding

 $^{^{34}}$ Kenneth J Arrow, Amartya Sen, and Kotaro Suzumura. Handbook of Social Choice and Welfare. Vol. 1 & 2. 2010

- Xudong Liu. "Modeling, Learning and Reasoning with Qualitative Preferences". Algorithmic Decision Theory, 2015.
- 2 Xudong Liu and Miroslaw Truszczynski. "Reasoning with Preference Trees over Combinatorial Domains". <u>Algorithmic Decision Theory</u>, 2015.
- Xudong Liu and Miroslaw Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". <u>AAAI</u> Conference on Artificial Intelligence, 2015.
- Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". <u>Multidisciplinary Workshop on Advances in Preference</u> <u>Handling</u>, 2014.

- Matthew Spradling, Judy Goldsmith, Xudong Liu, Chandrima Dadi, and Zhiyu Li. "Roles and Teams Hedonic Game". <u>Algorithmic</u> Decision Theory, 2013.
- Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". Algorithmic Decision Theory, 2013.
- Xudong Liu. "Aggregating Lexicographic Preference Trees Using Answer Set Programming: Extended Abstract". <u>International Joint</u> Conference on Artificial Intelligence Doctoral Consortium, 2013.
- 3 Xudong Liu and Miroslaw Truszczynski. "Learning Preference Trees and Forests". (In Preparation for IJCAI-16).

- Xudong Liu and Miroslaw Truszczynski. "Approximating Conditional Preference Networks Using Lexicographic Preference Trees". (In Preparation for AAMAS-17).
- Xudong Liu and Miroslaw Truszczynski. "Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees". (In Preparation for AAMAS-17).
- Xudong Liu and Miroslaw Truszczynski. "Reasoning About Lexicographic Preferences Over Combinatorial Domains". (In Preparation for journal submission).
- Xudong Liu and Christian Fritz. "On Personalizability and Extensibility of Multi-Modal Trip Planning". (In Preparation for ICAPS-17).

Related Work

- Quantitative:
 - Utility/Cost Functions
 - Possibilistic Logic³⁵
 - Fuzzy Preference Relations³⁶
 - Penalty Logic³⁷
- Qualitative:
 - Answer-Set Optimization Theories³⁸
 - Ceteris Paribus Networks (e.g., CP-nets³⁹, TCP-nets⁴⁰, CI-nets⁴¹)
 - Conditional Preference Theories⁴²

Preference Trees Related Work University of North Florida 67 / 68

 $^{^{35}}$ Didier Dubois, Jérôme Lang, and Henri Prade. "A Brief Overview of Possibilistic Logic". In: $\underline{\sf ECSQARU}$. 1991

³⁶SA Orlovsky. "Decision-making with a fuzzy preference relation". In: Fuzzy sets and systems (1978)

³⁷Gadi Pinkas. <u>Propositional non-monotonic reasoning and inconsistency in symmetric neural networks.</u> 1991

³⁸Gerhard Brewka, Ilkka Niemelä, and Miroslaw Truszczynski. "Answer Set Optimization". In: <u>IJCAI</u>. 2003

³⁹C. Boutilier et al. "CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements". In: <u>Journal of Artificial Intelligence Research</u> (2004)

⁴⁰Ronen I. Brafman and Carmel Domshlak. "Introducing Variable Importance Tradeoffs into CP-Nets". In: <u>UAI</u>. 2002

⁴¹Sylvain Bouveret, Ulle Endriss, and Jérôme Lang. "Conditional importance networks: A graphical language for representing ordinal, monotonic preferences over sets of goods". In: (2009)

⁴²Nic Wilson. "Extending CP-Nets with Stronger Conditional Preference Statements". In: <u>AAAI-04</u>. 2004

Questions?

Thank you!