Preference Trees over Combinatorial Domains

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Preferences Are Ubiquitous

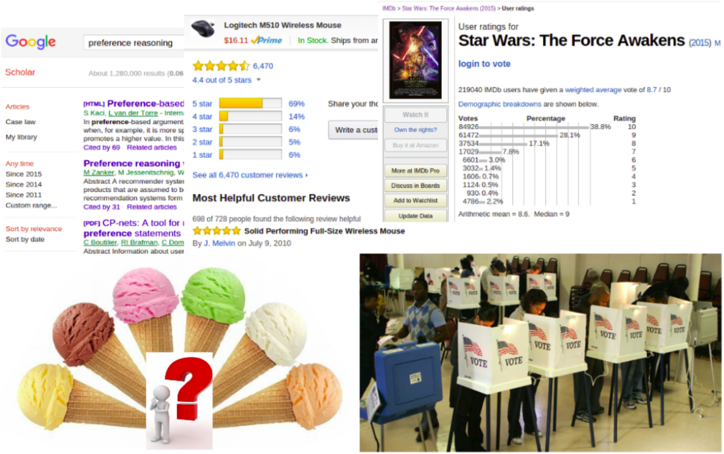


Figure : Preferences of different forms

Describing Preferences

Figure : How to express preferences?

1. On scale of 0 to 99, how will I rate these two cars?
   - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
2. Which one to me is better than the other?
   - I prefer Car1 to Car2. (Strict preference)
   - I like Car1 and Car2 equally. (Indifference/Equivalence)
   - I cannot decide. (Incomparability)

Describing Preferences

Figure : How to express preferences?

1. On scale of 0 to 99, how will I rate these two cars? (**Quantitative**)
    - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
2. Which one to me is better than the other? (**Qualitative**)
    - I prefer Car1 to Car2. (Strict preference)
    - I like Car1 and Car2 equally. (Indifference/Equivalence)
    - I cannot decide. (Incomparability)

Relations and Orderings

Binary Relations

Let O be a set of elements. A *binary relation* R over O is a collection of ordered pairs of elements in O; that is,

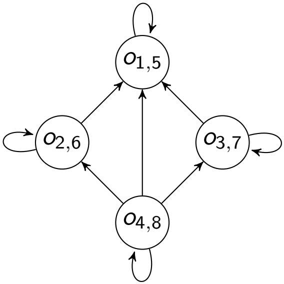
$$R \subseteq O \times O.$$

Properties of binary relations related to preferences:

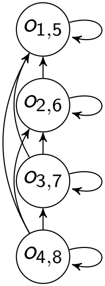
1. Reflexivity: $\forall o \in O$, $(o, o) \in R$.
2. Irreflexivity: $\forall o \in O$, $(o, o) \notin R$.
3. Totality: $\forall o_1, o_2$, $(o_1, o_2) \in R$ or $(o_2, o_1) \in R$.
4. Transitivity: $\forall o_1, o_2, o_3$, if $(o_1, o_2) \in R$ and $(o_2, o_3) \in R$, then $(o_1, o_3) \in R$.
5. Symmetry: $\forall o_1, o_2$, if $(o_1, o_2) \in R$, then $(o_2, o_1) \in R$.
6. Antisymmetry: $\forall o_1, o_2$, if $(o_1, o_2) \in R$ and $(o_2, o_1) \in R$, then $o_1 = o_2$.

Orderings

$\succeq$ is a *partial preorder* if it is reflexive and transitive, a *total preorder* if it is a partial preorder and total, a *partial order* if it is a partial preorder and antisymmetric, and a *total order* if it is a partial order and total.



(a) partial preorder



(b) total preorder

Relations and Orderings

Orderings

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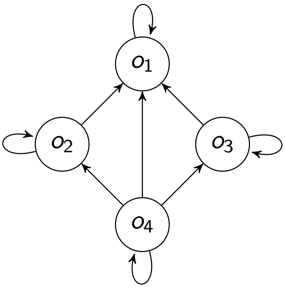
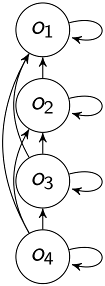


(a) partial order

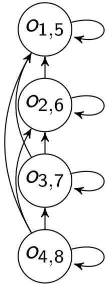


(b) total order

Relations and Orderings

Preference Relations

Let $\succeq$ be a preference relation that is a total preorder over O. We say that o_1 is *weakly preferred* to o_2 if $o_1 \succeq o_2$, that o_1 is *strictly preferred* ($\succ$) to o_2 if $o_1 \succeq o_2$ and $o_2 \not\succeq o_1$, and that o_1 is *indifferent* ($\approx$) from o_2 if $o_1 \succeq o_2$ and $o_2 \succeq o_1$.



(a) total preorder

$$o_1 \succeq o_5,$$
$$o_4 \succ o_2,$$
$$o_4 \approx o_8,$$

(b) preferences

Combinatorial Domains

Combinatorial Domains

Let V be a finite set of variables $\{X_1, \ldots, X_p\}$, D a set of finite domains $\{Dom(X_1), \ldots, Dom(X_p)\}$ for each variable X_i. A *combinatorial domain* $CD(V)$ is a set of *outcomes* described by combinations of values from $Dom(X_i)$:

$$CD(V) = \prod_{X_i \in V} Dom(X_i).$$

Combinatorial Domains: Example

Domain of cars over set V of p binary variables:

1. **BodyType**: $\{$mvan, sedan$\}$.
2. **Capacity**: $\{$5, 7m$\}$.
3. **Color**: $\{$blue, gray$\}$.

$\vdots$

$$CD(V) = \underbrace{\{\langle \text{sedan, 5, blue, } \ldots \rangle, \langle \text{mvan, 7m, gray, } \ldots \rangle, \ldots\}}_{2^p \text{ outcomes, too many!}}.$$

Computational Complexity

1. P, NP, $coNP$: We typically believe that $P \subset NP$ and $P \subset coNP$.
2. Δ_2^P: P^{NP}, Σ_2^P: NP^{NP}, and Π_2^P: $coNP^{NP}$.
3. C-complete: hardest decision problems in class C.

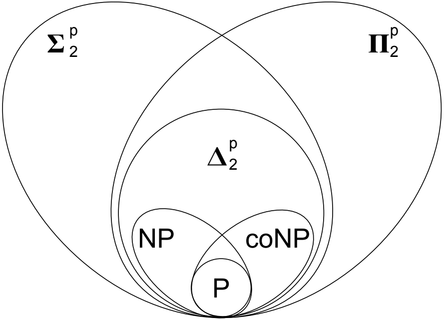


Figure : Computational complexity diagram

Combinatorial Domains: Example

Domain of cars (cf. the Car Evaluation Dataset[1])

1. **BodyType**: {mvan, sedan, sport, suv}.
2. **Capacity**: {2, 5, 7m}.
3. **Color**: {black, blue, gray, red, white}.
4. **LuggageSize**: {big, med, small}.
5. **Make**: {bmw, ford, honda, vw}.
6. **Price**: {low, med, high, vhigh}.
7. **Safety**: {low, med, high}.

[1] `http://www.cs.uky.edu/~liu/preflearnlib.php`, slightly adapted in the talk.

Qualitative Preferences

Individual:

Figure : Dominance Testing

Qualitative Preferences

Collective:

Figure : Social Choice and Welfare

Research Problems of Interest

1. Preference representation formalisms to compactly model qualitative preferences over combinatorial domains.
2. Preference elicitation and learning methods to cast preferences of agents in a formalism.
3. Preference reasoning tasks:
   - Dominance and optimization
   - Manipulation: better off by misreporting preferences untruthfully.

Q: How do we compactly represent qualitative preferences over combinatorial domains?

1. Preference Trees (P-trees)[2,14]
2. Partial Lexicographic Preference Trees (PLP-trees)[9]
3. Lexicographic Preference Trees (LP-trees)[5,13]

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[2] Niall M Fraser. "Ordinal preference representations". In: Theory and Decision (1994)

[3] Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

[4] Xudong Liu and Miroslaw Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

[5] Richard Booth et al. "Learning conditionally lexicographic preference relations". In: ECAI. 2010

[6] Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

Preference Learning

Q: How do we learn predictive qualitative preference models over combinatorial domains?

1. Partial Lexicographic Preference Trees (PLP-trees)[7,8,9]
    - Active and passive learning
    - Compute a (possibly small) PLP-tree consistent with all the data
    - Compute a PLP-tree that agrees with the data as much as possible
2. Preference Forests[10]
3. Preference Approximation[11]

[7] Michael Schmitt and Laura Martignon. "On the complexity of learning lexicographic strategies". In: The Journal of Machine Learning Research (2006)

[8] József Dombi, Csanád Imreh, and Nándor Vincze. "Learning lexicographic orders". In: European Journal of Operational Research (2007)

[9] Xudong Liu and Miroslaw Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

[10] Xudong Liu and Miroslaw Truszczynski. "Learning Preference Trees and Forests". In: IJCAI-16 (In Preparation)

[11] Xudong Liu and Miroslaw Truszczynski. "Approximating Conditional Preference Networks Using Lexicographic Preference Trees". In: AAMAS-17 (In Preparation)

Preference Reasoning

Q: How do we reason about preferences over combinatorial domains?

1. Preference Optimization[12,13,14,15]:
   - Dominance testing: $o_1 \succ_P o_2$?
   - Optimality testing: $o_1 \succ_P o_2$ for all $o_2 \neq o_1$?
   - Optimality computing: what is the optimal outcome wrt P?
   - Preference aggregation: which candidate wins the election?
2. Preference Misrepresentation[16,17]:
   - Manipulation

[12] Jérôme Lang, Jérôme Mengin, and Lirong Xia. "Aggregating Conditionally Lexicographic Preferences on Multi-issue Domains". In: CP. 2012

[13] Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

[14] Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

[15] Xudong Liu and Miroslaw Truszczynski. "Reasoning with Preference Trees over Combinatorial Domains". In: Proceedings of the 4th International Conference on Algorithmic Decision Theory (ADT). 2015

[16] Felix Brandt, Vincent Conitzer, and Ulle Endriss. "Computational social choice". In: Multiagent systems (2012)

[17] Xudong Liu and Miroslaw Truszczynski. "Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees". In: AAMAS-17 (In Preparation)

Preference Applications

Q: What fields can we apply preferences to?

1. Game Theory:
   - Hedonic games[18]
2. Automated Planning and Scheduling:
   - Trip planning[19]
3. Data-Driven Decision Making:
   - Predictive decisions[20]

[18] Matthew Spradling et al. "Roles and Teams Hedonic Game". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

[19] Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

[20] Xudong Liu and Miroslaw Truszczynski. "Learning Preference Trees and Forests". In: IJCAI-16 (In Preparation)

Outline

1. The languages of P-trees, PLP-trees, and LP-trees
2. Learning preference models in case of PLP-trees
3. Reasoning with preferences:
   - Computing winners and "strong" outcomes when votes are LP-trees
   - Application in trip planning
4. Future research directions

Outline

1. The languages of P-trees, PLP-trees, and LP-trees
2. Learning of preference models (PLP-trees and P-forests)
3. Reasoning with preferences:
   - Computing winners and “strong” outcomes when votes are LP-trees
   - Application in trip planning
4. Future research directions

Preference Trees

1. Let $\mathcal{I} = \{X_1, \dots, X_p\}$ be a set of attributes, and $D(\mathcal{I}) = \{Dom(X_1), \dots, Dom(X_p)\}$ a set of finite domains for $\mathcal{I}$.
2. A *literal* is an assignment to an attribute. We denote by $X_i := x_{i,j}$ the literal that assigns value $x_{i,j} \in Dom(X_i)$ to X_i. When no confusion, we write $x_{i,j}$, instead of $X_i := x_{i,j}$, as a literal. We then denote by $\mathcal{L} = \{x_{i,j} \in Dom(X_i) : X_i \in \mathcal{I}\}$ the set of literals given $\mathcal{I}$ and $D(\mathcal{I})$.
3. The combinatorial domain $CD(\mathcal{I})$ is defined as earlier.

Preference Trees

4. A **P-tree** T over $CD(\mathcal{I})$ is a binary tree, where non-leaf nodes are labeled with propositional formulas over $\mathcal{L}$.
5. Given an outcome $o \in CD(\mathcal{I})$, the **leaf** $l_T(o)$ is the leaf reached by traversing the tree T according to o. When at a node N labeled with φ, if $o \models \varphi$, we descend to the left child of N; otherwise, to the right.
6. For $o_1, o_2 \in CD(\mathcal{I})$, we have $o_1 \succ_T o_2$ if $l_T(o_1) \succ_T l_T(o_2)$, and $o_1 \approx_T o_2$ if $l_T(o_1) = l_T(o_2)$. Outcome o_1 is **optimal** if there exists no o_2 such that $o_2 \succ_T o_1$.

Example: The Cars Domain

1. **BodyType**(X_1): {mvan($x_{1,1}$), sedan($x_{1,2}$), sport($x_{1,3}$), suv($x_{1,4}$)}.
2. **Capacity**(X_2): {2, 5, 7m}.
3. **Color**(X_3): {black, blue, gray, red, white}.
4. **LuggageSize**(X_4): {big, med, small}.
5. **Make**(X_5): {bmw, ford, honda, vw}.
6. **Price**(X_6): {low, med, high, vhigh}.
7. **Safety**(X_7): {low, med, high}.

Example: Preference Trees over Cars

BodyType(X_1): {mvan($x_{1,1}$), sedan($x_{1,2}$), sport($x_{1,3}$), suv($x_{1,4}$)}.
Color(X_3): {black, blue, gray, red, white}.
Price(X_6): {low, med, high, vhigh}.

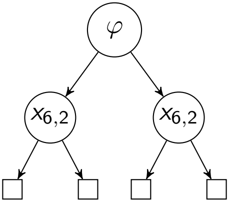


Figure : A P-tree over cars[21]

[21] $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$.

Example: Preferences over Cars

BodyType(X_1): {mvan($x_{1,1}$), sedan($x_{1,2}$), sport($x_{1,3}$), suv($x_{1,4}$)}.
Color(X_3): {black, blue, gray, red, white}.
Price(X_6): {low, med, high, vhigh}.

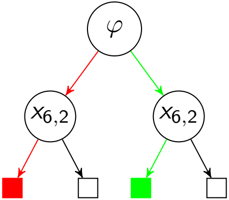
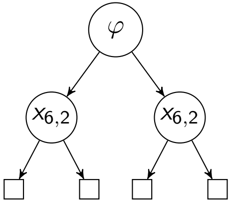


Figure : A P-tree over cars[21]

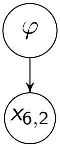
$Car2 \succ Car1$

[21] $\varphi = (x_{1,1} \wedge x_{3,5}) \vee (x_{1,2} \wedge x_{3,2})$.

Compact Representation of P-trees



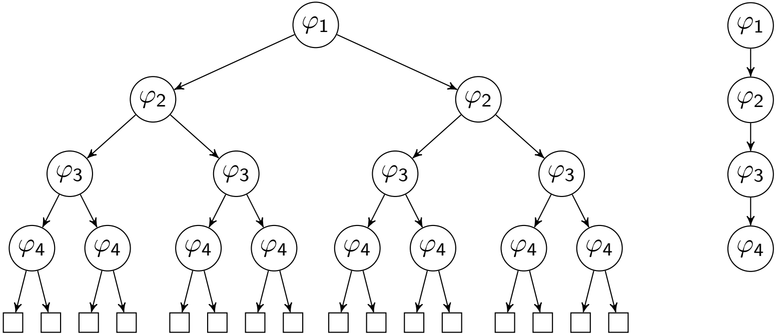
(a) Full



(b) Compact

Figure : Compact P-trees

Compact Representation of P-trees



(a) Full (b) Compact

Figure : Compact P-trees

Compact Representation of P-trees

A *compact P-tree* over $CD(\mathcal{I})$ is a binary tree where

1. every node is labeled with a Boolean formula over $\mathcal{I}$, and
2. every non-leaf node t labeled with φ has either two outgoing edges (Figure (a)), or one outgoing edge pointing left (Figure (b)), right (Figure (c)), or straight-down (Figure (d)).

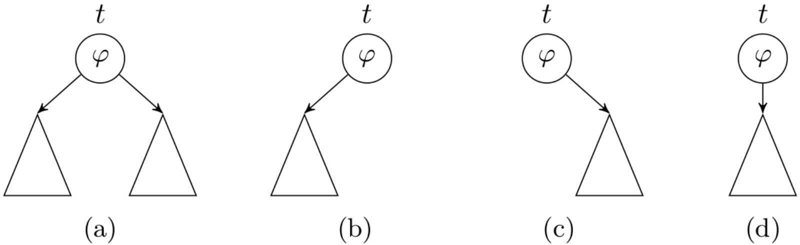


Figure : Compact P-trees

Computational Complexity Results

1. $\textsc{DomTest}$: is it that $o \succeq_T o'$ in P-tree T?
2. $\textsc{OptTest}$: is outcome o optimal w.r.t T?
3. $\textsc{OptProp}$: is there an optimal outcome o w.r.t T st $o \models \alpha$?

Partial Lexicographic Preference Trees (PLP-Tree)

A *PLP-tree* over $CD(\mathcal{I})$ is a tree, where

1. every node t is labeled with an attribute $Attr(t)$ in $\mathcal{I}$ and a conditional preference table $CPT(t)$,
2. every non-leaf node t has either one unlabeled outgoing edge or multiple outgoing edges labeled, each labeled by some value in $Dom(Attr(t))$, and
3. every attribute appears *at most* once on every branch.

Partial Lexicographic Preference Trees (PLP-Tree)

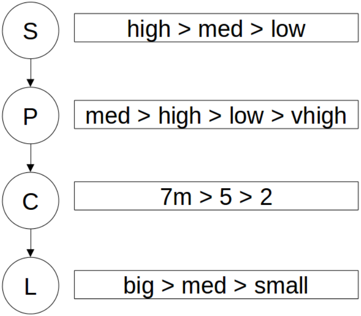


Figure : Unconditional Importance & Unconditional Preference (UIUP) PLP-tree

According to this UIUP PLP-tree, *Car1* is preferred to *Car2*.

Partial Lexicographic Preference Trees (PLP-Tree)

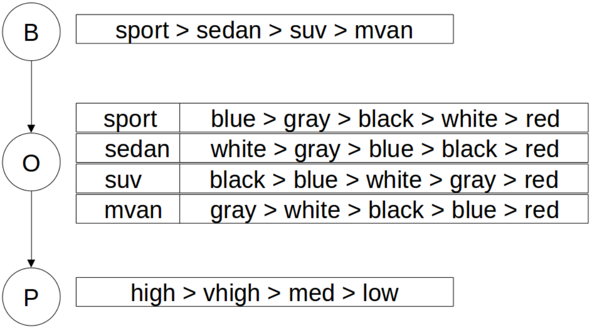


Figure : Unconditional Importance & Conditional Preference (UICP) PLP-tree

According to this UICP PLP-tree, *Car2* is preferred to *Car1*.

Partial Lexicographic Preference Trees (PLP-Tree)

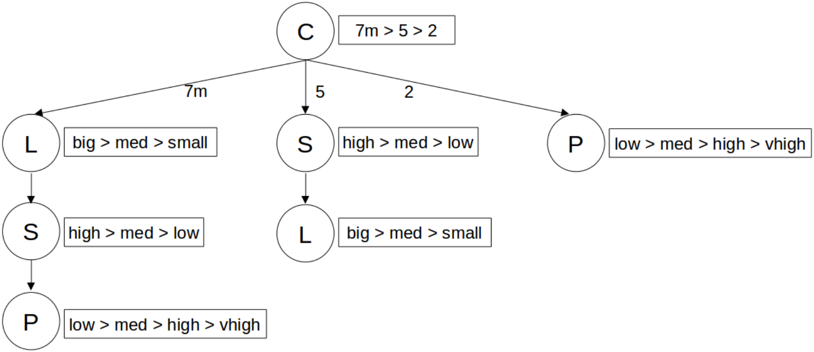


Figure : Conditional Importance & Unconditional Preference (CIUP) PLP-tree

According to this CICP PLP-tree, *Car1* is preferred to *Car2*.

Partial Lexicographic Preference Trees (PLP-Tree)

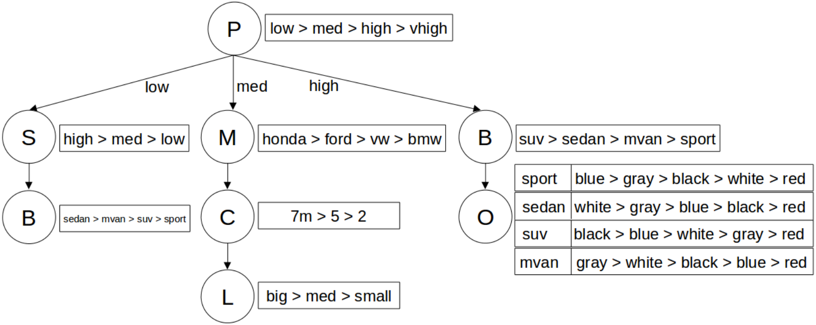


Figure : Conditional Importance & Conditional Preference (CICP) PLP-tree

According to this CICP PLP-tree, *Car1* is preferred to *Car2*.

Lexicographic Preference Trees (LP-Trees)

1. An *LP-tree* $\mathcal{L}$ over $CD(\mathcal{I})$ is a PLP-tree, where
   - each attribute appears **exactly once** on every path from the root to a leaf.

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1. The languages of P-trees, PLP-trees, and LP-trees
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4. Future research directions

Learning Problems on PLP-trees

Consistent Learning (ConsLearn)

Given an example set $\mathcal{E}$, decide whether there exists a PLP-tree T (of a particular type) such that T is consistent with $\mathcal{E}$.

Small Learning (SmallLearn)

Given an example set $\mathcal{E}$ and a positive integer l ($l \leq |\mathcal{E}|$), decide whether there exists a PLP-tree T (of a particular type) such that T is consistent with $\mathcal{E}$ and $|T| \leq l$.

Maixmal Learning (MaxLearn)

Given an example set $\mathcal{E}$ and a positive integer k ($k \leq m$), decide whether there exists a PLP-tree T (of a particular type) such that T satisfies at least k examples in $\mathcal{E}$.

Complexity Results on PLP-trees

	UP	CP
UI	P	P
CI	NPC[22]	P

(a) ConsLearn

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) SmallLearn

	UP	CP
UI	NPC[23]	NPC
CI	NPC	NPC

(c) MaxLearn

Figure : Complexity results for passive learning problems

[22] Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

[23] Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006.

Experimental Results on PLP-trees

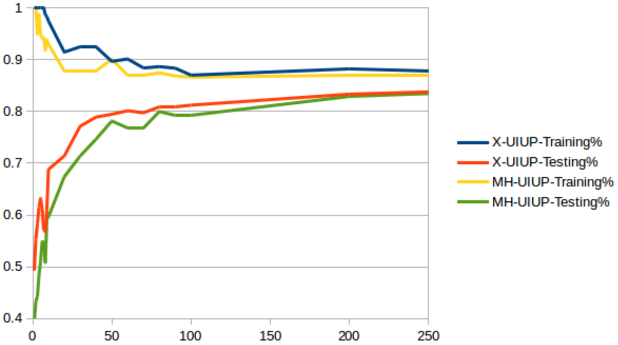

Figure : Learning curve for UIUP using ASP and greedy heuristic

Experimental Results on PLP-trees

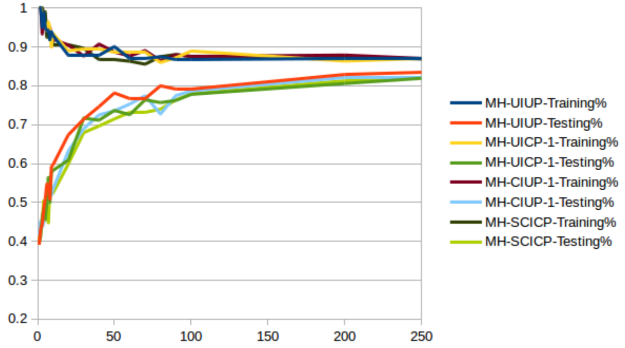

Figure : Learning curve for all four classes using greedy heuristic

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Positional Scoring Rules

- k-approval: $(1, \ldots, 1, 0, \ldots, 0)$ with k being the number of 1's and $m - k$ the number of 0's where $m = 2^p$.
- (k, l)-approval: $(a, \ldots, a, b, \ldots, b, 0 \ldots, 0)$, where a and b are constants ($a > b$) and the numbers of a's and b's equal to k and l, respectively.
- b-Borda: $(b, b - 1, \ldots, 0)$, where if $b > m - 1$, b-Borda is reduced to the regular Borda rule with $(m - 1, m - 2, \ldots, 1, 0)$.

The Evaluation and Winner Problems

The Evaluation Problem

Let r be a positional scoring rule with a scoring vector w, $\mathcal{C}$ a class of LP-trees. Given a $\mathcal{C}$-profile P of n LP-trees over p attributes and a positive integer R, the *evaluation* problem is to decide whether there exists an alternative $o \in \mathcal{X}$ such that $s_w(o, P) \geq R$.

The Winner Problem

Let r be a positional scoring rule with a scoring vector w, $\mathcal{C}$ a class of LP-trees. Given a $\mathcal{C}$-profile P of n LP-trees over p attributes, the *winner* problem is to compute an alternative $o \in \mathcal{X}$ with the maximum score $s_w(o, P)$.

Complexity of the Evaluation Problem: k-Approval

	UP	CP
UI	P	P
CI	P	P

(a) $k = 2^{p-1} \pm f(p)$, $f(p)$ is a poly

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $k = 2^{p-c}$, $c > 1$ is a const

Figure : k-Approval

Complexity of the Evaluation Problem: (k, l)-Approval

	UP	CP
UI	P	P
CI	P	P

(a) $k = l = 2^{p-1}$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $k = l = 2^{p-c}$, $c > 1$ is a const

Figure : (k, l)-Approval [24]

[24] Liu and Truszczynski, *Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming*, ADT, 2013.

Complexity of the Evaluation Problem: b-Borda

	UP	CP
UI	P	NPC
CI	NPC	NPC

(a) $b = 2^p - 1$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $b = 2^{p-c} - 1$, $c \geq 1$ is a const

Figure : b-Borda

Modeling the Problems in ASP

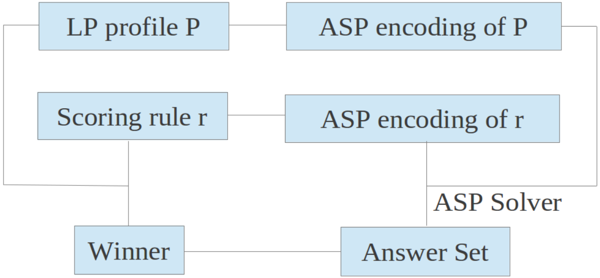


Figure : The winner problem

- Solvers: *clingo*[25], *clingcon*[26]

[25] M. Gebser et al. "Potassco: The Potsdam Answer Set Solving Collection". In: AI Communications (2011)

[26] Max Ostrowski and Torsten Schaub. "ASP modulo CSP: The clingcon system". In: TPLP (2012)

Modeling the Problems in W-MAXSAT

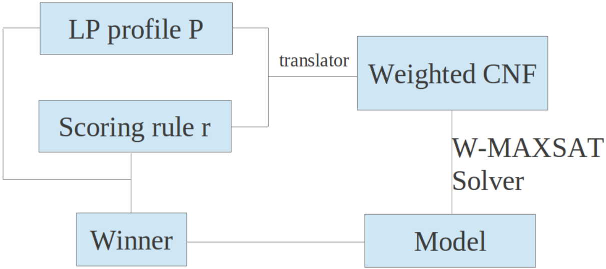


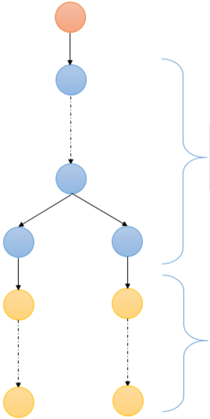
Figure : The winner problem

- Solver: *toulbar*[27]

[27] M Sanchez et al. "Max-CSP competition 2008: toulbar2 solver description". In: the Third International CSP Solver Competition (2008)

Random LP Profiles

- To experiment with LP profiles, we developed methods to randomly generate *encodings* of a special type of CI-CP LP-tree of size linear in the number of attributes

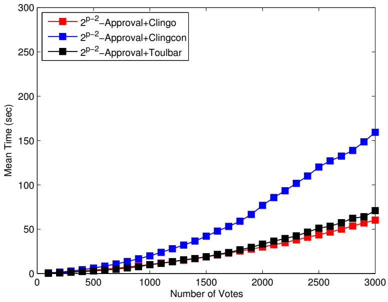


0_i	$0_r > 1_r$
1_i	$1_r > 0_r$

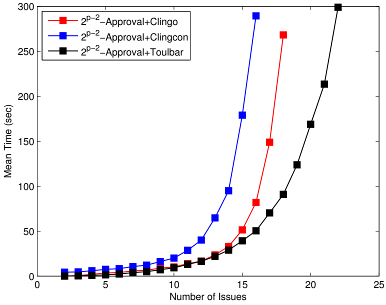
$0_j 0_k$	$0_s > 1_s$
$0_j 1_k$	$1_s > 0_s$
$1_j 0_k$	$0_s > 1_s$
$1_j 1_k$	$1_s > 0_s$

Figure : Random LP-tree

Varying p and n: 2^{p-2}-approval



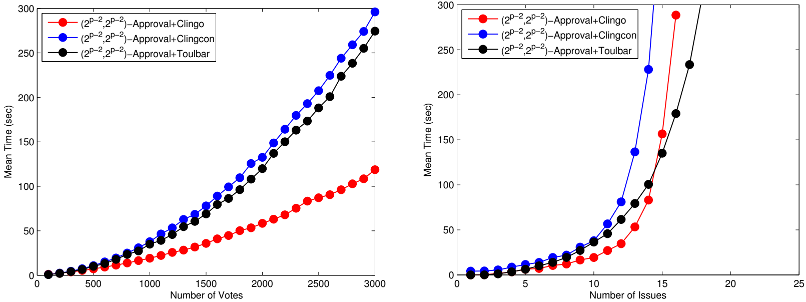
(a) Fixed #attributes (10)



(b) Fixed #votes (1000)

Figure : Solving the winner problem

Varying p and n: $(2^{p-2}, 2^{p-2})$-approval [28]

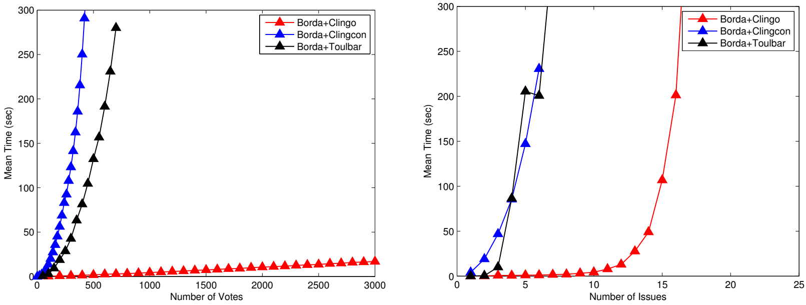


(a) Fixed #attributes (10) (b) Fixed #votes (1000)

Figure : Solving the winner problem

[28] scoring vector: $(2, \ldots, 2, 1, \ldots, 1, 0, \ldots, 0)$ with the numbers of 2's and 1's equal to 2^{p-2}

Varying p and n: Borda



(a) Fixed #attributes (10) (b) Fixed #votes (1000)

Figure : Solving the winner problem

Outline

1. The languages of P-trees, PLP-trees, and LP-trees
2. Learning of preference models (PLP-trees and P-forests)
3. Reasoning with preferences:
   - Computing winners and “strong” outcomes when votes are LP-trees
   - Application in trip planning
4. Future research directions

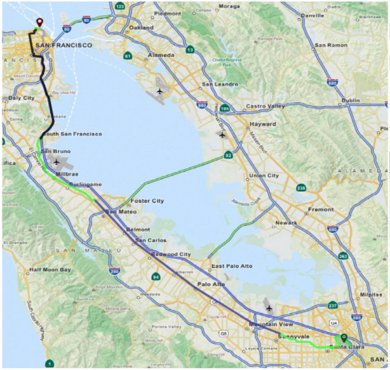
Personalization in Trip Planning

1. Important to incorporate user constraints and preferences into trip planning systems.
2. Collaboration with experts (in AI, planning, optimization, multi-agent systems) at PARC.
3. Developed a hipergraph-based trip planner that accommodates constraints specified as *linear temporal logic* and preferences expressed as *preferential cost function* to compute optimal routes using A*[29].
4. Available later for trip planning in the Bay Area, LA, and Denver.

[29] Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

Personalization in Trip Planning

1. From SJC, to Pier 39, Monday, 9am.
2. Constraints: never drive a car, and bike for 1 to 2 hours.
3. Preferences: bike = public (0.25) > wait(2) > walk(3), and 30\$/hr.



From:

San Jose Airport, San Jose, CA

To:

Pier 39, San Francisco, CA

Get directions

Route Summary
Start point: Terminal Drive
End point:
Driving time: 58 minutes
Distance: 48.6 miles

Segment	Mode	Time	Cost
1	bike	1 h 0 min 46 sec	\$0
2	transit	42 min	\$5.25
3	bike	56 min	\$0
4	uberx	15 min	\$23.92
5	bike	2 min	\$0
	Total:	2 h 57 min	\$29.17

Outline

Data-Driven Preference Engineering

1. Recommender Systems[30]:
    1. Collaborative
    2. Content-based
    3. Hybrid
2. Machine Learning:
    1. Supervised learning (e.g., decision trees, random forests)
    2. Label ranking[31]
3. Preference Elicitation (Human-in-the-Loop):
    1. Context-based
4. Preference Learning:
    1. Conditional Preference Networks, Preference Trees
    2. Stochastic Models (e.g., Choquet integral[32], TOPSIS-like models[33])

[30] Gediminas Adomavicius and Alexander Tuzhilin. "Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions". In: Knowledge and Data Engineering, IEEE Transactions on (2005)

[31] Eyke Hüllermeier et al. "Label ranking by learning pairwise preferences". In: Artificial Intelligence (2008)

[32] Agnes Leroy, Vincent Mousseau, and Marc Pirlot. "Learning the parameters of a multiple criteria sorting method". In: Algorithmic decision theory. 2011

[33] Manish Agarwal, Ali Fallah Tehrani, and Eyke Hüllermeier. "Preference-based Learning of Ideal Solutions in TOPSIS-like Decision Models". In: Journal of Multi-Criteria Decision Analysis (2014)

Preference Reasoning and Applications

1. Social Choice and Welfare[34]:
    1. Voting
    2. Fair devision
    3. Strategyproof Social Choice
2. Automated Planning and Scheduling:
    1. Travel scheduling
    2. Manufacturing
    3. Traffic control
3. Computer Vision and Image Processing:
    1. Image retrieval
    2. Image and video understanding

[34] Kenneth J Arrow, Amartya Sen, and Kotaro Suzumura. Handbook of Social Choice and Welfare. Vol. 1 & 2. 2010

Questions?

Thank you!