

Preference Trees over Combinatorial Domains

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Preferences Are Ubiquitous

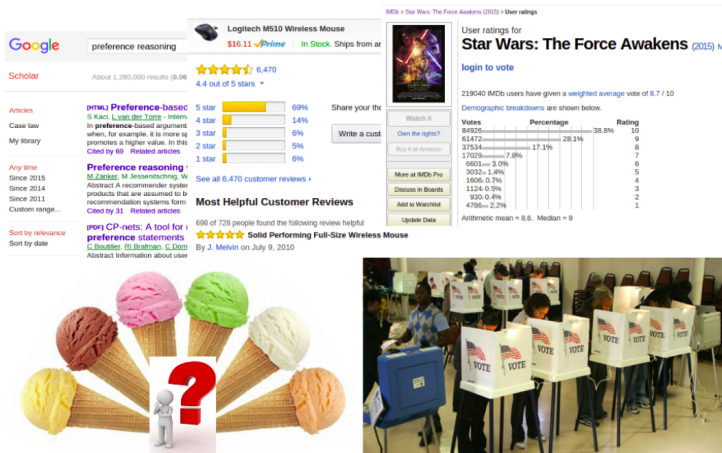


Figure : Preferences of different forms

Describing Preferences

Car1



Car2



Figure : How to express it?

- ① On scale of 0 to 99, how will I rate these two cars?
 - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- ② Which one to me is better than the other?
 - I prefer Car1 to Car2. (Strict preference)
 - I like Car1 and Car2 equally. (Indifference/Equivalence)
 - I cannot decide. (Incomparability)

Describing Preferences

Car1



Car2



Figure : How to express it?

- ① On scale of 0 to 99, how will I rate these two cars? (**Quantitative**)
 - I give Car1 44 points and Car2 78 points; thus, I prefer Car2 to Car1.
- ② Which one to me is better than the other? (**Qualitative**)
 - I prefer Car1 to Car2. (Strict preference)
 - I like Car1 and Car2 equally. (Indifference/Equivalence)
 - I don't know. (Incomparability)

Relations and Orderings

Binary Relations

Let O be a set of elements. A *binary relation* \preceq over O is a collection of ordered pairs of elements in O ; that is,

$$\preceq \subseteq O \times O.$$

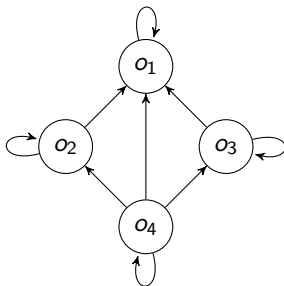
Properties of binary relations:

- ① Reflexivity: $\forall o \in O, o \preceq o$.
- ② Irreflexivity: $\forall o \in O, o \not\preceq o$.
- ③ Totality: $\forall o_1, o_2, o_1 \preceq o_2$ or $o_2 \preceq o_1$.
- ④ Transitivity: $\forall o_1, o_2, o_3$, if $o_1 \preceq o_2$ and $o_2 \preceq o_3$, then $o_1 \preceq o_3$.
- ⑤ Symmetricity: $\forall o_1, o_2$, if $o_1 \preceq o_2$, then $o_2 \preceq o_1$.
- ⑥ Antisymmetricity: $\forall o_1, o_2$, if $o_1 \preceq o_2$ and $o_2 \preceq o_1$, then $o_1 = o_2$.

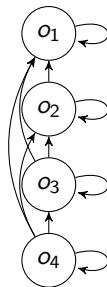
Relations and Orderings

Binary Relations

\preceq is a *partial preorder* if it is reflexive and transitive, a *total preorder* if it is a partial preorder and total, a *partial order* if it is a partial preorder and antisymmetric, and a *total order* if it is a partial order and total.



(a) partial (pre)order

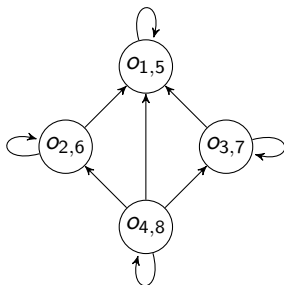


(b) total (pre)order

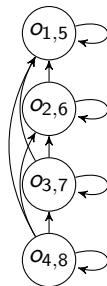
Relations and Orderings

Binary Relations

\preceq is a *partial preorder* if it is reflexive and transitive, a *total preorder* if it is a partial preorder and total, a *partial order* if it is a partial preorder and antisymmetric, and a *total order* if it is a partial order and total.



(a) partial preorder

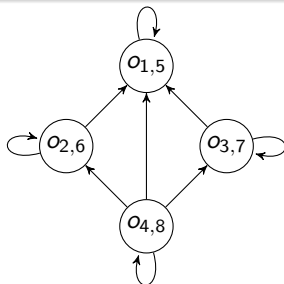


(b) total preorder

Relations and Orderings

Binary Relations

Let \preceq be a preference relation that is a partial preorder over O . We say that o_2 is *weakly preferred* to o_1 if $o_1 \preceq o_2$, that o_2 is *strictly preferred* (\prec) to o_1 if $o_1 \preceq o_2$ and $o_2 \not\preceq o_1$, that o_1 is *indifferent* (\approx) from o_2 if $o_1 \preceq o_2$ and $o_2 \preceq o_1$, and that o_1 is *incomparable* (\sim) with o_2 if $o_1 \not\preceq o_2$ and $o_2 \not\preceq o_1$.



(a) partial preorder

$$o_1 \preceq o_5,$$

$$o_4 \prec o_2,$$

$$o_4 \approx o_8,$$

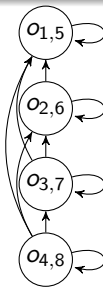
$$o_6 \sim o_7.$$

(b) preferences

Relations and Orderings

Binary Relations

Let \preceq be a preference relation that is a partial preorder over O . We say that o_2 is *weakly preferred* to o_1 if $o_1 \preceq o_2$, that o_2 is *strictly preferred* (\prec) to o_1 if $o_1 \preceq o_2$ and $o_2 \not\preceq o_1$, that o_1 is *indifferent* (\approx) from o_2 if $o_1 \preceq o_2$ and $o_2 \preceq o_1$, and that o_1 is *incomparable* (\sim) with o_2 if $o_1 \not\preceq o_2$ and $o_2 \not\preceq o_1$.



(a) total preorder

$$o_1 \preceq o_5,$$

$$o_4 \prec o_2,$$

$$o_4 \approx o_8,$$

(b) preferences

Combinatorial Domains

Combinatorial Domains

Let V be a finite set of variables $\{X_1, \dots, X_p\}$, D a set of finite domains $\{Dom(X_1), \dots, Dom(X_p)\}$ for each variable X_i . A *combinatorial domain* $CD(V)$ is a set of *outcomes* described by combinations of values from $Dom(X_i)$:

$$CD(V) = \prod_{X_i \in V} Dom(X_i).$$

Combinatorial Domains: Example

Domain of cars over set V of p binary variables:

① **BodyType**: {mvan, sedan}.

② **Capacity**: {5, 7m}.

③ **Color**: {blue, grey}.

⋮

$$CD(V) = \underbrace{\{\langle \text{sedan}, 4, \text{blue}, \dots \rangle, \langle \text{mvan}, 6\text{m}, \text{grey}, \dots \rangle, \dots \}}_{2^p \text{ outcomes, too many!}}$$

Computational Complexity

- ① P (NP): decision problems solvable by a deterministic (nondeterministic, resp.) TM in poly time in the size of the input.
 - We typically believe that $P \subset NP$.
- ② $coNP$: problems whose complements are in NP .
- ③ Δ_2^P : P^{NP} .
- ④ Σ_2^P : NP^{NP} .
- ⑤ PSPACE: decision problems solvable by a TM in poly space in the size of the input.
- ⑥ A decision problem L is C -hard if $L' \leq_p L$ for every L' in class C .
- ⑦ A decision problem L is C -complete if L is in class C and L is C -hard.

Computational Complexity

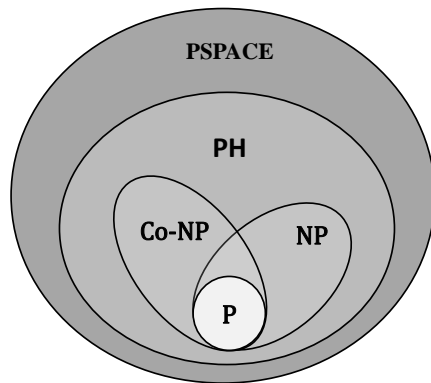


Figure : Computational complexity diagram

Combinatorial Domains: Example

Domain of cars (cf. the Car Evaluation Dataset¹)

- ① **BodyType**: {mvan, sedan, sport, suv}.
- ② **Capacity**: {2, 5, 7m}.
- ③ **Color**: {black, blue, grey, red, white}.
- ④ **LuggageSize**: {big, med, small}.
- ⑤ **Make**: {bmw, ford, honda, and vw}.
- ⑥ **Price**: {low, med, high, vhigh}.
- ⑦ **Safety**: {low, med, high}.

¹<http://www.cs.uky.edu/~liu/preflearnlib.php>, slightly adapted in the talk.

Qualitative Preferences

Individual:

Car1



<mvan, 6m, grey, big, honda, med, med>

Car2



<sedan, 4, blue, med, vw, med, med>



Figure : Dominance Testing

Qualitative Preferences

Collective:



Figure : Social Choice and Welfare

Preference Modeling

Q: How do we represent qualitative preferences over combinatorial domains?

- ① Answer-Set Optimization Theories²
- ② Ceteris Paribus Networks (e.g., CP-nets³, TCP-nets⁴, CI-nets⁵)
- ③ Conditional Preference Theories⁶

²Gerhard Brewka, Ilkka Niemelä, and Mirosław Truszczyński. "Answer Set Optimization". In: IJCAI. 2003

³C. Boutilier et al. "CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements". In: Journal of Artificial Intelligence Research (2004)

⁴Ronen I. Brafman and Carmel Domshlak. "Introducing Variable Importance Tradeoffs into CP-Nets". In: UAI. 2002

⁵Sylvain Bouveret, Ulle Endriss, and Jérôme Lang. "Conditional importance networks: A graphical language for representing ordinal, monotonic preferences over sets of goods". In: (2009)

⁶Nic Wilson. "Extending CP-Nets with Stronger Conditional Preference Statements". In: AAAI-04. 2004

Preference Modeling

Q: How do we represent qualitative preferences over combinatorial domains?

- ④ Preference Trees (e.g., **LP-trees**^{16,20}, **CLP-trees**¹⁷, **PLP-trees**¹⁸, **P-trees**^{11,21})

⁷Richard Booth et al. "Learning conditionally lexicographic preference relations". In: ECAI. 2010

⁸Xudong Liu and Mirosław Trzuszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

⁹Michael Bräuning and H Eyke. "Learning Conditional Lexicographic Preference Trees". In: Preference learning: problems and applications in AI (2012)

¹⁰Xudong Liu and Mirosław Trzuszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

¹¹Niall M Fraser. "Ordinal preference representations". In: Theory and Decision (1994)

¹²Xudong Liu and Mirosław Trzuszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

Preference Learning

Q: How do we learn predictive qualitative preference models over combinatorial domains?

- ① Ceteris Paribus Networks (e.g., CP-nets^{13,14,15})
- ② Preference Trees (e.g., LP-trees¹⁶, CLP-trees¹⁷, **PLP-trees**¹⁸)
- ③ **Preference Forests**¹⁹

¹³Jérôme Lang and Jérôme Mengin. "The complexity of learning separable ceteris paribus preferences". In: (2009)

¹⁴Frédéric Koriche and Bruno Zanuttini. "Learning conditional preference networks". In: Artificial Intelligence (2010)

¹⁵Yann Chevaleyre et al. "Learning ordinal preferences on multiattribute domains: The case of CP-nets". In: Preference learning. 2011

¹⁶Richard Booth et al. "Learning conditionally lexicographic preference relations". In: ECAI. 2010

¹⁷Michael Bräuning and H Eyke. "Learning Conditional Lexicographic Preference Trees". In: Preference learning: problems and applications in AI (2012)

¹⁸Xudong Liu and Mirosław Truszczynski. "Learning Partial Lexicographic Preference Trees over Combinatorial Domains". In: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI). 2015

¹⁹Xudong Liu and Mirosław Truszczynski. "Learning Preference Trees and Forests". In: IJCAI-16 (In Preparation)

Preference Reasoning

Q: How do we reason about preferences over combinatorial domains?

1 Preference Optimization^{20,21,22}:

- Dominance testing: $o_1 \succ_P o_2$?
- Optimality testing: $o_1 \succ_P o_2$ for all $o_2 \neq o_1$?
- Optimality computing: what is the optimal outcome wrt P ?
- Ranking: how are the outcomes ordered wrt P ?

2 Preference Misrepresentation²³:

- Control
- **Manipulation**²⁴
- Bribery

²⁰Xudong Liu and Miroslaw Truszczynski. "Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

²¹Xudong Liu and Miroslaw Truszczynski. "Preference Trees: A Language for Representing and Reasoning about Qualitative Preferences". In: Proceedings of the 8th Multidisciplinary Workshop on Advances in Preference Handling (MPREF). 2014

²²Xudong Liu and Miroslaw Truszczynski. "Reasoning with Preference Trees over Combinatorial Domains". In: Proceedings of the 4th International Conference on Algorithmic Decision Theory (ADT). 2015

²³Felix Brandt, Vincent Conitzer, and Ulle Endriss. "Computational social choice". In: Multiagent systems (2012)

²⁴Xudong Liu and Miroslaw Truszczynski. "Complexity of Manipulation in Elections Where Votes Are Lexicographic Preference Trees". In: AAMAS-17 (In Preparation)

Preference Applications

Q: What fields can we apply preferences to?

- ① Game Theory:
 - **Hedonic games**²⁵
- ② Automated Planning and Scheduling:
 - **Trip planning**²⁶
- ③ Data-Driven Decision Making:
 - **Predictive decisions**²⁷

²⁵Matthew Spradling et al. "Roles and Teams Hedonic Game". In: Proceedings of the 3rd International Conference on Algorithmic Decision Theory (ADT). 2013

²⁶Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

²⁷Xudong Liu and Mirosław Truszczynski. "Learning Preference Trees and Forests". In: IJCAI-16 (In Preparation)

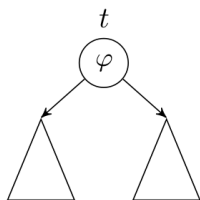
Preference Trees

- Let \mathcal{I} be a set of binary issues. The **combinatorial domain** $CD(\mathcal{I})$ is the set of *outcomes* represented by complete and consistent sets of literals over \mathcal{I} .
- A **P-tree** T over $CD(\mathcal{I})$ is a binary tree whose nodes, other than the leaves, are labeled with propositional formulas over \mathcal{I} .
- Given an outcome $M \in CD(\mathcal{I})$, the **leaf** $l_T(M)$ is the leaf reached by traversing the tree T according to M . When at a node N labeled with φ , if $M \models \varphi$, we descend to the left child of N ; otherwise, to the right.
- For $M, M' \in CD(\mathcal{I})$, we have $M \succ_T M'$ if $l_T(M) \succ_T l_T(M')$, and $M \approx_T M'$ if $l_T(M) = l_T(M')$. Outcome M is **optimal** if there exists no M' such that $M' \succ_T M$.

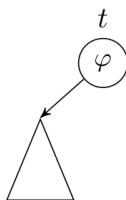
Compact Representation of P-trees

A *compact P-tree* over $CD(\mathcal{I})$ is a tree where

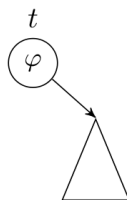
- ① every node is labeled with a Boolean formula over \mathcal{I} , and
- ② every non-leaf node t labeled with φ has either two outgoing edges (Figure (a)), or one outgoing edge pointing left (Figure (b)), right (Figure (c)), or straight-down (Figure (d)).



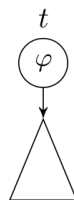
(a)



(b)



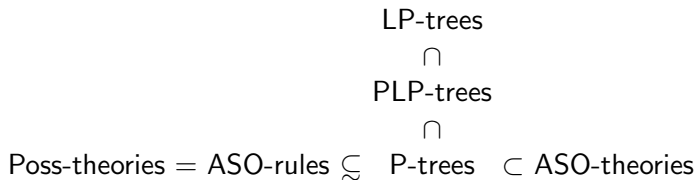
(c)



(d)

Figure : Compact P-trees

Relative Expressivity of Preference Languages



Computational Complexity Results

DOMTEST: is it that $o \succeq_T o'$ in P-tree T ?

OPTTEST: is outcome o optimal w.r.t T ?

OPTPROP: is there an optimal outcome o w.r.t T st $o \models \alpha$?

	DOMTEST	OPTTEST	OPTPROP
LP-tree	P	P	P
ASO-rule/ Poss-theory	P	coNP-c	$\Delta_2^P(P^{NP})$
P-tree	P	coNP-c ²⁸	$\Delta_2^P(P^{NP})$ -c ²⁹
ASO-theory	P	coNP-c	$\Sigma_2^P(NP^{NP})$ -c

Figure : Computational complexity results

²⁸The complement problem is reduced from the SAT problem.

²⁹The problem is reduced from the Maximum Satisfying Assignment (MSA) problem.

Partial Lexicographic Preference Trees (PLP-Tree)

A *PLP-tree* over $CD(\mathcal{I})$ is a labeled tree, where

- 1 every node t is labeled with a attribute $Attr(t)$ in \mathcal{I} and a conditional preference table $CPT(t)$,
- 2 every non-leaf node t has either one unlabeled outgoing edge or multiple outgoing edges labeled, each labeled by some value in $Dom(Attr(t))$, and
- 3 every attribute appears *at most* once on every branch.

Partial Lexicographic Preference Trees (PLP-Tree)

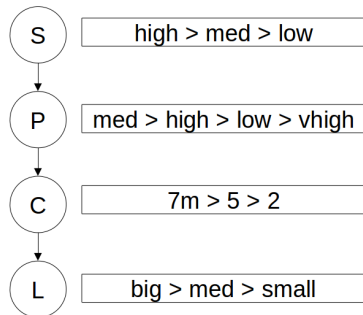


Figure : A UIUP PLP-tree

According to this UIUP PLP-tree, *Car1* is preferred to *Car2*.

Partial Lexicographic Preference Trees (PLP-Tree)

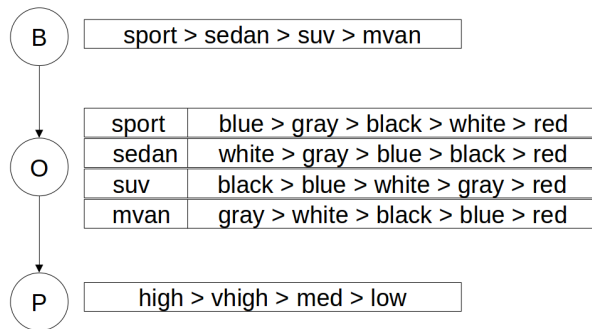


Figure : A UICP PLP-tree

According to this UICP PLP-tree, *Car2* is preferred to *Car1*.

Partial Lexicographic Preference Trees (PLP-Tree)

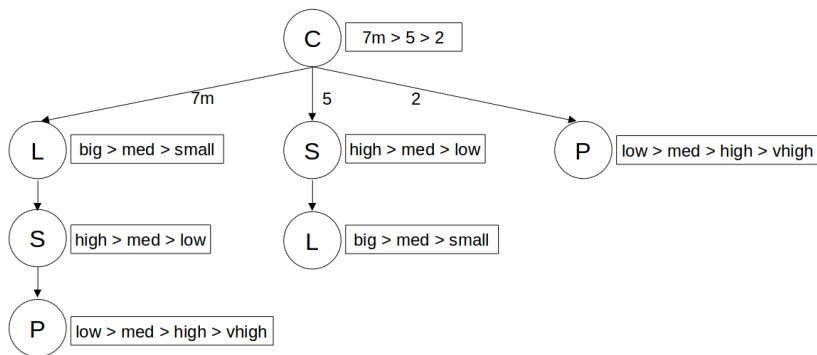


Figure : A CIUP PLP-tree

According to this CIUP PLP-tree, *Car1* is preferred to *Car2*.

Partial Lexicographic Preference Trees (PLP-Tree)

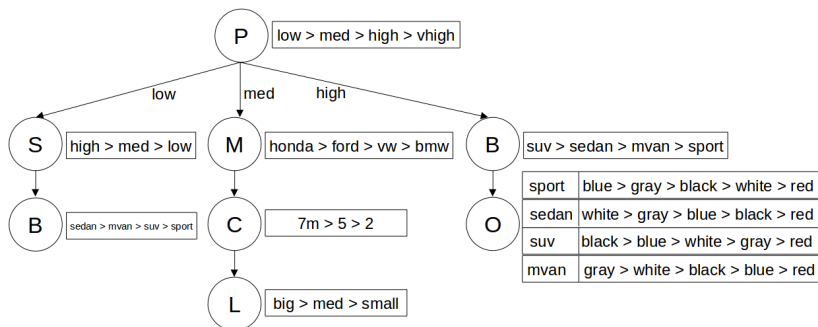


Figure : A CICIP PLP-tree

According to this CICIP PLP-tree, *Car1* is preferred to *Car2*.

Complexity Results

	UP	CP
UI	P	P
CI	NPC ³⁰	P

(a) CONSLearn

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) SMALLLearn

	UP	CP
UI	NPC ³¹	NPC
CI	NPC	NPC

(c) MAXLearn

Figure : Complexity results for passive learning problems

³⁰Booth et al., *Learning Conditionally Lexicographic Preference Relations*, 2010.

³¹Schmitt and Martignon, *On the Complexity of Learning Lexicographic Strategies*, 2006.

Experimental Results on Trees

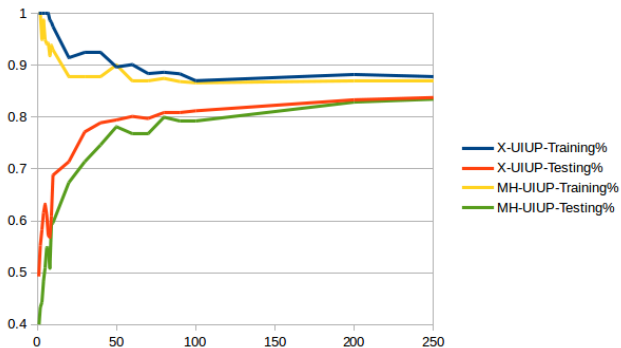


Figure : Learning curve for UIUP using ASP and greedy heuristic

Experimental Results on Trees

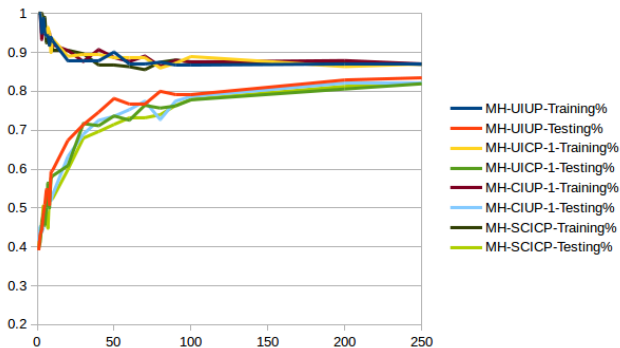


Figure : Learning curve for all four classes using greedy heuristic

Preference Forests

- ① A *preference forest* F is a collection of PLP-trees $F = \{T_1, \dots, T_n\}$.
- ② Denote by $N_F(o_1, o_2) = |\{T \in F : o_1 \succ_T o_2\}|$.
- ③ Given a preference forest F , and two outcomes o_1 and o_2 , we say that $o_1 \succ_F^{Maj} o_2$ iff $N_F(o_1, o_2) > N_F(o_2, o_1)$, and that $o_1 \approx_F^{Maj} o_2$ iff $N_F(o_1, o_2) = N_F(o_2, o_1)$.
 - Pro: intuitive, decided in polynomial time.
 - Con: Condorcet paradox.
 - Other aggregating rules: positional scoring rules, Copeland's method, etc.

Experimental Results on Forests

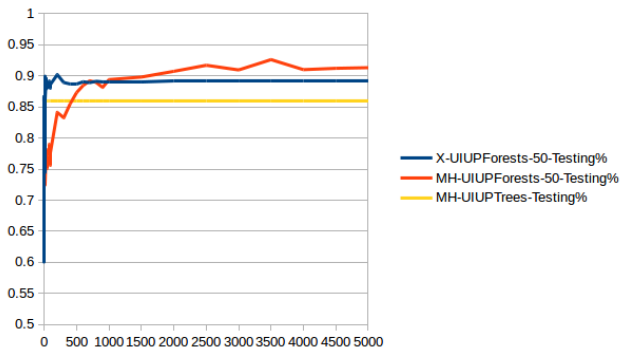


Figure : Learning UIUP using ASP and greedy heuristic

Experimental Results on Forests

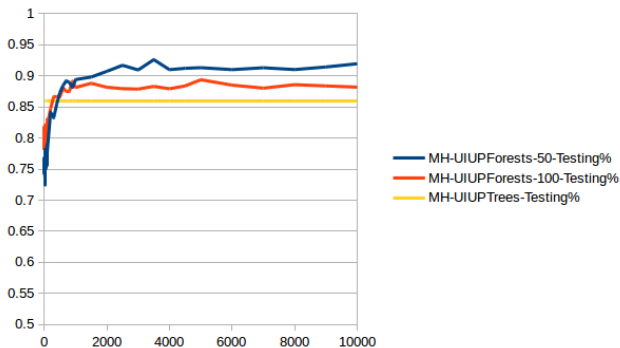


Figure : Learning all four classes using greedy heuristic

Lexicographic Preference Trees (LP-Trees)

- ① An *LP tree* \mathcal{L} over $\mathcal{I} = \{X_1, \dots, X_p\}$ is a (*binary*) tree, where
- each node t in \mathcal{L} is labeled by an issue from \mathcal{I} and with *preference information*, and
 - each issue appears **exactly once** on every path from the root to a leaf.

Positional Scoring Rules

- k -approval: $(1, \dots, 1, 0, \dots, 0)$ with k being the number of 1's and $m - k$ the number of 0's where $m = 2^P$.
- (k, l) -approval: $(a, \dots, a, b, \dots, b, 0, \dots, 0)$, where a and b are constants ($a > b$) and the numbers of a 's and b 's equal to k and l , respectively.
- b -Borda: $(b, b - 1, \dots, 0)$, where if $b > m - 1$, b -Borda is reduced to the regular Borda rule with $(m - 1, m - 2, \dots, 1, 0)$.

The Evaluation and Winner Problems

The Evaluation Problem

Let r be a positional scoring rule with a scoring vector w , \mathcal{C} a class of LP trees. Given a \mathcal{C} -profile P of n LP trees over p issues and a positive integer R , the *evaluation* problem is to decide whether there exists an alternative $o \in \mathcal{X}$ such that $s_w(o, P) \geq R$.

The Winner Problem

Let r be a positional scoring rule with a scoring vector w , \mathcal{C} a class of LP trees. Given a \mathcal{C} -profile P of n LP trees over p issues, the *winner* problem is to compute an alternative $o \in \mathcal{X}$ with the maximum score $s_w(o, P)$.

Complexity of the Evaluation Problem: k -Approval

	UP	CP
UI	P	P
CI	P	P

(a) $k = 2^{p-1} \pm f(p)$, $f(p)$ is a poly

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $k = 2^{p-c}$, $c > 1$ is a const

Figure : k -Approval

Complexity of the Evaluation Problem: (k, l) -Approval

	UP	CP
UI	P	P
CI	P	P

(a) $k = l = 2^{p-1}$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $k = l = 2^{p-c}$, $c > 1$ is a const

Figure : (k, l) -Approval ³²

³² Liu and Truszczynski, *Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming*, ADT, 2013.

Complexity of the Evaluation Problem: b -Borda

	UP	CP
UI	P	NPC
CI	NPC	NPC

(a) $b = 2^P - 1$

	UP	CP
UI	NPC	NPC
CI	NPC	NPC

(b) $b = 2^{P-c} - 1$, $c \geq 1$ is a constFigure : b -Borda

Modeling the Problems in ASP

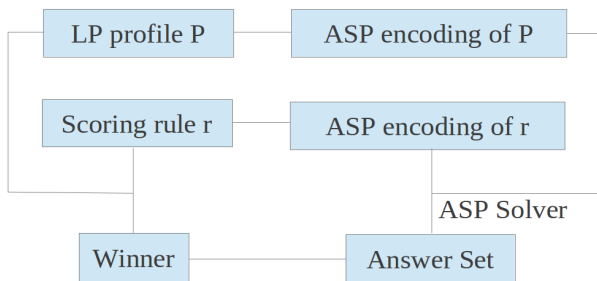


Figure : The winner problem

- Solvers: *clingo*³³, *clingcon*³⁴

³³M. Gebser et al. "Potassco: The Potsdam Answer Set Solving Collection". In: AI Communications (2011)

³⁴Max Ostrowski and Torsten Schaub. "ASP modulo CSP: The clingcon system". In: TPLP (2012)

Modeling the Problems in W-MAXSAT

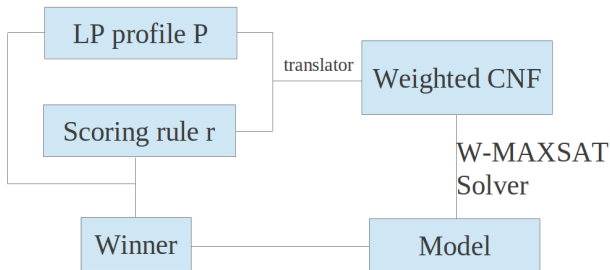


Figure : The winner problem

- Solver: *toulbar*³⁵

³⁵ M Sanchez et al. "Max-CSP competition 2008: *toulbar2* solver description". In: the Third International CSP Solver Competition (2008)

Random LP Profiles

- To experiment with LP profiles, we developed methods to randomly generate *encodings* of a special type of CI-CP LP tree of size linear in the number of issues

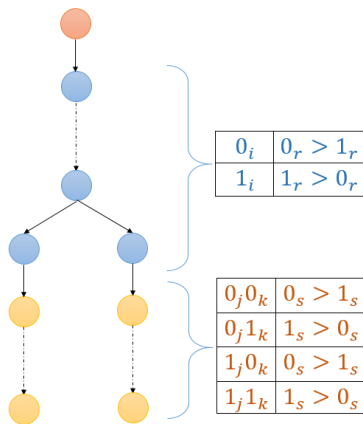
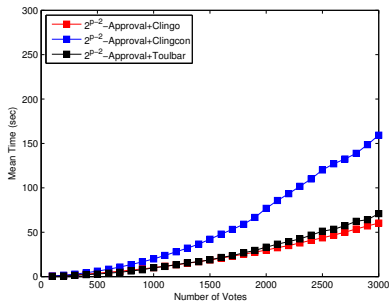
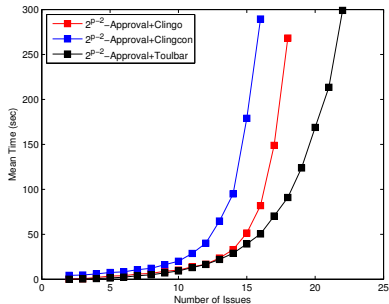


Figure : Random LP tree

Varying p and n : 2^{p-2} -approval



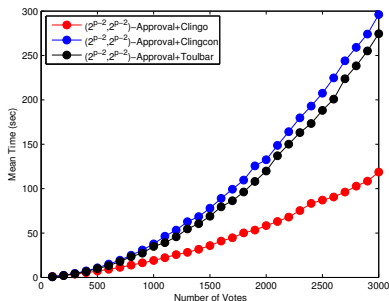
(a) Fixed #issues (10)



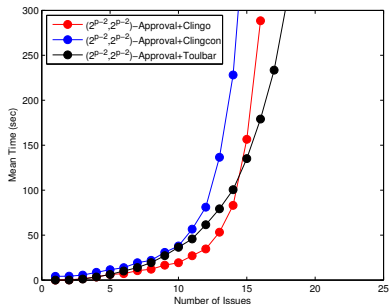
(b) Fixed #votes (1000)

Figure : Solving the winner problem

Varying p and n : $(2^{p-2}, 2^{p-2})$ -approval ³⁶



(a) Fixed #issues (10)

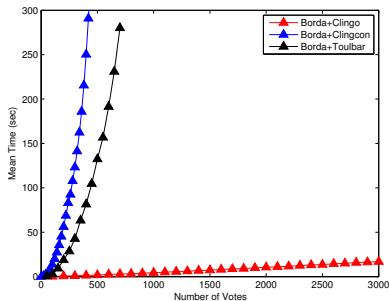


(b) Fixed #votes (1000)

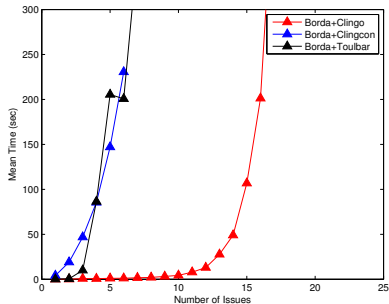
Figure : Solving the winner problem

³⁶ scoring vector: $(2, \dots, 2, 1, \dots, 1, 0, \dots, 0)$ with the numbers of 2's and 1's equal to 2^{p-2}

Varying p and n : Borda



(a) Fixed #issues (10)



(b) Fixed #votes (1000)

Figure : Solving the winner problem

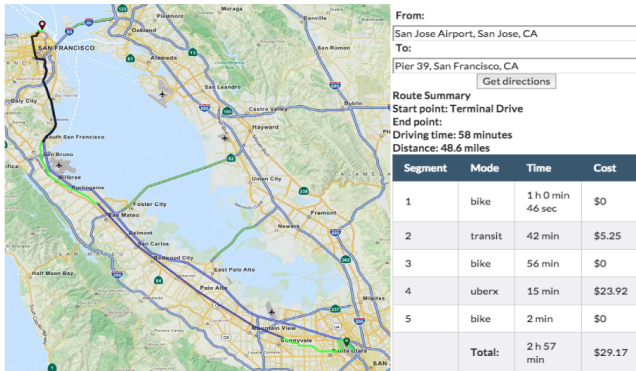
Personalization in Trip Planning

- ➊ Important to incorporate user constraints and preferences into trip planning systems.
- ➋ Collaboration with experts (in AI, planning, optimization, multi-agent systems) at PARC.
- ➌ Developed a hipergraph-based trip planner that accommodates constraints specified as *linear temporal logic* and preferences expressed as *preferential cost function* to compute optimal routes using A^* ³⁷.
- ➍ Available later for trip planning in the Bay Area, LA, and Denver.

³⁷Xudong Liu et al. "On Personalizability and Extensibility of Multi-Modal Trip Planning". In: PARC Symposium. 2015

Personalization in Trip Planning

- 1 From SJC, to Pier 39, Monday, 9am.
- 2 Constraints: never drive a car, and bike for 1 to 2 hours.
- 3 Preferences: bike = public (0.25) > wait(2) > walk(3), and 30\$/hr.



Preference Learning

Goal: preference predicting and feasibility of various preference models.

① Recommender Systems:

- ① Collaborative
- ② Content-based
- ③ Hybrid

② Machine Learning:

- ① Supervised learning
- ② Label ranking

③ Preference Elicitation:

- ① Positive learning
- ② Human-in-the-loop

④ Preference Learning:

- ① Conditional Preference Networks
- ② Preference Trees (e.g., LP-trees, CLP-trees, PLP-trees)
- ③ Stochastic Models (e.g., Choquet integral, TOPSIS-like models)

Preference Reasoning

Goal: personalized optimization and collaborative decision making.

① Preference Optimization:

- ① Consistency testing
- ② Dominance testing
- ③ Optimality testing

② Preference Aggregation:

- ① Social choice and welfare
- ② Voting systems

③ Preference Misrepresentation:

- ① Control
- ② Manipulation
- ③ Bribery

Preference Applications

Goal: exploring possibilities of collaboration with experts in other areas.

- ① Automated Planning and Scheduling:
 - ① Travel scheduling
 - ② Manufacturing
 - ③ Traffic control
- ② Computer Vision and Image Processing:
 - ① Image retrieval
 - ② Image and video understanding

Questions?

Thank you!