JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 53, No. 1, Feb. 2018, pp. 269–298

doi:10.1017/S0022109017001041

# Pitfalls in the Use of Systemic Risk Measures

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## Abstract

We examine pitfalls in the use of return-based measures of systemic risk contributions (SRCs). For both linear and nonlinear return frameworks, assuming normal and heavy-tailed distributions, we identify nonexotic cases in which a change in a bank's systematic risk, idiosyncratic risk, size, or contagiousness increases the risk of the system but lowers the measured SRC of the bank. Assessments based on estimated SRCs could thus produce false interpretations and incentives. We also identify potentially adverse side effects: A change in a bank's risk structure can make the measured SRC of its competitors increase more strongly than its own.

## I. Introduction

A measure of systemic risk aims to quantify how much an entity, be it a bank or hedge fund or sovereign, contributes to the vulnerability of the financial system. Recent years have seen a strong interest in refining such measures. Judging from the citation frequency, the two most influential concepts seem to be the conditional value at risk (CoVaR) family of measures proposed by Adrian and Brunnermeier (2016) and the marginal expected shortfall (MES) of Acharya, Pedersen, Philippon, and Richardson (2017).[1] Originally intended for use in bank regulation, the literature now not only discusses these measures in conjunction with regulation but also employs them for a variety of purposes: to examine whether systemic risk is priced (Meine, Supper, and Weiß (2015), Nucera, Schwaab, Koopman, and Lucas (2016)), to measure whether banks benefit from their too-big-to-fail

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[1]On Jan. 17, 2018, Google Scholar showed 1,788 citations for Adrian and Brunnermeier (2016) and 1,493 for Acharya et al. (2017).

status (Barth and Schnabel (2013)), to examine which funding channels or instruments are most important for systemic risk (López-Espinosa, Moreno, Rubia, and Valderrama (2012), Battaglia and Gallo (2013)), or to measure the contagion potential of sovereigns (Fong and Wong (2012)).

The literature on value at risk (VaR) has shown that the properties of risk measures and the consequences of choosing a specific measure for a particular purpose are not immediately obvious (cf. Artzner, Delbaen, Eber, and Heath (1999), Basak and Shapiro (2001)). With this article, we want to contribute to a better understanding of systemic risk measures by pointing out pitfalls that can arise in typical applications. We examine whether the measures can give conflicting or misleading signals, for example, indicate that a change in a bank's characteristics lowers its systemic risk contribution even though the change increases the risk of the system. Although we mainly interpret our results for a situation in which the measures are used in bank regulation, conclusions carry over to other applications. Wherever researchers build an analysis on measures of systemic risk, the reliability of their results depends on the quality of the measures, and on how well their properties are understood.

The measures that we examine are the MES, the $\Delta$CoVaR, the exposure $\Delta$CoVaR, and the BETA. They have in common that stock returns are the key input for empirical measurement. MES (Acharya et al. (2017)) is defined as an institution's average equity return on days in which the market return is below its 5% quantile. Originating in portfolio theory and suggested by its name (but not necessarily obvious from the definition), MES is a marginal risk measure in that it equals the gradual change in a risk measure, that is, the expected shortfall of the whole system's risk, when an institution gains weight in the system.

Risk contribution can also be understood as the change in systemic risk when an institution gets into distress rather than when it is added to the system. This is the intuition behind the $\Delta$CoVaR measure introduced by Adrian and Brunnermeier (2016). It is the VaR of the system conditional on an institution being in distress, compared to the system's VaR conditional on the institution being in normal condition. Adrian and Brunnermeier also consider the VaR of an institution conditional on the system being in distress. The corresponding measure, called exposure $\Delta$CoVaR, is more akin to portfolio theoretic concepts such as MES.

Several authors (e.g., Acharya et al. (2017), Benoit, Colletaz, Hurlin, and Pérignon (2013)) also analyze the BETA, not necessarily as a candidate systemic risk measure but rather as a benchmark for the proposed measures. We include it for the same reason.

We start by examining within a linear market model framework how the measures respond to differences in systematic and idiosyncratic risk as well as in size. We find that the $\Delta$CoVaR responds to idiosyncratic risk in an ambiguous way. When applied in regulation, the use of $\Delta$CoVaR could create incentives for banks to increase idiosyncratic risk in order to lower their estimated systemic risk contribution. The way in which each of the four measures reacts to systematic risk (as measured through the exposure to a common factor) can cause similar problems, provided that an institution has a large weight in the system. With respect to size we find that each measure is susceptible to situations in which a change in a bank's size increases the estimated risk contribution even though the system risk is decreased.

For the sake of exposition, we focus on analytic derivations that assume a multivariate normal distribution for returns. Simulations with multivariate $t$-distributions as well as with a dynamic structural model show that the effects also appear in the presence of heavy tails and time-varying variances and sensitivities, and that they do not depend much on whether equity returns or asset returns are taken as a basis for the analysis.

Next, we examine a contagion framework in which negative shocks to one bank spill over to other banks. Even simple contagion structures can lead to a complex behavior of the four systemic risk measures. Some measures, notably the $\Delta$CoVaR, have a tendency to assign a low systemic risk to infectious banks; others tend to do the opposite.

Our results are fundamental in the sense that we point out possible pitfalls that can arise in typical situations, with respect to key characteristics such as idiosyncratic risk or infectiousness. This should help in improving the use of existing measures, as well as in refining them. Deriving an exhausting set of axiomatic requirements for systemic risk measures and benchmarking existing proposals against them, though desirable, is beyond the scope of our article. To illustrate why this is difficult to achieve, consider the case of contagion. Not only are there many definitions of contagion, as discussed by Pericoli and Sbracia (2003), it is also not evident which properties the measures should have. Although it may seem intuitive to require that the systemic risk contribution of an infectious bank should, ceteris paribus, be larger than that of an infected bank, this is not obvious if the measure is used to identify risk control strategies. A regulator trying to increase the stability of a system could do so by either making banks more resilient that are likely to get infected, or lowering the contagion intensity of contagious banks. Which option is preferred depends on the costs of bringing about changes in resilience or infectiousness, the way in which banks respond to incentives, and other criteria applied by the regulator.

Closely related to our work are papers that explore the properties and limitations of systemic risk measures. Mainik and Schaanning (2014) suggest a modification of $\Delta$CoVaR to improve its dependence consistency. We examine whether the modification would mitigate concerns we raise in our article. Benoit et al. (2013) examine the similarity of risk rankings produced by different measures but do not examine whether the use of the measures can create unwanted incentives. Boucher, Kouontchou, and Maillet (2013), Danielsson, James, Valenzuela, and Zer (2016), and Löffler and Raupach (2013) discuss measurement problems, from which we abstract in our analysis. Guntay and Kupiec (2014) suggest separating systemic risk from systematic risk. Although there may be situations in which it is useful to disentangle systematic risk from the effects of spillovers or interactions, we follow others (e.g., Bisias, Flood, Lo, and Valavanis (2012), Allen and Carletti (2013)) and employ an inclusive definition of systemic risk because a systematic shock such as the bursting of a bubble can be sufficient to create jeopardizing systemwide losses.

Apart from the measures we focus on because of their widespread application, there are several other return-based measures, many of which are related to the MES and the CoVaR; other branches of the literature employ holdings-based and network-based analyses of systemic risk. For an overview of the extensive literature, see Bisias et al. (2012), Benoit, Colliard, Hurlin, and Pérignon (2017), and Hüser (2015).

The remainder of the article is structured as follows: Section II introduces the systemic risk measures studied in this article. Section III discusses possible problems in a linear return setting, and Section IV introduces contagion. Section V concludes. The Internet Appendix, consisting of Sections A–D, is available at www.jfqa.org.

## II. Systemic Risk Measures Studied in This Article

### A. ΔCoVaR and Exposure ΔCoVaR

Adrian and Brunnermeier (2016) suggest measures based on what they call CoVaR, which is implicitly defined through

$$\mathbf{P}\Big( X^j < -\mathrm{CoVaR}_\alpha^{j|C(X^i)} \Big| C\big(X^i\big)\Big) \quad = \quad \alpha,$$

where $X^j$ is a return and $\alpha > 0$ is a typically small number.[2] CoVaR is the VaR of object $j$ conditional on event $C$ happening to object $i$. Adrian and Brunnermeier suggest examining

$$\Delta\mathrm{CoVaR}_\alpha^{j,i} \quad \equiv \quad \mathrm{CoVaR}_\alpha^{j|\{X^i=-\mathrm{VaR}_\alpha^i\}} - \mathrm{CoVaR}_\alpha^{j|\{X^i=\mathrm{MEDIAN}^i\}}.$$

$\Delta\mathrm{CoVaR}_\alpha^{j,i}$ measures the change in the $\alpha$-VaR of $j$ conditional on $i$ moving from its median state to its own $\alpha$-VaR. Adrian and Brunnermeier mostly examine the case in which $j$ is given by the overall system, that is, a market index or a collection of banks, and $i$ is an individual institution; this is called ΔCoVaR throughout this article. However, Adrian and Brunnermeier also consider the opposite direction in what they call exposure ΔCoVaR, which is defined through

$$\Delta\mathrm{CoVaR}_\alpha^{j,\mathrm{SYSTEM}} \equiv \mathrm{CoVaR}_\alpha^{j|\{X^{\mathrm{SYSTEM}}=-\mathrm{VaR}_\alpha^{\mathrm{SYSTEM}}\}} - \mathrm{CoVaR}_\alpha^{j|\{X^{\mathrm{SYSTEM}}=\mathrm{MEDIAN}^{\mathrm{SYSTEM}}\}}.$$

$\Delta\mathrm{CoVaR}_\alpha^{j,\mathrm{SYSTEM}}$ is the change in the VaR of object $j$ conditional on the system moving into distress. The measure is more akin to MES and BETA than $\Delta\mathrm{CoVaR}_\alpha^{\mathrm{SYSTEM},j}$.

Adrian and Brunnermeier (2016) estimate the CoVaR with a quantile regression over 25 years of weekly data, choosing a confidence level $\alpha$ of 1%. We abstract from estimation problems by deriving results through closed-form expression, or Monte Carlo simulations with a large number of observations.

[2] CoVaR definitions vary in the literature. Taking $\alpha$ to be close to 0 is consistent with Acharya et al. (2017) and earlier versions of Adrian and Brunnermeier (2016), but not with the published version. Also the tail events are not fully consistent, depending on whether the "$\le$" operator is applied to losses or returns, which may matter if the conditional cumulative distribution function (CDF) of $X^j$ has flat parts. Furthermore, the above definition implicitly excludes that the CDF crosses the level $\alpha$ by a jump. Our article is not affected by these issues as all return variables are continuously distributed with positive densities, including the conditional distributions considered.

Adrian and Brunnermeier (2016) suggest that in the presence of time-varying risk, the precision of CoVaR estimates can be improved by conditioning the return-based estimates on current fundamental information. In most of our analysis, we consider static return frameworks. As we also abstract from estimation error, unconditional return-based estimates are optimal and fundamental information would not increase precision. The static framework also implies that a state-dependent modeling as in Adams, Füss, and Gropp (2014) would not enhance the informativeness of the CoVaR analysis.

In the base case, we do not model differences between asset returns, the use of which is advocated by Adrian and Brunnermeier (2016), and equity returns. This is done for the sake of exposition, and seems justified given that for the short return horizons examined in the literature, the return distributions do not differ greatly except for their volatility. In a robustness check (Section III.C), we show that conclusions are preserved insofar as all problems identified in the base case continue to exist when we use a dynamic structural credit risk model to differentiate between asset and equity returns, and when we introduce heavy tails and tail dependence into the return distributions of the base-case model. However, further problematic effects appear that we do not observe in the base case.

## B. Marginal Expected Shortfall

The MES put forward by Acharya et al. (2017) is defined as

$$\mathrm{MES}_i \quad \equiv \quad -\mathrm{E}\big(R_i | R_S < Q_S^\alpha\big),$$

where $R_i$ denotes the net equity return of institution $i$, $R_S$ is the system return, and $Q_S^\alpha$ is the quantile of the system return on level $\alpha$. Acharya et al. examine daily returns with a confidence level of 5%.

In Acharya et al. (2017), the system return is proxied by the Standard & Poor's (S&P) 500; that is, the authors include nonfinancial firms in the system. We deviate from this approach and follow Adrian and Brunnermeier (2016), who empirically specify the system as consisting of financial institutions only. The system's scope matters in our analysis when we inspect the situation in which a single bank is very large compared to the system. Such a situation would be less likely if the system were taken to be the whole economy. Hence, our setup includes the perspective of a smaller country's financial system with a few big players (e.g., precrisis Iceland); size is one of the parameters a systemic risk charge might react to in an undesired way.

Acharya et al. (2017) combine the MES measure with other information such as capital and size to calculate the *systemic expected shortfall*, which approximates the expected dollar amount by which a bank's capital falls short of regulatory capital, conditional on system distress; this is also the idea behind the SRISK measure analyzed by Brownlees and Engle (2017) and Acharya et al. (2017). Although these extensions may help capture important aspects of systemic risk, we stay with MES, primarily because it is defined on the scale of returns, as are the other measures considered in this article. Comparing a measure in dollars such as SRISK with nondimensional measures of returns would require further assumptions on how the latter are scaled to dollar amounts; this is not necessary with MES. Furthermore, any problems we identify with the return measure MES would also matter for dollar measures that combine MES with other information.

### C. BETA

BETA, the classic way to measure systematic risk, is defined as the regression coefficient in

$$(1) \qquad R_i = a_i + \mathrm{BETA}_i R_S + u_i,$$

where $R_i$ is the return of an individual bank and $R_S$ is typically an index return. As with the other measures, we understand it as an index of banks. We write $\mathrm{BETA}_i$, not $\beta_i$, to highlight its role as a systemic risk measure, and reserve the Greek letter for the loading on a latent common factor.

Although the difference between systemic risk (the danger of a breakdown of the financial system) and systematic risk (the exposure to common risk factors) is well acknowledged, a greater amount of the latter is likely to increase the former. For this reason, BETA can also give an indication of systemic risk. Because of this and the widespread use of BETA in finance, we examine its properties and compare them to those of other measures, similar to Acharya et al. (2017) and Benoit et al. (2013).

Gauthier, Lehar, and Souissi (2012) test different capital allocation rules for their potential to improve system stability. Although their "component value-at-risk" allocation is also given the attribute "beta" in parentheses, it is more akin to MES and exposure ΔCoVaR (at least in a linear setup with normal distributions; cf. Section III.A). Also, and despite similar names, the concept of BETA introduced here and that of tail beta introduced by Straetmans, Verschoor, and Wolff (2008) only have the idea in common that individual returns (or losses) are traced back to common factors. The tail beta is actually a conditional probability and designed to be invariant to marginal distributions. These are large conceptual and numerical differences from BETA.

Before we proceed, a remark on stand-alone measures such as VaR or expected shortfall is in order. Systemic risk measures have been proposed to overcome shortcomings of stand-alone measures. For the return-generating models we study in the remainder of this article, VaR and expected shortfall increase with an increase in idiosyncratic or systematic risk, but their shortcomings are also manifest. Apart from the trivial fact that a bank's VaR or expected shortfall does not differentiate between systematic and idiosyncratic risk, these measures would also not change if the bank gained weight in the system or if it became more contagious.

## III. Systemic Risk Measures in the Linear Case

In this section, we use a linear factor model to examine whether the suggested measures for systemic risk fulfill elementary requirements with respect to a bank's choice of risk. We start with a linear combination of normally distributed factors, which we call the normal model. It allows analytical representations of the systemic risk measures, which we present first. Afterward, we investigate in which way the measures depend on risk parameters. Assuming that the parameters are under control of the banks, the sensitivities reveal potential incentives that a systemic risk measure would put on banks if applied as a systemic risk charge. Control over risk parameters will never be complete (and will have multiple other effects, e.g., on profitability), but clearly banks have more options to steer the risk of their business than average industrial firms.

As motivated in the introduction, we do not aim at judging systemic risk measures according to how useful they would be if systemic risk were regulated according to some optimum principle. We leave the way open in which a systemic risk charge or other regulatory measures are implemented but assume that banks desire to appear to be of low systemic importance, according to the systemic risk measure in place. Hence, we inspect whether sensitivities to risk parameters have appropriate signs. Of course, other incentives may exist, in particular, benefits from receiving a too-big-to-fail status. In cases in which such incentives are stronger than the opposite incentives arising from the regulation of systemic risk, a change in the interpretation of the results may be required.

What matters for a bank is not only its own systemic risk but also the systemic risk of its competitors. If a bank's actions increase the systemic risk of competitors, the bank may have a desire to take such actions to gain a competitive advantage. We therefore consider different types of sensitivities: the impact of a bank's risk parameter on its own systemic risk measure as well as side effects on other banks.

We consider a banking system consisting of $N$ banks. $R_i$, the return of bank $i$, is determined by the exposure to a common risk factor $F$ and idiosyncratic risk $\varepsilon_i$. As explained above, we leave it open whether we have equity or asset returns in mind. Whether this neglect is justified is discussed in Section III.C by means of a robustness check. Both factor returns and idiosyncratic components are assumed to be independent normal random variates. The size of bank $i$ relative to the whole system is assumed to be $w_i$. Corresponding to the return type, size may be understood as market capitalization or total bank assets. We follow Adrian and Brunnermeier (2016) and take this system return to be the one that takes the role of a general market index, a choice that we also make in the estimation of MES and BETA.[3]

The system and its components are described through the following equations:

$$\text{(2)} \qquad R_i = \beta_i F + \varepsilon_i, \quad R_S = \sum_{i=1}^{N} w_i R_i$$

$$\text{with} \quad F \sim \mathrm{N}\left(\mu, \sigma_F^2\right), \quad \varepsilon_i \sim \mathrm{N}\left(0, \sigma_i^2\right), \quad \sum_{i=1}^{N} w_i = 1,$$

where $F$ and all $\varepsilon_i$s are independent, $\beta_i$ denotes the exposure to the common factor, and $R_S$ is the return on the banking system index. We assume that all $\beta_i$s are positive.

[3]We considered an alternative definition of the system return index that excludes the bank for which a systemic risk measure is computed. This does not solve key incentive problems that we document below, and creates additional problems. Results are available in Internet Appendix C.

### A. Analytic Expressions for the Risk Measures

To calculate measures of systemic risk, we need to specify conditional distributions. Owing to the linearity of the system and the normality of the random variables, we can approach the problem in a linear regression framework. We start with an analysis of CoVaR measures. When we condition $R_S$ on $R_i$, we study an orthogonal representation

$$R_S \quad = \quad c_i + d_i R_i + v_i, \quad R_i \perp v_i,$$

and obtain:

$$d_i \;=\; \frac{\text{cov}(R_S, R_i)}{\sigma^2(R_i)}, \quad c_i \;=\; \text{E}(R_S) - d_i \text{E}(R_i); \quad \sigma^2(v_i) \;=\; \sigma^2(R_S) - d_i^2 \sigma^2(R_i).$$

When we use $\Delta$CoVaR to study how the system is affected by a distress of bank $i$, we obtain:

$$\begin{aligned} \Delta\text{CoVaR}_\alpha^{S,i} &= -Q_\alpha(R_S \,|\, R_i = Q_\alpha(R_i)) + Q_\alpha(R_S \,|\, R_i = Q_{0.5}(R_i)) \\ &= -[c_i + d_i Q_\alpha(R_i) + Q_\alpha(v_i)] + [c_i + d_i Q_{0.5}(R_i) + Q_\alpha(v_i)] \\ &= -d_i [Q_\alpha(R_i) - Q_{0.5}(R_i)] \;=\; -d_i \sigma(R_i) \Phi^{-1}(\alpha), \end{aligned}$$

with $\Phi$ denoting the standard normal CDF. Expanding $d_i$ gives

$$\Delta\text{CoVaR}_\alpha^{S,i} \quad = \quad \frac{\text{cov}(R_S, R_i)}{\sigma(R_i)} \Phi^{-1}(1-\alpha). \tag{3}$$

We now turn to what Adrian and Brunnermeier (2016) call exposure $\Delta$CoVaR. For this measure, $R_i$ has to be conditioned on $R_S$ rather than $R_S$ on $R_i$. We therefore study

$$R_i \quad = \quad a_i + b_i R_S + u_i \tag{4}$$

(which is also the definition equation (1) of the BETA measure) to obtain:

$$b_i = \frac{\text{cov}(R_S, R_i)}{\sigma^2(R_S)}, \quad a_i = \text{E}(R_i) - b_i \text{E}(R_S), \quad \sigma^2(u_i) = \sigma^2(R_i) - b_i^2 \sigma^2(R_S). \tag{5}$$

When we use exposure $\Delta$CoVaR to study how bank $i$ is affected by the system, similar calculations as for $\Delta$CoVaR give:

$$\Delta\text{CoVaR}_\alpha^{i,S} \quad = \quad \frac{\text{cov}(R_S, R_i)}{\sigma(R_S)} \Phi^{-1}(1-\alpha). \tag{6}$$

The next measure considered is the MES. As in exposure $\Delta$CoVaR, a bank's return is conditioned on the system return. We therefore start from equation (4) to derive:

$$\begin{aligned} \text{MES}_i &= -\text{E}\big(R_i \,\big|\, R_S < Q_S^\alpha\big) \;=\; -\text{E}\big(a_i + b_i R_S + u_i \,\big|\, R_S < Q_S^\alpha\big) \\ &= -a_i - b_i \text{E}\big(R_S \,\big|\, R_S < Q_S^\alpha\big) - \text{E}\big(u_i \,\big|\, R_S < Q_S^\alpha\big). \end{aligned}$$

By construction of an ordinary least squares regression, $u_i$ and $R_S$ are uncorrelated. Because their joint distribution is multivariate normal (as both are linear images of independent normals), they are independent, so that $\mathrm{E}\big(u_i \,\big|\, R_S < Q_S^\alpha\big)$ is 0. We therefore obtain

$$\mathrm{MES}_i = -a_i - b_i \mathrm{E}\big(R_S \,\big|\, R_S < Q_S^\alpha\big)$$

and, expanding $a_i$,

$$
\begin{aligned}
(7)\qquad \mathrm{MES}_i &= -\mathrm{E}(R_i) - b_i\,\mathrm{E}\big(R_S - \mathrm{E}(R_S) \,\big|\, R_S < Q_S^\alpha\big) \\
&= -\beta_i \mu - b_i\,\sigma(R_S)\,\mathrm{E}\big(Z \,\big|\, Z < \Phi^{-1}(\alpha)\big), \quad Z \sim \mathrm{N}(0,1) \\
&= -\beta_i \mu - \frac{\mathrm{cov}(R_S, R_i)}{\sigma(R_S)} \mathrm{E}\big(Z \,\big|\, Z < \Phi^{-1}(\alpha)\big) \\
&= -\beta_i \mu + \frac{\mathrm{cov}(R_S, R_i)}{\sigma(R_S)} \frac{\phi\big(\Phi^{-1}(\alpha)\big)}{\alpha}.
\end{aligned}
$$

The last transform is a familiar result for truncated normal distributions. Comparing equations (6) and (7), MES and exposure ΔCoVaR turn out to be essentially substitutes in a linear model of normals, in that they differ only by a constant factor and a shift that is typically small over a short-term horizon.

BETA has a very simple form in the linear setup. Comparing equations (1) and (4) and applying equation (5), $\mathrm{BETA}_i = \mathrm{cov}(R_S, R_i)/\sigma^2(R_S)$. Note that the weighted average of all $\mathrm{BETA}_i$s equals 1 by construction, which makes a difference in our sensitivity analyses where we let a single $\beta_i$ increase while keeping the others constant. This is impossible for $\mathrm{BETA}_i$, as there is always a compensating change in the $\mathrm{BETA}_j$ of other banks. The difference between $\mathrm{BETA}_i$ and $\beta_i$ is therefore not just a notational one, besides the influence of idiosyncratic risks on $\mathrm{BETA}_i$.

The following formulas summarize the systemic risk measures in our linear model:

$$
\begin{aligned}
(8)\qquad \Delta\mathrm{CoVaR}_\alpha^{S|i} &= \frac{\mathrm{cov}(R_S, R_i)}{\sigma(R_i)} \Phi^{-1}(1-\alpha), \\
\Delta\mathrm{CoVaR}_\alpha^{i|S} &= \frac{\mathrm{cov}(R_S, R_i)}{\sigma(R_S)} \Phi^{-1}(1-\alpha), \\
(9)\qquad \mathrm{MES}_i &= -\beta_i \mu + \frac{\mathrm{cov}(R_S, R_i)}{\sigma(R_S)} \frac{\phi\big(\Phi^{-1}(\alpha)\big)}{\alpha}, \\
\mathrm{BETA}_i &= \frac{\mathrm{cov}(R_S, R_i)}{\sigma^2(R_S)}.
\end{aligned}
$$

The covariance between the individual and the system return is common and central to all four measures; it is then scaled down by a specific variation measure of either individual or system returns. Only MES deviates from this pattern by an additive component, which we show in Internet Appendix A.8 to be usually of low influence.

## B. Sensitivities to Risk Parameters

With the analytic expressions at hand, we can now investigate the sensitivities of systemic risk measures to changes in a bank's risk parameters. We study not only the effects of bank-specific idiosyncratic ($\sigma_i$) and systematic risk ($\beta_i$), but also of size[4] ($w_i$). Size is not literally a risk parameter but has an impact on the risk of the system, even in a simple setup such as ours, where it affects only the weights of individual returns in the average return $R_S$.

The volatility $\sigma_F$ of the systematic factor is another parameter that affects correlation with the system. We do not report the sensitivities to it because banks can influence it much less than the other parameters, or not at all.

As argued above, two kinds of sensitivity appear to be relevant.

First, there is the effect that a bank's parameter has on its own systemic risk measure; we call it the *direct effect* and quantify it by the partial derivative.

Second, there is a potential side effect on other banks' systemic risk measures. A bank could benefit if its relative risk ranking compared to other banks improves. We measure this side effect through the change in the ratio of a bank's systemic risk to the systemic risk of another, representative bank. We call this sensitivity the *relative effect*. Summing up, for a parameter $p_i$ applying to bank $i$ and one of the systemic risk measures, SRM, considered in this article, we calculate

$$\frac{\partial\, \mathrm{SRM}_i}{\partial p_i} \text{ (direct effect) and } \frac{\partial}{\partial p_i}\left[\frac{\mathrm{SRM}_i}{\mathrm{SRM}_j}\right], \quad j \neq i \text{ (relative effect).}$$

We focus on two basic properties the sensitivities should fulfill to avoid unwanted incentives and interpretations. If a change in the parameter of a bank increases the risk of the whole system, it should also increase the systemic risk measure of the bank. In addition, it should not lower the systemic risk relative to other banks. To give the two properties a precise meaning, we must specify what we understand by the risk of the system. We define it as $\sigma(R_S)$ but note that in our multivariate normal setup, $\mathrm{VaR}_\alpha(R_S)$ or expected shortfall $\mathrm{ES}_\alpha(R_S)$ are almost synonymous with the standard deviation, as regards monotonicity.[5]

Both for idiosyncratic and systematic risk, there is no ambiguity about what sensitivity is appropriate. If $\beta_i$ or $\sigma_i$ rises, ceteris paribus, this always increases $\sigma(R_S)$, so that bank $i$ should be assigned more systemic risk. Positive partial derivatives to $\beta_i$ and $\sigma_i$ are therefore appropriate.

By contrast, the case of size is ambiguous. If a bank's weight in the system increases, this can make $\sigma(R_S)$ rise or fall. For instance, when a bank with low $\beta_i$ and moderate idiosyncratic risk gains weight, $\sigma(R_S)$ can fall because of the low $\beta_i$ even though idiosyncratic risks become less diversified. Assigning such a

[4]A change in size ceteris paribus is understood as a dollar change (of total assets or market capitalization, depending on the return used), while the dollar value of the other banks remains constant. When the weight $w_i$ of a bank increases, the other banks' weights are therefore assumed to shrink proportionally, keeping the sum of all weights at 1.

[5]An opposite situation in which a parameter shift lets $\sigma(R_S)$ rise while $\mathrm{VaR}_\alpha(R_S) = -Q_\alpha(R_S)$ shrinks would require the VaR to be negative. Even for a particularly moderate tail (10%), low system volatility (10%), and long risk horizon (1 year), the annual drift of $R_S$ would have to exceed 12.8% to make the system VaR negative.

growing bank less systemic risk would then be appropriate, at least as long as the role of size is limited to its impact on the joint distribution of returns.[6]

Describing the signs that sensitivities can take is complex. We examine partial derivatives to determine the sensitivities to a certain parameter, thus keeping all other parameters fixed. However, the sign of the partial derivative can depend on the value of the other parameters that we take to be fixed. To get a quick but informative overview of the sensitivities, we provide the following information in Table 1:

i) In Panel A, we report the range of signs. A superscript $n$ marks cases where the stated range applies under normal conditions. Dark-gray cells mark cases where the sensitivities of the measure and the system risk can take opposite signs under realistic conditions.

ii) In Panel B, we report the signs of partial derivatives for a base-case parameterization where parameters are set to $\mathrm{E}(F)=0.05$, $\sigma_F=0.2$, $\sigma_i=0.2$, which are expressed on an annual basis; to translate them to daily returns, we divide $\mathrm{E}(F)$ by 260 and the standard deviations by $\sqrt{260}$. Further parameters are: $N=50$, $\beta_i=1$, $w_i=1/50$ for all $i$s, quantile levels $\alpha=0.01$ for the CoVaR measures, and 0.05 for MES. Light-gray cells mark cases in which the system's total risk is in a local minimum at $w_i=0.02$. We therefore have to distinguish upward changes of $w_i$ from downward changes.

iii) In Panel C, we report signs for a system with a dominant bank of high systematic risk. Parameters are: $\beta_1=1.5$, $w_1=0.3$, $w_j=0.7/49$ for $j>1$; other parameters are as in the base case.

All sensitivities are analyzed in various subsections of Internet Appendix A; see also Panel D for specific references.

Table 1 provides a mixed impression. On one hand, there are many cases in which a positive (direct or relative) effect combines with a positive effect on the system's total risk $\sigma(R_S)$. On the other hand, there are also many cases in which a negative effect on the measure can combine with a positive effect on $\sigma(R_S)$, or vice versa, under realistic conditions. We now look at some of the effects in more detail.

If a bank increases its idiosyncratic risk, there are two opposing effects on $\Delta$CoVaR: Inspecting the fraction in the representation of $\Delta\mathrm{CoVaR}_\alpha^{S|i}$ found in equation (8), the numerator $\mathrm{cov}(R_S, R_i)$ increases in idiosyncratic risk because the bank $i$ is part of the system. The relevant addend to the covariance is given by $w_i\sigma_i^2$ so that the effect on the numerator becomes smaller, the lower the weight of the bank within the system. The denominator, which equals $\sqrt{\beta_i^2\sigma_F^2+\sigma_i^2}$, is unaffected by the bank's weight, and the dependency on $\sigma_i$ has a substantial linear part. Taken together, assuming moderate values for $w_i$ and $\sigma_i$, the denominator's sensitivity in $\sigma_i$ will be stronger than that of the numerator so that $\Delta$CoVaR can

[6]Changing size may have additional effects, which, however, cannot be analyzed in our simple linear model. For example, an increase in size could lower system stability because a larger banking sector makes it more likely that systemic threats cannot be contained through government intervention.

TABLE 1

**Effect of Risk Parameters on Systemic Risk Measures**

In Table 1, we analyze sensitivities of systemic risk measures to certain risk parameters in a linear setting. Returns are described through $R_i = \beta_i F + \varepsilon_i$, $R_S = \sum_{j=1}^{N} w_j R_j$, with independent $F \sim \mathrm{N}(\mu, \sigma_F^2)$ and $\varepsilon_i \sim \mathrm{N}(0, \sigma_i^2)$. A *direct effect* of a parameter is understood as the partial derivative of a systemic risk measure. A *relative effect* refers to the ratio between the systemic risk measures of two banks. It is the ratio's partial derivative to a parameter of the bank in the numerator, for example, $\partial(\mathrm{MES}_i/\mathrm{MES}_j)/\partial\sigma_i$. Panel A presents possible signs of the derivatives. A superscript $n$ marks cases where the sign applies under normal conditions. Only very implausible parameter combinations, specified in Internet Appendix A, would generate the opposite sign. Panel B reports the partial derivatives' signs for the base case where parameters (per annum for drift and volatility) are set to $N=50$, $\mathrm{E}(F)=0.05$, $\sigma_F=0.2$, $\sigma_j=0.2$ , $\beta_j=1$, $w_j=1/50$ for all $j$; quantile level $\alpha=0.01$ for the conditional value at risk (CoVaR) measures and 0.05 for the marginal expected shortfall (MES). Panel C reports signs for a system with a dominant bank of high systematic risk. Parameters are: $\beta_i=1.5$, $w_i=0.3$, $w_j=0.7/49$ for $j \neq i$; other parameters as in the base case. Dark-gray cells mark cases where the sensitivities of the measure and the system's risk $\sigma(R_S)$ can take on opposite signs under realistic conditions (Panel A) or actually do so (Panels B and C). Light-gray cells in Panel B mark an ambiguous case where the sensitivity of $\sigma(R_S)$ to $w_i$ switches at $w_i=0.02$. A plus sign is then appropriate for $w_i>0.02$ and inappropriate otherwise.

| Parameter | Effect Type | ΔCoVaR | Exp. ΔCoVaR | MES | BETA |
|---|---|---|---|---|---|
| *Panel A. Range of the Sign of Partial Derivative* | | | | | |
| Idiosyncratic risk $\sigma_i$ | Direct | ± | + | + | + |
| | Relative | ± | + | $+^n$ | + |
| Systematic risk $\beta_i$ | Direct | + | + | $+^n$ | ± |
| | Relative | ± | ± | ± | ± |
| Size $w_i$ | Direct | ± | + | + | ± |
| | Relative | $+^n$ | $+^n$ | $+^n$ | $+^n$ |
| *Panel B. Sign of Partial Derivative in Base-Case Parameterization* | | | | | |
| Idiosyncratic risk $\sigma_i$ | Direct | – | + | + | + |
| | Relative | – | + | + | + |
| Systematic risk $\beta_i$ | Direct | + | + | + | + |
| | Relative | + | + | + | + |
| Size $w_i$ | Direct | + | + | + | + |
| | Relative | + | + | + | + |
| *Panel C. Sign of Partial Derivative in a Parameterization with One Risky and Dominant Bank* | | | | | |
| Idiosyncratic risk $\sigma_i$ | Direct | – | + | + | + |
| | Relative | – | + | + | + |
| Systematic risk $\beta_i$ | Direct | + | + | + | + |
| | Relative | + | + | + | + |
| Size $w_i$ | Direct | + | + | + | – |
| | Relative | + | + | + | + |
| *Panel D. References to Figures and Internet Appendix A (analytic results)* | | | | | |
| Idiosyncratic risk $\sigma_i$ | Direct | Base case, A.1 | A.5 | A.5 | A.4 |
| | Relative | Base case, A.11 | A.12 | A.12 | A.12 |
| Systematic risk $\beta_i$ | Direct | A.2 | A.7 | A.8 | A.6 |
| | Relative | Figure 3, A.14 | A.13 | A.13 | A.13 |
| Size $w_i$ | Direct | A.3 | A.10 | A.10 | Figure 4, A.9 |
| | Relative | A.16 | A.15 | A.15 | A.15 |

shrink while idiosyncratic risk rises. A systemic risk charge based on ΔCoVaR would therefore create an incentive for banks to increase their idiosyncratic risk, which would increase the volatility of the system return.

The partial derivative can be positive if the bank's weight or the idiosyncratic risk is large enough. To see which case probably prevails in practice, we analyze the partial derivative of ΔCoVaR more deeply in Internet Appendix A.1.

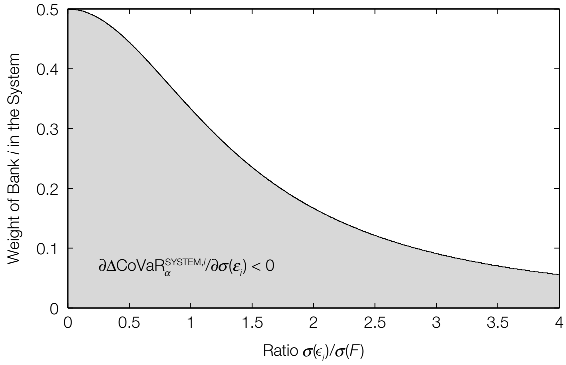
In Figure 1 we pick a special case where all $\beta_i$s equal 1. Then,

$$\frac{\partial \mathrm{CoVaR}_\alpha^{S,i}}{\partial \sigma_i} \propto \frac{w_i}{1-w_i} - \left(1 + \frac{\sigma_i^2}{\sigma_F^2}\right)^{-1}, \tag{10}$$

FIGURE 1

Regions in Which ΔCoVaR Is a Falling Function of Idiosyncratic Risk

In Figure 1, we examine a system with $N=50$ banks. We focus on the idiosyncratic risk of bank $i$, which has weight $w_i$ in the system. Returns are described through $R_i=\beta_i F+\varepsilon_i$, $R_S=\sum_{j=1}^{N} w_j R_j$, with independent $F\sim\mathrm{N}(\mu,\sigma_F^2)$ and $\varepsilon_i\sim\mathrm{N}(0,\sigma_i^2)$. For the graph, banks are assumed to have a uniform $\beta$ of 1. The $x$-axis is given by the ratio of idiosyncratic to systematic risk. The $y$-axis plots $w_i$; other banks have a uniform weight $(1-w_i)/49$. The figure shows those points as a gray area where the partial derivative $\partial\Delta\mathrm{CoVaR}_\alpha^{\mathrm{SYSTEM},i}/\partial\sigma_i<0$ is negative. CoVaR stands for conditional value at risk.




which gives rise to setting the ratio $\sigma_i/\sigma_F$ on the $x$-axis of Figure 1 and $w_i$ on the $y$-axis.

Points where the partial derivative is negative are displayed in gray. The area marks cases in which a bank striving for a low systemic risk measure might be tempted to increase its idiosyncratic risk. The area seems to cover most of the practically relevant cases. For instance, if the weight of a bank in the system is below 10%, the idiosyncratic risk can be up to 2.8 times larger than its exposure to the systematic factor and the bank still has an incentive to increase idiosyncratic risk. Even if the bank makes up one-third of the system, it still may be tempted to increase $\sigma_i$ as long as this is not larger than $\sigma_F$.

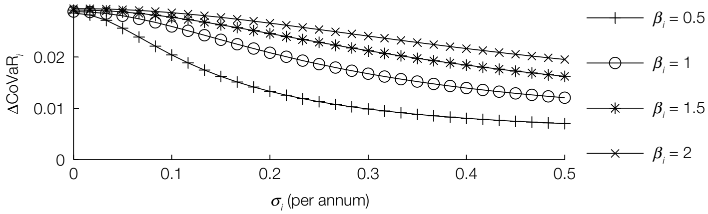
To give an idea of the strength of the effect, we provide an example in Figure 2 where ΔCoVaR is plotted against the per annum (p.a.) values of $\sigma_i$ between 0 and ½. Each line refers to a certain $\beta_i$; all other banks' $\beta_j$s are held constant at 1 and their idiosyncratic risk at 0.2 (p.a.). All banks have the same weight; further parameters are given in the figure's notes. The graph confirms the negative dependency on $\sigma_i$. Bank $i$ would benefit from large idiosyncratic risk most if its $\beta_i$ is low. For the example of $\beta_i=0.5$, assume the bank raises $\sigma_i$ from 0.15 to 0.25. It could lower its ΔCoVaR from 0.0162 to 0.0113, which is a reduction of 30.4%. For $\beta_i=1.5$, where the sensitivity is weaker, the reduction would amount to 13%.

To examine whether this adverse sensitivity can be mitigated by a recently suggested modification, we change the ΔCoVaR definition as suggested by Mainik and Schaanning (2014): When determining $\mathrm{CoVaR}_{0.01}$, we condition on $\left\{R_i\le-\mathrm{VaR}_{0.01}^i\right\}$ rather than on $\left\{R_i=-\mathrm{VaR}_{0.01}^i\right\}$. We use 100 million trials to simulate ΔCoVaR with this definition and find that the effects of idiosyncratic risk have the same sign and magnitude as with the standard definition. For example, if we start with the base parameters and increase idiosyncratic risk from

FIGURE 2

How ΔCoVaR Responds to Idiosyncratic Risk

In Figure 2, we examine the same system of $N=50$ banks as in Figure 1, with the following additional assumptions. Banks have equal weights 1/50. Except bank $i$, all banks are uniform. They have a $\beta_j=1$ and a standard deviation of idiosyncratic risk of 0.2 on an annual basis, the same as that of the systematic factor. On the $x$-axis, the idiosyncratic risk $\sigma_i$ of bank $i$ varies from 0 to 0.5. The figure plots the ΔCoVaR of bank $i$ on a daily basis at a quantile level of 0.01, for different exposures $\beta_i$ to the systematic factor. CoVaR stands for conditional value at risk.




20% to 30%, standard ΔCoVaR decreases by 22%, whereas ΔCoVaR modified according to Mainik and Schaanning decreases by 21%.[7]

The idiosyncratic risk of bank $i$ has no side effect on the ΔCoVaR of another bank $j$ as the term $\mathrm{cov}\left(R_S, R_j\right)/\sigma_j$ in equation (3) (applied to bank $j$) is insensitive to $\sigma_i$. The relative effect on ΔCoVaR is therefore just proportional to the direct effect.

However, direct and relative effects can also have opposite directions, as we show using the example of how ΔCoVaR reacts to systematic risk if general systematic risk is low ($\sigma_F=0.1$) and bank $i$ bears large idiosyncratic risks ($\sigma_i=0.4$). In Figure 3, $\beta_i$ varies between 0.5 and 2. Graph A indicates, regardless of the bank's weight, that ΔCoVaR always grows with $\beta_i$, as it should; in Internet Appendix A.2, we show that this holds in general. However, the side effect (Graph B) can be so strong that ΔCoVaR relative to other banks decreases. This is observed in Graph C in cases where bank $i$ has a very large weight in the system. If a bank were increasing its exposure to systematic risk, a hypothetical ΔCoVaR-based systemic risk charge would therefore "punish" this bank, but even more so its competitors.

To reveal the conditions under which the sensitivity to $\beta_i$ is so strange, we perform an approximation, assuming that all banks have negligible weights, except bank $i$. Defining $\kappa \equiv w_i/(1-w_i)$ as a monotonic function of $w_i$, $\beta_* \equiv (1-w_i)\sum_{j\neq i} w_j\beta_j$ as the other banks' weighted average of systematic risks, and $\gamma \equiv \beta_i/\beta_*$, we obtain

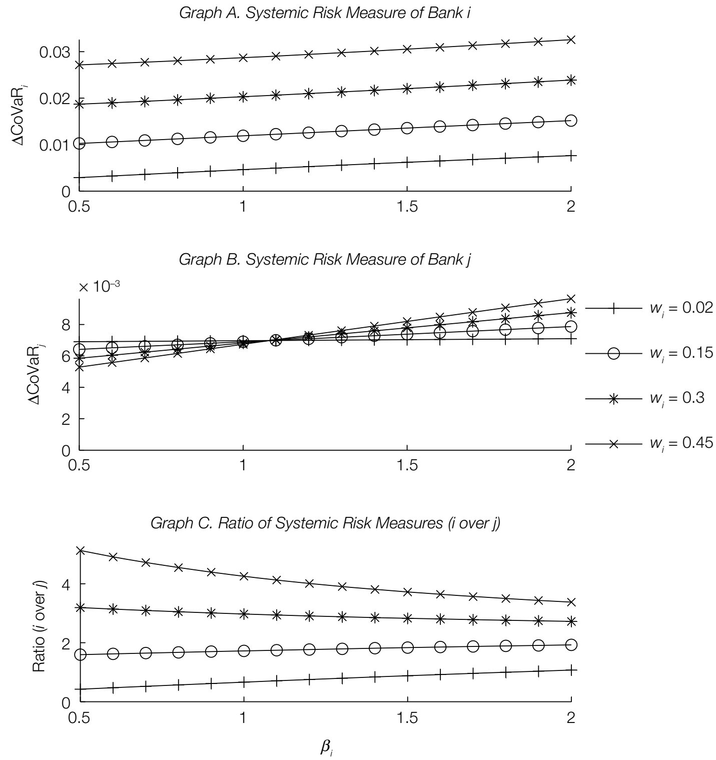
$$(11)\qquad \frac{\partial}{\partial\beta_i}\left[\frac{\Delta\mathrm{CoVaR}_\alpha^{S,i}}{\Delta\mathrm{CoVaR}_\alpha^{S,j}}\right] \quad\propto\quad \ldots \approx 1-\kappa\left(\kappa\frac{\sigma_i^2}{\sigma_F^2\beta_*^2}+\gamma(\kappa\gamma-1)\right),$$

[7]The finding appears intuitive. Mainik and Schaanning (2014) find that the modification is better able to capture dependence structures, but the problem driving our results is basically an errors-in-variables problem: We can regard ΔCoVaR as being based on a regression of the system return on the return of one bank (cf. Section III.A). In this regression, the bank return proxies for the systematic factor, with the measurement error being given by the idiosyncratic risk. The larger the idiosyncratic risk, the larger the measurement error and the larger the ensuing downward bias in the regression coefficient.

FIGURE 3

How ΔCoVaR Responds to Systematic Risk

In Figure 3, we examine the same system of $N = 50$ banks as in Figure 2, with the following modifications: $\sigma_i = 0.4$, $\sigma_F = 0.1$ (both per annum), $\beta_j = 1$ for $j \neq i$. On the $x$-axis, the systematic risk $\beta_i$ of bank $i$ varies from 0.5 to 2. Graph A plots the ΔCoVaR of bank $i$ on a daily basis at a quantile level of 0.01, for different exposure weights $w_i$ in the system. Graph B shows the ΔCoVaR for one of the other (uniform) banks. Graph C shows the ΔCoVaR of bank $i$ divided by that of another bank. CoVaR stands for conditional value at risk.




which is derived in Internet Appendix A.14. This expression tends to be negative when i) bank $i$ is large relative to the other banks; ii) its idiosyncratic risk is large relative to the other banks' average systematic risk, given by $\sigma_F \beta_*$; and iii) its $\beta_i$ is above the average.

In other words, if ΔCoVaR were used to define a systemic risk charge, the effect of $\beta_i$ on a bank's own risk charge would be weaker than that on its competitors' charge, and this disproportion would be most pronounced if the bank were already riskier and larger than the others. In a particularly competitive environment, the relative effect might be beneficial enough to outweigh the increased costs involved with the own risk charge. In this way, ΔCoVaR could set a risk-increasing incentive.

A similar relative effect can be observed for the other measures. Using the same numerical example as for Figure 3, where $\sigma_i$ is high and $\sigma_F$ is low, the ratio[8] $\text{SRM}_i/\text{SRM}_j$ is falling in $\beta_i$ if the weight of bank $i$ is exceptionally high.

[8]Here, SRM stands for exposure ΔCoVaR, MES, or BETA.

The partial derivative calculated in Internet Appendix A.13 gives the same result: Assuming for simplicity that the weight of bank $j$ in the system can be neglected, we find

$$\lim_{w_j \to 0} \frac{\partial}{\partial \beta_i} \left[ \frac{\text{SRM}_i}{\text{SRM}_j} \right] \propto 1 - \frac{w_i^2 \sigma_i^2}{\bar{\beta}^2 \sigma_F^2}, \tag{12}$$

where $\bar{\beta} \equiv \sum_{k=1}^{N} w_k \beta_k$ is a weighted average over all banks. Hence, $\beta_i$ has a negative relative effect under the condition that idiosyncratic risk, multiplied by the weight of bank $i$, exceeds average systematic risk, given by $\bar{\beta}\sigma_F$. There is a commonality with ΔCoVaR in that the relative effect on all four systemic risk measures tends to be negative if size and idiosyncratic risk are high and average systematic risk is low.

The same condition $w_i \sigma_i > \bar{\beta}\sigma_F$ can also turn BETA into a decreasing function of $\beta_i$ (see Internet Appendix A.6).

The parameters considered so far, $\sigma_i$ and $\beta_i$, have in common that a systemic risk measure should positively depend on them because the system's total risk $\sigma(R_S)$ always becomes larger if $\sigma_i$ or $\beta_i$ increases. As size can have the opposite effect, we now take a closer look at the factors that make $\sigma(R_S)$ grow or shrink when a bank gains size (recall that we treat size independently of its particular meaning in this part of the article, be it total assets or market capitalization; it is simply the weight $w_i$ in the definition of $R_S$).

In addition to average $\bar{\beta}$, which covers all banks, and $\beta_*$, which excludes bank $i$, we denote the part of $\sigma^2(R_S)$ that comes from idiosyncratic risks by $\sigma_*^2$. Mainly interested in the sign of partial derivatives, we find:

$$\frac{\partial \sigma^2(R_S)}{\partial w_i} \propto \bar{\beta}\sigma_F^2 [\beta_i - \beta_*] + w_i \sigma_i^2 - (1 - w_i)\sigma_*^2. \tag{13}$$

For the following arguments, it suffices to discuss a standard situation, assuming that bank $i$ is not particularly large and idiosyncratic risks are not excessive in general. We can then ignore the last two addends in equation (13) such that it mainly depends on the difference $\beta_i - \beta_*$ whether $\sigma^2(R_S)$ is a rising or falling function of $w_i$.

That either case is equally realistic reveals many of the systemic risk measures' size sensitivities in a different light. Except for the direct effects on ΔCoVaR and BETA, these effects are all positive (Panel A of Table 1). Each of them can thus easily co-occur with negative effects on $\sigma^2(R_S)$, which is inappropriate in that a systemic risk measure would "punish" a growing bank for making the system safer.

The size effect on ΔCoVaR is different because it can be positive or negative (technical details are found in Internet Appendix A.3). It could therefore switch the sign just when the effect on $\sigma^2(R_S)$ switches such that, in the end, ΔCoVaR might depend on size appropriately. This is partly true, as we can show that the inequality $\partial \Delta\text{CoVaR}_i / \partial w_i < 0$ implies $\partial \sigma^2(R_S)/\partial w_i < 0$ or, in words, that growth would be "rewarded" by ΔCoVaR only if this also makes the system safer. The opposite is not true, however. As with other measures that are positively dependent on size throughout, a positive size effect on ΔCoVaR can combine with a negative effect on $\sigma^2(R_S)$ under realistic conditions.

We now discuss the other ambiguous size effect, which is the direct effect on BETA. The general formula for the partial derivative is too complicated for a general analysis. We therefore make the simplifying assumption that only bank $i$ has a non-negligible weight in the system, whereas all other banks are infinitesimally small. Then they do not contribute idiosyncratic risks to the system anymore. As the resulting formula is still difficult to interpret, we consider the limiting case where $w_i$ is also small. Equation (A-8) in Internet Appendix A.9 gives

$$\lim_{w_i \to 0} \frac{\partial\,\mathrm{BETA}_i}{\partial w_i} \quad \propto \quad \sigma_i^2 - \beta_i[\beta_i - \beta_*]\sigma_F^2.$$

The derivative is negative if the bank's exposure to systematic risk is above the average and the idiosyncratic risk is comparably small. Even for arbitrarily large $\sigma_i$, there is always a $\beta_i$ above which the derivative becomes negative. If a systemic risk charge were based on $\mathrm{BETA}_i$ without further corrections, the direct effect of size would be particularly undesirable because exactly those banks that bear more systematic risk than the others, meaning $\beta_i > \beta_*$, would be rewarded for growth, whereas this growth would increase the variance of the system return.[9]

Numerical examples suggest similar effects in the general case where bank $i$ has a considerable size and the idiosyncratic risks of other banks are non-negligible. Figure 4 illustrates the extent to which a bank might benefit from growth if BETA were used for a systemic risk charge. We assume the base-case parameters for the systematic factor as well as for idiosyncratic risk, and take all banks except bank $i$ to have a $\beta_j$ equal to 1. Bank $i$'s weight in the system varies from 0 to 0.5 so that it can become very dominant relative to the other 49 banks with weight $w_j = (1 - w_i)/49$.

For different exposures $\beta_i$ to systematic risk, Graph A of Figure 4 shows how $\mathrm{BETA}_i$ depends on bank $i$'s weight. Up to $\beta_i = 1.5$, the bank would not benefit from growth, in sharp contrast to the case $\beta_i = 3$.[10] If bank $i$ grows from 0% to 29% of the system, ceteris paribus, it lowers its BETA by a full integer. At the same time, the index becomes more volatile; $\sigma(R_S)$ is increased from 20.2% to 32.2% p.a. Turning to the side effect (Graph B), we observe that the BETAs of the other banks decrease as well.[11] This decrease is so strong that the ratios $\mathrm{BETA}_i/\mathrm{BETA}_j$ are monotonic in $w_i$ (Graph C); that is, the relative size effect on BETA is

[9] Growth of $\sigma^2(R_S)$ can be concluded from equation (13), where infinite diversification makes $\sigma_*$ vanish.

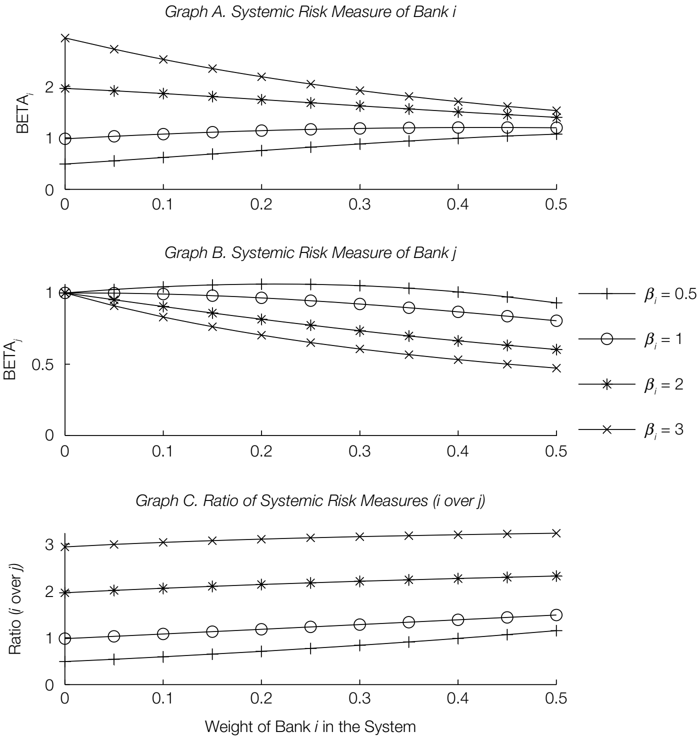
[10] This parameter, corresponding to BETAs between 2 and 3, appears fairly large. Here are some empirical observations for comparison. Stiroh (2006) reports an average BETA for banks of 0.45 at a standard deviation of 0.42. He finds a maximum BETA of 3.41. Note that these figures are BETAs for an index that covers all industries, unlike in our setup where only banks are included. A regression on an index of banks only would give an average BETA of 1, so that we might expect standard deviations for the BETAs at the order of $0.42/0.45 = 0.93$, as a rough approximation. BETAs above 2 or even 3 thus do not appear implausible. Related year-specific BETA estimates reported by Baele, De Jonghe, and Vander Vennet (2007) have a mean of 0.61 and annual standard deviations around 0.50, which suggests a standard deviation of BETAs around 0.82 ($= 0.5/0.61$) for a bank-based index.

[11] The finding that all BETAs decrease may be surprising at first glance, as their weighted average must equal 1. The decrease can be explained by noting that the high-BETA bank gains weight along with the fall in BETAs.

FIGURE 4

How BETA Responds to Size

In Figure 4, we examine the same system of $N = 50$ banks as in Figure 2. On the $x$-axis, the weight of bank $i$ varies from 0 to ½. Graph A shows, for different exposures $\beta_i$ to the systematic factor, the BETA of bank $i$, which is the regression coefficient $b_i$ in $R_i = a_i + b_i R_S + u_i$. It is calculated without estimation error according to equation (9). Graph B shows BETA for one of the other (uniform) banks. Graph C shows the BETA of bank $i$ divided by that of another bank.




positive. In Internet Appendix A.15, we analytically show that this is always the case.

Observing that at least the relative size effect on BETA is positive (albeit not necessarily appropriate, as shown above), we could try to align BETA with system risk by introducing a sensible scaling factor, for which our measure of system risk, $\sigma(R_S)$, is the most natural candidate. Doing so is in line with the "component value-at-risk" approach introduced by Gauthier et al. (2012) as one of their candidate rules for capital allocation.

Looking at equations (8) and (9) for representations of the other systemic risk measures, we find our scaled BETA to be $\text{cov}(R_S, R_i)/\sigma(R_S)$, which is essentially the same as exposure ΔCoVaR and MES, apart from constant factors and a typically small offset in the case of MES. As they both positively depend on size, we conclude that rescaling BETA would indeed help a bit but end in a concept equivalent to exposure ΔCoVaR.

To give an intermediate summary, a fairly clear message can be taken away from the sensitivities to single parameters. If a systemic risk charge for banks were defined as a monotonic function of either of the systemic risk measures, the charge could set adverse incentives with regard to banks' risk parameters under many realistic conditions. MES and $\Delta$CoVaR are nearly equivalent in the linear normal setup. $\Delta$CoVaR stands out because of its negative sensitivity to idiosyncratic risk.

Although sensitivities to single parameters are informative, the picture is not complete, as a bank could also make simultaneous parameter changes. For example, consider a joint variation of $\sigma_i$ and $\beta_i$. Table 1 does not report undesired direct effects of these parameters on exposure $\Delta$CoVaR or MES if only one parameter is changed. Inappropriate effects may arise if $\sigma_i$ and $\beta_i$ can simultaneously vary such that a systemic risk measure SRM$_i$ decreases while the system risk $\sigma(R_s)$ increases, or vice versa. With the help of a first-order approximation, we can find sufficient conditions for such effects by inspecting:

$$\begin{aligned}
\Delta \mathrm{SRM}_i &= \frac{\partial \mathrm{SRM}_i}{\partial \sigma_i}\Delta\sigma_i + \frac{\partial \mathrm{SRM}_i}{\partial \beta_i}\Delta\beta_i, \\
\Delta\sigma(R_S) &= \frac{\partial \sigma(R_S)}{\partial \sigma_i}\Delta\sigma_i + \frac{\partial \sigma(R_S)}{\partial \beta_i}\Delta\beta_i.
\end{aligned}$$

This approximation is a system of two linear equations. For any given values of $\Delta\mathrm{SRM}_i$ and $\Delta\sigma(R_S)$, including those that have different signs, it is solvable through a choice of $\Delta\sigma_i$ and $\Delta\beta_i$, provided the vectors of first derivatives are linearly independent and the parameters remain in the permissible range, in particular, $\sigma_i \geq 0$. Given the complex structure of the first derivatives, one would expect linear independence to be the rule rather than the exception. For the parameter combinations of the base case and the combination with a risky dominant bank (cf. Table 1), we determine the derivatives and find them to be linearly independent for each of the four measures.

How important the possible adverse effects of joint variation are relative to the effects brought about by misaligned partial derivatives depends on the situation.

If, to consider one case, a bank's sole objective is to minimize its systemic risk measure, and if the parameters under the bank's control are free of other incentives or restrictions, partial derivatives will guide the bank in how to achieve this aim, at least for marginal changes. A bank would not explicitly seek to exploit the socially unwanted effects of joint variation unless it had an additional interest in increasing system risk, or keeping it above some level.

For our example with changes in $\sigma_i$ and $\beta_i$, banks that do not care about system risk would only care about the first equation in the system from above. Regulators or researchers could predict the banks' actions based on the SRM's partial derivatives and check with the help of the partial derivatives of $\sigma(R_S)$ whether they lead to undesirable outcomes, in the same way that we proceed in the analysis of Table 1.

If, to consider another case, a bank's parameters vary for reasons that are unrelated to systemic risk measurement, there will be situations in which a joint variation of some parameters leads to a higher system risk and a lower systemic risk contribution even though this possibility is not indicated by an analysis of single partial derivatives. In consequence, the informativeness of a systemic risk measure, that is, how well it ranks institutions according to their contribution to system risk, can suffer through effects that partial derivatives do not reveal.

These two cases indicate that it is difficult to judge how important joint effects are relative to partial effects in practice. For such a judgment, we would need to model how the parameters affect the overall objective function of the bank, not just how they affect systemic risk contributions and system risk. In any case, the observation that a joint variation can lead to undesired effects should receive attention.

In this section as well as in the rest of the article, we focus on partial derivatives because they help identify problems that arise if banks actively respond to the way their risks are measured. Such a response is widely regarded as a first-order problem in financial regulation (e.g., Acharya, Cooley, Richardson, and Walter (2009)), whereas there is little research on how the risk of the system enters the objective function of a bank.[12] Another motivation for our focus is that partial derivatives reveal conceptual differences in the problems that systemic risk measures bring about. In comparison, problems associated with joint variation appear to be less measure specific. They can arise under fairly general conditions, for instance, if partial derivatives are linearly independent as described above.

## C. Robustness to Distributional Assumptions

So far, we assume returns follow a multivariate normal distribution. By contrast, empirical returns may exhibit heavy tails as well as tail dependence, features to which systemic risk measures might react sensitively because they focus on tail behavior. Heavy tails can be due to time-varying volatilities, another empirical feature of returns that we have not modeled so far; this is central to the approaches of Brownlees and Engle (2017) and Acharya, Engle, and Richardson (2017). Furthermore, the choice between asset and equity returns as the basis for systemic risk measures is something we cannot consistently study with a multivariate normal framework.

To test whether the previous results are robust to variations in distributional assumptions, we take two parallel approaches: a structural credit risk model and a multivariate $t$-distribution.

For the first robustness test, we use a multivariate extension of the structural model of Collin-Dufresne and Goldstein (2001), which features heavy-tailed and tail-dependent equity returns. Internet Appendix B provides the details of the model; the main characteristics are as follows. Asset values follow correlated geometric Brownian motions. While asset returns have static parameters, the amount of debt is continuously adjusted by bank management to keep leverage close to a static target level. In consequence, leverage becomes a mean-reverting process, and so does the instantaneous equity volatility as it is nearly proportional to leverage. Equity returns are thus heteroscedastic.

[12] In Acharya (2009), banks have an incentive to take correlated risks, which increase system risk, but they do not have a direct interest in increasing system risk.

We use simulations[13] to perform three tests that closely correspond to the scenarios in Figures 2–4, using risk parameters that are broadly consistent with bank asset volatilities from Moody's KMV (own calculations) and estimates from Memmel and Raupach (2010). We give only an overview of the outcome here; the details and a summary table are provided in Internet Appendix B.2.

In the first test, we vary the idiosyncratic risk of asset returns as in Figure 2. For both asset and equity returns, we observe that an increase in idiosyncratic risk lowers $\Delta$CoVaR, as in the normal model. Bank $i$'s equity returns get heavier distribution tails when idiosyncratic risk rises, meaning that the decrease in $\Delta$CoVaR goes along with both increased individual variance and heavier tails. Interestingly, there is a direct and relative negative effect on MES that did not exist in the normal model. It appears only with equity returns, which suggests that tail thickness plays a role here.

Second, we test the sensitivity to $\beta_i$ as in Figure 3. For both asset and equity returns, an increasing $\beta_i$ (which also increases tail thickness) can have a negative relative effect on all four systemic risk measures if bank $i$ is very large relative to the other banks. This finding conforms to the results of the normal model.

In a third test, we vary the size of bank $i$ as in Figure 4. The results of the normal model are confirmed, but further negative sensitivities appear that did not exist in the normal model: For high $\beta_i$, there is a negative relative size effect on $\Delta$CoVaR and both direct and relative negative effects on exposure $\Delta$CoVaR. Apparently, tail thickness matters, as no such effect is observed for asset returns and auxiliary tests suggest that the higher volatility of equity does not explain the difference.

We conclude the first robustness test with the observation that all undesired sensitivities found in the normal model are confirmed for lognormal asset returns and for equity returns with heavier tails. However, further problematic sensitivities show up if distribution tails are thicker than in the normal model.

As an alternative way of generating heavy-tailed returns, we consider a multivariate $t$-distribution. To this end, we sample correlated normals from equation (2) and multiply them with an independent common factor drawn from a $\chi^2$ distribution with $\nu$ degrees of freedom; scaling down by another (constant) factor preserves base-case variances.

We consider degrees of freedom $\nu$ between 4 and 10 for different parameterizations of equation (2) and thus obtain a large number of simulation results, which we do not report in full here. The outcome is similar to the structural model in that key results from the previous section continue to be found in typical cases, and there are some variations.

For example, when we start from the base case and then increase $\sigma_i$ from 0.2 to 0.3, $\beta_i$ from 1 to 1.5, or $w_i$ from 0.02 to 0.25, the simulations show negative

[13] We run 10 million independent simulations of bank and system returns. In CoVaR calculations, simulated returns do not fall into the conditioning event because its probability is 0. Instead, we condition on a superset of positive probability. $\text{CoVaR}_{0.01}^{S|\{R_1=\text{VaR}_{0.01}^1\}}$, for example, is determined as follows: Select the simulation runs in which the return $R_1$ lies between the 0.8% and 1.2% quantile of $R_1$, and determine the 1% quantile of $R_S$ for this selection. MES is determined as the average simulated return of a bank under the condition that the simulated system return is below its 5% quantile. BETA is estimated through a regression of a bank's simulated returns on the simulated system returns.

effects on the measures in each of the two cases where Panel B of Table 1 has a negative sign; in the case of a positive sign, the simulated effects are also positive except for $\nu=4$, which shows additional negative effects for ΔCoVaR and exposure ΔCoVaR.

There are further situations in which the degrees of freedom matter. When we lower $\sigma_i$ from 0.2 to 0.1, ΔCoVaR increases for $\nu \geq 5$ as in the normal model, but it decreases for $\nu=4$. This observation does not question the general conclusion, though. As just described, lowering $\nu$ not only eliminates undesired sensitivities but also generates new undesired sensitivities.

Summing up, deviating from a multivariate normal distribution leads to more complex results but confirms the key conclusion of the previous section: There are nonexotic situations in which a parameter change that increases the risk of the system lowers the estimated systemic risk contribution of the bank for which the parameter was changed.

## IV. Systemic Risk Measures in the Contagion Case

After studying linear return relations within a simple 1-factor model, we now turn to examining contagion effects. An overview of different contagion definitions is given in Pericoli and Sbracia (2003). The main definition we examine is one in which contagion is brought about by spillovers of idiosyncratic shocks.

Assume that the returns of the banks and the system evolve according to

$$(14) \qquad R_i = \beta_i F + \varepsilon_i + \sum_{j \neq i} \gamma_j I_{\{\varepsilon_j < \kappa\}} \varepsilon_j, \quad R_S = \sum_i w_i R_i.$$

That is, there can be contagion from one bank to other banks in the system. If bank $j$ is afflicted by a realization of idiosyncratic risk that is worse than $\kappa$, other banks are partially affected, too. As in the previous section, we assume that $F$ and all $\varepsilon_j$s are independent normal variates. Because the dependence structure is now considerably more involved, we resort to 100 million Monte Carlo simulations and estimation procedures as described in footnote 13.

As before, the analysis is conducted using assumptions typical of daily returns. In the base case, banks are equally weighted ($w_i = 1/N$), factor $\beta_i$s are uniformly set to 1, and the following p.a. drift and volatility and parameters are chosen: $\mathrm{E}(F)=0.05$, $\sigma_F=0.2$, and $\sigma_i=0.2$ for all $i$s. As before, dividing these parameters by 260 or $\sqrt{260}$ translates them to daily returns. The number of banks is set to $N=50$.

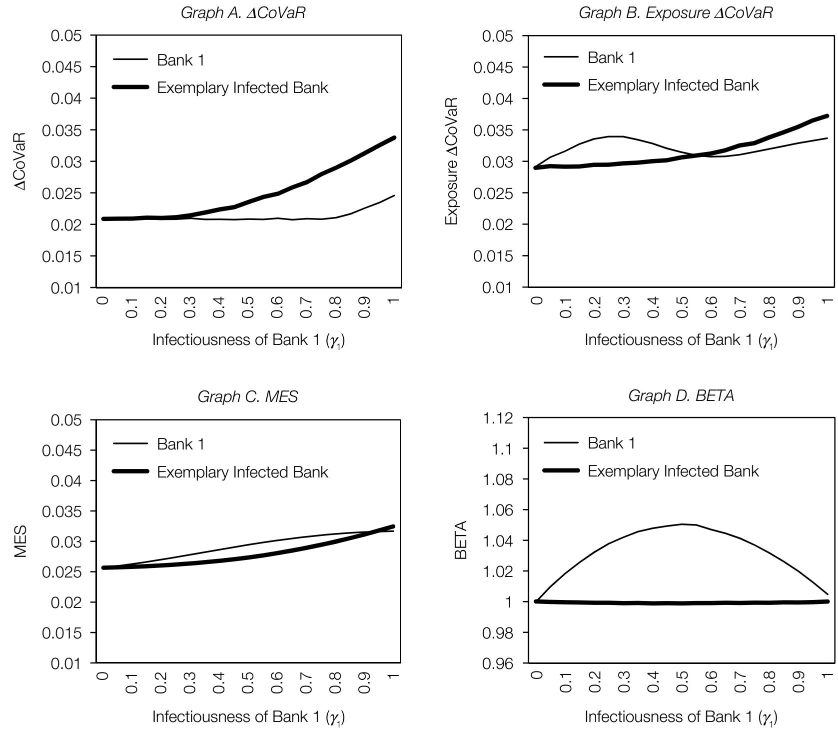
For the first analysis, we assume that only bank 1 is infectious, that is, $\gamma_j=0$ for $j>1$. We set the contagion threshold $\kappa$ to $-0.0204$, which corresponds to the 5% quantile of $\varepsilon_1$. The only parameter that is left to specify is the contagion intensity $\gamma_1$. Figure 5 shows the risk figures that result if we vary $\gamma_1$ from 0 to 1 and keep the other parameters at the values just described. The way the risk measures depend on the contagion intensity differs markedly.

In several cases, the systemic risk measure of the infectious bank decreases with an increasing contagion intensity, or it increases at a lower rate than the systemic risk measure of an infected bank. Such patterns could create undesirable incentive effects. If a systemic risk charge were based on ΔCoVaR, for example,

FIGURE 5

**Systemic Risk Measures in the Contagion Case**

In Figure 5, we simulate returns for $N$ equal-sized banks. The banking system return is the average of bank returns. Bank returns are driven by a common factor $F$, idiosyncratic risk, and a spillover of idiosyncratic risk from bank 1 to the other banks: $R_1 = \beta_1 F + \varepsilon_1$, $R_j = \beta_j F + \varepsilon_j + \gamma_1 I_{\{\varepsilon_1 < \kappa\}} \varepsilon_1$ for $j > 1$, $R_S = N^{-1} \sum_{j=1}^{N} R_j$. Parameters (per annum for drift and volatility) are set to $N = 50$, $\kappa = -0.0204$, $\mathrm{E}(F) = 0.05$, $\sigma_F = 0.2$, $\sigma_i = 0.2$, $\beta_i = 1$ for all $i$. The contagion intensity of bank 1 is varied from $\gamma_1 = 0$ to $\gamma_1 = 1$. The measures are estimated through Monte Carlo simulation with 100 million trials. Conditional value at risk (CoVaR$_\alpha$) measures are computed on a daily basis for $\alpha = 1\%$, using observations between the ($\alpha - 0.2\%$) and ($\alpha + 0.2\%$) quantiles of the conditioning variable. The marginal expected shortfall (MES) relates to $\alpha = 5\%$.




a bank would have a marginal incentive to become infectious. Its charge would not change, though the charge of its competitors would increase. This would create an advantage for the bank that becomes infectious because its competitors would be required to hold more capital, pay an insurance premium, or obey some other restriction.

An explanation of the patterns is complicated by the fact that a change in the contagion intensity can have several direct and indirect effects on an individual measure. We therefore relegate the full discussion to Internet Appendix D. Below we give only an intuition for cases in which the systemic risk contribution of the infectious bank does not increase with the contagion intensity or increases at a lower rate than that of an infected bank.

## A. ΔCoVaR

To understand how ΔCoVaR is affected by the contagion intensity $\gamma_1$, we express the system return as a function of the return of an individual bank.

For ease of exposition, we incorporate the choice of uniform unit $\beta_j$, $1 \leq j \leq N$, and equal-weighted banks that we made for the simulation. When applied to bank 1, the left-hand side of equation (14) then implies $F = R_1 - \varepsilon_1$, which can be plugged into the representation of $R_S$ to eliminate the factor return $F$:

$$(15) \qquad R_S \quad = \quad R_1 - \frac{N-1}{N}\left[1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right]\varepsilon_1 + \frac{1}{N}\sum_{j=2}^{N}\varepsilon_j.$$

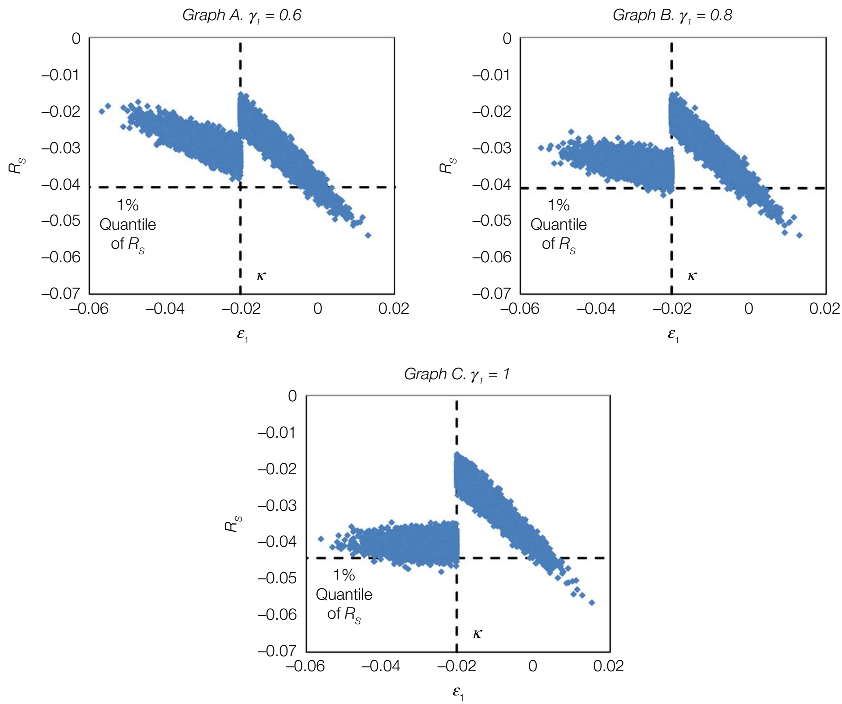
The equation shows that one way of arriving at a low conditional realization of $R_S$ is to have a very positive realization of $\varepsilon_1$. Intuitively, given that $R_1$ is at the 1% quantile, a positive $\varepsilon_1$ implies that the factor realization is very negative. If $\varepsilon_1$ is positive, however, there is no contagion.

The extent to which low realizations of $R_S$ are associated with contagion is illustrated in Figure 6. It plots the simulated $R_S$ against the simulated $\varepsilon_1$ conditional

FIGURE 6

System Return and Idiosyncratic Risk of the Infectious Bank If the Return of the Latter Is at Its 1% Quantile

In Figure 6, we simulate returns for the same system as in Figure 5 in which bad outcomes of $\varepsilon_1$ can spill over to the returns of the other banks. Eliminating the systematic factor implies: $R_S = R_1 + (N-1)\,N^{-1}\left[I_{\{\varepsilon_1<\kappa\}}\gamma_1 - 1\right]\varepsilon_1 + N^{-1}\sum_{j=2}^{N}\varepsilon_j$. In the graph, we plot simulated $R_S$ against simulated $\varepsilon_i$ for a selection of observations in which $R_1$ is in a close interval ($\pm$0.2%) around its 1% quantile. The contagion intensity $\gamma_1$ is set to either 0.6, 0.8, or 1.



on $R_1$ being near its 1% quantile.[14] With $\gamma_1 = 0.6$ (Graph A), none of the simulated cases of contagion is associated with a system return that is below its 1% quantile. In consequence, a change in the contagion intensity does not affect ΔCoVaR. Moving on to $\gamma_1 = 0.8$ (Graph B) and $\gamma_1 = 1$ (Graph C), contagion increasingly matters for the 1% quantile of the system return. A higher $\gamma_1$ thus leads to a higher ΔCoVaR because it magnifies the negative effects of contagion. Together, this explains why ΔCoVaR of the infectious bank in Graph A of Figure 5 does not change until $\gamma_1$ reaches a value around 0.75.

## B. Exposure ΔCoVaR

Comparing the four measures in Figure 5, the exposure ΔCoVaR of the infectious bank in Graph B exhibits the most complex pattern. At first, it becomes larger. Then, it shrinks, but this new tendency is again reversed. Rearranging equation (15), we can examine how the return of the infectious bank depends on the system return:

$$R_1 \quad = \quad R_S + \frac{N-1}{N}\left(1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right)\varepsilon_1 - \frac{1}{N}\sum_{j=2}^{N}\varepsilon_j .$$

A change in $\gamma_1$ affects the quantiles of $R_S$ as well as the conditional distribution of $\varepsilon_1$. We focus on the effects that can make the exposure ΔCoVaR decline as $\gamma_1$ increases.

If we condition on the 1% quantile of $R_S$ to determine $\text{CoVaR}_{1\%}$, an increase in $\gamma_1$ makes it more likely that contagion has led to the extreme realization of $R_S$. More contagion implies more extremely negative realizations of $\varepsilon_1$, but what matters for $\text{CoVaR}_{1\%}$ is $\left(1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right)\varepsilon_1$, which is pulled toward 0 as $\gamma_1$ increases. To get an intuition why the second effect is stronger, consider the extreme case in which $\gamma_1$ equals 1. Then, the smallest value that $\left(1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right)\varepsilon_1$ can take is $\kappa$, even though there will be many realizations with an $\varepsilon_1$ smaller than $\kappa$. The overall trough-shaped pattern in the exposure ΔCoVaR arises because this effect is at some point outsized by another effect of $\gamma_1$ that works in the opposite direction: Increasing $\gamma_1$ lowers the 1% quantile of $R_S$ on which we condition.

## C. MES

For the infectious bank, we again inspect:

$$R_1 \quad = \quad R_S + \frac{N-1}{N}\left(1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right)\varepsilon_1 - \frac{1}{N}\sum_{j=2}^{N}\varepsilon_j .$$

MES and exposure ΔCoVaR are similar in that we condition on a tail event of the system return. Increasing $\gamma_1$ makes it more likely that contagion has occurred in the conditioning event that $R_S$ is below its 5% quantile. A higher probability of contagion means that the conditional means of both $R_S$ and $\varepsilon_1$ are more negative. But the direct effect of $\gamma_1$ works in the opposite direction. Through $\left(1 - I_{\{\varepsilon_1 < \kappa\}}\gamma_1\right)\varepsilon_1$, an increase in $\gamma_1$ makes MES less extreme. The concave shape

[14] Recall that selecting observations in which the return $R_1$ lies between its 0.8% and 1.2% quantile is our numerical procedure for conditioning on the 1% quantile of $R_1$.

of MES arises because the first effect dominates for small $\gamma_1$, whereas the second effect gains weight when $\gamma_1$ gets larger.

## D. BETA

Equation (14) shows that the return of the infectious bank does not depend on the contagion intensity $\gamma_1$. For infected banks, by contrast, the return linearly depends on $\gamma_1$ in case of contagion, which carries over to the system return. Hence, $\text{cov}(R_1, R_S)$ is linear in $\gamma_1$, whereas both $\text{cov}(R_2, R_S)$ and $\sigma^2(R_S)$ are quadratic in $\gamma_1$. These observations explain why $\text{BETA}_i = \text{cov}(R_i, R_S)/\sigma^2(R_S)$ is hump-shaped for the infectious bank and almost flat for infected banks.

## E. Robustness

To examine whether the results are robust to the parameter choices and distributional assumptions made above, we examine several modifications:

(1) Because of the spillover term $-\gamma_1 I_{\{\varepsilon_1 < \kappa\}}\varepsilon_1$, the infected banks have a lower expected return and a higher volatility than the infectious bank. We harmonize the moments by adding a constant to the infected banks' returns and scaling down their idiosyncratic risk.

(2) We change the contagion threshold $\kappa$ from $-0.0204$ to $-0.0289$, the 1% quantile of the idiosyncratic risk component.

(3) We change the contagion threshold $\kappa$ to $-0.0383$, the 0.1% quantile of the idiosyncratic risk component.

(4) We change the weight of the infectious bank from 2% in the base case to 25%. The other weights are changed to $(1-0.25)/49$.

(5) We change the systematic factor loading of the infectious bank from $\beta_1 = 1$ to 1.25.

(6) In the base case, there is just 1 infectious bank. In this variation, the number of infectious banks is increased to 5, each of them having the same contagion intensity.

(7) We assume the factor return $F$ to follow a generalized autoregressive conditionally heteroscedastic (GARCH(1,1)) process (see Bollerslev (1986)) with the same unconditional volatility as in the base case. We choose $\delta = 0.04$ and $\lambda = 0.95$ and set

$$\sigma^2(F_t) \quad = \quad (1-\delta-\lambda) \times 0.2^2/260 + \delta F_{t-1}^2 + \lambda\sigma^2(F_{t-1}).$$

(8) Instead of assuming a normal distribution for the factor returns and the idiosyncratic components, we assume they follow $t$-distributions with 4 degrees of freedom.

(9) We replace the spillover of return shocks by a spillover of volatility: If the idiosyncratic risk of the contagious bank falls below $\kappa$, the idiosyncratic risk of the infected banks is $m$ times higher than in the base case. The base level of idiosyncratic risk is chosen such that the total volatility of the infected

banks equals the volatility of the infectious bank. Based on return behavior surrounding the Lehman collapse,[15] we assume $m=3$.

(10) We introduce a time delay in the spillover by shifting 50% of it to the next day:

$$R_{it} = \beta_i F_t + \varepsilon_{it} + 0.5 \sum_{j \neq i} \gamma_j I_{\{\varepsilon_{jt} < \kappa\}} \varepsilon_{jt} + 0.5 \sum_{j \neq i} \gamma_j I_{\{\varepsilon_{j,t-1} < \kappa\}} \varepsilon_{j,t-1}.$$

Because of the large number of variations, we cannot present the full set of results for varying contagion intensities. Although the variations may shift the risk measure curves or flatten or elevate them, patterns are mostly very similar to those shown in Figure 5. This outcome is exemplified in Table 2, which lists the risk measures for a contagion intensity $\gamma_1$ of 0.75. For this choice, Figure 5 shows that

TABLE 2

Systemic Risk Measures in the Presence of Contagion: Robustness Tests

In Table 2, we simulate returns of $N$ banks. The aggregate system return is the value-weighted average of bank returns. Individual bank returns are driven by a common factor $F$, idiosyncratic risk, and bank-to-bank spillovers of idiosyncratic risk: $R_i = \beta_i F + \varepsilon_i + \sum_{j \neq i} \gamma_j I_{\{\varepsilon_j < \kappa\}} \varepsilon_j$, $R_S = \sum_{j=1}^{N} w_j R_j$. In the base case, parameters (per annum for drift and volatility) are set to $N=50$, $\kappa=-0.0204$, $\mathrm{E}(F)=0.05$, $\sigma_F=0.2$, $\sigma_i=0.2$, $\beta_i=1$, $w_i=1/50$ for all $i$s. Banks 2 to 50 are not infectious ($\gamma_j=0$, $\forall j>1$), whereas the contagion intensity of bank 1 is set to $\gamma_1=0.75$. For the variations, the table lists the changes relative to the base case. The measures are estimated through Monte Carlo simulation with 100 million trials. Conditional value at risk (CoVaR$_\alpha$) measures are computed for $\alpha=1\%$ with observations between the ($\alpha-0.2\%$) and ($\alpha+0.2\%$) quantiles of the conditioning variable. The marginal expected shortfall (MES) relates to $\alpha=5\%$. GARCH stands for generalized autoregressive conditionally heteroscedastic.

| | Specification | Bank | ΔCoVaR | Exp. ΔCoVaR | MES | BETA |
|---|---|---|---|---|---|---|
| | Base case | Infectious | 0.0208 | 0.0315 | 0.0310 | 1.0369 |
| | | Infected | 0.0280 | 0.0329 | 0.0294 | 0.9993 |
| 1 | Equal return mean and volatility across banks | Infectious | 0.0209 | 0.0515 | 0.0310 | 1.0392 |
| | | Infected | 0.0286 | 0.0593 | 0.0284 | 0.9993 |
| 2 | Threshold $\kappa=-0.0289$ ($\mathrm{P}(\varepsilon_1<\kappa)=0.01$) | Infectious | 0.0208 | 0.0553 | 0.0277 | 1.0112 |
| | | Infected | 0.0256 | 0.0594 | 0.0270 | 0.9996 |
| 3 | Threshold $\kappa=-0.0383$ ($\mathrm{P}(\varepsilon_1<\kappa)=0.001$) | Infectious | 0.0207 | 0.0573 | 0.0260 | 1.0020 |
| | | Infected | 0.0224 | 0.0579 | 0.0259 | 1.0002 |
| 4 | Factor loading of infectious bank $\beta_1=1.25$ | Infectious | 0.0233 | 0.0394 | 0.0363 | 1.2510 |
| | | Infected | 0.0278 | 0.0333 | 0.0294 | 0.9947 |
| 5 | Weight of infectious bank $w_1=25\%$ | Infectious | 0.0257 | 0.0342 | 0.0337 | 1.1430 |
| | | Infected | 0.0287 | 0.0323 | 0.0290 | 0.9522 |
| 6 | Five infectious banks | Infectious | 0.0373 | 0.0428 | 0.0418 | 1.0240 |
| | | Infected | 0.0398 | 0.0436 | 0.0412 | 0.9971 |
| 7 | GARCH(1,1) for systematic factor | Infectious | 0.0262 | 0.0327 | 0.0318 | 1.0370 |
| | | Infected | 0.0303 | 0.0339 | 0.0302 | 0.9991 |
| 8 | $t$-distribution with 4 degrees of freedom | Infectious | 0.0403 | 0.0388 | 0.0355 | 1.0442 |
| | | Infected | 0.0447 | 0.0397 | 0.0331 | 0.9992 |
| 9 | Volatility spillover instead of return spillover | Infectious | 0.0208 | 0.0275 | 0.0259 | 1.0000 |
| | | Infected | 0.0237 | 0.0320 | 0.0257 | 1.0003 |
| 10 | 50% of spillover is shifted to next day | Infectious | 0.0201 | 0.0335 | 0.0262 | 0.9848 |
| | | Infected | 0.0227 | 0.0307 | 0.0276 | 1.0001 |

[15]For the 29 depositary institutions listed in Appendix A of Acharya et al. (2017), we examine the idiosyncratic volatility over the 30 days ending Sept. 12, 2008, as well as over the 30 days starting Sept. 15, 2008 (Lehman collapse). Using a 1-factor model with the S&P 500 as the factor, the median idiosyncratic volatility increases by a factor of 2.98.

$\Delta$CoVaR assigns a higher risk to the infected banks; exposure $\Delta$CoVaR does as well but with a smaller difference, whereas MES and BETA assign a lower risk to the infected banks. In most variations, the sign of the differences is not changed. For the variations considered here, it never changes for $\Delta$CoVaR, once for MES, and twice for BETA. Exposure $\Delta$CoVaR shows the largest number of changes, which does not appear surprising given that it shows the most complex behavior in Figure 5. The findings from Figure 5 therefore seem fairly robust to variations in the parameters and the modeling framework. As in the previous section, they are also robust to distributional assumptions.

Finally, we consider the alternative definition of Mainik and Schaanning (2014) for $\Delta$CoVaR; that is, we condition on $\left\{R_i \leq \text{VaR}^i_{0.01}\right\}$ when determining $\text{CoVaR}^{S|i}_{0.01}$, and on $\left\{R_S \leq \text{VaR}^S_{0.01}\right\}$ when determining $\text{CoVaR}^{i|S}_{0.01}$ for the calculations of exposure $\Delta$CoVaR. With this modification, $\Delta$CoVaR now assigns a higher systemic risk to the infectious bank. Exposure $\Delta$CoVaR, however, continues to ascribe a lower risk to the infectious bank; the gap even widens for the base-case parameterization: The modified exposure $\Delta$CoVaR of the infectious bank is 0.034 versus 0.038 for the infected bank. We also determine modified $\Delta$CoVaR measures for the volatility spillover case and find unchanged patterns. Both $\Delta$CoVaR and exposure $\Delta$CoVaR continue to assign a larger systemic risk to the infected bank, which shows that the change in conditioning does not have a consistent effect.

## V. Conclusion

We examine possible pitfalls in the use of return-based measures of systemic risk contributions. Specifically, we check for cases in which a change in an entity's systematic risk, idiosyncratic risk, size, or contagiousness increases the risk of the system but lowers the systemic risk contribution of the entity. In such cases, rankings based on estimated systemic risk contributions could produce false interpretations and incentives. In particular, if banks benefit from having a lower estimated systemic contribution in the eyes of their regulators, the use of such measures could motivate banks to take actions that increase the risk of the system rather than reduce it.

Although the link between the measured systemic risk contribution and the actual impact on system risk is often appropriate, we identify many nonexotic cases in which it is not. In a linear factor model framework with multivariate normal risk factors, we find that the $\Delta$CoVaR measure proposed by Adrian and Brunnermeier (2016) can imply a lower systemic risk contribution if a bank increases its idiosyncratic risk. BETA can lead to unwanted effects if a bank with a large systematic risk increases its size.

There are two situations in which all considered measures fail. First, neither of the measures is able to capture the ambiguous effect that the size of a bank can have on system risk. Second, if a bank is very dominant, its risk contribution may fall relative to the contribution of another bank if the bank increases its systematic risk. The last example highlights that a bank can favorably alter its systemic risk contribution relative to those of other banks even though its own risk contribution increases. A study of sensitivities should therefore not be limited to the change in the systemic risk contribution of the bank under analysis but should include side effects.

The problematic sensitivities we identify are not limited to the return model with multivariate normal risk factors that we start with. They also appear when we examine heavy-tailed equity returns generated by a dynamic structural model, or returns drawn from a multivariate $t$-distribution. In addition, we find further nonexotic cases of undesired sensitivities that did not appear in the normal model.

Once we introduce contagion into the analysis, we also find differences among the four measures, but again, none of them is immune against creating false incentives. Stronger spillovers can be associated with a lower systemic risk contribution even though they increase the risk of the system.

Our results should not be interpreted in the sense that measure A is to be preferred to measure B if it shows a lower number of unwanted effects in the analysis. Within our framework, we cannot weigh or compare the importance of the specific problems we identify. In addition, we abstract from estimation error, whereas in practice, systemic risk measures need to be estimated with limited data. Some measures may be less sensitive to estimation error than others. Finally, we focus on undesired effects of ceteris paribus changes in individual parameters. As we show by means of an example, joint parameter variations can lead to additional problems that are not visible through the lens of a ceteris paribus analysis.

Despite these caveats, our results are of general relevance and applicability because we show that systemic risk measures can exhibit undesirable properties in standard return frameworks. Knowledge about such limitations is important for the practical measurement of systemic risk as well as for the development of new measurement approaches.

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