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A theory of risk disclosure

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Abstract In this paper, we consider the price effects of risk disclosure. We develop a model in which investors are uncertain about the variance of a firm's cash flows and the firm releases an imperfect signal regarding this variance. In our model, uncertainty over the riskiness of a firm's cash flows leads to a variance uncertainty premium in its price. We demonstrate that risk disclosure decreases the firm's cost of capital by reducing this premium and that the market response to risk disclosure is small when the expected level of risk is high. Moreover, we find that firms acquire and disclose more risk information when their cash flow risk is greater than expected. Finally, we demonstrate that in a multi-asset setting, only risk disclosure concerning systematic risks will impact the cost of capital.

Keywords Risk disclosure · Variance uncertainty · Cost of capital

JEL Classification C11 · D53 · D83 · G12 · M41

1 Introduction

In the wake of the recent financial crisis, regulatory authorities have increased the pressure on firms to disclose information about the riskiness of their cash flows. For example, in 2009 the SEC approved rules that require firms to issue disclosures on

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compensation practices that could lead employees to take on excessive risks. The regulatory changes originated in investors' demand for risk-related disclosures; in 2012 the FASB stated that "users of financial statements overwhelmingly indicated that ... understanding a reporting entity's exposures to risks that are inherent in financial instruments and the ways in which reporting entities manage these risks is integral to making informed decisions about capital allocation."[1] The literature on disclosure in capital markets, however, has focused on the disclosure of information that mainly informs investors about expected cash flows, rather than the uncertainty surrounding cash flows. Based on this focus, the theoretical literature that studies the impact of disclosure on stock prices and the cost of capital assumes that the riskiness of cash flows is known.[2] In this setting, earnings disclosures change the market's expectation of future cash flows and result in a reduction in investors' perception of the riskiness of these cash flows (for example, Christensen et al. 2010; Holthausen and Verrecchia 1988; Lambert et al. 2007). We extend this literature to study the capital market effects of information that directly concerns the riskiness of firms' cash flows.

We first develop a single-firm version of Lambert et al. (2007) where we relax the assumption that investors know the cash flow variance. Within this framework, we investigate how investors respond to the disclosure of an imperfect signal about the uncertain variance and the cost of capital implications of such disclosure. To establish a baseline for our study of risk disclosure, we derive a closed form expression for price when investors are uncertain of the true cash flow variance. We show that the variance uncertainty premium studied in the dynamic asset pricing literature (see Buraschi and Jiltsov 2006) and in Gron et al. (2012) emerges in our setting. The intuition for this result is that variance uncertainty creates fat tails in the distribution of cash flows. The disutility that investors with negative exponential utility experience from risk increases at an increasing rate, analogous to the standard concavity result for the mean of cash flows. As a result, these investors are averse to fat-tailed distributions. In the economics literature that studies risk attitudes, such an aversion is referred to as temperance (Ebert 2013; Kimball 1992). The existence of a variance uncertainty premium suggests that, if risk disclosure reduces uncertainty over the variance of the firm's cash flows, it also reduces the cost of capital, which provides an economic rationale for the FASB's statement that understanding a firm's riskiness is critical to efficient capital allocation.

With this foundation, we introduce a disclosure of risks by the firm into our model and study how price responds to this disclosure. In particular, we assume that the firm commits to the truthful disclosure of a signal regarding the variance of its cash flows. Some intuitive results from the prior disclosure literature also materialize in our setting: investors respond more strongly to variance information when there is greater prior uncertainty over the variance and when the disclosure is more precise.

[1] FASB (2012). More recently, the Enhanced Disclosure Task Force issued an extensive report recommending several improvements in the risk disclosure of banks, claiming that "investors and other public stakeholders are demanding better access to risk information from banks; information that is more transparent, timely and comparable across institutions."

[2] See Verrecchia (2001) and Beyer et al. (2010), or Bertomeu and Cheynel (2016) for surveys of the disclosure literature.

Variance disclosure always reduces the variance uncertainty premium that investors place on the firm, since investors' perceived distribution over the variance narrows. While this increases the firm's price on average, the net effect of a variance disclosure on share price depends on the realization of that disclosure. For example, a risk disclosure that suggests the firm is facing a very uncertain economic climate reduces uncertainty about the variance but increases the risk premium itself.

Critically, however, we find that the intuition derived from traditional earnings disclosure models such as those in Holthausen and Verrecchia (1988) and Lambert et al. (2007) does not completely carry over to disclosures regarding the second moment. In standard earnings disclosure models, the amount of uncertainty resolved by any earnings signal is constant and hence any earnings disclosure leads to the same reduction in the risk premium. In contrast, the amount of uncertainty resolved by a variance signal is a function of the realization of that signal; high risk signals leave investors with greater residual uncertainty about the true variance. We demonstrate that this is a natural consequence of the fact that distributions over the variance must be nonnegative. Next, we find that investors' response to variance information is a function of the prior expectation over the amount of risk faced by the firm. Specifically, when the expected level of risk is very high (low), the market response to disclosure is small (high). This is the case because variance signals tend to be noisier when the expected risk is higher, which leads to a lower amount of uncertainty resolved by the disclosure and thus an attenuated market response to the disclosure. Finally and intuitively we find that greater levels of risk aversion increase the magnitude of the response to risk disclosure. This implies that the results derived from Bayesian models of earnings disclosure cannot be applied to risk disclosures, where information signals are inherently non-normal. In general, our model suggests that modelling information through normal distributions may not fully explain capital market responses to the diversity of information communicated in firms' financial statements.

Next, we study the impact of risk disclosure from an ex-ante perspective and find that risk disclosures reduce the variance uncertainty premium that investors impose on the firm but have expected no impact on the risk premium as the cash flow risk itself does not change in expectation. This implies that firms can expect to reduce their total cost of capital by making a commitment to disclose information concerning cash flow variance. Thus we offer theoretical evidence in support of the Enhanced Disclosure Taskforce's statement that "by enhancing investors' understanding of banks' risk exposures and risk management practices, high-quality risk disclosures may reduce uncertainty premiums and contribute to broader financial stability." We perform comparative statics on this cost of capital reduction and show that it is affected by the same parameters that affect the size of the price response to risk disclosure: prior uncertainty over the variance, the prior mean of the variance distribution, and investors' risk aversion.

As our main focus is on the price effects of risk disclosure, we do not explicitly consider a firm's optimal choice of disclosure precision in the main model. In the absence of disclosure costs, firms in our model would commit to perfect risk disclosure, since this minimizes investor uncertainty. On the other hand, if firms chose the optimal risk disclosure precision in the face of a cost to more precise disclosure, they would simply trade off the cost of risk disclosure and its impact on the cost of capital. Thus, in the static setting considered so far, our model offers little insight over the extant disclosure literature regarding firms' optimal disclosure choices. However, we find that in a dynamic setting, firms' optimal risk disclosure choices exhibit a feature not found in models of earnings disclosure. In particular, we next consider a setting where a firm has multiple opportunities to disclose risk information over time and assume that its goal is to maximize the long-term price (price conditional on all disclosures). In standard earnings disclosure models, any disclosed earnings report leaves investors with the same amount of uncertainty. This implies that firms' decision to disclose at any point in time is independent of prior earnings reports. On the other hand, as discussed above, the residual uncertainty following a risk disclosure depends on the realized report. For this reason, a firm's interest to disclose risk information depends on the information it has previously disclosed. More precisely, since information that suggests the firm faces a higher level of risk leaves investors with more residual uncertainty, firms are more likely to provide additional disclosures when the initial news suggests that their risks are high. This implies that firms' financial statements contain more risk disclosures when their cash flows are riskier than expected and that firms are likely to follow up high disclosed risks with supplemental risk disclosures down the line. In line with our predictions, Campbell et al. (2014) and Hope et al. (2016) find that firms with greater realized variances disclose more risk information in their 10-Ks.

Finally, we consider a multi-asset market. In this model, we show that prices continue to contain an additional risk premium for variance uncertainty over the common factor; however, the variance uncertainty premium for idiosyncratic risk vanishes as the economy grows large. We show that, for disclosure to impact the cost of capital in this setting, it must contain information on systematic risk. Moreover, when disclosure contains information regarding the common risk factor, it reduces the risk discount of *all* firms and thus has positive externalities.

We emphasize that our work is just a first step to understand the economic forces of risk disclosures. For example, in the spirit of prior work on disclosure and the cost of capital (for example, Christensen et al. 2010; Lambert et al. 2007; Gao 2010), our model takes as exogenous the incentives of firms and their managers to disclose variance information and assumes away the ability of managers to bias this information. Our model thus best speaks to settings where manipulating or concealing information causes firms to incur significant legal penalties. Jørgensen and Kirschenheiter (2003), on the other hand, focus on the discretionary disclosure of perfect information about the variance. Jørgensen and Kirschenheiter find that a discretionary disclosure equilibrium exists in which only firms with low variances are willing to separate themselves by incurring a disclosure cost. We assume away such strategic incentives and study the impact of risk disclosure on the cost of capital when the firm has made a commitment to truthfully disclosing a signal regarding its risks. Moreover, we consider a setting of imperfect risk disclosure, which enables us to demonstrate that the Bayesian updating applied by investors to a variance distribution results in economic forces that do not appear in standard models of earnings disclosures. We also demonstrate that the channel through which risk disclosure impacts firms' costs of capital is through its impact on the variance uncertainty premium.

A stream of accounting literature has considered the impact of earnings disclosures when investors face uncertainty over the variance of cash flows (Beyer 2009) or the precision of the accounting signal (Hughes and Pae 2004; Kirschenheiter and Melumad 2002; Subramanyam 1996). These models are based on the statistical literature on Bayesian updating with uncertain precisions and typically depict information as true cash flows plus a noise term.[3] Investors in these models can use noisy signals of cash flows to indirectly update on the variance of cash flows or the precision of the firm's signal. In particular, when a firm discloses a signal that substantially deviates from the market's prior, investors infer that the variance of cash flows is high. Our results build on this literature in two dimensions. First, rather than considering the impact of an earnings announcement in the face of uncertainty over cash flow variance or information precision, we consider the impact of a signal that directly concerns the cash flow variance. The type of disclosure we consider better matches the risk disclosures that appear in section 1A of the 10-K and that are increasingly studied by empirical researchers and emphasized by regulators. Second, this literature largely assumes that prices are linear in the first and second moments of cash flows, rather than explicitly deriving price from trade among risk averse investors.[4] This assumption implies that there cannot exist a variance uncertainty premium and thus assumes away any ex-ante impact of second-moment information on the cost of capital.

Recent empirical work examines the information content of risk disclosure in firms' 10-K filings (for example, Bao and Datta 2014; Bischof et al. 2016; Campbell et al. 2014; Hope et al. 2016). Our paper offers a theoretical rationale for the existing findings that prices respond to risk disclosure (Campbell et al. 2014). Furthermore, we predict that prices react more strongly to risk disclosure when prior uncertainty is high and when the expected level of uncertainty is low, and that firms with greater uncertainty over the variance have increased incentives to disclose variance information. Our model also predicts that firms that commit to disclose information regarding their risks earn returns closer to the risk-free rate, ceteris paribus. In addition, we predict that the effects of risk disclosure are attenuated for firms with very precise earnings, and that firms with unexpectedly high risk disclosures exert additional effort to provide market participants with information regarding these risks. Our large economy results should motivate empirical researchers to distinguish between risk disclosure regarding systematic risks and idiosyncratic risks, as, from a theoretical perspective, systematic risk disclosure should have a greater impact on market prices.

[3] Neururer et al. (2016) and Sridharan (2015) provide empirical studies of the variance information in disclosed earnings.

[4] See, for example, Beyer (2009), Hughes and Pae (2004), Kirschenheiter and Melumad (2002), Penno (1996), and Subramanyam (1996). Modeling variance disclosure as a direct signal regarding the variance demands a suitable nonnegative distribution for the variance, a conjugate prior for that distribution, and a utility function that yields a closed form solution with these distributions. We believe that the prior literature assumes risk neutral or mean variance pricing for tractability purposes. While this is suitable for the settings these papers examine, our focus is on the pricing of variance uncertainty and the effect of risk disclosures.

Our model is related to the literature on ambiguity aversion, which investigates uncertainty over the distribution of cash flows. A common assumption in this literature is that investors apply a discount to the expected cash flows either by operating under the most pessimistic distribution from a specified set of possible distributions (Garlappi et al. 2007; Gibloa and Schmeidler 1993; Illeditsch 2011) or by applying a concave transformation to a specified set (Caskey 2009). For example, Illeditsch (2011) considers the effect of ambiguity over the precision of a public signal on investors' portfolio allocation problem. Our paper supplements this literature because we endogenously derive the discount that investors apply when distributional uncertainty is over the variance of cash flows instead of assuming that such a discount exists and takes a particular form. This allows us to analyze what drives the discount and show that the discount may be reduced through risk disclosure.

Our study is also related to the literature on uncertainty about risk factor loadings, for example, Armstrong et al. (2013). Because investors are uncertain about the exposure of a firm's returns to a risk factor, they are uncertain about the risk of the firm's return. Using a pricing kernel that is convex in the factor loading, Armstrong et al. (2013) predict that factor loading uncertainty decreases expected returns. We develop prices in a market where investors have a negative exponential utility function such that prices decrease (expected returns increase) in the variance uncertainty.

Finally, our paper relates to the literature on estimation risk (Barry and Brown 1985; Coles et al. 1995). Barry and Brown (1985) examine the difference in betas that results when investors must estimate the mean and covariance matrix of returns. They find that betas are higher for firms with greater variance uncertainty. Barry and Brown (1985) do not directly model price formation but rather assume that returns are exogenous and that beta is the metric of importance when evaluating the effect of variance uncertainty. On the contrary, our multiasset model shows that beta alone is not sufficient to capture the effects of variance uncertainty on prices. Coles et al. (1995) show that the CAPM does not hold in its traditional form when investors face estimation risk over the mean and variance of cash flows. Coles et al. (1995) assumes that an investor's expected utility is increasing in the mean and decreasing in the variance of cash flows, which, again, implies that uncertainty over the variance is not priced.

The remainder of the paper is organized as follows. Section 2 develops our core model and derives prices under variance uncertainty, price responses to disclosure, and the effect of an ex-ante commitment to disclosure on the cost of capital. Section 3 considers dynamic learning and disclosure. Finally, Section 4 extends the single asset model to a multiple asset setting.

2 A single asset model

2.1 Setup

The economy we consider mirrors a single-firm version of Lambert et al. (2007), with the exception that we allow the variance of cash flows to be uncertain. In particular, we consider a single period economy with two assets: a risk-free asset with a price normalized to 1 and a risky asset with a per-share price of P. The economy in our model is populated by a continuum of homogeneous risk-averse investors who have negative exponential utility with risk aversion parameter ρ, i.e., $u(w) = -\exp[-\rho w]$. The risk-free asset is in unlimited supply, while we normalize the per capita supply of the risky asset to 1. Conditional on the variance, the per-share payoff to the risky asset, $\tilde{x}$, is normally distributed with mean μ and variance $\tilde{V}$.[5] In contrast to the existing literature on disclosure and the cost of capital, we assume that $\tilde{V}$ is unknown to investors and follows a gamma distribution. This is similar to the literature in finance and statistics that builds on the Variance-Gamma Model for returns developed by Madan and Seneta (1990). The gamma distribution is typically parameterized by a shape parameter a and a scale parameter b, with mean $\mu_V \equiv \frac{a}{b}$ and variance $\sigma_V^2 \equiv \frac{a}{b^2}$. In the main text, we parameterize the gamma distribution by its mean and variance to provide better intuition for our comparative statics. In particular, we assume that $\tilde{V}$ has the following density function:

$$f(V) = \frac{\left(\frac{\mu_V}{\sigma_V^2}\right)^{\frac{\mu_V^2}{\sigma_V^2}} V^{\frac{\mu_V^2}{\sigma_V^2}-1} e^{-V\frac{\mu_V}{\sigma_V^2}}}{\Gamma\left(\frac{\mu_V^2}{\sigma_V^2}\right)} \text{ for} V \geq 0. \tag{1}$$

It is easily checked that $E(\tilde{V}) = \mu_V$ and $Var(\tilde{V}) = \sigma_V^2$.[6] We assume a gamma distribution for the variance as it has been widely used in the statistics literature.[7] Furthermore, as we will show, the gamma distribution yields a closed form solution for prices when combined with negative exponential utility.

For prices before and after the disclosure to have the same structural form, the likelihood function of the disclosure must have the gamma distribution as a conjugate prior. There exist two well recognized distributions that have this property: the Poisson distribution with unknown mean parameter and the gamma distribution with known shape and unknown rate parameters (Fink 1995). We employ the Poisson likelihood as the gamma likelihood does not yield analytically tractable results for the expected price.

In particular, we assume that the firm discloses a signal $\tilde{S}$ which is equal to the mean of τ Poisson distributed random variables, which are centered around $\tilde{V}$ and are independent conditional on $\tilde{V}$.[8] The number of signals received by investors, τ, corresponds to the precision of $\tilde{S}$ since the amount of noise in the $\tilde{S}$ is decreasing

[5] We denote random variables with a tilde "~".

[6] Characterizing the gamma distribution by its mean and variance creates the following restriction: $\mu_V = 0 \iff \sigma_V^2 = 0$. This occurs because a zero mean implies the distribution is degenerate at zero.

[7] The inverse gamma is widely used as a conjugate prior for the variance of a normal distribution when signals are drawn from a normal-gamma distribution (see DeGroot 1970). We choose to examine the gamma distribution rather than the inverse gamma distribution as the moment generating function for an inverse gamma does not exist.

[8] In the Appendix, we show that the mean of these signals is a sufficient statistic for their individual realizations.

in τ.[9] In sum, the firm's disclosure $\tilde{S}$ is a random variable centered around the true variance, which exhibits a degree of noise that is decreasing in τ.

Applying results from Bayesian statistics, one can show that $\tilde{V}|\tilde{S}$ is again gamma distributed. In the following lemma, we express the conditional mean and variance in terms of the prior mean and variance, the signal, and the precision parameter.

Lemma 1 *The conditional mean and conditional variance of the variance distribution given the signal $\tilde{S}$ are equal to:*

$$E(\tilde{V}|\tilde{S}) = E(\tilde{V}) + \frac{Cov(\tilde{V}, \tilde{S})}{Var(\tilde{S})}(\tilde{S} - E(\tilde{S})) \text{ and} \tag{2}$$

$$Var(\tilde{V}|\tilde{S}) = Var(\tilde{V}) - \frac{Cov(\tilde{V}, \tilde{S})^2}{Var(\tilde{S})} + \left(\frac{Cov(\tilde{V}, \tilde{S})}{Var(\tilde{S})}\right)^2 \frac{\tilde{S} - \mu_V}{\tau} \tag{3}$$

where $Var(\tilde{V}) = Cov(\tilde{V}, \tilde{S}) = \sigma_V^2$ *and* $Var(\tilde{S}) = \sigma_V^2 + \tau^{-1}\mu_V$.

As in the case of the normal prior and normal likelihood, Lemma 1 shows that the expected variance is linear in the signal and that the signal receives greater weight as precision τ increases. Furthermore, the coefficient on $\tilde{S}$ is equal to the regression coefficient $\frac{Cov(\tilde{V},\tilde{S})}{Var(\tilde{S})}$. If the signal is equal to its prior mean, μ_V, there is no updating on the mean, but the variance is reduced by $\frac{Cov(\tilde{V},\tilde{S})^2}{Var(\tilde{S})}$.

In what follows, we first derive and discuss the price before disclosure, then turn to the price following the variance disclosure, and finally analyze the expected effects of disclosure on price.

2.2 Ex-ante Price

Our assumption of an uncertain variance implies that the unconditional distribution of cash flows exhibits excess kurtosis (or "fat tails"), relative to a normal distribution with a known variance.[10] This implies that the probability that cash flows take on extreme values is greater when uncertainty about the variance exists.[11] Figure 1 compares a normal distribution with uncertain, gamma distributed variance to a normal distribution with known variance.

As noted above, several prior papers in accounting that feature uncertainty over the second moment have assumed either risk neutral or mean-variance utility (for example, Beyer 2009; Subramanyam 1996; Kirschenheiter and Melumad 2002). Thus

[9] Technically, if the underlying Poisson signals are equal to $\{\tilde{s}_i\}_{i=1}^{\tau}$, we have that $Var(\tilde{S}|\tilde{V}) = Var\left(\tau^{-1}\Sigma_{i=1}^{\tau}\tilde{s}_i|\tilde{V}\right) = \tau^{-1}\tilde{V}$. This is decreasing in τ for any realization of $\tilde{V}$.

[10] This can be seen by computing excess kurtosis, defined as the fourth standardized moment minus the kurtosis of a normal distribution (which equals 3): $\frac{E[(\tilde{V}-\mu)^4]}{(E[(\tilde{V}-\mu)^2])^2} - 3 = 3\frac{\sigma_V^2}{\mu_V^2}$.

[11] While the fat tails that follow from the uncertain variance seemingly map to the empirical findings in Mandelbrot (1963) and Fama (1965), those studies suggest that stock *returns* exhibit fat tails, whereas our result implies that cash flows themselves exhibit fat tails.

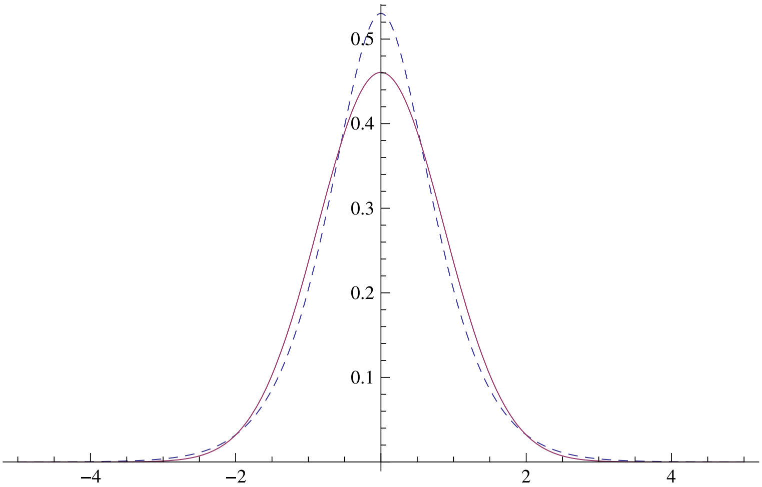

Fig. 1 Dashed - Normal distribution with variance uncertainty; Solid - Normal distribution conditional on the variance equal to its expectation

these papers exogenously impose that prices do not have a component related to the uncertainty over variance. In contrast, price in our model is the result of trade among investors with negative exponential utility, which yields a premium for uncertainty over the variance similar to that found in the dynamic asset pricing literature (for example, Buraschi and Jiltsov 2006). The intuition for this premium is as follows: investors with negative exponential utility apply a discount to cash flow distributions that exhibit fat tails, as their utility is concave in the variance of cash flows,[12] a notion that is distinct from risk aversion. This is generically true for utility functions that have a negative 4^{th} derivative, including power utility. The economics literature on risk preferences refers to agents with such utility functions as "temperate" and refers to the negative ratio of the 4^{th} and 3^{rd} derivative, $-u''''(x)/u'''(x)$, as the coefficient of absolute temperance.[13] In the case of negative exponential utility, the coefficient of absolute temperance is equal to the risk aversion parameter ρ, and hence we should expect that the premium related to variance uncertainty increases in ρ.

To show more explicitly how temperance manifests for the negative exponential utility function, note that an investor's certainty equivalent given demand D and price P for an arbitrary distribution over the cash flow variance reduces to the following:[14]

$$CE(D, P) = \rho D(\mu - P) - \ln\left(E\left(e^{D^2 \frac{\rho^2}{2} \tilde{V}}\right)\right). \tag{4}$$

[12]It is easily seen that $\frac{\partial^2}{\partial V^2} E(-e^{-\rho \tilde{x}}) = \frac{\partial^2}{\partial V^2}\left(-e^{-\rho\mu - \frac{\rho^2}{2} V}\right) < 0$.

[13]See, for example, Eeckhoudt et al. (1996), Gollier and Pratt (1996), and Noussair et al. (2014). Noussair et al. (2014) also present experimental evidence that suggests that individuals are indeed temperate.

[14]This statement is shown in the proof of Lemma 2.

Jensen's inequality implies that $\ln\left(E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right)\right) > D^2\frac{\rho^2}{2}E(\tilde{V})$. Thus an investor's certainty equivalent is reduced when a mean-preserving spread is applied to the variance. As a result, we should also expect that the firm's share price decreases as uncertainty over the variance increases.

To provide the market clearing price, we have to assume that $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$.[15] The condition prevents a situation where investors have infinite negative utility when they hold their share of the per capita endowment; if the condition did not hold, no price would allow for the market to clear.[16] Lemma 2 verifies investors' temperance leads to a variance uncertainty premium in price.

Lemma 2 *Assume that* $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$. *The firm's price can be expressed as:*

$$P = \mu - RP_0 - VUP_0, \tag{5}$$

$$\textit{where } RP_0 = \rho\mu_V \tag{6}$$

$$\textit{and } VUP_0 = \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V. \tag{7}$$

The price function in Lemma 2 reduces to the expected cash flows minus the standard risk premium minus an additional term for uncertainty over the variance, which we label the variance uncertainty premium or VUP_0. Note that the variance uncertainty premium equals the risk premium multiplied by an inflation factor. When there is no risk in the asset, i.e., $\mu_V = 0$, which also requires that $\sigma_V^2 = 0$, both the risk premium and the variance uncertainty premium vanish, and prices are equal to the mean. When there is no variance uncertainty in the asset, i.e., when σ_V^2 is set to 0, the variance uncertainty premium reduces to zero, but the risk premium remains.

Recall that, for a negative exponential utility function, the coefficient of absolute risk aversion, ρ, is equal to the coefficient of absolute temperance, $-u''''(x)/u'''(x) = \rho$. Note further that, while σ_V^2 increases excess kurtosis, μ_V decreases excess kurtosis. The variance uncertainty premium behaves accordingly, increasing in ρ and σ_V^2 and decreasing in μ_V. Intuitively, as ρ grows, investors become more averse to "low tail" events, and as σ_V^2 grows, the probability of these extreme events increases. Furthermore, an increase in μ_V increases the kurtosis of the underlying normal distribution, and the premium that investors demand for this is embedded in the standard risk premium, $\rho\mu_V$.

[15]The gamma distribution with shape parameter a and a scale parameter b is only defined for $a > 0$ and $b > 0$. To derive an investor's certainty equivalent, $b > \frac{D^2\rho^2}{2}$ has to hold. That is, the scale parameter has to be sufficiently large or the equilibrium demand (that is, the shares per capita) has to be sufficiently small. We derive an investor's certainty equivalent with the standard parameterization in the proof to Lemma 2.

[16]If the per capita endowment were an arbitrary constant e rather than 1, the condition becomes $\frac{1}{2}\rho^2e^2\frac{\sigma_v^2}{\mu_v} < 1$.

We discuss comparative statics in the following corollary, which takes into account not only the effect of these parameters on the variance uncertainty premium but also on the risk premium:

Corollary 1 *Prices are (i) decreasing in the variance of the cash flow variance σ_V^2 and risk aversion ρ, (ii) decreasing in the mean of the variance μ_V for $\mu_V > \rho\sigma_V^2$ and increasing in the mean of the variance μ_V for $\mu_V < \rho\sigma_V^2$, and (iii) uniformly decreasing in a location shift in the variance.*

The results in Corollary 1 (i) are straightforward given the previous discussion. The results in part (ii) are non-monotonic because, holding σ_V^2 constant, an increase in the expected variance, μ_V, has two effects. First, as usual, an increase in μ_V directly increases the risk premium. Second, however, it decreases the excess kurtosis, which decreases the variance uncertainty premium. The direct effect on the risk premium dominates for $\mu_V > \rho\sigma_V^2$ such that the sum of the risk premium and variance uncertainty premium is increasing in the expected variance. For $\mu_V < \rho\sigma_V^2$, the reduction in the variance uncertainty premium dominates, and price increases in μ_V. Note that, for a distribution that only takes on positive values for $V \in [0, \infty)$, changing the mean while keeping the variance constant requires a change in the shape of the distribution. That is, μ_V is not a simple location parameter as it is for normal distributions and thus does not uniformly reduce the investor's valuation of the distribution.[17] On the other hand, as we show in part (iii), a location shift of the form $\tilde{V}' = \tilde{V} + k$ for $k > 0$ strictly reduces prices as it only impacts the risk premium; this occurs because the location shift increases the distribution of $\tilde{V}$ in the sense of first-order stochastic dominance.[18]

[17] Increases in the mean holding the variance fixed reduce the degree of positive skew in the distribution.

[18] To understand more generally how prices respond to shifts in the distribution, consider changes in the variance distribution in the sense of first- and second- order stochastic dominance (FSD and SSD respectively). We should expect that distributional shifts in $\tilde{V}$ in the sense of FSD reduce price, and distributional shifts in $\tilde{V}$ in the sense of SSD increase price. Ali (1975) derives the following necessary and sufficient conditions for FSD and SSD for the gamma distribution characterized by shape and rate a and b:

$$\tilde{V}_1 \underset{FSD}{\succ} \tilde{V}_2 \text{ when } a_1 \geq a_2 \text{ and } b_1 \leq b_2 \text{ with one equality strict;} \tag{8}$$

$$\tilde{V}_1 \underset{SSD}{\succ} \tilde{V}_2 \text{ when} \frac{a_1}{a_2} \geq Max\left(1, \frac{b_1}{b_2}\right). \tag{9}$$

Expressing prices in terms of a and b, we find:

$$P = \mu - \frac{a}{b}\rho - \frac{\rho^2}{2b - \rho^2}\frac{a}{b}\rho. \tag{10}$$

A shift in the distribution of $\tilde{V}$ in the sense of FSD involves either increasing a or decreasing b; in either case, price falls. A shift in the distribution of $\tilde{V}$ in the sense of SSD is achieved by either increasing b and increasing a by at least the same percentage or by decreasing b and weakly increasing a. In either case, price increases as expected. The comparative static with respect to σ_V^2 in effect increases b while increasing a at the same rate. Equation 10 indicates that this increases prices only through its impact on the variance uncertainty premium.

Our results imply that empirically, both assets with higher variance and assets with more uncertainty about their variance should earn higher returns. This can act as a correlated omitted variable in empirical studies that consider the pricing of information. In the next section, we analyze how prices react to information announcements regarding their variance.

2.3 Price after risk disclosure

Now that we have established how investors price variance uncertainty in our model, we turn to the effects of risk disclosure by the firm. Our goal is to model the sort of risk disclosure that appears in section 1A and elsewhere in firms' 10-K reports, which appears to give investors some imperfect information directly regarding the riskiness of the firm's cash flows. The firm's price in Eq. 5 clearly suggests that such a risk disclosure has a direct impact on the firm's price, both through the risk premium and the variance uncertainty premium.

In a setting with a normally distributed cash flows and normally distributed earnings noise, the variance conditional on an information signal does not depend upon that signal's realization. However, Lemma 1 shows that the conditional uncertainty about the variance is not constant but, similar to the mean, is linearly increasing in the signal realization. To understand this phenomenon, note that we use a Poisson distribution, which exhibits greater noise for higher realizations of the fundamental variance $\tilde{V}$ (note that $Var(\tilde{S}|\tilde{V}) = \tau^{-1}\tilde{V}$). This is an intuitive property for the signalling distribution of a variance distribution to possess, since any variance distribution must be non-negative. Intuitively, the amount of noise that can be exhibited by low realizations of the variance $\tilde{V}$ is constrained by the bound at zero. To demonstrate this point, consider what happens in the knife-edged case where investors receive a signal $\tilde{S} = 0$ with infinitely large precision τ. As a zero mean has to imply a zero variance for a non-negative distribution, all variance uncertainty disappears. Furthermore, note that this result corresponds to the empirical pattern of increased return volatility accompanying bad news or, in our case, high variance news (for example, Black 1976). We will see that this feature differentiates our results on risk disclosure from those in standard models of earnings disclosure.

Nevertheless, as one would expect, providing investors with a variance signal *on average* reduces the amount of uncertainty over that variance. Technically, since $E(\tilde{S}) = \mu_V$, we have that:

$$E(Var(\tilde{V}|\tilde{S})) = Var(\tilde{V}) - \frac{Cov(\tilde{V}, \tilde{S})^2}{Var(\tilde{S})}. \tag{11}$$

In other words, the expected conditional variance is strictly lower than the unconditional variance, and the difference is increasing in the covariance between the signal and the true variance. As $\tau \to \infty$, the expected conditional variance approaches 0. Figure 2 depicts the variance distribution as the risk disclosure becomes increasingly precise, given that the risk disclosure equals the market's

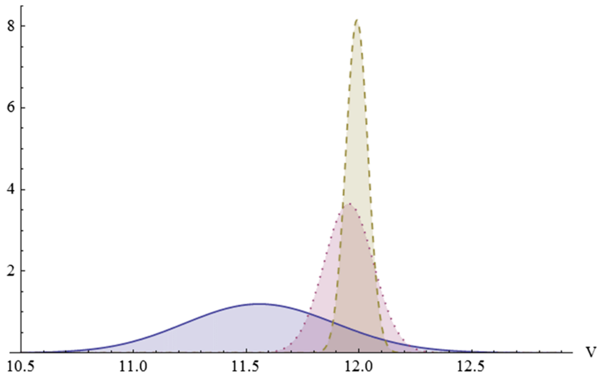

Fig. 2 This figure depicts the variance distribution for $\tau = 100$, 1000, and 5000. The tighter distributions reflect greater τ.

prior.[19] Note that, as the variance distributions tighten, the underlying distributions of cash flows correspondingly exhibit less kurtosis.

Proposition 1 derives the firm's price conditional on the risk disclosure. After the risk disclosure, the necessary condition from Proposition (2), $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$, is relaxed and becomes $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1 + \tau\frac{\sigma_V^2}{\mu_V}$.

Proposition 1 *Assume that* $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1 + \tau\frac{\sigma_V^2}{\mu_V}$. *The firm's price conditional on the risk disclosure can be expressed as:*

$$P(\tilde{S}) = \mu - RP_0 - \phi(\tau)VUP_0 - \alpha(\tau)(\tilde{S} - E(\tilde{S})), \tag{12}$$

$$\textit{where } \phi(\tau) = \frac{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}} \tag{13}$$

$$\textit{and } \alpha(\tau) = \rho\frac{\tau\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}}. \tag{14}$$

To demonstrate the effect of a disclosure on price, consider the price reaction to a risk disclosure that is equal to the market's prior, i.e., $\tilde{S} = E(\tilde{S})$. While the risk premium does not change, the variance premium is multiplied by a factor that is less than one and decreasing in τ. Intuitively, even when a risk disclosure has no mean effect, it reduces the ex-post uncertainty about the variance in proportion to its

[19] Setting the mean of the signals equal to their prior mean isolates the uncertainty reduction effect of information from any effect due to a change in the posterior expectation of the variance distribution. Although not apparent from the diagram, all three distributions have the same mean; the skewness of the gamma distribution obscures this fact.

precision and thus increases price. In the limit, as the variance disclosure becomes perfectly revealing, i.e., as $\tau \to \infty$, the variance uncertainty premium disappears.

Next, consider the price reaction to a signal that deviates from the prior mean. Similar to the classic result from Bayesian updating with normal distributions, price responds linearly to the deviation of a signal from its prior mean. This implies that there exists a response coefficient to the variance disclosure, $\alpha(\tau)$. As expected, the market's response $\alpha(\tau)$ is increasing in the precision τ. Furthermore, note that $\alpha(\tau)$ is increasing in risk aversion ρ, as higher risk aversion increases the importance of changes in the expected variance, and increasing in the ratio of the prior variance to the prior mean $\frac{\sigma_V^2}{\mu_V}$. An increase in $\frac{\sigma_V^2}{\mu_V}$ increases the signal to noise ratio of $\tilde{S}$. That is, the total amount of uncertainty to be resolved divided by the expected noise in the signal, $\frac{Var(\tilde{V})}{E(Var(\tilde{S}|\tilde{V}))} = \frac{1}{\tau}\frac{Var(\tilde{V})}{E(\tilde{V})} = \frac{1}{\tau}\frac{\sigma_V^2}{\mu_V}$ increases in $\frac{\sigma_V^2}{\mu_V}$. The expected noise in the signal, $E(Var(\tilde{S}|\tilde{V}))$, is increasing in μ_V since, holding fixed τ, when the expected cash flow variance is high, the market expects the risk disclosure to be less precise. On the other hand, when the prior variance σ_V^2 is greater, there is more uncertainty left to be resolved by the risk disclosure.

Finally, note that, for standard models of earnings disclosure in which cash flows and earnings follow normal distributions, the signal's *realization* influences price through the market's expectation of future cash flows but has no effect on the risk premium, which is reduced by a fixed constant (for example, Lambert et al. 2007). In contrast, in our setting, the realization of the signal itself affects both the risk premium and the variance uncertainty premium. To illustrate, we rewrite $P(\tilde{S})$ from Proposition (1) to absorb the term $\alpha(\tau)(\tilde{S} - E(\tilde{S}))$ into the risk and variance uncertainty premia:

$$P(\tilde{S}) = \mu - \rho E(\tilde{V}|\tilde{S}) - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}}\rho E(\tilde{V}|\tilde{S}). \tag{15}$$

Note that $\tilde{S}$ affects both the risk premium and the variance premium through its impact on $E(\tilde{V}|\tilde{S})$. This arises because investors' residual uncertainty after the risk disclosure is increasing in the realization of that disclosure.

2.4 Risk disclosure and the cost of capital

The discussion of Proposition 1 suggests that there is a price response even when the signal merely confirms the prior expectations. This implies that risk disclosure should have an effect on the firm's expected cost of capital. As an extension to the last section, we now examine the *ex-ante* effects of risk disclosure, i.e., the impact on the cost of capital. Following prior literature, we define the cost of capital as the discount that is applied to price relative to expected cash flows, that is, $E[\tilde{x}] - E(P|\Theta)$, where Θ is all information available to the market. Taking the expectation of price conditional on the signal, we find that:

$$E[\tilde{x}] - E(P(\tilde{S})) = RP_0 + \phi(\tau)VUP_0. \tag{16}$$

The cost of capital is a decreasing function of τ since $\phi'(\tau) < 0$. Moreover, the effect of τ on the cost of capital is increasing in risk aversion and increasing in the ratio of the prior variance to the prior mean of the variance distribution, $\frac{\sigma_V^2}{\mu_V}$. Intuitively, the value of reducing uncertainty over the variance is greater when the price responsiveness to the signal is larger.

As one would expect, $E(P(\tilde{S}))$ is increasing in τ at a decreasing rate, such that, as $\tau \to \infty$, the variance uncertainty premium disappears and expected price is exactly equal to the mean, less a discount for the prior expectation of risk. Concavity of the price effect in τ is a desirable attribute, as it suggests that, generally, firms that act to maximize their expected price do not find it optimal to fully acquire and disclose risk information even if the cost of doing so is linear. Our results therefore imply that firms acquire great benefit from acquiring and disclosing at least some variance information. Moreover, our results provide a theoretical rationale for the regulatory efforts of the SEC and FASB if legal mandates provide a mechanism for firms to commit to disclosure.

The following corollary summarizes the cost of capital effects of risk disclosure.

Corollary 2 *A firm's cost of capital is decreasing at a decreasing rate in the precision τ of its disclosure. The effect of τ on the cost of capital is increasing in the ratio $\frac{\sigma_V^2}{\mu_V^2}$ and increasing in risk aversion ρ.*

While our model captures disclosure about the uncertain variance of cash flows, an earnings disclosure with an uncertain precision (as in Subramanyam 1996) has a similar outcome, even in a setting with a constant cash flow variance. A random earnings precision implies that, from the investors' perspective, there is uncertainty about the conditional cash flow variance. That is, even though investors know the cash flow variance *before* the earnings disclosure, they are uncertain about how much residual cash flow variance remains *after* the disclosure. Corollary 2 suggests that, ex ante, it is beneficial for firms interested in reducing their cost of capital to commit to a constant earnings precision.

Finally, note that the quality of firms' earnings and other financial disclosures may impact the market's reaction to their risk disclosures. Fundamentally, earnings disclosures act to remove uncertainty about cash flows and therefore reduce the absolute uncertainty about the risk of the firm's cash flows. Intuitively, consider the extreme case in which earnings fully reveal the firm's future performance: in this case, cash flows are risk free and there cannot be risk uncertainty. Additionally, as noted in the literature on earnings disclosures in the presence of uncertain variances (for example, Subramanyam 1996; Neururer et al. 2016), the magnitude of an earnings signal may enable the market to update on the riskiness of the cash flow distribution. That is, earnings information itself may communicate information regarding the underlying cash flow variance. Both arguments imply that more precise earnings disclosures may act as a substitute for risk disclosures, causing risk disclosure to result in smaller market reactions and to have a reduced impact on the cost of capital. Empirically, this suggests that firms with less precise earnings face a greater marginal impact of risk disclosure on the cost of capital, potentially leading them to increase their efforts to provide such information. Moreover, it suggests that the price reaction to risk disclosure decreases in the quality of disclosed earnings.

3 Dynamic risk disclosure

The analysis up to this point focuses on the capital market effects of risk disclosure, rather than the firm's choice of disclosure quality. For that reason, we ignore any costs of disclosure. Extending the model to allow the firm to choose the quality of its disclosures yields straightforward results. For example, suppose the firm pays $c(\tau)$, where $c(\tau)$ is weakly concave and increasing, in return for a disclosure precision of τ. Following Corollary 2 the firm chooses a higher τ for higher values of $\frac{\sigma_V^2}{\mu_V^2}$ and ρ, which is similar to results in the existing disclosure literature. However, the similarity to prior literature changes when we allow the firm to make a dynamic information acquisition and disclosure decision.

In this section, we extend the model to include a cost of disclosure and consider the decision of a firm that has multiple opportunities over time to disclose information on its risks. Specifically, we assume that the firm may acquire and disclose an information signal regarding $\tilde{V}$ at each of several dates $t = 1, 2, ..., T$. For simplicity, we assume that the firm has to disclose any signal it acquires and that the firm acts to maximize its price at date T, net of costs. The first assumption ensures that our results are not driven by strategic discretionary disclosure incentives, as in Verrecchia (1983) and Jørgensen and Kirschenheiter (2003). The second assumption ensures that every disclosure has the same weight in the firm's objective function. In standard disclosure models, such as Lambert et al. (2007), where cash flows and the noise in earnings are normally distributed, the benefit to continuing disclosure at any point in time is *independent* of the prior disclosures. In other words, the firm would choose the same level of disclosure ex post as it would ex ante (before any signal is realized). In these models, the benefit to releasing information is a function only of the amount of uncertainty faced by investors, the precision of the signal acquired, and investors' risk aversion.[20] None of these determinants are affected by the performance revealed by prior disclosure; importantly, under the assumption of normal distributions, residual uncertainty is not a function of the signal acquired.[21]

[20] An exception to this is the work of Zhou (2016), where the firm's discretionary disclosure decision depends on past realizations. In the model, the market does not know the true expected value, which causes the perceived distribution and therefore the discretionary disclosure threshold to be a function of past disclosures.

[21] To see this formally, consider the setup of our model with no uncertainty over the variance of cash flows and consider the sequential disclosure of mean signals $\tilde{m}_\tau$. In particular, let $\tilde{m}_\tau = \tilde{x} + \tilde{\varepsilon}_\tau$, where $\tilde{\varepsilon}_\tau \sim N(0, \eta)$, $Cov(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j) = 0 \; \forall i, j$, and $Cov(\tilde{\varepsilon}_\tau, \tilde{x}) = 0$. Let $\tilde{M}_\tau$ be the mean of the first τ disclosed signals, and let V equal the known variance of cash flows. Then, after disclosing $\tau - 1$ signals with mean $\tilde{M}_{\tau-1}$, the expected benefit from receiving another signal is:

$$E(P(\tilde{M}_\tau)|\tilde{M}_{\tau-1}) - P(\tilde{M}_{\tau-1}) = \delta,$$

where $\delta = \rho\left(\frac{1}{\tau\eta^{-1}+V^{-1}} - \frac{1}{(\tau+1)\eta^{-1}+V^{-1}}\right)$. Since the benefit is not a function of $\tilde{M}_{\tau-1}$,the decision to disclose an additional signal never depends on prior disclosures.

The result that the firm's incentives to engage in disclosure are independent of previous disclosures is striking and seems specific to the assumption of normality. In our model of risk disclosure, which mandates the consideration of a non-normal distribution, we find that this result no longer holds. Intuitively, recall from Eq. 2 that the residual uncertainty over the firm's cash flow variance after a risk disclosure is increasing in the value of the disclosure, S. This implies that, if firms disclose preliminary information that suggests risks are high, they have strong incentives to continue disclosing risk information. As an example, consider a firm whose mandatory risk disclosure suggests that risks are high. Equation 2 suggests that residual uncertainty over their risks is heightened, such that the firm has a greater incentive to provide supplemental risk disclosures in the future.

Specifically, we assume that in period 1 the firm can pay a cost k to learn and disclose (truthfully) a variance signal s_1. In period 2, the firm again chooses whether to learn and disclose a signal at an additional cost k; this repeats until time T, where T is an arbitrary integer. If the firm does not pay the cost k at time t, it does not learn a variance signal. Since the firm observes the realization of the first τ signals before choosing whether to learn and disclose the $(\tau+1)^{th}$ signal, our setup is very similar to the classic statistical problem of sequential sampling (see DeGroot 1970, ch. 12).

To understand the firm's incentives in our model, recall that the cost of capital benefit of a risk disclosure is increasing in the amount of uncertainty over the variance. Additionally, when choosing whether to acquire an additional signal, the firm must take into account that there exists an option value to continuing to learn and disclose. An option value arises since if the signal in the next period is sufficiently high, the firm finds it optimal to disclose yet another signal. Since the continuation decision is a function of the random signal realizations, the number of signals a firm acquires before stopping, which we refer to as $\tilde{\varsigma}$, is a random variable.

Clearly, if the firm does not disclose in period τ, it continues to abstain from disclosing, as it recieves no new information and hence the nature of its decision is unchanged in each subsequent period. Now, let $\tilde{\Gamma}_\tau$ denote the sum of the disclosed signals received up to any time $\tau \in \{1, ..., T\}$, given that the firm has disclosed in each period. In the Appendix, we show that the immediate benefits to disclosing an additional signal after acquiring τ signals, $E(P(\tilde{\Gamma}_{\tau+1})|\tilde{\Gamma}_\tau) - P(\tilde{\Gamma}_\tau)$, can be written as $\tilde{\Gamma}_\tau f(\tau) + \gamma_\tau$ for a positive function $f(\tau)$ with $f'(\tau) < 0$ and some constant γ_τ. Furthermore, let C_τ be the event such that the firm discloses in period τ. The event C_{T-1} is equivalent to $\tilde{\Gamma}_{T-1} \in [\frac{k-\gamma_{T-1}}{f(T-1)}, \infty)$. The continuation value in period $T-2$, the first period in which there exists a real option, thus takes the following form:

$$\tilde{\Gamma}_{T-2} f(T-2) + \gamma_\tau + \Pr(C_{T-1}|\tilde{\Gamma}_{T-2}) E(P(\tilde{\Gamma}_T) - P(\tilde{\Gamma}_{T-1}) - k | C_{T-1}, \tilde{\Gamma}_{T-2}). \quad (17)$$

Clearly, the immediate benefit $\tilde{\Gamma}_{T-2} f(T-2) + \gamma_{T-2}$ increases in $\tilde{\Gamma}_{T-2}$. In the Appendix, we show a higher signal $\tilde{\Gamma}_{T-2}$ increases the distribution of $\tilde{\Gamma}_{T-1}$ in the sense of first-order stochastic dominance, and that both the probability of continuing next period and the expected benefit to continuing are increasing in $\tilde{\Gamma}_{T-1}$. Together, these results imply that the continuation value also increases in $\tilde{\Gamma}_{T-2}$. Continuing via backwards induction, we prove the following proposition:

Proposition 2 *The firm discloses in any given period τ if the sum of the signals it has learned thus far, $\tilde{\Gamma}_\tau$, belongs to an interval of the form $[c_\tau, \infty)$. This implies that, after disclosing high variance news, the firm is more likely to continue acquiring and disclosing additional signals. When the underlying variance $\tilde{V}$ is higher, the expected number of signals disclosed, $E(\tilde{\varsigma}|\tilde{V})$, is greater.*

The proposition states that, holding as fixed the expected level of risk faced by the firm, μ_V, if the firm has a higher underlying variance $\tilde{V}$, it acquires and discloses more information signals to the market. Hence Proposition 2 suggests that firms that disclose riskier cash flows than expected invest additional effort in continuing to inform the market regarding those risks. Furthermore, the proposition predicts that increased risk disclosures follow unexpected economy-wide increases in average risk such as the 2008 financial crisis. This result is distinct from our previous finding that σ_V^2 increased the impact of disclosure on prices; here we find that the true variance, in addition to the degree of variance uncertainty, impacts the amount of variance information a firm discloses.

As discussed above, a setting with known cash flow variance but uncertain precision of earnings disclosure leads to an uncertain conditional cash flow variance (Subramanyam 1996). The intuition offered in this section suggests that, in such a setting, investors' incentive to acquire information is a function of the earnings signal revealed. In equilibrium, investors respond to signals that lead to a high residual variance with their own information gathering efforts, resulting in a relatively constant conditional variance. For example, a conservative disclosure policy that provides more precise information about a firm's downside risk may lead to increased incentives to acquire information about a firm's upside potential.

4 Multiple asset extension

In this section, we consider an extension of our model to a multi-asset setting. Our single asset model leaves open the question of whether risk disclosures impact the cost of capital when investors hold diversified portfolios. Furthermore, we would like to address whether there exist positive externalities to risk disclosure. To address these questions, we develop a factor model where both idiosyncratic and systematic variances are unknown.

Assume that there are N firms in the economy with a per capita supply of $\frac{1}{N}$ whose cash flows $\tilde{x}_i$ are equal to a common factor loading plus idiosyncratic noise, i.e., $\tilde{x}_i = \beta_i \tilde{F} + \tilde{\varepsilon}_i$, where $\tilde{F}$ and $\tilde{\varepsilon}_i$ are independent. Furthermore, assume that both the common factor and the idiosyncratic components of cash flows have uncertain variances: $\tilde{F}|\tilde{V}_F \sim N(\mu_F, \tilde{V}_F)$ and $\tilde{\varepsilon}_i|\tilde{V}_i \sim N(\mu_{\varepsilon_i}, \tilde{V}_i)$ where $\tilde{V}_F$ is gamma distributed with mean μ_F and variance σ_F^2 and $\tilde{V}_i$ is gamma distributed with mean μ_{V_i} and variance $\sigma_{V_i}^2$. Finally, assume that the uncertain variance distributions are independent. Without loss of generality, we scale the factor such that the average beta is one: $\bar{\beta} \equiv \frac{1}{N} \sum_{i=1}^{N} \beta_i = 1$.

By assuming that the per capita supply of each asset is equal to $\frac{1}{N}$, the total risk in the economy remains constant when we vary the number of firms in the economy. There are two possible interpretations of this assumption. First, we could argue that, as the economy grows, the numerator of per capita supply shrinks. In this case, we are letting each firm become an arbitrarily small portion of the economy while keeping the total size of the economy the same. An alternative interpretation is that the investor base grows with the economy, and hence the per capita supply decreases because the denominator grows (see Lambert et al. (2007) for an in depth discussion of this issue.) In either case, in the limit, each individual investor holds an arbitrarily small amount of any given asset.

Let $\mu_i = E(\tilde{x}_i) = \beta_i \mu_F + \mu_{\varepsilon_i}$. Then, as a baseline, the price of firm k in the standard case when there is no variance uncertainty, that is, $\tilde{V} = \mu_{V_k}$ with certainty, equals:

$$P_k = \mu_k - \frac{\rho \mu_F}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\rho \mu_{V_k}}{N}. \tag{18}$$

We define $RP_S \equiv \frac{\rho \mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right)$ as the risk premium associated with the systematic component and $RP_I \equiv \frac{\rho \mu_{V_k}}{N}$ as the risk premium associated with the idiosyncratic component. As $N \to \infty$, we find that $RP_S \to \rho \beta_k \mu_{V_F}$ and $RP_I \to 0$, i.e., the idiosyncratic risk premium vanishes, and the systematic risk premium converges to risk aversion times the covariance of the firm's cash flows with the common factor. Next, we perform an analysis similar to the single asset case to derive prices under variance uncertainty.

Proposition 3 *Assume that* $\frac{\rho^2}{2}\frac{\sigma_{V_F}^2}{\mu_{V_F}} < 1$ *and* $\frac{\rho^2}{2}\frac{\sigma_{V_k}^2}{\mu_{V_k}} < N^2$.[22] *Then the price of the* k^{th} *asset is equal to the mean less the systematic and idiosyncratic risk premia less systematic and idiosyncratic variance uncertainty premia:*

$$P_k = \mu_k - RP_S - RP_I - VUP_S - VUP_I \tag{19}$$

$$\textit{where } VUP_S = \frac{\frac{1}{2}\rho^2 \frac{\sigma_{V_F}^2}{\mu_{V_F}}}{1 - \frac{1}{2}\rho^2 \frac{\sigma_{V_F}^2}{\mu_{V_F}}} RP_S \tag{20}$$

$$\textit{and } VUP_I = \frac{\frac{1}{2}\rho^2 \frac{\sigma_{V_k}^2}{\mu_{V_k}}}{N^2 - \frac{1}{2}\rho^2 \frac{\sigma_{V_k}^2}{\mu_{V_k}}} RP_I. \tag{21}$$

In essence, the systematic and idiosyncratic components of cash flows are valued separately, and price is equal to the sum of their values. Thus there exist variance uncertainty premia for both components. When $\beta_k > 0$, as in the single asset case,

[22] These conditions mirror the condition from the single asset case and imply that investors are willing to hold shares at any finite price.

the systematic variance uncertainty premium is increasing in $\sigma^2_{V_F}$, ambiguous with respect to μ_{V_F}, but decreasing for first-order stochastic dominant shifts in the factor variance. As $\sigma^2_{V_F} \to 0$ and $\sigma^2_{V_k} \to 0$, price converges to the baseline price with no variance uncertainty. Prices continue to be quadratic in the factor loading. For $\beta_k < 0$, all of the comparative statics are reversed, since the firm serves as a hedge.

The portion of the variance uncertainty premium associated with idiosyncratic risk is identical to that in the single asset case if the endowment had been $\frac{1}{N}$. As such, it carries the same intuition as in the single asset case for finite N. We summarize these results in the following corollary.

Corollary 3 *As $\sigma^2_{V_F} \to 0$ and $\sigma^2_{V_k} \to 0$, the price of firm k converges to the baseline price with no variance uncertainty. For $\beta_k > 0$, the price of firm k is decreasing in the variance of the idiosyncratic and systematic variances, $\sigma^2_{V_F}$ and $\sigma^2_{V_k}$, is decreasing in location shifts in the idiosyncratic or systematic variances, and is decreasing in the risk aversion ρ. For $\beta_k < 0$, the comparative statics on the systematic component are reversed.*

Next, consider what happens as the number of assets in the economy approaches infinity. Recall that, as supply per capita equals $\frac{1}{N}$, the total risk in the economy is constant in N. Thus, when we let $N \to \infty$ in Eq. 19, the idiosyncratic component vanishes, and price becomes:

$$\lim_{N\to\infty} P_k = \mu_k - \rho\mu_{V_F}\beta_k - \frac{\frac{1}{2}\rho^2 \frac{\sigma^2_{V_F}}{\mu_{V_F}}}{1 - \frac{1}{2}\rho^2 \frac{\sigma^2_{V_F}}{\mu_{V_F}}} \rho\mu_{V_F}\beta_k. \tag{22}$$

Similar to a setting with known variance, idiosyncratic risk factors are not priced, and investors are willing to pay a premium for firms whose cash flows are negatively correlated with the market portfolio.

4.1 Systematic and idiosyncratic disclosure

Equation 22 implies that, in a large economy, idiosyncratic disclosure has no impact on the cost of capital. Only disclosure that contains information on the systematic factor affects the cost of capital. For demonstration purposes, suppose that each firm issues a unique signal that can be broken down into two components, a systematic component and an idiosyncratic component. Mathematically, we assume this implies that the firm discloses two signals of the sort we discussed in the single asset case:

$$\begin{aligned} \tilde{S}_{kI} &= \sum_{j=1}^{\tau_I} s_{jIk};\, s_{jIk} \sim Poisson(\tilde{V}_k) \text{ and} \\ \tilde{S}_{kF} &= \sum_{i=1}^{\tau_F} s_{jFk};\, s_{jFk} \sim Poisson(\tilde{V}_F), \end{aligned} \tag{23}$$

where the s_{jIk}'s and s_{jFk}'s are mutually independent conditional on $\tilde{V}_k$ and $\tilde{V}_F$.[23] Let $\tilde{\mathbf{S}}_I = (\tilde{S}_{1I}, ..., \tilde{S}_{NI})$ be the vector of firms' idiosyncratic signals and $\tilde{\mathbf{S}}_F = (\tilde{S}_{1F}, ..., \tilde{S}_{NF})$ be the vector of firms' systematic signals. Then, as $N \to \infty$, the k^{th} firm's price conditional on the signals is equal to:

$$P_k|\tilde{\mathbf{S}}_I, \tilde{\mathbf{S}}_F = \mu_k - RP_S - \phi(\tau'\mathbf{1})VUP_S - \alpha(\tau'\mathbf{1})(\bar{S}_F - E(\bar{S}_F)), \tag{24}$$

where $\bar{S}_F = (\tau'\mathbf{1})^{-1}\tau'\tilde{\mathbf{S}}_F$, $\phi(\tau'\mathbf{1}) = \left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}}\right)\left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}} + \frac{\sigma^2_{V_F}}{\mu_{V_F}}\tau'\mathbf{1}\right)^{-1}$, and $\alpha(\tau'\mathbf{1}) = \rho\beta_k\frac{\sigma^2_{V_F}}{\mu_{V_F}}\tau'\mathbf{1}\left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}} + \frac{\sigma^2_{V_F}}{\mu_{V_F}}\tau'\mathbf{1}\right)^{-1}$. This implies the cost of capital is equal to:

$$\mu_k - E(P_k|\tilde{S}_{kF}) = RP_S - \phi(\tau'\mathbf{1})VUP_S. \tag{25}$$

The market uses a weighted average $\bar{S}_F = \left(\tau'\mathbf{1}\right)^{-1}\tau'\tilde{\mathbf{S}}_F$ of the firms' signals when updating on the factor variance and takes into account the total precision of all disclosures, $\tau'\mathbf{1}$. The cost of capital is decreasing in τ_{iF}, but idiosyncratic disclosure τ_{ik} plays no role. As long as disclosure contains some information regarding the common factor, the results from all of our single asset models continue to hold in an economy with multiple assets.

Similar to the analysis of an earnings disclosure in Admati and Pfleiderer (2000) and Lambert et al. (2007), this implies that any firm's incentive to disclose information is reduced because a single firm's disclosure is unlikely to be very informative about the systematic risk. However, risk disclosure that has a systematic component generates a positive externality in that it reduces uncertainty for all firms in the economy and reduces the aggregate cost of capital. This provides a potential rationale for the attention regulatory authorities have been giving to mandated risk disclosures.

Finally, note that, in a setting without variance uncertainty, firms in a large economy have no incentive to commit to disclose information about idiosyncratic cash flows, as the associated risk is not priced. However, after receiving this type of information, firms have an incentive to disclose good news as in, for example, Verrecchia (1983). The reason is that, while idiosyncratic risks are not priced, all expected cash flows are priced such that firms prefer to disclose positive information regarding idiosyncratic cash flows *even* in a large economy. In our setting, this latter result does not apply. In a large economy, neither idiosyncratic risk uncertainty nor idiosyncratic risks are priced. For that reason, firms have no incentive to disclose even low idiosyncratic risk information.

[23] Mathematically, modeling variance disclosure by multiple firms in our setup is not straightforward since the systematic components of firms' disclosures likely overlap. In particular, multiple firms may aggregate signals that contain the same $\tilde{s}_{ik}$. Nevertheless, it is intuitive that firms' information disclosures can be jointly used to assess the uncertain variance of the factor, and thus we assume that investors can tease apart the novel information in a firm's disclosures.

5 Conclusion

In recent years, the SEC and FASB have taken action to increase firms' risk disclosure. However, the theoretical literature on disclosure has offered sparse evidence on the effect of risk disclosure on prices. In particular, it has either assumed away pricing of variance uncertainty or has focused on the strategic disclosure setting with perfect disclosure of variance uncertainty. In this paper, we address how risk disclosure impacts market prices and its impact on the cost of capital. Our results emphasize that the intuition derived from earnings disclosure models are only partially applicable to the setting of risk disclosure.

We begin by deriving price when investors have negative exponential utility and cash flows are normal conditional on a gamma distributed variance. We show that investors penalize firms with variance uncertainty. We next consider how an ex-ante commitment to risk disclosure can affect a firm's cost of capital. Analogous to the standard result for mean-based disclosure, risk disclosure reduces investors perceived riskiness of the variance distribution. This leads to a decrease in the cost of capital. Technically, this results from that fact that variance uncertainty implies that the cash flow distribution exhibits positive excess kurtosis and that investors with a negative exponential utility are kurtosis averse (that is, they are "temperate"). Risk disclosure reduces the excess kurtosis and thus increases expected price.

To explore some differences between earnings disclosures and risk disclosure, we endogenize the acquisition of information by the firm, and show that firms tend to acquire and disclose more information after receiving high variance signals. This arises due to the correlation between mean and variance for non-negative distributions. Finally, we show that the results of mean disclosure in a multi-asset environment carry over to risk disclosure.

A notable contribution of our paper is that it provides a set of tools for examining risk disclosure that can be applied to other research settings. In particular, we have derived expected utility given negative exponential utility functions with a well known distribution over the variance. This could be applied to study a setting in which investors have private information. We have also abstracted completely from agency issues by assuming the firm exogenously makes decisions to maximize prices. Future research could extend our results on variance uncertainty to study agency problems.

Furthermore, we investigate disclosures that inform investors exclusively about the variance of cash flows. This implies that the risk disclosure in our model is symmetric in that it informs investors equally about upside potential and downside risks. There are multiple avenues for future research to build on our model. First, a disclosure that informs investors about downside risk would be interesting to investigate when cash flows do not have a symmetric distribution. Second, when investors are uncertain about the variance of cash flows, an earnings disclosure likely affects all moments of the cash flow distribution. It would be interesting to investigate the capital market effects of an earnings disclosure when investors have preferences over more than the first two moments. Finally, firms disclose information about their exposure to macroeconomic risk factors. Naturally, the effects of such a disclosure should depend on the distribution of the risk factor. Furthermore, fluctuation of the firm's exposure should introduce correlation between the expected value and the variance of cash flows when the risk factor itself is random (as a high exposure leads to a high mean *and* a high variance). This implies that the cash flow distribution exhibits skewness. Because investors with CARA utility have preferences over all moments of the distribution, skewness should be priced and should impact the price effects of disclosure.

Unfortunately, our study shows that the expected utility of an investor with a negative exponential utility is relatively complex. This naturally limits the ability to answer several questions with our setting. However, future research could be based on a mean-variance-kurtosis utility function and a state contingent variance with a binary state.[24] Consistent with our results, one can show that variance uncertainty increases kurtosis more generally. For example, consider two possible lotteries, A and B, whose payoff distributions depend on whether the state is 1 or 2. Lottery A pays off $\varepsilon_1 \sim N\left(0, \sigma_1^2\right)$ in state 1 and $\varepsilon_2 \sim N\left(0, \sigma_2^2\right)$ in state 2, whereas lottery B pays off 0 in state 1 and $\varepsilon_1 + \varepsilon_2$ in state 2. When $\sigma_1^2 = \sigma_2^2$, lottery A does not exhibit excess kurtosis, but lottery B, by "grouping" the risks, does. Noussair et al. (2014) show that agents with a positive coefficient of absolute temperance, $-u''''(x)/u'''(x)$, prefer lottery A over B. In this setting, risk disclosure is equivalent to a signal regarding the state of nature.

In summary, while our model provides some insight into the capital market impact of risk disclosure, it is but a first step toward developing regulatory recommendations. It should be clear from our model that many of the same trade-offs that have been highlighted for mean disclosure (for example, Beyer et al. 2010) hold for risk disclosure, and that regulators must take into consideration the multitude of effects that mandated disclosure may have.

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Appendix

Proof of Lemma 1

The well known result from Bayesian statistics is couched in terms of the shape and scale parameters, a and b (for example, DeGroot (1970) pg. 164). Let $\tilde{V} \sim Gamma(a, b)$ and $\tilde{s}_i \sim Poisson(\tilde{V})$ where $\tilde{s}_1, ..., \tilde{s}_\tau$ are independent conditional on $\tilde{V}$. Then,

$$\tilde{V}|\tilde{s}_1, ..., \tilde{s}_\tau \sim Gamma\left(a + \sum_{i=1}^{\tau} \tilde{s}_i, b + \tau\right). \tag{26}$$

[24] A mean-variance-kurtosis utility function is consistent with the fourth-order development of the Arrow-Pratt expression for the risk premium (see Le Courtois 2012).

We next show that the mean of the signals is a sufficient statistic for their individual realizations by using the Fisher-Neyman factorization theorem:

$$\begin{aligned} f(s_1, ..., s_\tau, V) &= f(s_1, ..., s_\tau | V) f(V) \\ &= \prod_{i=1}^{\tau} f(s_i|V) f(V) = \prod_{i=1}^{\tau} \frac{V^{s_i} e^{-V}}{s_i!} \frac{V^{a-1} e^{-Vb}}{b^{-a}\Gamma(a)} \\ &= \frac{V^{\Sigma_{i=1}^{\tau} s_i} e^{-V}}{\prod_{i=1}^{\tau} s_i!} \frac{V^{a-1} e^{-Vb}}{b^{-a}\Gamma(a)} = \frac{1}{\prod_{i=1}^{\tau} s_i!} \frac{V^{a-1+\Sigma_{i=1}^{\tau} s_i} e^{-V(b+1)}}{b^{-a}\Gamma(a)} \\ &\equiv h(s_1, ..., s_\tau) g\left(\Sigma_{i=1}^{\tau} s_i, V\right). \end{aligned} \tag{27}$$

Proving the lemma is simply a matter of using these results and performing algebraic manipulations.

$$\begin{aligned} E(\tilde{V}|\tilde{S}) &= E(\tilde{V}|\tilde{s}_1, ..., \tilde{s}_\tau) = \frac{a + \sum_{i=1}^{\tau} \tilde{s}_i}{b+\tau} = \frac{a + \tau\tilde{S}}{b+\tau} \\ &= \frac{\tau\tilde{S}\sigma_V^2 + \mu_V^2}{\tau\sigma_V^2 + \mu_V} = \mu_V + \frac{\sigma_V^2}{\sigma_V^2 + \tau^{-1}\mu_V}(\tilde{S} - \mu_V) \text{ and} \\ Var(\tilde{V}|\tilde{S}) &= Var(\tilde{V}|\tilde{s}_1, ..., \tilde{s}_\tau) = \frac{a + \sum_{i=1}^{\tau} \tilde{s}_i}{(b+\tau)^2} = \frac{a + \tau\tilde{S}}{(b+\tau)^2} \\ &= \frac{\frac{\mu_V^2}{\sigma_V^2} + \tau\tilde{S}}{\left(\frac{\mu_V}{\sigma_V^2} + \tau\right)^2} = \frac{\left(\tilde{S}\tau\sigma_V^2 + \mu_V^2\right)\sigma_V^2}{\left(\tau\sigma_V^2 + \mu_V\right)^2}. \end{aligned} \tag{28}$$

Note that:

$$\begin{aligned} Cov(\tilde{V}, \tilde{S}) &= E(\tilde{V}\tilde{S}) - E(\tilde{V})E(\tilde{S}) \\ &= E(\tilde{V}E(\tilde{S}|\tilde{V})) - \mu_V^2 \\ &= E(\tilde{V}^2) - \mu_V^2 = \sigma_V^2 \text{ and} \\ Var(\tilde{S}) &= Var(E(\tilde{S}|\tilde{V})) + E(Var(\tilde{S}|\tilde{V})) \\ &= \sigma_V^2 + \tau^{-1}\mu_V. \end{aligned} \tag{29}$$

Hence we can alternatively write:

$$\begin{aligned} E(\tilde{V}|\tilde{S}) &= E(\tilde{V}) + \frac{Cov(\tilde{V}, \tilde{S})}{Var(\tilde{S})}(\tilde{S} - E(\tilde{S})) \text{ and} \\ Var(\tilde{V}|\tilde{S}) &= Var(\tilde{V}) - \frac{Cov(\tilde{V}, \tilde{S})^2}{Var(\tilde{S})} + (\frac{Cov(\tilde{V}, \tilde{S})}{Var(\tilde{S})})^2 \frac{\tilde{S} - \mu_V}{\tau}. \end{aligned} \tag{30}$$

Proof of Lemma 2

For simplicity, we first find price for a gamma distribution parameterized by shape and scale, a and b. We then convert this to price for a gamma characterized by μ_V and σ_V^2 by using the fact that $a = \frac{\mu_V^2}{\sigma_V^2}$ and $b = \frac{\mu_V}{\sigma_V^2}$. Investors solve

$$\begin{aligned} &\arg\max_D E(E(u(D(\tilde{x} - P))|\tilde{V})) \qquad (31)\\ &= \arg\max_D E\left(-e^{-\rho D(\mu - P) + D^2\frac{\rho^2}{2}\tilde{V}}\right)\\ &= \arg\max_D -e^{-\rho D(\mu - P)} E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right), \end{aligned}$$

where the simplifications follow from the law of iterated expectations and the MGF of a normal distribution. After applying a monotone transformation, this is equal to:

$$\arg\max_D \rho D(\mu - P) - \ln\left(E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right)\right). \qquad (32)$$

Solving for the latter term, we find:

$$\begin{aligned} E\left(e^{\frac{D^2\rho^2}{2}\tilde{V}}\right) &= \int_0^\infty e^{\frac{D^2\rho^2}{2}x}\frac{x^{a-1}e^{-xb}}{b^{-a}\Gamma(a)}dx \qquad (33)\\ &= \int_0^\infty \left(\frac{b - \frac{D^2\rho^2}{2}}{b}\right)^{-a} \frac{x^{a-1}e^{-x\left(b-\frac{D^2\rho^2}{2}\right)}}{\left(b - \frac{D^2\rho^2}{2}\right)^{-a}\Gamma(a)}dx\\ &= \left(\frac{b - \frac{D^2\rho^2}{2}}{b}\right)^{-a} \underbrace{\int_0^\infty \frac{x^{a-1}e^{-x\left(b-\frac{D^2\rho^2}{2}\right)}}{\left(b - \frac{D^2\rho^2}{2}\right)^{-a}\Gamma(a)}dx}_{\text{Gamma PDF}}\\ &= \left(1 - \frac{D^2\rho^2}{2b}\right)^{-a}. \end{aligned}$$

Note that the integral of the gamma PDF only exists for $b > \frac{D^2\rho^2}{2}$. Otherwise, the integral is equal to negative infinity. We conjecture an equilibrium where $b > \frac{D^2\rho^2}{2}$ holds. Then, in equilibrium, since all investors are homogenous, all investors hold the per capita endowment of 1 share. Thus, given our assumption $b > \frac{\rho^2}{2} \iff \rho^2\sigma_V^2 < 2\mu_V^2$, the conjecture is verified. Each investor's first-order condition is:

$$\begin{aligned} &\frac{\partial}{\partial D}\left(\rho D(\mu - P) + a\ln\left(1 - \frac{D^2\rho^2}{2b}\right)\right) = 0 \qquad (34)\\ \implies &(\mu - P)\left(b - \frac{1}{2}\rho^2 D^2\right) = a\rho D. \end{aligned}$$

Again, in equilibrium, $D = 1$ since all investors are homogeneous. This implies

$$P = \mu - \frac{a\rho}{b - \frac{1}{2}\rho^2}. \tag{35}$$

Substituting $a = \frac{\mu_V^2}{\sigma_V^2}$ and $b = \frac{\mu_V}{\sigma_V^2}$, we get:

$$P = \mu - \frac{1}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V = \mu - \rho\mu_V - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V. \tag{36}$$

Proof of Corollary 1

The result that prices are decreasing in σ_V^2 and ρ can easily be seen by taking the respective derivatives of Eq. 5. The derivative with respect to μ_V is equal to:

$$-4\rho\frac{\mu_V}{\left(2\mu_V - \rho^2\sigma_V^2\right)^2}\left(\mu_V - \rho^2\sigma_V^2\right), \tag{37}$$

which has the same sign as $-\left(\mu_V - \rho^2\sigma_V^2\right)$. Finally, to show that prices uniformly decrease in a location shift in the variance, note that repeating the analysis for the proof of Proposition 1 yields:

$$\begin{aligned}\arg\max_D E(E(u(D(\tilde{x} - P))|\tilde{V})) &= \arg\max_D E\left(-e^{-\rho D(\mu - P) + D^2\frac{\rho^2}{2}(\tilde{V}+k)}\right)\\ &= \arg\max_D -e^{-\rho D(\mu-P)+\frac{D^2\rho^2}{2}k}E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right),\end{aligned} \tag{38}$$

such that investor's first-order condition is:

$$\frac{\partial}{\partial D}\left(\rho D(\mu - P) + \frac{D^2\rho^2}{2}k + a\ln\left(1 - \frac{D^2\rho^2}{2b}\right)\right) = 0, \tag{39}$$

which implies

$$P = \mu - k\rho - \frac{2a\rho}{2b - \rho^2} = \mu - (k + \mu_V)\rho - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V. \tag{40}$$

Proof of Proposition 1

Proving the proposition is simply a matter of plugging the results from the lemma into Eq. 5:

$$\begin{aligned}P|S &= \mu - \rho E(\tilde{V}|\tilde{S}) - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\tau\sigma_V^2 + \mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\tau\sigma_V^2+\mu_V}}\rho E(\tilde{V}|\tilde{S})\\ &= \mu - \rho E(\tilde{V}|\tilde{S}) - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}}\rho E(\tilde{V}|\tilde{S}).\end{aligned} \tag{41}$$

Proof of Corollary 2 Taking the respective derivatives of Eq. 16 proves the claims.

Proof of Proposition 2

We start by establishing two useful lemmas. Again, define $\tilde{\Gamma}_\tau$ to be the sum of all disclosed signals up to time τ and let $P(\tilde{\Gamma}_\tau)$ denote the price of the firm given that the firm has disclosed in every period until time τ.

Lemma 3 *The immediate benefit to disclosing an additional signal, $E(P(\tilde{\Gamma}_{\tau+1})|\tilde{\Gamma}_\tau) - P(\tilde{\Gamma}_\tau)$ takes the form $\tilde{\Gamma}_\tau f(\tau) + \gamma_\tau$ where $f(\tau) > 0$ and $f'(\tau) < 0$ and γ_τ is a constant independent of $\tilde{\Gamma}_\tau$.*

Proof of Lemma 3

The firm chooses to disclose an additional signal $\tilde{s}_{\tau+1}$ when its expectation of the change in price given the additional signal is greater than the cost of acquiring an additional signal:

$$E(P(\tilde{\Gamma}_{\tau+1}) - P(\tilde{\Gamma}_\tau)|\tilde{\Gamma}_\tau) > k. \tag{42}$$

Writing out the left hand side and applying the law of iterated expectations, we find:

$$\begin{aligned}
&E(P(\tilde{\Gamma}_{\tau+1}) - P(\tilde{\Gamma}_\tau)|\tilde{\Gamma}_\tau) &(43)\\
&= -E(E(\tilde{V}|\tilde{\Gamma}_{\tau+1})|\tilde{\Gamma}_\tau)\rho - E\left[\frac{\frac{1}{2}\rho^2\frac{E(Var(V|\tilde{\Gamma}_\tau, s_{\tau+1})|\tilde{\Gamma}_\tau)}{E(V|\tilde{\Gamma}_\tau)}}{1-\frac{1}{2}\rho^2\frac{E(Var(V|\tilde{\Gamma}_\tau, s_{\tau+1})|\tilde{\Gamma}_\tau)}{E(V|S_\tau)}} E(V|\tilde{\Gamma}_{\tau+1})|\tilde{\Gamma}_\tau\right]\rho\\
&\quad + E(\tilde{V}|\tilde{\Gamma}_\tau)\rho + \frac{\frac{1}{2}\rho^2\frac{Var(V|\tilde{\Gamma}_\tau)}{E(V|\tilde{\Gamma}_\tau)}}{1-\frac{1}{2}\rho^2\frac{Var(V|\tilde{\Gamma}_\tau)}{E(V|\tilde{\Gamma}_\tau)}} E(\tilde{V}|\tilde{\Gamma}_\tau)\rho\\
&= E(\tilde{V}|\tilde{\Gamma}_\tau)\rho\left(\frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}} - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + (\tau+1)\frac{\sigma_V^2}{\mu_V}}\right) &(44)\\
&\equiv \tilde{\Gamma}_\tau f(\tau) + \gamma_\tau, &(45)
\end{aligned}$$

where we used the fact that

$$\begin{aligned}
E(Var(\tilde{V}|\tilde{\Gamma}_{\tau+1})|\tilde{\Gamma}_\tau) &= E\left(\frac{\left((\tilde{\Gamma}_\tau + \tilde{s}_{\tau+1})\sigma_V^2 + \mu_V^2\right)\sigma_V^2}{\left((\tau+1)\sigma_V^2 + \mu_V\right)^2}|\tilde{\Gamma}_\tau\right) &(46)\\
&= \frac{\left(\left(\tilde{\Gamma}_\tau + \frac{\tilde{\Gamma}_\tau\sigma_V^2 + \mu_V^2}{\tau\sigma_V^2 + \mu_V}\right)\sigma_V^2 + \mu_V^2\right)\sigma_V^2}{\left((\tau+1)\sigma_V^2 + \mu_V\right)^2}\\
&= E(\tilde{V}|\tilde{\Gamma}_\tau)\frac{\sigma_V^2}{(\tau+1)\sigma_V^2 + \mu_V}.
\end{aligned}$$

We next prove the following lemma, which will be used several times.

Lemma 4 *An increase in* $\tilde{\Gamma}_{\tau-1}$ *increases the distribution of* $\tilde{\Gamma}_{\tau}|\tilde{\Gamma}_{\tau-1}$ *in the sense of first-order stochastic dominance.*

Proof of Lemma 4

Define $F_{\tau}(\cdot|\tilde{\Gamma}_{\tau-1})$ to be the CDF of the τ^{th} signal $\tilde{s}_{\tau}$ conditional on the sum of the first $\tau-1$ signals. We start by showing $\frac{\partial}{\partial\tilde{\Gamma}_{\tau-1}}F_{\tau}(\cdot|\tilde{\Gamma}_{\tau-1}) < 0$;

$$F_{\tau}(c|\tilde{\Gamma}_{\tau-1}) = \Pr(\tilde{s}_{\tau} < c|\tilde{\Gamma}_{\tau-1}) \tag{47}$$

$$= \int \Pr(\tilde{s}_{\tau} < c|V)\Pr(V|\tilde{\Gamma}_{\tau-1})dV. \tag{48}$$

Recall the following general result: if some random variable $\tilde{x}$ leads to an increase in the conditional distribution $f(\cdot|\tilde{x})$ in the sense of first-order stochastic dominance and $u' < 0$, then $\frac{\partial}{\partial x}\int u'(\cdot)f(\cdot|\tilde{x}) < 0$. This can be shown using integration by parts. Furthermore, recall that $\tilde{\Gamma}_{\tau-1}$ increases the distribution of $\tilde{V}$ in the sense of first-order stochastic dominance and that $\frac{\partial}{\partial\tilde{V}}\Pr(\tilde{s}_{\tau} < c|\tilde{V}) < 0$. This implies $\frac{\partial}{\partial\tilde{\Gamma}_{\tau-1}}F_{\tau}(\cdot|\tilde{\Gamma}_{\tau-1}) < 0$. Combining these results:

$$\Pr(\tilde{\Gamma}_{\tau} \leq c|\tilde{\Gamma}_{\tau-1}) = \Pr(\tilde{\Gamma}_{\tau-1} + \tilde{s}_{\tau} \leq c|\tilde{\Gamma}_{\tau-1}) \tag{49}$$

$$= \Pr(\tilde{s}_{\tau} \leq c - \tilde{\Gamma}_{\tau-1}|\tilde{\Gamma}_{\tau-1}) \tag{50}$$

$$= F_{\tau}(g(\tilde{\Gamma}_{\tau-1})|\tilde{\Gamma}_{\tau-1}), \tag{51}$$

$$\text{where } g(\tilde{\Gamma}_{\tau-1}) \equiv c - \tilde{\Gamma}_{\tau-1}. \tag{52}$$

Using the chain rule,

$$\frac{d}{d\tilde{\Gamma}_{\tau-1}}\Pr(\tilde{\Gamma}_{\tau} < c|\tilde{\Gamma}_{\tau-1}) = \frac{\partial F_{\tau}(g(\tilde{\Gamma}_{\tau-1})|\tilde{\Gamma}_{\tau-1})}{\partial\tilde{\Gamma}_{\tau-1}} + \frac{\partial F_{\tau}}{\partial g}\frac{\partial g(\tilde{\Gamma}_{\tau-1})}{\partial\tilde{\Gamma}_{\tau-1}} < 0. \tag{53}$$

To simplify notation, call the event of continuing in period τ, C_{τ}. Our strategy in proving the proposition is as follows. First, it is easily verified that the firm never discloses after period τ if it did not disclose in a previous period, such that we can look only at disclosure policies in which the firm discloses up to some period and then stops. Now, we start by considering a firm that has disclosed until the last period with the option to disclose, $T-1$. We show that the value to disclosing additional information is of the form $\tilde{\Gamma}_{T-1}f(T-1) + \gamma_{T-1} - k$ and thus is increasing in $\tilde{\Gamma}_{T-1}$. We then look at period $T-2$ and show that, given the firm has disclosed in every period, the expected benefit to disclosing is equal to $\tilde{\Gamma}_{T-2}f(T-2) + \gamma_{T-2} +$ $\text{OptVal}(\tilde{\Gamma}_{T-2}) - k$. We show that the option value has the property that it is increasing in $\tilde{\Gamma}_{T-2}$, which implies that the expected benefit to disclosure is increasing in $\tilde{\Gamma}_{T-2}$. This process can then be repeated iteratively.

Consider terminal period $T-1$. The value to continuing is:

$$ContVal_{T-1} = \tilde{\Gamma}_{T-1}f(T-1) + \gamma_{T-1} - k. \tag{54}$$

There is clearly no option value, as the next period is the terminal period. In period $T-2$, the firm's continuation value is:

$$\begin{aligned} ContVal_{T-2} &= \Pr(C_{T-1}|\tilde{\Gamma}_{T-2})E(P(\tilde{\Gamma}_T) - P_{T-2} - 2k|C_{T-1}, \tilde{\Gamma}_{T-2}) & (55)\\ &\quad +(1-\Pr(C_{T-1}|\tilde{\Gamma}_{T-2}))E\left(P(\tilde{\Gamma}_{T-1}) - P_{T-2} - k|C^C_{T-1}, \tilde{\Gamma}_{T-2}\right) & (56)\\ &= \tilde{\Gamma}_{T-2}f(T-2) + \gamma_{T-2} - k + OptVal(T-2, \tilde{\Gamma}_{T-2}), & (57)\end{aligned}$$

where $OptVal(T-2, \tilde{\Gamma}_{T-2}) = \Pr(C_{T-1}|\tilde{\Gamma}_{T-2})E(P(\tilde{\Gamma}_T) - P(\tilde{\Gamma}_{T-1}).$ $-k|C_{T-1}, \tilde{\Gamma}_{T-2})$. This can be seen by adding and subtracting $\Pr(C_{T-1}|\tilde{\Gamma}_{T-2})$ $E(P(\tilde{\Gamma}_{T-1}) - P_{T-2} - k|C_{T-1}, \tilde{\Gamma}_{T-2})$ and simplifying. The option value is nonnegative since the firm would never choose to continue unless doing so yielded positive returns in expectation. We next show that option value is increasing in $\tilde{\Gamma}_{T-2}$. Clearly, $\Pr(C_{T-1}|\tilde{\Gamma}_{T-2})$ is increasing in $\tilde{\Gamma}_{T-2}$. Furthermore, by the smoothing law:

$$\begin{aligned} E(P(\tilde{\Gamma}_T) - P(\tilde{\Gamma}_{T-1})|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}) &= E(E(P(\tilde{\Gamma}_T) - P(\tilde{\Gamma}_{T-1})|\tilde{\Gamma}_{T-1})|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2})\\ &= f(T-1)E(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}) + \gamma_{T-1}.\end{aligned}$$

Next, we show that this expectation is increasing in $\tilde{\Gamma}_{T-2}$. The argument is nontrivial since first-order stochastic dominance is not a sufficient condition for *truncated* expectations to be increasing (for example, Shaked and Shanthikumar 2006).

Lemma 5 $\frac{\partial}{\partial \tilde{\Gamma}_{T-2}} E(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}) > 0.$

Proof of Lemma 5

Note that

$$\begin{aligned} E(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}) &= E(E(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}, \tilde{V})|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2})\\ &\equiv E\left[h^*(\tilde{\Gamma}_{T-2}, \tilde{V})|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right],\\ \text{where} h^*(\tilde{\Gamma}_{T-2}, \tilde{V}) &= E(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}, \tilde{V}).\end{aligned}$$

We first show that $h^*(\tilde{\Gamma}_{T-2}, \tilde{V})$ is increasing in $\tilde{\Gamma}_{T-2}$ and $\tilde{V}$. We finish the proof by combining this with the fact that $\tilde{\Gamma}_{T-2}$ shifts the distribution of $\tilde{V}$ in the sense of first-order stochastic dominance and using an integration by parts argument as was done previously.

Note that $\tilde{V}$ and $\tilde{\Gamma}_{T-2}$ shift the distribution of $\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-2}, \tilde{V}$ upwards in the sense of monotone likelihood ratio dominance. To see this, suppose $\Gamma_{t-1} > \Gamma'_{t-1}$. For monotone likelihood ratio dominance to hold, we must show the following increases in V and Γ_{t-2}:

$$\begin{aligned} \frac{\Pr(\Gamma_{t-1}|\Gamma_{t-2}, V)}{\Pr(\Gamma'_{t-1}|\Gamma_{t-2}, V)} &= \frac{\frac{e^{-V}V^{(\Gamma_{t-1}-\Gamma_{t-2})}}{(\Gamma_{t-1}-\Gamma_{t-2})!}}{\frac{e^{-V}V^{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)}}{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}}\\ &= \frac{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}{(\Gamma_{t-1}-\Gamma_{t-2})!}V^{\left(\Gamma_{t-1}-\Gamma'_{t-1}\right)}.\end{aligned}$$

This is clearly increasing in V and is increasing in Γ_{t-2} since

$$\frac{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}{\left(\Gamma_{t-1}-\Gamma_{t-2}\right)!}=\left(\prod_{i=1}^{\Gamma'_{t-1}-\Gamma_{t-1}}\left(\Gamma'_{t-1}-\Gamma_{t-2}+i\right)\right)^{-1}$$

increases in Γ_{t-2}.

Monotone likelihood ratio dominance implies that truncated expectations are increasing, that is, if z increases the expectation of $\tilde{x}$ in the sense of MLR, then $\forall T$, $\frac{\partial}{\partial z}E\left[\tilde{x}|\tilde{x}>T,z\right]$ (see chapter 1 of Shaked and Shanthikumar (2006) for a proof in the continuous case; a proof in the discrete case follows similarly).

Finalizing the proof of Proposition 2

Thus we have that the option value is increasing in $\tilde{\Gamma}_{T-2}$, and hence the continuation value in time $T-2$ is increasing in $\tilde{\Gamma}_{T-2}$. Backing up one more step to show how the inductive argument would continue, consider the continuation value at time $T-3$. This is given by:

$$ContVal_{T-3}(\tilde{\Gamma}_{T-3})=\tilde{\Gamma}_{T-3}f(T-3)+\gamma_{T-3}-k+OptVal(T-3), \tag{58}$$

where

$$\begin{aligned}OptVal(T-3) &= \Pr(C_{T-2}|\tilde{\Gamma}_{T-3})E(ContVal_{T-2}(\tilde{\Gamma}_{T-2})|\tilde{\Gamma}_{T-3})\\ &= \Pr(C_{T-2}|\tilde{\Gamma}_{T-3})E(\tilde{\Gamma}_{T-2}f(T-2)+\Pr(C_{T-1}|\tilde{\Gamma}_{T-2})\\ &\qquad\times ContVal_{T-1}(\tilde{\Gamma}_{T-1})|\tilde{\Gamma}_{T-3}).\end{aligned} \tag{59}$$

We know that $\frac{\partial}{\partial\tilde{\Gamma}_{T-3}}\Pr(C_{T-2}|\tilde{\Gamma}_{T-3})>0$. In addition, $\frac{\partial}{\partial\tilde{\Gamma}_{T-2}}ContVal_{T-2}(\tilde{\Gamma}_{T-2})$ >0; combined with the first lemma that says increases in $\tilde{\Gamma}_{T-3}$ increase the distribution of $\tilde{\Gamma}_{T-2}$ in the sense of first-order stochastic dominance, this implies that an increase in $\tilde{\Gamma}_{T-3}$ increases the expected continuation value. Continuing by induction, this shows that the continuation value in any period τ is increasing in $\tilde{\Gamma}_{\tau}$, and thus the firm continues for values of $\tilde{\Gamma}_{\tau}\in[c_{\tau},\infty)$ for some $c_{\tau}\geq 0$.

Finally, we show that $E(\tilde{\varsigma}|\tilde{V})$ is increasing in $\tilde{V}$. Note that:

$$E(\tilde{\varsigma}|\tilde{V}) = \sum_{i=1}^{T}\Pr(\tilde{\varsigma}\geq i|\tilde{V}) \tag{60}$$

$$= \sum_{\tilde{s}_1>c_1}\sum_{\tilde{s}_2>c_2-\tilde{s}_1}\cdots\sum_{s_i>c_i-\tilde{\Gamma}_{i-1}}\Pr(\tilde{s}_1|\tilde{V})\cdots\Pr(\tilde{s}_2|\tilde{V}) \tag{61}$$

$$= \sum_{\tilde{s}_1>c_1}\Pr(\tilde{s}_1|\tilde{V})\sum_{\tilde{s}_2>c_2-\tilde{s}_1}\Pr(\tilde{s}_2|\tilde{V})\cdots\sum_{s_i>c_i-\tilde{\Gamma}_{i-1}}\Pr(\tilde{s}_2|\tilde{V}). \tag{62}$$

Since $\tilde{V}$ increases the distribution of $\tilde{s}_i$ in the sense of first-order stochastic dominance, for any $q>0$, $\frac{\partial}{\partial V}\sum_{\tilde{s}_i>q}\Pr(\tilde{s}_i|\tilde{V})>0$. This implies the result.

Proof of Proposition 3

We prove the proposition in terms of a_i and b_i and then transform the result into the parameterization in terms of μ_V and σ_V^2. The proof is very similar to the case for a single asset. First, apply the MGF of a normal distribution, we get:

$$\begin{aligned} & E(E(U(D(x-P))|\tilde{V}_F,\tilde{V}_i)) \\ &= -e^{-\sum_{i=1}^{N}\rho D_i(\mu_i-P_i)} E\left(e^{\frac{\rho^2\tilde{V}_F}{2}\left(\sum_{i=1}^{N} D_i^2\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)}\right)\prod_{i=1}^{N} E\left(e^{\frac{\rho^2}{2}D_i^2\tilde{V}_i}\right). \end{aligned} \tag{63}$$

Applying monotone transformations and then using the MGF of the gamma distribution, we find:

$$\begin{aligned} & \arg\max_{\tilde{D}} E(E(U(D(x-P))|\tilde{V}_F,\tilde{V}_i)) \\ &= \arg\max_{\tilde{D}} \sum_{i=1}^{N}\rho D_i(\mu_i-P_i) - \ln E\left(e^{\frac{\rho^2\tilde{V}_F}{2}\left(\sum_{i=1}^{N} D_i^2\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)}\right) - \sum_{i=1}^{N}\ln E\left(e^{\frac{\rho^2}{2}D_i^2\tilde{V}_i}\right) \\ &= \arg\max_{\tilde{D}} \sum_{i=1}^{N}\rho D_i(\mu_i-P_i) + a_F\ln\left(1-\frac{\rho^2}{2b_F}\left(\sum_{i=1}^{N} D_i^2\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)\right) \\ &\quad + \sum_{i=1}^{N} a_i \ln\left(1-\frac{\rho^2 D_i^2}{2b_i}\right). \end{aligned} \tag{64}$$

Note that, as in the single asset case, a conjecture and verify approach and the assumption that $\frac{\rho^2}{N^2}\left(\sum_{i=1}^{N}\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j\right) < 2b_F$ ensures that expected utility exists. The first-order conditions for the k^{th} demand imply:

$$\rho\,(\mu_k-P_k) + a_F\frac{-\frac{\rho^2}{b_F}D_k\beta_k^2 - \frac{\rho^2}{b_F}\sum_{i=1}^{N} D_i\beta_i\beta_j}{1-\frac{\rho^2}{2b_F}\left(\sum_{i=1}^{N} D_i^2\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)} + a_k\frac{-\frac{\rho^2 D_k}{b_k}}{1-\frac{\rho^2 D_k^2}{2b_k}} = 0. \tag{65}$$

In equilibrium, we know that all demands are $\frac{1}{N}$. Plugging this in and solving, we get

$$\begin{aligned} P_k &= \mu_k - \frac{a_F\rho\left(\beta_k^2+\sum_{i=1}^{N}\beta_i\beta_k\right)}{Nb_F-\frac{\rho^2}{2N}\left(\sum_{i=1}^{N}\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j\right)} - \frac{a_k\rho}{Nb_k-\frac{\rho^2}{2N}} \\ &= \mu_k - \frac{a_F\rho\left(\beta_k^2+N\beta_k\right)}{Nb_F-N\frac{\rho^2}{2}} - \frac{a_k\rho}{Nb_k-\frac{\rho^2}{2N}}. \end{aligned} \tag{66}$$

After substituting $a_k = \frac{\mu^2_{V_k}}{\sigma^2_{V_k}}$, $b_k = \frac{\mu_{V_k}}{\sigma^2_{V_k}}$, $a_F = \frac{\mu^2_{V_F}}{\sigma^2_{V_F}}$, and $b_F = \frac{\mu_{V_F}}{\sigma^2_{V_F}}$, and performing algebraic manipulations, we find:

$$P_k = \mu_k - \frac{\frac{\mu^2_{V_F}}{\sigma^2_{V_F}}\rho\left(\beta_k^2 + N\beta_k\right)}{N\frac{\mu_{V_F}}{\sigma^2_{V_F}} - N\frac{\rho^2}{2}} - \frac{\frac{\mu^2_{V_i}}{\sigma^2_{V_i}}\rho}{N\frac{\mu_{Vi}}{\sigma^2_{V_i}} - \frac{\rho^2}{2N}} \tag{67}$$
$$= \mu_k - \frac{\rho\mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\rho\mu_{Vk}}{N}$$
$$- \frac{\frac{1}{2}\rho^2 N\frac{\sigma^2_{V_F}}{\mu_{V_F}}}{N - \frac{1}{2}\rho^2 N\frac{\sigma^2_{V_F}}{\mu_{V_F}}}\frac{\rho\mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\frac{1}{2}\rho^2\frac{\sigma^2_{V_k}}{\mu_{V_k}}}{N^2 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_k}}{\mu_{V_k}}}\frac{\rho\mu_{V_k}}{N}.$$

It can be shown that a first-order stochastic dominant shift in $\tilde{V}_F$ reduces the price of all firms by adding $k_F > 0$ to $\tilde{V}_F$ as was done in the proof of Proposition 2.

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