Binary Search Trees: Applications

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Compute order statistics in binary search trees.
- Use trees to store and manipulate sequential lists of elements.

Outline

Problem

Things you might want to do:

- Return the 7^{th} largest element.
- Return the median element.
- Return the 25% percentile element.

Order Statistics

Order Statistics

Input: The root of a tree T and a number k

Output: The k^{th} smallest element in T

Idea

- Need to know which subtree to look in.
- Need to know how many elements are in left subtree.

New Field

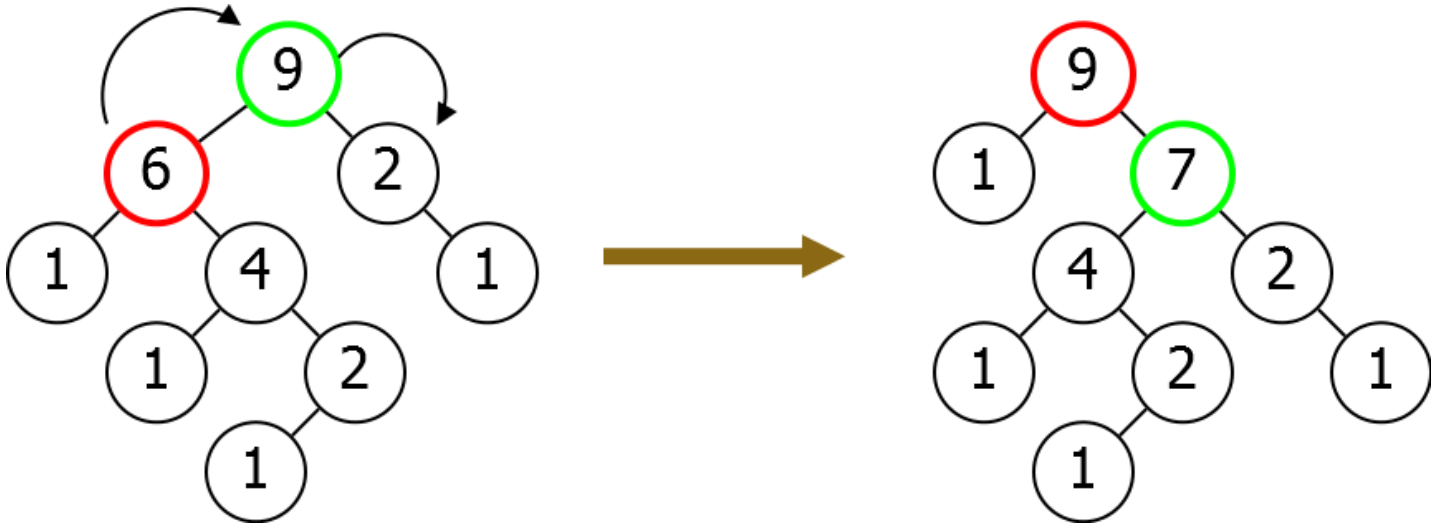
N.`Size` returns the number of elements in the subtree of N. Should satisfy:

$$N.\texttt{Size} = N.\texttt{Left.Size} + N.\texttt{Right.Size} + 1,$$

where null nodes have size zero.

Maintaining Value

When you rotate, you need to recompute sizes.



Recompute

RecomputeSize(N)

N.Size $\leftarrow$ N.Left.Size+N.Right.Size+1

Rotate

As before

RecomputeSize(Old root)

RecomputeSize(New root)

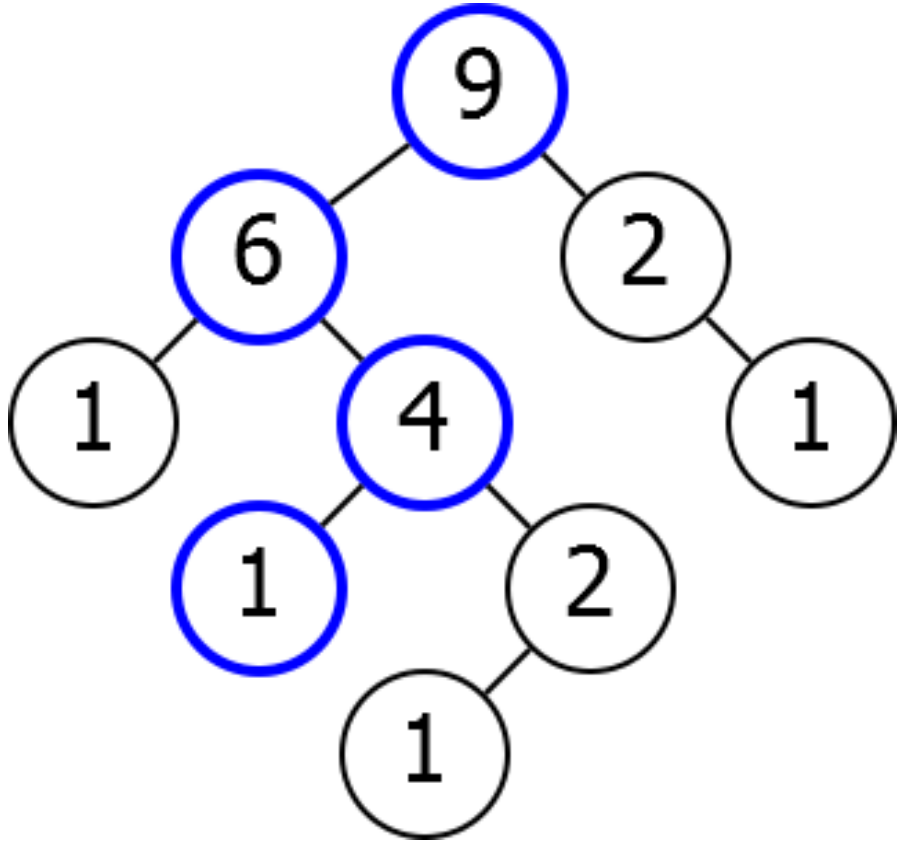
Order Statistics

OrderStatistic(R, k)

```
s ← R.Left.Size
if k = s + 1:
  return R
else if k < s + 1:
  return OrderStatistic(R.Left, k)
else if k > s + 1:
  return OrderStatistic(R.Right, k − s − 1)
```

Analysis

Runtime $O(h)$.



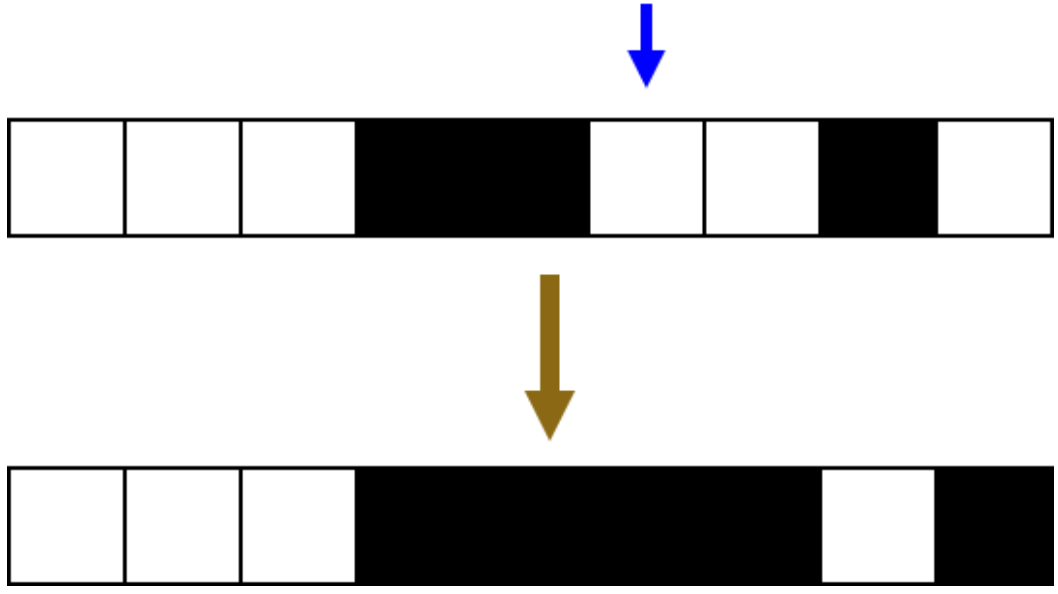
Puzzle

How do you compute the rank of the node with a given key?

Outline

Problem

- Array of squares.
- Each black or white.
- Want to be able to flip colors of all squares after index x.

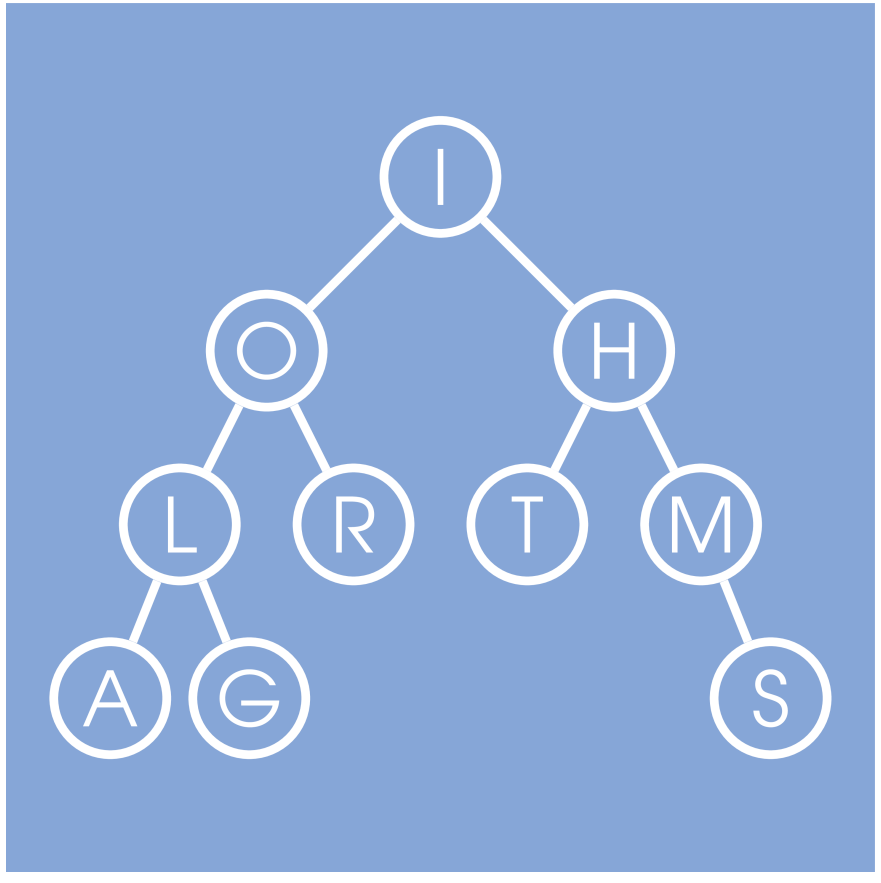


Operations

- `NewArray(`n`)`-Creates an array with n white squares.
- `Color(`m`)` - Returns color of m^{th} square.
- `Flip(`x`)` - Flips the color of all squares of index $> x$.

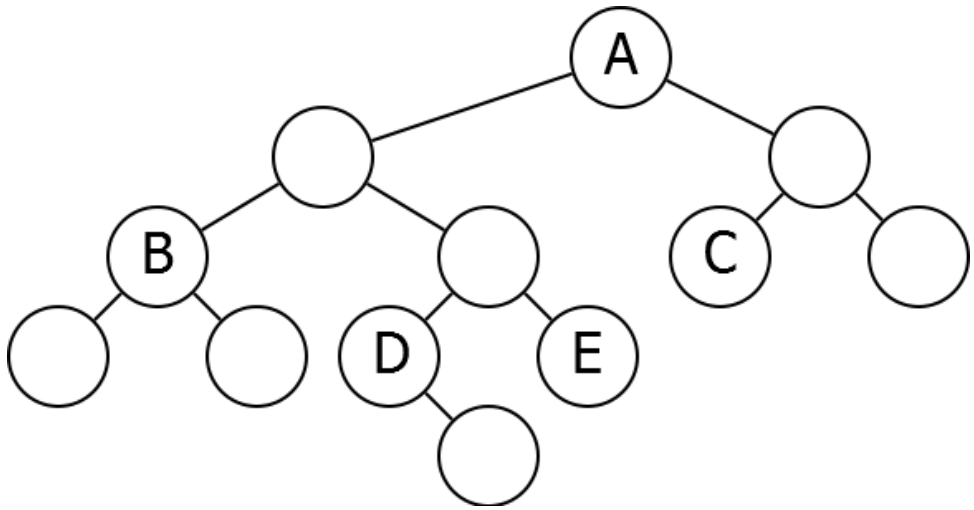
New Use For Trees

Store elements in sorted order.



Problem

Which node represents the 5^{th} smallest element in this tree?



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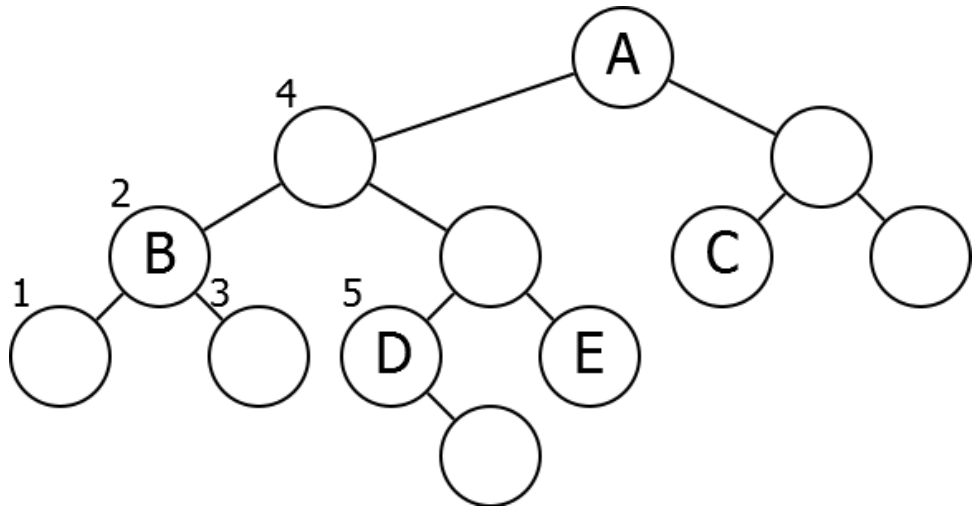
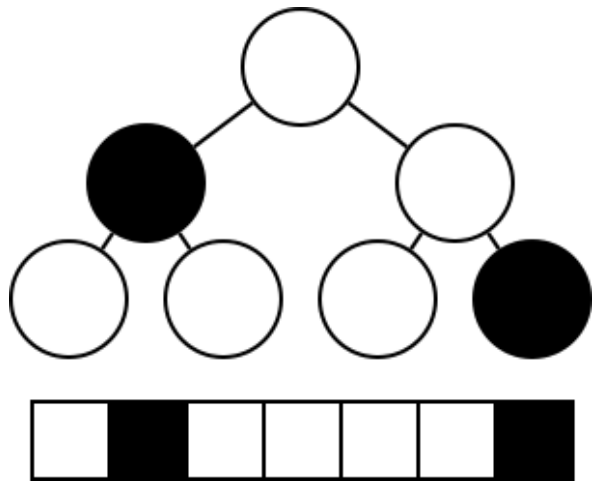


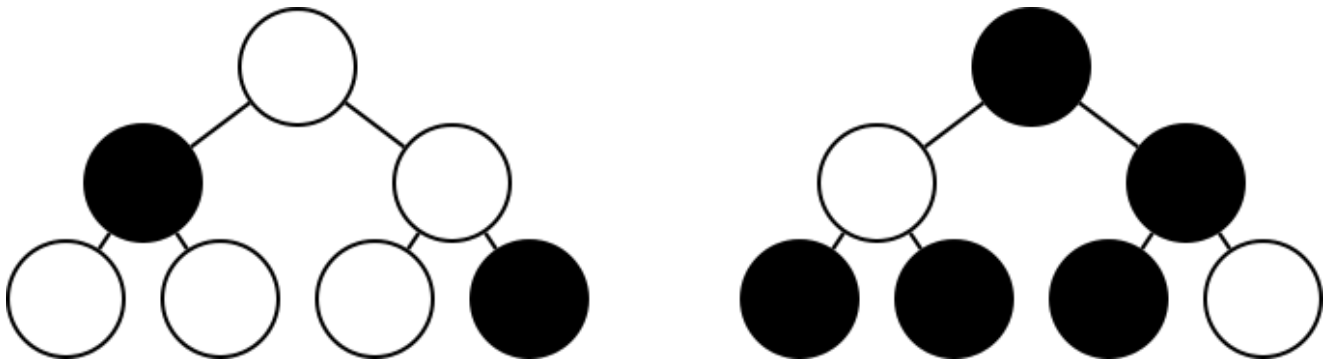
Idea

Store tree with nodes corresponding to list colors:



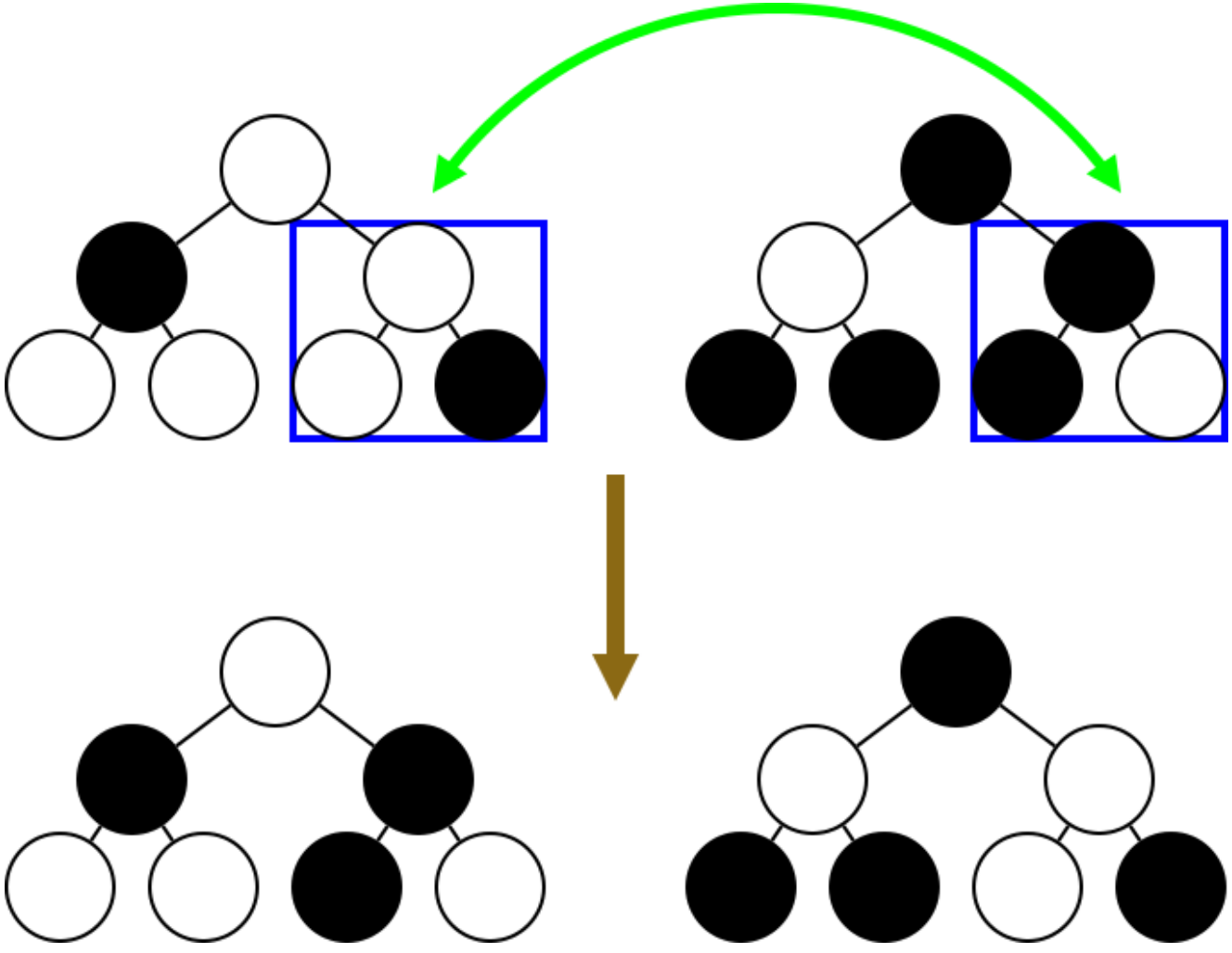
Idea II

Two trees- one with opposite colors:



Idea

Flip using merge and split



Create

NewArray(n)

Create two trees T_1, T_2 with keys $1 \ldots n$.

Give nodes extra Color field.

All in T_1 have color White

All in T_2 have color Black

Find

```
Color(m)
```

$N \leftarrow \texttt{Find}(m, T_1)$
return N.Color

Flip

Flip(x)

$(L_1, R_1) \leftarrow \texttt{Split}(T_1, x)$
$(L_2, R_2) \leftarrow \texttt{Split}(T_2, x)$
$\texttt{Merge}(L_1, R_2) \rightarrow T_1$
$\texttt{Merge}(L_2, R_1) \rightarrow T_2$

Moral

Trees can be used for more than searching.

Can be used to store lists.