

Linear Programming: Introduction

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Advanced Algorithms and Complexity
Data Structures and Algorithms

Learning Objectives

- See an example of the type of problem solved by linear programming.

Factory

You are running a widget factory and trying to optimize your production procedures to save money.

Machines vs. Workers

Can use combination of machines and workers.

- Have only 100 machines.
- Unlimited workers.
- Each machine requires 2 workers to operate.

Production

- Each machine makes 600 widgets a day.
- Each worker makes 200 widgets a day.

Limited Demand

Total demand for only 100,000 widgets a day.

Algebra

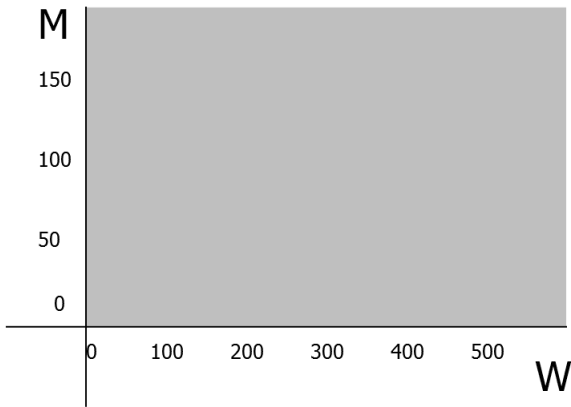
Let W be the number of workers and M the number of machines.

Constraints:

- $W \geq 0$.
- $100 \geq M \geq 0$.
- $W \geq 2M$.
- $100,000 \geq 200(W - 2M) + 600M$.

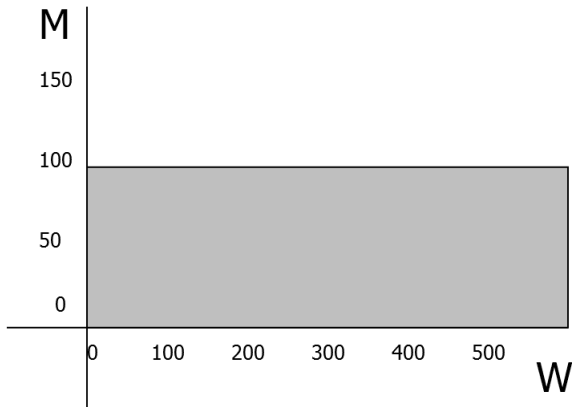
Graph

$$M, W \geq 0$$



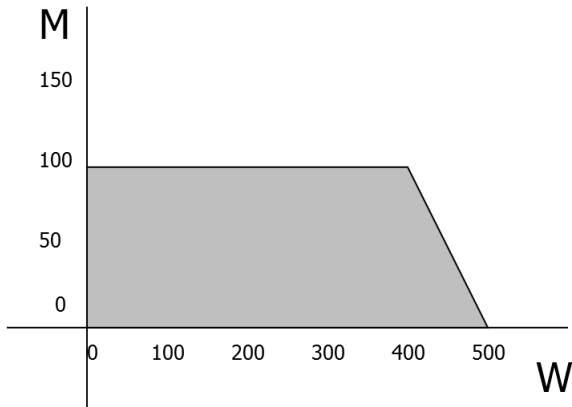
Graph

$$M \leq 100$$



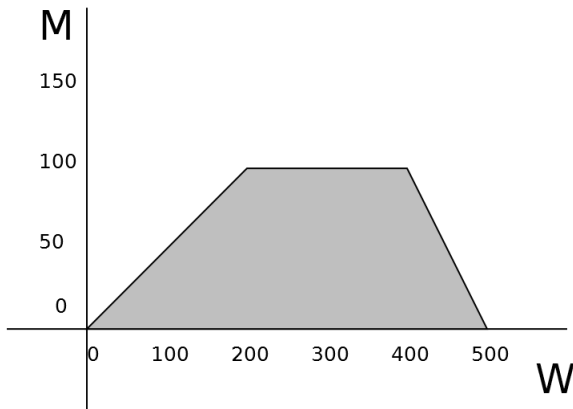
Graph

$$M + W \leq 500$$



Graph

Diagram of possible configurations:



Profits

Profits are determined as follows:

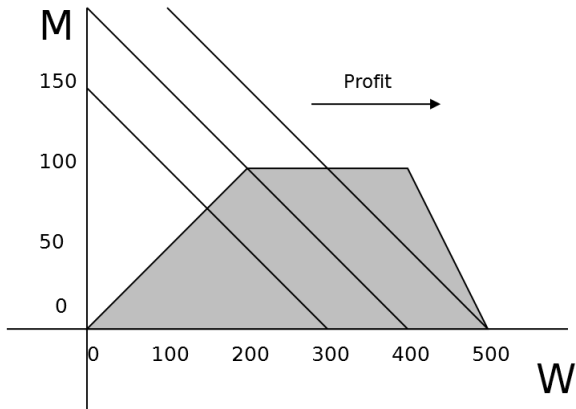
- Each widget earns you \$1.
- Each worker costs you \$100/day.

Total profits (in dollars per day):

$$200(W - 2M) + 600M - 100W = 100W + 200M.$$

Graph

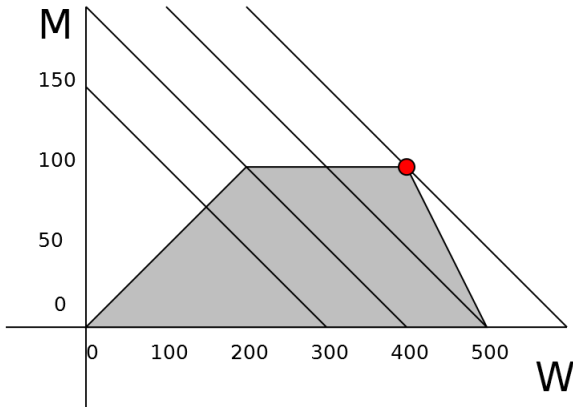
Profit mapped on graph:



Optimum

Best: $M = 100$, $W = 400$ [NB: A corner]

Profit = \$60,000/day.



Proof of Optimality

$$\begin{array}{rcl} 100 \cdot [& 001 \cdot M + 000 \cdot W & \leq 100] \\ +0.5 \cdot [& 200 \cdot M + 200 \cdot W & \leq 100,000] \\ \hline & 200 \cdot M + 100 \cdot W & \leq 60,000. \end{array}$$

Summary

Maximized:

$$200M + 100W$$

subject to constraints:

$$0M + 1W \geq 0$$

$$1M + 0W \geq 0$$

$$-1M + 0W \geq -100$$

$$-2M + 1W \geq 0$$

$$-1M - 1W \geq -500$$