

Binary Search Trees: Basic Operations

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Implement basic operations on Binary Search Trees.
- Understand some of the difficulties with making updates.

Outline

- 1 Find
- 2 Next Element
- 3 Search
- 4 Insert
- 5 Delete

Find

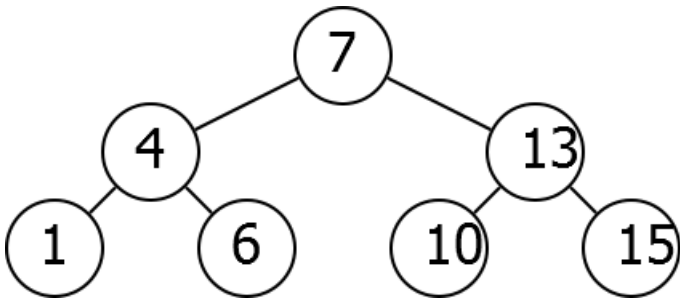
Find

Input: Key k , Root R

Output: The node in the tree of R with key k

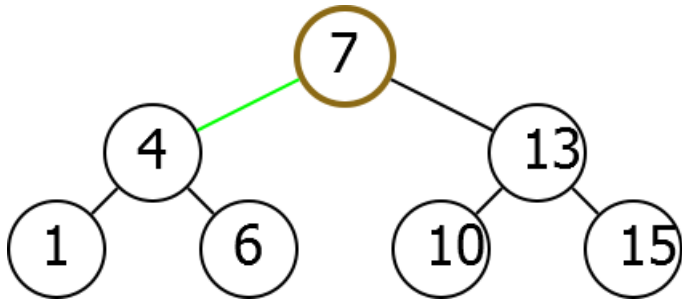
Idea

Find(6)



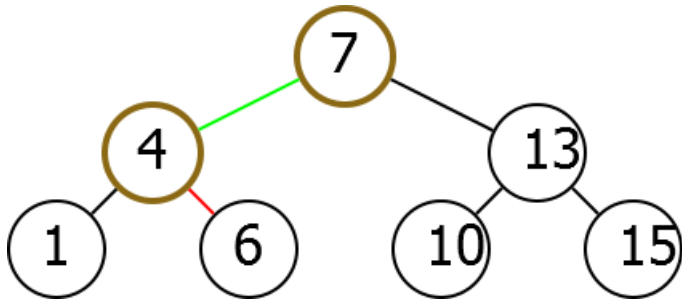
Idea

Find(6)



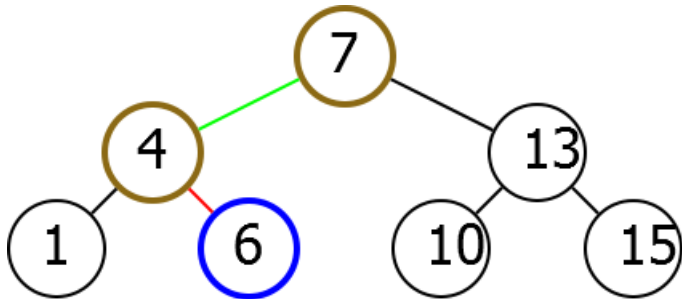
Idea

Find(6)



Idea

Find(6)



Algorithm

Find(k, R)

if $R.\text{Key} = k$:

 return R

else if $R.\text{Key} > k$:

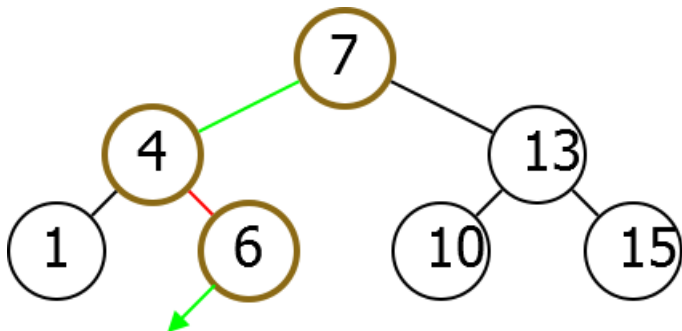
 return Find($k, R.\text{Left}$)

else if $R.\text{Key} < k$:

 return Find($k, R.\text{Right}$)

Missing Key

Run Find(5).



Key not in tree. Did find point where it should be.

Missing Key

If you stop before reaching a null pointer, you find the place in the tree where k would fit.

Modification

Find (modified)

```
else if  $R.\text{Key} > k$ :  
    if  $R.\text{Left} \neq \text{null}$ :  
        return Find( $k, R.\text{Left}$ )  
    return  $R$ 
```

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Adjacent Elements

Given a node N in a Binary Search Tree, would like to find adjacent elements.

Next

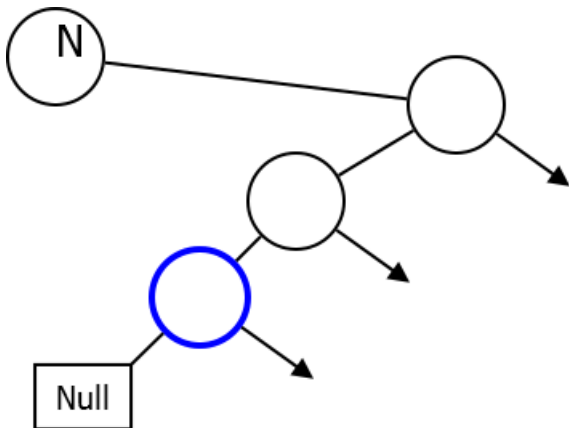
Next

Input: Node N

Output: The node in the tree with the next largest key.

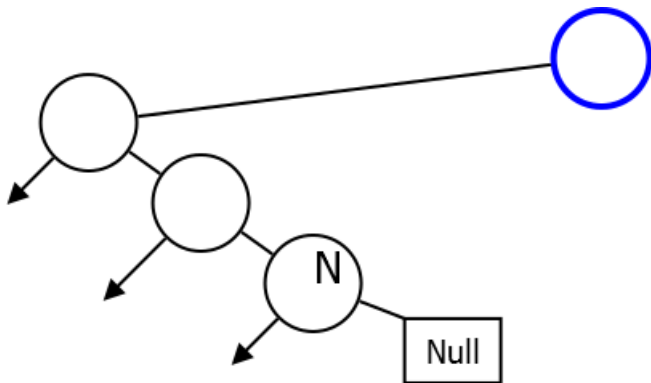
Case I

If you have right child.



Case II

No right child.



Next

Next(*N*)

```
if N.Right  $\neq$  null:  
    return LeftDescendant(N.Right)  
else:  
    return RightAncestor(N)
```

Left Descendant

LeftDescendant(*N*)

```
if N.Left = null  
    return N  
else:  
    return LeftDescendant(N.Left)
```

Right Ancestor

RightAncestor(N)

```
if  $N.$ Key <  $N.$ Parent.Key  
    return  $N.$ Parent  
else:  
    return RightAncestor( $N.$ Parent)
```

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Range Search

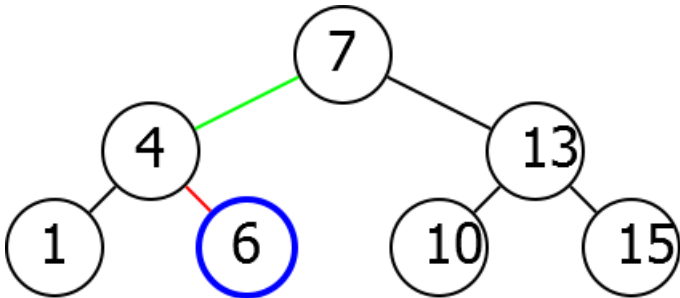
Range Search

Input: Numbers x, y , root R

Output: A list of nodes with key between x and y

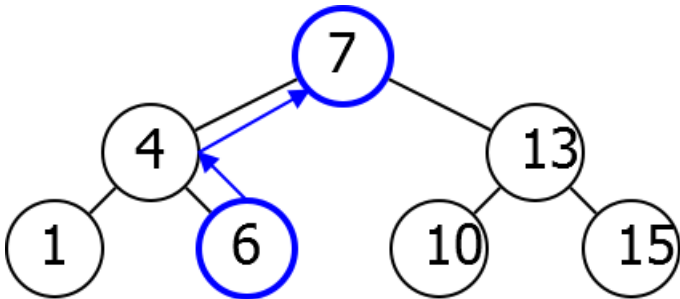
Idea

RangeSearch(5, 12).



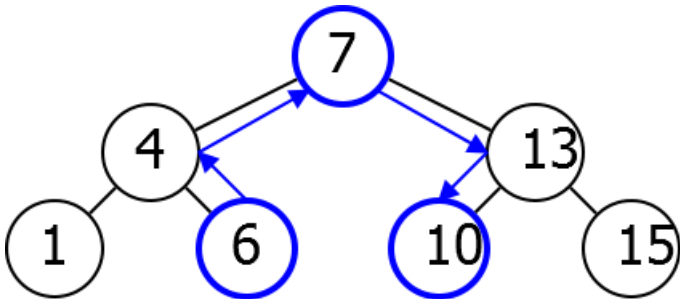
Idea

RangeSearch(5, 12).



Idea

RangeSearch(5, 12).



Implementation

RangeSearch(x, y, R)

$L \leftarrow \emptyset$

$N \leftarrow \text{Find}(x, R)$

while $N.\text{Key} \leq y$

 if $N.\text{Key} \geq x$:

$L \leftarrow L.\text{Append}(N)$

$N \leftarrow \text{Next}(N)$

return L

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Insert

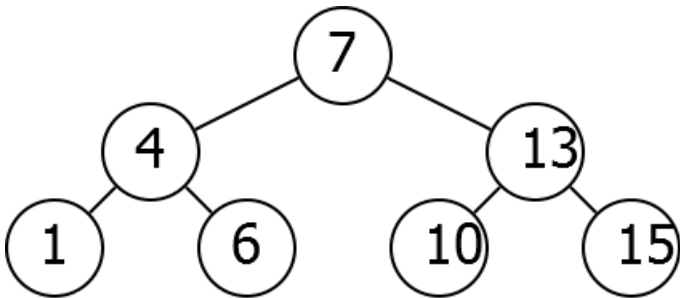
Insert

Input: Key k and root R

Output: Adds node with key k to the tree

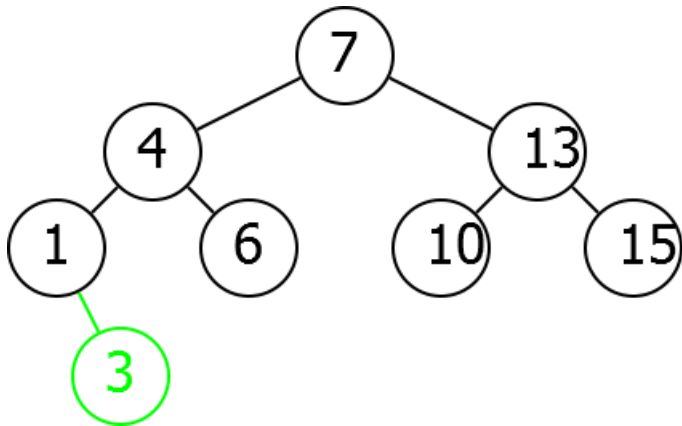
Insert Idea

Insert(3)



Insert Idea

Insert(3)



Implementation

$\text{Insert}(k, R)$

$P \leftarrow \text{Find}(k, R)$

Add new node with key k as child of
 P

Outline

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Delete

Delete

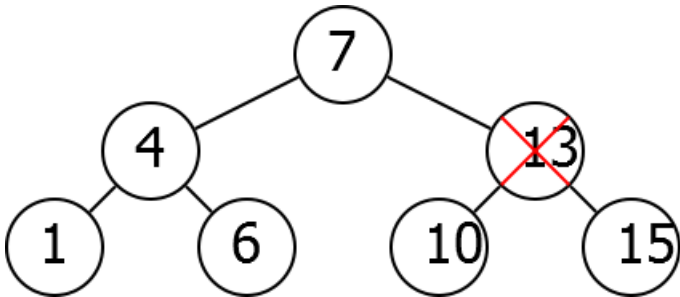
Input: Node N

Output: Removes node N from the tree

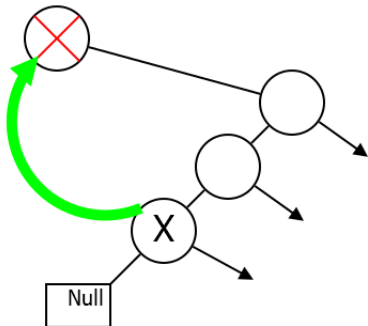
Difficulty

Cannot simply remove.

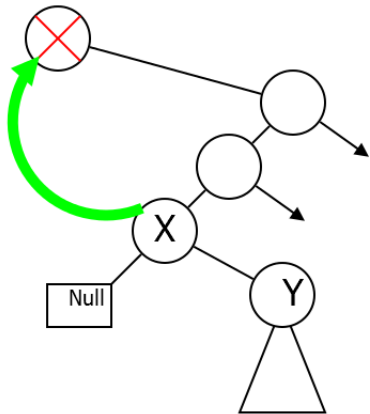
Delete(13)



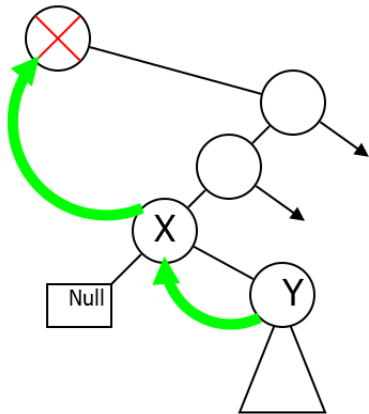
Idea



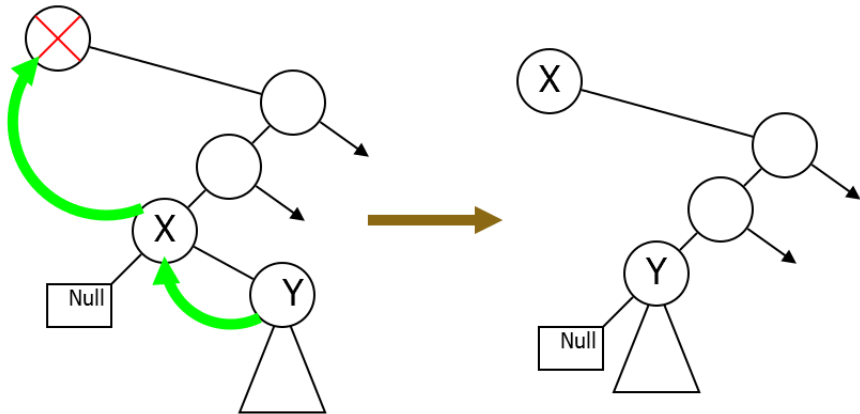
Idea



Idea



Idea



Implementation

Delete(N)

```
if  $N$ .Right = null:
```

```
    Remove  $N$ , promote  $N$ .Left
```

```
else:
```

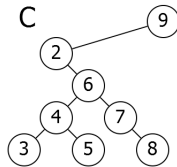
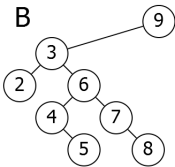
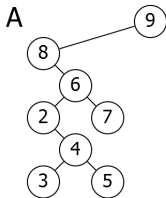
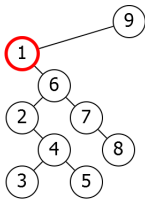
```
     $X \leftarrow \text{Next}(N)$ 
```

```
     $X$ .Left = null
```

```
    Replace  $N$  by  $X$ , promote  $X$ .Right
```

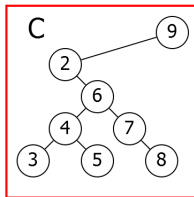
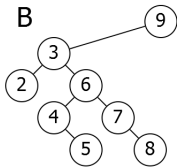
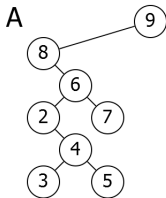
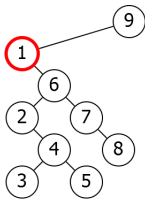
Problem

Which of the following trees is obtained when the selected node is deleted?



Problem

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Next Time

Runtime and balance.