

Multiagent Reinforcement Learning



DeepMind

Marc Lanctot

RLSS @ Lille, July 11th 2019

Joint work with many great collaborators!



Talk plan

1. What is Multiagent Reinforcement Learning (MARL)?
2. Foundations & Background
3. Basic Formalisms & Algorithms
4. Advanced Topics

Part 1: What is MARL?

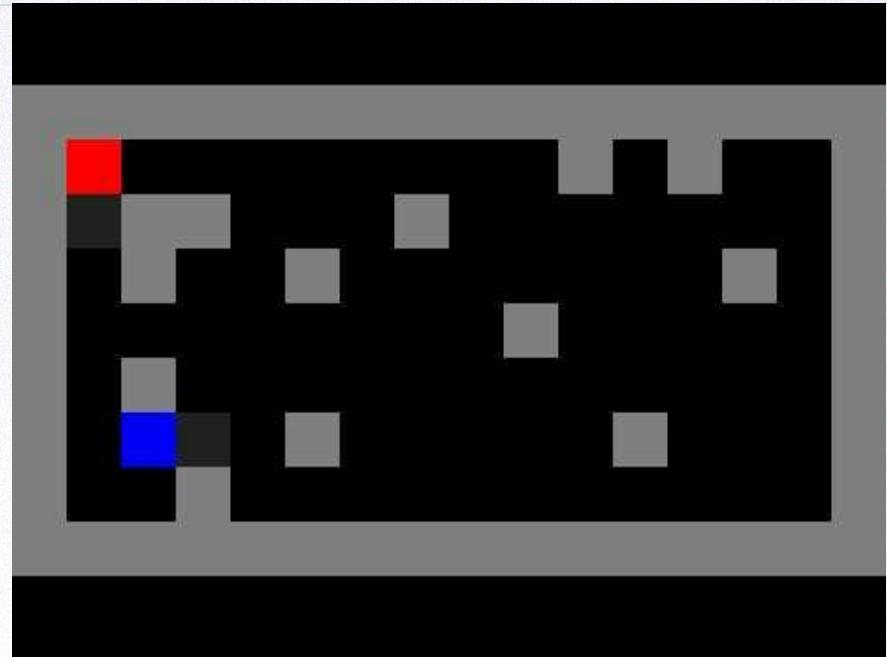
Multiagent Reinforcement Learning

Pommerman

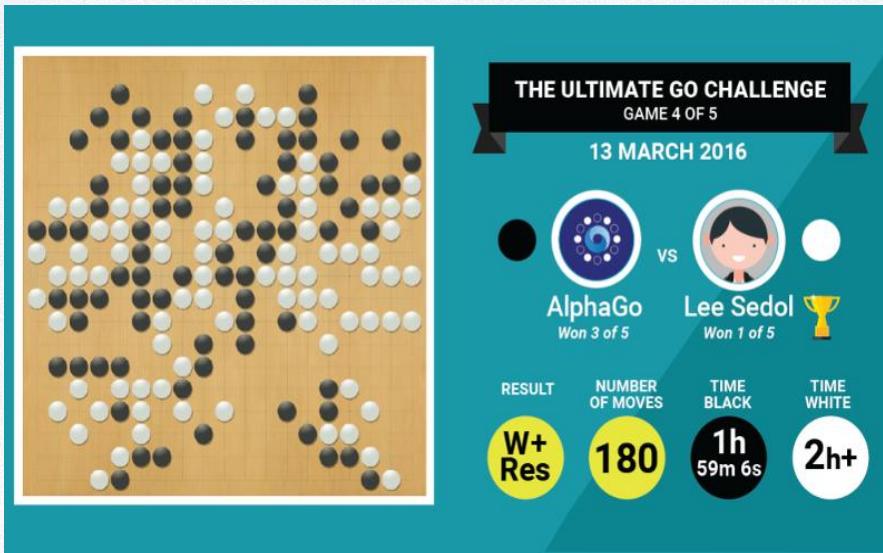


pommerman.com

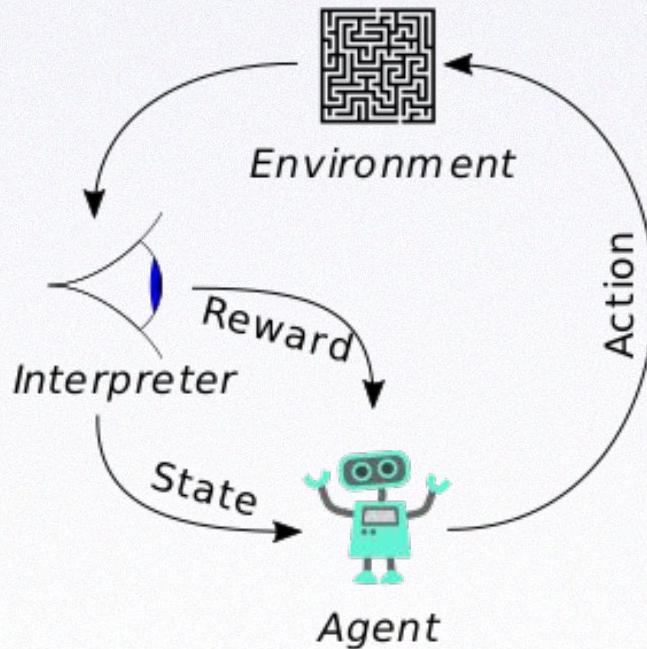
Laser Tag



Multiagent Reinforcement Learning

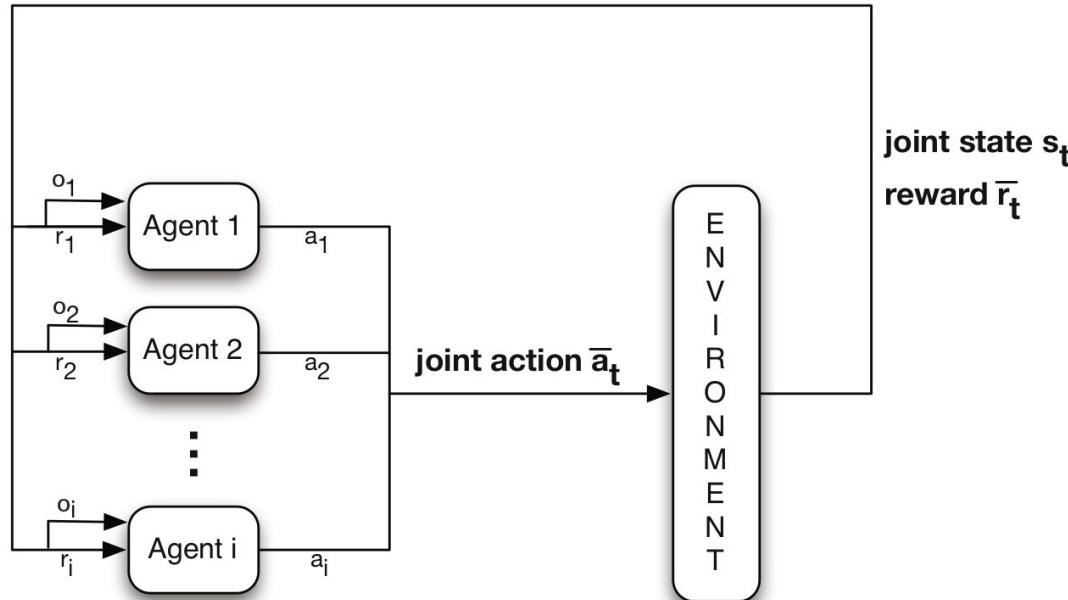


Traditional (Single-Agent) RL



Source: Wikipedia

Multiagent Reinforcement Learning



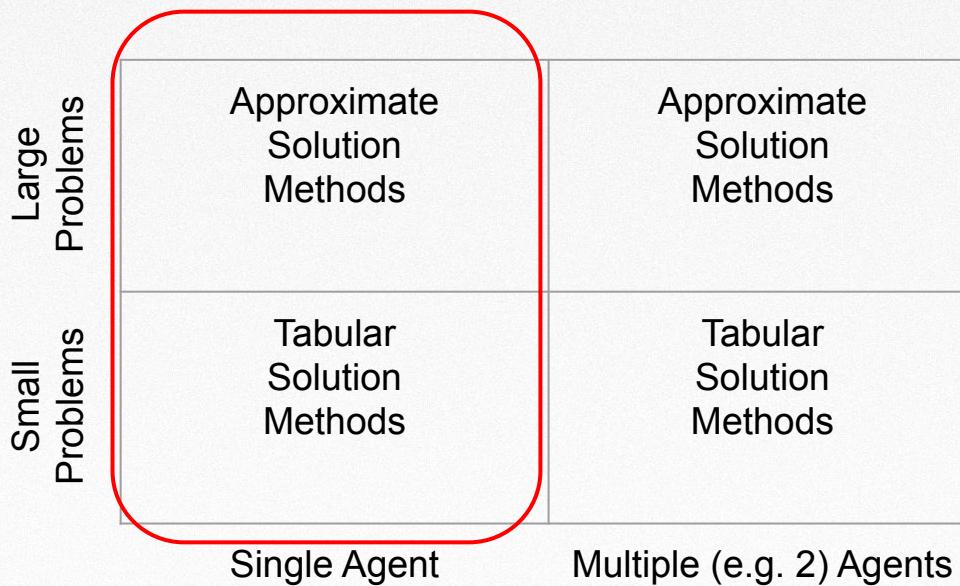
Source: Nowe, Vrancx & De Hauwere 2012

Motivations: Research in Multiagent RL

	Large Problems	Approximate Solution Methods	Approximate Solution Methods
	Small Problems	Tabular Solution Methods	Tabular Solution Methods
Single Agent		Multiple (e.g. 2) Agents	

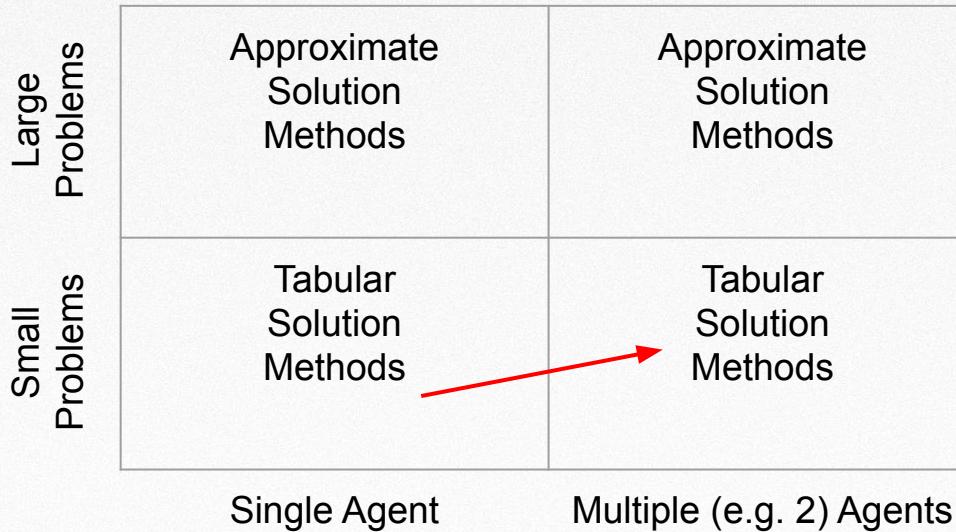
Motivations: Research in Multiagent RL

Sutton & Barto '98, '18



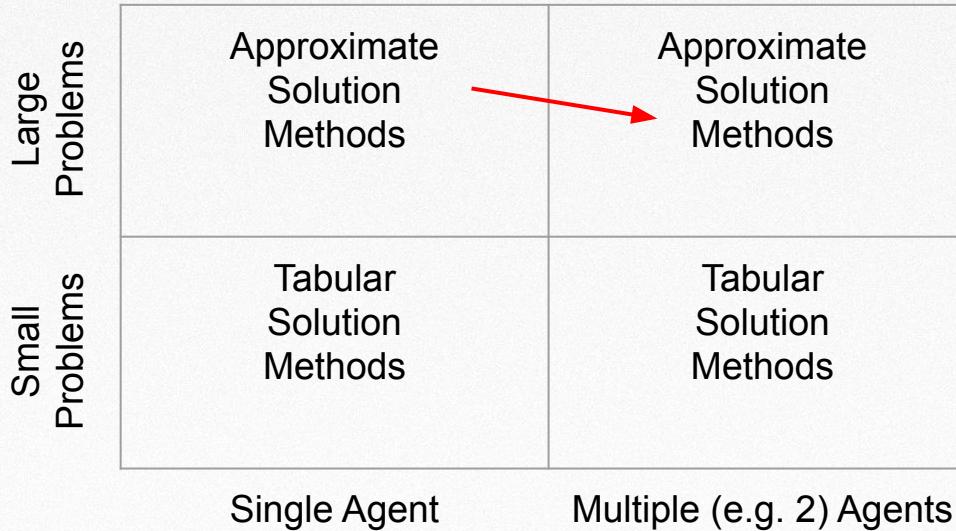
Motivations: Research in Multiagent RL

First era of multiagent RL



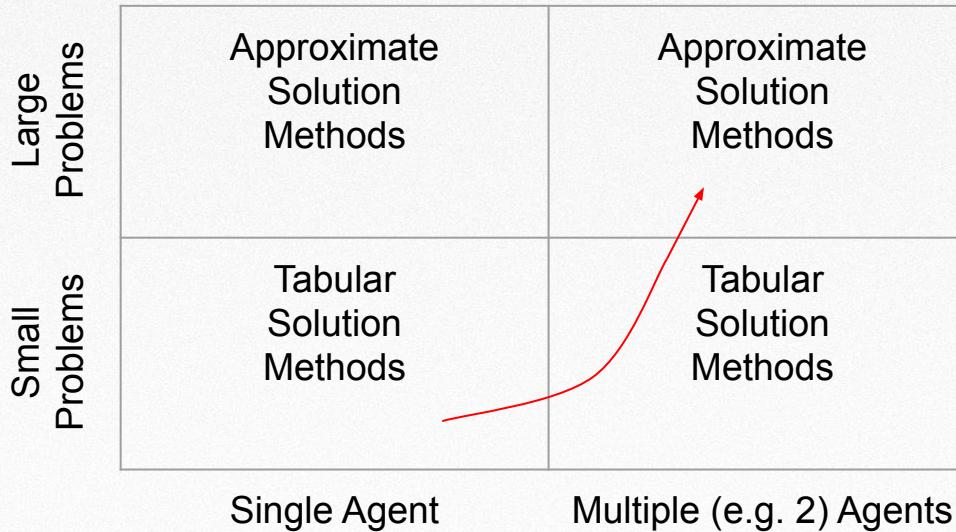
Motivations: Research in Multiagent RL

Multiagent Deep RL era ('16 - now)



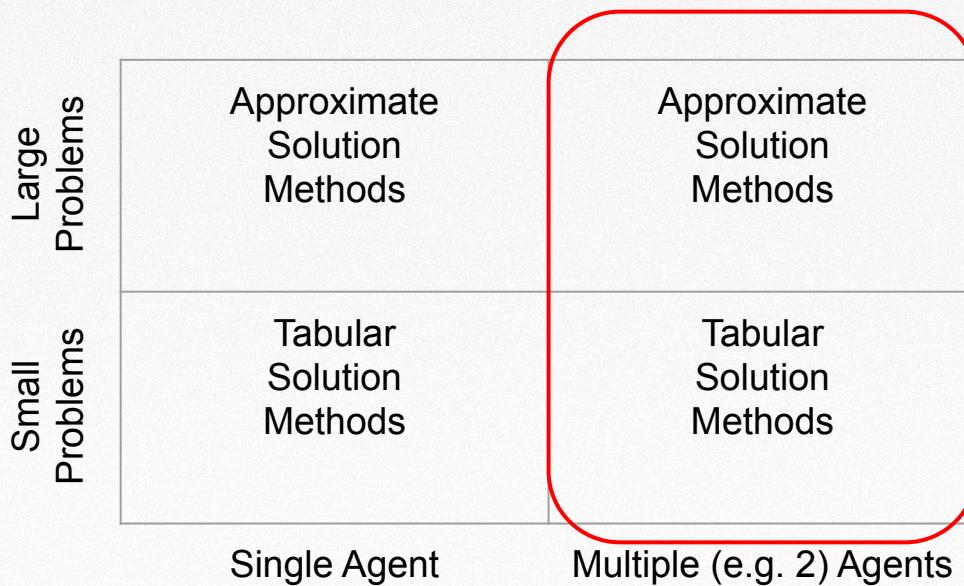
Motivations: Research in Multiagent RL

Talk focus



Motivations: Research in Multiagent RL

My 10-year mission



Important Historical Note

If multi-agent learning is the answer,
what is the question?

Yoav Shoham, Rob Powers, and Trond Grenager
Stanford University
`{shoham,powers,grenager}@cs.stanford.edu`

February 15, 2006

Artificial Intelligence, Volume 171, Issue 7

Foundations of multi-agent learning: Introduction to the special issue

Rakesh V. Vohra, Michael P. Wellman

Pages 363-364

An economist's perspective on multi-agent learning

Drew Fudenberg, David K. Levine

Pages 378-381

Perspectives on multiagent learning

Tuomas Sandholm

Pages 382-391

Artificial Intelligence, Volume 171, Issue 7

Agendas for multi-agent learning

Geoffrey J. Gordon

Pages 392-401

Multiagent learning is not the answer. It is the question

Peter Stone

Pages 402-405

What evolutionary game theory tells us about multiagent learning

Karl Tuyls, Simon Parsons

Pages 406-416

Artificial Intelligence, Volume 171, Issue 7

Multi-agent learning and the descriptive value of simple models

Ido Erev, Alvin E. Roth

Pages 423-428

The possible and the impossible in multi-agent learning

H. Peyton Young

Pages 429-433

No regrets about no-regret

Yu-Han Chang

Pages 434-439

Artificial Intelligence, Volume 171, Issue 7

A hierarchy of prescriptive goals for multiagent learning

Martin Zinkevich, Amy Greenwald, Michael L. Littman

Pages 440-447

Learning equilibrium as a generalization of learning to optimize

Dov Monderer, Moshe Tennenholtz

Pages 448-452

Some Specific Axes of MARL

Centralized:

- One brain / algorithm deployed across many agents

Decentralized:

- All agents learn individually
- Communication limitations defined by environment

Some Specific Axes of MARL

Prescriptive:

- Suggests how agents *should* behave

Descriptive:

- Forecast how agent *will* behave

Some Specific Axes of MARL

Cooperative: Agents cooperate to achieve a goal

Competitive: Agents compete against each other

Neither: Agents maximize their utility which may require cooperating and/or competing

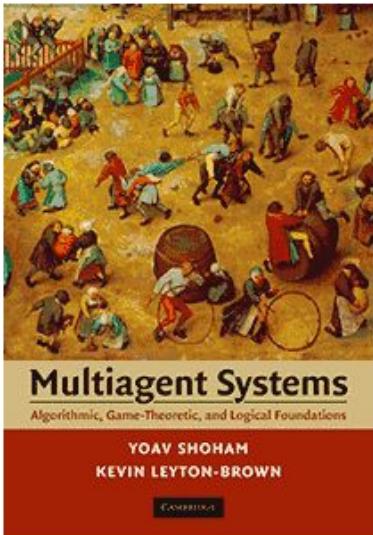
Our Focus Today

1. Centralized training for decentralized execution
(very common)
2. Mostly prescriptive
3. Mostly competitive; sprinkle of cooperative and neither

Part 2: Foundations & Background

Shoham & Leyton-Brown '09

[Main Page](#) [Table of Contents](#) [Instructional Resources](#) [Errata](#) [eBook Download](#) new!



Multiagent Systems Algorithmic, Game-Theoretic, and Logical Foundations

Yoav Shoham
Stanford University
Kevin Leyton-Brown
University of British Columbia

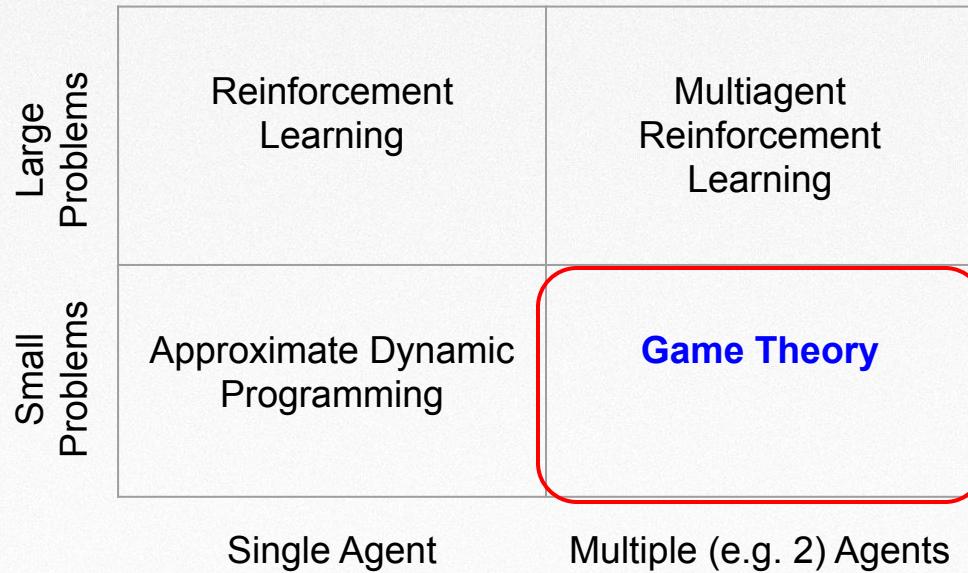
Cambridge University Press, 2009
Order online: [amazon.com](#).

masfoundations.org

Foundations of (MA)RL



Foundations of Multiagent RL



Biscuits vs Cookies

A Note on Terminology

Player	Agent
Game	Environment
Strategy	Policy
Best Response	Greedy Policy
Utility	Reward
State	(Information) State

Normal-form “One-Shot” Games

- Set of **players** $i \in \mathcal{N} = \{1, 2, \dots, n\}$

Normal-form “One-Shot” Games

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- Set of **joint actions** $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_n$
- A **utility** function $u : \mathcal{N} \times \mathcal{A} \rightarrow U \subseteq \mathbb{R}$

Example: (Bi-)Matrix Games ($n = 2$)

column player

		A	B
		0 , 0	1 , -1
row player	a	-1 , 1	0 , 0
	b		

Example: (Bi-)Matrix Games (n = 2)

actions

row player

column player

		A	B
row player	a	0 , 0	1 , -1
	b	-1 , 1	0 , 0

Example: (Bi-)Matrix Games ($n = 2$)

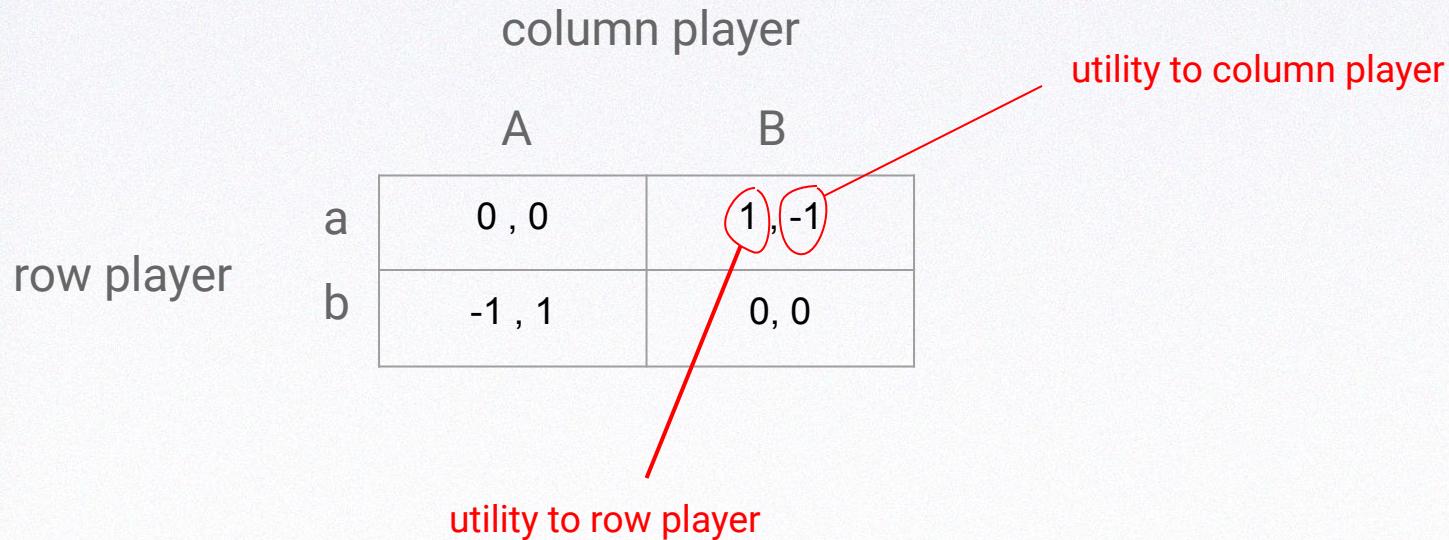
column player

		A	B
		0 , 0	1 , -1
row player	a	-1 , 1	0 , 0
	b	0 , 0	1 , -1

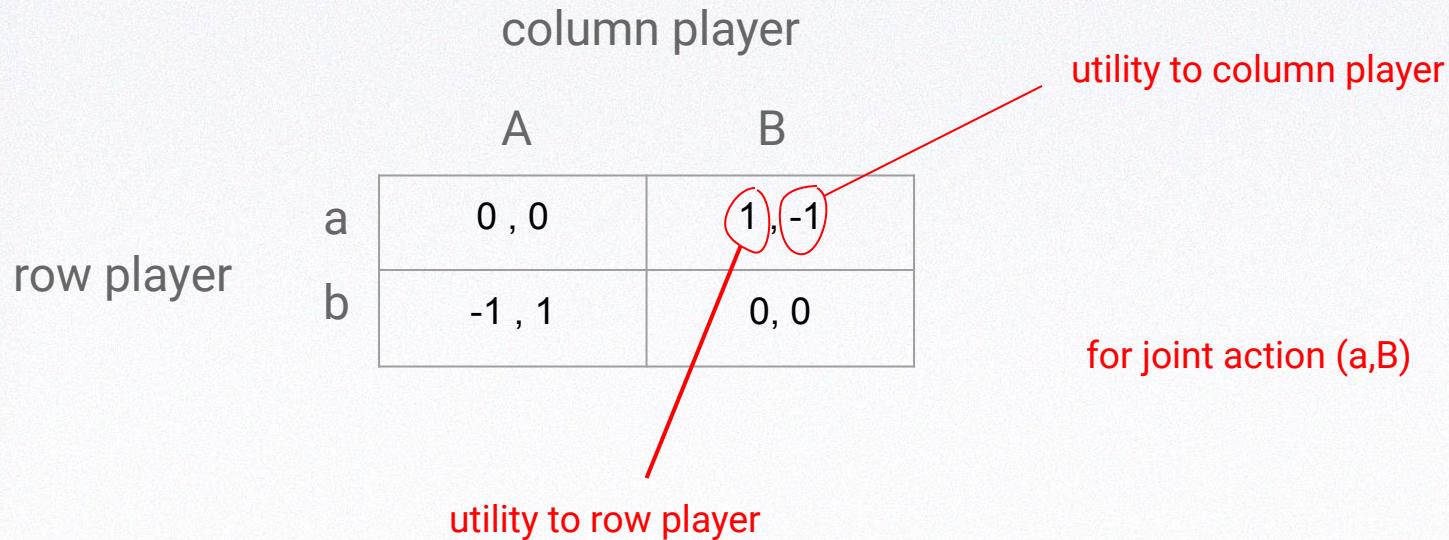
utility to row player



Example: (Bi-)Matrix Games (n = 2)



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Normal-form “One-Shot” Games

- Set of **players** $i \in \mathcal{N} = \{1, 2, \dots, n\}$
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Each player: $\pi_i \in \Delta(\mathcal{A}_i)$, maximize $\mathbb{E}_{a \sim \pi}[u_i(a)]$

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Problem! This is a *joint policy*



Best Response

Suppose we are player i and we fix policies of other players

Best Response

Suppose we are player i and we fix policies of other players ($-i = \mathcal{N} - \{i\}$)

Best Response

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Best Response

Suppose we are player i and we fix policies of other players ($-i = \mathcal{N} - \{i\}$)

$$\pi_i \in \Delta(\mathcal{A}_i), \text{ maximize } \mathbb{E}_{a \sim \pi} [u_i(a)]$$

$$\pi_i \in BR(\pi_{-i}) \Leftrightarrow u_i(\pi_i, \pi_{-i}) = \max_{\pi'_i} \mathbb{E}_{a \sim (\pi'_i, \pi_{-i})} [u_i(a)]$$

Best Response

Suppose we are player i and we fix policies of other players ($-i = \mathcal{N} - \{i\}$)

$$\pi_i \in \Delta(\mathcal{A}_i), \text{ maximize } \mathbb{E}_{a \sim \pi} [u_i(a)]$$

$$\pi_i \in BR(\pi_{-i}) \Leftrightarrow u_i(\pi_i, \pi_{-i}) = \max_{\pi'_i} \mathbb{E}_{a \sim (\pi'_i, \pi_{-i})} [u_i(a)]$$

π_i is a **best response** to π_{-i}

Solving a Matrix Game

column player

		A	B
		0 , 0	1 , -1
row player	a	0 , 0	1 , -1
	b	-1 , 1	0 , 0

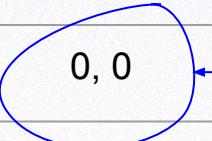
Solving a Matrix Game

column player

row player

		A	B
		0 , 0	1 , -1
a	a	0 , 0	1 , -1
	b	-1 , 1	0 , 0

Let's start here



Solving a Matrix Game

column player

	A	B
a	0 , 0	1 , -1
b	-1 , 1	0 , 0

row player

The matrix shows the payoffs for each combination of strategies:

- Row player a, Column player A: Payoff (0, 0)
- Row player a, Column player B: Payoff (1, -1)
- Row player b, Column player A: Payoff (-1, 1)
- Row player b, Column player B: Payoff (0, 0)

Arrows indicate best responses:

- An arrow points from the value 1 (in the cell (a, B)) to the column player's strategy B.
- An arrow points from the value -1 (in the cell (b, A)) to the row player's strategy b.

Both players have *incentive to deviate* (assuming the opponent stays fixed)

Solving a Matrix Game

column player

		A	B
		0 , 0	1 , -1
row player	a	0 , 0	1 , -1
	b	-1 , 1	0 , 0

The cell containing the value $-1, 1$ is highlighted with a blue oval.

Solving a Matrix Game

column player

		A	B
		0 , 0	1 , -1
row player	a	0 , 0	1 , -1
	b	-1 , 1	0 , 0



Solving a Matrix Game

column player

row player

		A	B
		0 , 0	1 , -1
a	A	0 , 0	1 , -1
	b	-1 , 1	0 , 0

(a,A) is a *fixed point* of this process

Solving a Matrix Game

column player

	A	B
a	0 , 0	1 , -1
b	-1 , 1	0 , 0

(a,A) is a *fixed point* of this process

$$\pi_i \in \Delta(\mathcal{A}_i), \text{ maximize } \mathbb{E}_{a \sim \pi} [u_i(a)]$$

Let's Try Another....

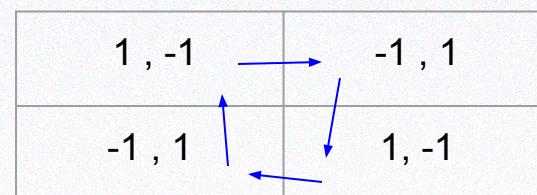
column player

		A	B
		1 , -1	-1 , 1
row player	a	1 , -1	-1 , 1
	b	-1 , 1	1, -1

Let's Try Another....

column player

		A	B
		1 , -1	-1 , 1
row player	a	1 , -1	-1 , 1
	b	-1 , 1	1, -1



Nash equilibrium

A Nash equilibrium is a **joint policy** π such that no player has incentive to deviate *unilaterally*.

Nash equilibrium: A Solution Concept

A Nash equilibrium is a **joint policy** π such that no player has incentive to deviate *unilaterally*.

$$\forall i \in \mathcal{N}, \pi_i \in BR(\pi_{-i})$$

Some Facts

- Nash equilibrium always exists in finite games
- Computing a Nash eq. is PPAD-Complete
 - One solution is to focus on tractable subproblems
 - Another is to compute approximations
- Assumes players are (unbounded) rational
- Assumes knowledge:
 - Utility / value functions
 - Rationality assumption is common knowledge

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$

column player

	A	B	
row player	a	1 , -1	-1 , 1
	b	-1 , 1	1, -1

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$

column player

$\max V$

	A	B	
row player	a	$1, -1$	$-1, 1$
	b	$-1, 1$	$1, -1$

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$

column player

$\max V$

$\pi(a) - \pi(b) \geq V$ (vs. A)

row player

	A	B
a	1, -1	-1, 1
b	-1, 1	1, -1

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$

column player

row player

	A	B
a	1, -1	-1, 1
b	-1, 1	1, -1

$\max V$

$$\pi(a) - \pi(b) \geq V \quad (\text{vs. A})$$

$$-\pi(a) + \pi(b) \geq V \quad (\text{vs. B})$$

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$

column player

row player

	A	B
a	1, -1	-1, 1
b	-1, 1	1, -1

$$\max V$$

$$\pi(a) - \pi(b) \geq V \quad (\text{vs. A})$$

$$-\pi(a) + \pi(b) \geq V \quad (\text{vs. B})$$

$$\pi(a) + \pi(b) = 1$$

$$0 \leq \pi(a), \pi(b) \leq 1$$

Best Response Condition

For any (possibly stochastic) joint policy π_{-i} ,

There exists a **deterministic** best response:

$$\pi_i^b \in BR(\pi_{-i})$$

Best Response Condition

For any (possibly stochastic) joint policy π_{-i} ,

There exists a **deterministic** best response:

$$\pi_i^b \in BR(\pi_{-i})$$

Proof: Assume otherwise. The values of each deterministic policy (action) must be the same, by def. of BR. Then we can put full weight on any of them.

Two-Player Zero-Sum Games

Matching Pennies: $u_1(\cdot) = -u_2(\cdot)$
column player

row player

	A	B
a	1, -1	-1, 1
b	-1, 1	1, -1

$$\max V$$

$$\pi(a) - \pi(b) \geq V \quad (\text{vs. A})$$

$$-\pi(a) + \pi(b) \geq V \quad (\text{vs. B})$$

$$\pi(a) + \pi(b) = 1$$

$$0 \leq \pi(a), \pi(b) \leq 1$$

This is a Linear Program!

- Solvable in polynomial time (!)
 - Easy to apply off-the-shelf solvers
- Will find one solution
- Matching Pennies: $\pi(a) = \pi(b) = \frac{1}{2}, V = 0$

Minimax



John von Neumann 1928

Max-min: P1 looks for a π_1 such that

$$v_1 = \max_{\pi_1} \min_{\pi_2} u_1(\pi_1, \pi_2)$$

Min-max: P1 looks for a π_1 such that

$$v_1 = \min_{\pi_2} \max_{\pi_1} u_1(\pi_1, \pi_2)$$

In **two-player, zero-sum** these are the same!

---> **The Minimax Theorem**

Consequences of Minimax

The optima $\pi^* = (\pi_1^*, \pi_2^*)$

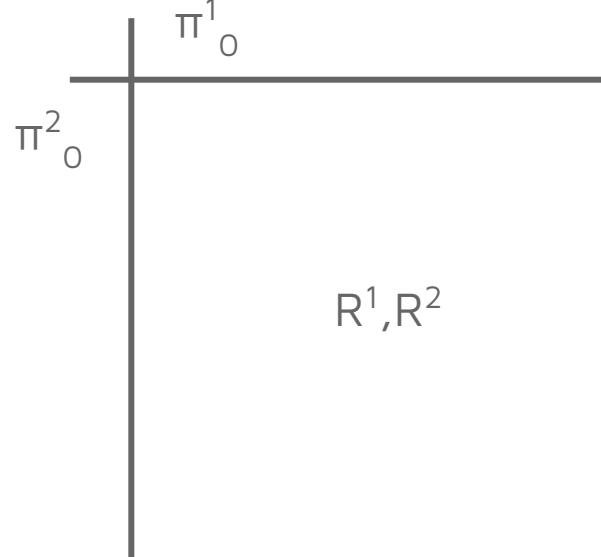
- These exist! (They sometimes might be stochastic.)
- Called a **minimax-optimal joint policy**. Also, a **Nash equilibrium**.
- They are **interchangeable**:

$$\forall \pi^*, \pi^{*\prime} \Rightarrow (\pi_1^*, \pi_2^{*\prime}), (\pi_1^{*\prime}, \pi_2^*) \text{ also minimax-optimal}$$

- Each policy is a **best response** to the other.

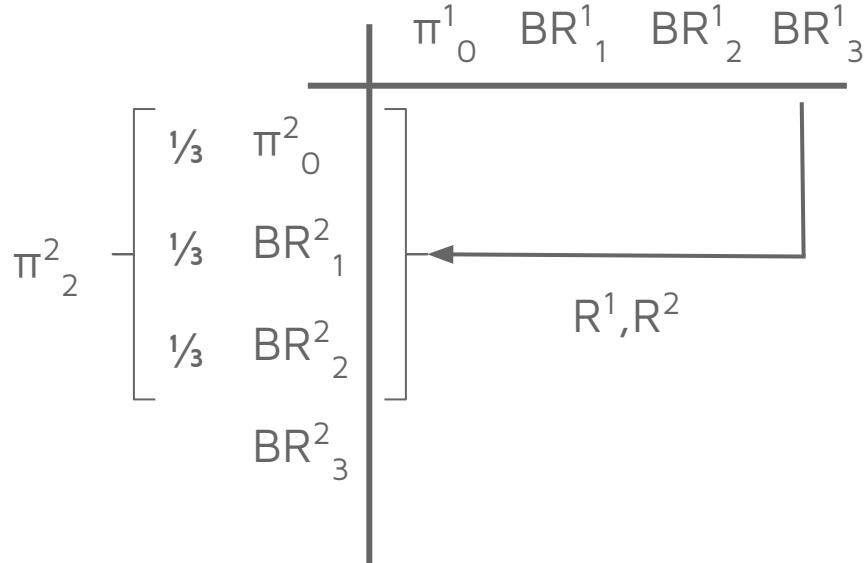
Normal Form Games: Algorithms

- Fictitious Play:
 - Start with an arbitrary policy per player (π_0^1, π_0^2),



Normal Form Games: Algorithms

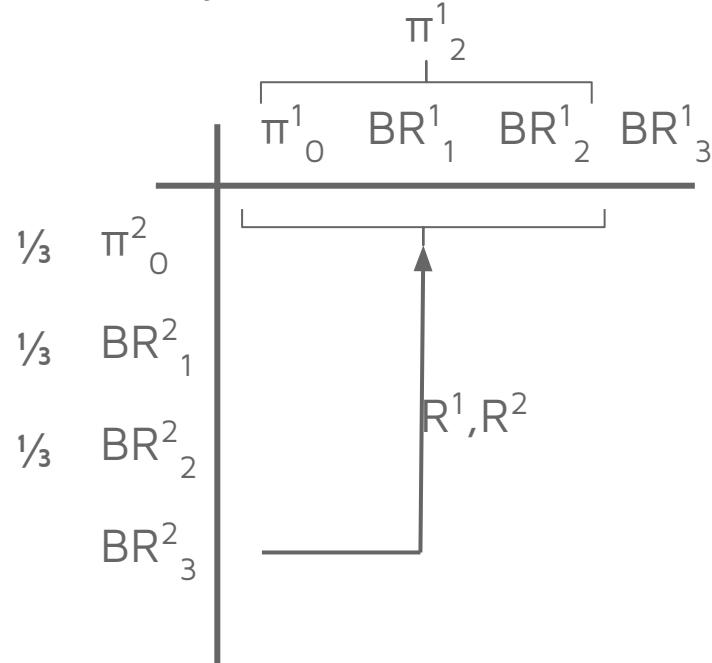
- Fictitious Play:



- Start with an arbitrary policy per player (π^1_0, π^2_0) ,
 - Then, play best response against a uniform distribution over the past policy of the opponent (BR^1_n, BR^2_n) .

Normal Form Games: Algorithms

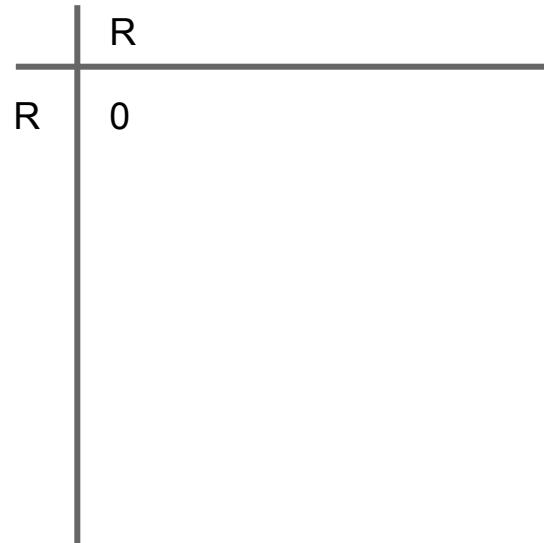
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Normal Form Games: Algorithms

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 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$



Normal Form Games: Algorithms

- Fictitious Play:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:
 - $BR_1^1, BR_1^2 = P, P$
 - $(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2}, 0)$

	R	P
R	0	1
P	-1	0

Normal Form Games: Algorithms

- Fictitious Play:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:

	R	P	P
R	0	1	1
P	-1	0	0
P	-1	0	0

- Iteration 2:
 - $BR_1^1, BR_1^2 = P, P$
 - $(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2}, 0)$

Normal Form Games: Algorithms

- Fictitious Play:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:

	R	P	P	S
R	0	1	1	-1
P	-1	0	0	1
P	-1	0	0	1
S	1	-1	-1	0

- Iteration 2:
 - $BR_1^1, BR_1^2 = P, P$
 - $(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2}, 0)$
- Iteration 3:
 - $BR_2^1, BR_2^2 = P, P$
 - $(\frac{1}{3}, \frac{2}{3}, 0), (\frac{1}{3}, \frac{2}{3}, 0)$
- Iteration 4:
 - $BR_3^1, BR_3^2 = S, S$
 - $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$

Normal Form Games: Algorithms

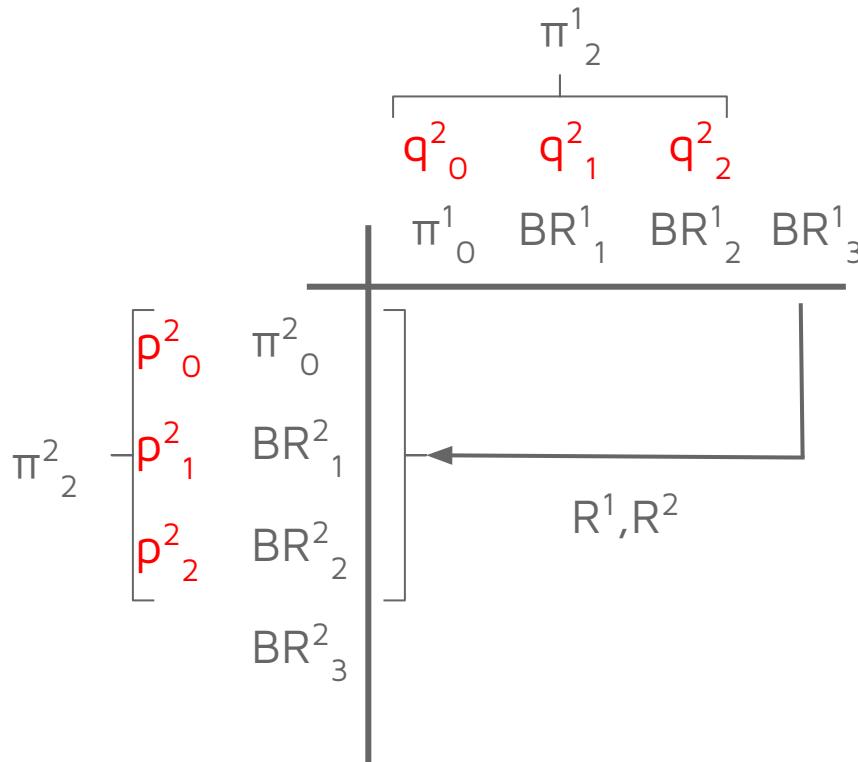
- Fictitious Play:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:

	R	P	P	S	S
R	0	1	1	-1	-1
P	-1	0	0	1	1
P	-1	0	0	1	1
S	1	-1	-1	0	0
S	1	-1	-1	0	0

- Iteration 2:
 - $BR_1^1, BR_1^2 = P, P$
 - $(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2}, 0)$
- Iteration 3:
 - $BR_2^1, BR_2^2 = P, P$
 - $(\frac{1}{3}, \frac{2}{3}, 0), (\frac{1}{3}, \frac{2}{3}, 0)$
- Iteration 4:
 - $BR_3^1, BR_3^2 = S, S$
 - $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$

Normal Form Games: Algorithms

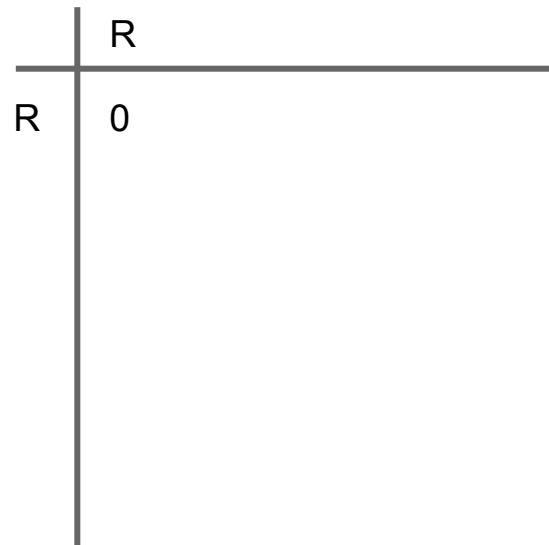
- double oracle [HB McMahan 2003]:



- Start with an arbitrary policy per player (π^1_0, π^2_0) ,
 - Compute (p^n, q^n) by solving the game at iteration n
 - Then, best response against (p^n, q^n) and get a new best response (BR^1_n, BR^2_n) .

Normal Form Games: Algorithms

- double oracle:
- Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$



Normal Form Games: Algorithms

- double oracle:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:
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 - Solve the game : $(0, 1, 0), (0, 1, 0)$

	R	P
R	0	1
P	-1	0

Normal Form Games: Algorithms

- double oracle:
 - Start with $(R, P, S) = (1, 0, 0), (1, 0, 0)$
 - Iteration 1:

	R	P	S
R	0	1	-1
P	-1	0	1
S	1	-1	0

- Iteration 1:
 - $BR_1^1, BR_1^2 = P, P$
 - Solve the game : $(0, 1, 0), (0, 1, 0)$
- Iteration 2:
 - $BR_2^1, BR_2^2 = S, S$
 - $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

Cooperative Games

$$u_i(\cdot) = u_j(\cdot)$$

column player

		A	B	C
		1, 1	0, 0	0, 0
row player	a	0, 0	2, 2	0, 0
	b	0, 0	0, 0	5, 5
	c	0, 0	0, 0	5, 5

Cooperative Games

$$u_i(\cdot) = u_j(\cdot)$$

column player

		A	B	C
row player	a	1, 1	0, 0	0, 0
	b	0, 0	2, 2	0, 0
	c	0, 0	0, 0	5, 5

These are all Nash equilibria!

General-Sum Games

No constraints on utilities!

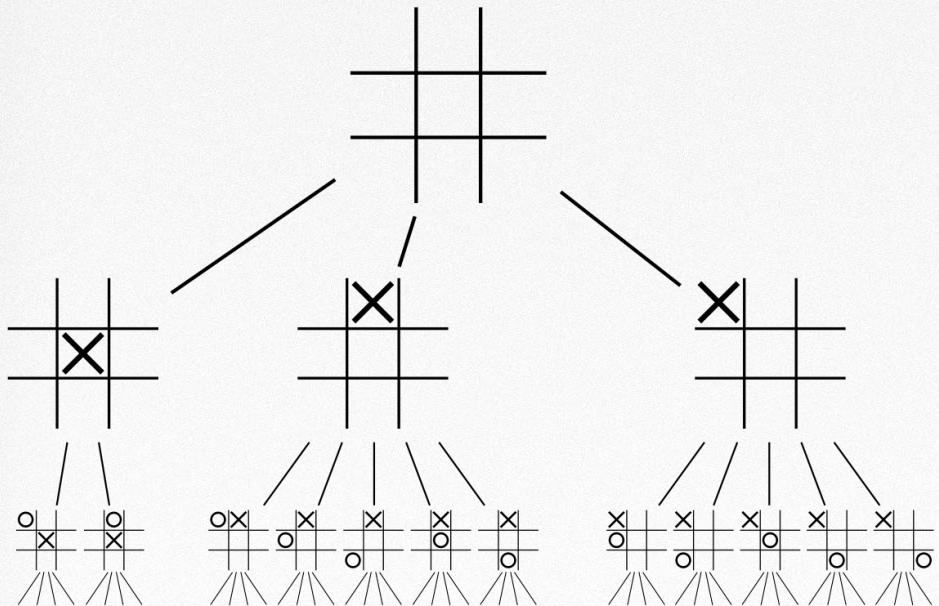
column player

		A	B
		3, 2	0, 0
row player	a	3, 2	0, 0
	b	0, 0	2, 3

The Sequential Setting: Extensive-Form Games

What about sequential games...?

Perfect Information Games



(Finite) Perfect Information Games: Model

- Start with an *episodic MDP*

(Finite) Perfect Information Games: Model

- Start with an *episodic MDP*
- Add a **player identity** function:

$$\tau(s) \in \mathcal{N} \cup \{(s)\}$$

Simultaneous move node (many players play simultaneously)

(Finite) Perfect Information Games: Model

- Start with an *episodic MDP*
- Add a **player identity** function:

$$\tau(s) \in \mathcal{N} \cup \{s\}$$

- Define rewards *per player*:

$$r_i(s, a, s') \text{ for } i \in \mathcal{N}$$

(Finite) Perfect Information Games: Model

- Start with an *episodic MDP*
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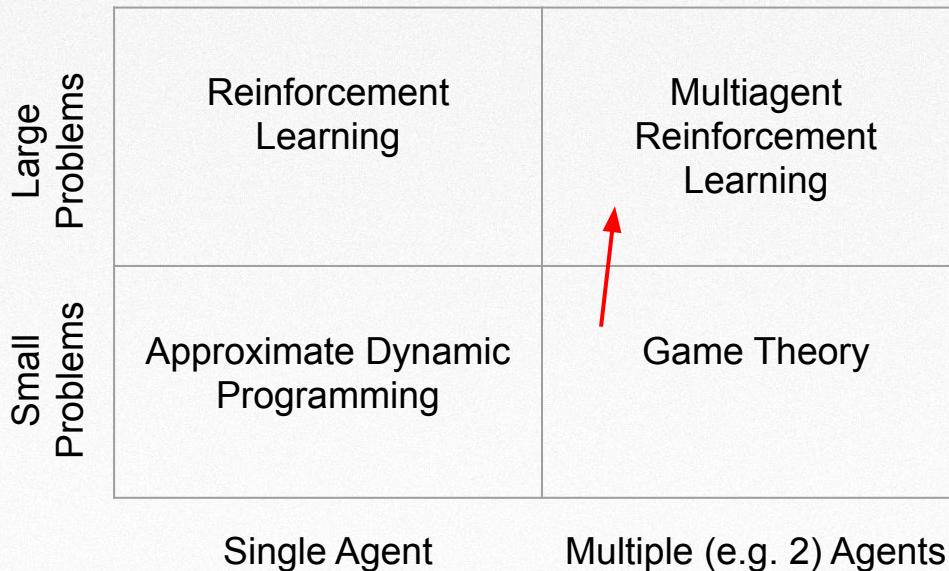
- Define rewards *per player*:

$$r_i(s, a, s') \text{ for } i \in \mathcal{N}$$

- (Similarly for returns: $G_{t,i}$ is the return to player i from s_t)

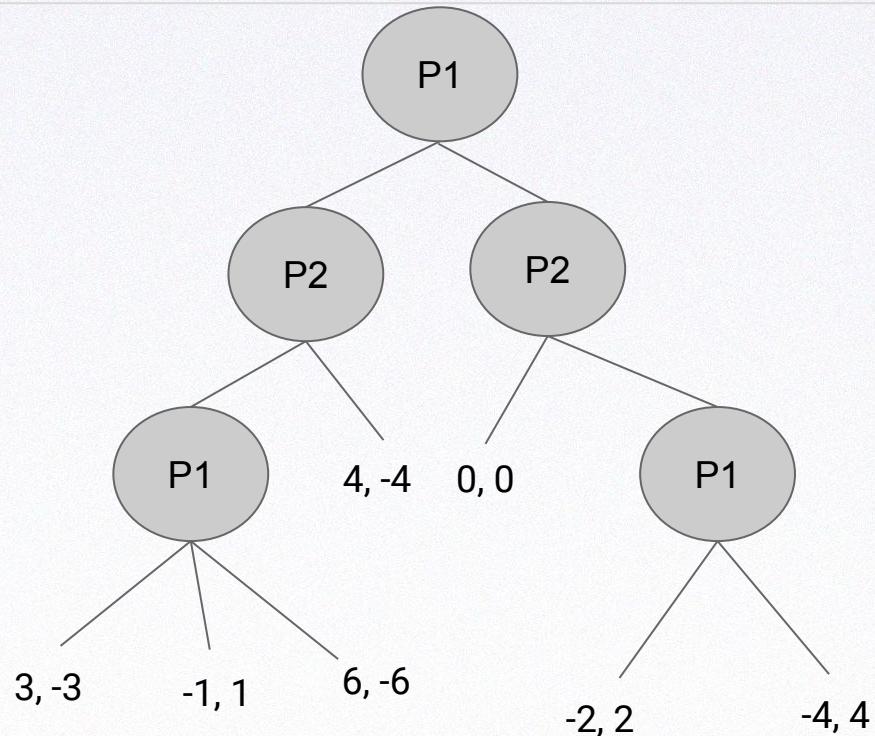
Part 3: Basic Formalisms & Algorithms

Foundations of RL



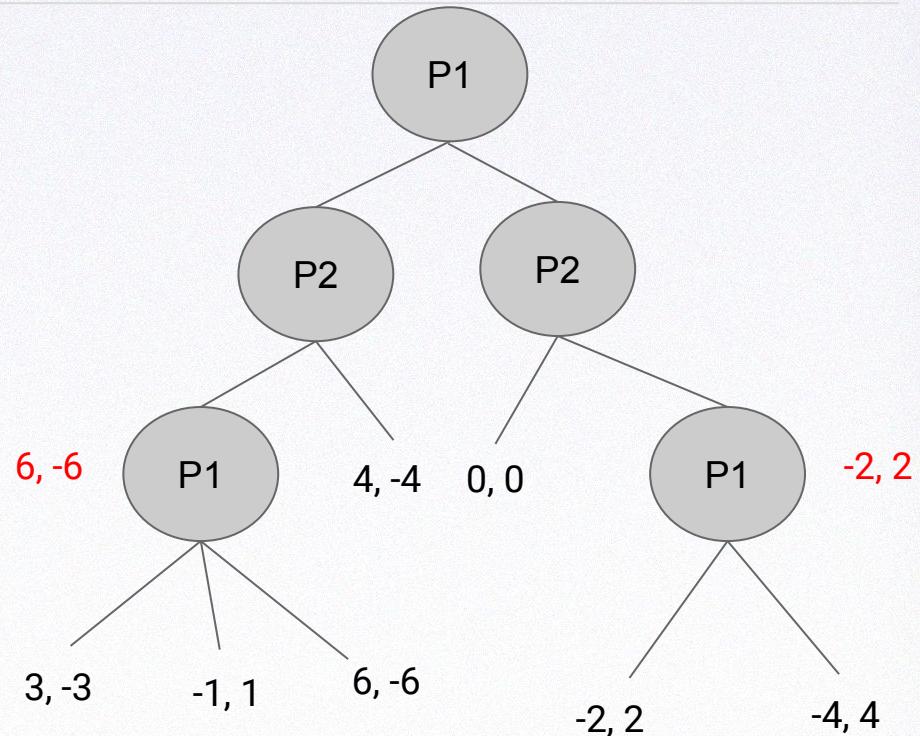
Backward Induction

Solving a *turn-taking* perfect information game



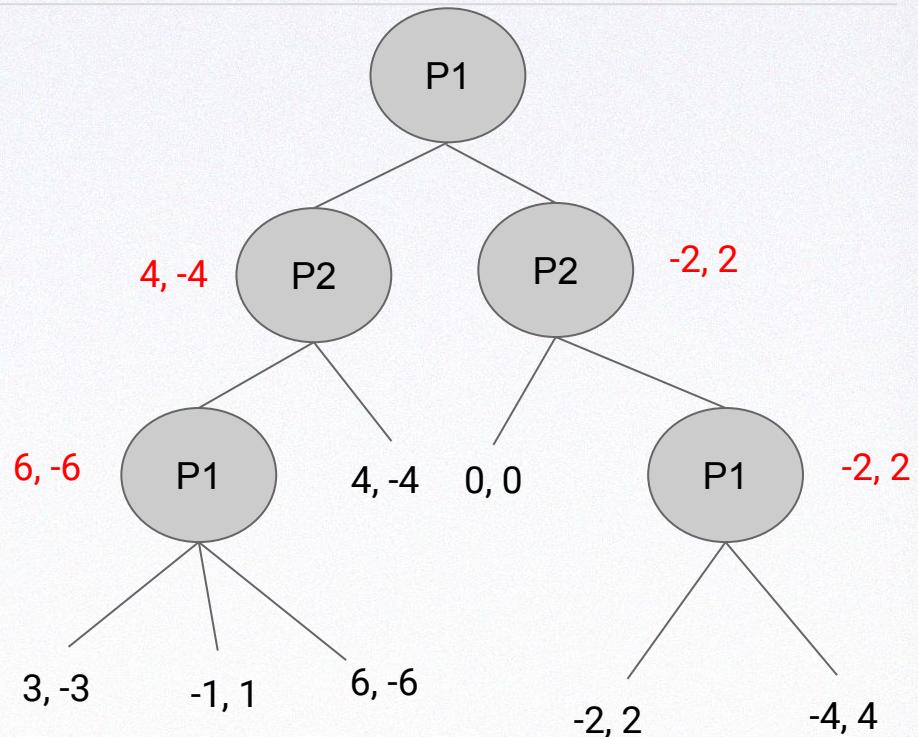
Backward Induction

Solving a *turn-taking* perfect information game



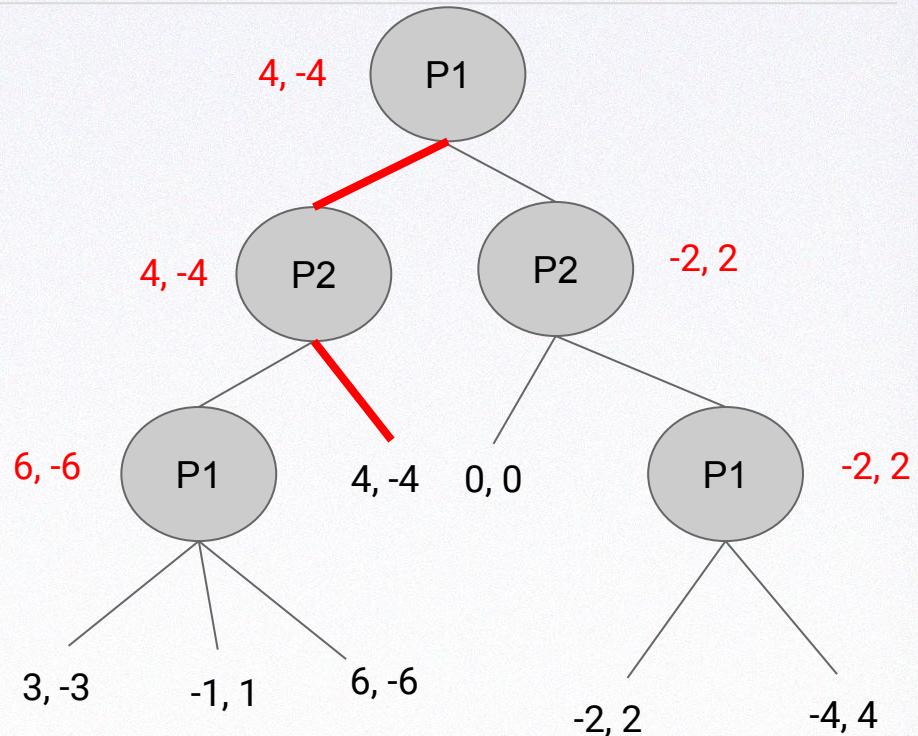
Backward Induction

Solving a *turn-taking* perfect information game



Backward Induction

Solving a *turn-taking* perfect information game



Intro to RL: Tabular Approximate Dyn. Prog.

Value iteration

Initialize array V arbitrarily (e.g., $V(s) = 0$ for all $s \in \mathcal{S}^+$)

Repeat

$$\Delta \leftarrow 0$$

For each $s \in \mathcal{S}$:

$$v \leftarrow V(s)$$

$$V(s) \leftarrow \max_a \sum_{s',r} p(s',r|s,a) [r + \gamma V(s')]$$

$$\Delta \leftarrow \max(\Delta, |v - V(s)|)$$

until $\Delta < \theta$ (a small positive number)

Output a deterministic policy, $\pi \approx \pi_*$, such that

$$\pi(s) = \arg \max_a \sum_{s',r} p(s',r|s,a) [r + \gamma V(s')]$$

Turn-Taking 2P Zero-sum Perfect Info. Games

- Player to play at s : $\tau(s)$
- Reward to player i : r_i
- Subset of legal actions $\text{LEGAL ACTIONS}(s)$
- Often assume episodic and $\gamma = 1$

Values of a state **to player i**: $V_i(s)$

Identities:

$$\forall s, a, s' : r_1 = -r_2, \quad V_1(s) = -V_2(s)$$

2P Zero-Sum Perfect Info. Value Iteration

Value iteration

Initialize array V_i arbitrarily (e.g., $V_i(s) = 0$ for all $s \in \mathcal{S}^+$)

Repeat

$$\Delta \leftarrow 0$$

For each $s \in \mathcal{S}$:

$$v \leftarrow V_i(s)$$

$$V_i(s) \leftarrow \max_a \sum_{s', r_i} p(s', r_i | s, a) [r_i + \gamma V_i(s')]$$

$$\Delta \leftarrow \max(\Delta, |v - V_i(s)|)$$

until $\Delta < \theta$ (a small positive number)

Let $i = t(s)$

Output a deterministic policy, $\pi \approx \pi_*$, such that *$i = t(s)$*

$$\pi(s) = \arg \max_a \sum_{s', r_i} p(s', r_i | s, a) [r_i + \gamma V_i(s')]$$

Minimax

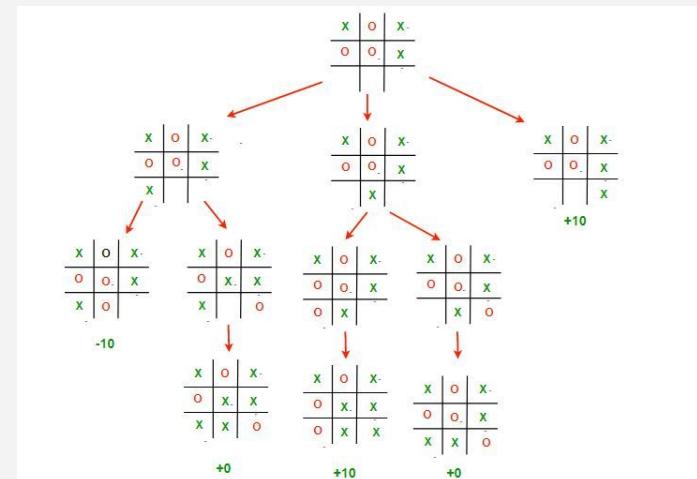
A.K.A. Alpha-Beta, Backward Induction, Retrograde Analysis, etc...

Start from search state S ,

Compute a depth-limited approximation:

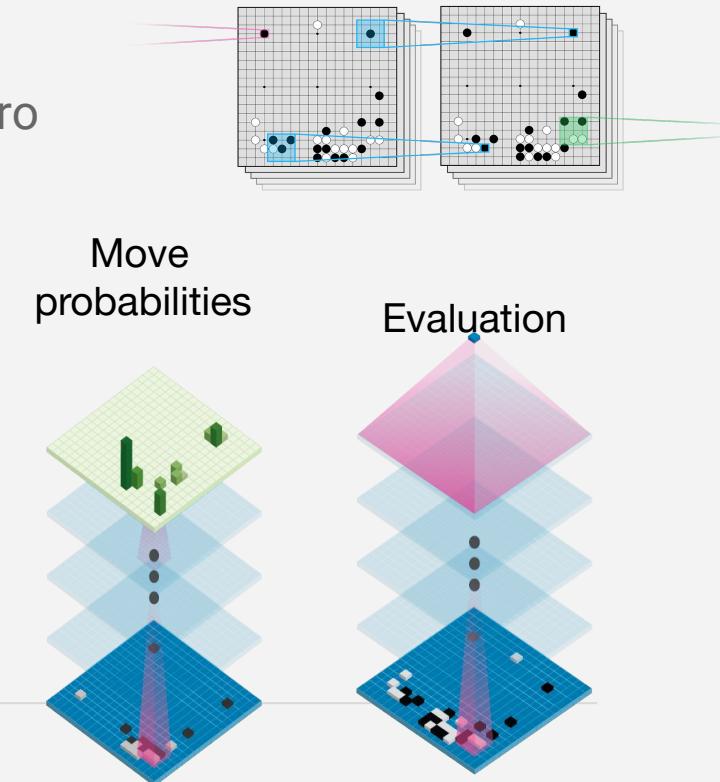
$$V_{i,d}(s) = \begin{cases} u_i(s) & \text{if } s \text{ is terminal,} \\ h_i(s) & \text{if } d = 0, \\ \sum_{s'} p(s, a, s') V_{i,d-1}(s') & \text{otherwise.} \end{cases}$$

---> Minimax Search



Two-Player Zero-Sum Policy Iteration

- Analogous to adaptation of value iteration
- Foundation of AlphaGo, AlphaGo Zero, AlphaZero
 - Better policy improvement via MCTS
 - Deep network func. approximation
 - Policy prior cuts down *breadth*
 - Value network cuts the *depth*



2P Zero-Sum Games with Simultaneous Moves

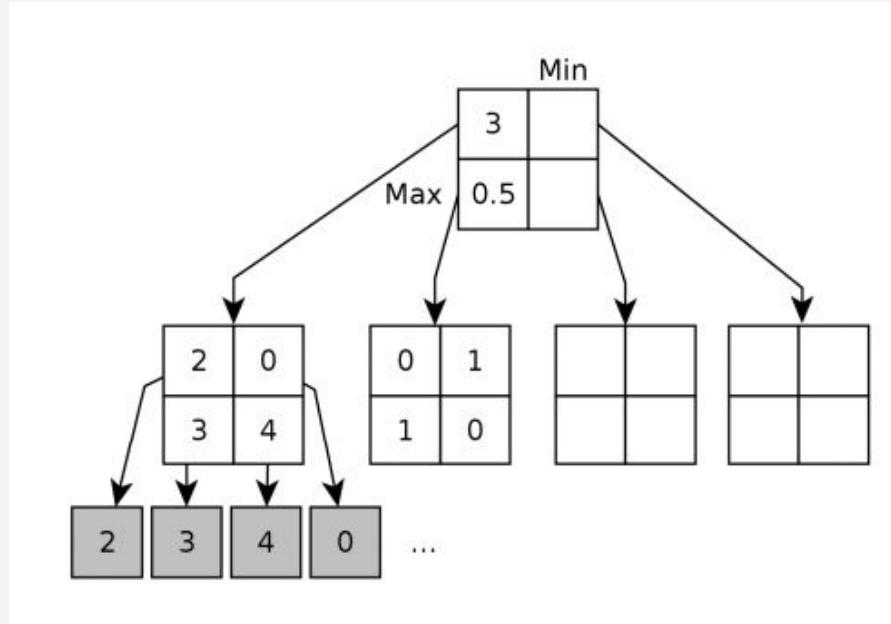


Image from [Bozansky et al. 2016](#)

Markov Games

“Markov Soccer”

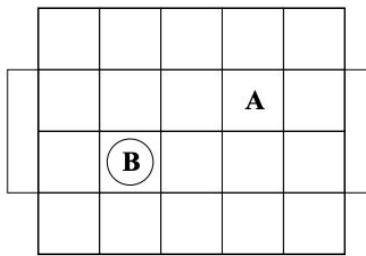
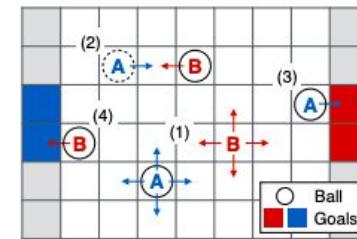
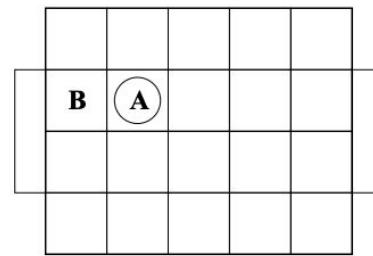


Figure 2: An initial board (left) and a situation requiring a probabilistic choice for A (right).



	Defensive	Offensive
w/ ball	Avoid opponent	Advance to goal
w/o ball	Defend goal	Intercept the ball

Figure 3. Left: Illustration of the soccer game. Right: Strategies of the hand-crafted rule-based agent.

Littman '94

Also: Lagoudakis & Parr '02, Uther & Veloso '03, Collins '07

He et al. '16

Value Iteration for Zero-Sum Markov Games

Value iteration

Initialize array V arbitrarily (e.g., $V(s) = 0$ for all $s \in \mathcal{S}^+$)

Repeat

$$\Delta \leftarrow 0$$

For each $s \in \mathcal{S}$:

$$v \leftarrow V(s)$$

$$V(s) \leftarrow \min_{\pi_2(s)} \max_{\pi_1(s)} \mathbb{E}_{a \sim \pi(s), s'} [r_1(s, a, s') + \gamma V_1(s')]$$

$$\Delta \leftarrow \max(\Delta, |v - V(s)|)$$

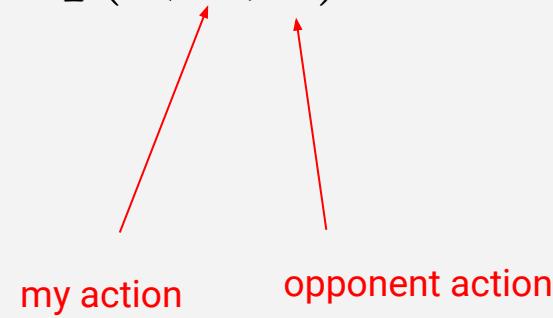
until $\Delta < \theta$ (a small positive number)

Output a ~~deterministic~~ policy, $\pi \approx \pi_*$, such that ~~computed above~~

$$\pi(s) = \arg \max_a \sum_{s', r} p(s', r | s, a) [r + \gamma V(s')]$$

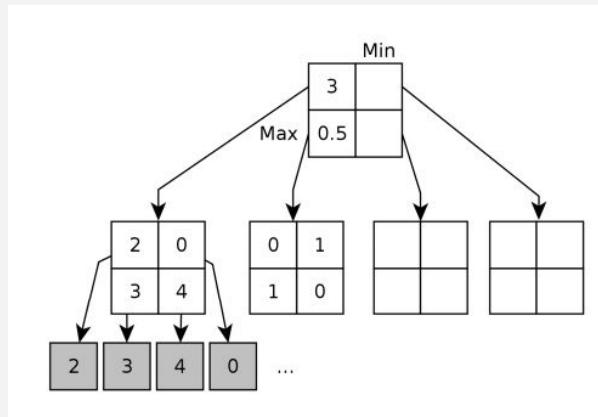
First MARL Algorithm: Minimax-Q (Littman '94)

1. Start with arbitrary joint value functions $q(s, a, o)$



First MARL Algorithm: Minimax-Q (Littman '94)

1. Start with arbitrary joint value functions $q(s, a, o)$



my action opponent action

Induces a matrix of values

The diagram features several red annotations:

- Two red arrows originate from the text "my action" and "opponent action" and point upwards towards the tree structure.
- A red wavy line starts below the text "Induces a matrix of values" and curves upwards, ending near the top of the tree, suggesting a mapping or flow from the induced values back to the actions.

First MARL Algorithm: Minimax-Q (Littman '94)

1. Start with arbitrary joint value functions $q(s, a, o)$
2. Define policy π as in value iteration (by solving an LP)

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First MARL Algorithm: Minimax-Q (Littman '94)

1. Start with arbitrary joint value functions $q(s, a, o)$
2. Define policy π as in value iteration (by solving an LP)
3. Generate trajectories of tuple (s, a, o, s') using
behavior policy $\pi' = \epsilon \text{UNIF}(\mathcal{A}) + (1 - \epsilon)\pi$
4. Update $q(s, a, o) = (1 - \alpha)q(s, a, o) + \alpha(r(s, a, o, s') + \gamma v(s'))$

First Era of MARL

Follow-ups to Minimax Q:

- Friend-or-Foe Q-Learning (Littman '01)
- Correlated Q-learning (Greenwald & Hall '03)
- Nash Q-learning (Hu & Wellman '03)
- Coco-Q (Sodomka et al. '13)

Function approximation:

- LSPI for Markov Games (Lagoudakis & Parr '02)

First Era of MARL

Nash Convergence of Gradient Dynamics in General-Sum Games

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Tel Aviv University
Tel Aviv, Israel
mansour@math.tau.ac.il

Singh, Kearns & Mansour '03, [Infinitesimal Gradient Ascent \(IGA\)](#)

First Era of MARL

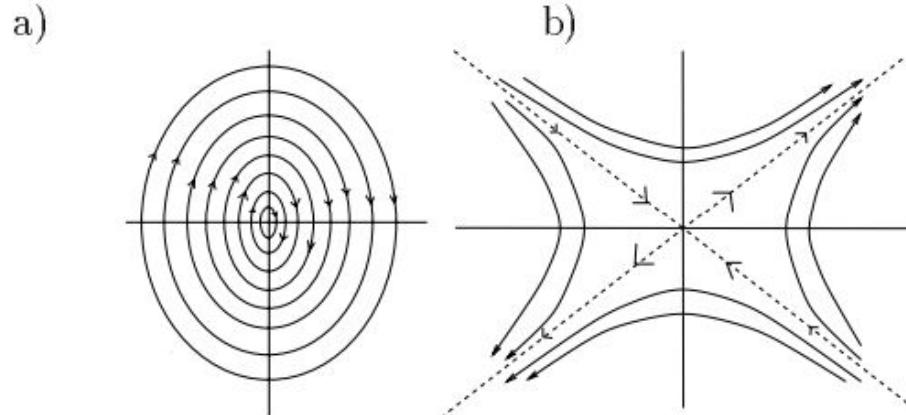


Figure 1: The general form of the dynamics: a) when U has imaginary eigenvalues and b) when U has real eigenvalues.

Image from Singh, Kearns, & Mansour '03

Formalize optimization as a dynamical system:

policy gradients

Analyze using well-established techniques

First Era of MARL

→ Evolutionary Game Theory: **replicator dynamics**

$$\dot{\pi}_t(a) = \pi_t(a) [u(a, \boldsymbol{\pi}_t) - \bar{u}(\boldsymbol{\pi}_t)]$$



time derivative

First Era of MARL

→ Evolutionary Game Theory: **replicator dynamics**

$$\dot{\pi}_t(a) = \pi_t(a) [u(a, \boldsymbol{\pi}_t) - \bar{u}(\boldsymbol{\pi}_t)]$$



time derivative



utility of action a against
the joint policy / population
of other players

First Era of MARL

→ Evolutionary Game Theory: **replicator dynamics**

$$\dot{\pi}_t(a) = \pi_t(a) [u(a, \boldsymbol{\pi}_t) - \bar{u}(\boldsymbol{\pi}_t)]$$



time derivative



utility of action a against
the joint policy / population
of other players



Expected / average utility
of the joint policy /
population

First Era of MARL

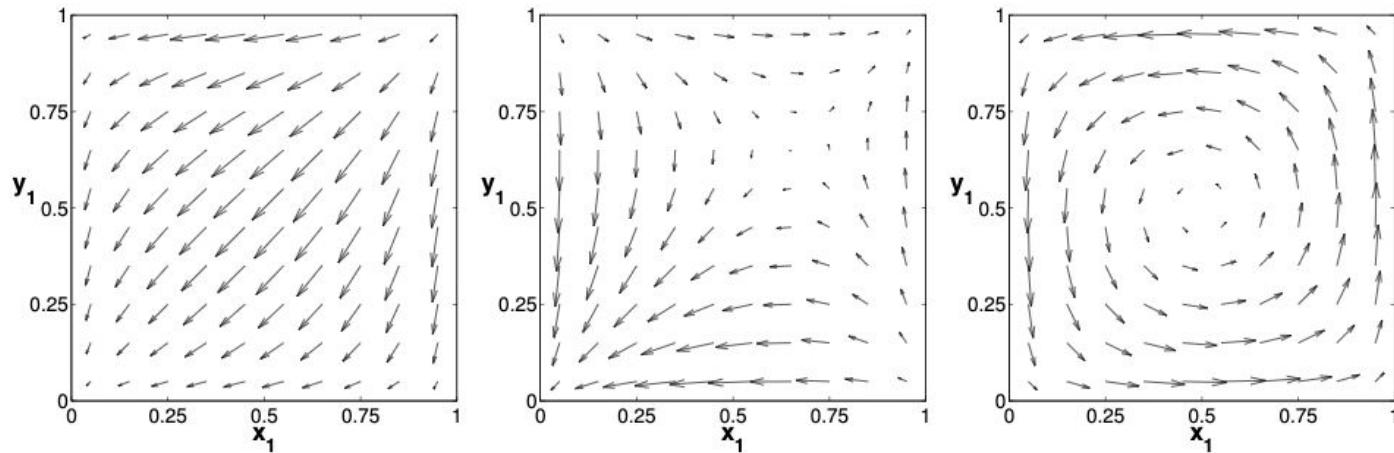


Figure 4: The replicator dynamics, plotted in the unit simplex, for the prisoner's dilemma (left), the stag hunt (center), and matching pennies (right).

[Bloembergen et al. 2015](#)

First Era of MARL

WoLF: Win or Learn Fast. (Bowling & Veloso '01).

IGA is **rational** but not **convergent**!

- *Rational*: opponents converge to a fixed joint policy
→ learning agent converges to a best response of joint policy
- *Convergent*: learner necessarily converges to a fixed policy

Use specific *variable learning rate* to ensure convergence (in 2x2 games)

First Era of MARL

Follow-ups to policy gradient and replicator dynamics:

- WoLF-IGA, WoLF-PHC
 - WoLF-GIGA (Bowling '05)
 - Weighted Policy Learner (Abdallah & Lesser '08)
 - Infinitesimal Q-learning (Wunder et al. '10)
 - Frequency-Adjusted Q-Learning (Kaisers et al. '10, Bloembergen et al. '11)
 - Policy Gradient Ascent with Policy Prediction (Zhang & Lesser '10)
 - Evolutionary Dynamics of Multiagent Learning (Bloembergen et al. '15)
-

So.....

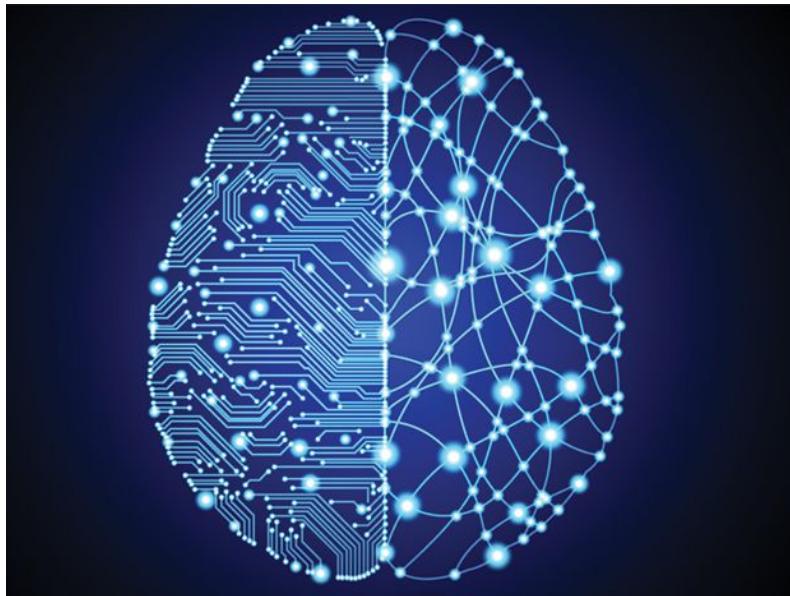
Why call it “the first era”?

So.....

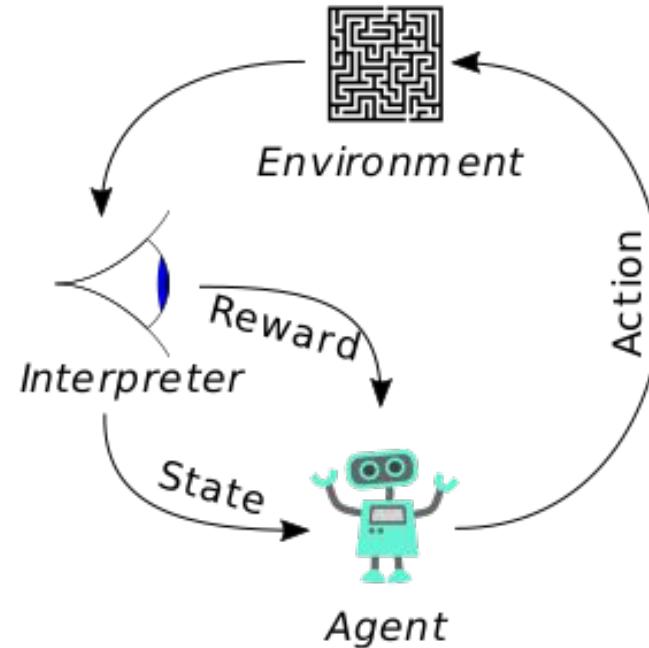
Why call it “the first era”?

Scalability was a major problem.

Second Era: Deep Learning meets Multiagent RL



Source: spectrum.ieee.org

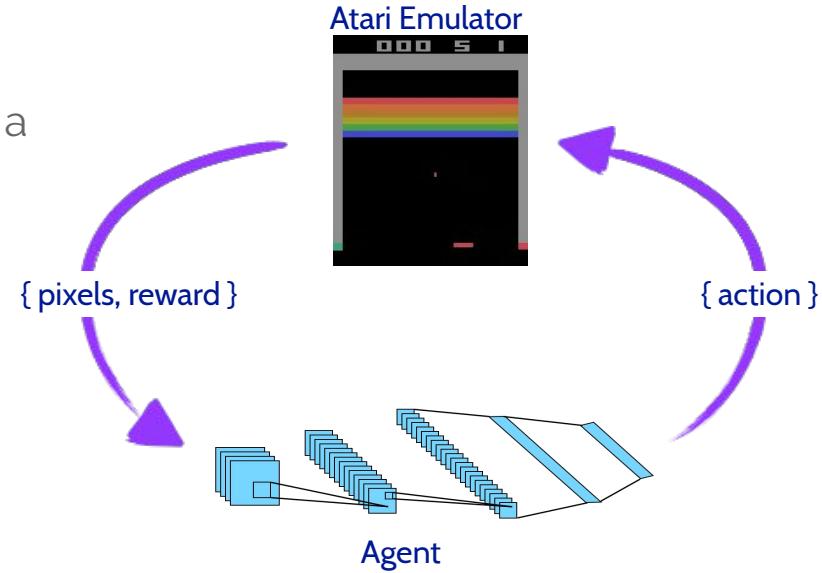


Source: [wikipedia.org](https://en.wikipedia.org)

Deep Q-Networks (DQN) Mnih et al. 2015

“Human-level control through deep reinforcement learning”

- Represent the action value (Q) function using a convolutional neural network.
- Train using end-to-end Q-learning.
- Can we do this in a stable way?

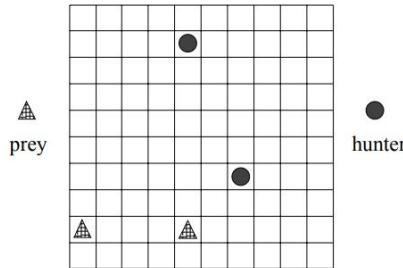


Independent Q-Learning Approaches

Independent Q-learning [Tan, 1993]

$$Q(x, a) \leftarrow Q(x, a) + \beta(r + \gamma V(y) - Q(x, a))$$

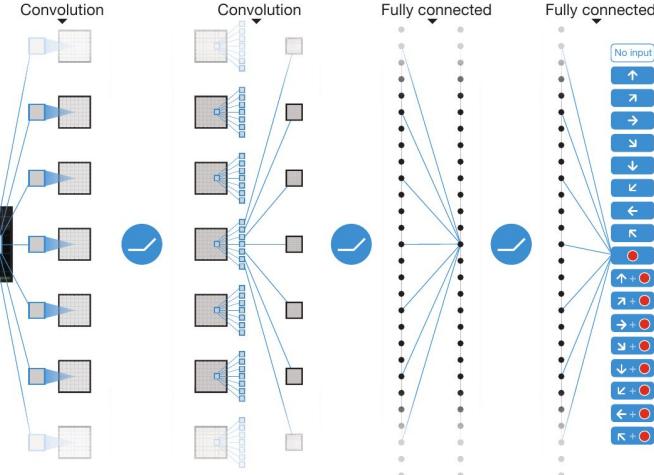
$$V(x) = \max_{b \in actions} Q(x, b)$$



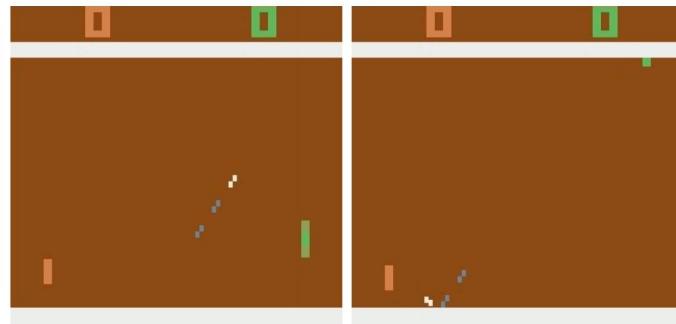
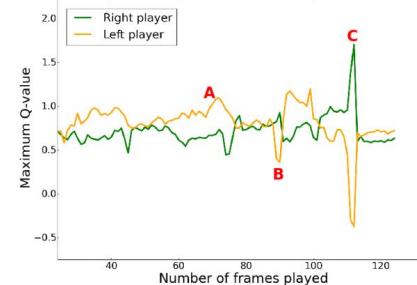
N-of-prey/N-of-hunters	1/1	1/2
Random hunters	123.08	56.47
Learning hunters	25.32	12.21

Table 1: Average Number of Steps to Capture a Prey

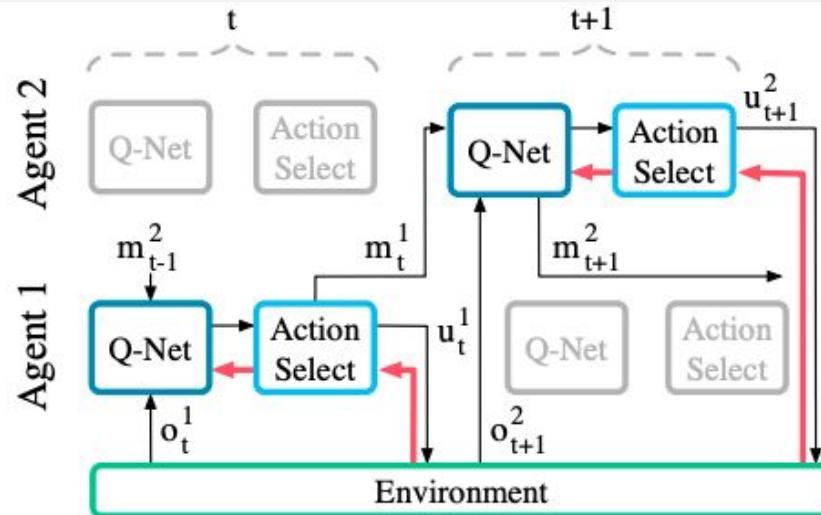
Independent Deep Q-Networks [Tampuu et al., 2015]



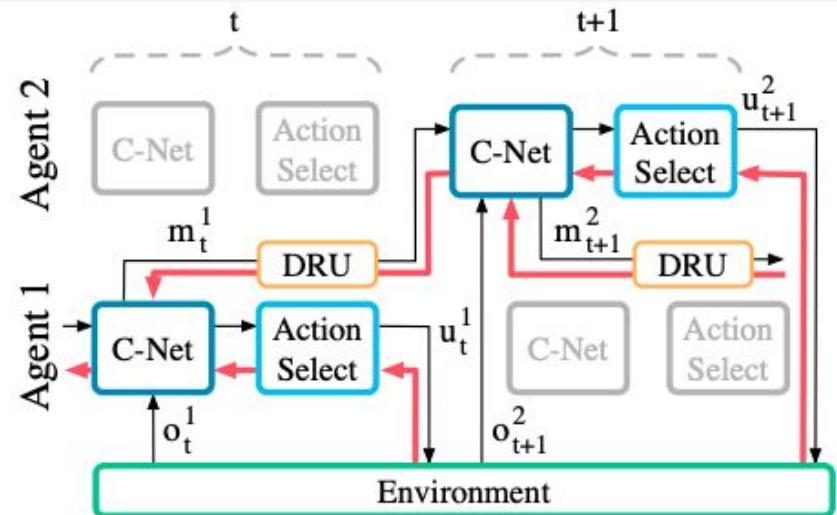
Evolution of Q-value



Learning to Communicate



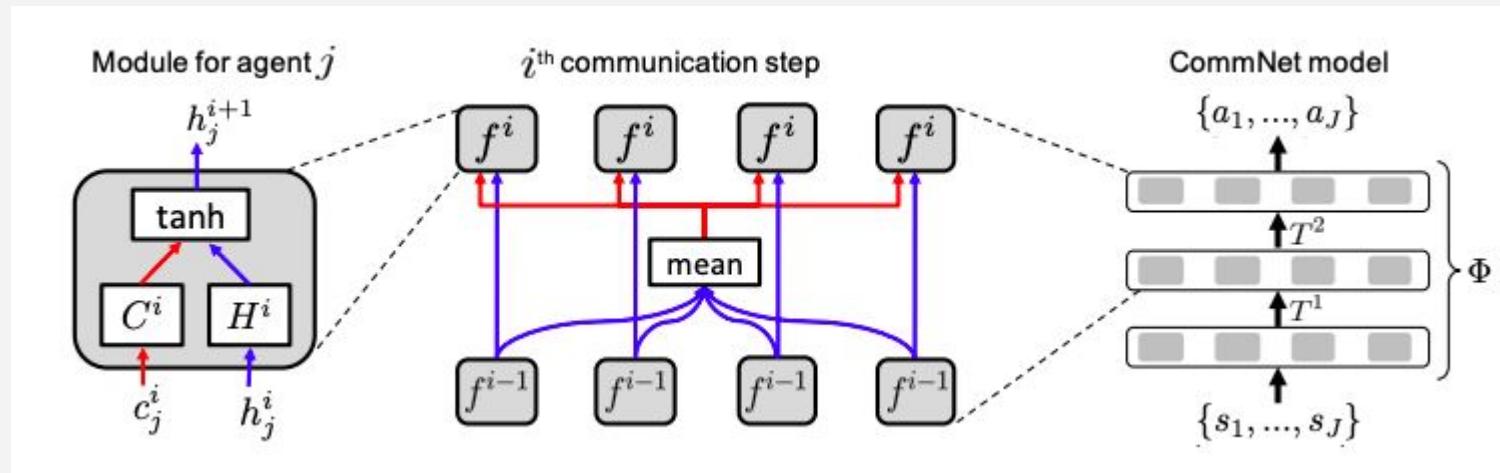
(a) RIAL - RL based communication



(b) DIAL - Differentiable communication

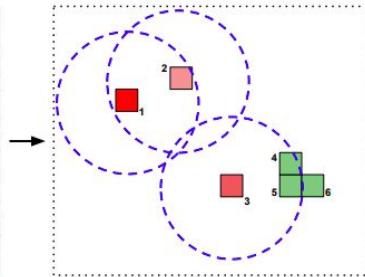
Foerster et al. '16

Learning to Communicate



Sukhbaatar et al. '16

Cooperative Multiagent Tasks



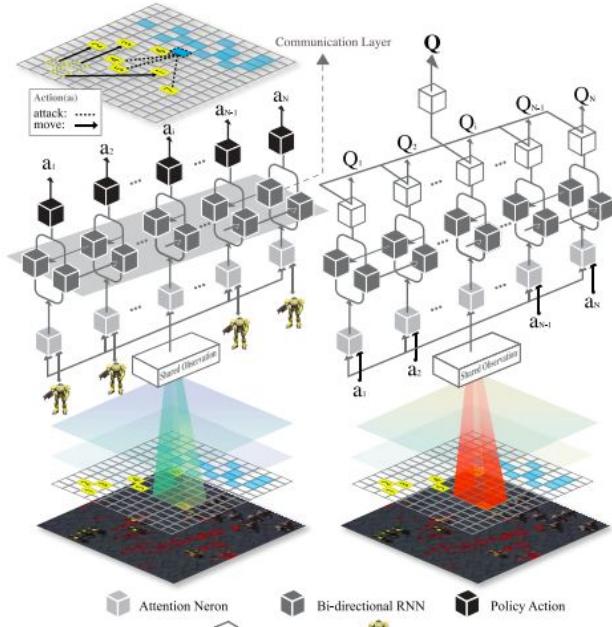
$$\begin{aligned} \rightarrow O(s_t, 1) &= f(\blacksquare_1, 1) \oplus f(\blacksquare_2, 1) \\ \rightarrow O(s_t, 2) &= f(\blacksquare_1, 2) \oplus f(\blacksquare_2, 2) \\ \rightarrow O(s_t, 3) &= f(\blacksquare_1, 3) \oplus f(\blacksquare_2, 3) \\ &\quad \oplus f(\blacksquare_3, 3) \end{aligned}$$

Foerster et al. '18

Episodic Exploration for Deep Deterministic Policies:
An Application to StarCraft Micromanagement Tasks

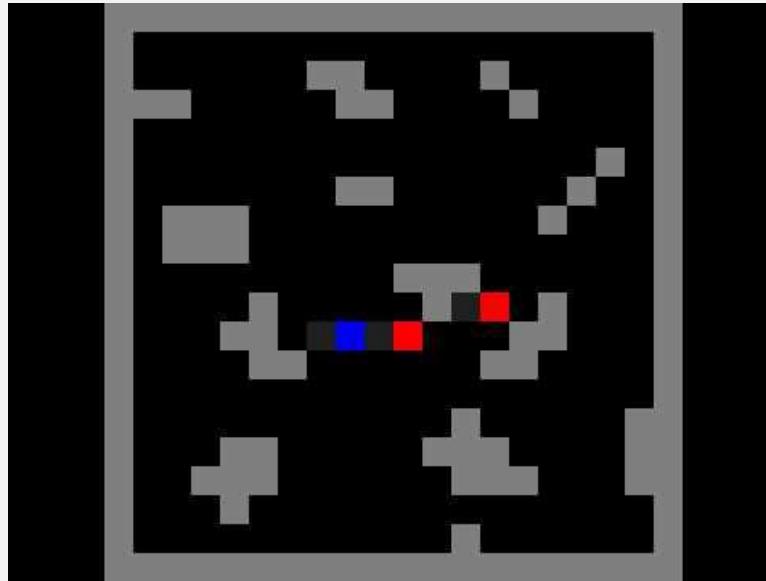
Nicolas Usunier*, Gabriel Synnaeve*, Zeming Lin, Soumith Chintala
Facebook AI Research
usunier,gab,zlin,soumith@fb.com

November 29, 2016

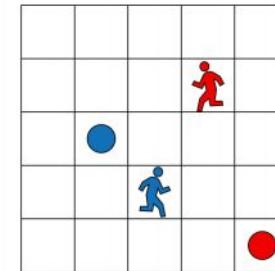


BIC-Net (Peng et al.'17)

Sequential Social Dilemmas



Leibo et al. '17



- + +1
- + +1 -2
- + +1 -2
- + +1

(a) Coins



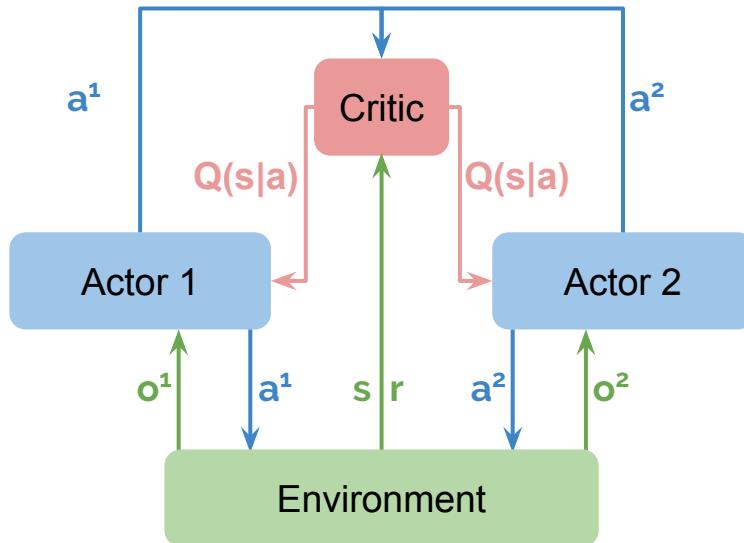
Orange Misses: +1 -2
Green Misses: +1 -2

(b) PPD

Lerer & Peyskavich '18

Centralized Critic Decentralized Actor Approaches

- **Idea:** reduce nonstationarity & credit assignment issues using a central critic
- **Examples:** MADDPG [Lowe et al., 2017] & COMA [Foerster et al., 2017]
- Apply to both cooperative and competitive games



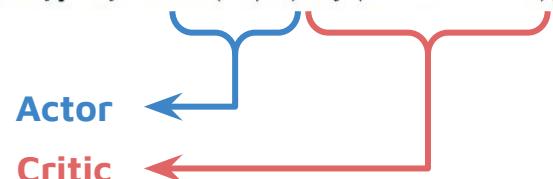
Centralized critic trained to minimize loss:

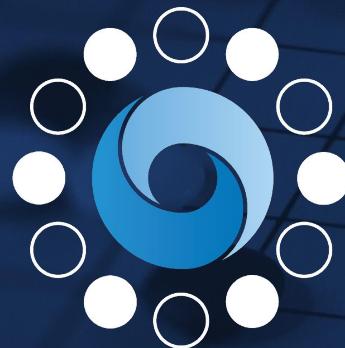
$$\mathcal{L}(\theta_i) = \mathbb{E}_{\mathbf{x}, a, r, \mathbf{x}'} [(Q_i^\pi(\mathbf{x}, a_1, \dots, a_N) - y)^2],$$

$$y = r_i + \gamma Q_i^{\pi'}(\mathbf{x}', a'_1, \dots, a'_N) \Big|_{a'_j = \pi'_j(o_j)}$$

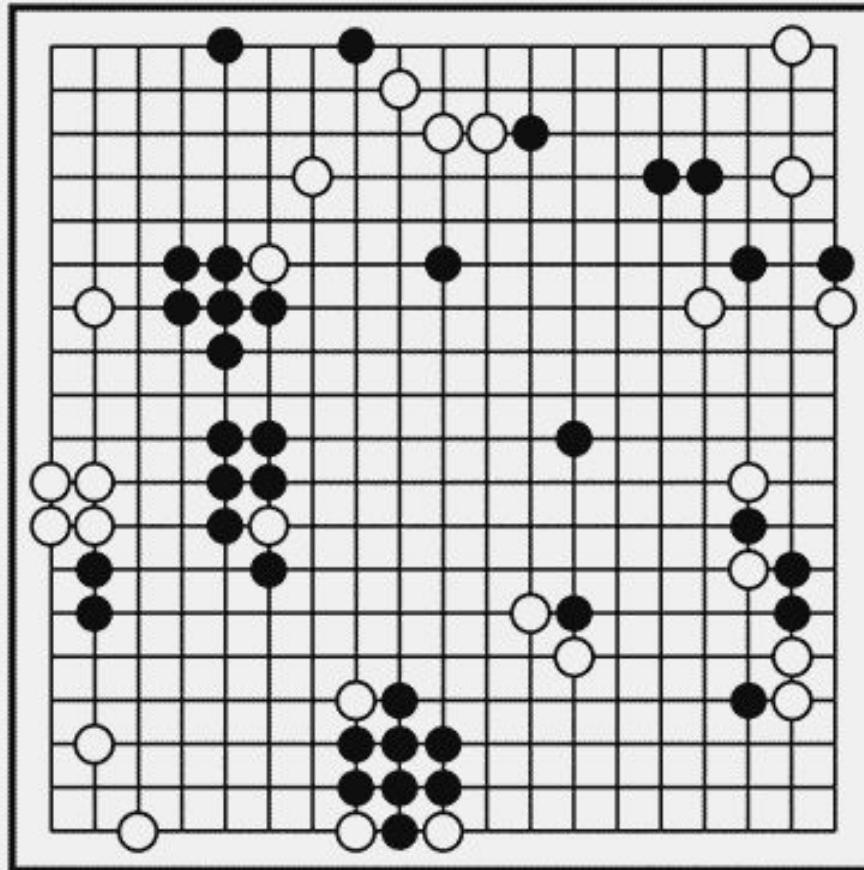
Decentralized actors trained via policy gradient:

$$\nabla_{\theta_i} J(\theta_i) = \mathbb{E}_{s \sim p^\mu, a_i \sim \pi_i} [\nabla_{\theta_i} \log \pi_i(a_i | o_i) Q_i^\pi(\mathbf{x}, a_1, \dots, a_N)]$$





AlphaGo



AlphaGo vs. Lee Sedol

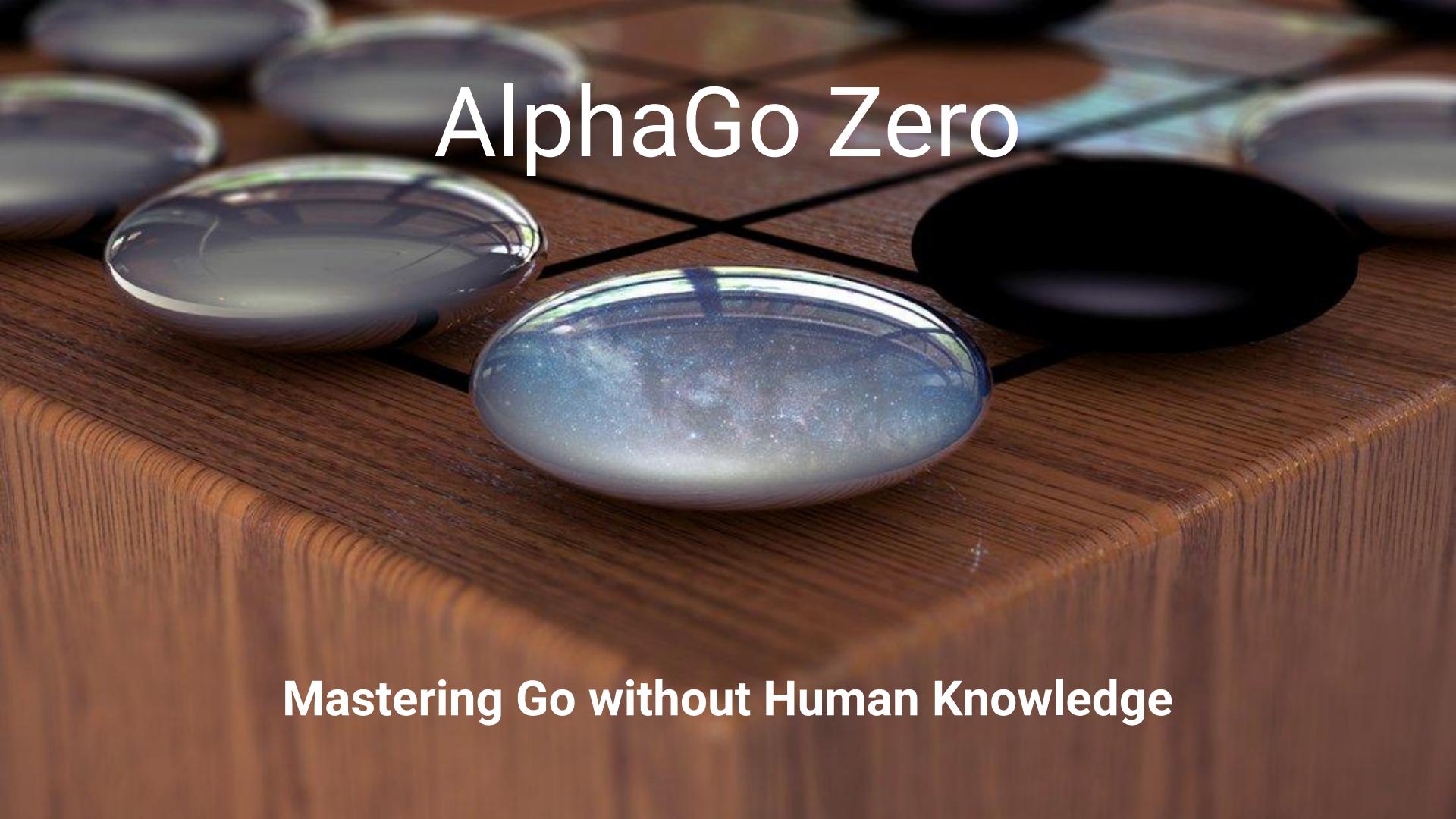
Lee Sedol (gp): winner of 18 world titles

Match was played in Seoul, March 2016

AlphaGo won the match 4-1

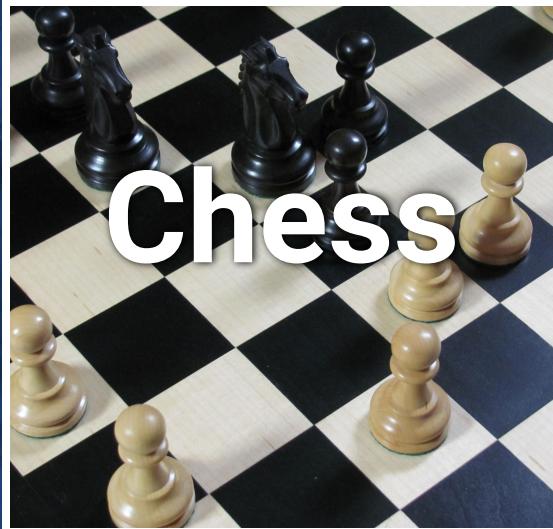


AlphaGo Zero

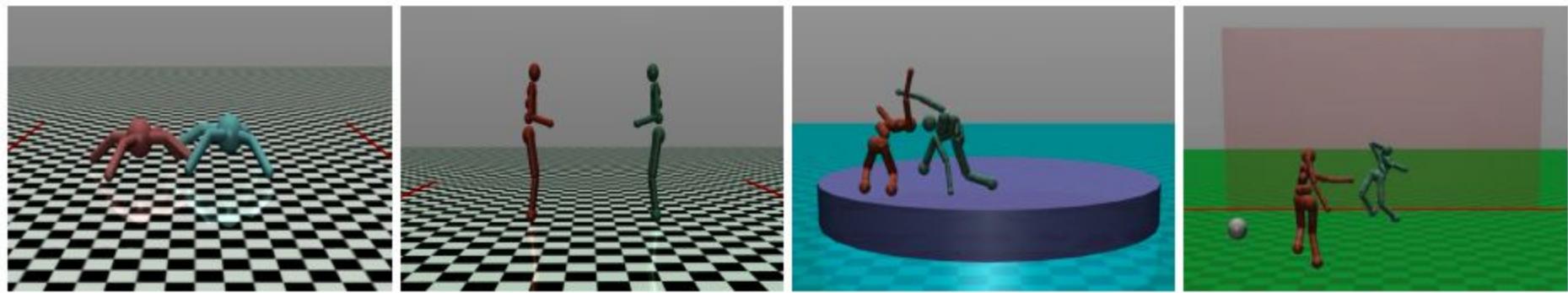
A close-up photograph of a wooden Go board. Several black and white circular stones are placed on the board, forming a partial cross shape. One stone, located in the lower center, has a unique, translucent blue surface with a visible starry galaxy or nebula pattern, suggesting a connection to the AI theme.

Mastering Go without Human Knowledge

AlphaZero: One Algorithm, Three Games

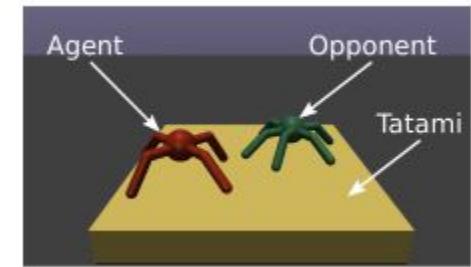
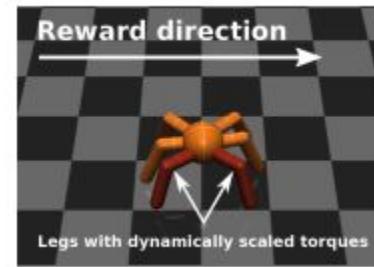


3D Worlds



[Bansal et al. '18](#)

Meta-Learning in RoboSumo



[AI-Shedivat et al. '17](#)

Emergent Coordination Through Competition

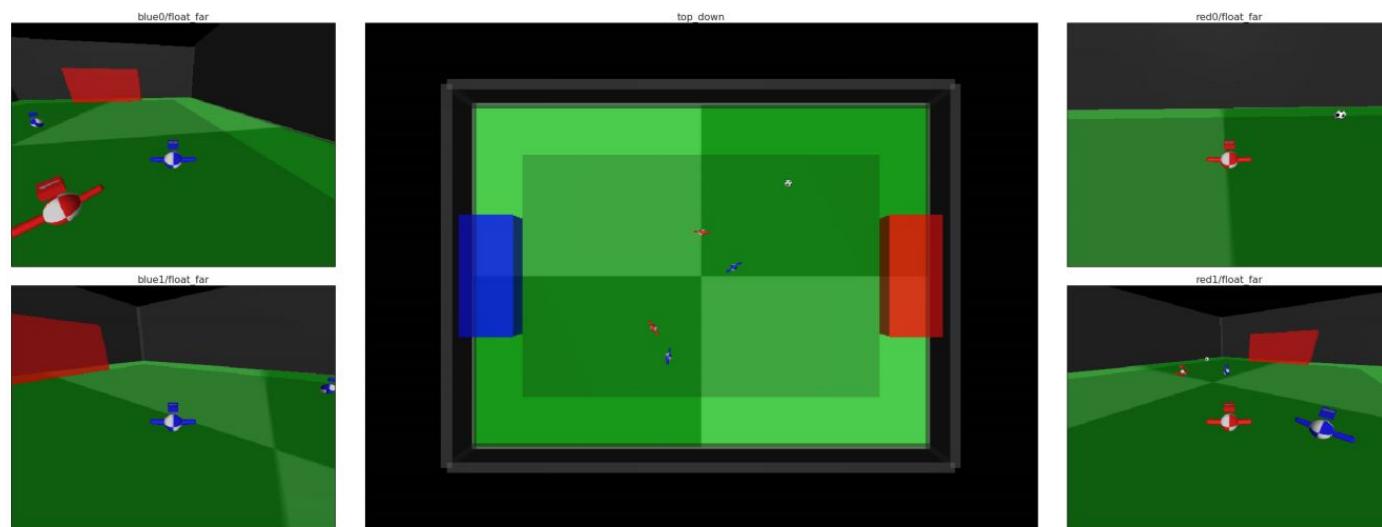
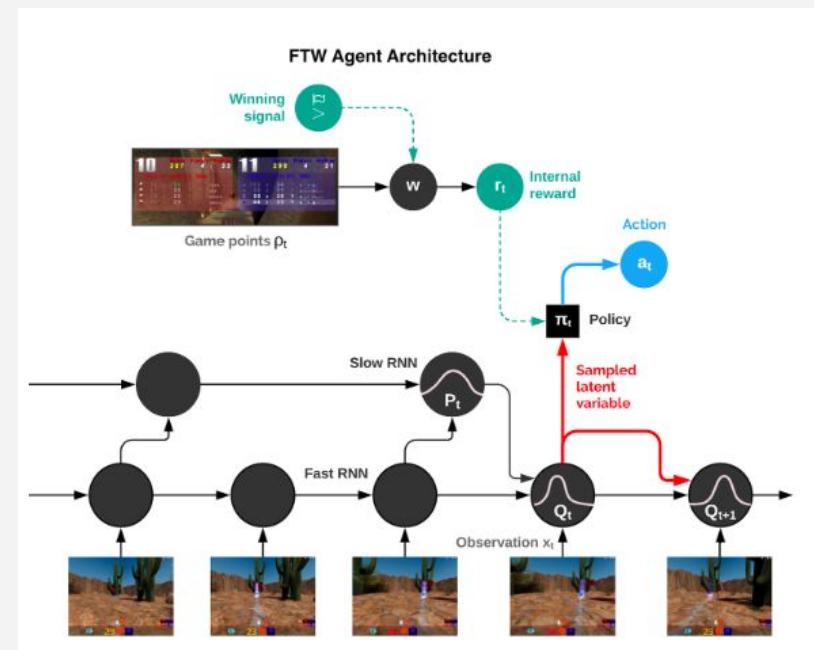
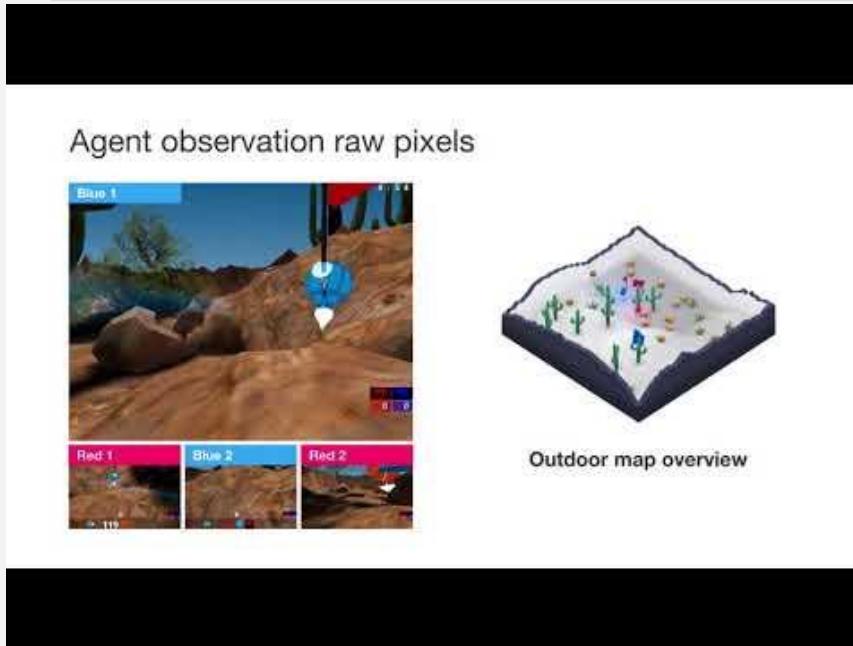


Figure 1: Top-down view with individual camera views of 2v2 multi-agent soccer environment.

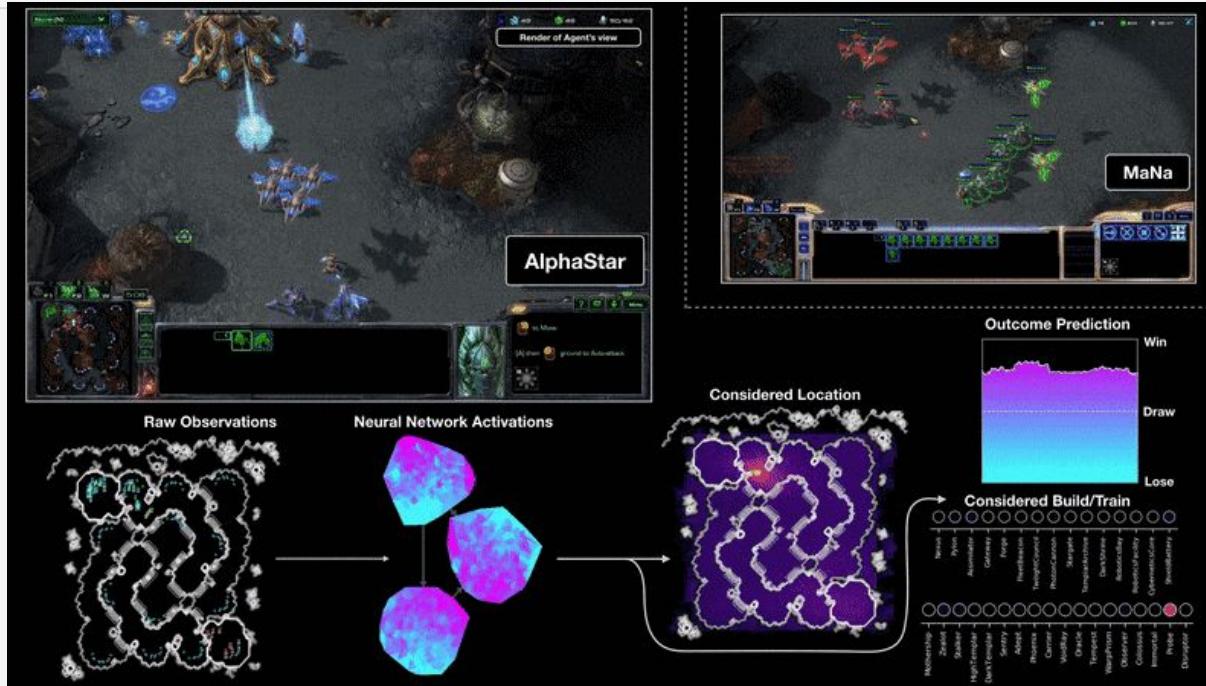
[Liu et al. '19](#) and http://git.io/dm_soccer

Capture-the-Flag (Jaderberg et al. '19)



<https://deepmind.com/blog/capture-the-flag-science/>

AlphaStar (Vinyals et al. '19)



<https://deepmind.com/blog/alphastar-mastering-real-time-strategy-game-starcraft-ii/>

Dota 2: OpenAI Five



<https://openai.com/blog/openai-five-finals/>

Deep Multiagent RL Survey

Is multiagent deep reinforcement learning the answer or the question? A brief survey

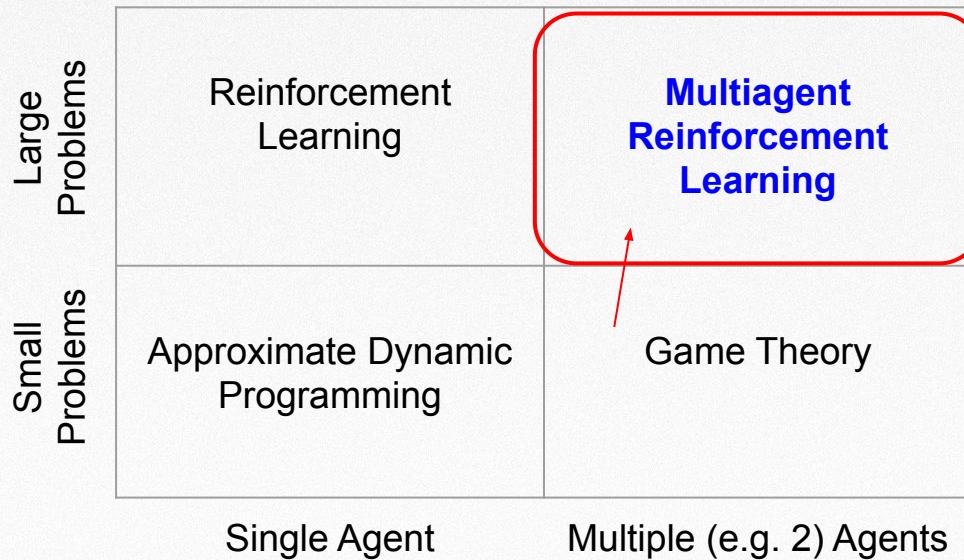
Pablo Hernandez-Leal , Bilal Kartal and Matthew E. Taylor
`{pablo.hernandez,bilal.kartal,matthew.taylor}@borealisai.com`

*Borealis AI
University of Alberta CCIS 3-232
Edmonton, Canada*

<https://arxiv.org/abs/1810.05587>

Part 4: Partial Observability

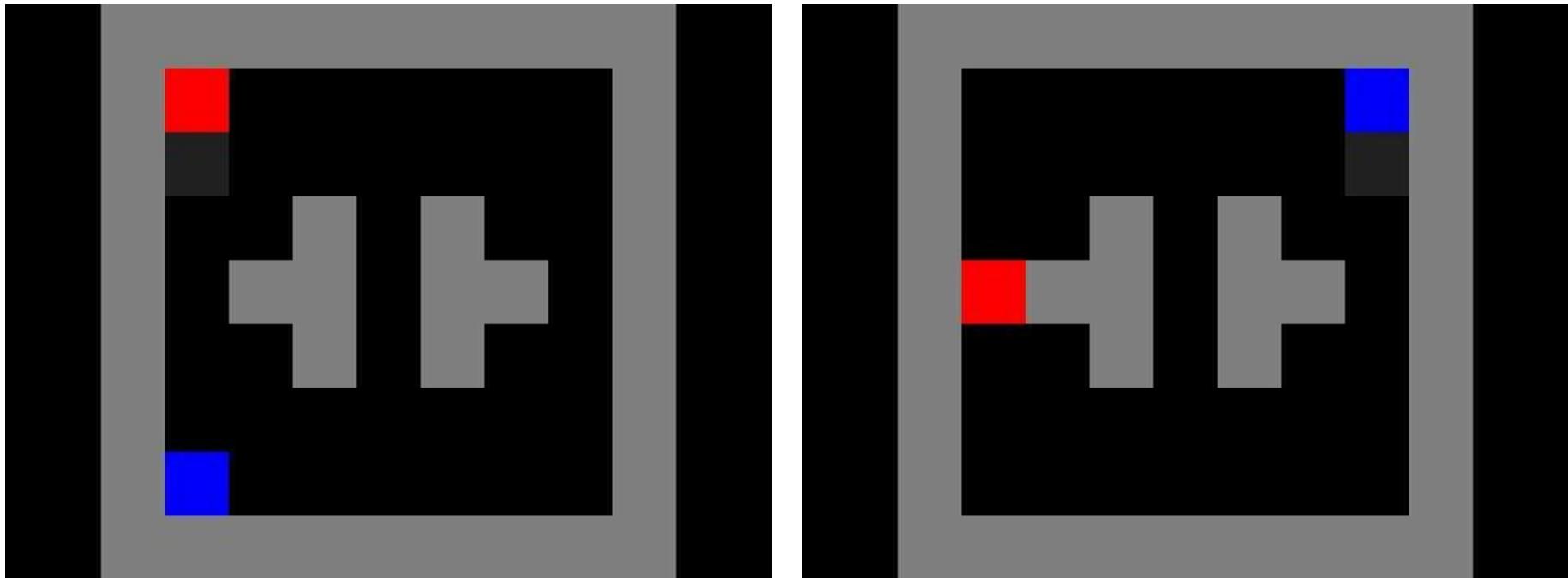
Foundations of Multiagent RL



Independent Deep Q-networks

(Mnih et al. '15)

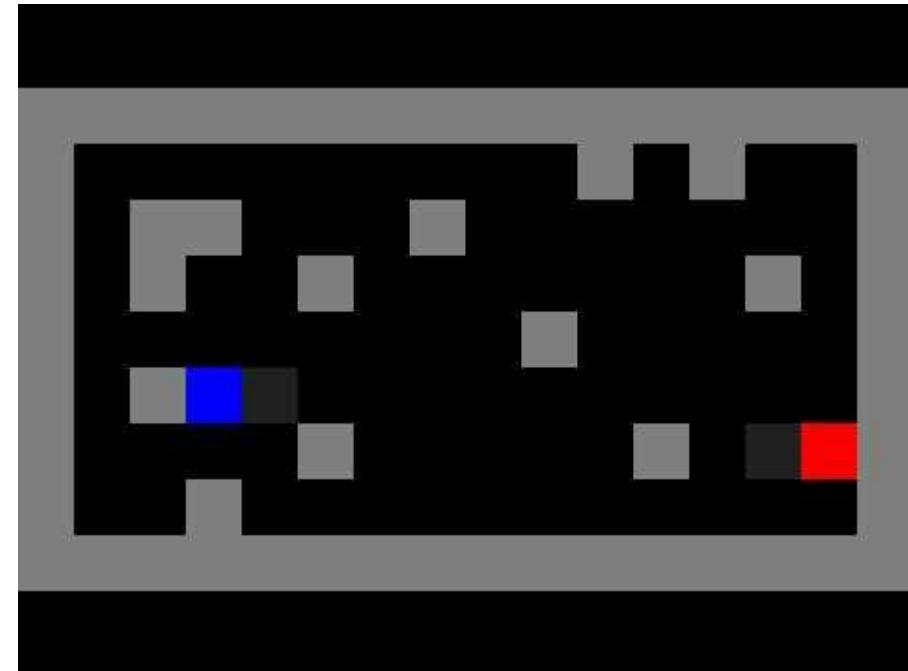
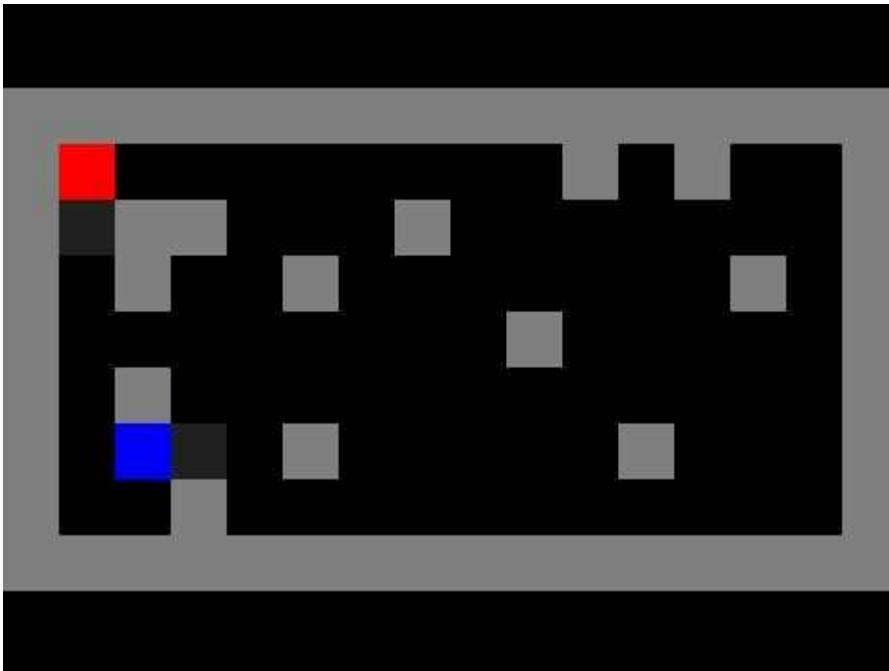
"small2" map



Independent Deep Q-networks

(Mnih et al. '15)

"small3" map



Fictitious Self-Play [Heinrich et al. '15, Heinrich & Silver 2016]

- **Idea:** Fictitious self-play (FSP) + reinforcement learning
- Update rule in sequential setting *equivalent* to standard fictitious play (matrix game)
- Approximate NE via two neural networks:

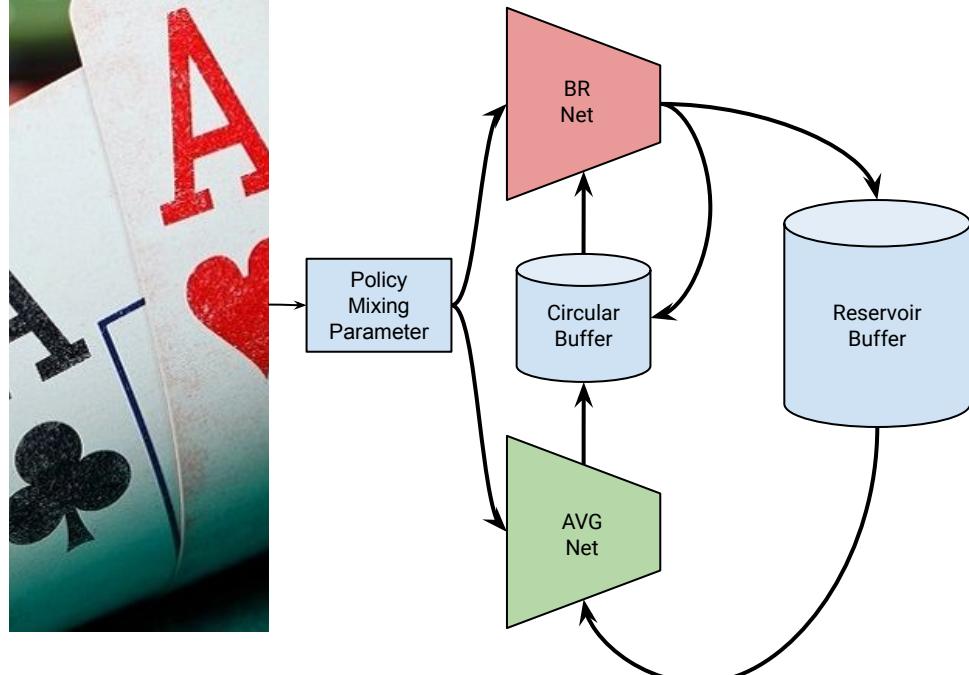
1. Best response net (BR):

- Estimate a best response
- Trained via RL



2. Average policy net (AVG):

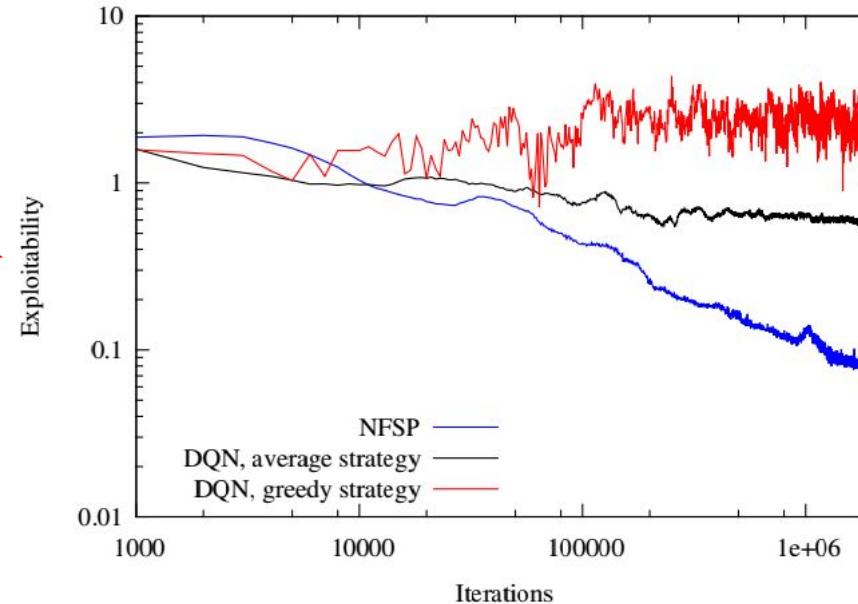
- Estimate the time-average policy
- Trained via supervised learning



Neural Fictitious Self-Play [Heinrich & Silver 2016]

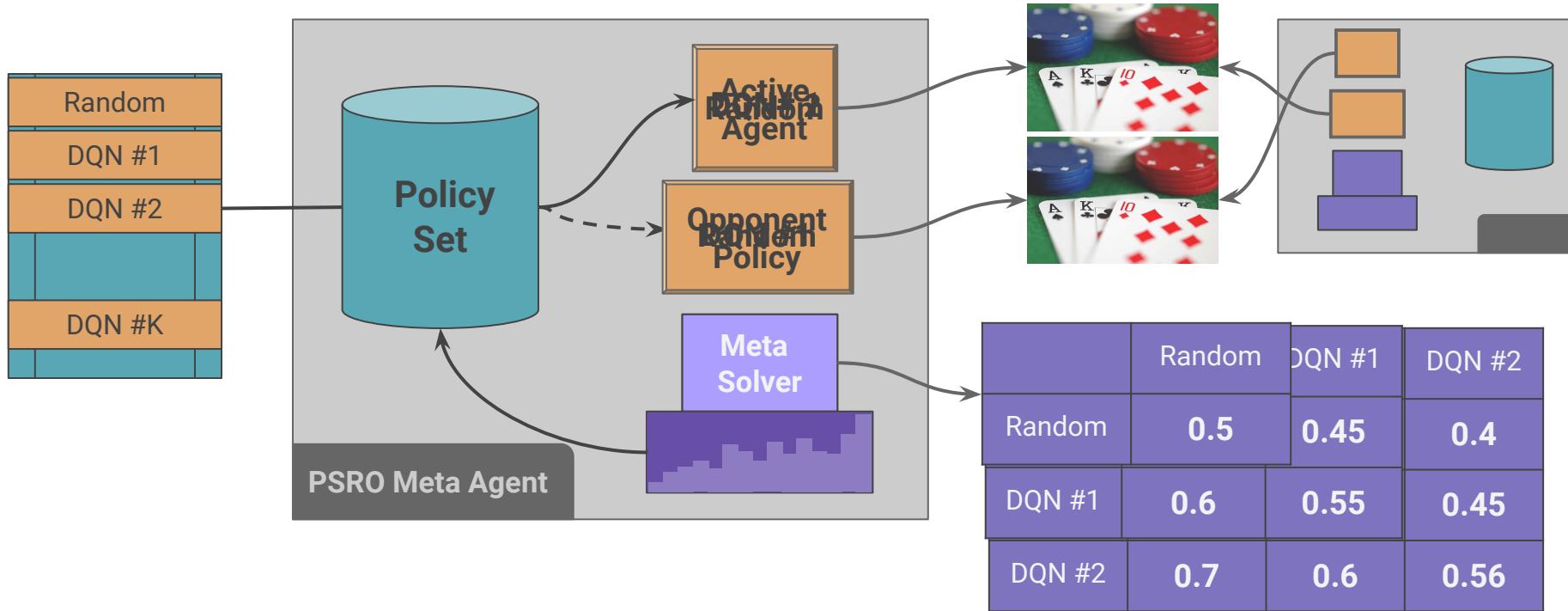
- Leduc Hold'em poker experiments:

“Closeness” to Nash



- 1st scalable end-to-end approach to learn **approximate Nash equilibria w/o prior domain knowledge**
 - Competitive with superhuman computer poker programs when it was released

Policy-Space Response Oracles (Lanctot et al. '17)



Quantifying “Joint Policy Correlation”

In RL:

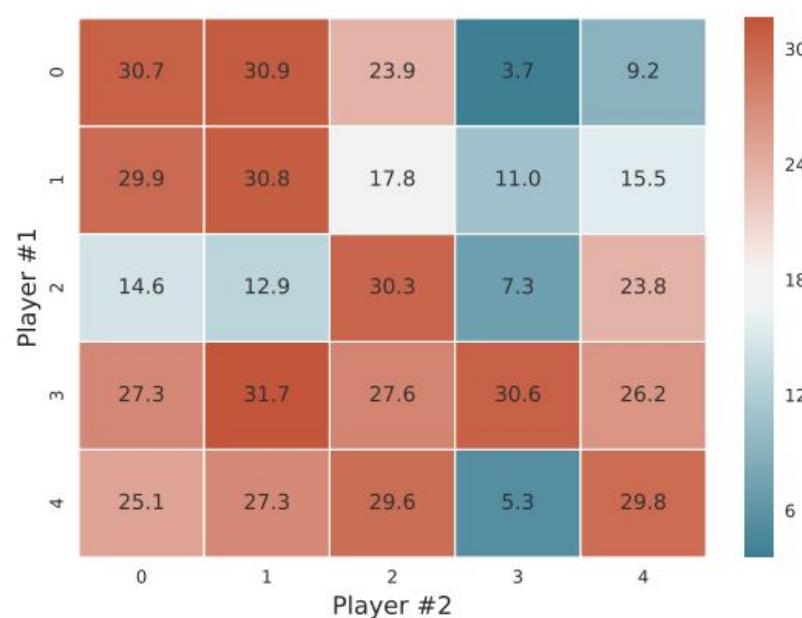
- Each player uses optimizes independently
- After many steps, joint policy (π_1, π_2) co-learned for players 1 & 2

Computing **JPC**: start **5 separate instances of the same experiment**, with

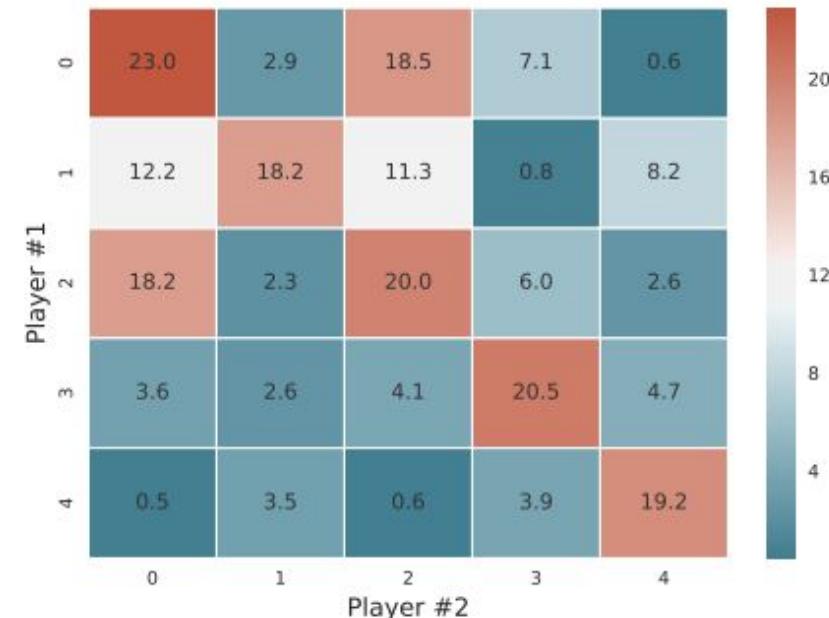
- Same hyper-parameter values
- Differ **only by seed** (!)
- Reload all 25 combinations and play π_1^i with π_2^j for instances i, j

Joint Policy Correlation in Independent RL

InRL in small2 (first) map



InRL in small4 map



JPC Results - Laser Tag

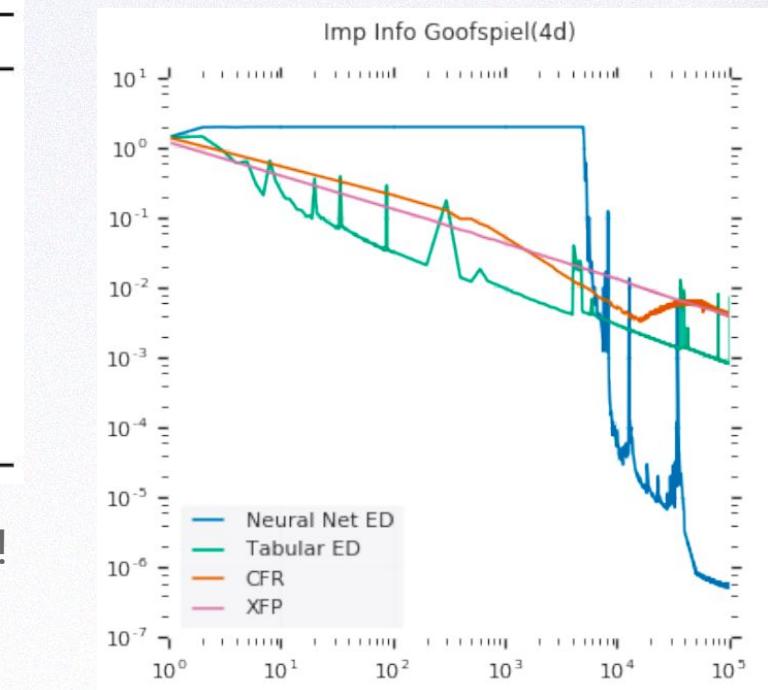
Game	Diag	Off Diag	Exp. Loss
LT small2	30.44	20.03	34.2 %
LT small3	23.06	9.06	62.5 %
LT small4	20.15	5.71	71.7 %
Gathering field	147.34	146.89	none
Pathfind merge	108.73	106.32	none

Exploitability Descent [\(Lockhart et al. '19\)](#)

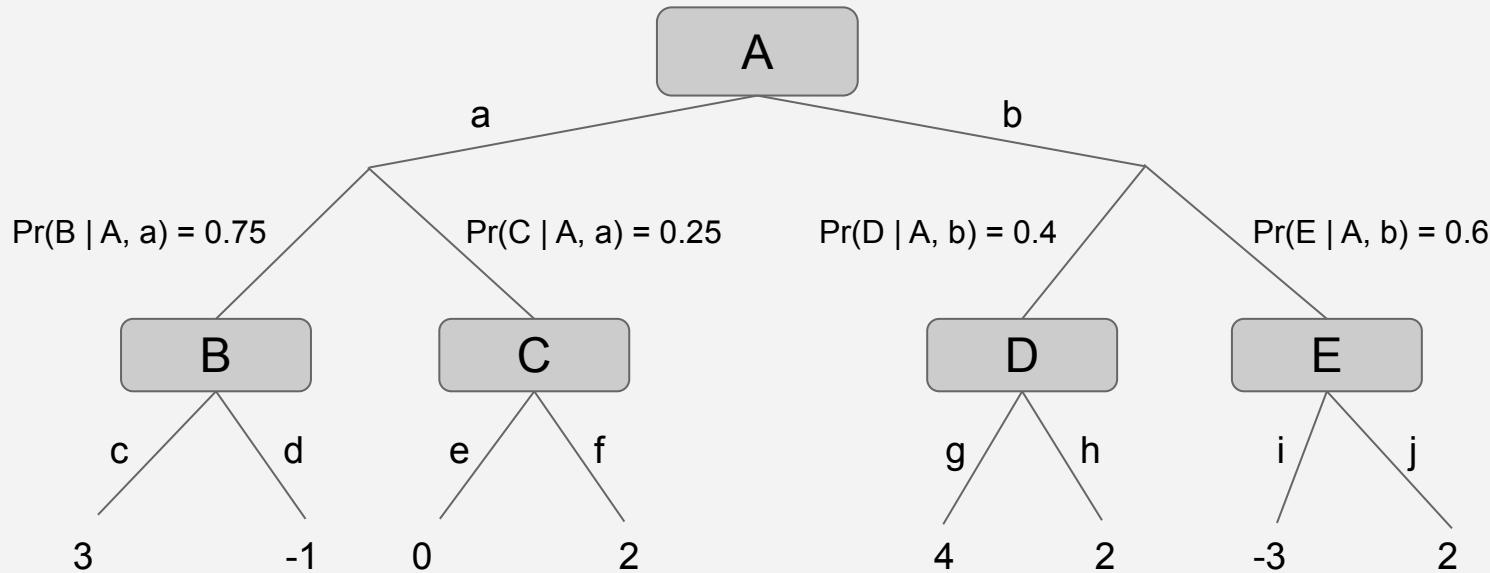
Algorithm 2: Exploitability Descent (ED)

```
input :  $\pi^0$  — initial joint policy
1 for  $t \in \{1, 2, \dots\}$  do
2   for  $i \in \{1, \dots, n\}$  do
3     Compute a best response  $b_i^t(\pi_{-i}^{t-1})$ 
4     for  $i \in \{1, \dots, n\}, s \in \mathcal{S}_i$  do
5       Define  $b_{-i}^t = \{b_j^t\}_{j \neq i}$ 
6       Let  $\mathbf{q}^b(s) = \text{VALUESVSBRS}(\pi_i^{t-1}(s), b_{-i}^t)$ 
7        $\pi_i^t(s) = \text{GRADASCENT}(\pi_i^{t-1}(s), \alpha^t, \mathbf{q}^b(s))$ 
```

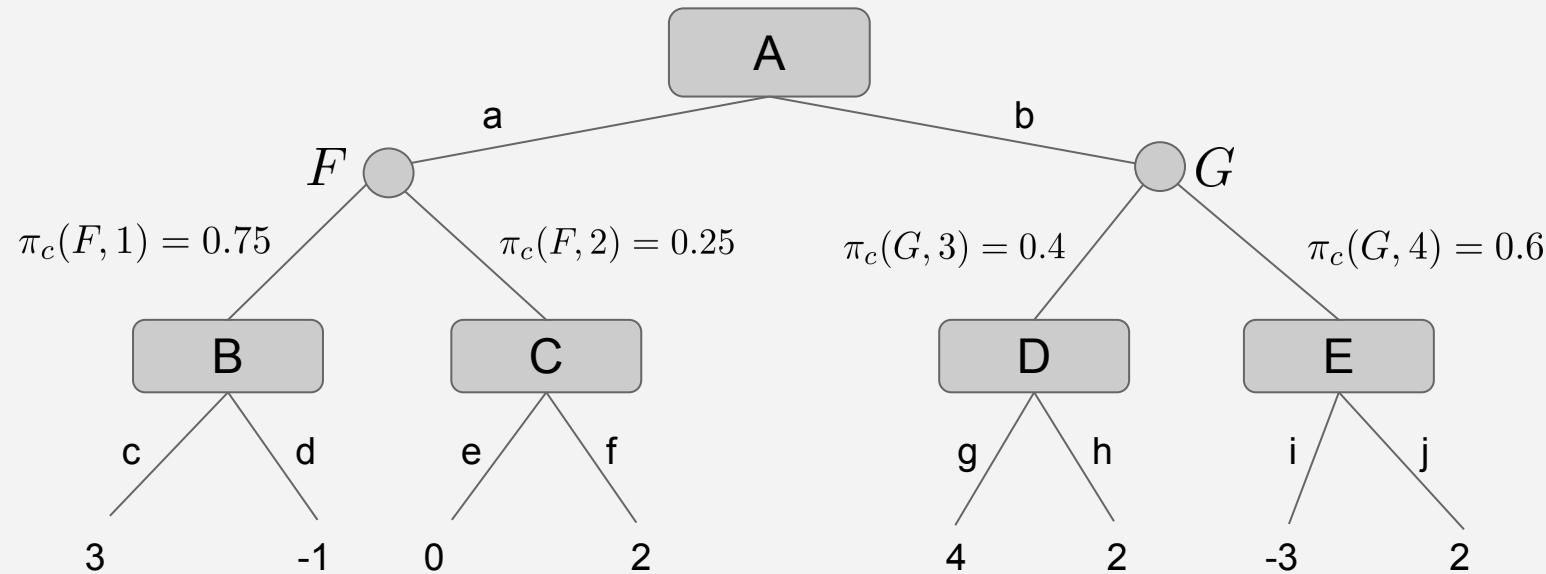
- A FP-like algorithm conv. *without averaging!*
- Amenable to function approximation



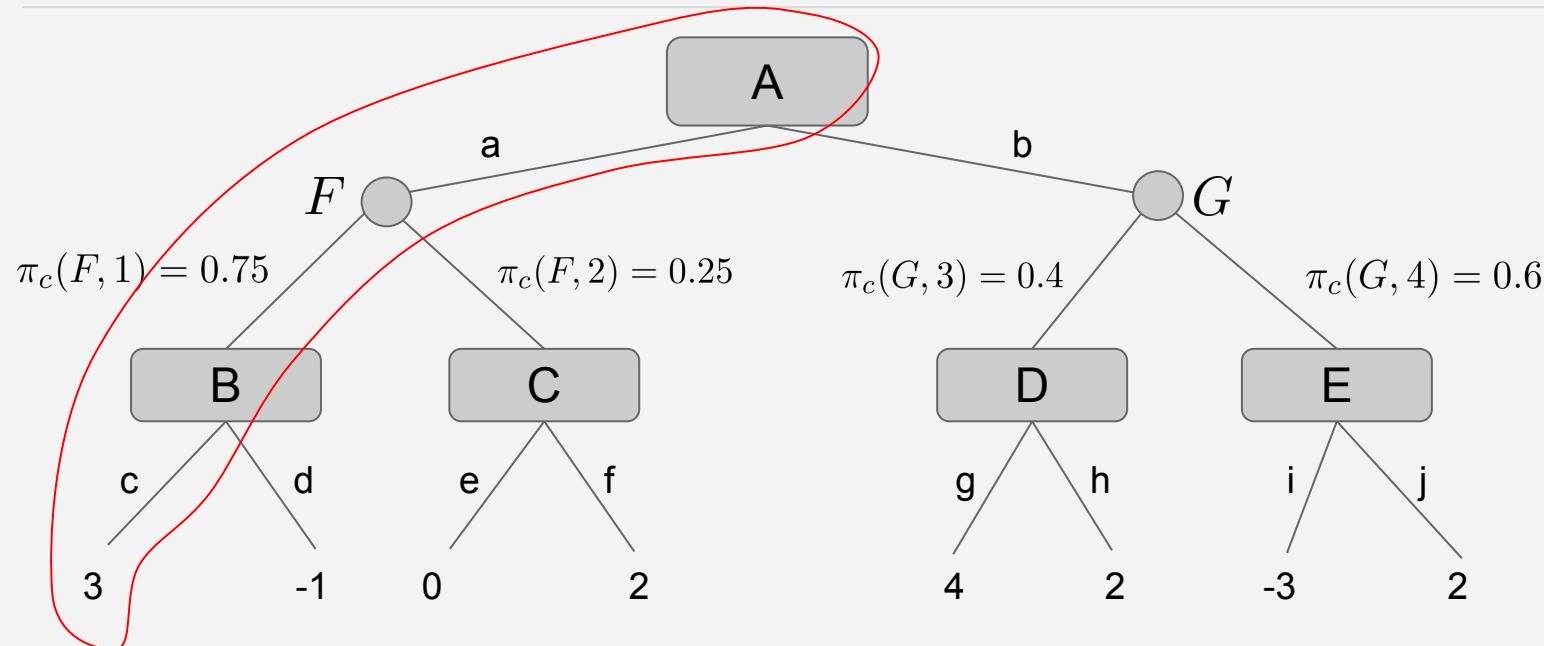
A simple MDP



A simple MDP Multiagent System

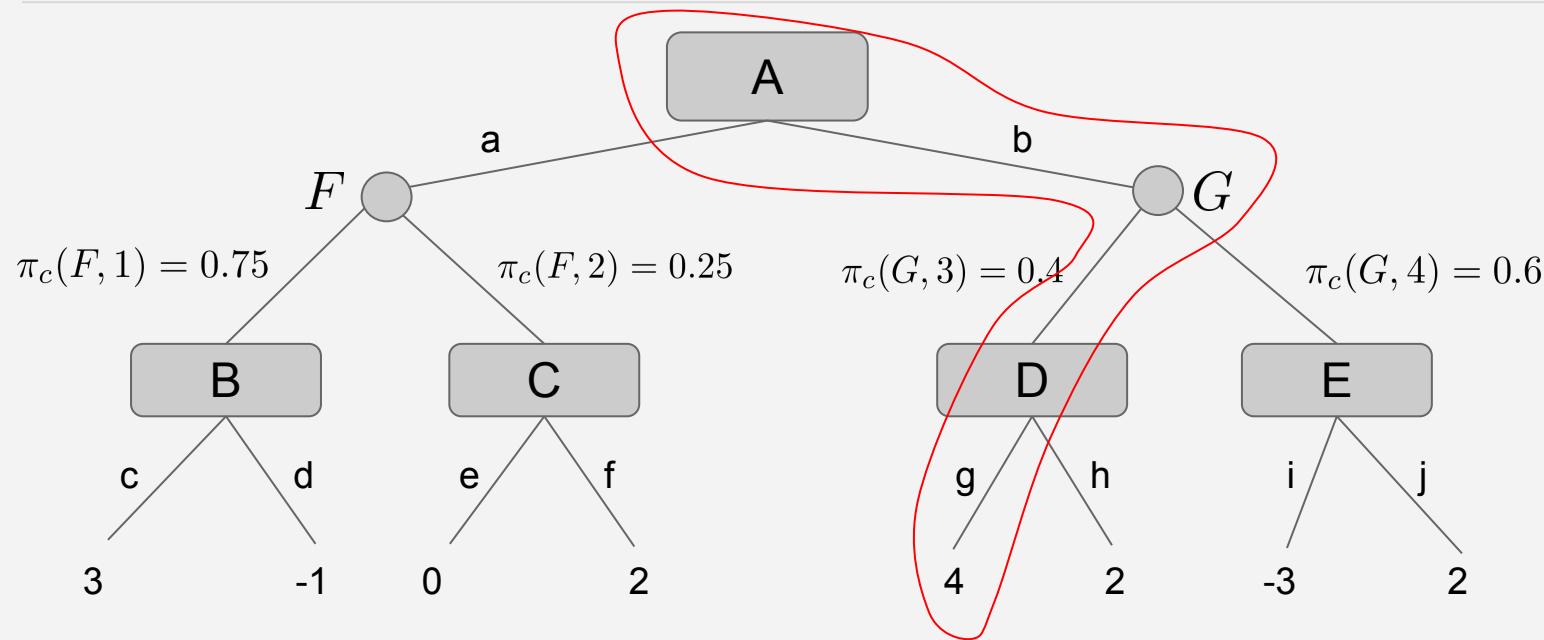


Terminal history A.K.A. Episode



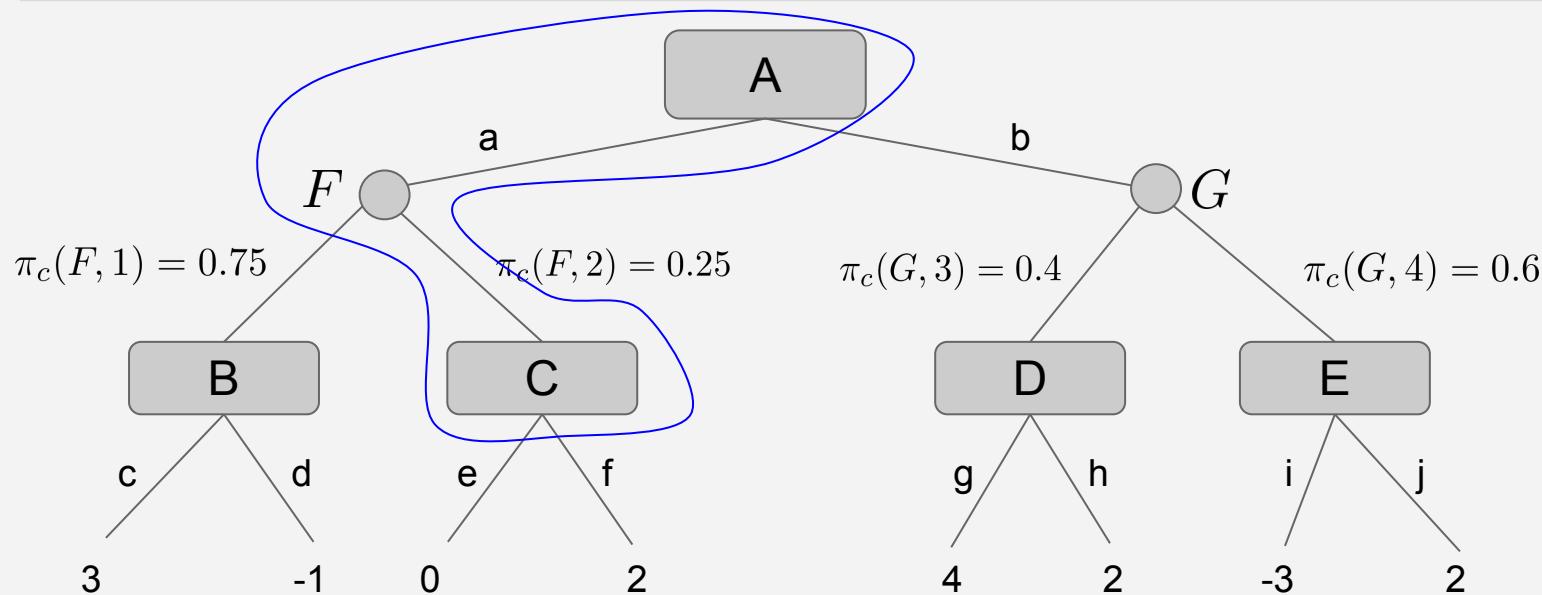
(A, a, F, 1, B, c) is a terminal history.

Terminal history A.K.A. Episode



(A, a, F, 1, B, c) is a *terminal history*. **(A, b, G, 3, D, g)** is another terminal history.

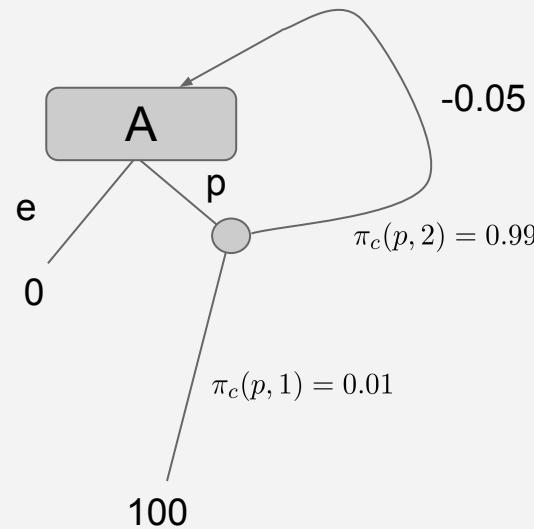
Prefix (non-terminal) Histories



(**A, a, F, 2, C**) is a history. It is a *prefix* of (**A, a, F, 2, C, e**) and (**A, a, F, 2, C, f**).

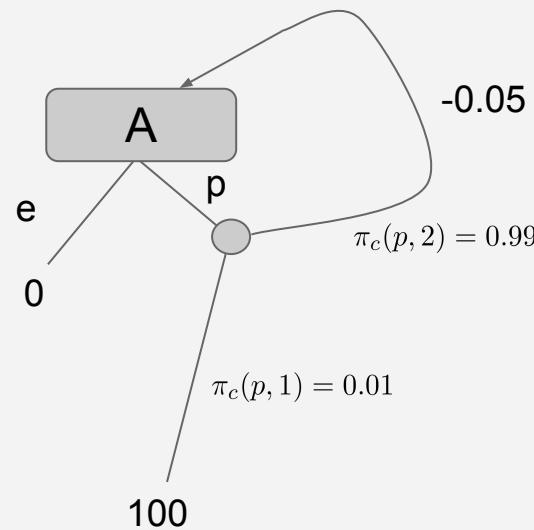
Perfect Recall of Actions and Observations

Another simple MDP:

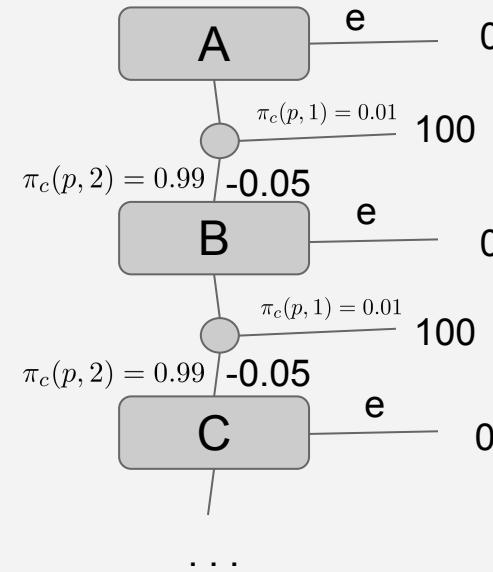


Perfect Recall of Actions and Observations

Another simple MDP:



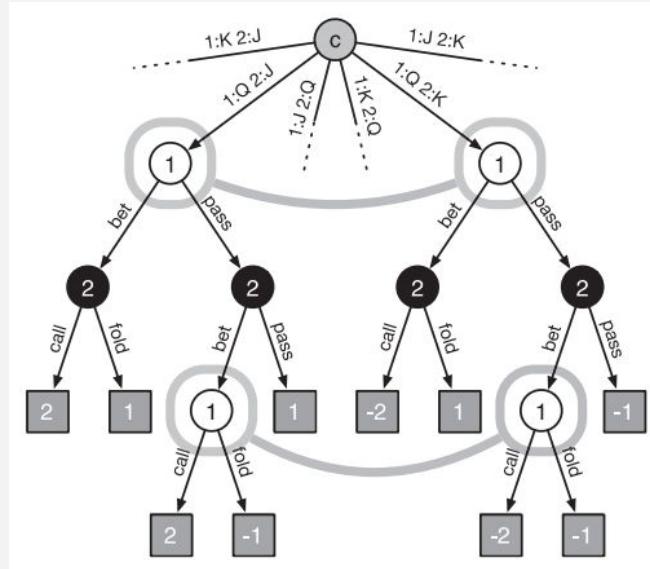
A different MDP:



Partially Observable Environment

An **information state** is a set of histories consistent with an agent's observations.

3-card Poker deck:
Jack, Queen, King



Terminology

What is a “state”?

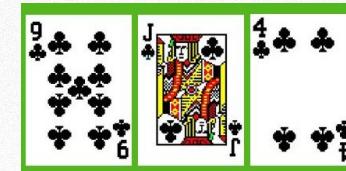
- An **information state** s corresponds to sequence of *observations*
 - with respect to the player to act at s

Example information state in Poker:

Ante: 1 chip per player,

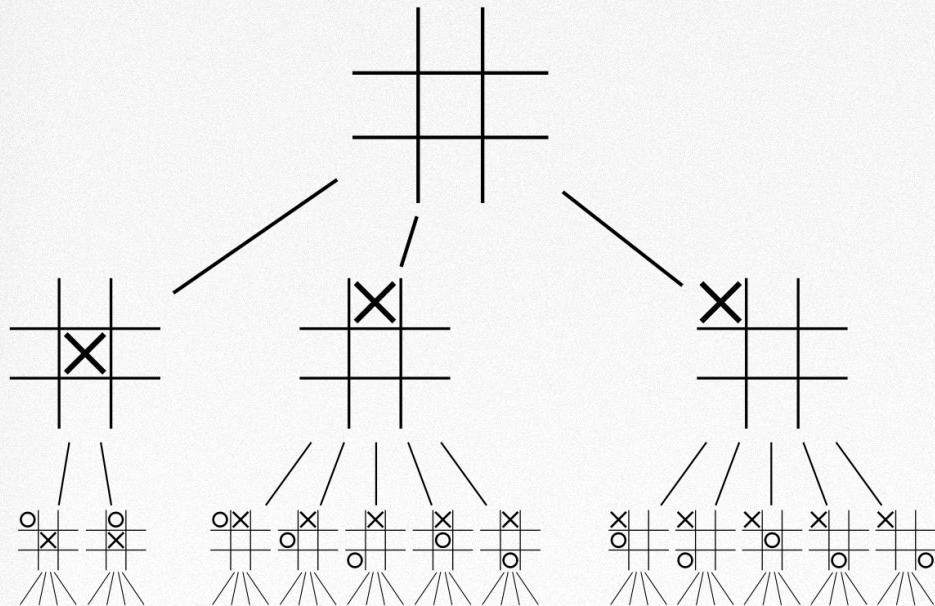


, P1 bets 1 chip, P2 calls,



Environment is in one of many **world/ground states** $h \in s$

Recall: Turn-Taking Perfect Information Games

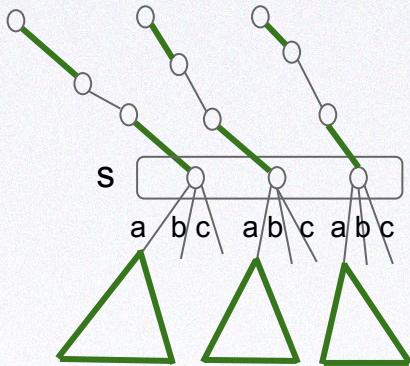


→ Exactly one history per information state!

$\{Q, V\}$ -values and Counterfactual Values

What..... is a counterfactual value?

$$v_i^c(\pi, s, a)$$

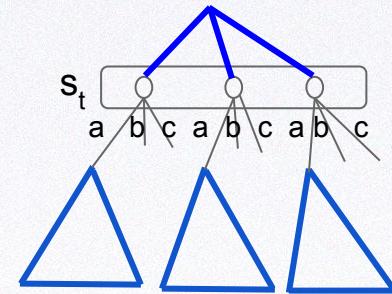


The portion of the expected return (under s) from the start state, given that:

player i plays to reach information state s (then plays a).

Q-values in Partially Observable Environments

What..... is a q-value?



$$q_{\pi,i}(s_t, a_t) = \mathbb{E}_{\rho \sim \pi}[G_t \mid S_t = s_t, A_t = a_t]$$

Q-values in Partially Observable Environments

$$= \sum_{h, z \in \mathcal{Z}(s_t, a_t)} \Pr(h \mid s_t) \eta^\pi(ha, z) u_i(z)$$

All terminal histories z reachable from s , paired with their prefix histories ha , where h is in s

Reach probabilities: product of all policies' state-action probabilities along the portion of the history between ha and z

Return achieved over terminal history z

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\Pr(s_t \mid h) \Pr(h)}{\Pr(s_t)} \eta^\pi(ha, z) u_i(z)$$

By Bayes rule

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\Pr(h)}{\Pr(s_t)} \eta^\pi(ha, z) u_i(z)$$

Since h is in s_t and h is unique to s_t

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\eta^\pi(h)}{\sum_{h' \in s_t} \eta^\pi(h')} \eta^\pi(ha, z) u_i(z)$$

Q-values in Partially Observable Environments

$$\sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\eta_i^\pi(h) \eta_{-i}^\pi(h)}{\sum_{h' \in s_t} \eta_i^\pi(h') \eta_{-i}^\pi(h')} \eta^\pi(ha, z) u_i(z)$$

Only player i's reach probabilities

Player i's opponent's probabilities (*inc. chance!*)

Similarly here

and here

The diagram illustrates the components of a Q-value update in a partially observable environment. It shows the formula for calculating the Q-value of a state-action pair (ha, z) based on the sum of all possible histories h and observations z given the current state s_t and action a_t . The formula is weighted by the product of the player's own reach probabilities $\eta_i^\pi(h)$ and the opponent's probabilities $\eta_{-i}^\pi(h)$, normalized by the total number of histories h' in the state space s_t . Arrows from text labels point to the terms $\eta_i^\pi(h)$, $\eta_{-i}^\pi(h)$, $\eta_i^\pi(h')$, and $\eta_{-i}^\pi(h')$.

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\eta_i^\pi(h) \eta_{-i}^\pi(h)}{\eta_i^\pi(h) \sum_{h' \in s_t} \eta_{-i}^\pi(h')} \eta^\pi(ha, z) u_i(z)$$

Due to perfect recall (!!)

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\eta_{-i}^\pi(h)}{\sum_{h' \in s_t} \eta_{-i}^\pi(h')} \eta^\pi(ha, z) u_i(z)$$

Q-values in Partially Observable Environments

$$= \sum_{h,z \in \mathcal{Z}(s_t, a_t)} \frac{\eta_{-i}^\pi(h)}{\sum_{h' \in s_t} \eta_{-i}^\pi(h')} \eta^\pi(ha, z) u_i(z)$$

This is a counterfactual value!

Q-values in Partially Observable Environments

$$= \frac{1}{\sum_{h \in s_t} \eta_{-i}^\pi(h)} v_i^c(\pi, s_t, a_t)$$

$$= \frac{1}{\mathcal{B}_{-i}(\pi, s_t)} v_i^c(\pi, s_t, a_t)$$

For full derivation, see Sec 3.2 of [Srinivasan et al. '18](#)

Yeah.. so.... ?

\(ツ)

Counterfactual Regret Minimization (CFR)

Zinkevich et al. '08

- Algorithm to compute approx Nash eq. In 2P zero-sum games
- **Hugely successful in Poker AI**
- Size traditionally reduced apriori based on expert knowledge
- **Key innovation: counterfactual values:** $v_i^c(\pi, s, a)$ $v_i^c(\pi, s)$

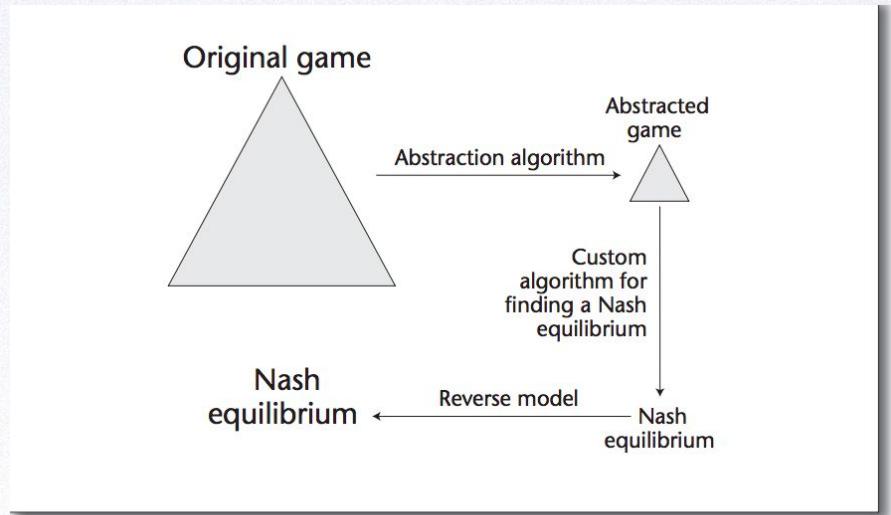


Figure 1. Current Paradigm for Solving Large Incomplete-Information Games.

Image form Sandholm '10

CFR is policy iteration!

- Policy evaluation is analogous
- Policy improvement: use regret minimization algorithms
 - Average strategies converge to Nash in self-play
- Convergence guarantees are on the *average policies*



DeepStack

(Moravcik et al. '17)

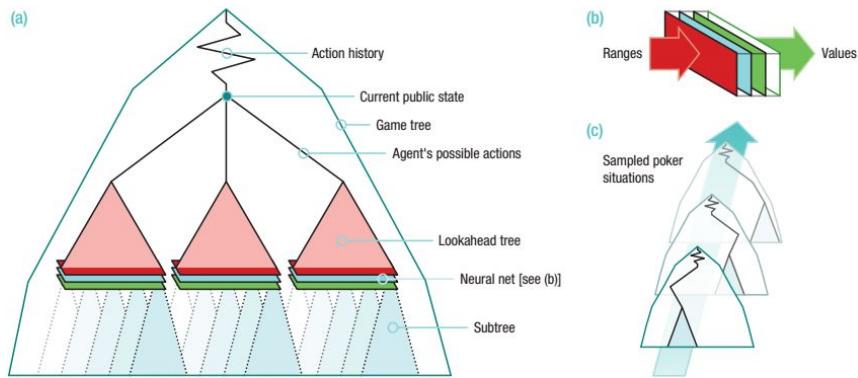
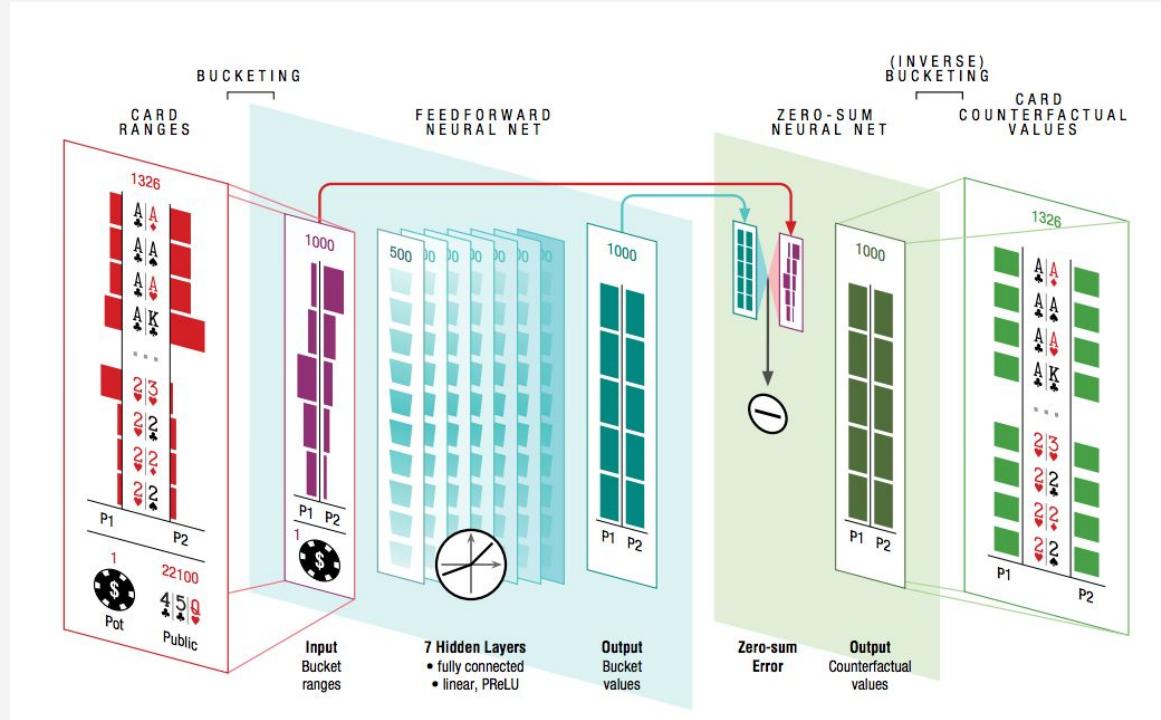


Figure 2: DeepStack overview. (a) DeepStack re-solves for its action at every public state it is to act, using a depth limited lookahead where subtree values are computed using a trained deep neural network (b) trained before play via randomly generated poker situations (c).



DeepStack

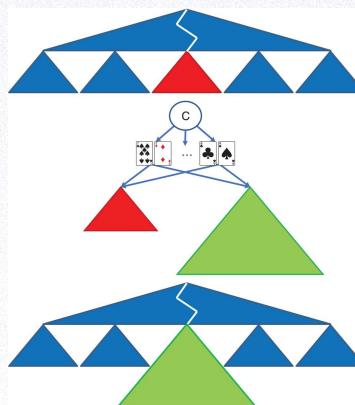
(Moravcik et al. '17)



Libratus ([Brown & Sandholm '18](#))

RESEARCH ARTICLE

Superhuman AI for heads-up no-limit poker: Libratus beats top professionals



Policy Gradient Algorithms

Parameterized policy π_θ with parameters θ (e.g. a neural network)

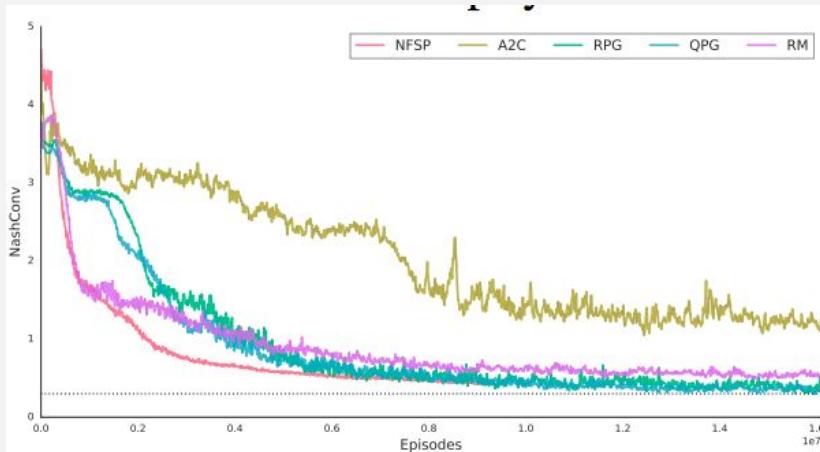
Define a score function $J(\pi_\theta) = v_\pi(s_0) = \mathbb{E}_\pi[G_0]$

Main idea: do gradient ascent on J.

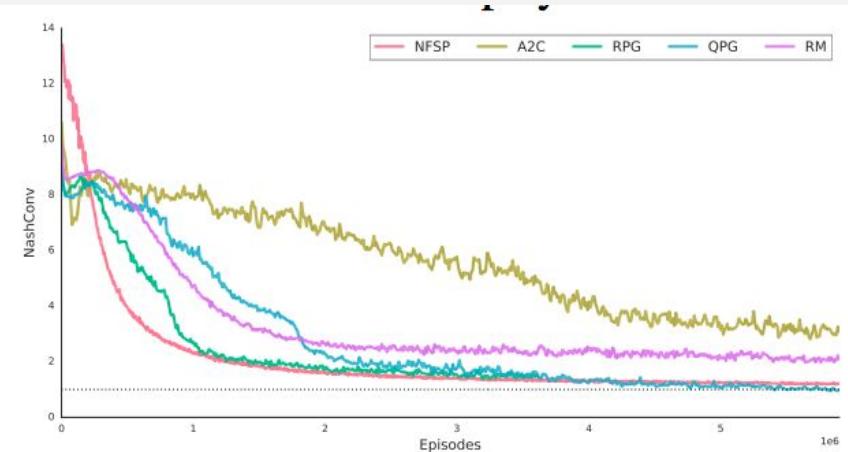
1. **REINFORCE** (Williams '92, see RL book ch. 13) + PG theorem: you can do this via estimates from sample trajectories.
2. **Advantage Actor-Critic (A2C)** (Mnih et al '16): you can use deep networks to estimate the policy and baseline value $v(s)$

Regret Policy Gradients [\(Srinivasan et al. '18\)](#)

- Policy gradient is doing a form of CFR minimization!
- Several new policy gradient variants inspired connection to regret



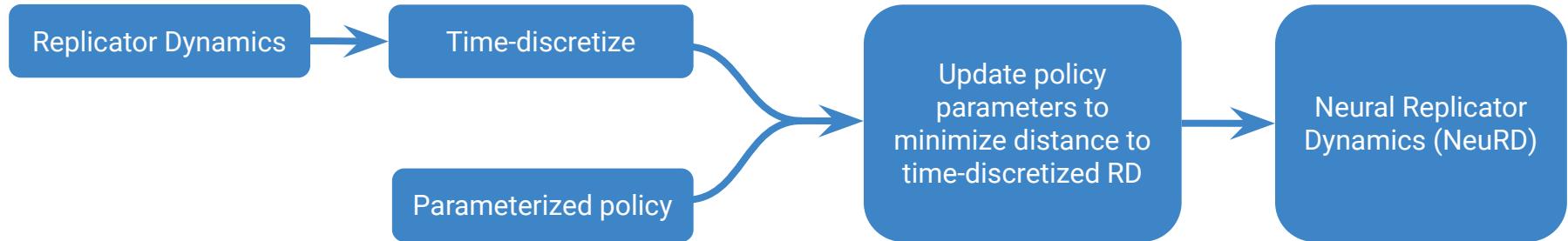
NASHCONV in 2-player Leduc



NASHCONV in 3-player Leduc

Neural Replicator Dynamics (NeuRD)

[Omidshafiei, Hennes, Morrill et al. '19](#)



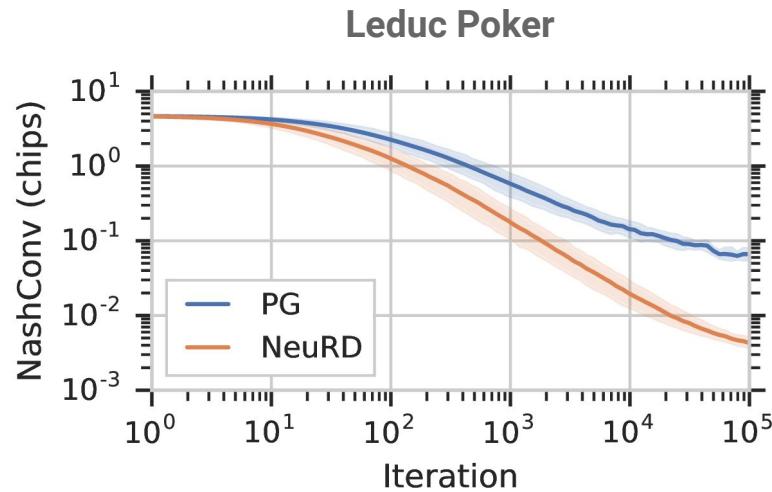
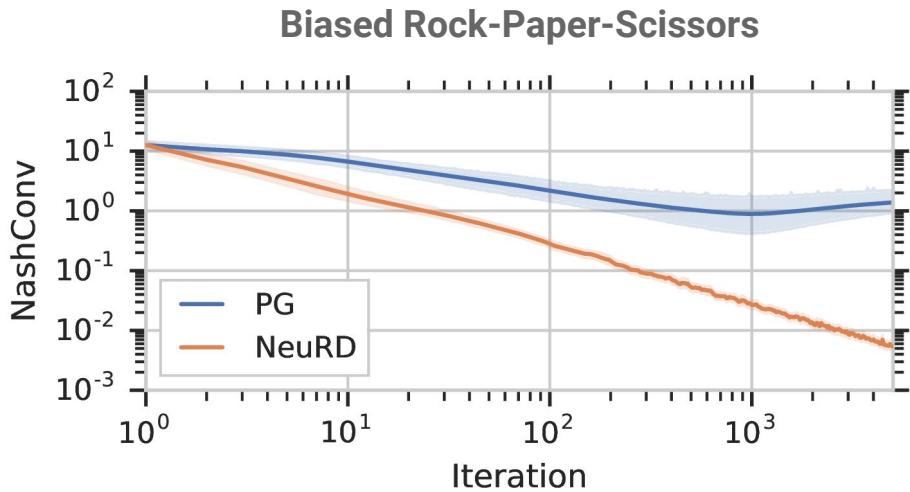
$$\theta_t = \theta_{t+1} + \eta \sum_{s,a} \nabla_{\theta} y_{t-1}(s_t, a_t; \theta) A(s_t, a_t; \theta, w)$$

Logits, where policy is
 $\pi = \text{softmax}(y)$

Advantage $q(s,a)-v(s)$

The diagram shows the calculation of the gradient term in the update equation. It consists of two main parts: a blue bracket labeled "Logits, where policy is $\pi = \text{softmax}(y)$ " and a red bracket labeled "Advantage $q(s,a)-v(s)$ ". These two components are combined by a horizontal line that splits into two vertical lines, one for each bracket, which then meet at a single point below the equation.

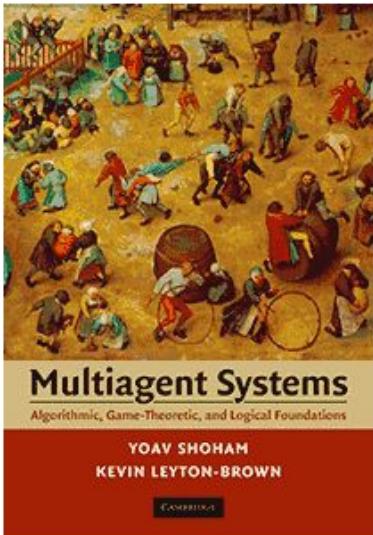
NeuRD: Results



Where to Go From Here?

Shoham & Leyton-Brown '09

[Main Page](#) [Table of Contents](#) [Instructional Resources](#) [Errata](#) [eBook Download](#) new!



Multiagent Systems Algorithmic, Game-Theoretic, and Logical Foundations

Yoav Shoham
Stanford University
Kevin Leyton-Brown
University of British Columbia

Cambridge University Press, 2009
Order online: [amazon.com](#).

masfoundations.org

Surveys and Food for Thought

- If multi-agent learning is the answer, what is the question?
 - Shoham et al. '06
 - Hernandez-Leal et al. '19
- A comprehensive survey of MARL (Busoniu et al. '08)
- Game Theory and Multiagent RL (Nowé et al. '12)
- Study of Learning in Multiagent Envs (Hernandez-Leal et al. '17)

The Hanabi Challenge

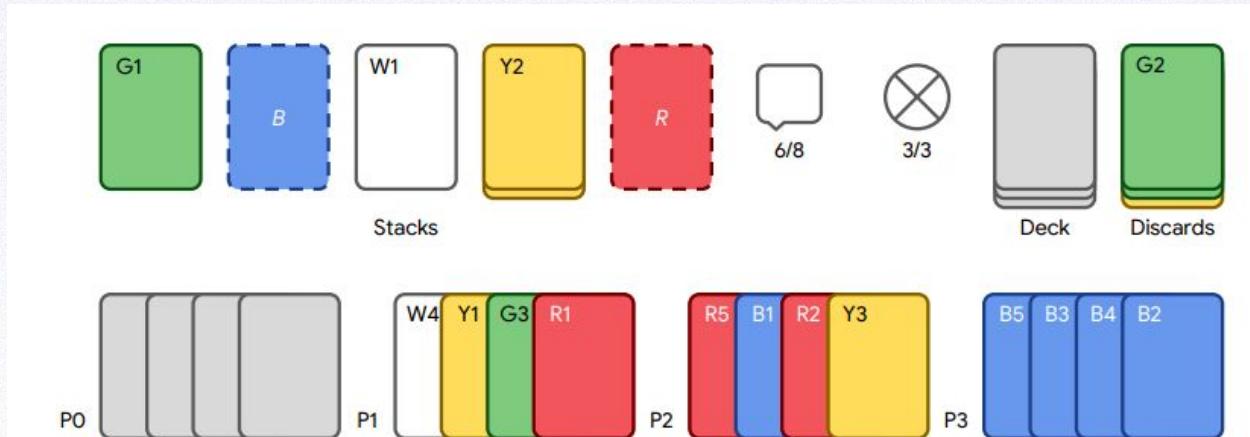


Figure 1: Example of a four player Hanabi game from the point of view of player 0. Player 1 acts after player 0 and so on.

[Bard et al. '19](#)

Also Competition at IEEE Cog (ieee-cog.org)

OpenSpiel: Coming Soon!

- Open source framework for research in RL & Games
- C++, Python, and Swift impl's
- 25+ games
- 10+ algorithms
- Tell all your friends! (Seriously!)

→ August 2019

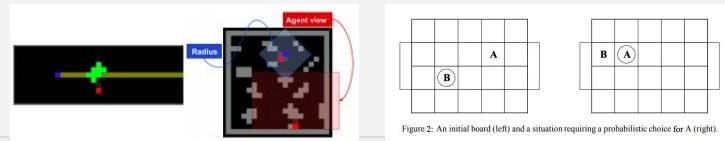
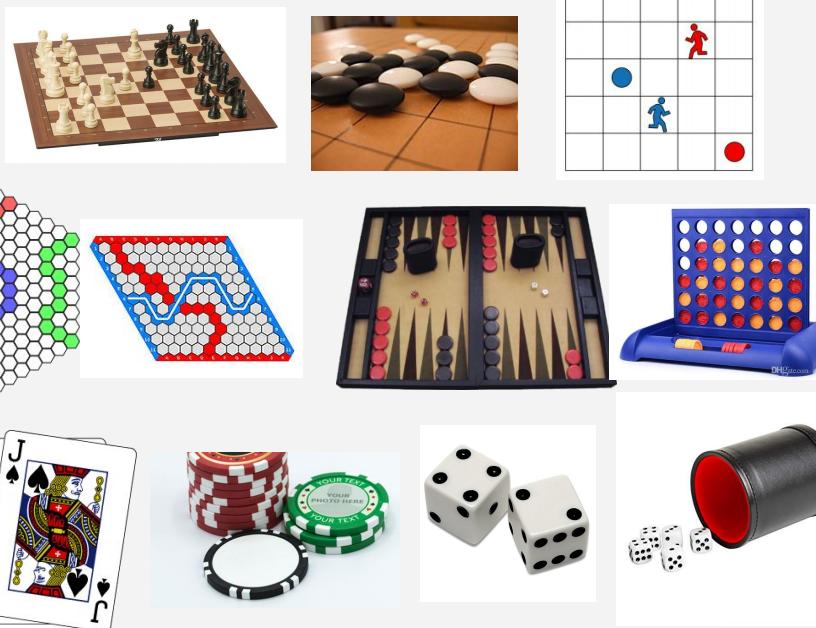


Figure 2: An initial board (left) and a situation requiring a probabilistic choice for A (right).



AAAI 2020 Workshop on RL in Games?



AAAI19-RLG Summary:

- 39 accepted papers
 - 4 oral presentations
 - 35 posters
- 1 “Mini-Tutorial”
- 3 Invited Talks
- Panel & Discussion

<http://aaai-rlg.mlanctot.info/>

Questions?

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