McCulloch-Pitts Neuron

Neuroinformatics Tutorial 2

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Content

- Revision
- Biological Motivation for ANNs
- McCulloch-Pitts Neuron
- Tasks

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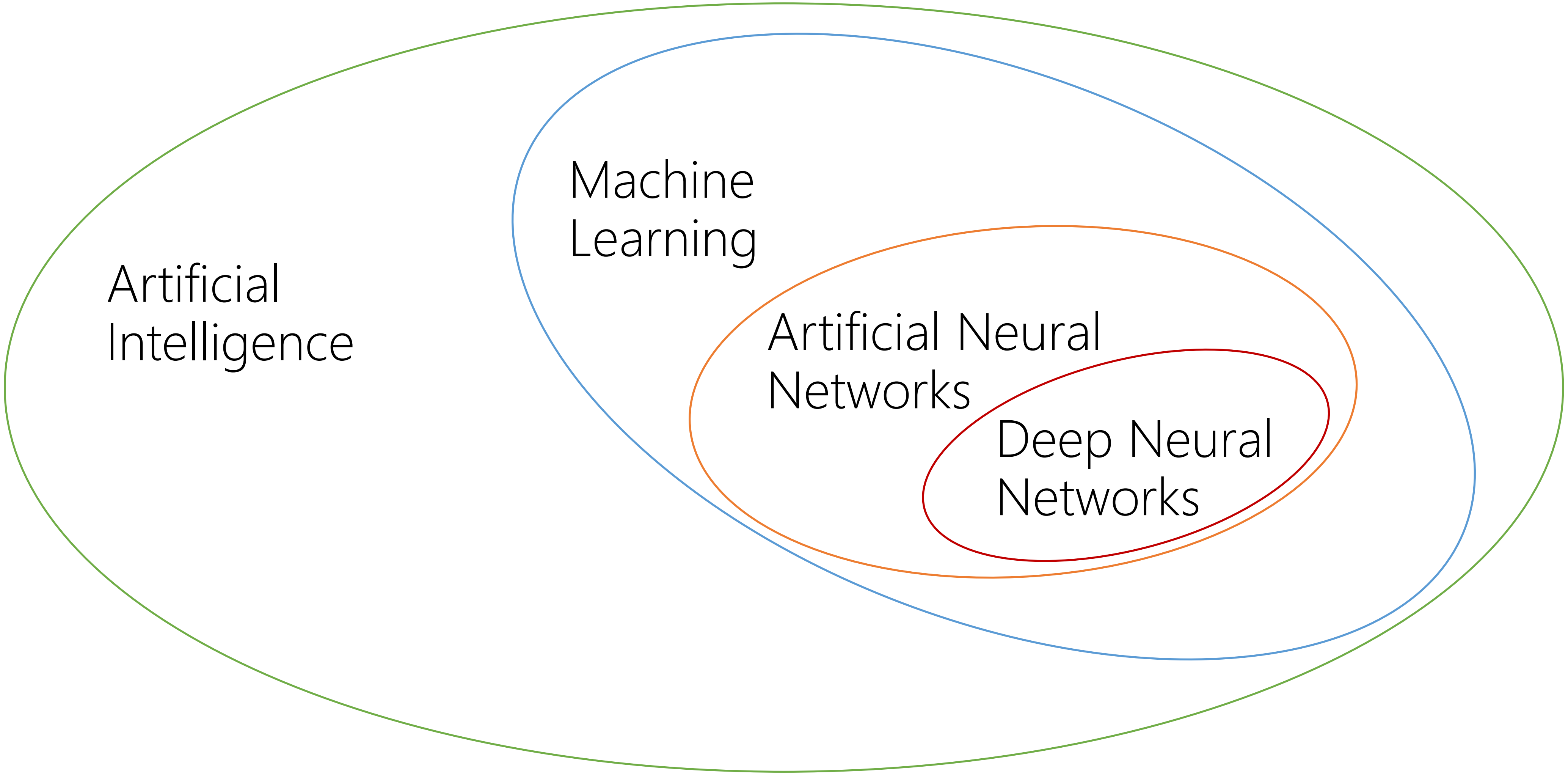
Relation to AI



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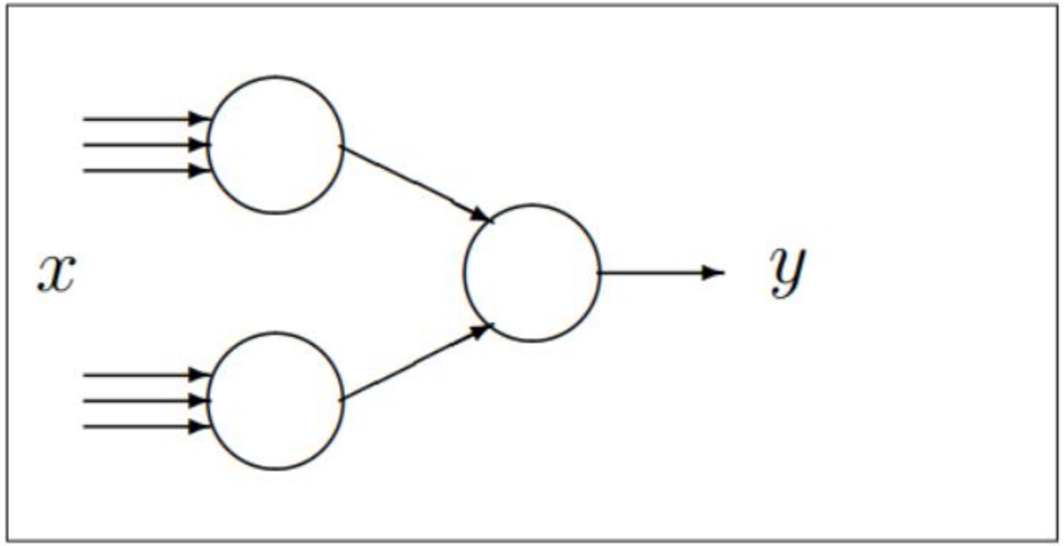

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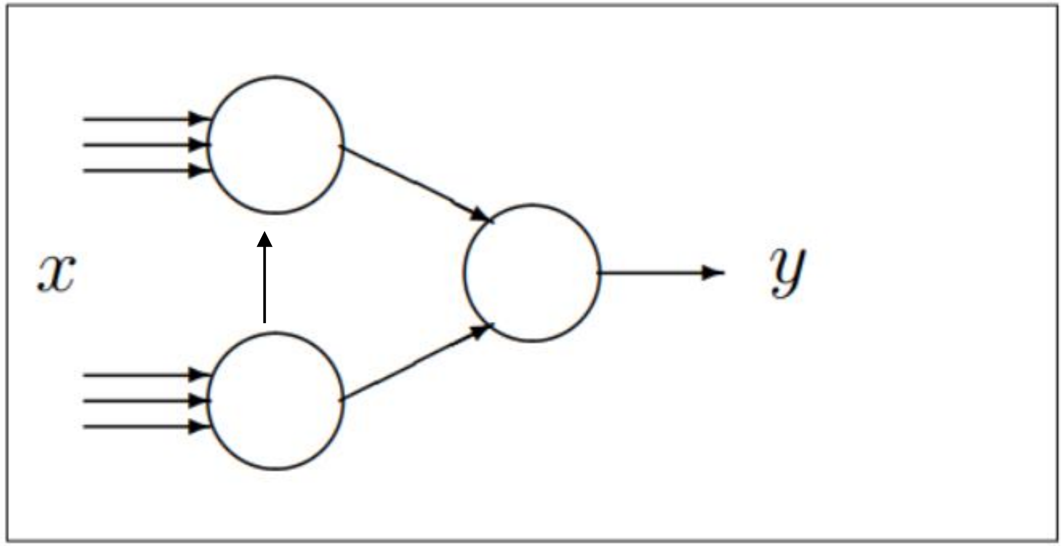

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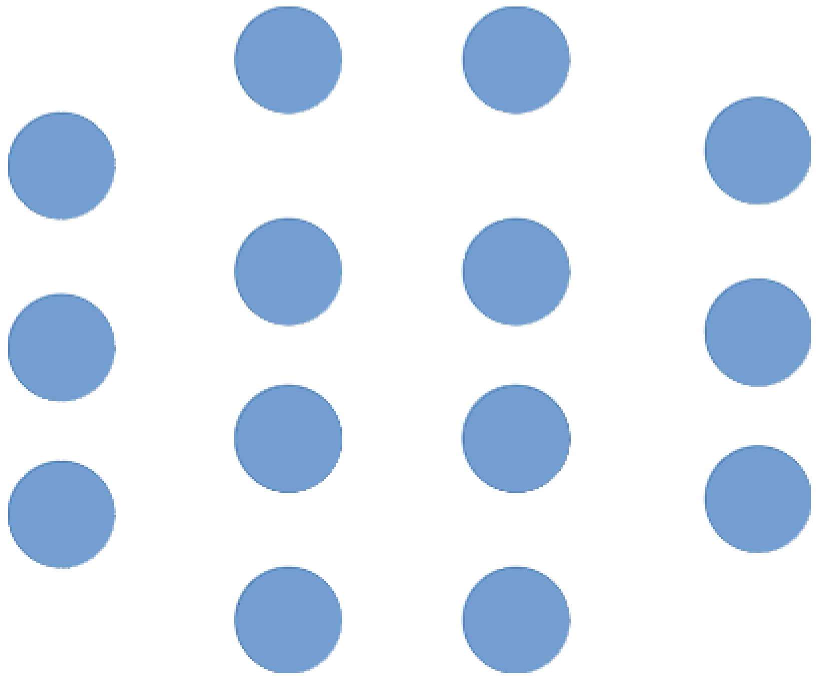
$\mathcal{K}$: Knotenmenge



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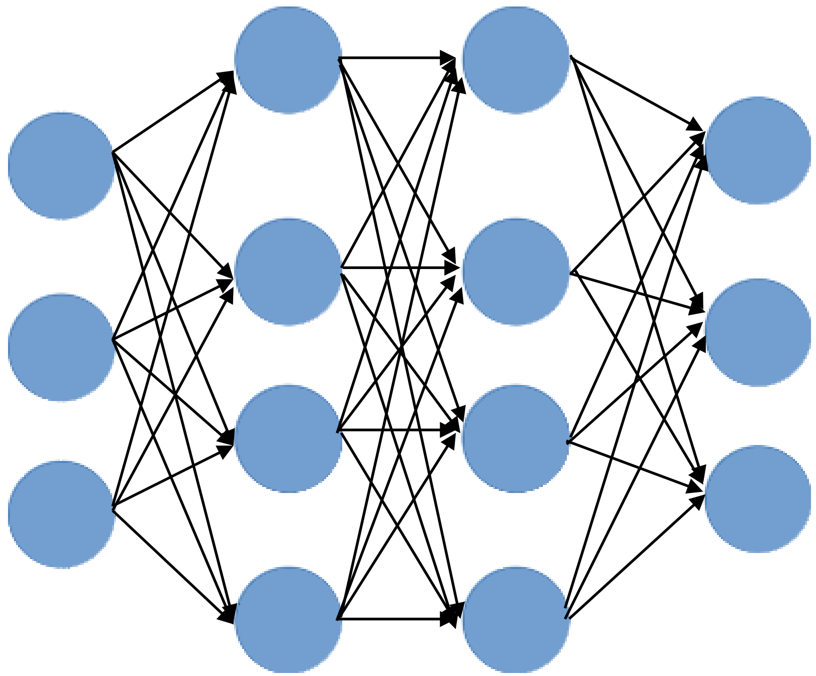

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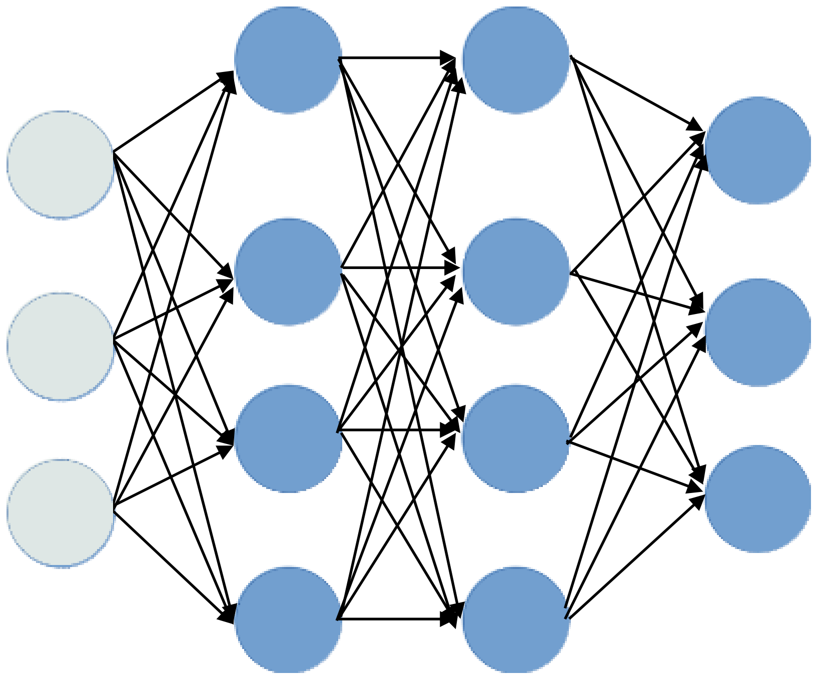

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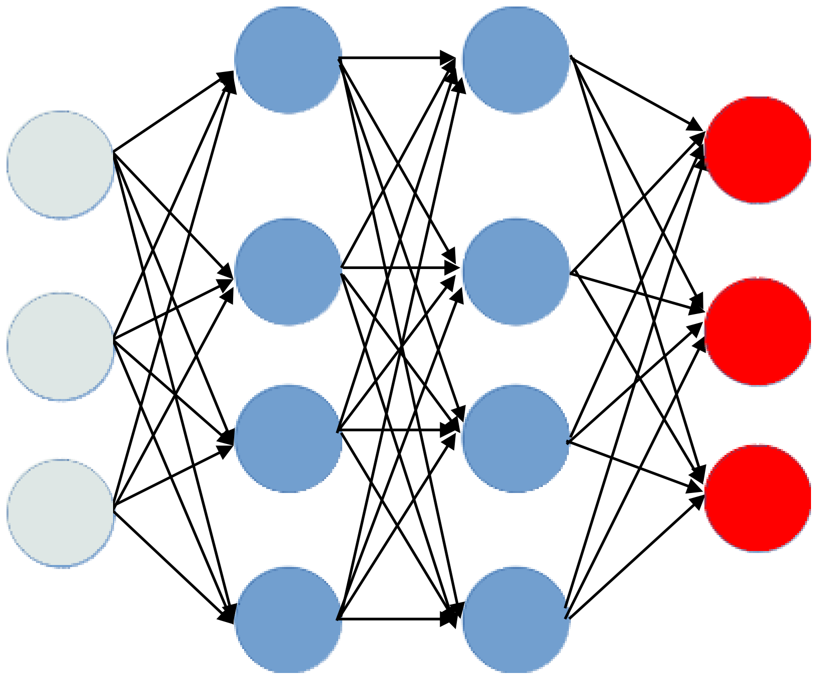

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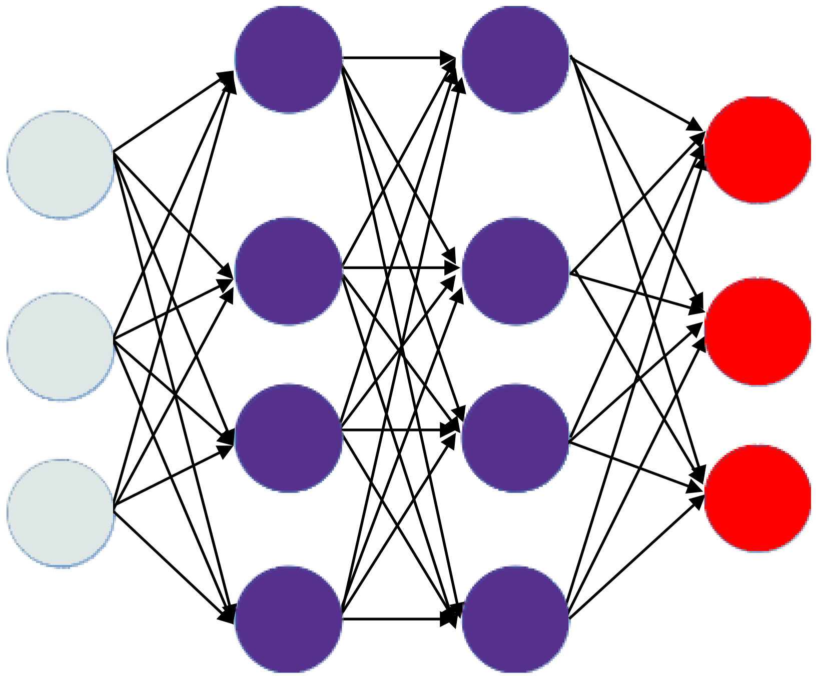
$\mathcal{H}$: Verdeckte Knoten: $\mathcal{H} := \mathcal{K} \backslash (\mathcal{I} \cup \mathcal{O})$



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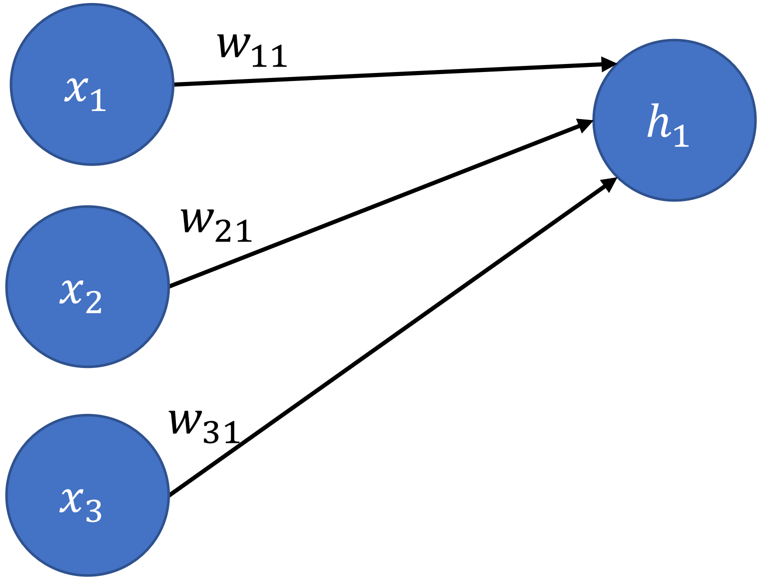

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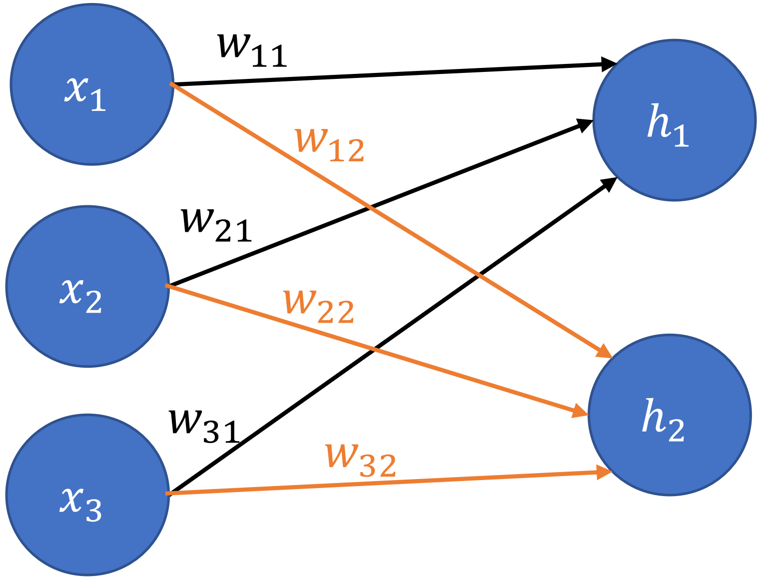

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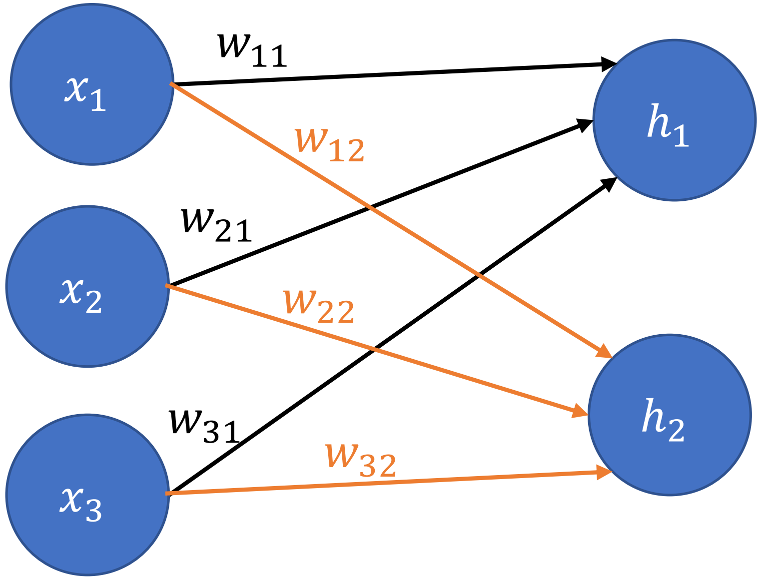

$$W = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

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Beispiele für Propagierungsfunktionen:

- $u_j := \sum_i w_{ij} x_i$ (linearer Assoziator)
- $u_j := \prod_i w_{ij} x_i$ (nicht-linearer Assoziator)
- $u_j := \max_i \{w_{ij} x_i\}$ (Maximum gewichtete Eingaben)
- $u_j := \sum_i s_i$, mit $s_i := \begin{cases} +1 & : \text{falls } w_{ij} x_i > 0 \\ -1 & : \text{sonst} \end{cases}$

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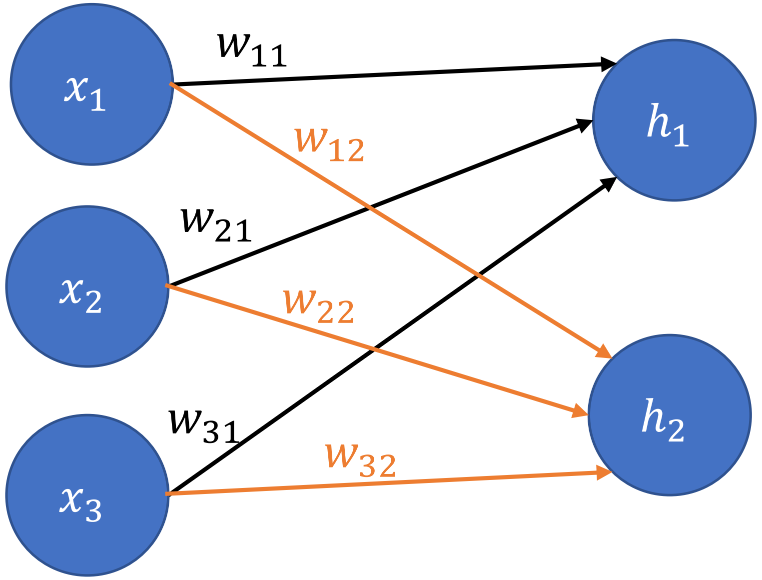

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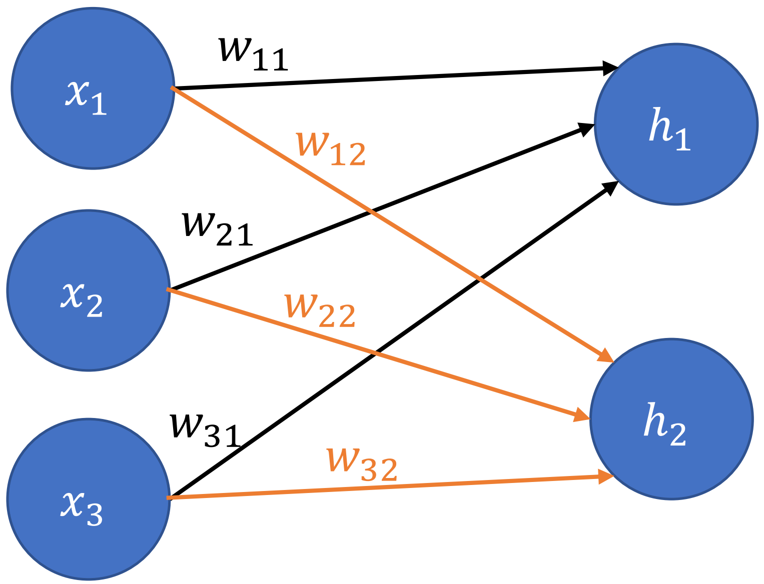

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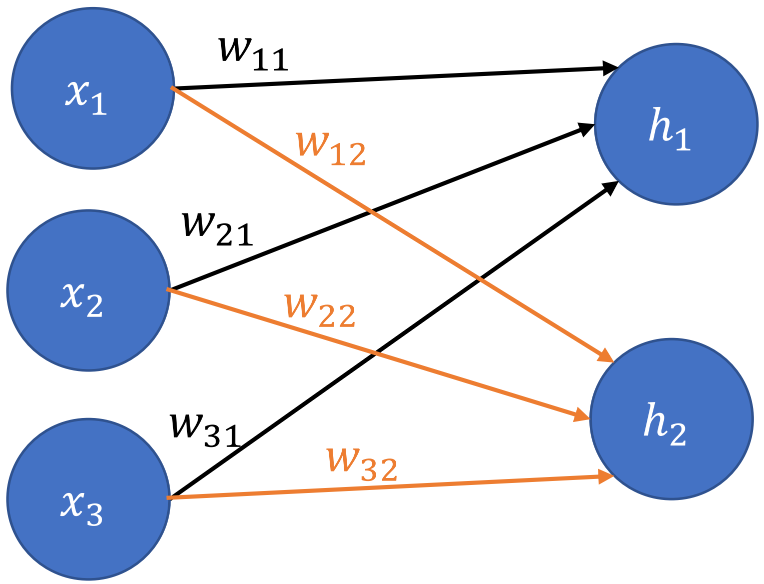

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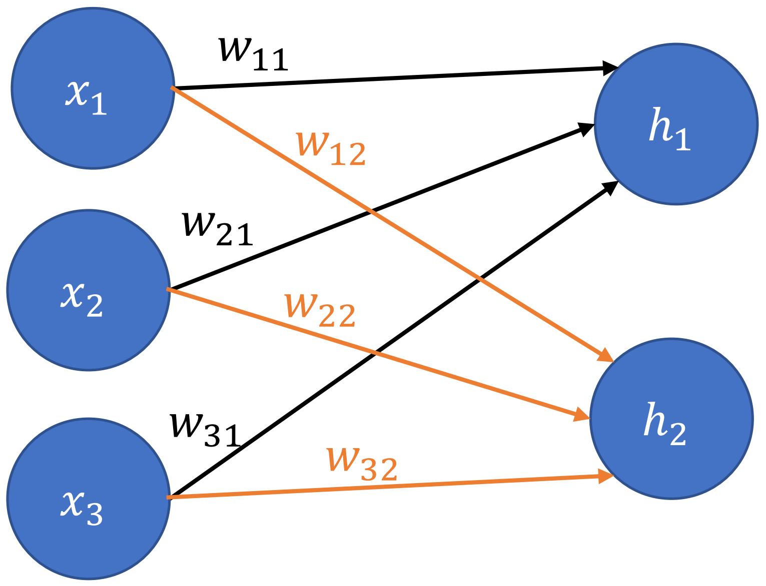

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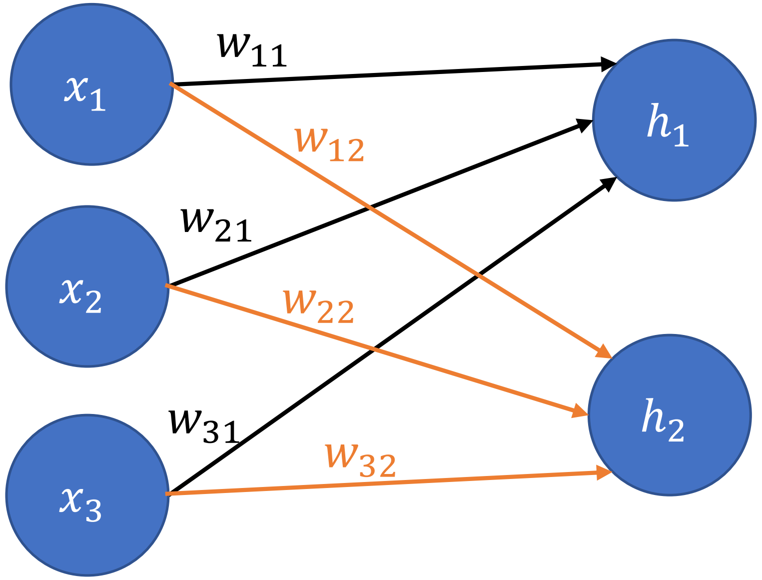

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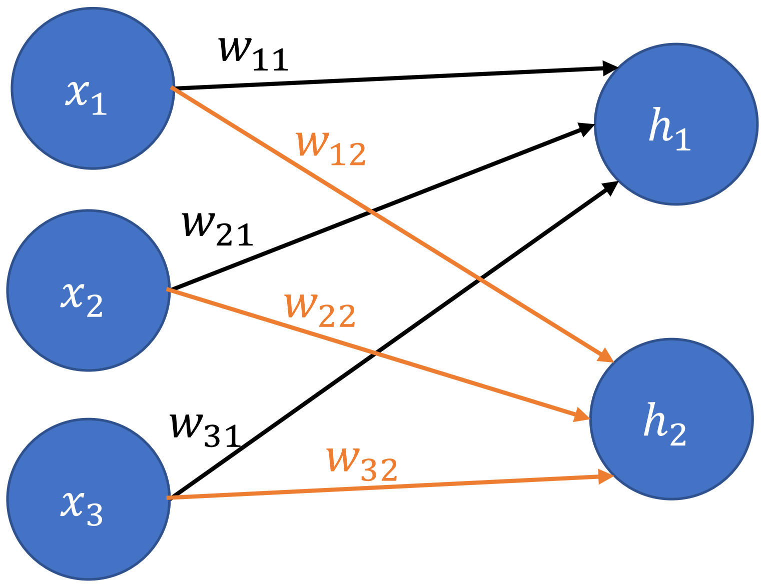

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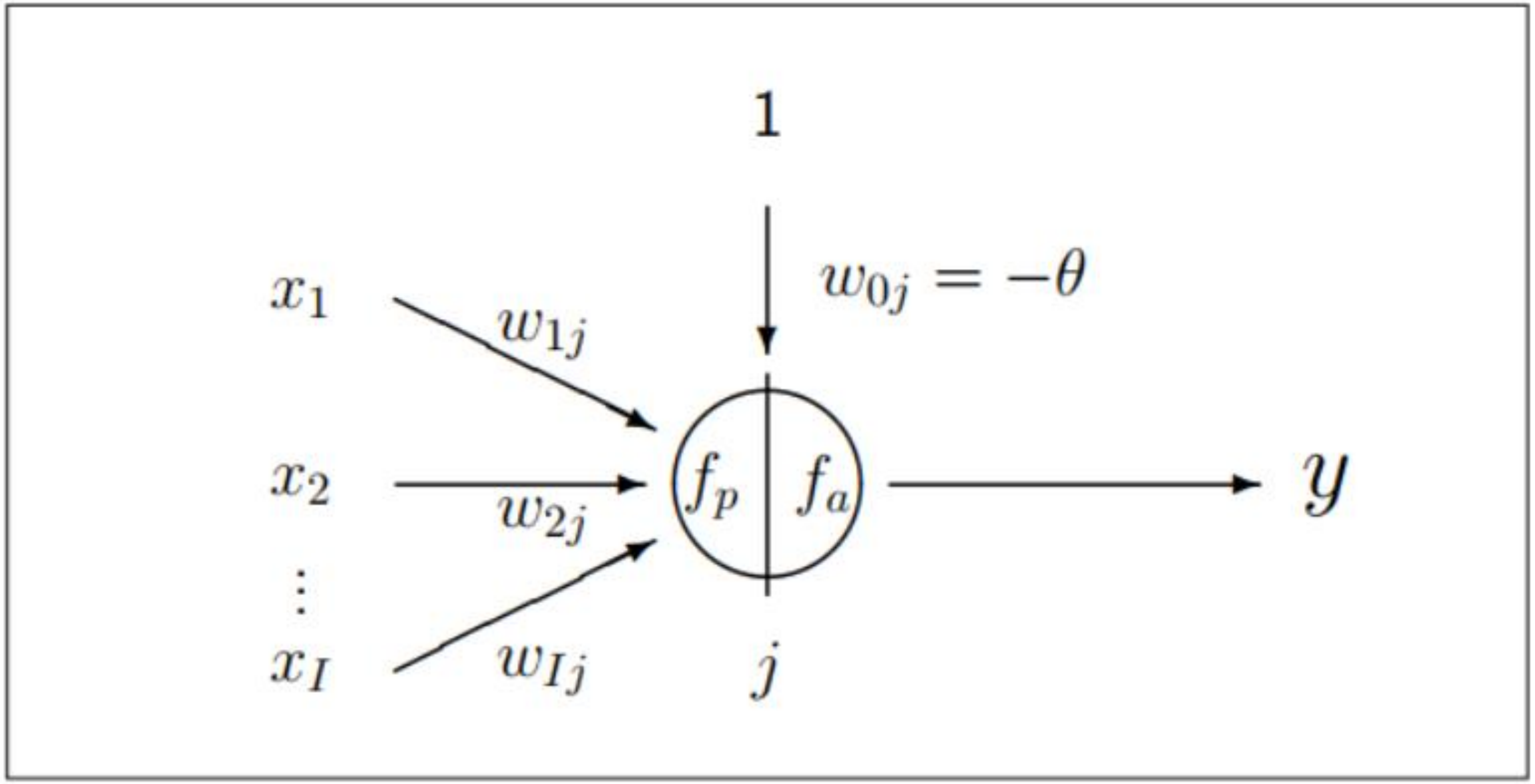

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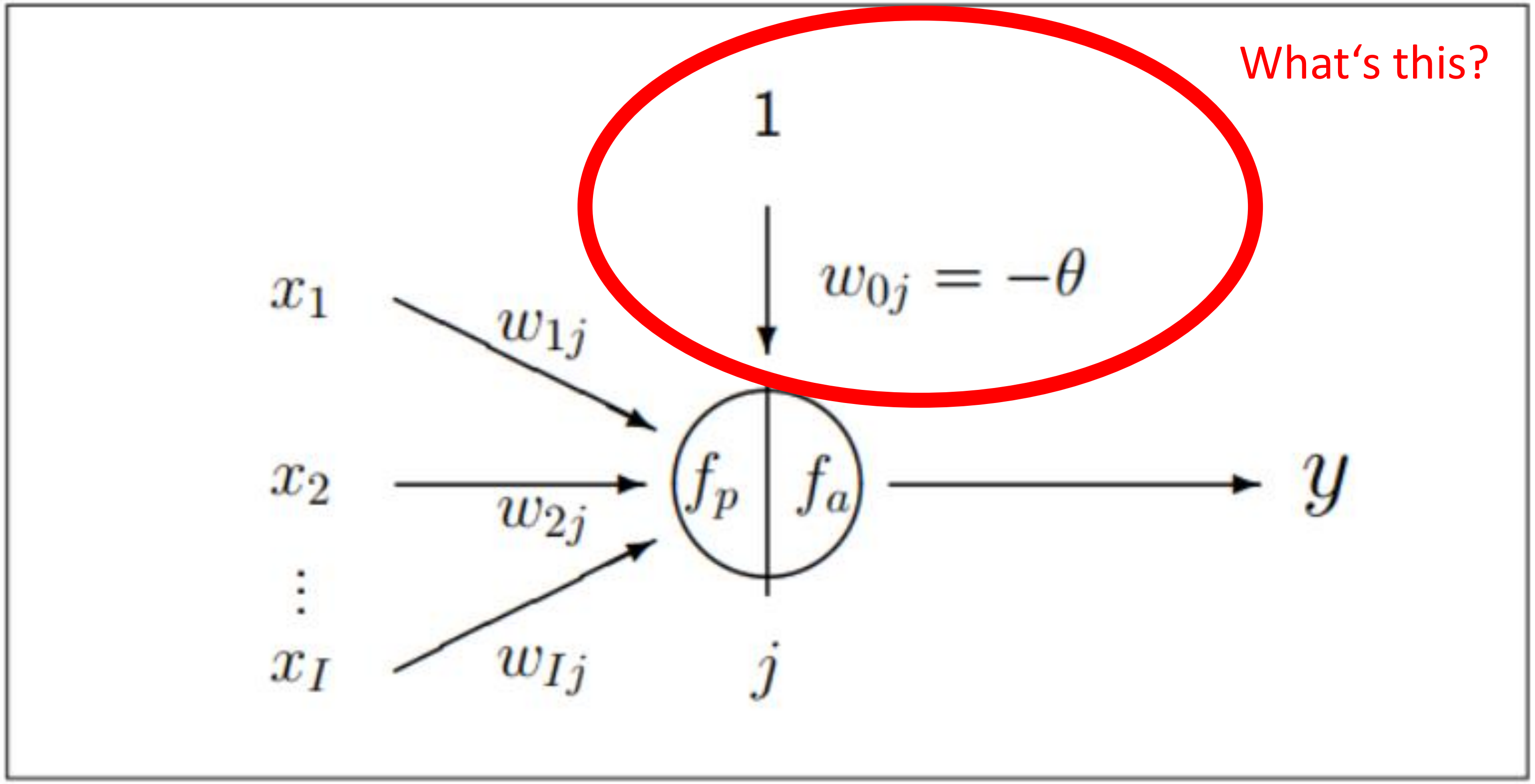

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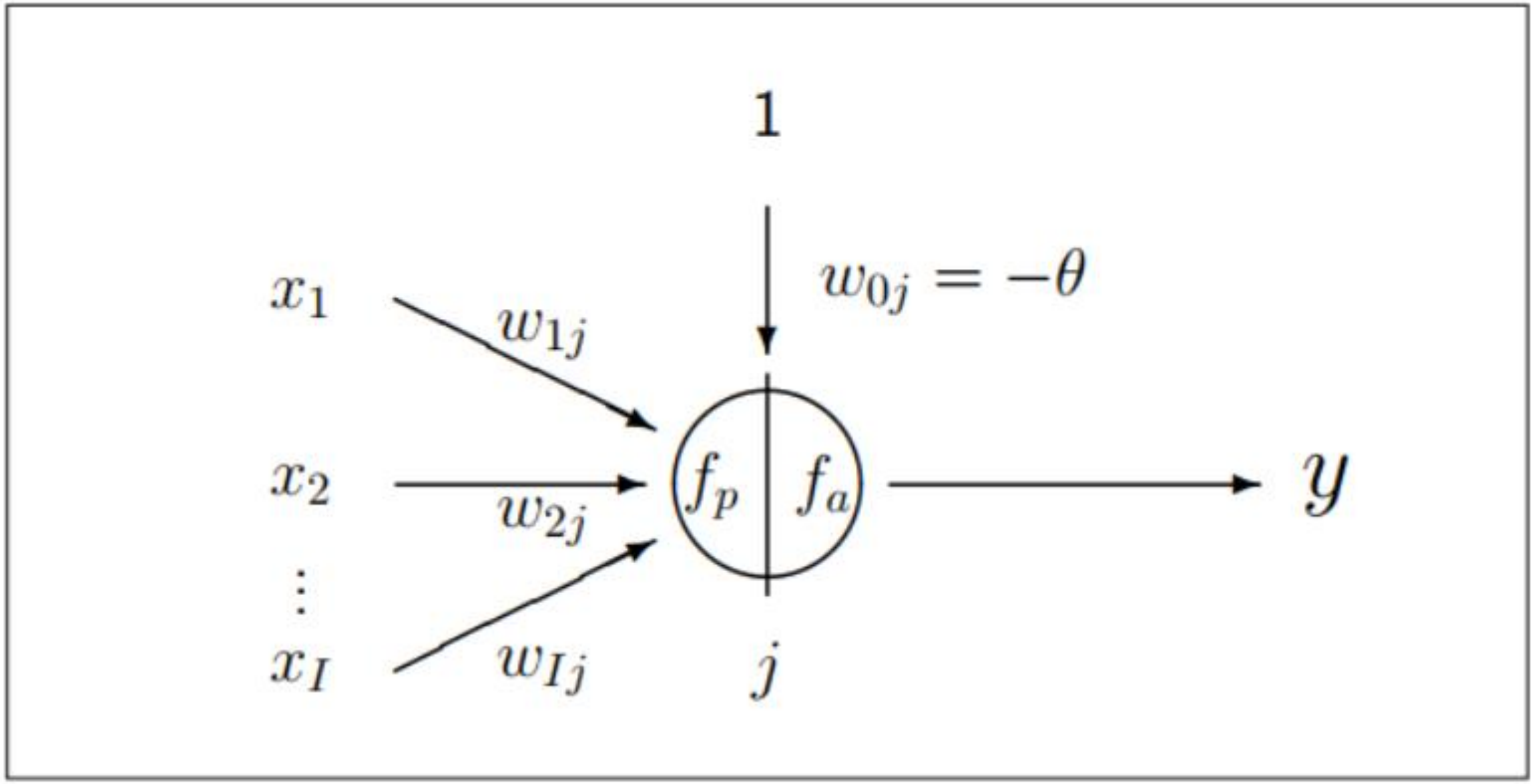
$$\mathrm{f}_a(\hat{x}, \Theta) := \begin{cases} 1, & \text{if } \hat{x} > \Theta \\ 0, & \text{else} \end{cases} \Rightarrow \mathrm{f}_a(\hat{x}) := \begin{cases} 1, & \text{if } \hat{x} > 0 \\ 0, & \text{else} \end{cases}$$

- Output of a neuron:

$$\mathrm{f}_a(f_p(x_1, x_2, ..., x_n), \Theta) := \begin{cases} 1, & \text{if } f_p(x_1, x_2, ..., x_n) > \Theta \\ 0, & \text{else} \end{cases} \Rightarrow \mathrm{f}_a(f_p(x_0, x_1, ..., x_n)) := \begin{cases} 1, & \text{if } f_p(x_0, x_1, x_2, ..., x_n) > 0 \\ 0, & \text{else} \end{cases}$$

Scheme of Artificial Neuron



$f_p | f_a$ wird oft weggelassen, wenn aus dem Zusammenhang klar.

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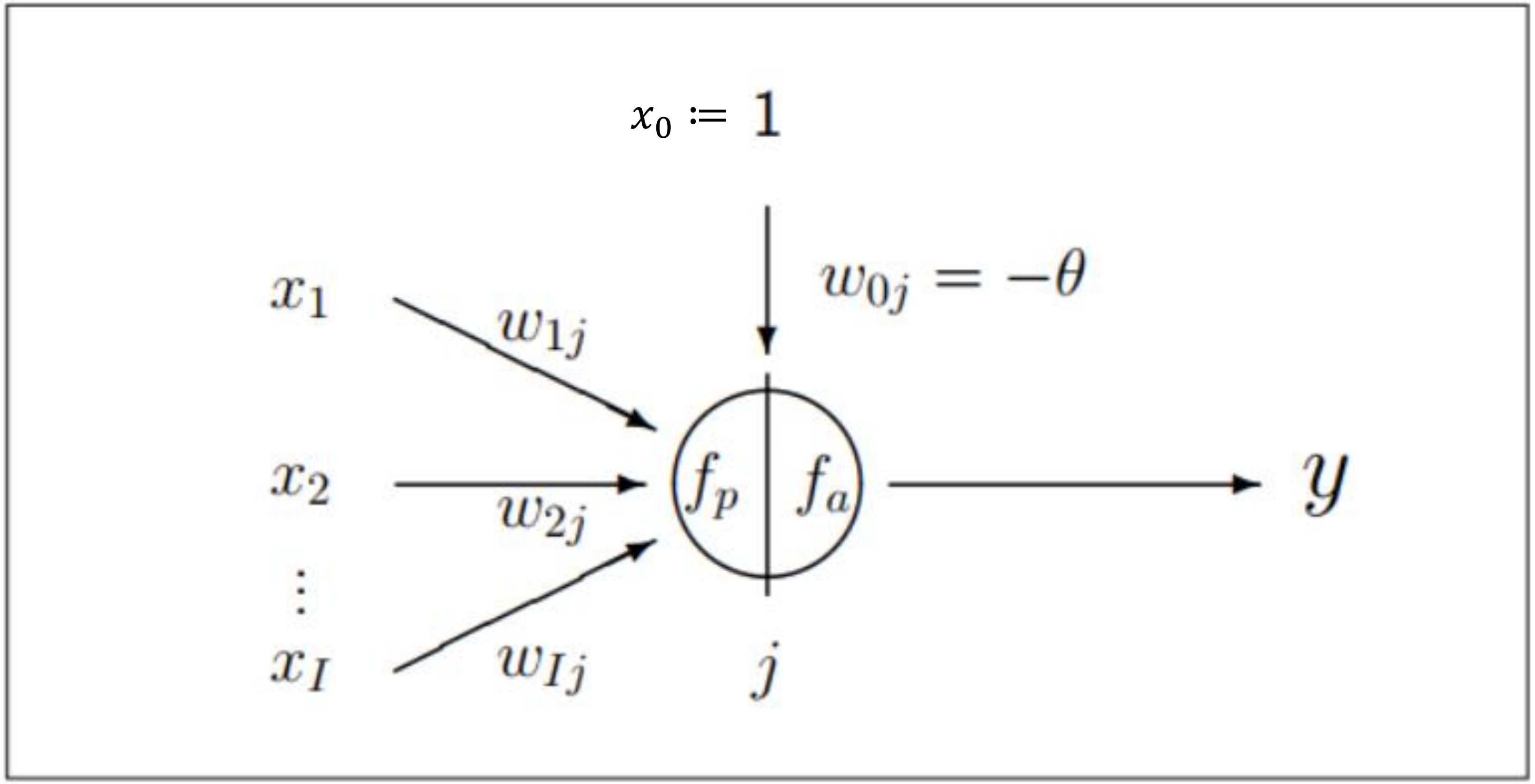

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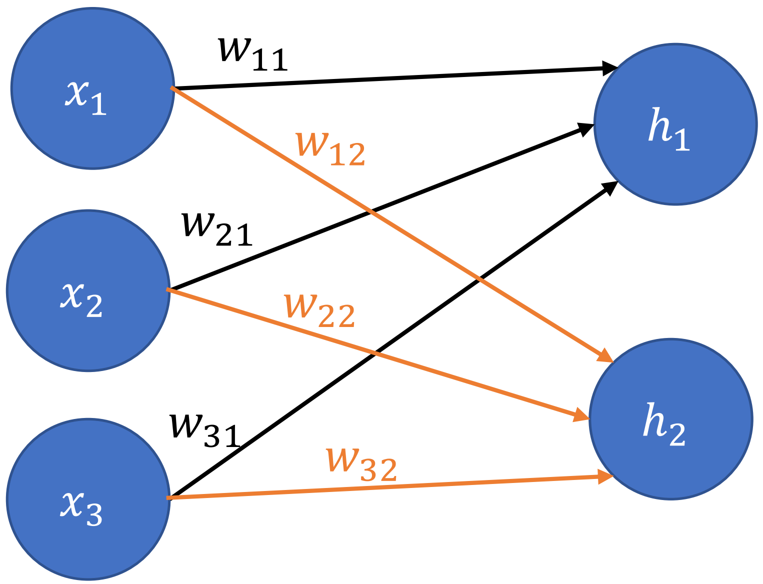
Calculation of propagated value



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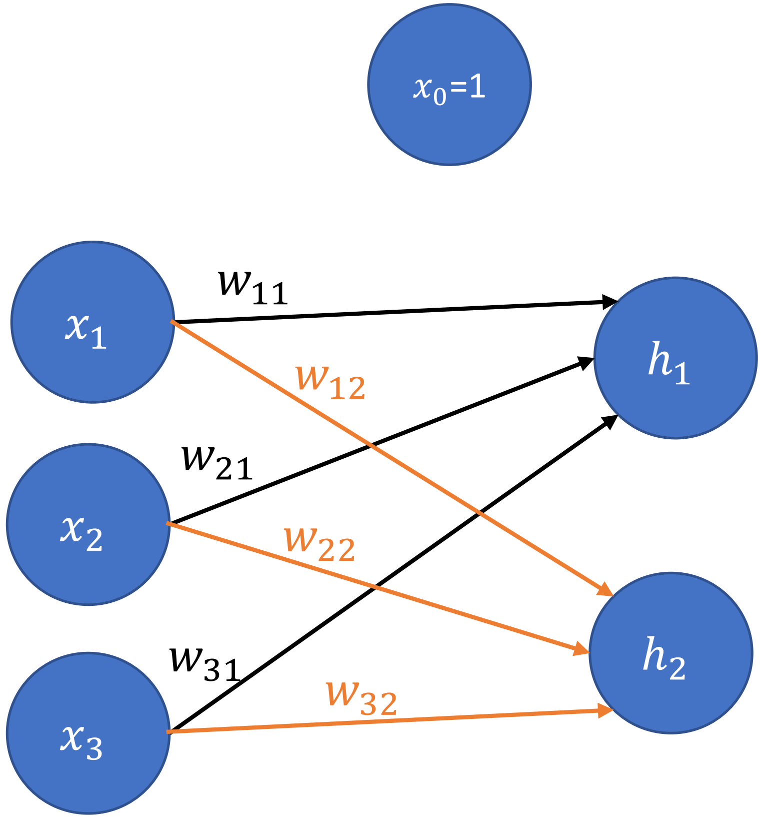

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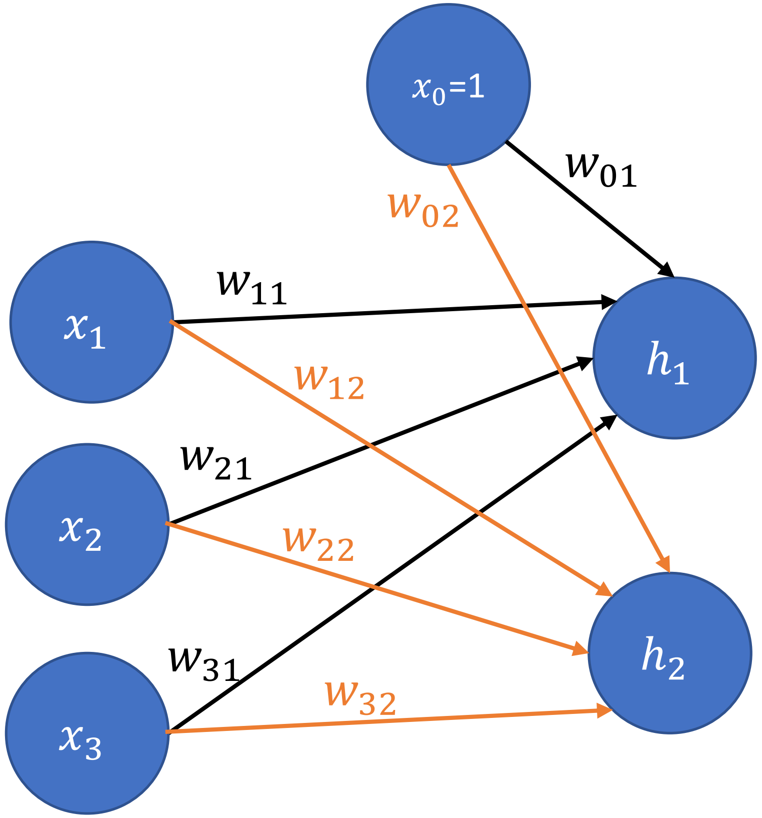

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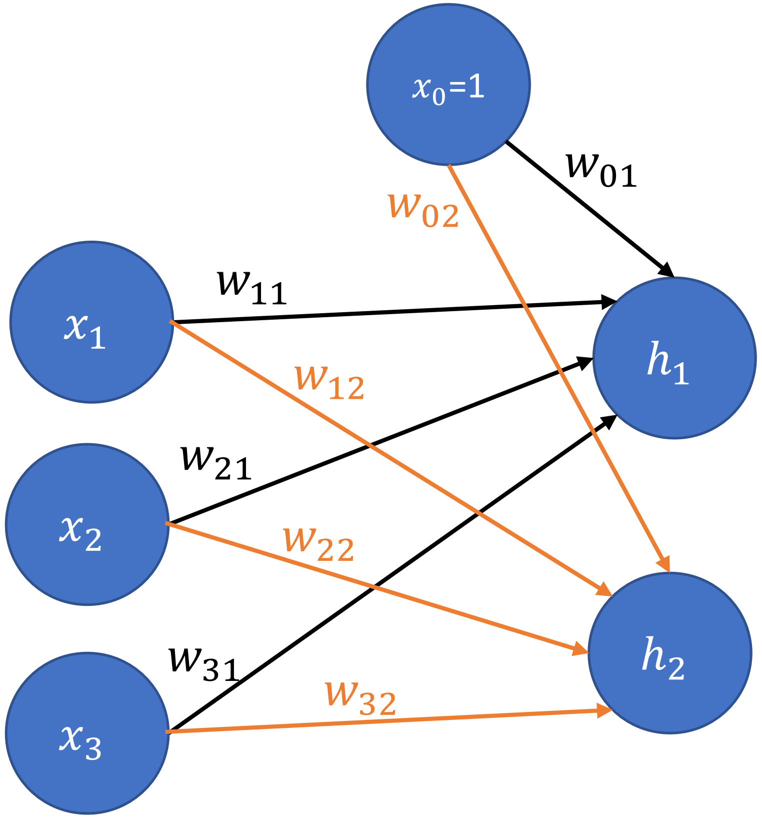

$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^{3} w_{i2} x_i$$

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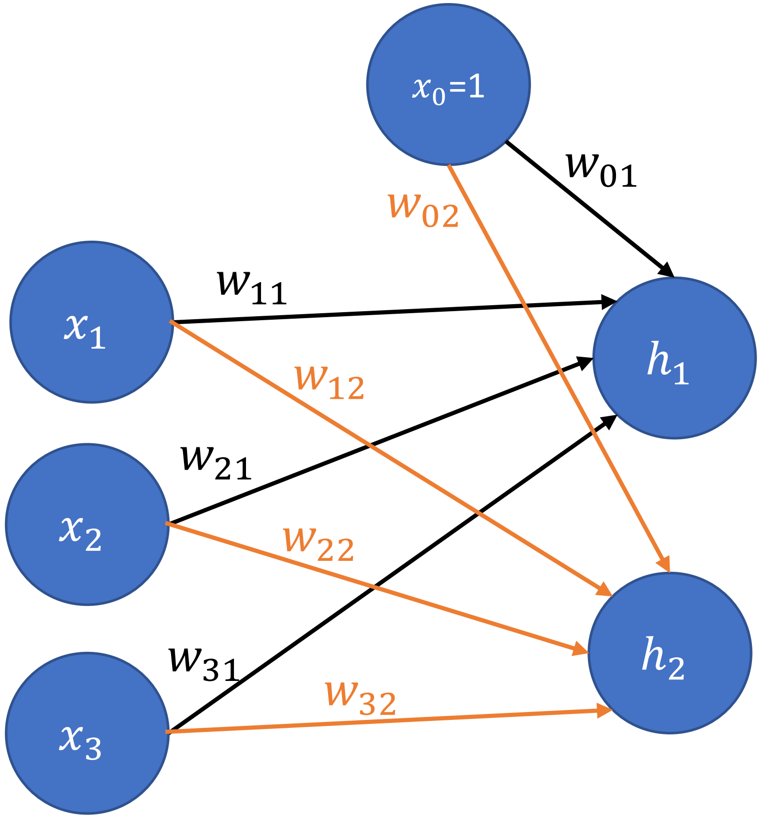

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \qquad \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$

Revision

- Which of the following terms describe a parts of a biological neuron?

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- Which of the following terms describe a parts of a biological neuron?
  1. Perikaryon
  2. Dentrites
  3. Axon
  4. Axon hillhock
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A: all

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C: 1-3, 5, 6

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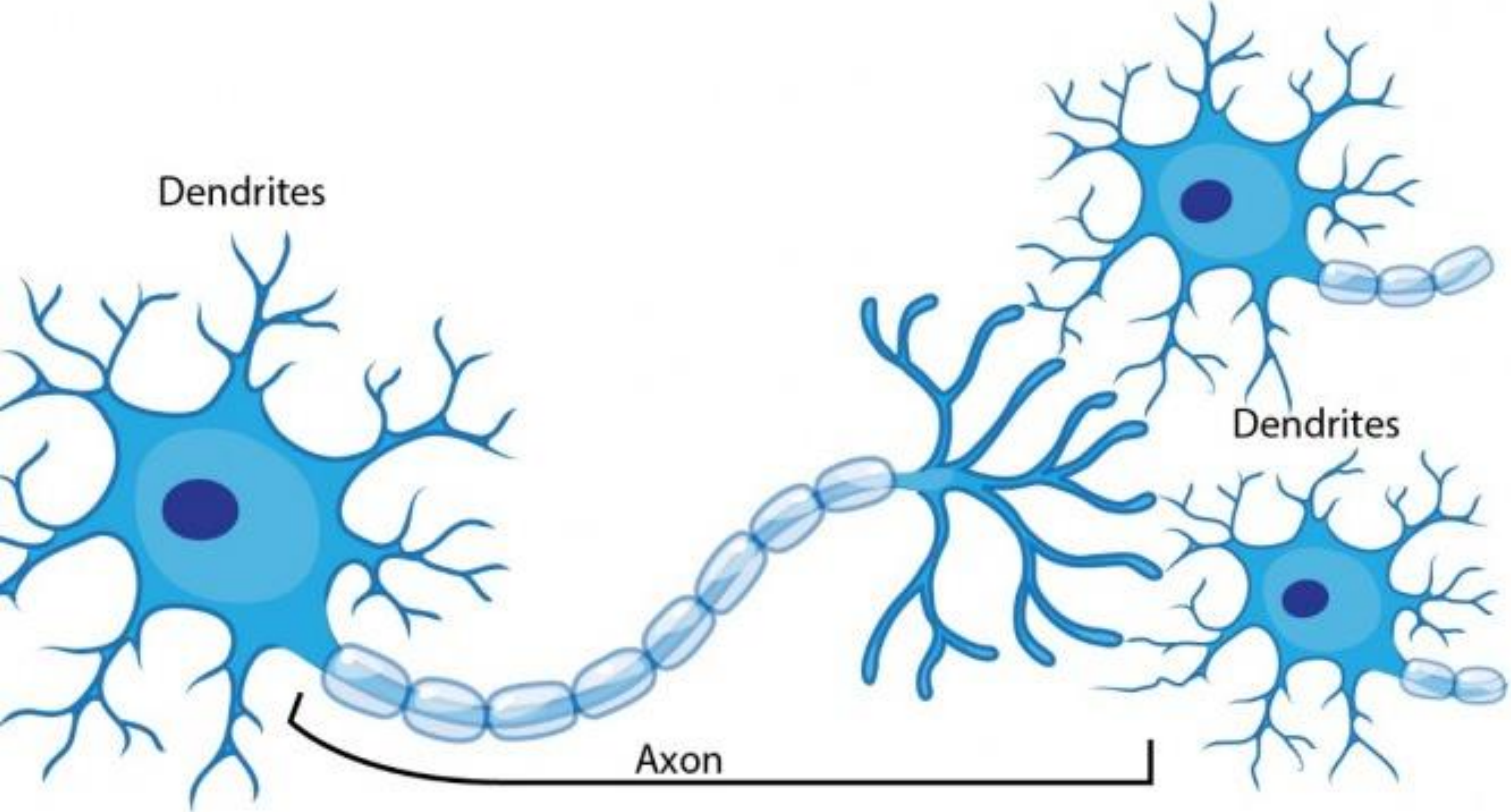
Dendrites
Dendrites
Axon

Content

- Revision
- Biological Motivation for ANNs
- McCulloch-Pitts Neuron
- Tasks

Biological Motivation for ANNs

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  - Summation of postsynaptic potentials
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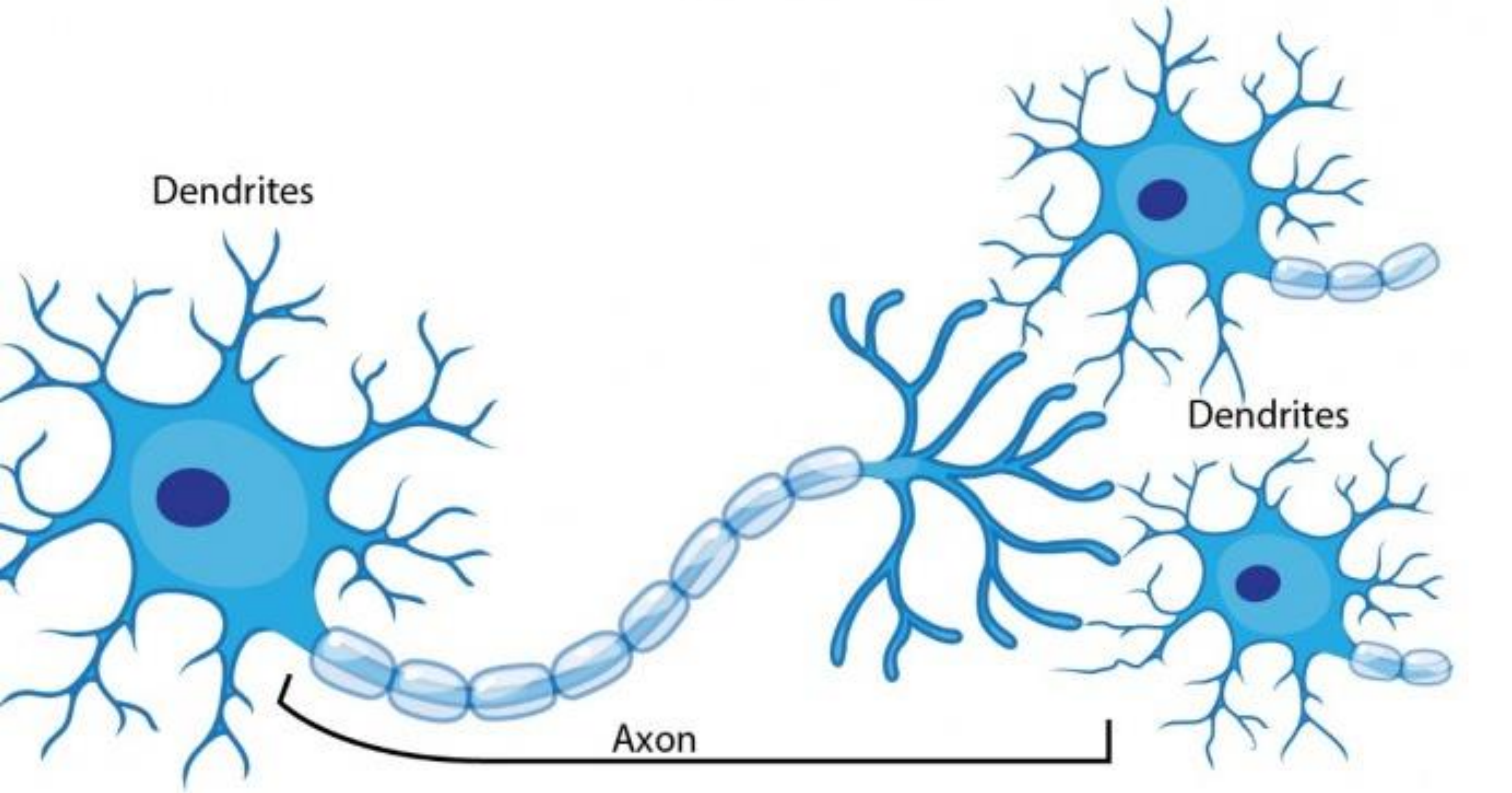
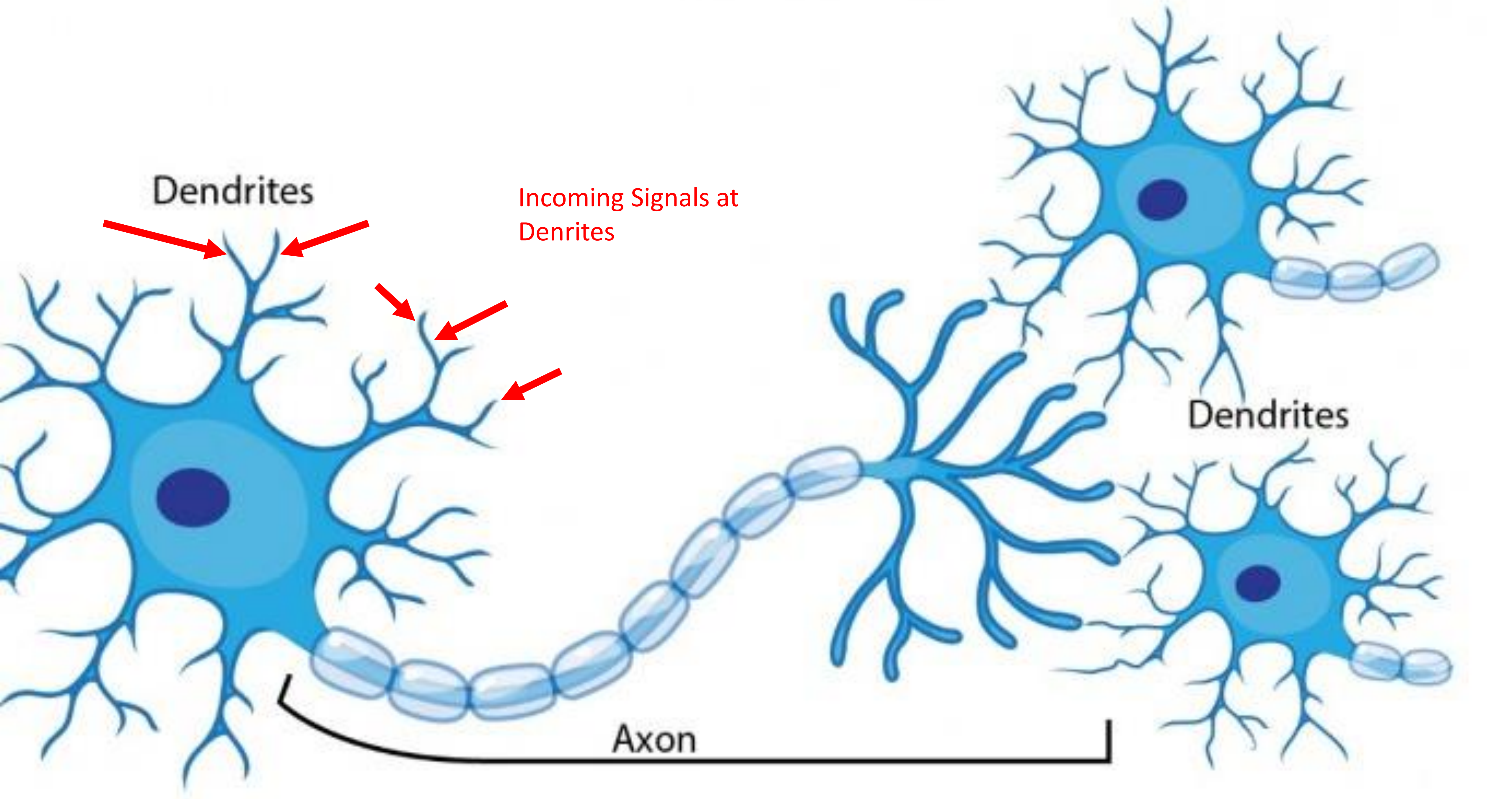
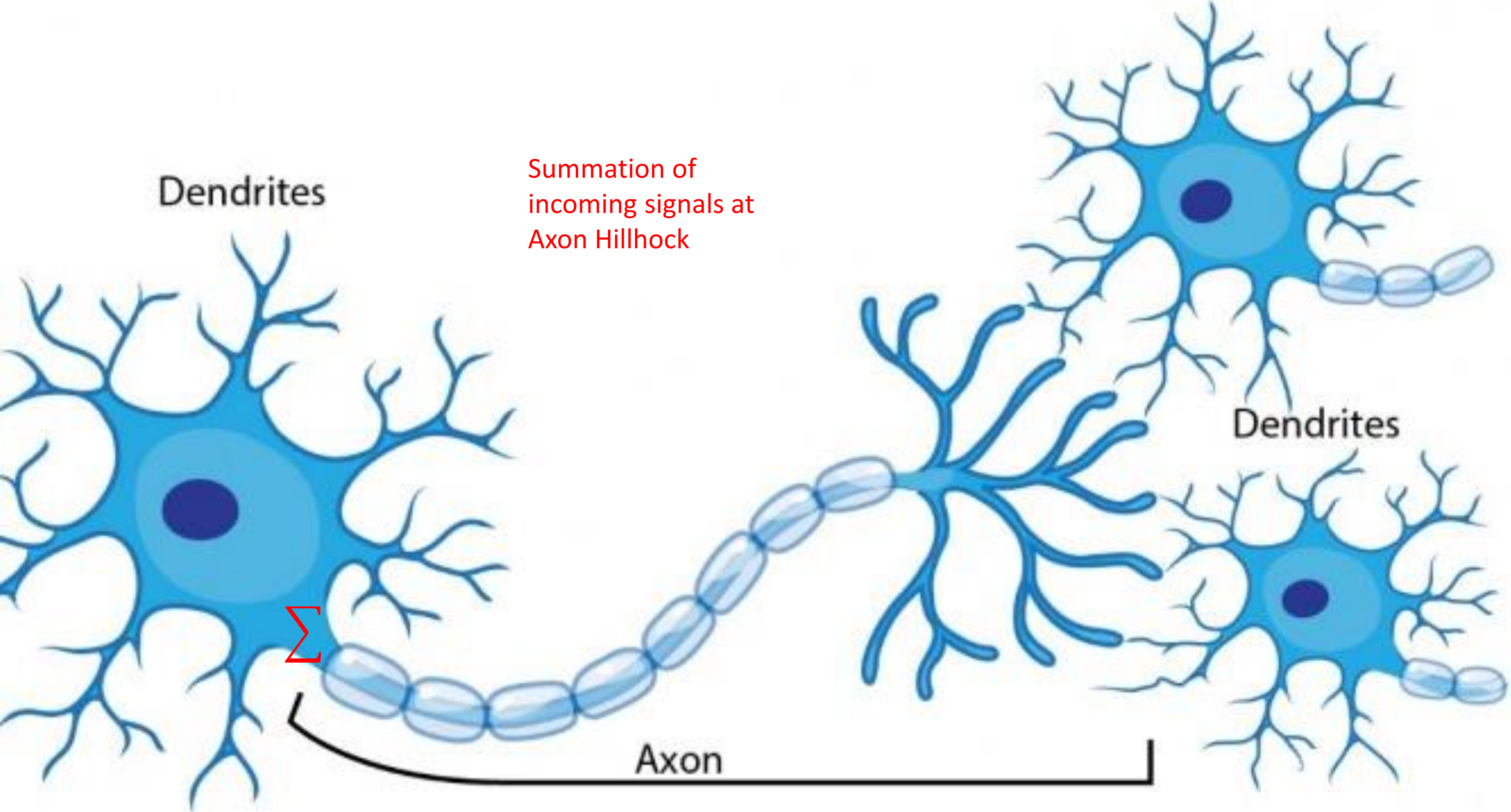
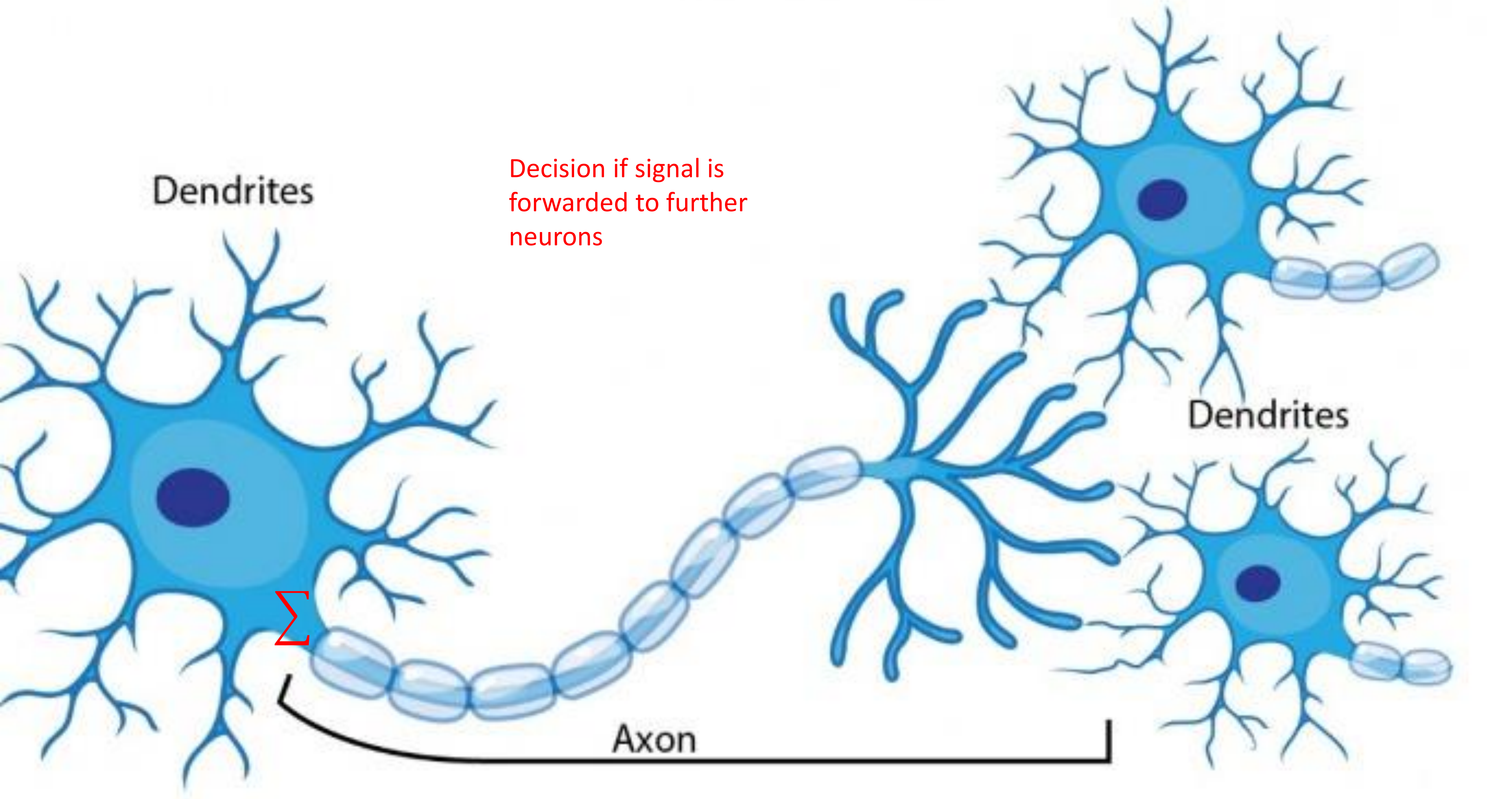
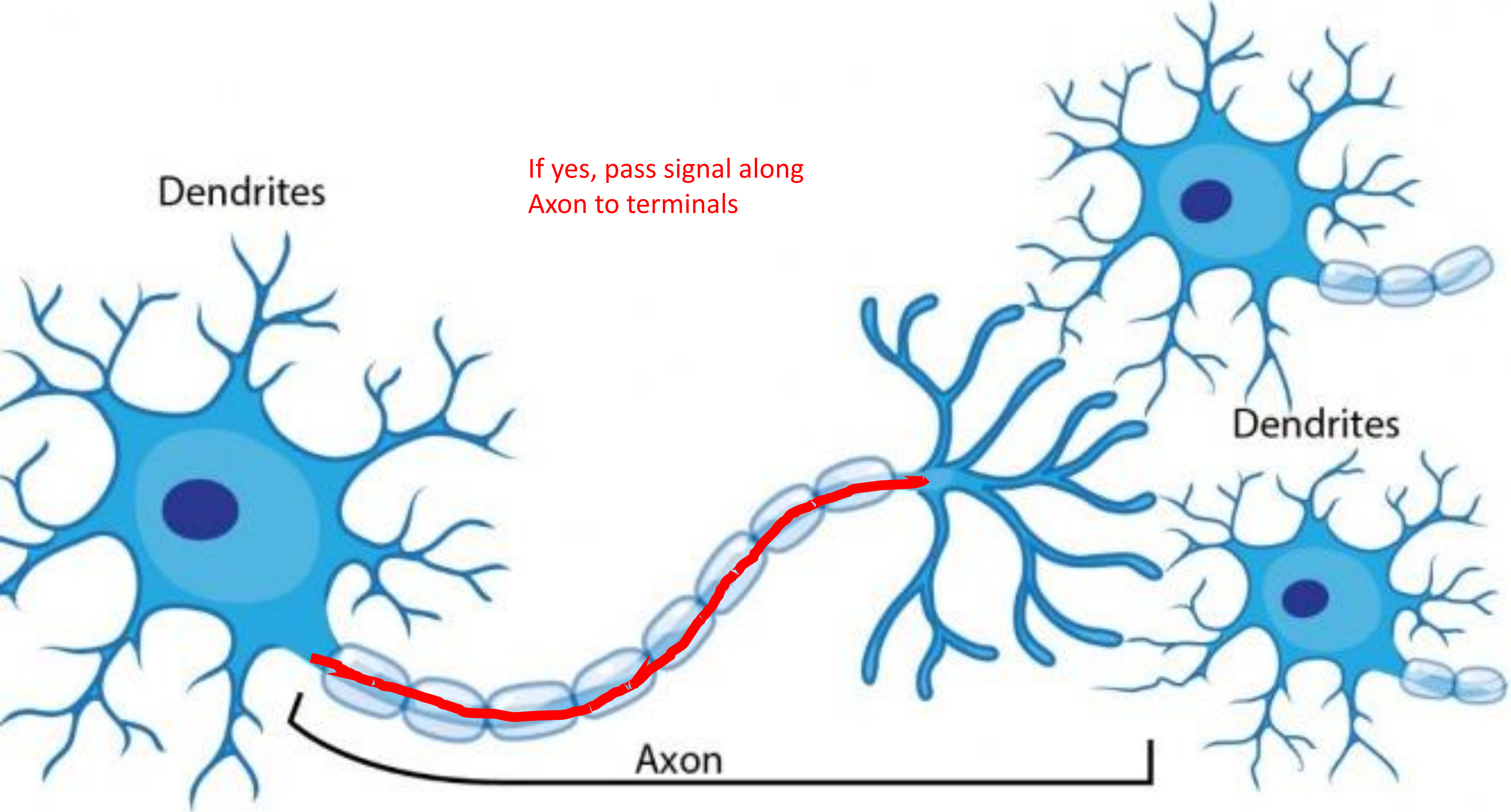

Dendrites
Dendrites
Axon


Dendrites
Incoming Signals at
Denrites
Dendrites
Axon

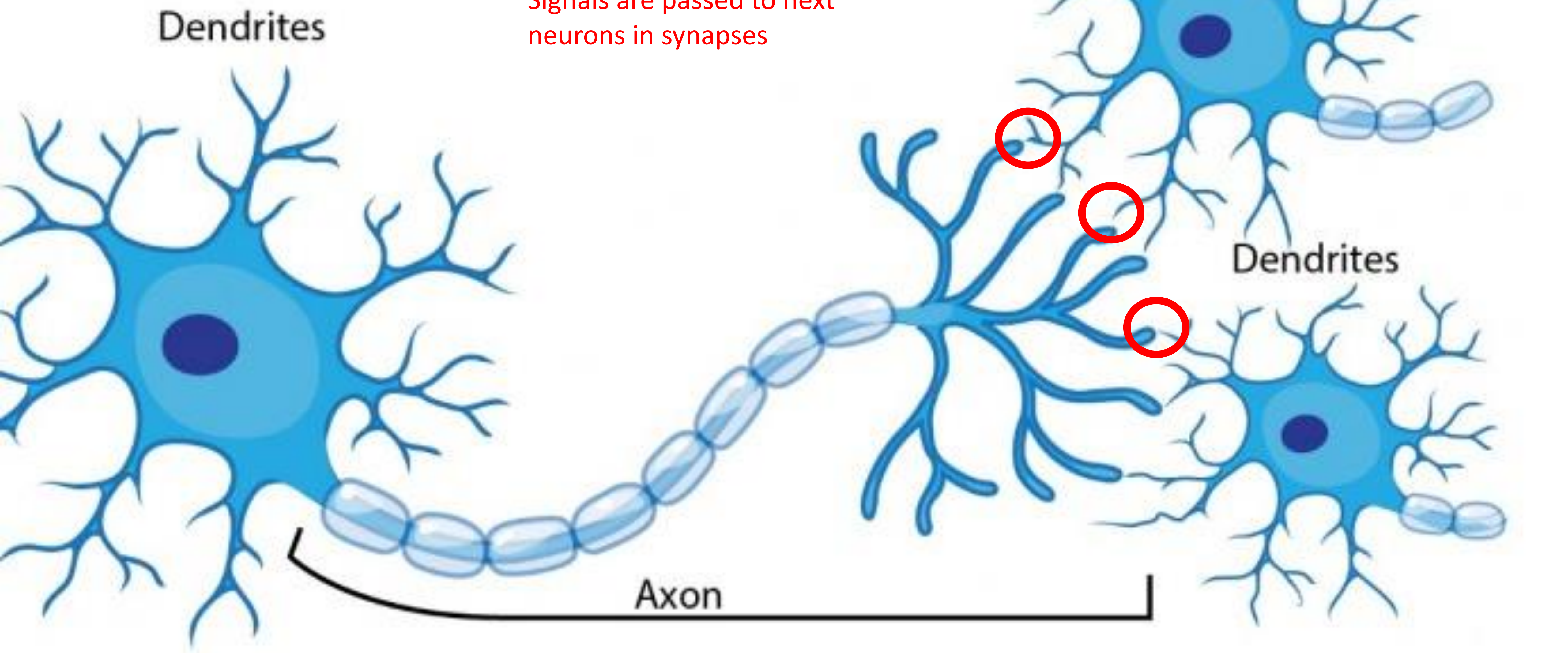

Summation of incoming signals at Axon Hillhock
Dendrites
Σ
Axon
Dendrites


Dendrites
Decision if signal is forwarded to further neurons
Σ
Dendrites
Axon

If yes, pass signal along
Axon to terminals



Signals are passed to next neurons in synapses



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The McCulloch Pitts Neuron

- Very simple model of neuron by McCulloch and Pitts

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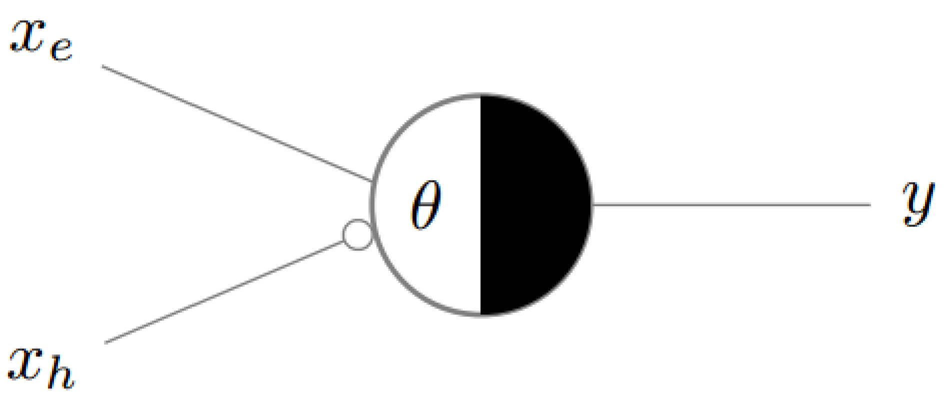
  - Each signal can be either:
    - Excitatory or
    - Inhibiting
- Output (binary):
  1. Sum over all excitatory signals
  2. If sum is greater than (or equal to) a threshold $\Theta \in \mathbb{R}$
     AND if **all** inhibiting signals are zero,
     then return 1, elso return 0

The McCulloch Pitts Neuron



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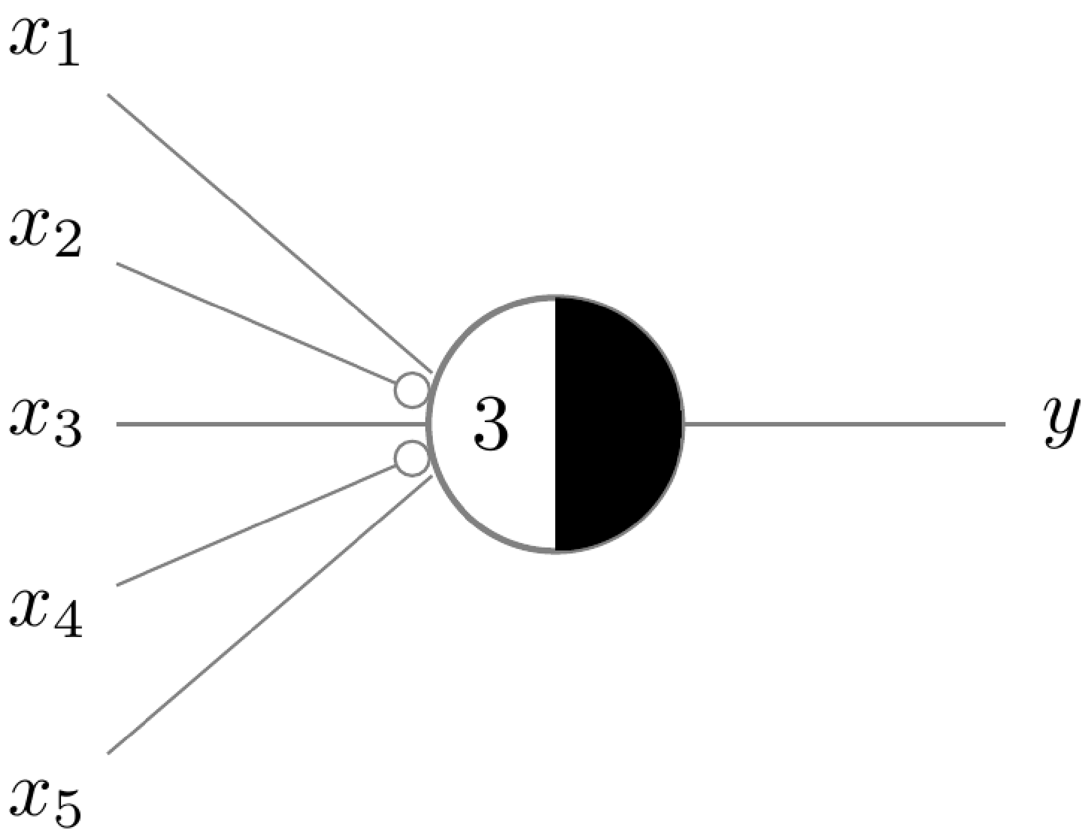
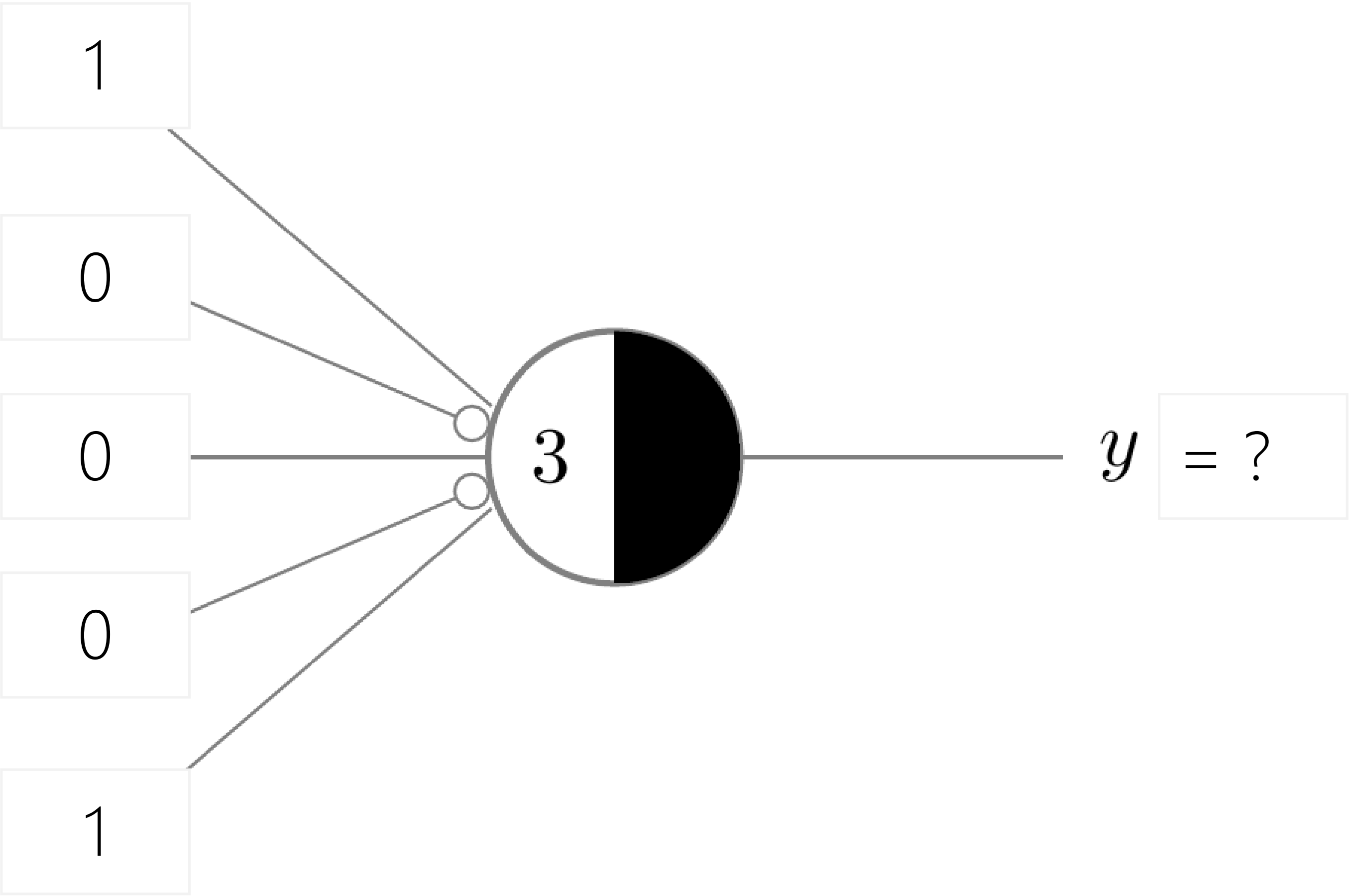
The McCulloch Pitts Neuron



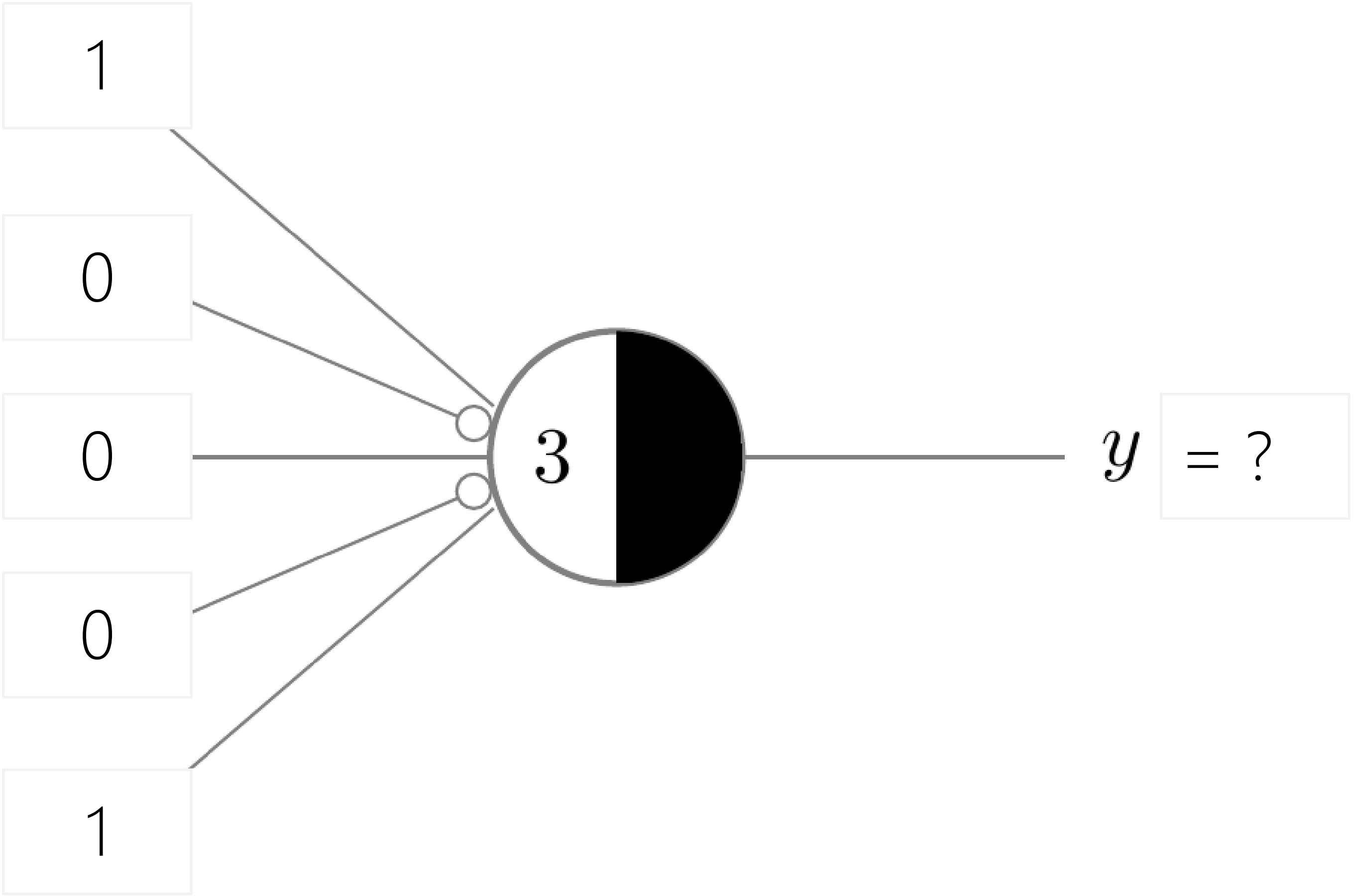
The McCulloch Pitts Neuron



A: y = 0

B: y = 1

The McCulloch Pitts Neuron



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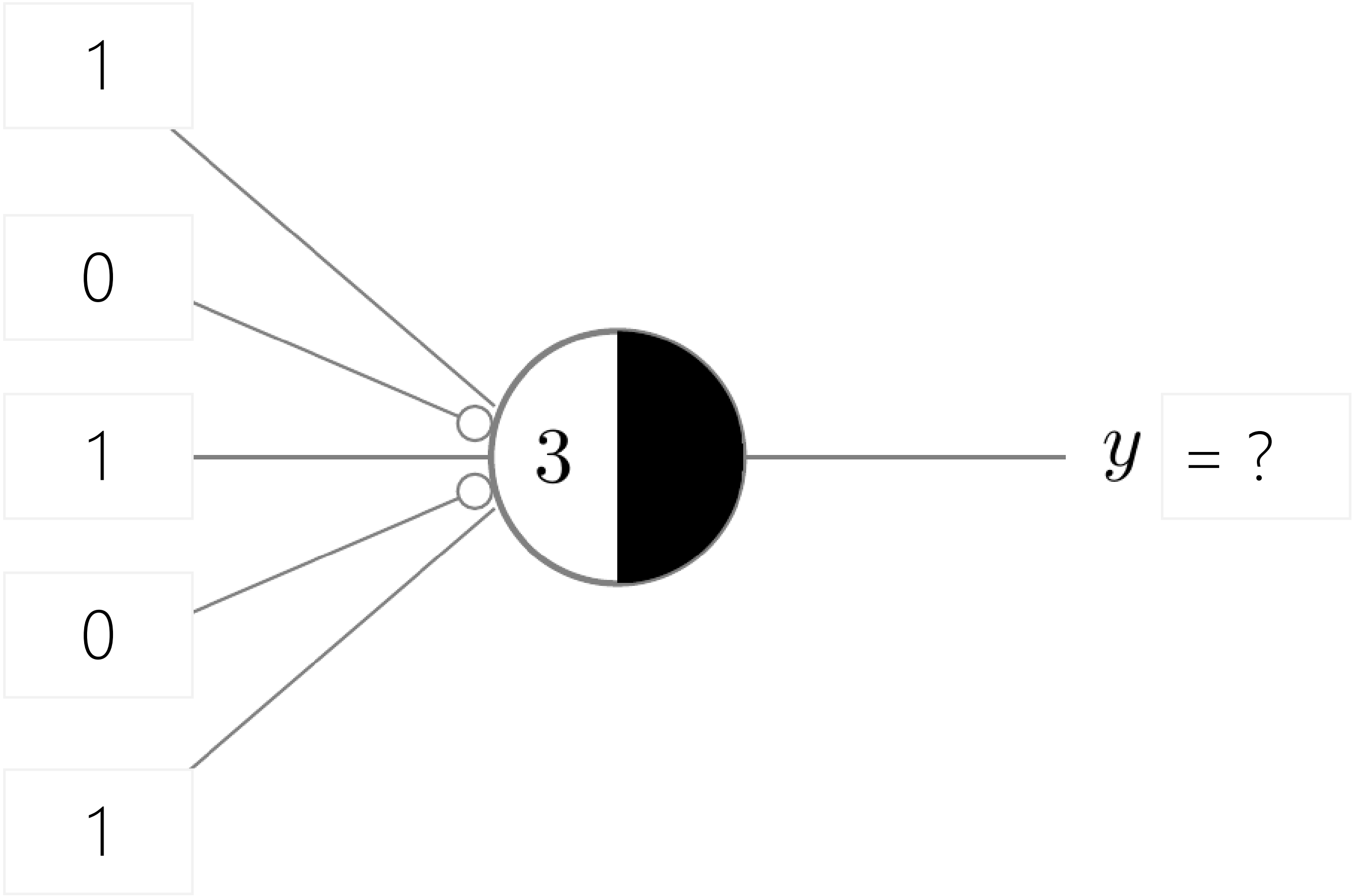

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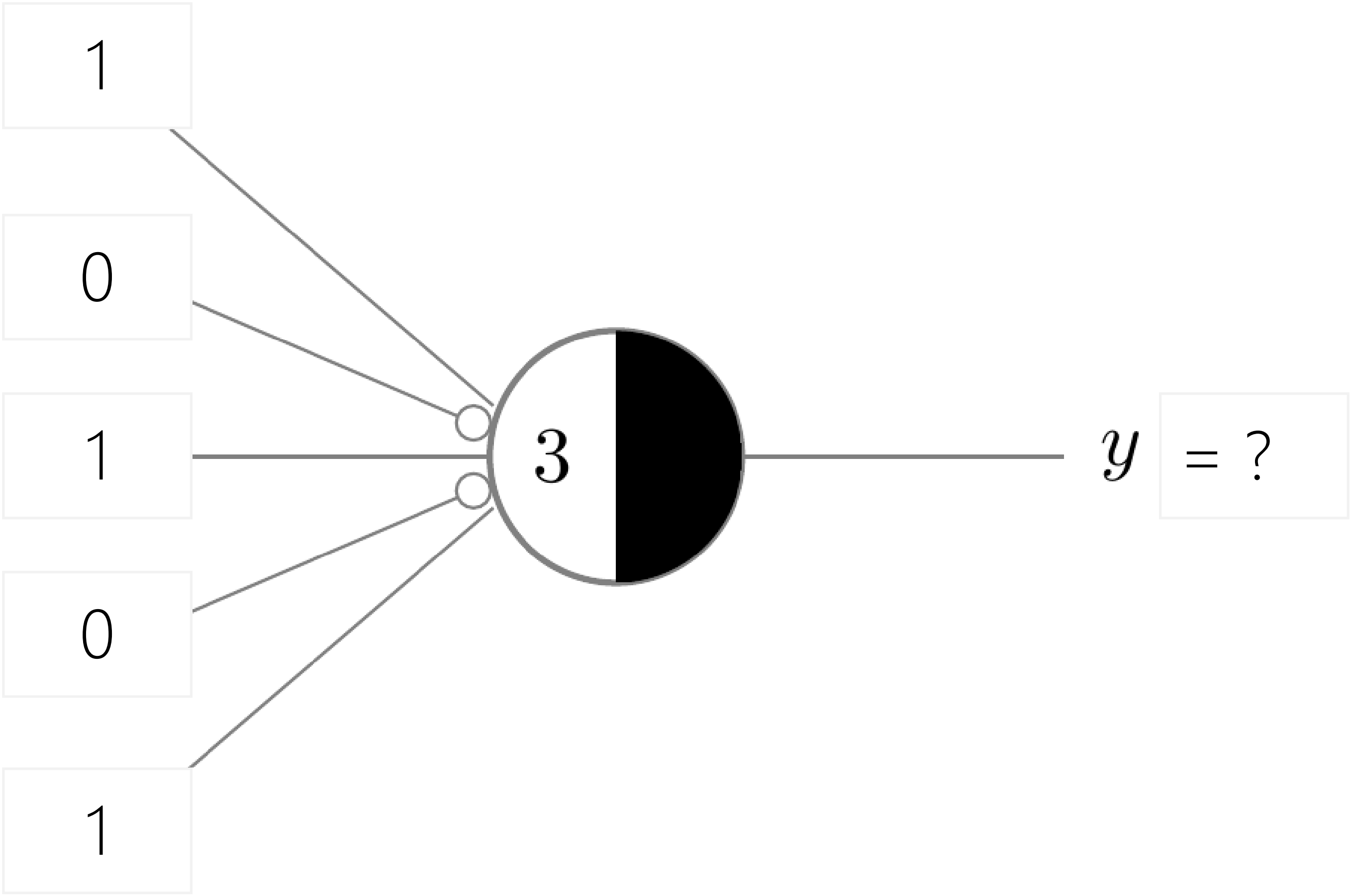

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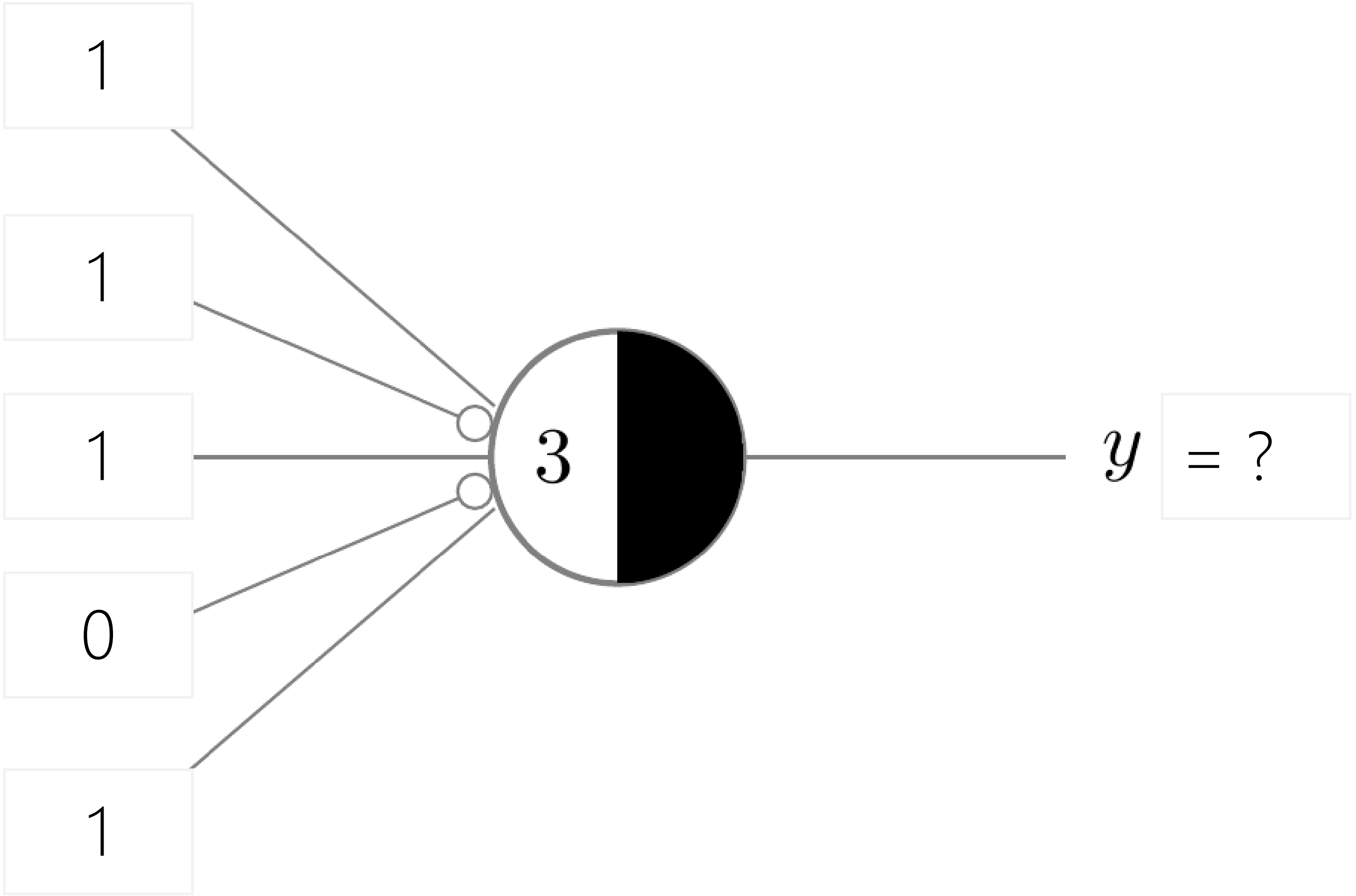

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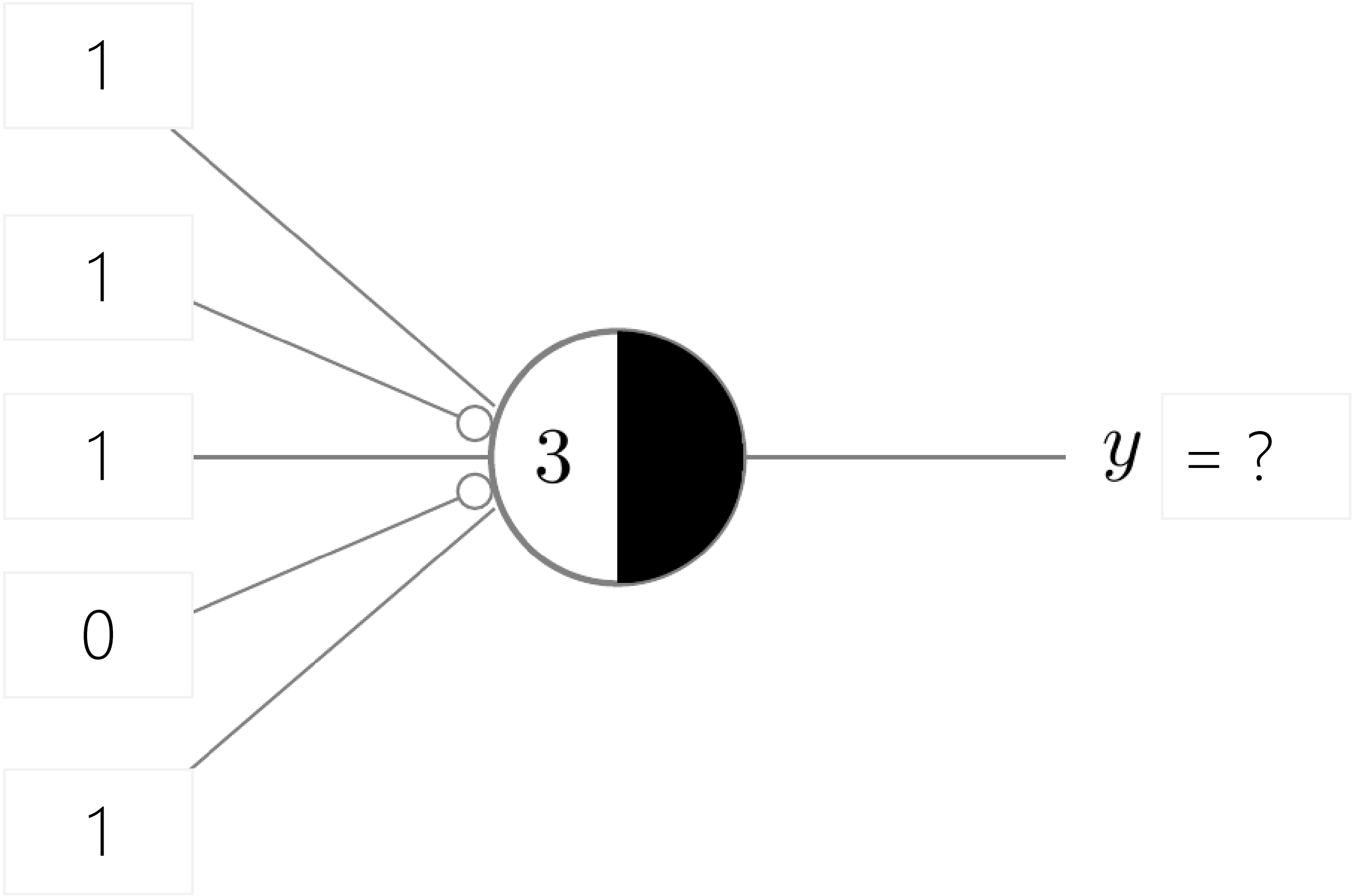

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Excursion: Boolean Functions

- Given a binary input vector, assign a binary output, i.e.:

$$f: \{0,1\}^N \rightarrow \{0,1\}$$

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- Example: AND function:

$$f_{AND} : \{0,1\}^2 \rightarrow \{0,1\}$$

with:

$$
\begin{aligned}
f_{AND}(0,0) &\coloneqq 0 \\
f_{AND}(0,1) &\coloneqq 0 \\
f_{AND}(1,0) &\coloneqq 0 \\
f_{AND}(1,1) &\coloneqq 1
\end{aligned}
$$

Excursion: Boolean Functions

- Representation of binary function as truth table:

x_1	x_2	f_{AND}
0	0	0
0	1	0
1	0	0
1	1	1

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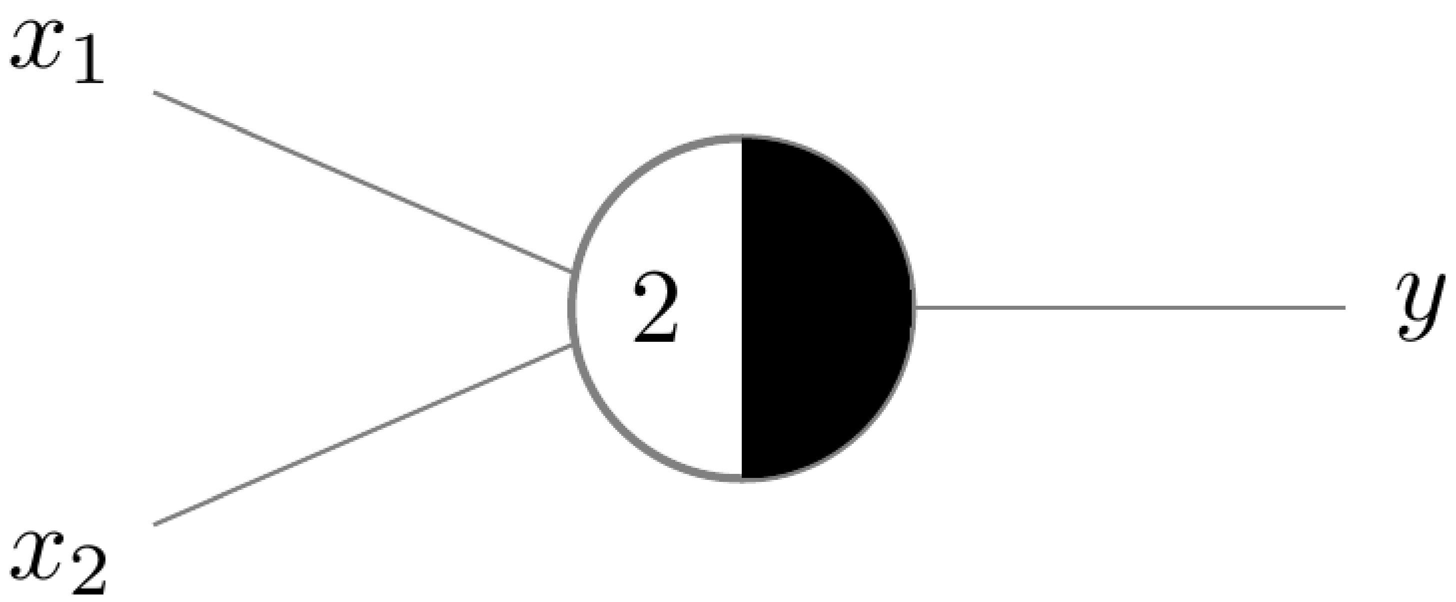

Excursion: Boolean Functions

- Given a binary input vector, assign a binary output, i.e.:

$$f: \{0,1\}^N \rightarrow \{0,1\}$$

- Example: OR function:

$$f_{OR} : \{0,1\}^2 \rightarrow \{0,1\}$$

with:

$$f_{OR}(0,0) \coloneqq 0$$
$$f_{OR}(0,1) \coloneqq 1$$
$$f_{OR}(1,0) \coloneqq 1$$
$$f_{OR}(1,1) \coloneqq 1$$

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- Representation of binary function as truth table:

x_1	x_2	f_{OR}
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0	1	1
1	0	1
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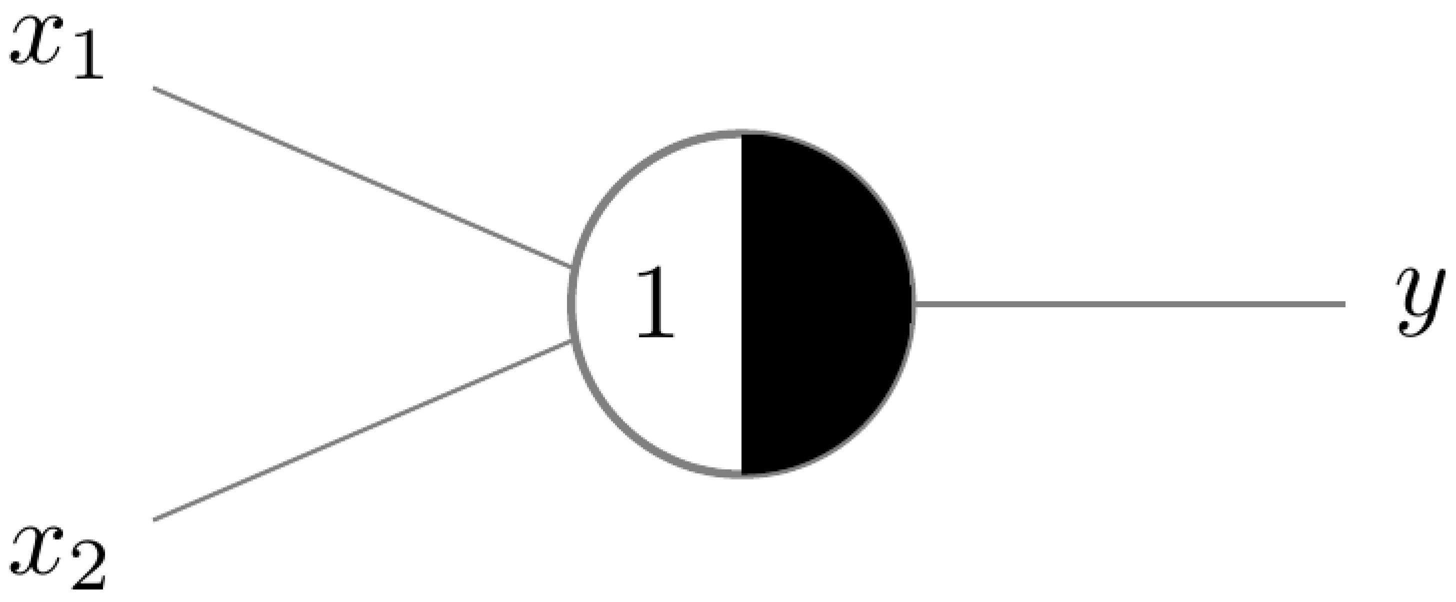

Excursion: Boolean Functions

- Notation:

$$f(x_1, x_2, x_3) \coloneqq x_1 \neg x_2 x_3$$

  means:

$$„\ x_1 \text{and (not } x_2) \text{ and } x_3\ “$$

Excursion: Boolean Functions

x_1	x_2	x_3	$x_1 \neg x_2 x_3$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

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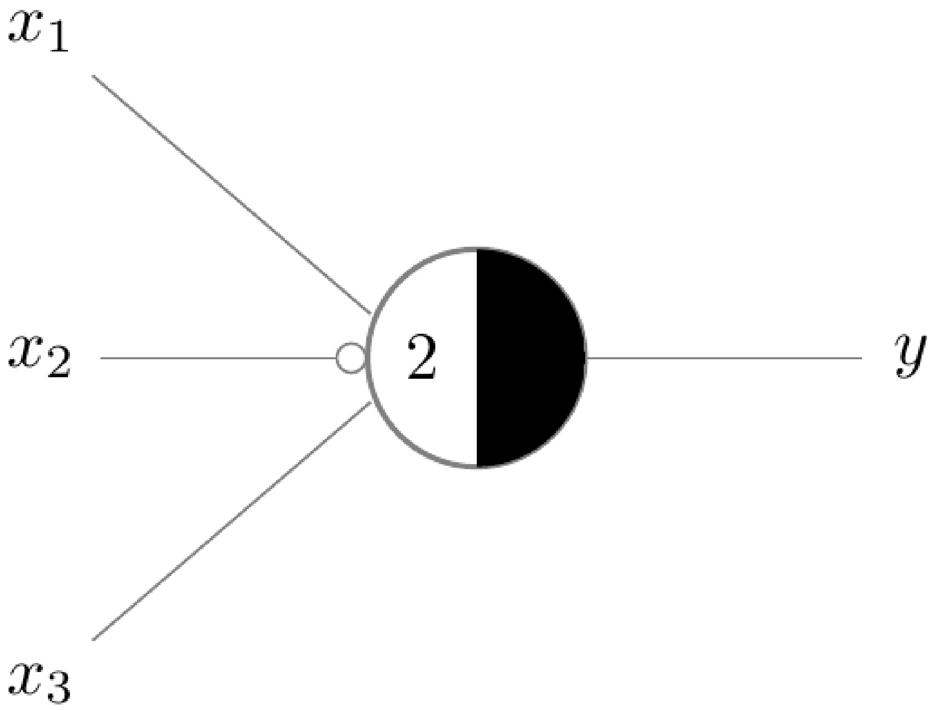
$$f(x_1, x_2, x_3) := x_1 \neg x_2 x_3$$



Excursion: Boolean Functions

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$$f(x_1, x_2, x_3) := x_1 \neg x_2 x_3 \vee x_2$$

  means:

$$„\,[\, x_1 \text{and (not } x_2) \text{ and } x_3]\; or\; x_2 \quad "$$

Excursion: Boolean Functions

$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$x_1 \neg x_2 x_3 \vee x_2$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Excursion: Boolean Functions

x_1	x_2	x_3	$x_1 \neg x_2 x_3 \vee x_2$
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0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
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Excursion: Boolean Functions

x_1	x_2	x_3	$x_1 \neg x_2 x_3 \lor x_2$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	1
1	1	0	
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