UNIVERSITÄT DUISBURG ESSEN

Open-Minded

AdaLinE

Neuroinformatics Tutorial 7

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Content

- Revision: Practical Task
- Revision: Lecture
- Tensorflow
- New Practical Task

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- **Revision: Practical Task**
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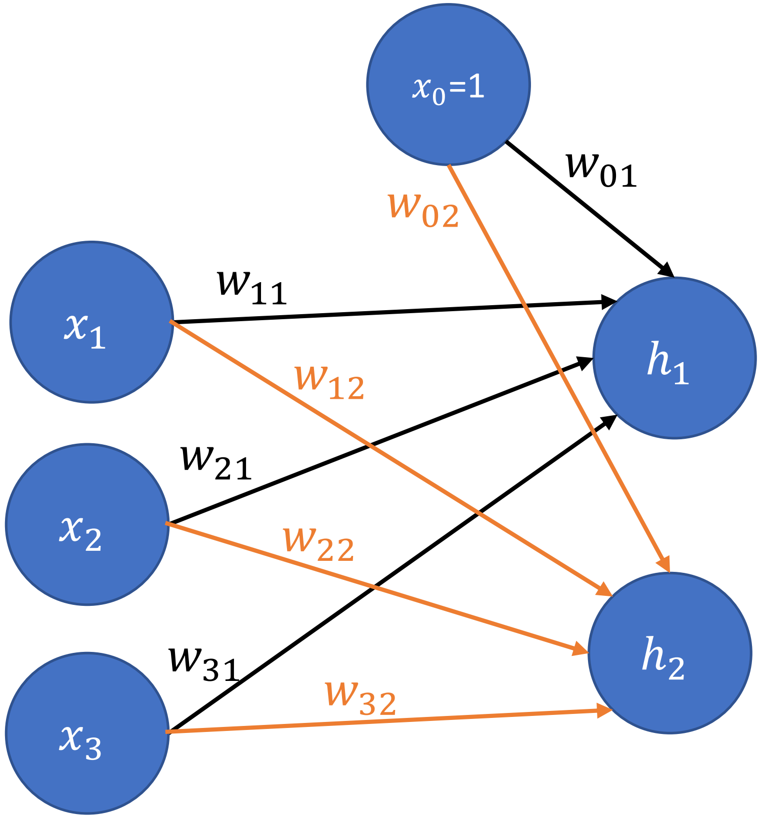
Calculation of propagated value



$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^{3} w_{i2} x_i$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \qquad \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$

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  - Activation function: Identity!

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  - Let $w(i)$ denote the weight vector at iteration i
  - Let $x := (1, x_1, \ldots, x_n)^T \in \Omega \subset \mathbb{R}^{n+1}$ denote an arbitrary extended sample point from the training data set

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- Let $\tilde{y}_{w(i)}(x) := f_a(f_p(x))$ denote the actual output of the AdaLinE with weight vector $w(i)$

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    - Add fraction (depending on learning rate and error) of sample $\boldsymbol{x}$ to current weight vector!

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$$w(i+1) \leftarrow w(i) + \alpha \frac{\rho(x,i)x}{||x||^2}$$

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Amount of error correction dependent on weight update!

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- Why?

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    for any weight update $\boldsymbol{\Delta \tilde{w}}$
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$$|\Delta \tilde{w}^T x| = |\Delta \rho(x, i)| =: \zeta$$

To prove this claim we will use the Cauchy Schwarz Inequality:

$$|\Delta \tilde{w}^T x|^2 \leq ||\Delta \tilde{w}||^2 ||x||^2$$

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- Finally:

$$||\gamma x|| \leq ||\Delta\tilde{w}||$$

Reminder

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- I.e. parallel to x !

Revision: Lecture

- Alternative to Proportional Learning rule?
  - Gradient Descent on some loss function
    (In Lecture: MSE)

  - Basically any optimization approach could work!
    - Optimization Problem:
      Find weight vector, such that loss is minimal

Drawing

Content

- Revision: Practical Task
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- Tensorflow
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Tensorflow

- Deep Learning Library (from Google)
- Widely used for Deep Learning Applications
- Some (popular) alternatives:
  - PyTorch (Facebook, before: (also) NYU)
  - Caffe2 (Facebook, before: UC Berkeley)
  - CNTK (Microsoft)
  - MXNet (U Washington, MIT, Hong Kong U, etc..., associated with Amazon)
  - Theano (U Montréal -> development discontinued)
  - Keras (High Level Interface for Tensorflow, CNTK, Theano, MXNet)
    - Integrated in Tensorflow >= 2.0

Tensorflow

- Works with computational graphs
- Processes tensors

Computational Graph

- ```
  x = 5;
  y = 4;
  w = 3;
  a = x-y;
  b = y*w;
  c = b+w;
  d = a*c;
  ```
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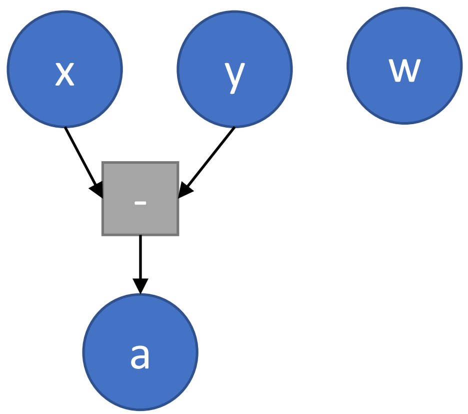

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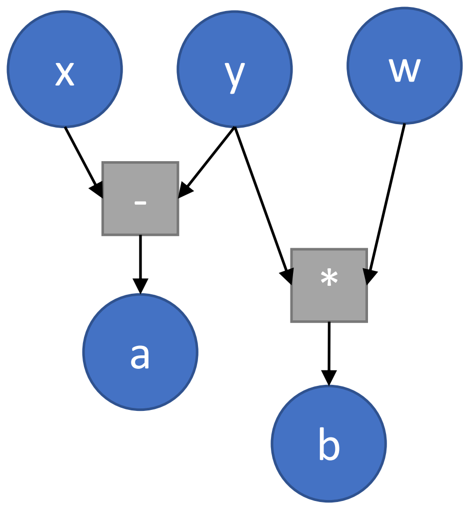
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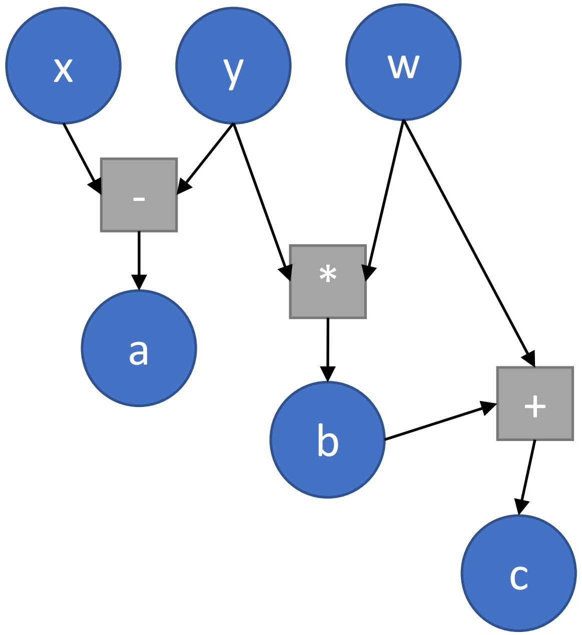

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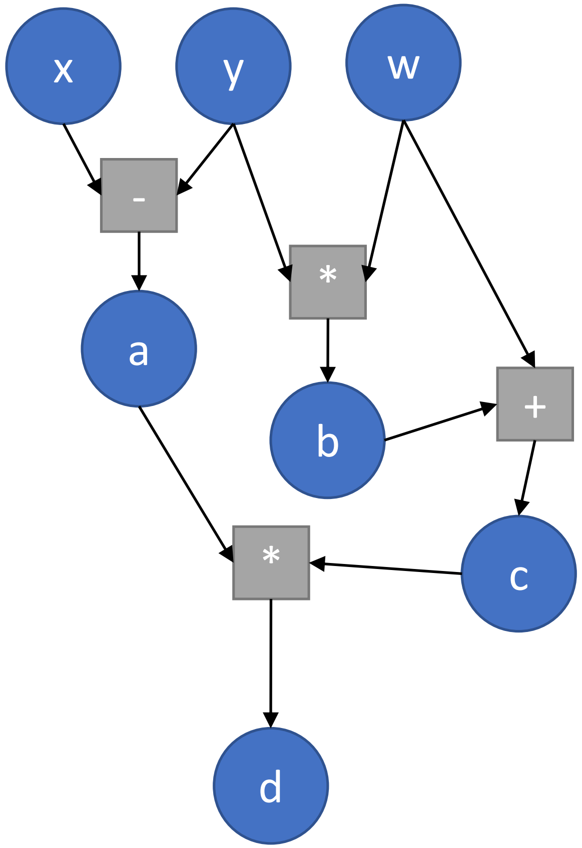
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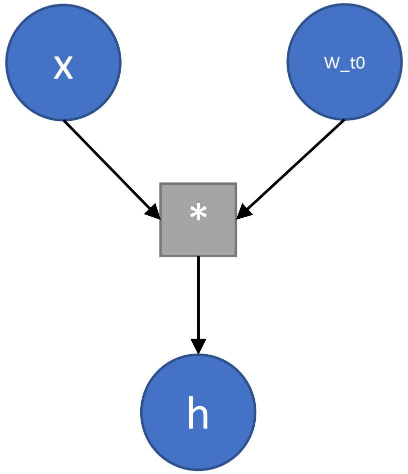
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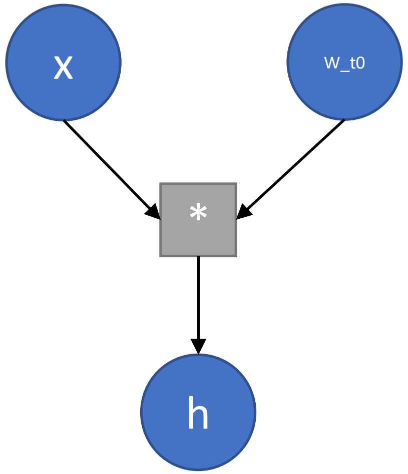
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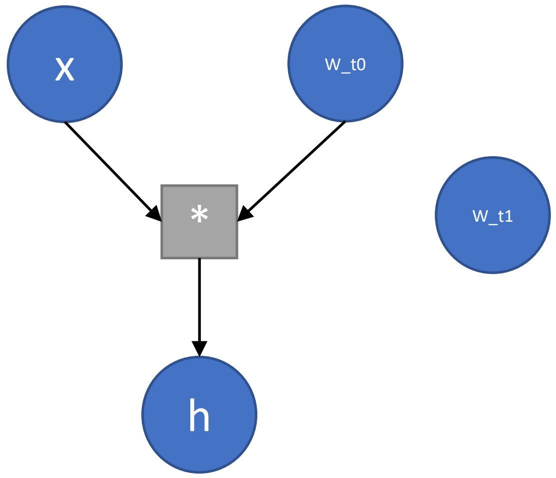

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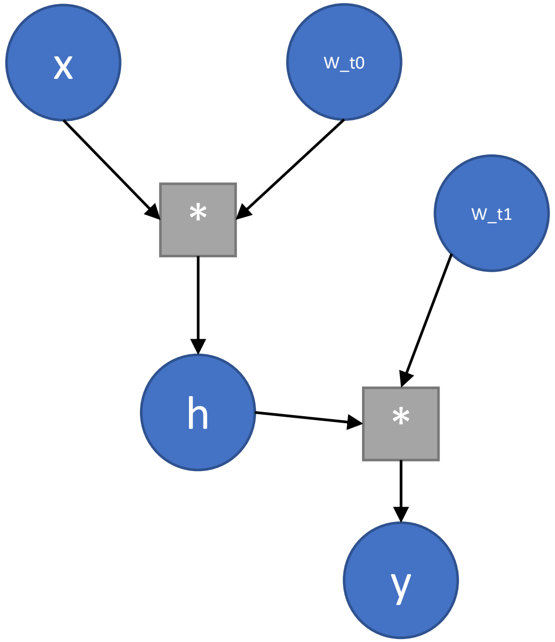

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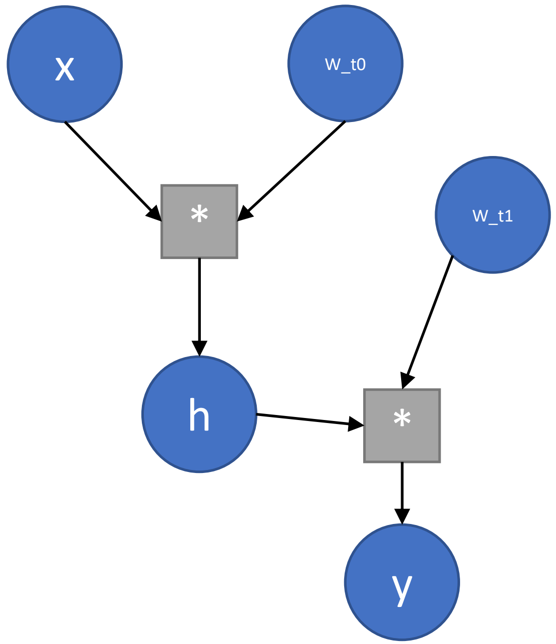
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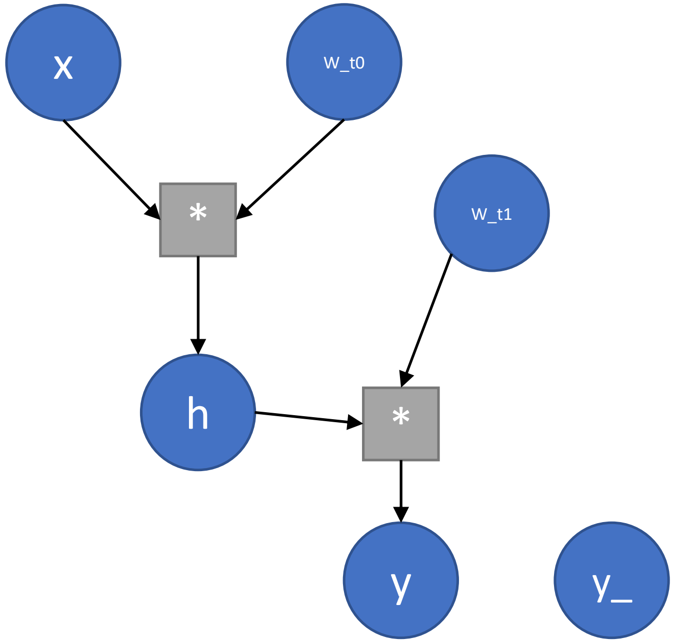

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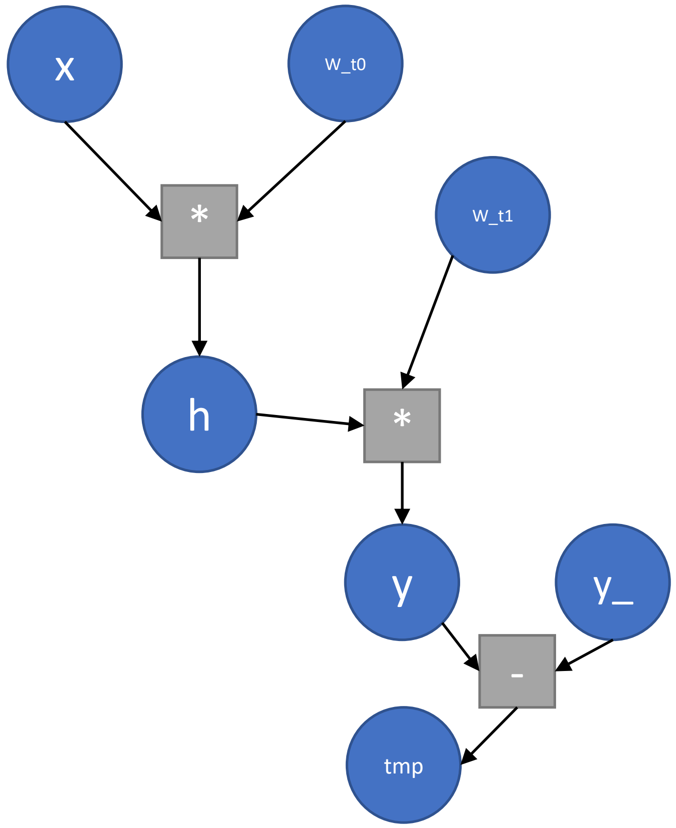

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h    = tf.matmul(W_t0, x)                 #3x1

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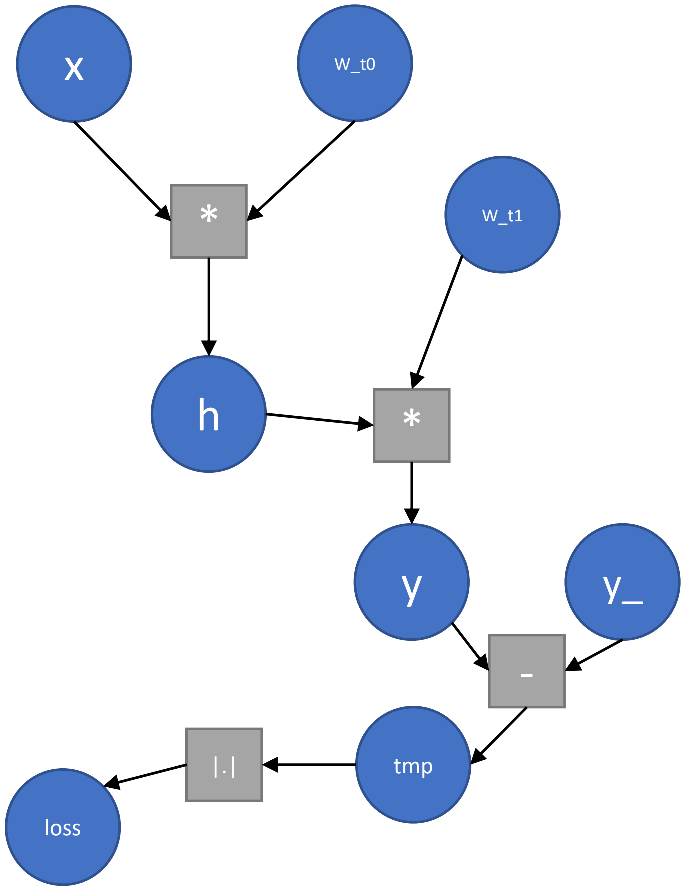
y    = tf.matmul(W_t1, x)                 #1x1



y_   = np.array([[3.3]])
loss = tf.abs(y-y_)
```



Computational Graph – loss

```
x    = np.array([[1.0],[2.1],[3.1]])      #3x1
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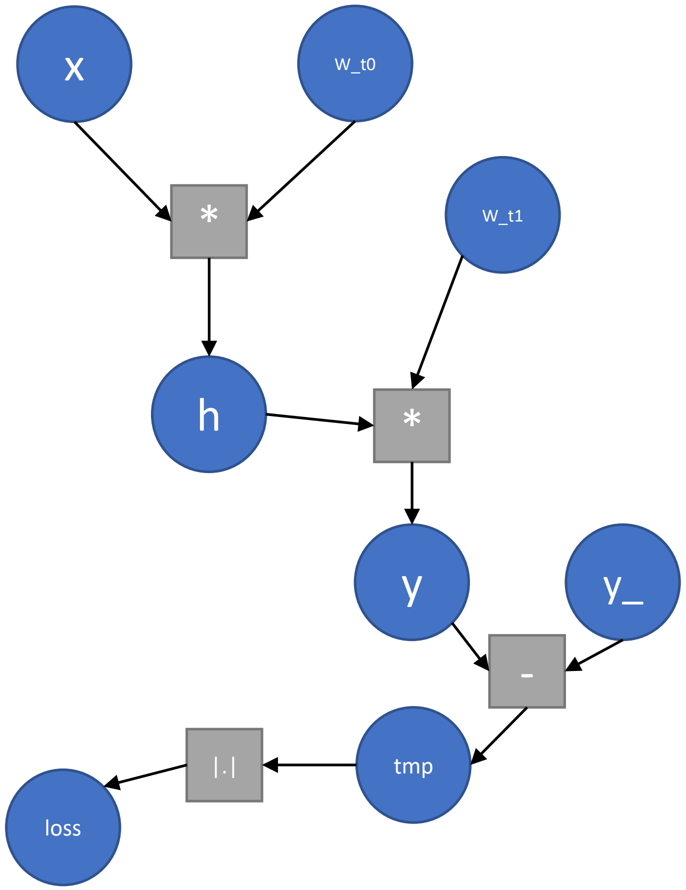
y_   = np.array([[3.3]])
loss = tf.abs(y-y_)

log_loss = tf.log(loss)
```



Computational Graph – loss

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W_t0 = tf.Variable(np.random.rand(3,3))   #3x3
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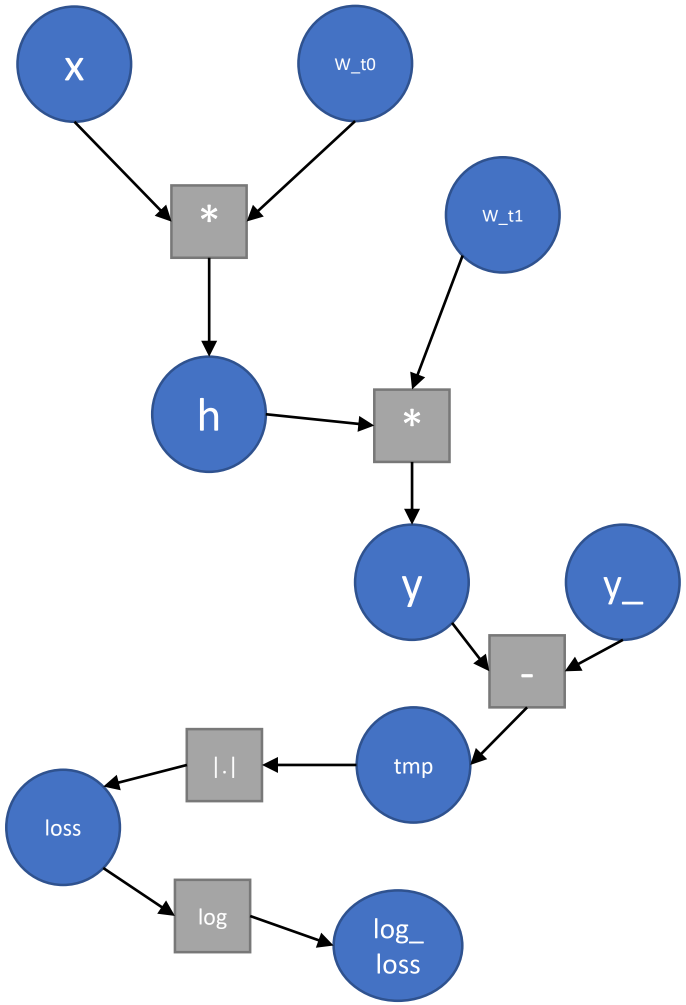

Computational Graph – loss

```
• x    = np.array([[1.0],[2.1],[3.1]])     #3x1
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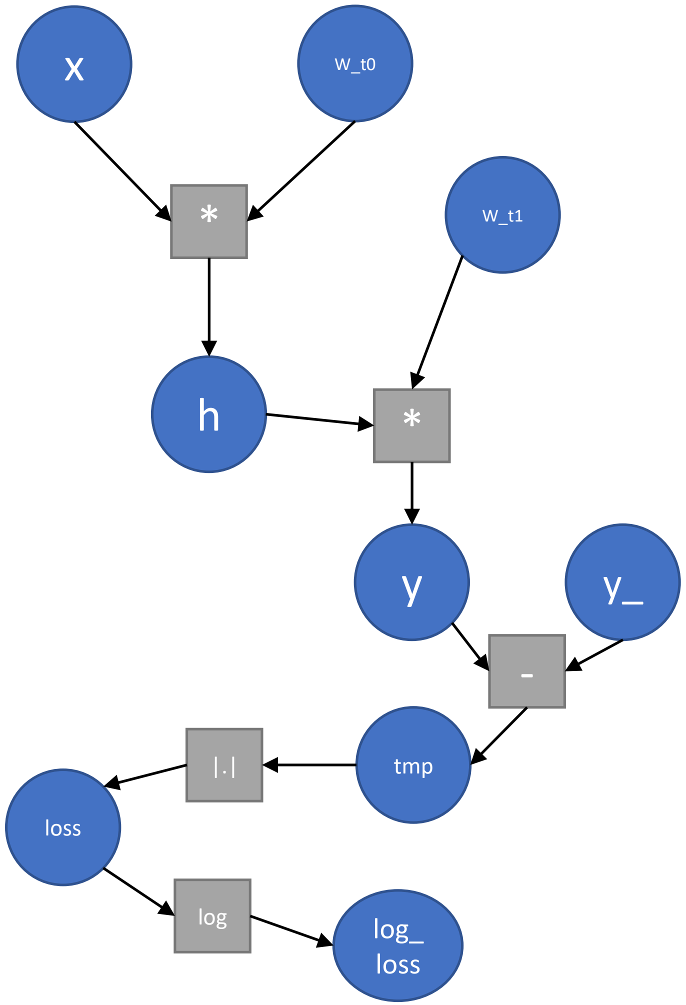

Computational Graph – optimization

```
x    = np.array([[1.0],[2.1],[3.1]])       #3x1
W_t0 = tf.Variable(np.random.rand(3,3))    #3x3
h    = tf.matmul(W_t0, x)                  #3x1

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```

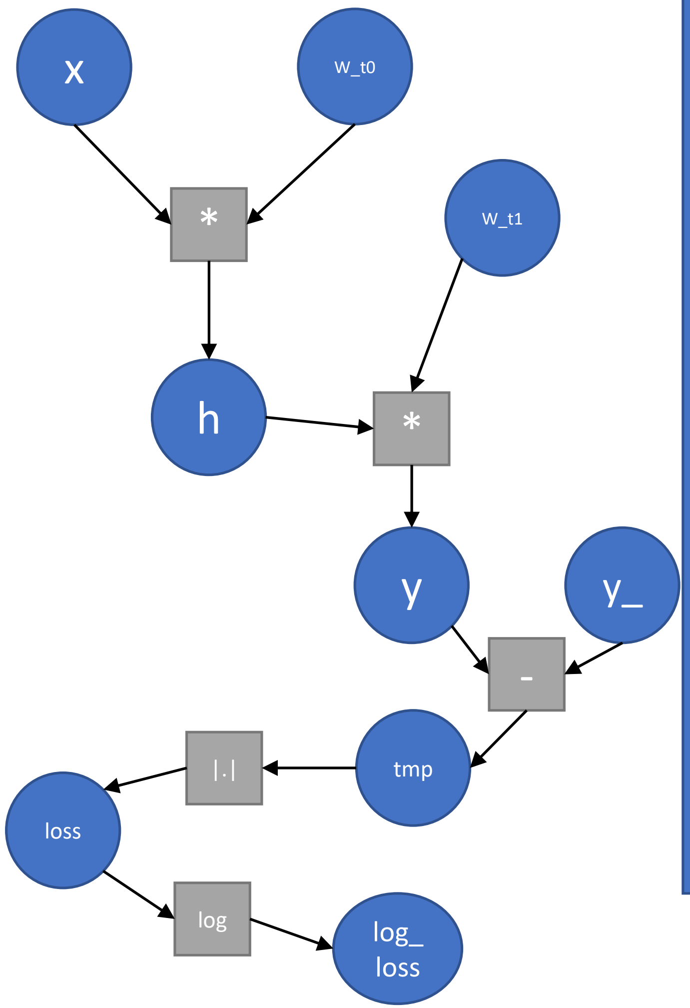

Goal:

Computational Graph – optimization

```
x    = np.array([[1.0],[2.1],[3.1]])      #3x1
W_t0 = tf.Variable(np.random.rand(3,3))   #3x3
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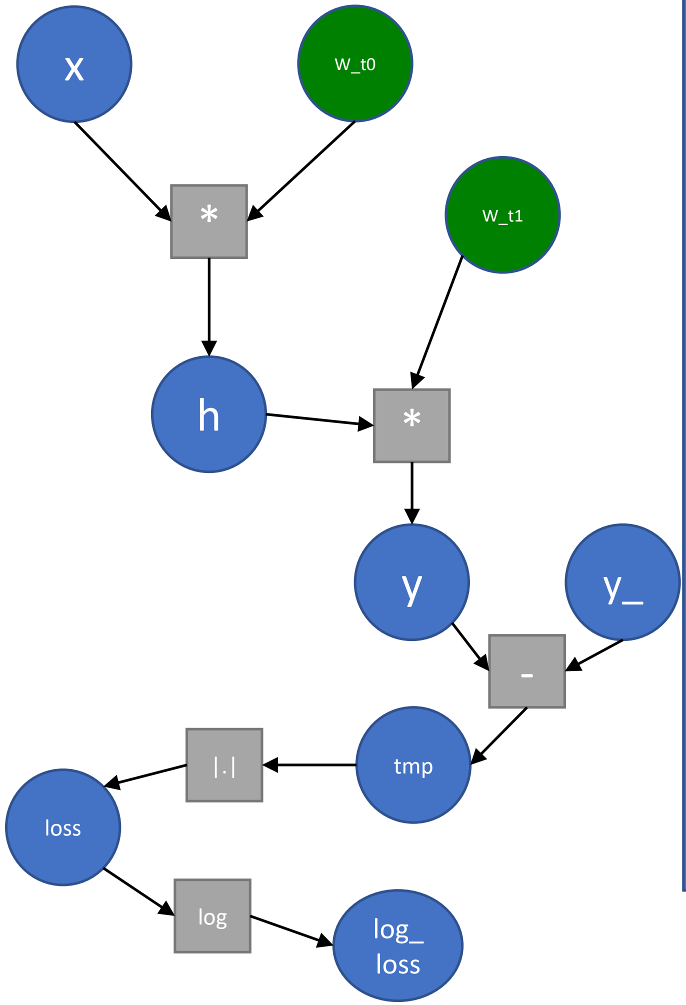

Goal:
Adapt variables

Computational Graph – optimization

```
x    = np.array([[1.0],[2.1],[3.1]])       #3x1
W_t0 = tf.Variable(np.random.rand(3,3))    #3x3
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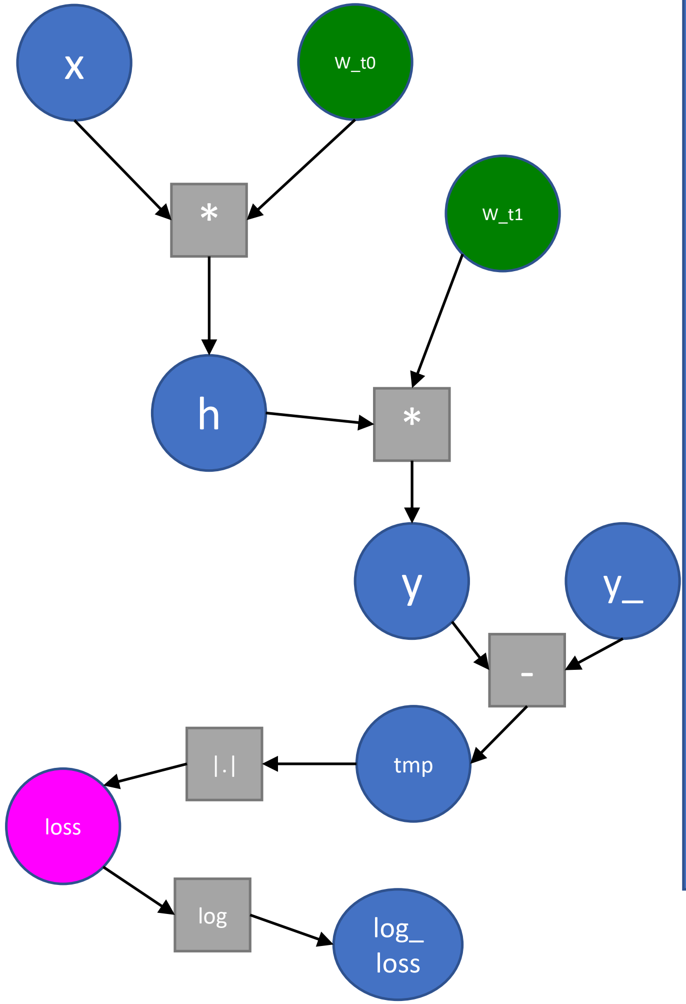
```



Goal:
Adapt variables
to minimize loss!

Computational Graph – optimization

```
• x    = np.array([[1.0],[2.1],[3.1]])      #3x1
  W_t0 = tf.Variable(np.random.rand(3,3))   #3x3
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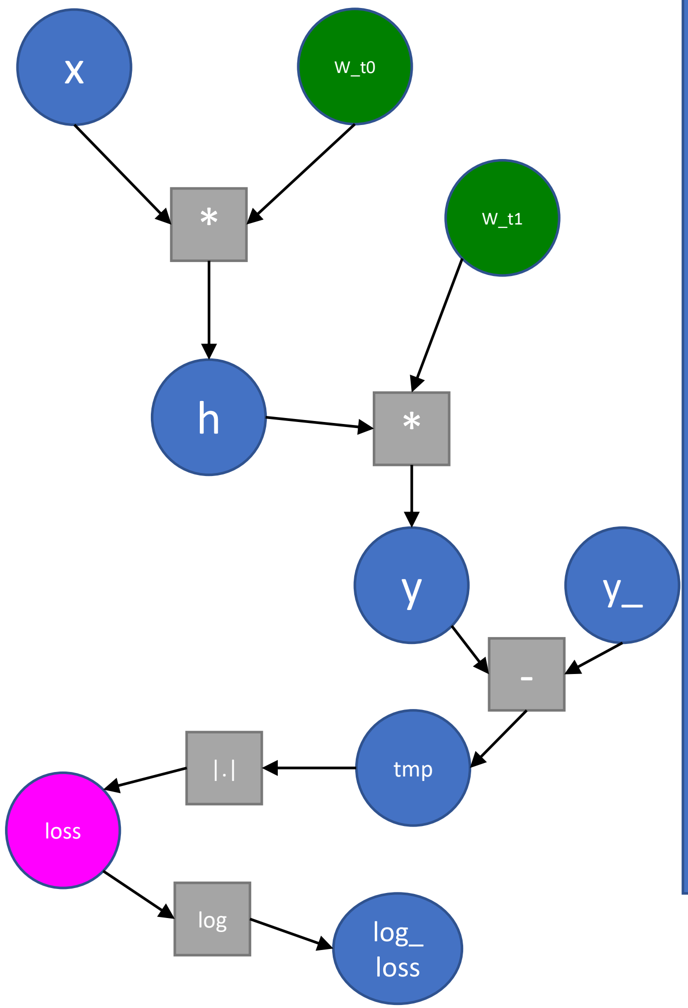
```



Goal:
Adapt variables
to minimize loss!

Use gradient based
optimizer!

Computational Graph – optimization

```
x    = np.array([[1.0],[2.1],[3.1]])       #3x1
W_t0 = tf.Variable(np.random.rand(3,3))    #3x3
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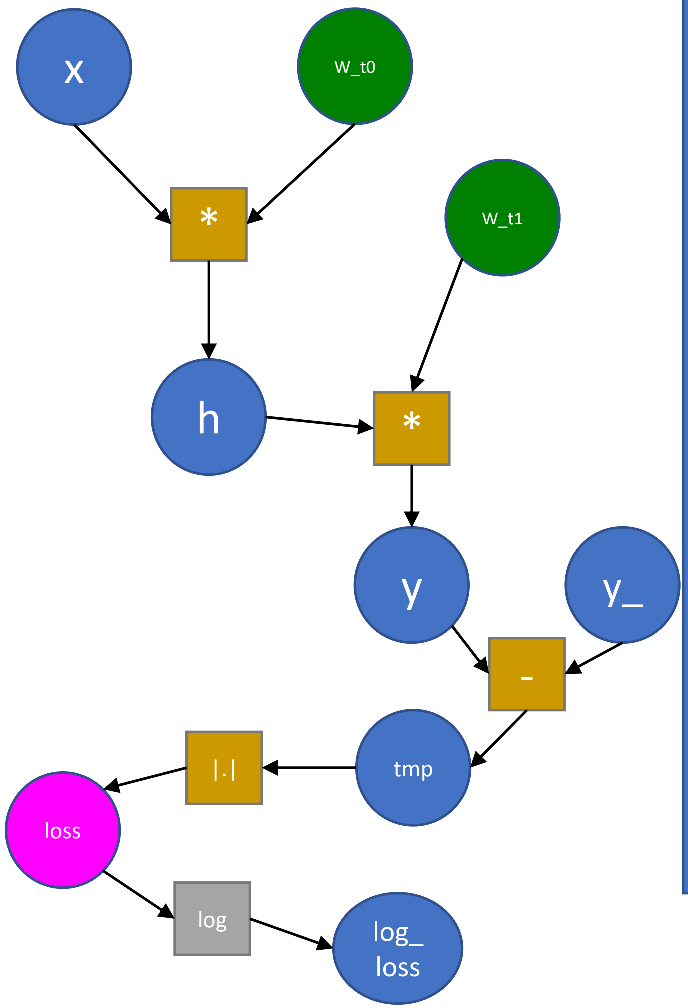

Goal:
Adapt variables
to minimize loss!

Use gradient based optimizer!

Need to differentiate along a chain of operations (remember chain rule)!

Computational Graph – optimization

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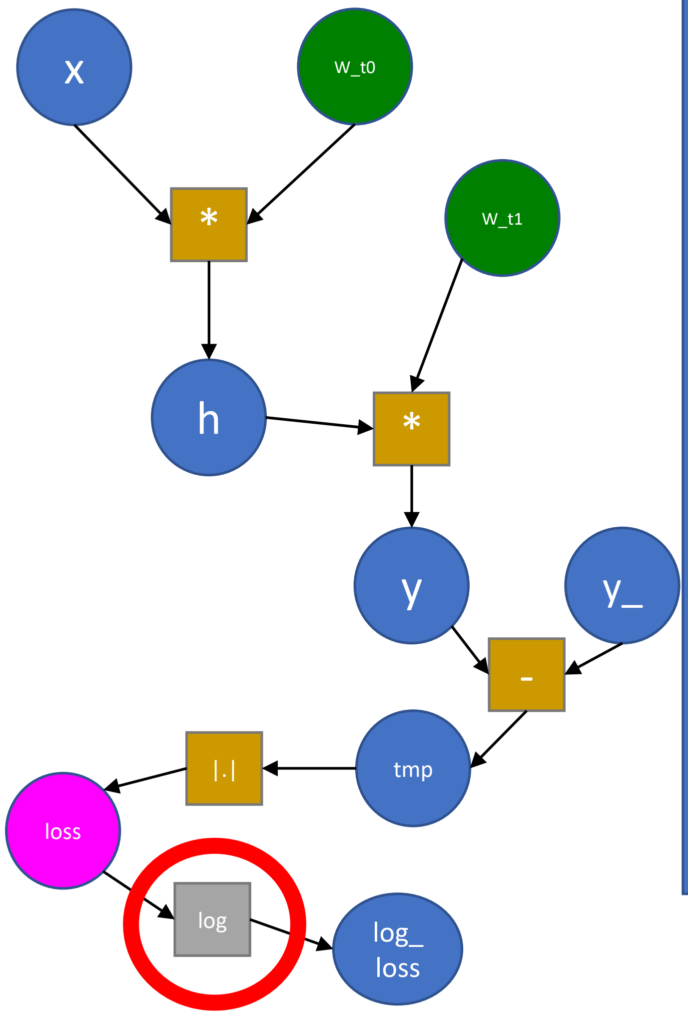

Goal:
Adapt variables
to minimize loss!

Use gradient based optimizer!

Need to differentiate along a chain of operations (remember chain rule)!

Observation:
Some operations within graph are not needed!

Computational Graph – optimization

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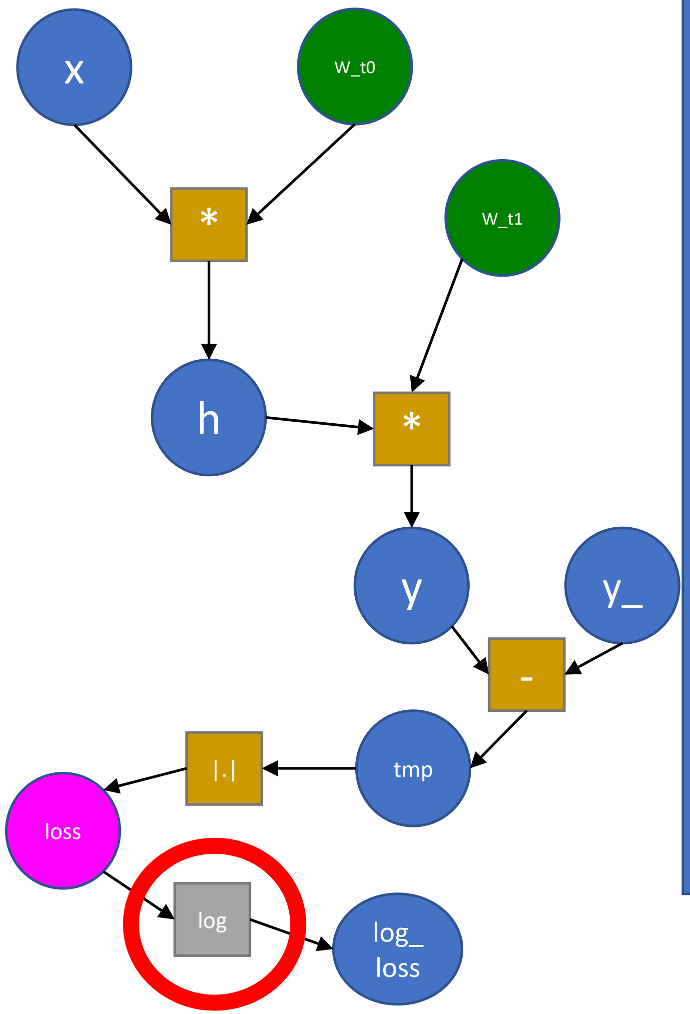

Therefore:
„Tape" only relevant portion of graph for optimization!

Tensorflow - Example

```
import numpy as np
import tensorflow as tf

class myModel():
    def __init__(self, num_inputs, num_outputs):
        self.num_inputs   = num_inputs
        self.weights_t0   = tf.Variable(np.random.rand(3,num_inputs))
        self.weights_t1   = tf.Variable(np.random.rand(num_outputs,3))

        self.optimizer    = tf.keras.optimizers.Adam(learning_rate=0.01)
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    def force_col_vec(self, new_input):
        new_input   = np.array(new_input)
        vec_length = np.prod(new_input.shape)

        return np.reshape(new_input, [vec_length, 1])
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    def get_output(self, new_input):
        new_input = self.force_col_vec(new_input)
        h = tf.matmul(self.weights_t0, new_input)
        y = tf.matmul(self.weights_t1, h)

        return y
```

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    def get_loss(self, new_input, target_output):
        model_output = self.get_output(new_input)
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    def get_loss(self, new_input, target_output):
        model_output = self.get_output(new_input)
        loss = tf.abs(model_output- target_output)

        return loss

    def _get_gradient(self, new_input, target_output):
        with tf.GradientTape() as tape:
            loss = self.get_loss(new_input, target_output)
            grad = tape.gradient(loss,[self. weights_t0, self.weights_t1])

        return grad
```

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        return grad

    def update_weights(self, new_input, target_output):
        grad = self._get_gradient(new_input, target_output)
        self.optimizer.apply_gradients(zip(grad ,[self. weights_t0, self.weights_t1]))
```

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Content

- Revision: Practical Task
- Revision: Lecture
- Tensorflow
- New Practical Task