

**Open-**Minded

### AdaLinE

Neuroinformatics Tutorial 7

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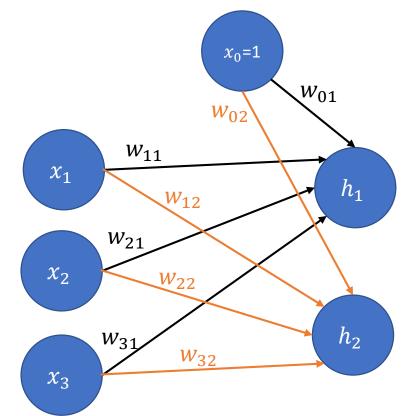


### Content

- Revision: Practical Task
- Revision: Lecture
- Tensorflow
- New Practical Task



# Calculation of propagated value



$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^3 w_{i2} x_i$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$



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  - Real input:
  - Real weights
  - Propagation function (linear associator)

 $(1, x_1, \dots, x_n)^T \in \mathbb{R}^{n+1}$  $(-\Theta, w_1, \dots, w_n)^T \in \mathbb{R}^n$ 

$$\sum_{i=0}^n w_i x_i$$

Activation function: Identity!



- Which statements regarding AdaLinE and RBP are true?
  - 1. AdaLinE is capable of regression
  - 2. RBP has same activation function as AdaLinE
  - 3. AdaLinE has same structure as RBP except for propagation function
  - 4. RBP has same structure as AdaLinE except for activation function



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  - Let  $w:=(-\Theta,w_1,\ldots,w_n)^T\in\mathbb{R}^{n+1}$  denote the extended weight vector including the bias
  - Let w(i) denote the weight vector at iteration i
  - Let  $x := (1, x_1, ..., x_n)^T \in \Omega \subset \mathbb{R}^{n+1}$  denote an arbitrary extended sample point from the training data set



- Let  $\hat{y}(x) \in \mathbb{R}$  denote the desired target output
- Let  $\tilde{y}_{w(i)}(x) := f_a(f_p(x))$  denote the actual output of the AdaLinE with weight vector w(i)



- Idea:
  - Draw a sample point  $\boldsymbol{x}$  randomly
  - Check if AdaLinE output is target output
  - If not, adjust the weights!
    - Calculate error (difference in desired output)

$$\rho(x,i) := \hat{y}(x) - \tilde{y}_{w(i)}(x)$$

• Add fraction (depending on learning rate and error) of sample  $m{x}$  to current weight vector!



• If 
$$\hat{y}(x) == \tilde{y}_{w(i)}(x)$$
  
Do nothing

• If 
$$\hat{y}(x) \neq \tilde{y}_{w(i)}(x)$$
 
$$w(i+1) \leftarrow w(i) + \alpha \frac{\rho(x,i)x}{||x||^2}$$



We can generalize the weight update rule to:

$$w(i+1) \leftarrow w(i) + \Delta w(i)$$

Therefore the amount of error reduction for AdaLinE is:

$$\begin{split} |\Delta \rho(x,i)| &:= |\rho(x,i+1) - \rho(x,i)| \\ &= |[\hat{y}(x) - \tilde{y}_{w(i+1)}(x)] - [\hat{y}(x) - \tilde{y}_{w(i)}(x)]| \\ &= |[\hat{y}(x) - w(i+1)^T x] - [\hat{y}(x) - w(i)^T x]| \\ &= |-(w(i+1) - w(i))^T x| \\ &= |\Delta w(i)^T x| \end{split}$$
 Amount of error correction dependent on weight update!



 There are many possible ways to choose the weight update, such that amount of error correction is the same

$$|\Delta \rho(x,i)| = |\Delta w(i)^T x|$$

• For proportional learning rule we choose weight update  $\Delta w(i)$  such that  $\Delta w(i)$  is parallel to sample point x, i.e.

$$\Delta w(i) := \gamma x, \quad \gamma \in \mathbb{R}$$

$$(\Delta w(i) := \alpha \frac{\rho(x,i)x}{||x||^2}, \quad \alpha \in \mathbb{R})$$

Why?



## Previous Learning Achievements

- Learning achievement is encoded in w(i)
- We want  $||\Delta w(i)||$  to be small
- Claim:
  - If  $\Delta w(i)$  is parallel to x , i.e.  $\Delta w(i) := \gamma x, \ \ \gamma \in \mathbb{R}$  then  $||\gamma x|| \leq ||\Delta \tilde{w}||$

for **any** weight update  $\Delta \tilde{w}$  with the same error reduction

$$|\Delta \tilde{w}^T x| = |\Delta \rho(x, i)| =: \zeta$$



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for **any** weight update  $\Delta \tilde{w}$  with the same error reduction

$$|\Delta \tilde{w}^T x| = |\Delta \rho(x, i)| =: \zeta$$

To prove this claim we will use the Cauchy Schwarz Inequality:

$$|\Delta \tilde{w}^T x|^2 \le ||\Delta \tilde{w}||^2 ||x||^2$$



## Previous Learning Achievements

Directly from Cauchy Schwarz

$$\zeta^2 = |\Delta \tilde{w}^T x|^2 \le ||\Delta \tilde{w}||^2 ||x||^2$$

• By design (choose scalar factor accordingly):

$$\zeta^2 = |\gamma x^T x|^2 = ||\gamma x||^2 ||x||^2$$

• Therefore:

$$||\gamma x||^2 ||x||^2 \le ||\Delta \tilde{w}||^2 ||x||^2$$

Finally:

$$||\gamma x|| \le ||\Delta \tilde{w}||$$



### Reminder

For AdaLinE the weight update is defined as:

$$\Delta w(i) := \alpha \frac{\rho(x,i)x}{||x||^2}, \quad \alpha \in \mathbb{R}$$

ullet I.e. parallel to  $oldsymbol{x}$  !



- Alternative to Proportional Learning rule?
  - Gradient Descent on some loss function (In Lecture: MSE)
  - Basically any optimization approach could work!
    - Optimization Problem: Find weight vector, such that loss is minimal

### Drawing





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### Tensorflow

- Deep Learning Library (from Google)
- Widely used for Deep Learning Applications
- Some (popular) alternatives:
  - PyTorch (Facebook, before: (also) NYU)
  - Caffe2 (Facebook, before: UC Berkeley)
  - CNTK (Microsoft)
  - MXNet (U Washington, MIT, Hong Kong U, etc..., associated with Amazon)
  - Theano (U Montréal -> development discontinued)
  - Keras (High Level Interface for Tensorflow, CNTK, Theano, MXNet)
    - Integrated in Tensorflow >= 2.0



### Tensorflow

- Works with computational graphs
- Processes tensors



```
• x = 5;
y = 4;
w = 3;
a = x-y;
b = y*w;
c = b+w;
d = a*c;
```

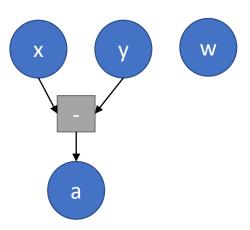


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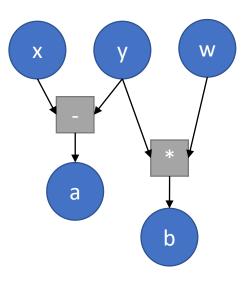


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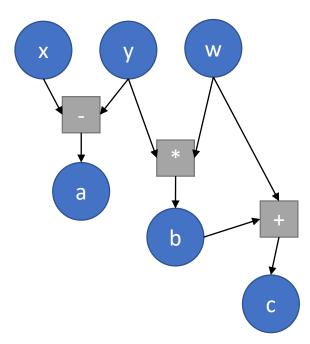


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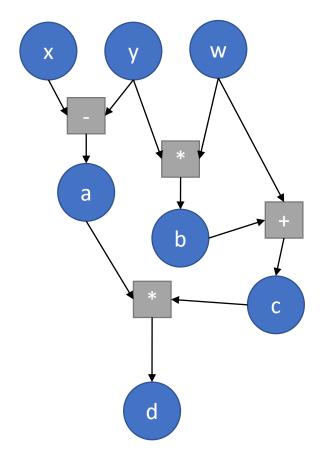


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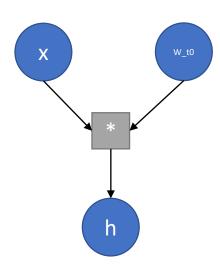
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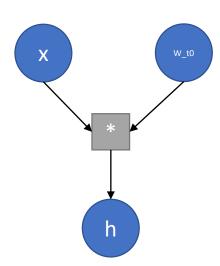
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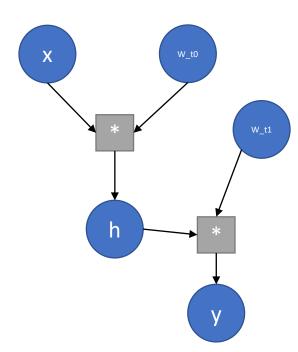
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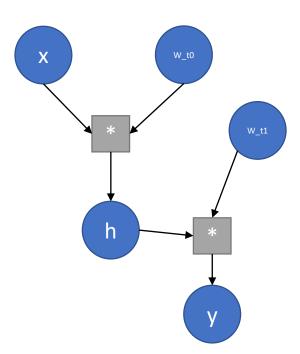




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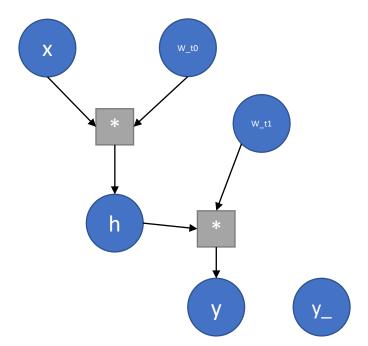




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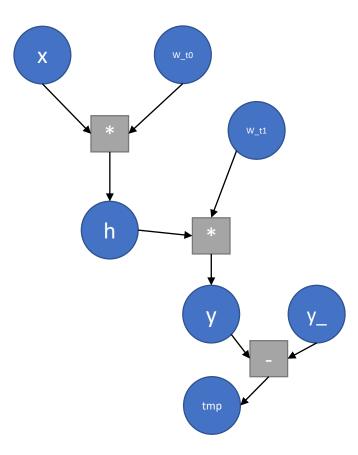




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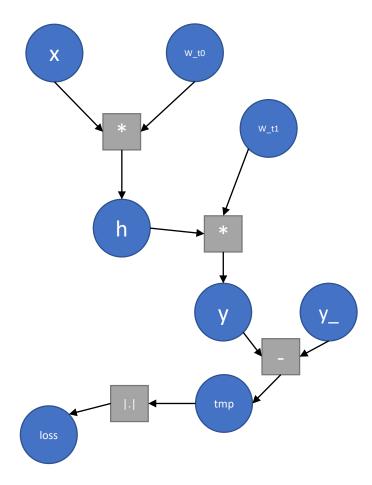




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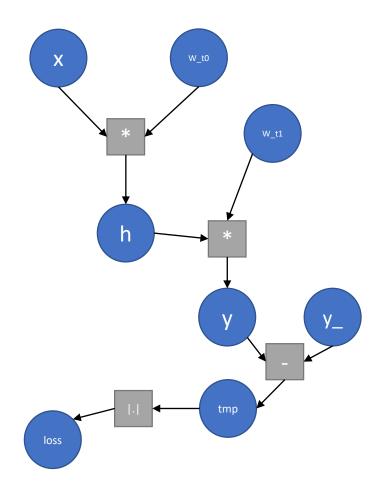


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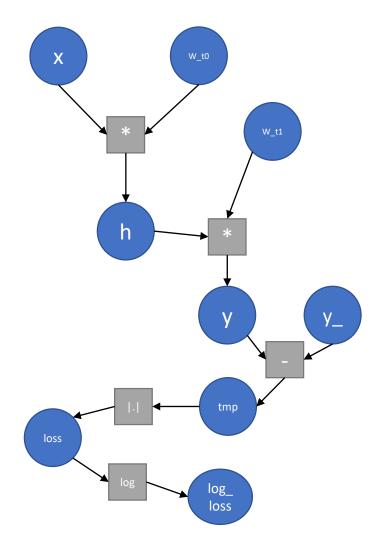
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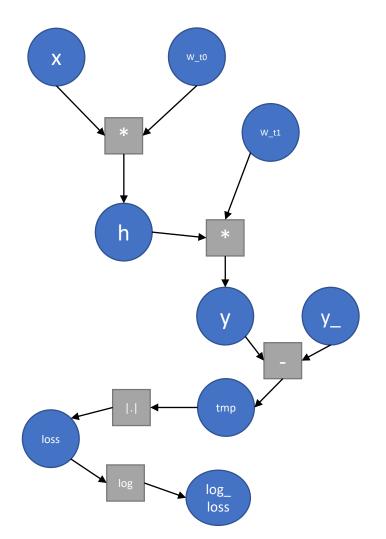


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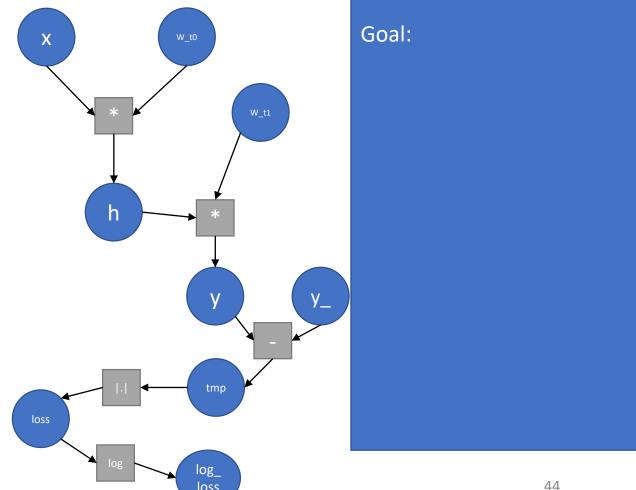
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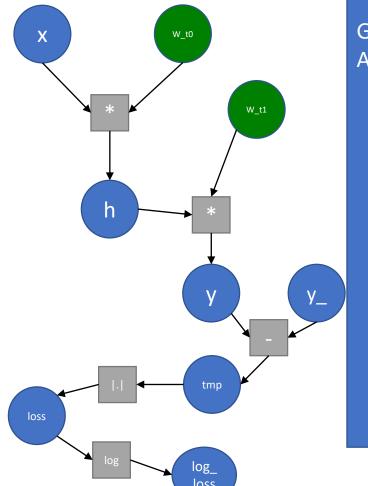


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Goal: Adapt variables

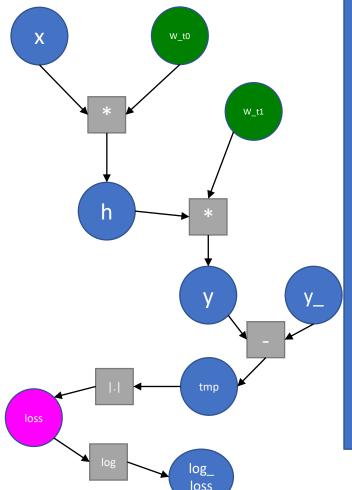


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Goal:
Adapt variables
to minimize loss!

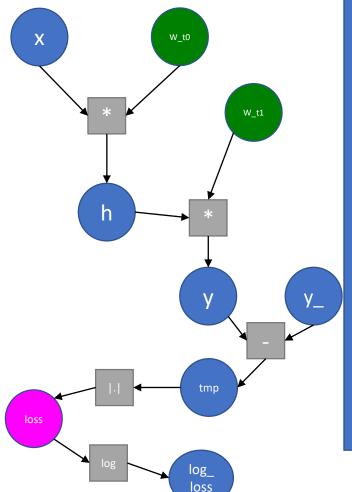


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Goal:
Adapt variables
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Use gradient based optimizer!



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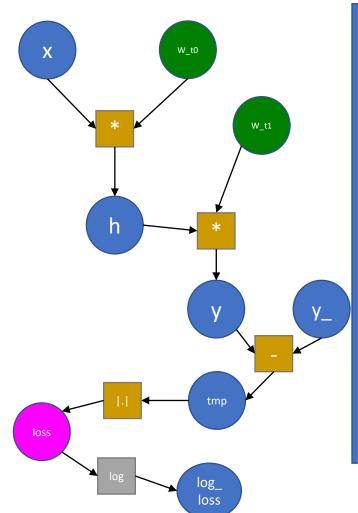
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Use gradient based optimizer!

Need to differentiate along a chain of operations (remember chain rule)!



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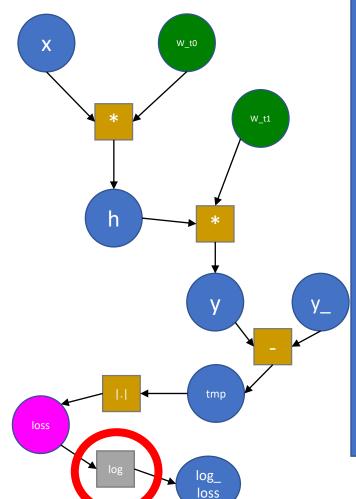
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#### Goal:

Adapt variables to minimize loss!

Use gradient based optimizer!

Need to differentiate along a chain of operations (remember chain rule)!

#### Observation:

Some operations within graph are not needed!



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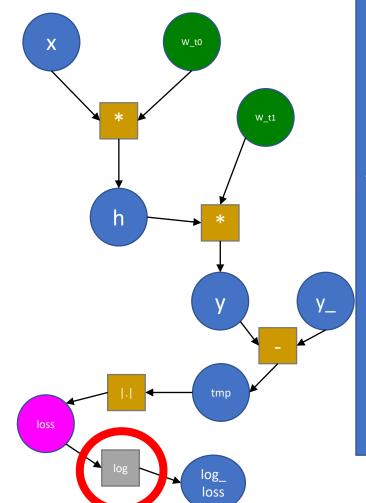
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Therefore: "Tape" only relevant portion of graph for optimization!



```
import numpy as np
import tensorflow as tf

class myModel():
    def __init__(self, num_inputs, num_outputs):
        self.num_inputs = num inputs
        self.weights_t0 = tf.Variable(np.random.rand(3,num_inputs))
        self.weights_t1 = tf.Variable(np.random.rand(num_outputs,3))

    self.optimizer = tf.keras.optimizers.Adam(learning_rate=0.01)
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def force_col_vec(self, new_input):
        new_input = np.array(new_input)
        vec_length = np.prod(new_Input.shape)
        return np.reshape(new_input, [vec_length, 1])
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def get_output(self, new_input):
        new_input = self.force_col_vec(new_input)
        h = tf.matmul(self.weights_t0, new_input)
        y = tf.matmul(self.weights_t1, h)

        return y
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    new_input = self.force_col_vec(new_input)
    h = tf.matmul(self.weights_t0, new_input)
    y = tf.matmul(self.weights_t1, h)
            return y
      def get_loss(self, new_input, target_output):
    model_output = self.get_output(new_input)
            loss = tf.abs(model output- target_output)
             return loss
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import tensorflow as tf
class myModel():
     def init (self, num inputs, num outputs):
          self.num inputs = num inputs
          self.weights_t0 = tf.Variable(np.random.rand(3,num_inputs))
self.weights_t1 = tf.Variable(np.random.rand(num_outputs,3))
          self.optimizer = tf.keras.optimizers.Adam(learning rate=0.01)
     def force col vec(self, new input):
          new input = np.array(new input)
          vec length = np.prod(new Input.shape)
          return np.reshape(new input, [vec length, 1])
    def get_output(self, new_input):
    new_input = self.force_col_vec(new_input)
    h = tf.matmul(self.weights_t0, new_input)
    y = tf.matmul(self.weights_t1, h)
          return y
     def get loss(self, new input, target_output):
          model output = self.get output(new input)
          loss = tf.abs(model output- target_output)
          return loss
     def get gradient(self, new input, target output):
          with tf.GradientTape() as tape:
                loss = self.get loss(new_input, target output)
grad = tape.gradient(loss,[self.weights_t0, self.weights_t1])
          return grad
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```
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### Content

- Revision: Practical Task
- Revision: Lecture
- Tensorflow
- New Practical Task

