Hebbian Learning

Neuroinformatics Tutorial 5

Duc Duy Pham[1]

[1]Intelligent Systems, Faculty of Engineering,
University of Duisburg-Essen, Germany

Content

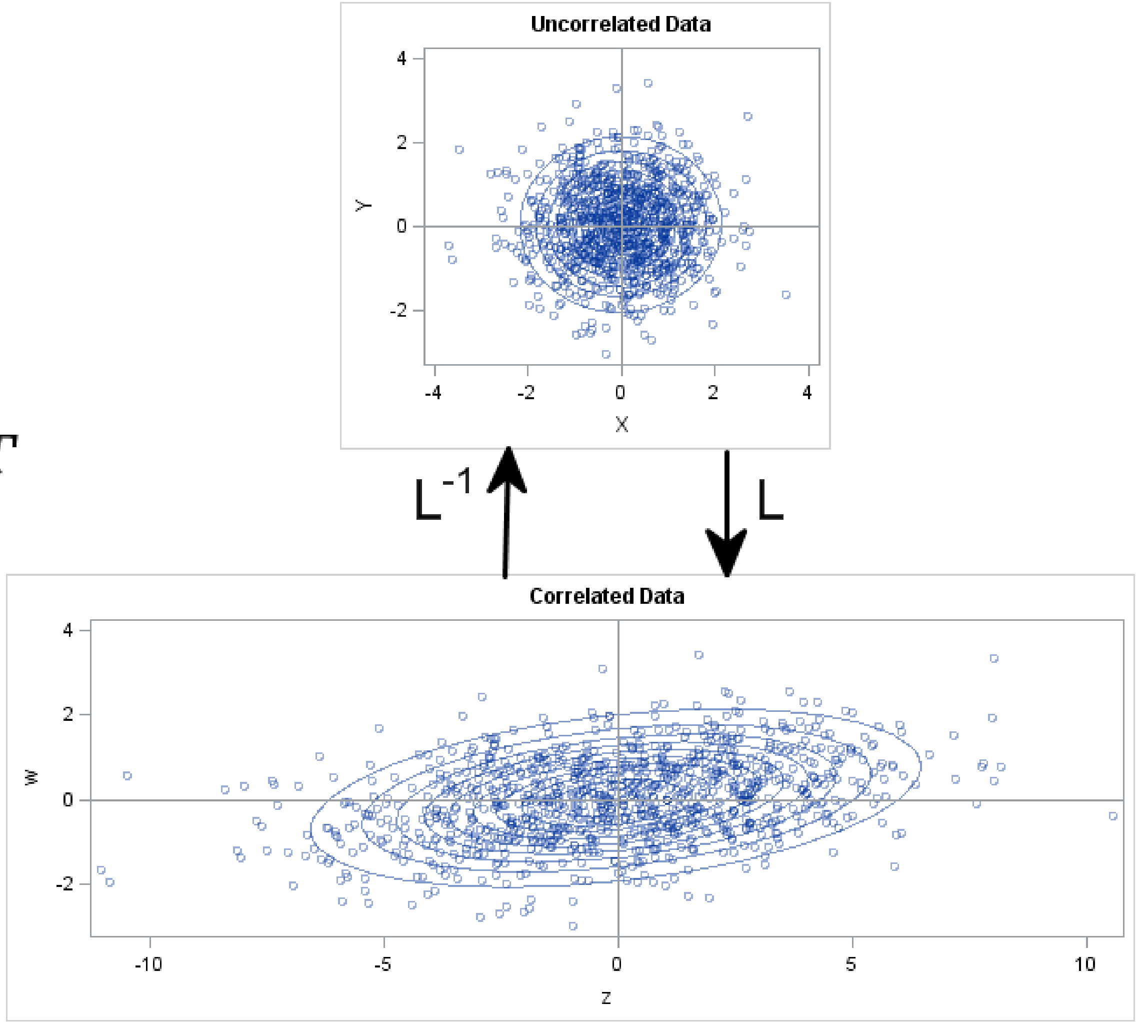
- **Revision: Mahalanobis Classifier**
- Revision: Lecture
- Hebbian Learning

Revision: Mahalanobis Distance

- What is the main idea of the Mahalanobis distance?

Inverse Cholesky Transform

$$L := U^T$$



Revision: Mahalanobis Distance

- What statement regarding the Mahalanobis distance is true?

Revision: Mahalanobis Distance

- What do you need to calculate the Mahalanobis distance in practice?

Revision: Mahalanobis Distance

- How can you use the Mahalanobis distance for classification?

Revision: Mahalanobis Distance

- How do you use the Mahalanobis distance for classification (step by step)?

Revision: Mahalanobis Distance

- „Training: "
  - Calculate mean for each class
  - Calculate covariance matrix for each class

$$\mathrm{Cov}(\mathrm{X}_k) = \frac{1}{N_k} \sum_{i=1}^{N_k} (x_i - \mu_k)(x_i - \mu_k)^T$$

  - Invert covariance matrices

- Inference:
  - Calculate Mahalanobis distance to each class mean
  - Choose class with least distance

Mahalanobis Classifier: Jupyter

Content

- Revision: Mahalanobis Classifier
- **Revision: Lecture**
- Hebbian Learning

Revision: Lecture

- When are two sets $\mathcal{P}, \mathcal{N} \subset \mathbb{R}^n$ linearly separable?

Revision: Lecture

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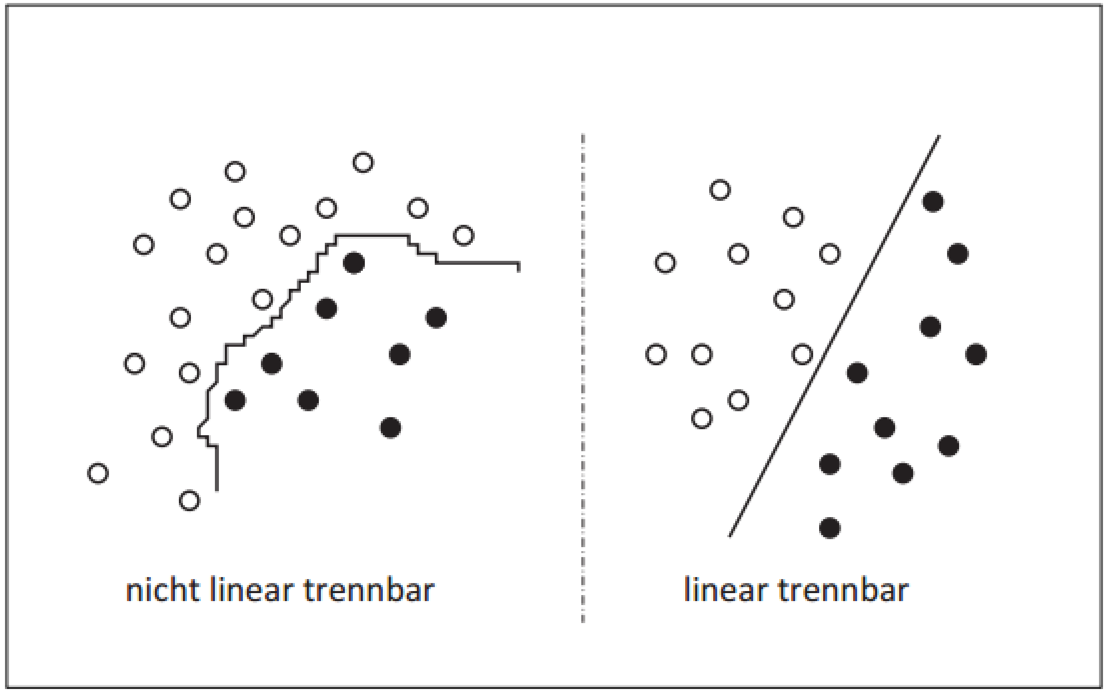

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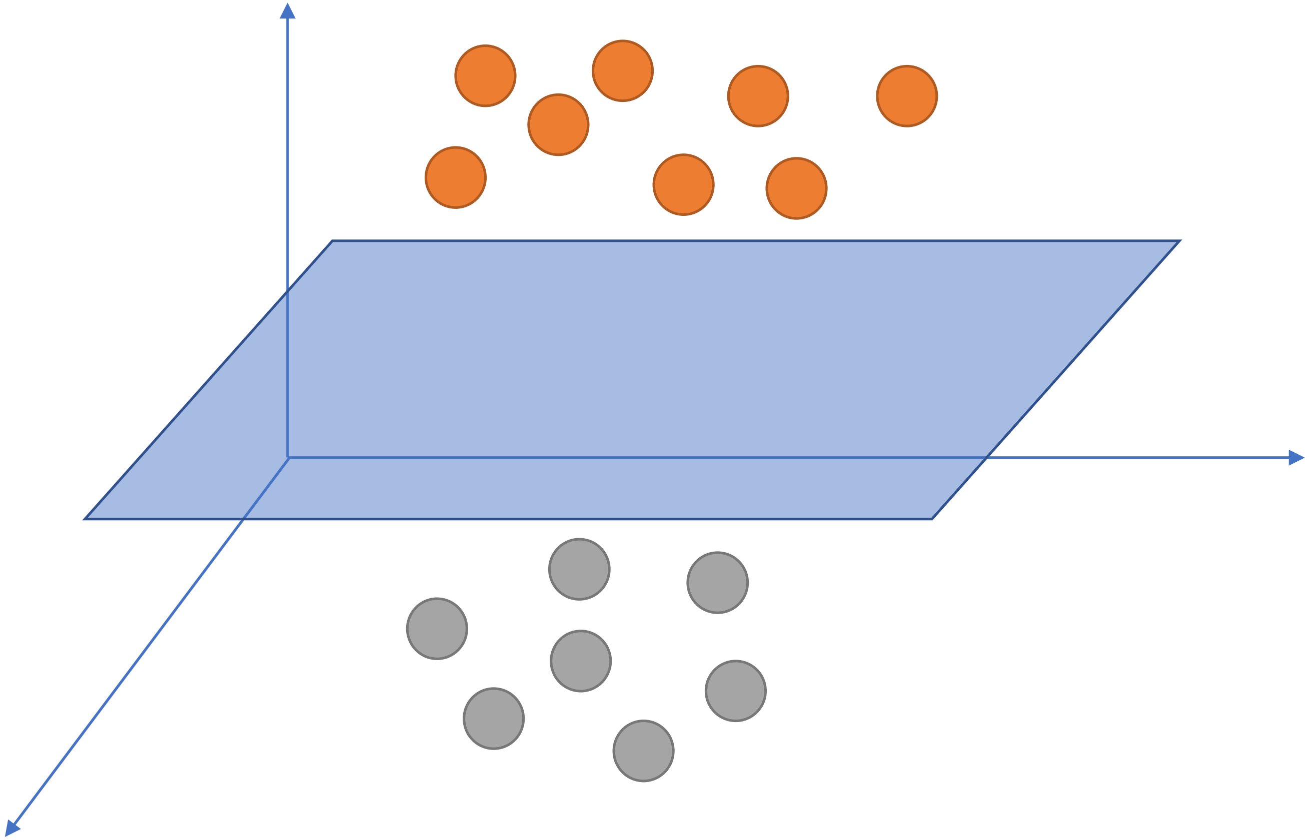


Revision: Lecture

- Formal definition of (absolute) linear separability of two sets $\mathcal{P}, \mathcal{N} \subset \mathbb{R}^n$ (must know!):

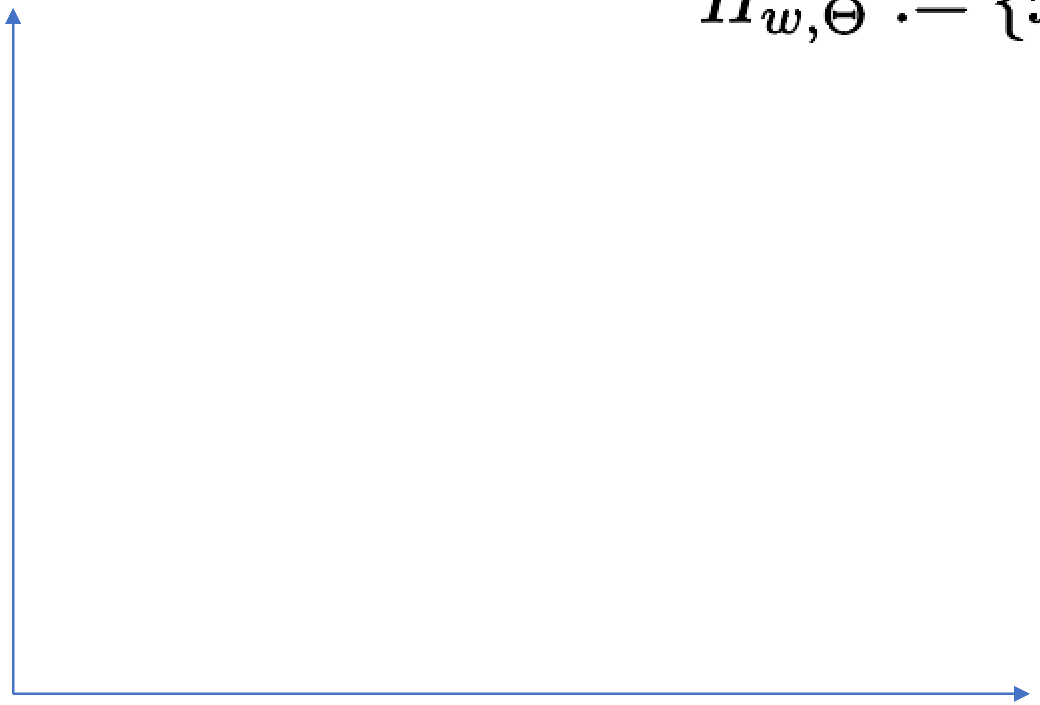
Revision: Lecture

- Interpretation of formulas

$$H_{w,\Theta} := \{x \in \mathbb{R}^n | w^T x = \Theta\}, w \in \mathbb{R}^n, \Theta \in \mathbb{R}$$

$$\Rightarrow \Theta = ||w|| \cdot ||x|| \cdot \cos(\varphi)$$

$$\Rightarrow ||x|| \cdot \cos(\varphi) = \frac{\Theta}{||w||}$$



Revision: Lecture

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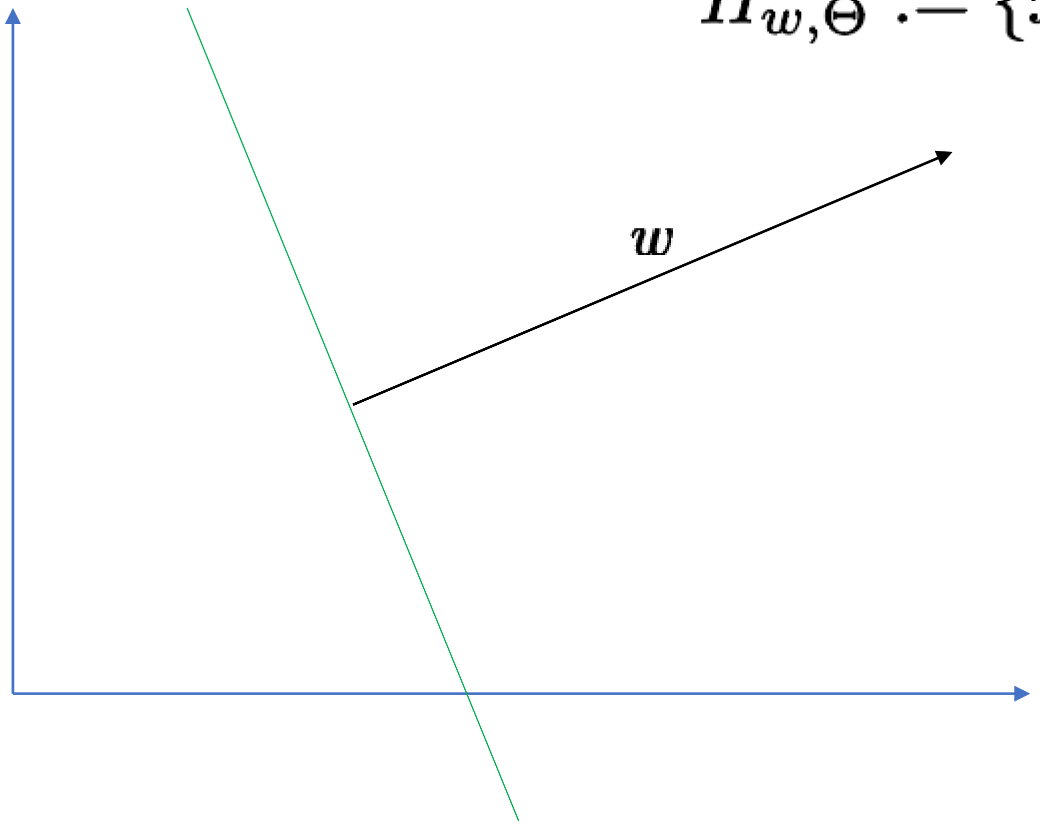

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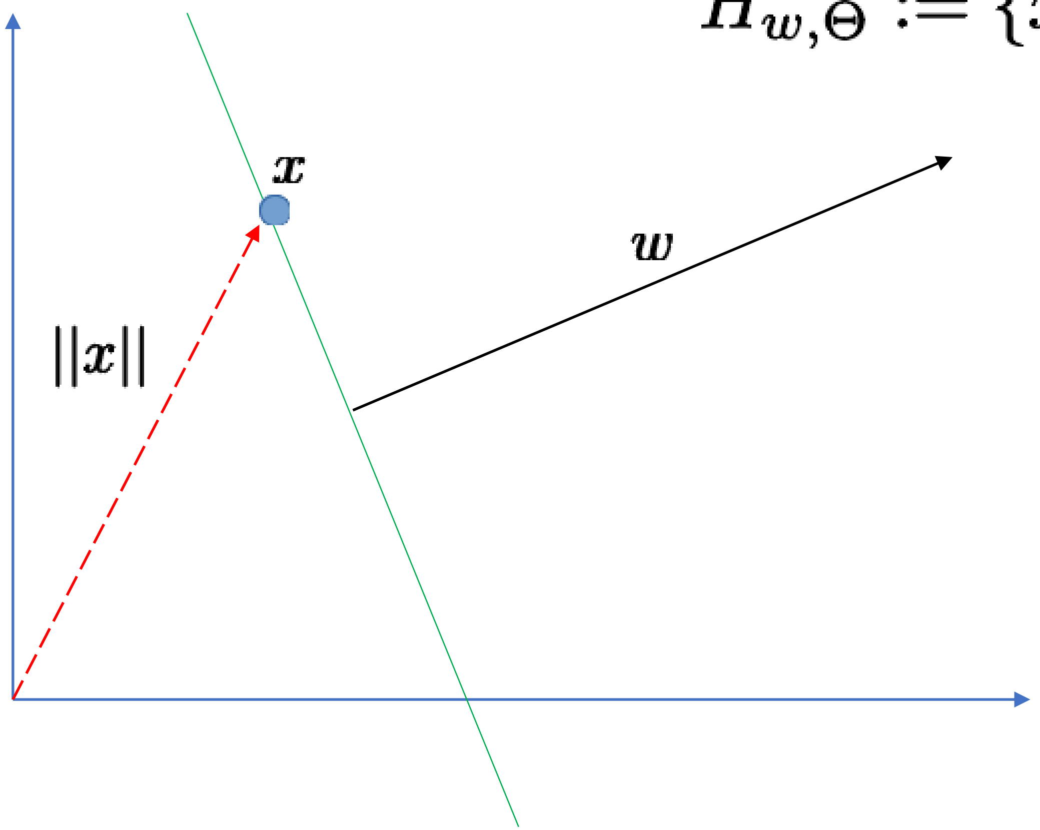

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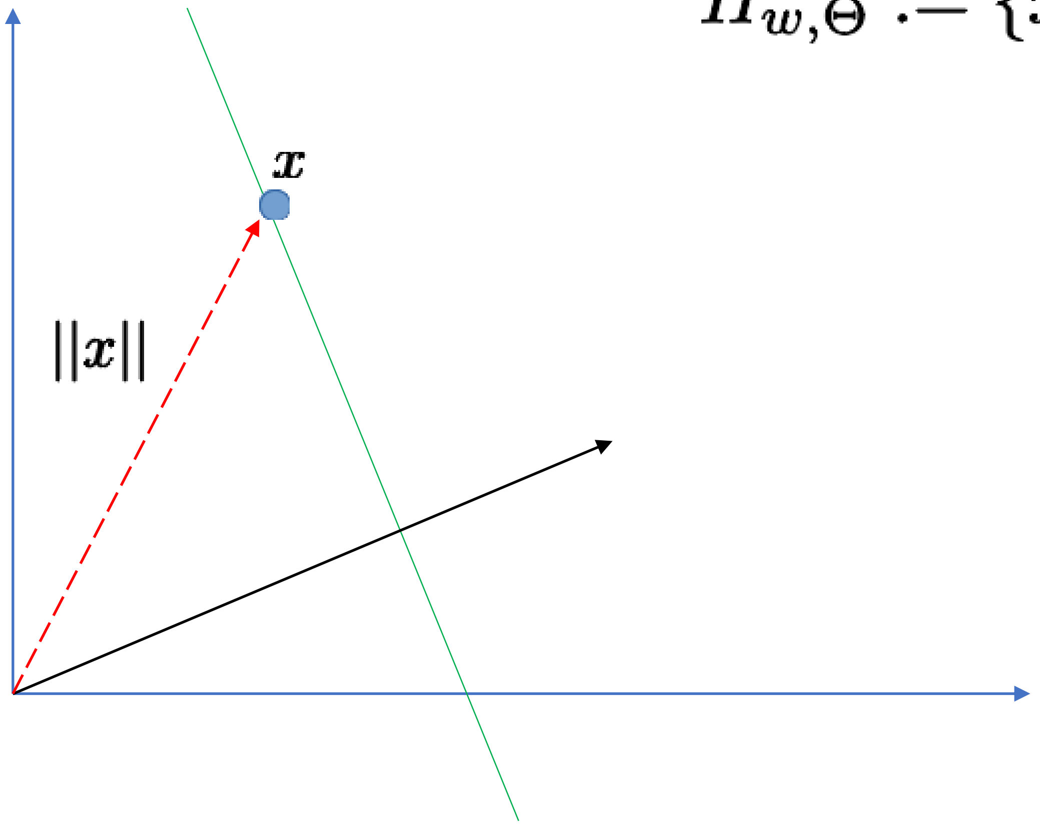

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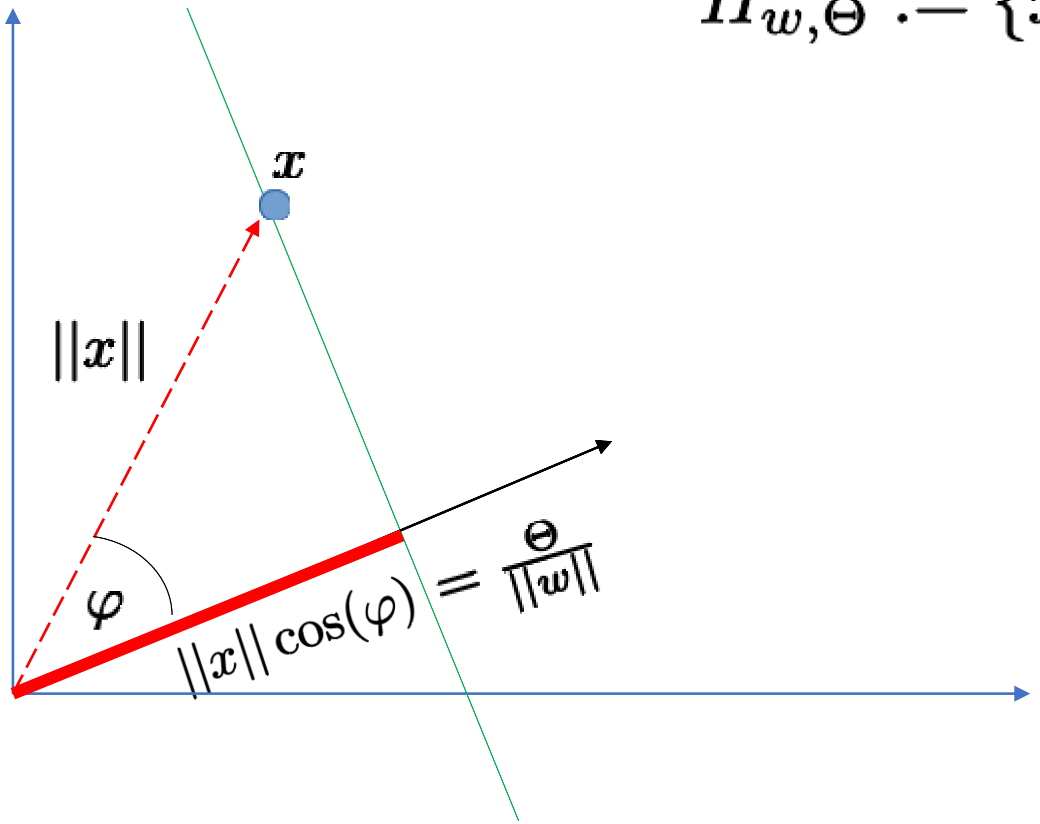

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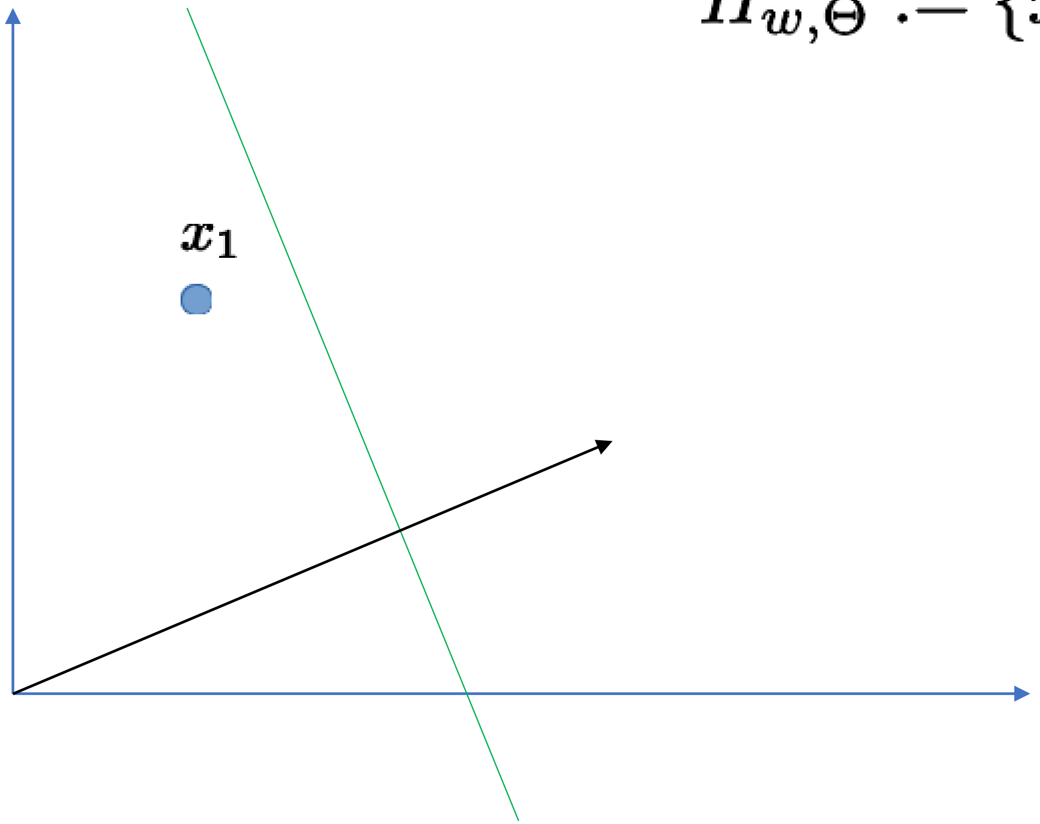

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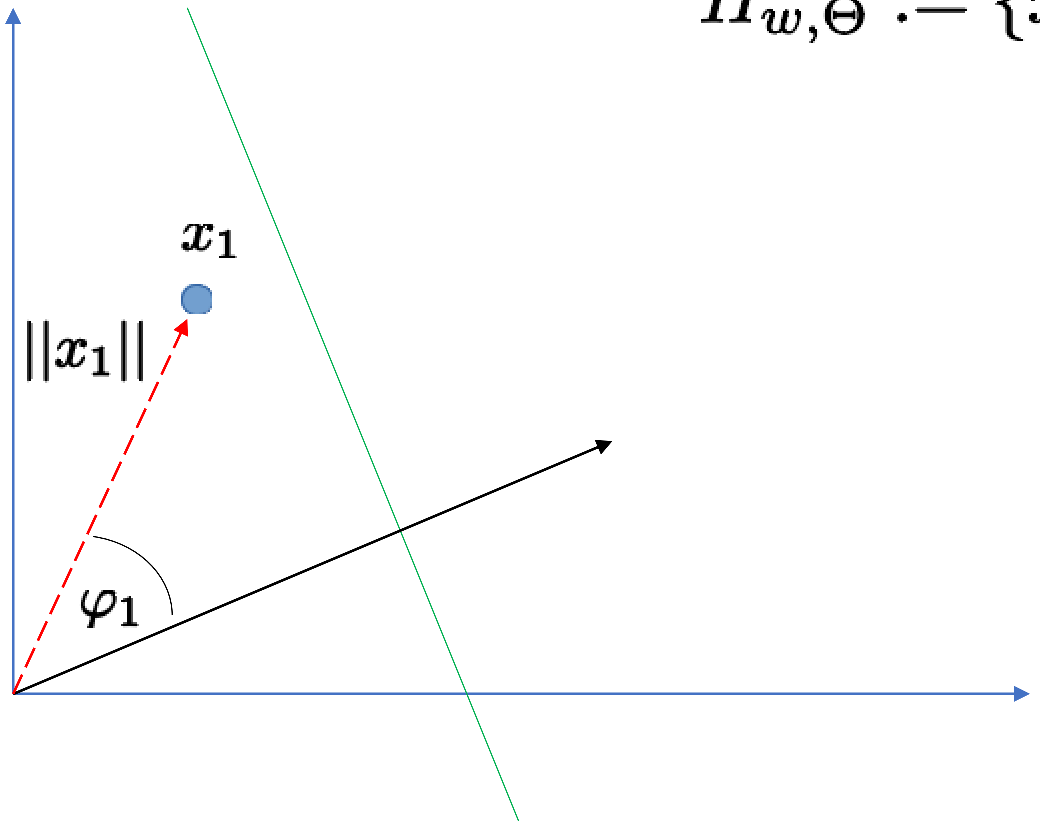

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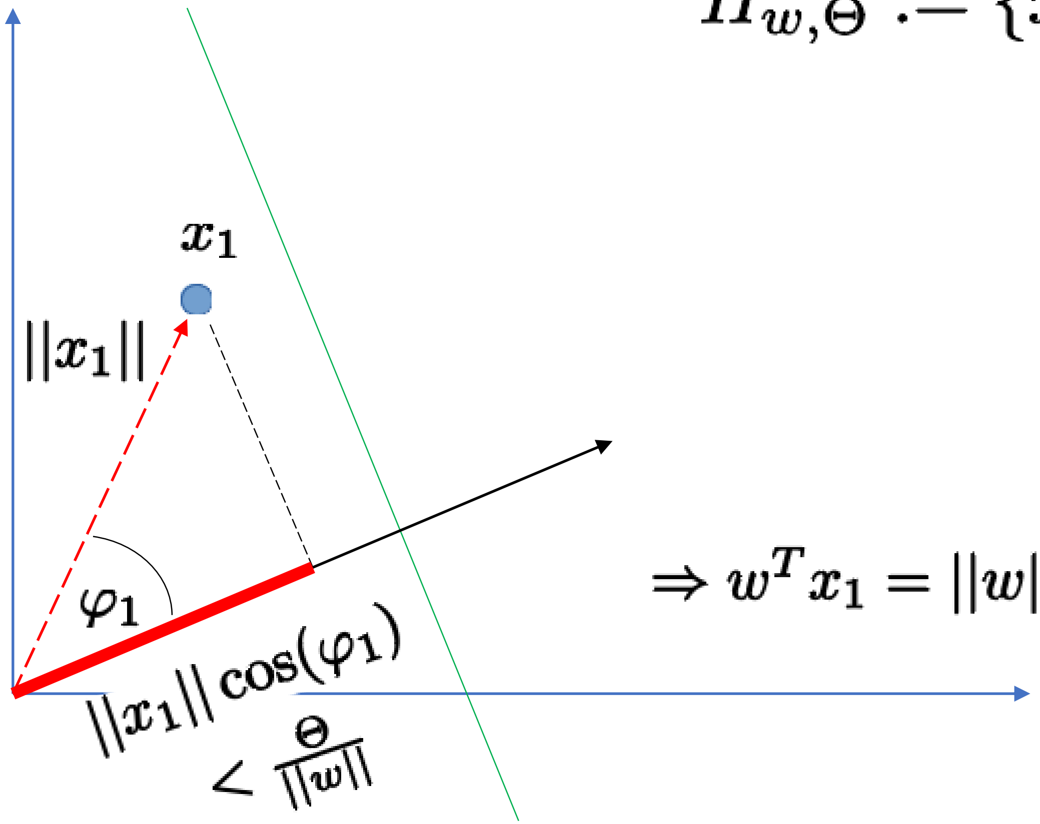

$$\Rightarrow w^T x_1 = ||w|| ||x_1|| \cos(\varphi_1) < \Theta$$

Revision: Lecture

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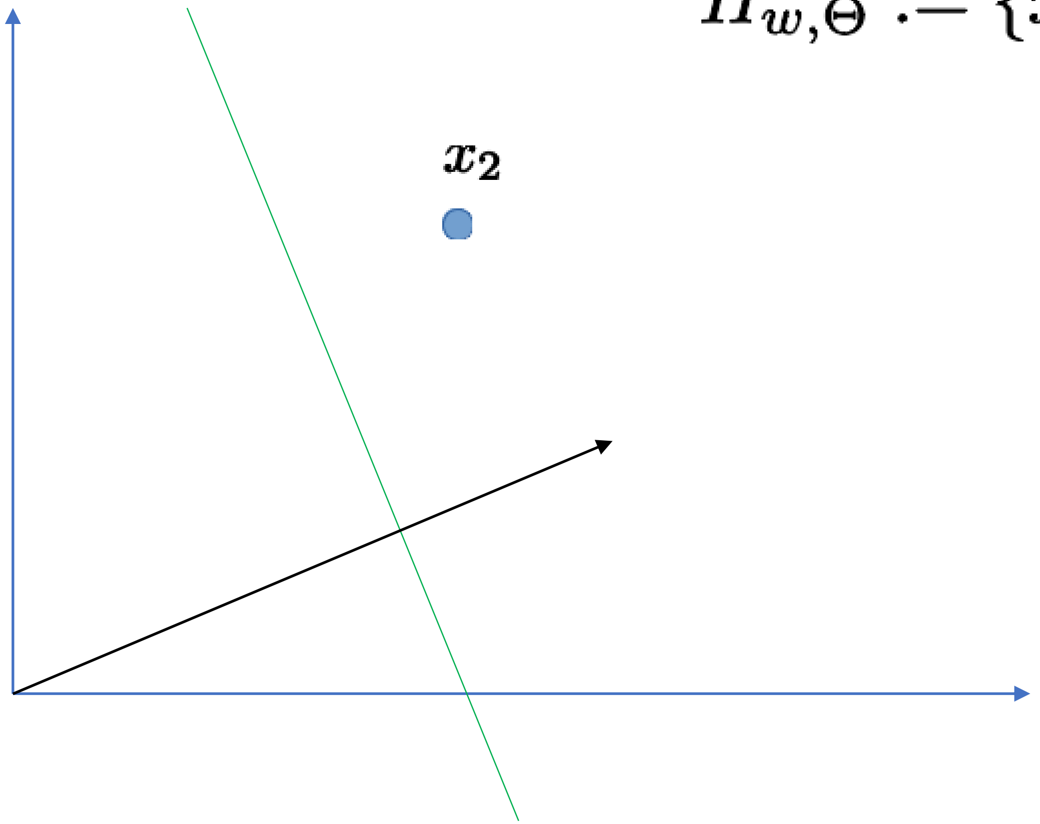

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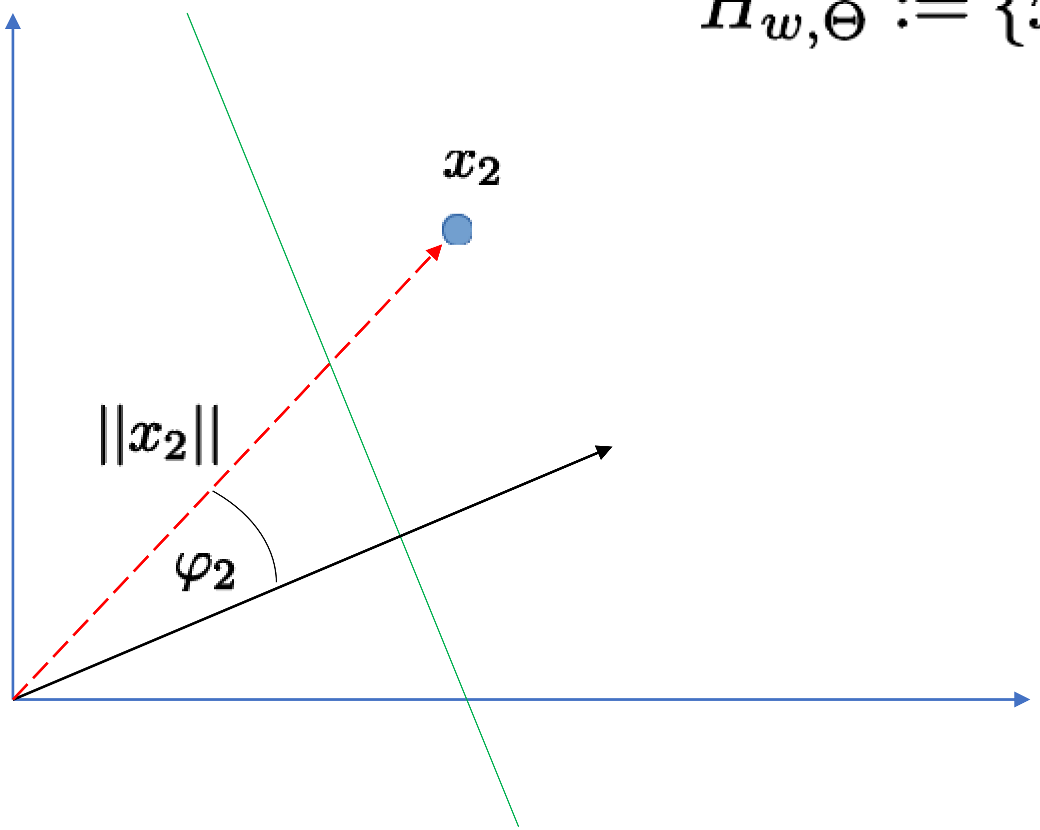

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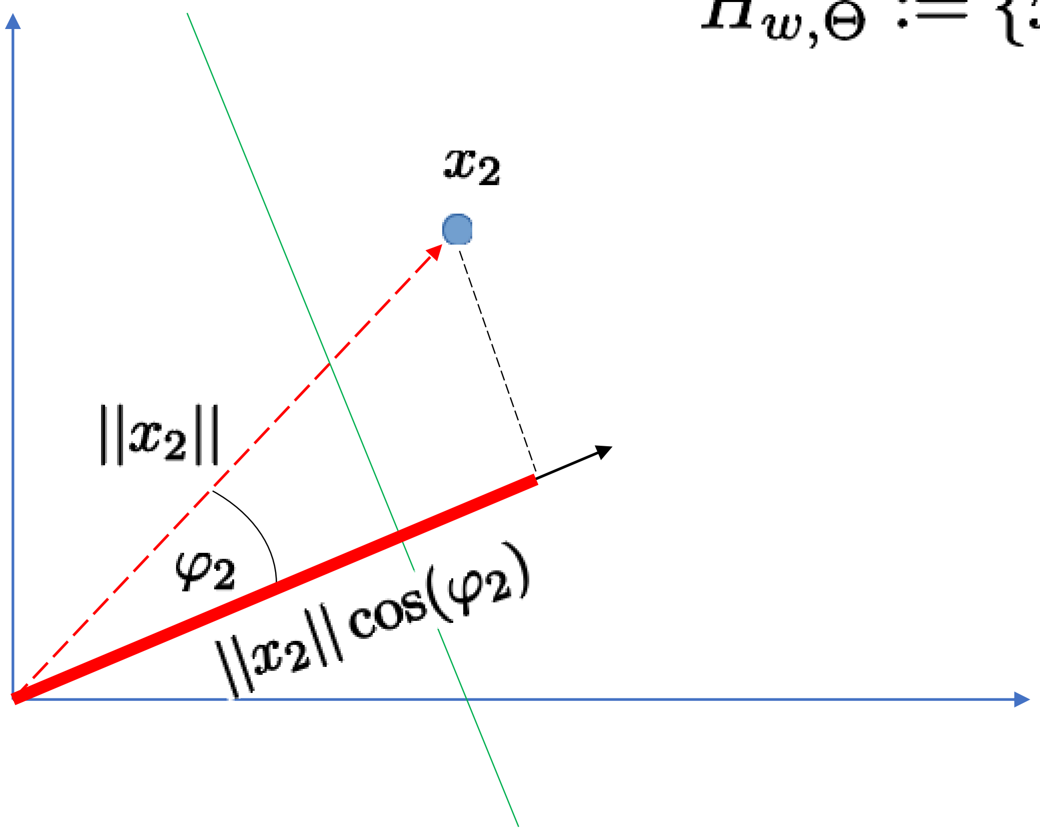

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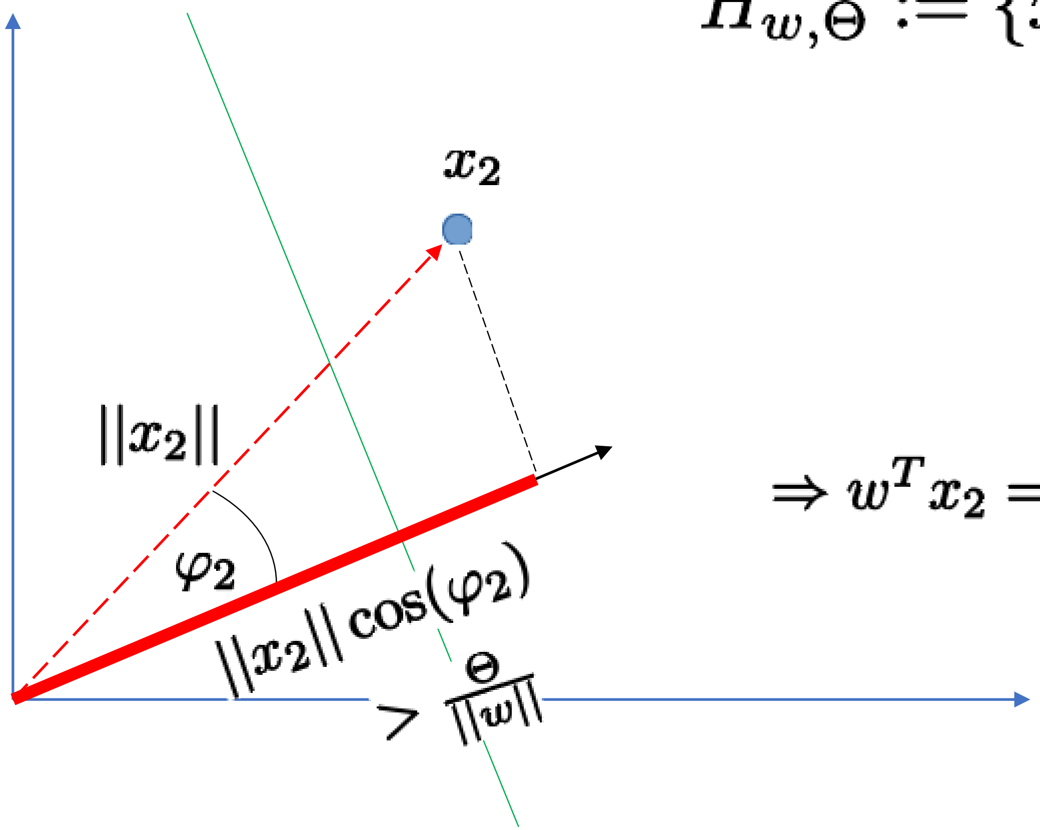

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$$\Rightarrow ||x|| \cdot \cos(\varphi) = \frac{\Theta}{||w||}$$

$$\Rightarrow w^T x_2 = ||w|| ||x_2|| \cos(\varphi_2) > \Theta$$

Revision: Lecture

- Formal definition of (absolute) linear separability of two sets $\mathcal{P}, \mathcal{N} \subset \mathbb{R}^n$(must know!):

- If there exist $(w_1, \dots w_n)^T \in \mathbb{R}^n, \Theta \in \mathbb{R}$

  such that $\sum_{i=1}^{n} w_i x_i \geq (>) \Theta \;\; \forall (x_1, \dots, x_n)^T \in \mathcal{P}$

  and $\sum_{i=1}^{n} w_i x_i < \Theta \;\; \forall (x_1, \dots, x_n)^T \in \mathcal{N}$

Revision: Lecture

- What is the formal definition of a Rosenblatt Perceptron?

Revision: Lecture

- Reformulation of step function

$$x_0 = 1$$
$$w_0 = -\Theta$$

$$y := f_a := \begin{cases} 1 \;\; if \sum_{i=0}^{n} w_i x_i \geq 0 \\ 0 \;\; else \end{cases}$$

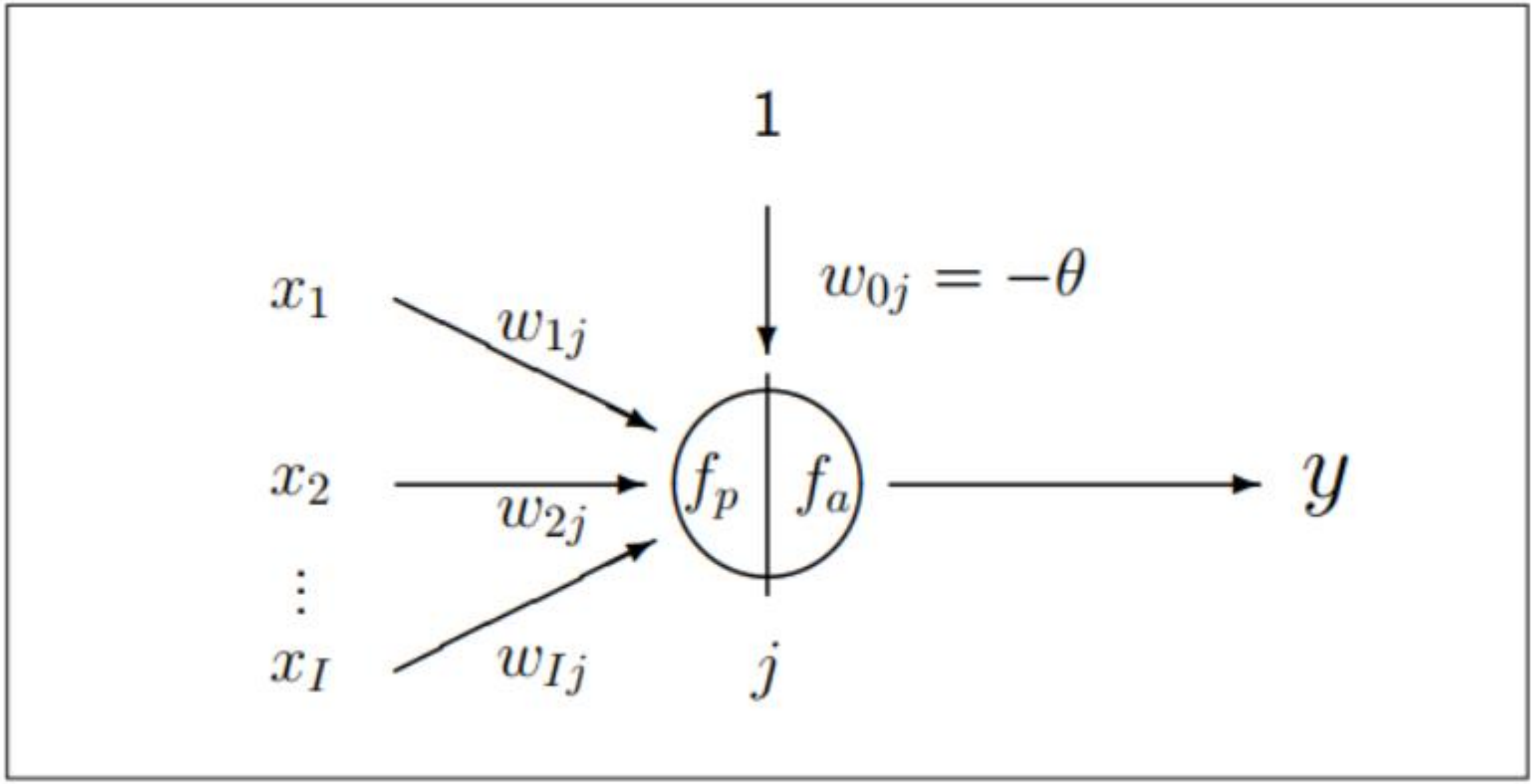
Revision: Lecture

- Reformulation of step function

$$x_0 = 1$$
$$w_0 = -\Theta$$

$$y := f_a := \begin{cases} 1 & if \sum_{i=0}^{n} w_i x_i \geq 0 \\ -1 & else \end{cases}$$

Scheme of Artificial Neuron



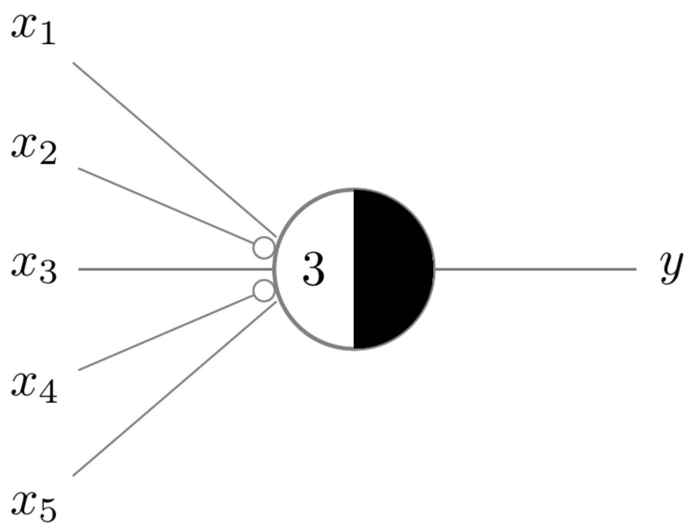
$f_p|f_a$ wird oft weggelassen, wenn aus dem Zusammenhang klar.

Revision: Lecture

- Which statements regarding Rosenblatt Perceptron and McCulloch Pitts Neuron are true?
    1. The McCulloch Pitts Neuron can process real valued input
    2. The Rosenblatt Perceptron can process real valued input
    3. Both neuron models have inhibiting and excitatory edges
    4. The Rosenblatt Perceptron can model McColloch Pitts Neurons

Revision: Lecture

- How can you use a Rosenblatt Perceptron to represent a McCulloch Pitts Neuron?



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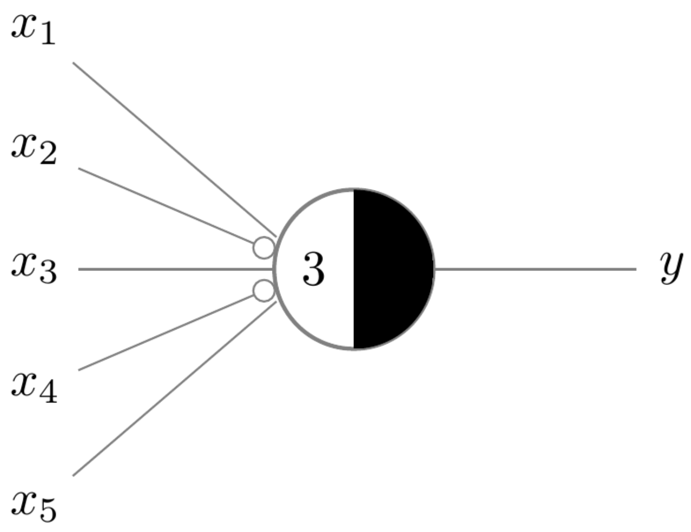

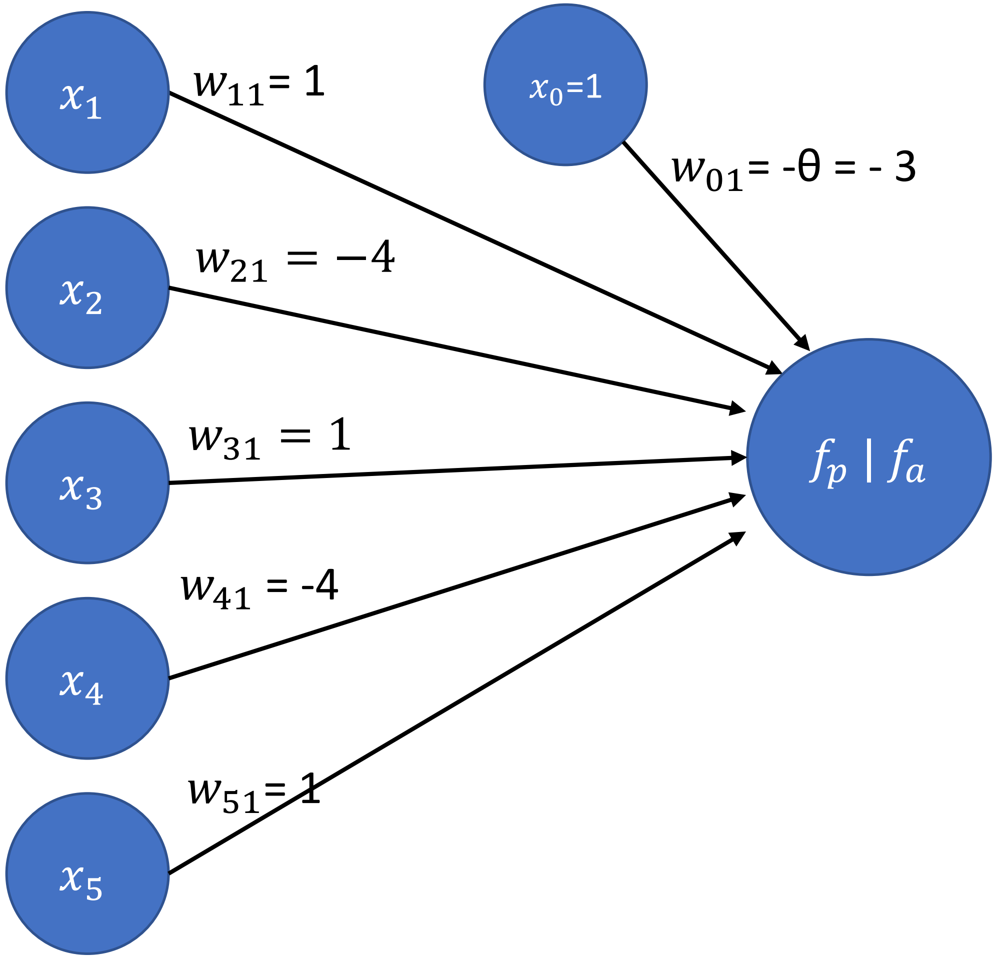

Content

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- Hebbian Learning

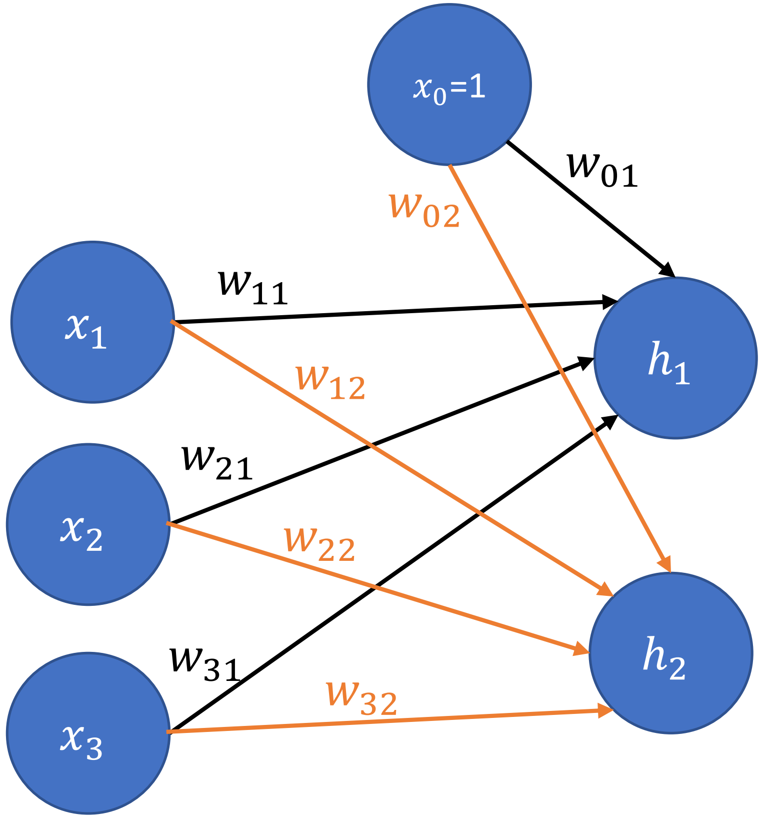
Hebbian Learning

- Postulated by Donald Olding Hebb in 1949
- Foundation of many learning rules

- Idea:
  - If neuron j receives a signal from neuron i and both neurons are strongly activated, then the connection of j and i should be strong!

  - In artificial neural networks the strength og the connection is usually represented by the edge weight

Calculation of propagated value



$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^{3} w_{i2} x_i$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \qquad \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$

Hebbian Learning Rule

- Given:
  - Outputs of previous neurons (can also be input neurons)
  - In case of scheme : input neurons, i.e:

    $x_0, x_1, x_2, \dots x_n \in \{-1, 1\}$

  - Output of current neurons
  - In case of scheme: $h_1, h_2 \in \{-1, 1\}$

  - Weights in between all neurons

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  - Output of current neurons
  - In case of scheme: $h_1, h_2 \in \{-1, 1\}$
  - Weights in between all neurons
  - Learning rate α
- Learning Rule:
  - Update weights by comparing similarity of outputs
  - $\Delta w_{i,j} \coloneqq \alpha \cdot x_i \cdot h_j$
  - $w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$