Hebbian Learning

Neuroinformatics Tutorial 5

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Content

- Revision: Mahalanobis Classifier
- Revision: Lecture
- Hebbian Learning

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Revision: Mahalanobis Distance

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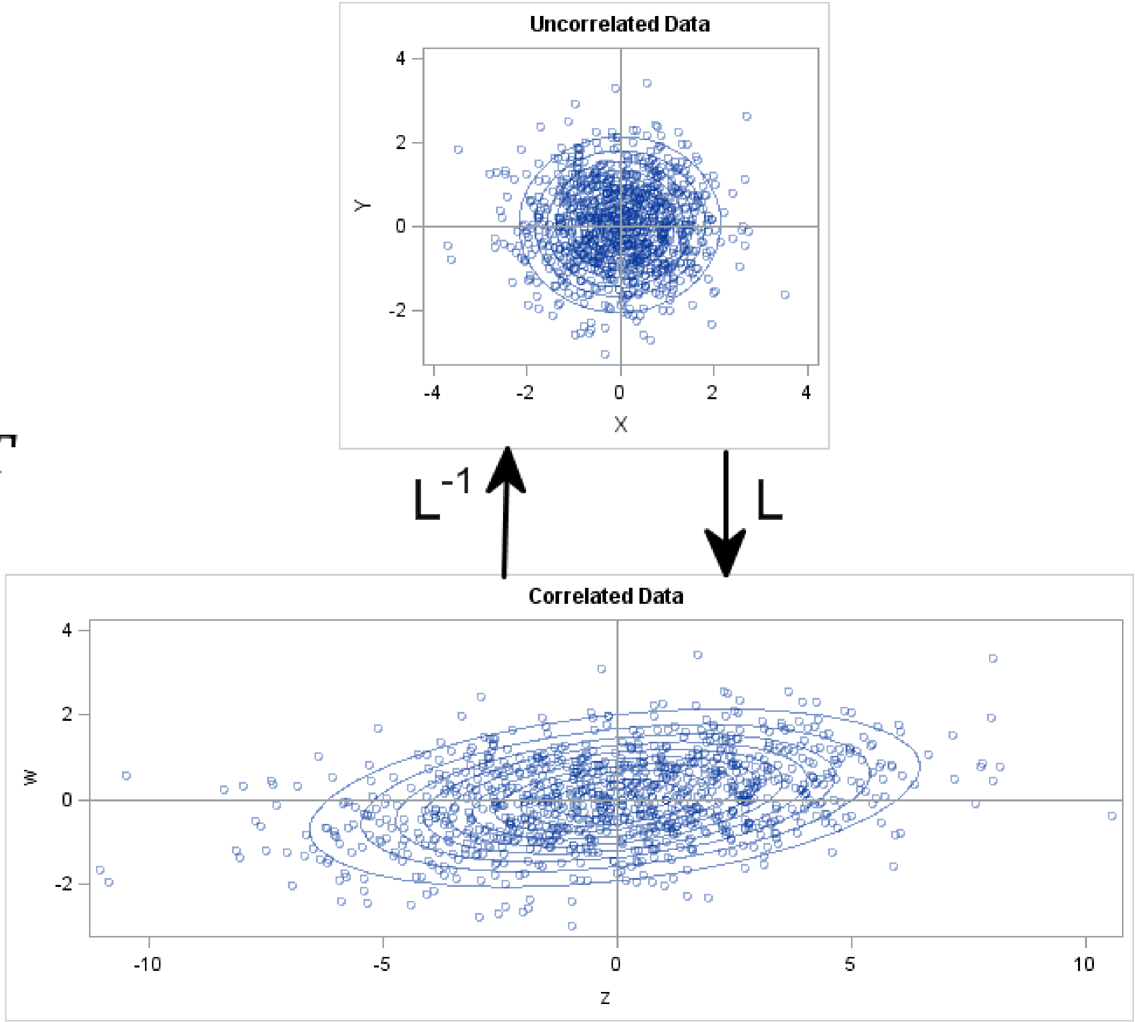
Inverse Cholesky Transform

$$L := U^T$$



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- Inference:
  - Calculate Mahalanobis distance to each class mean
  - Choose class with least distance

Mahalanobis Classifier: Jupyter

Content

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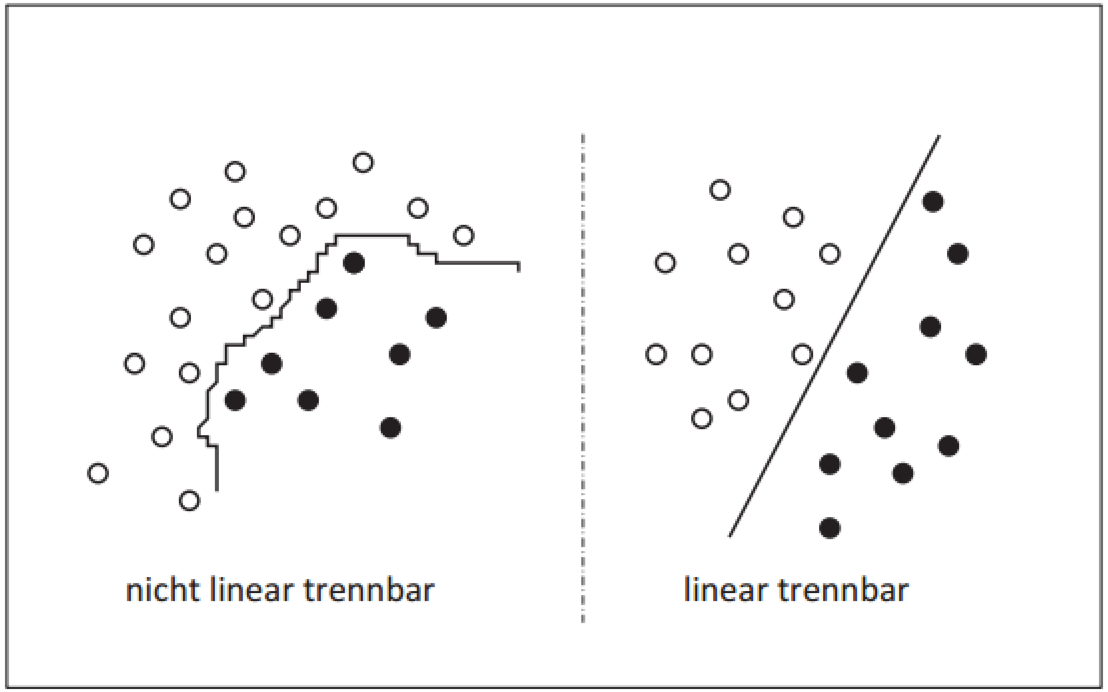

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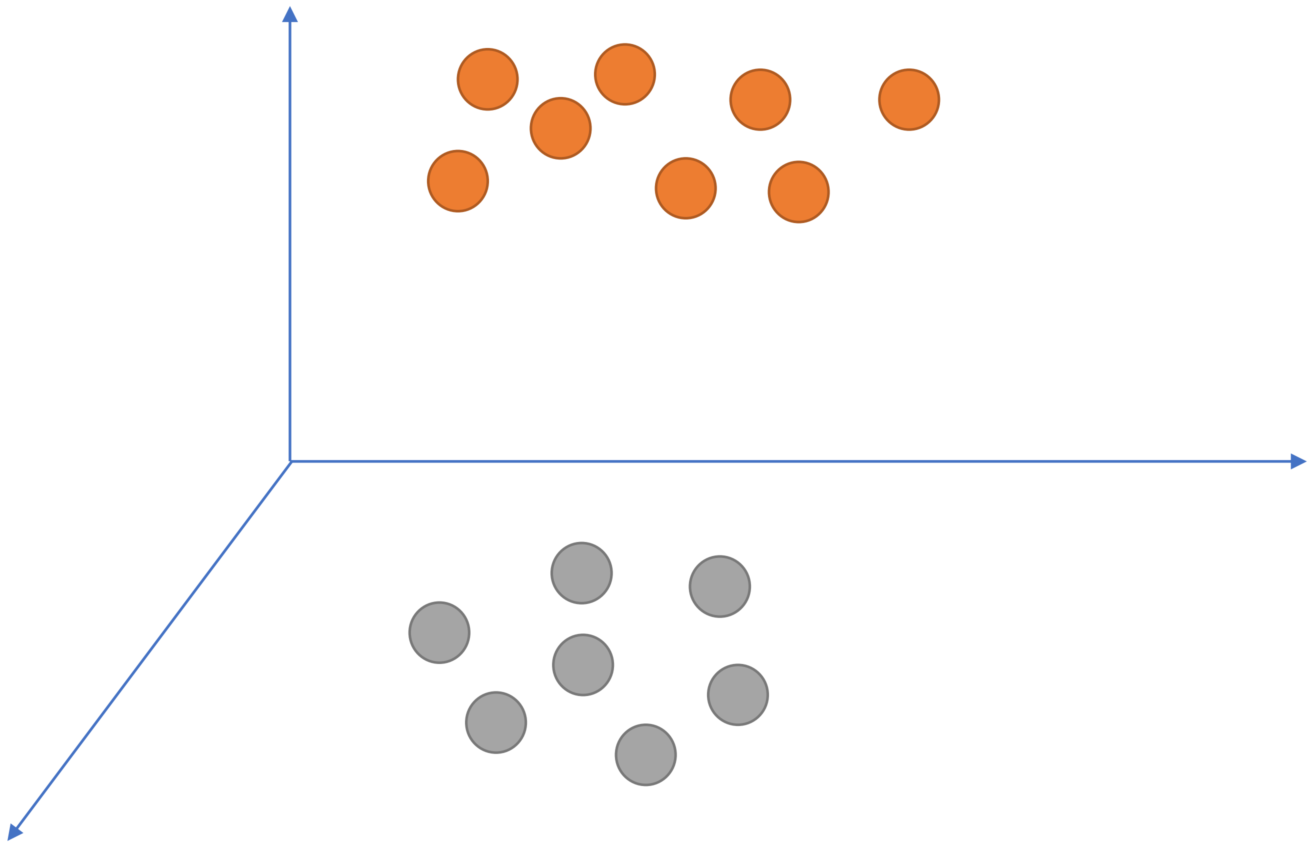

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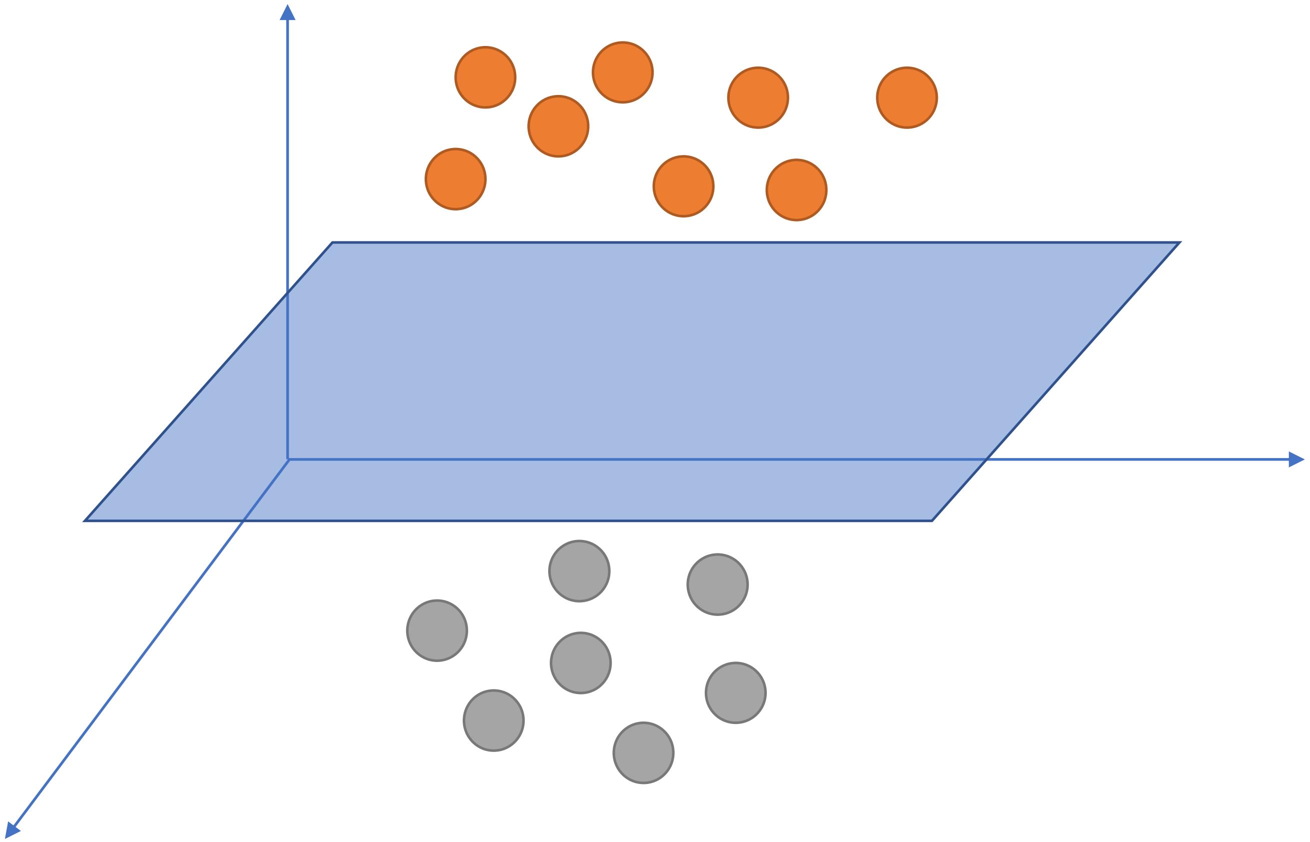

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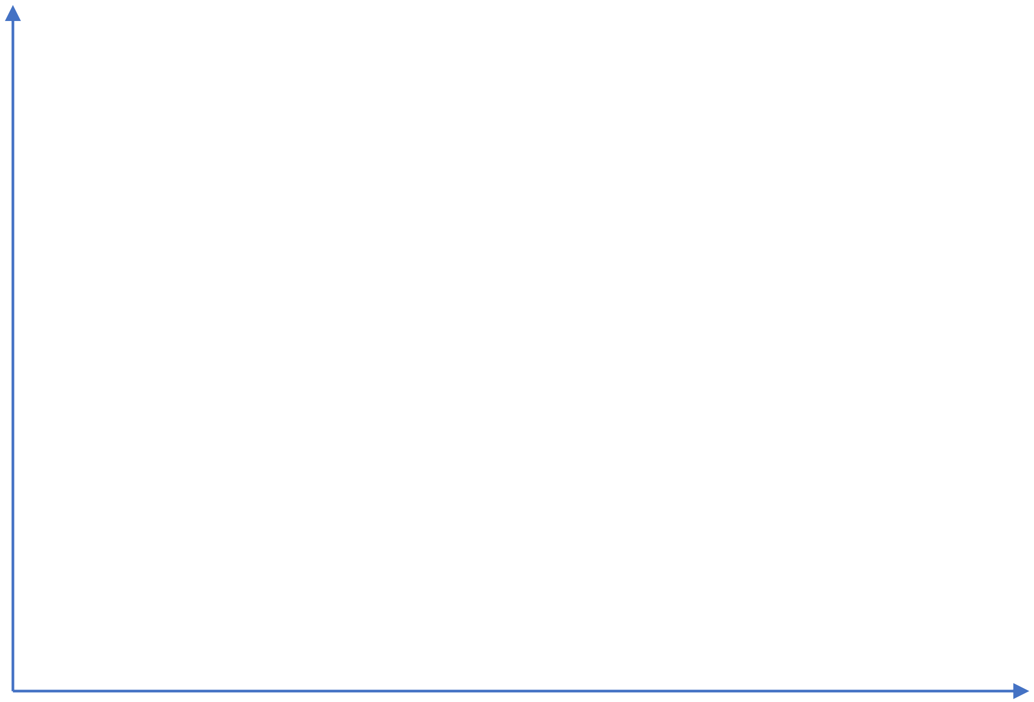

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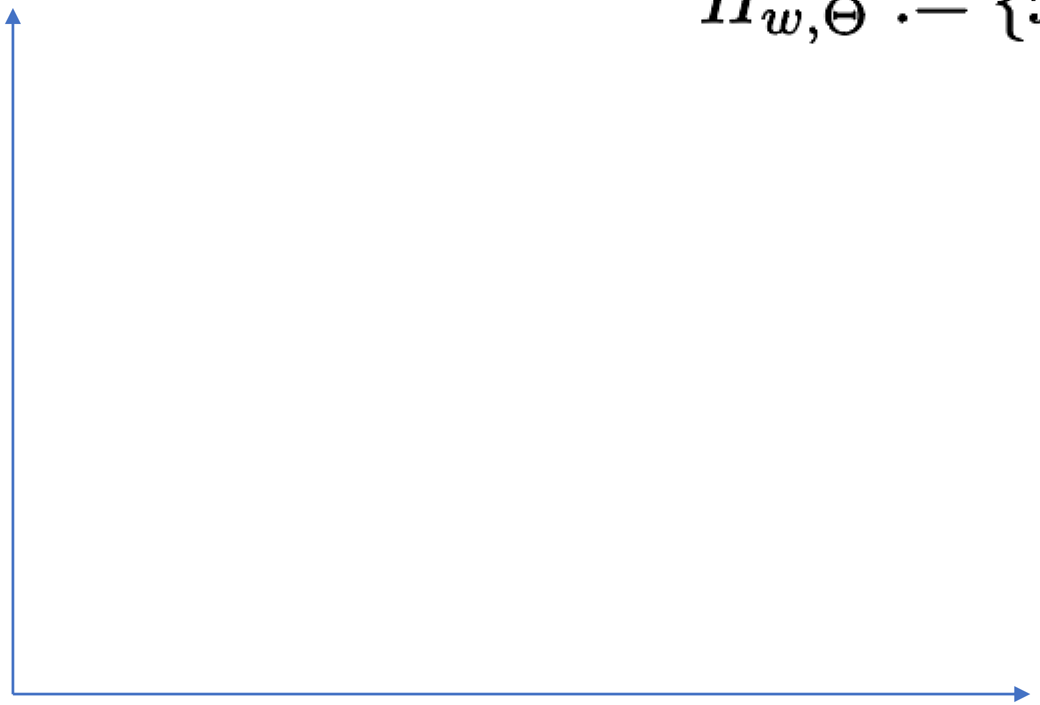

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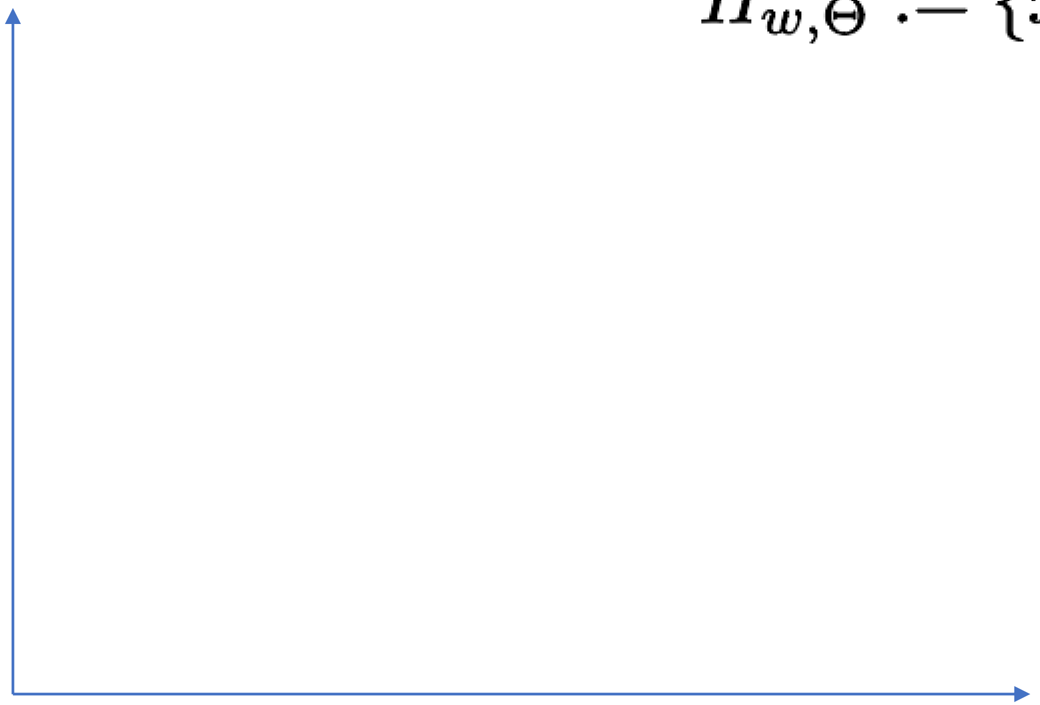

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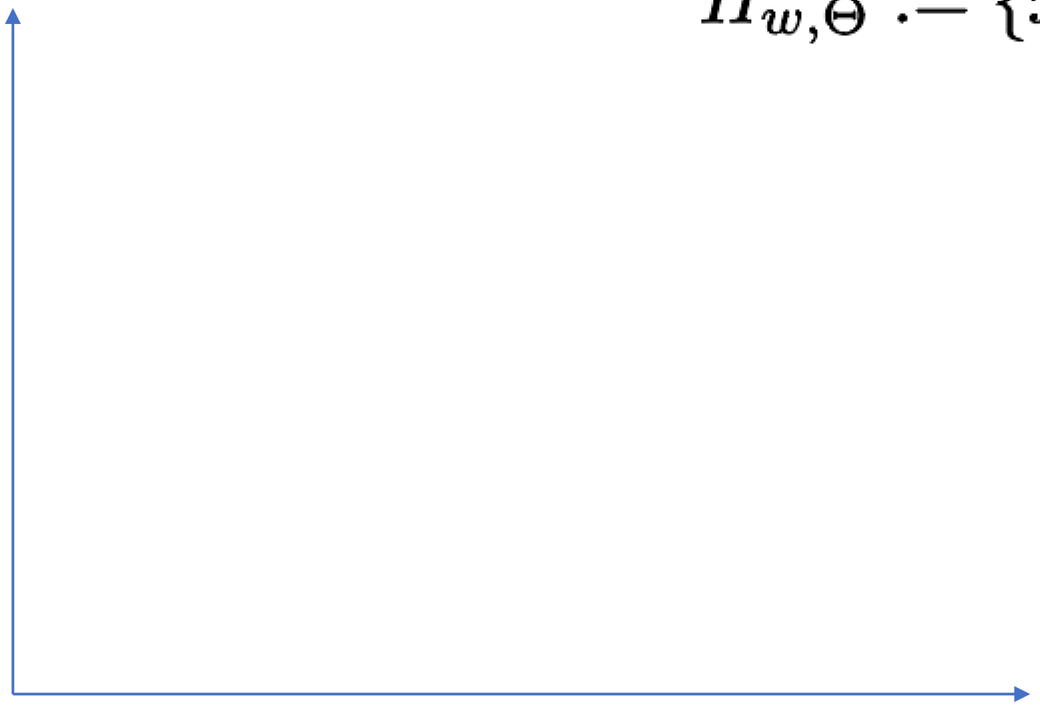

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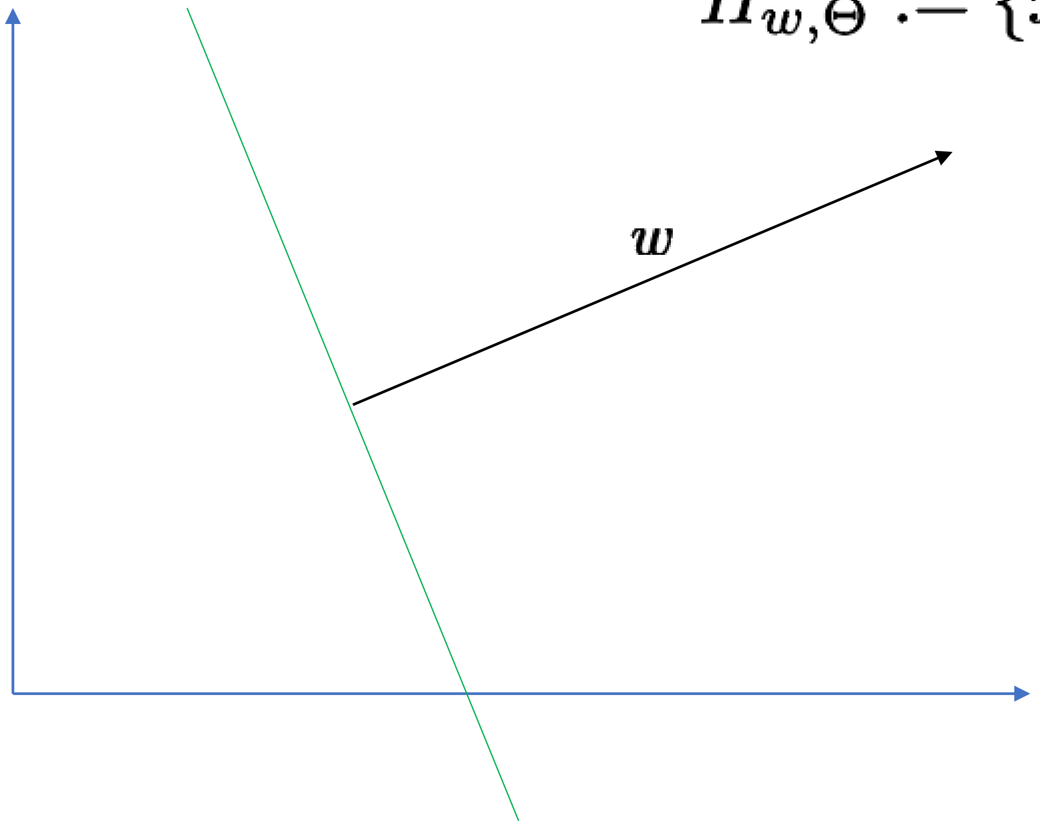

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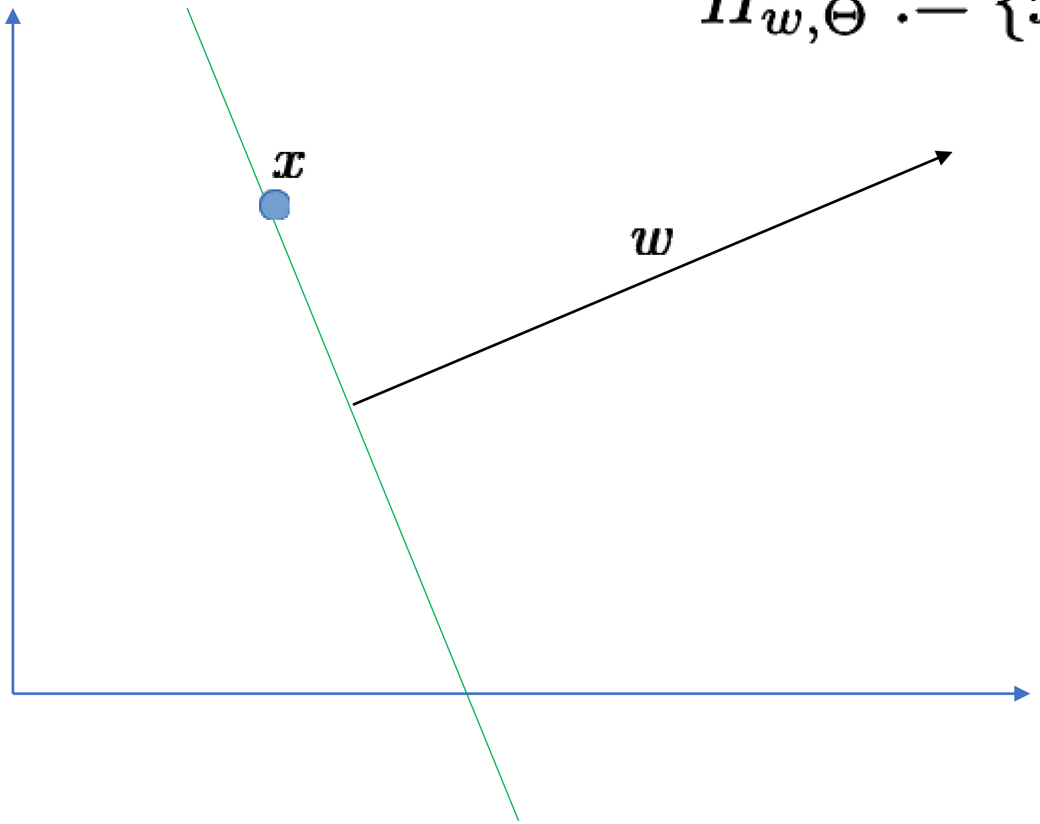

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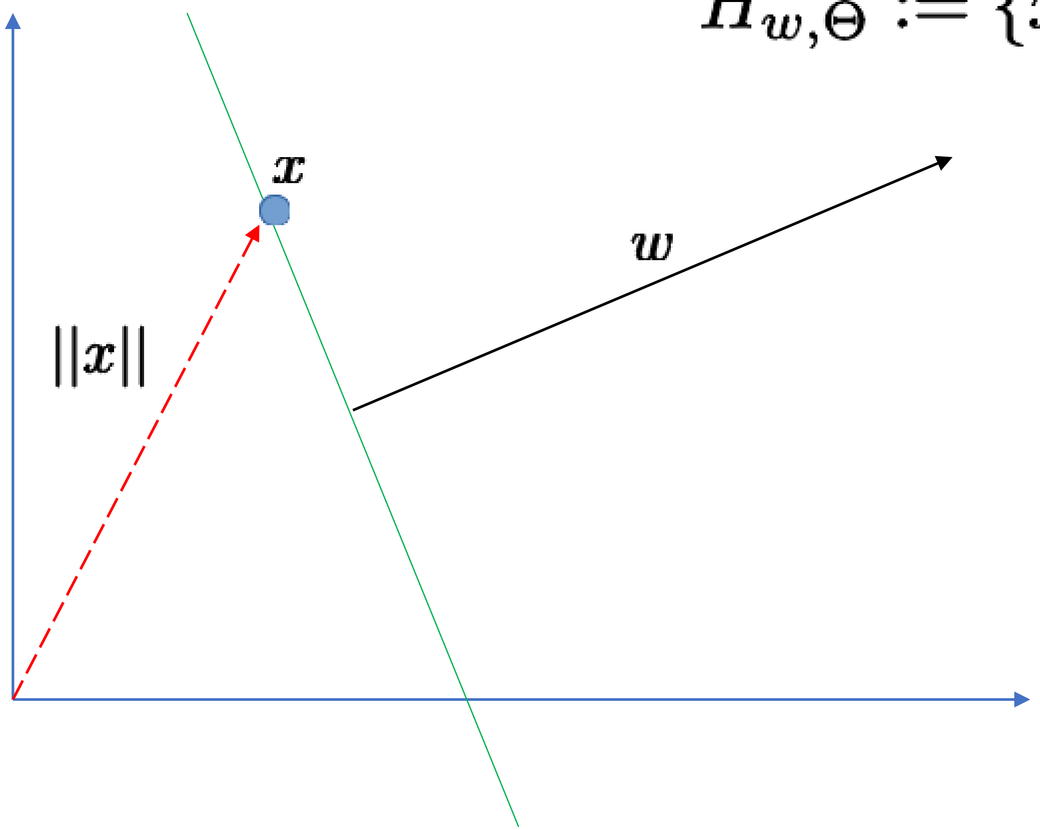

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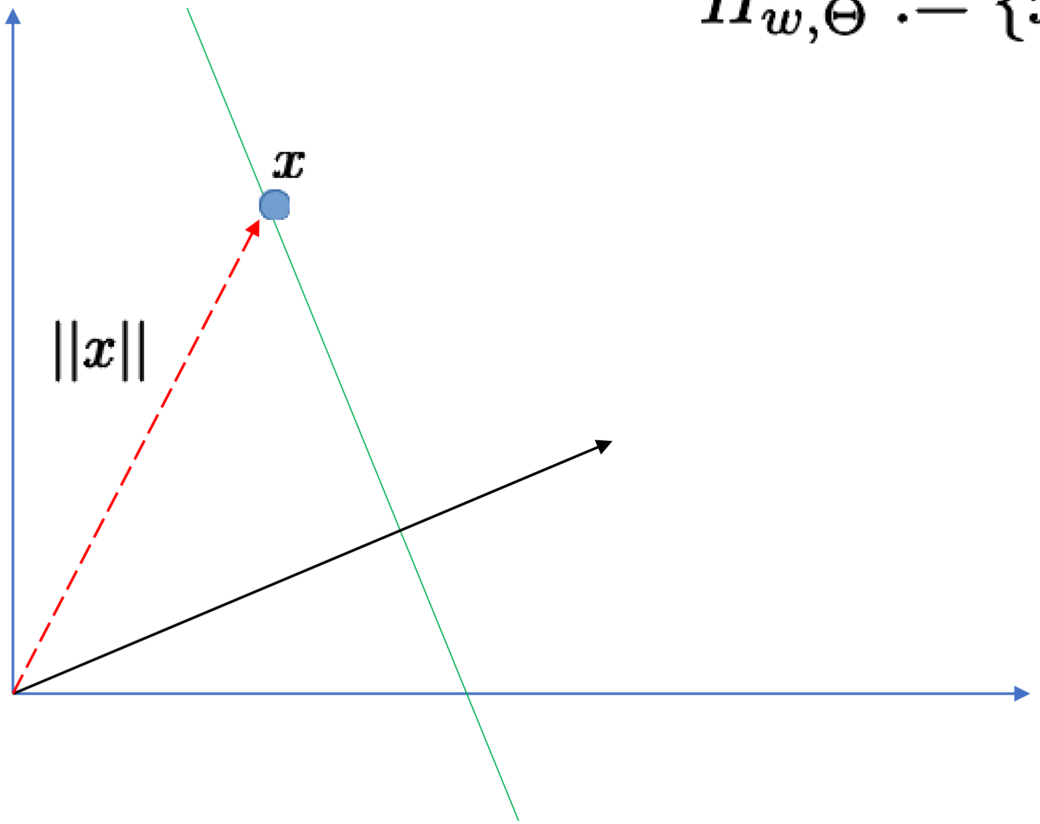

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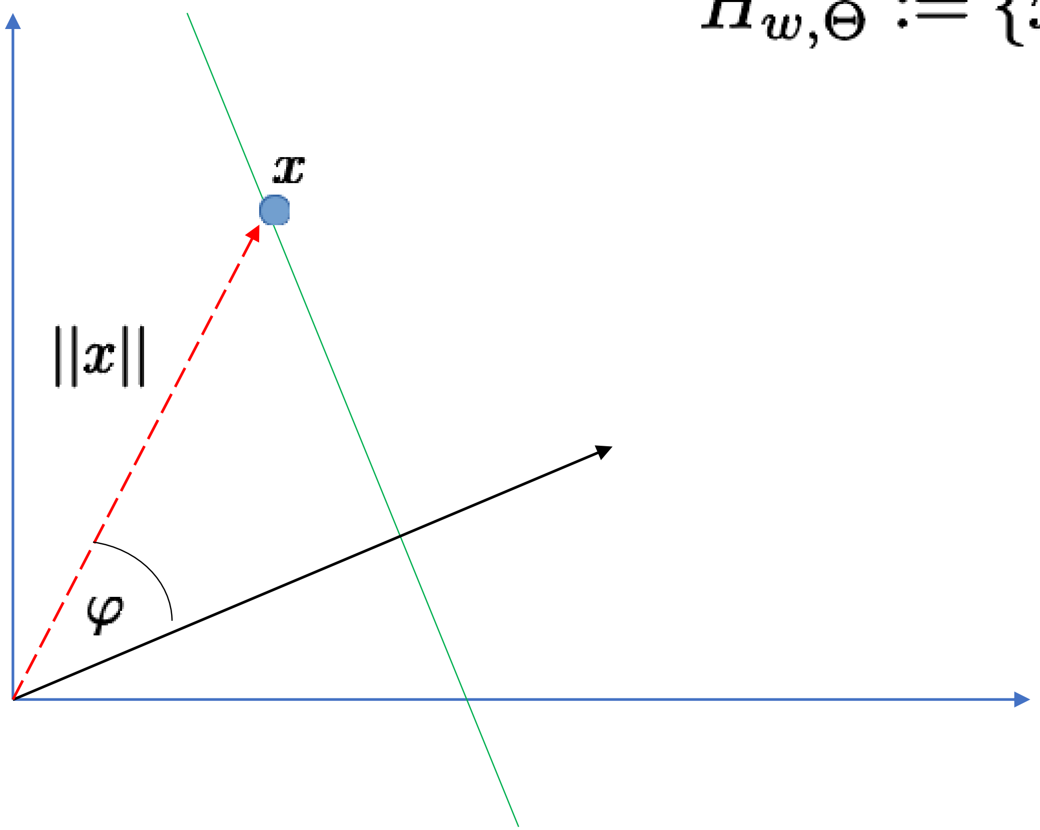

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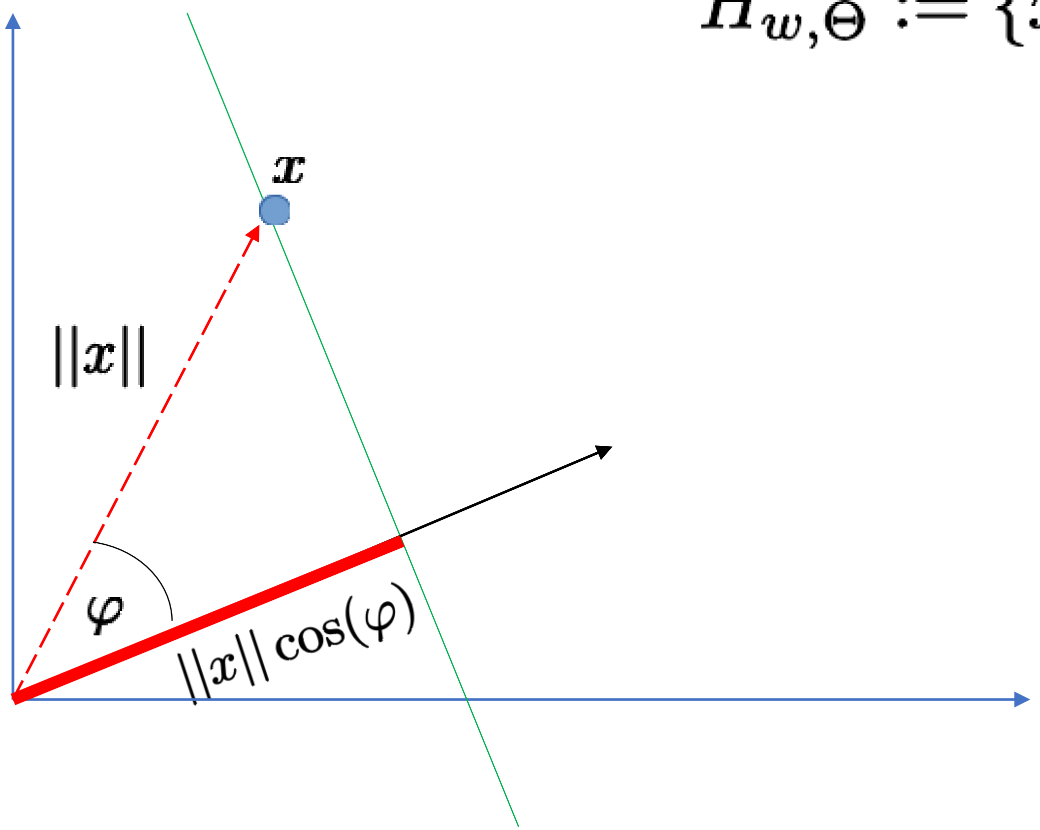

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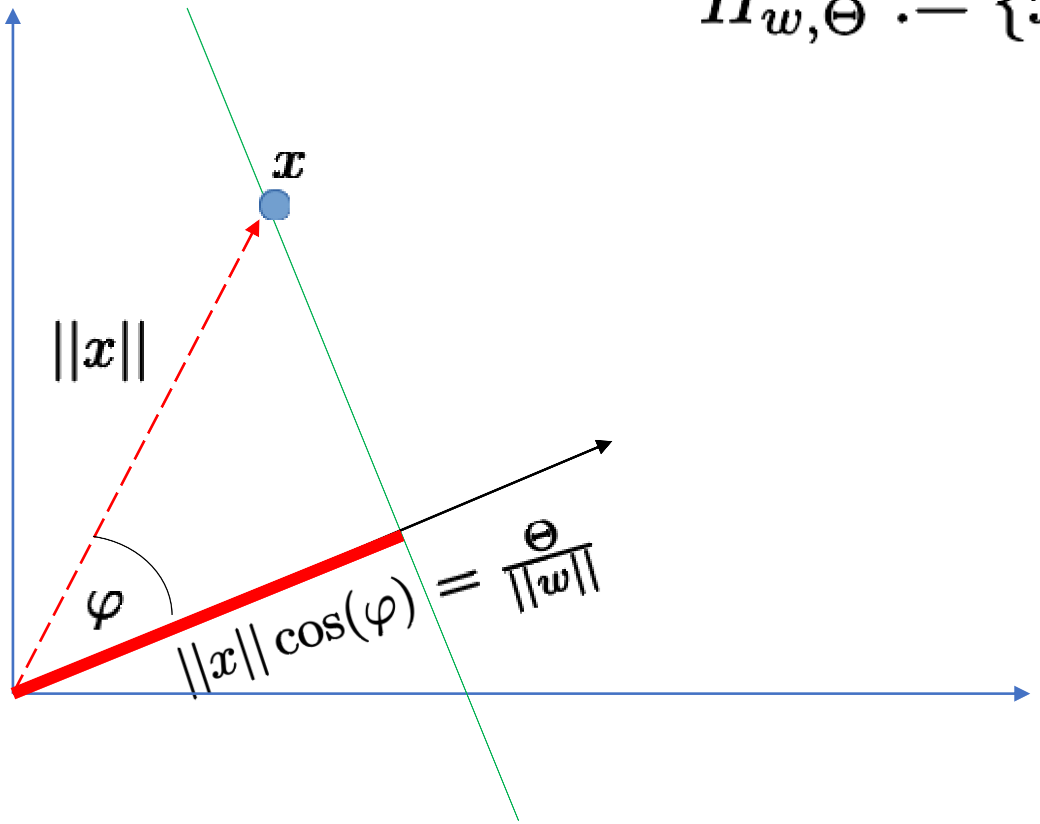

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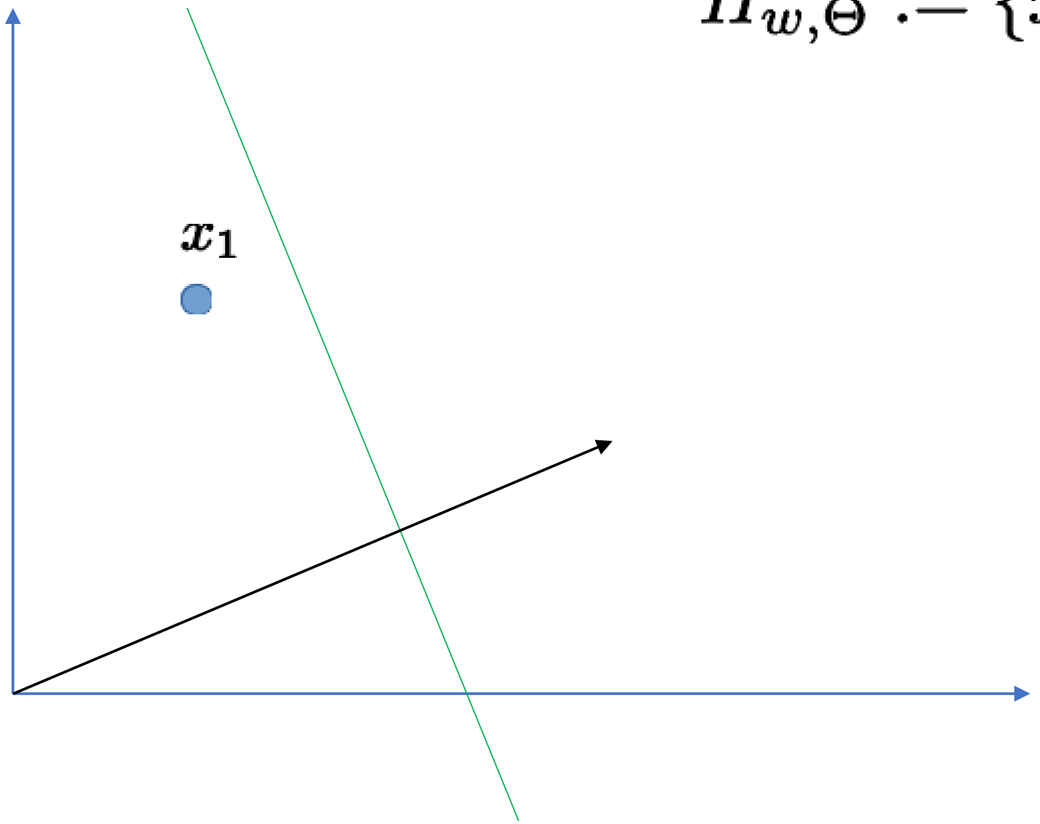

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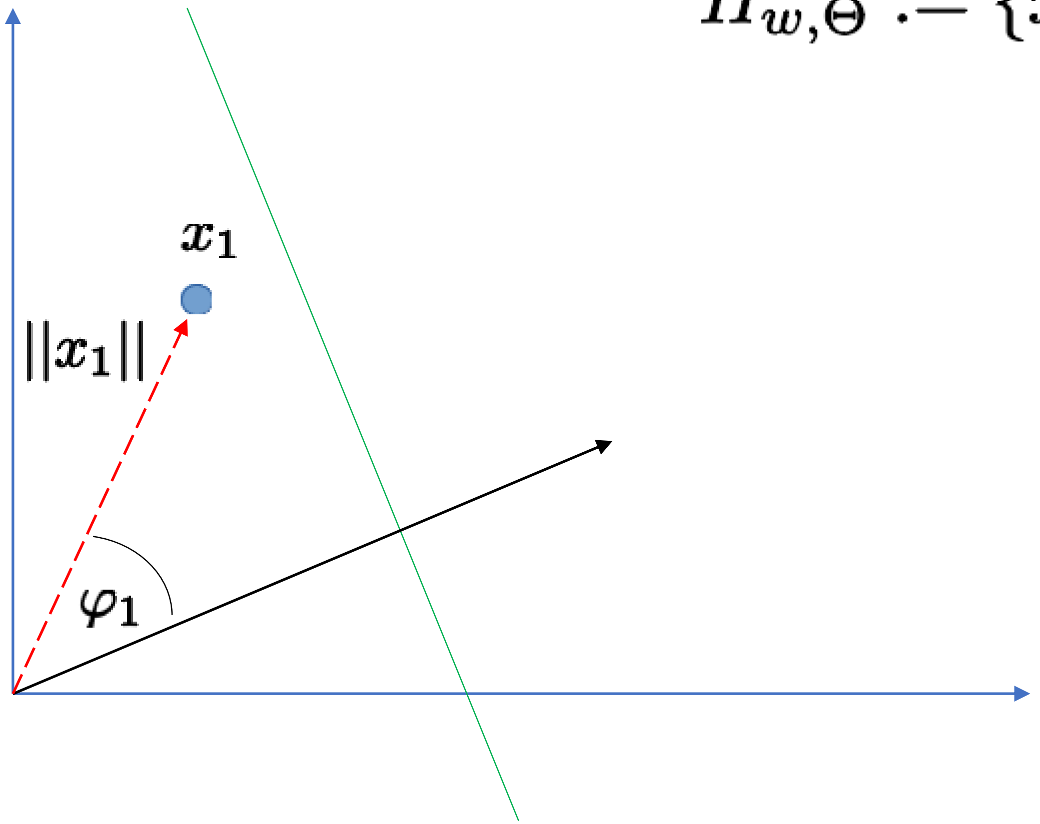

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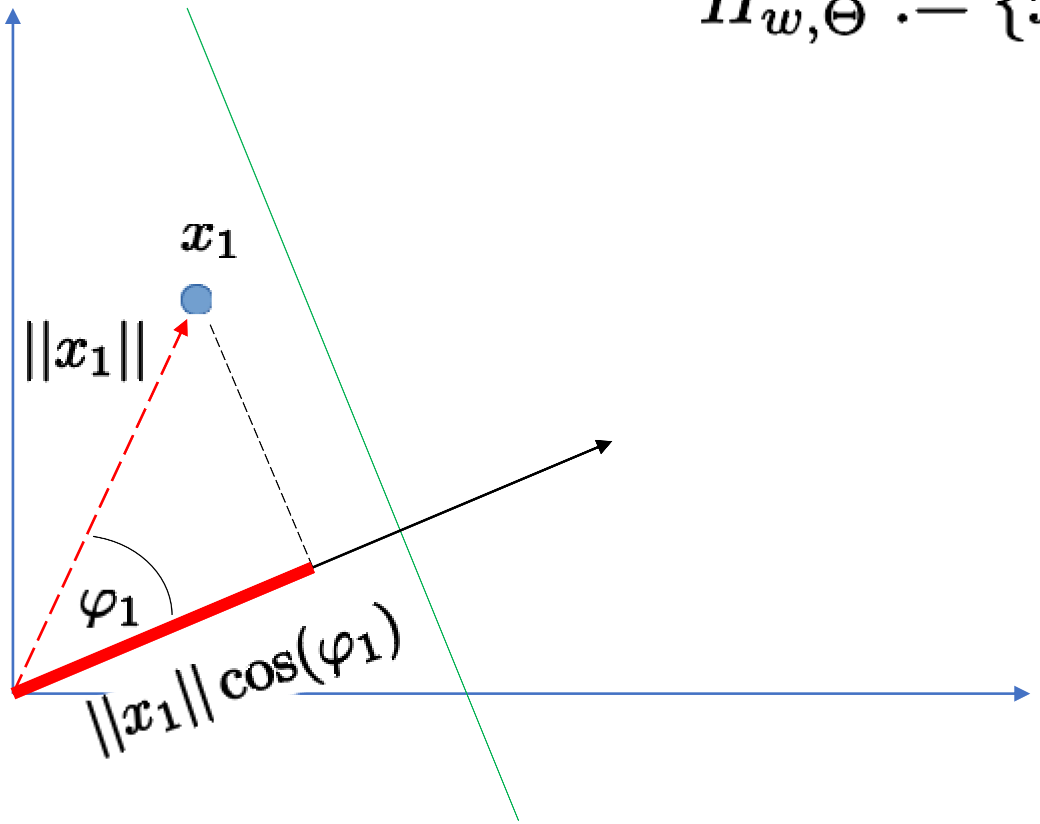

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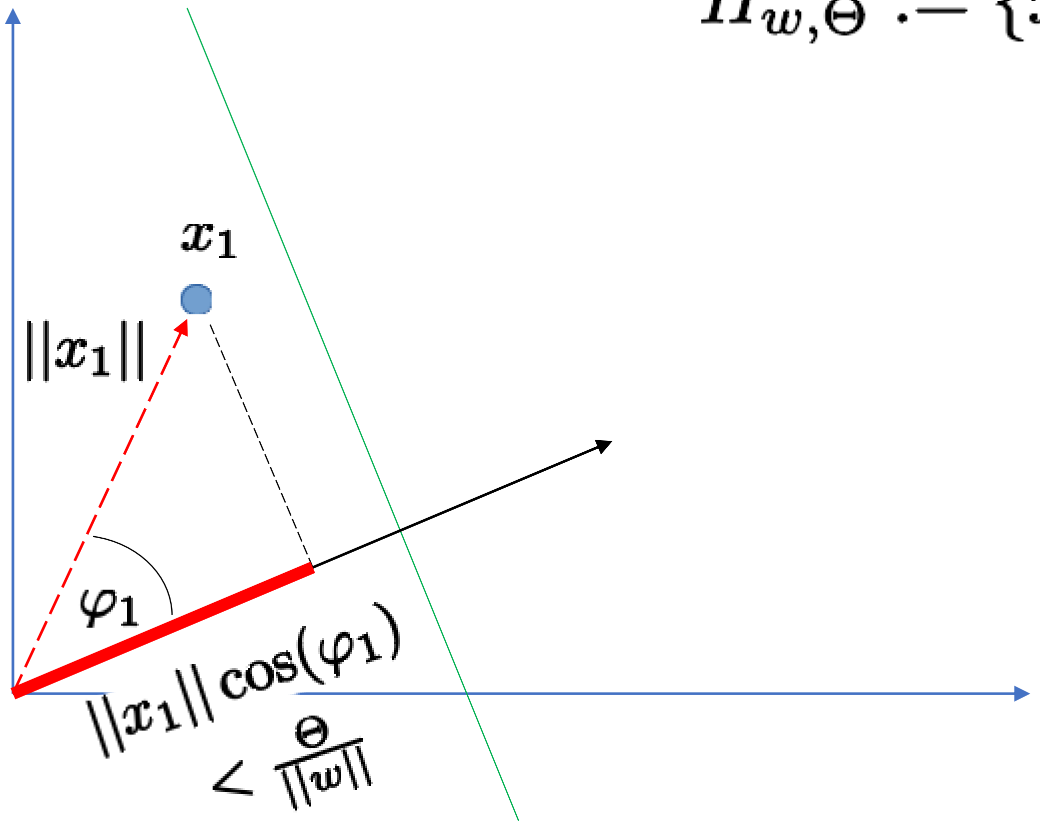

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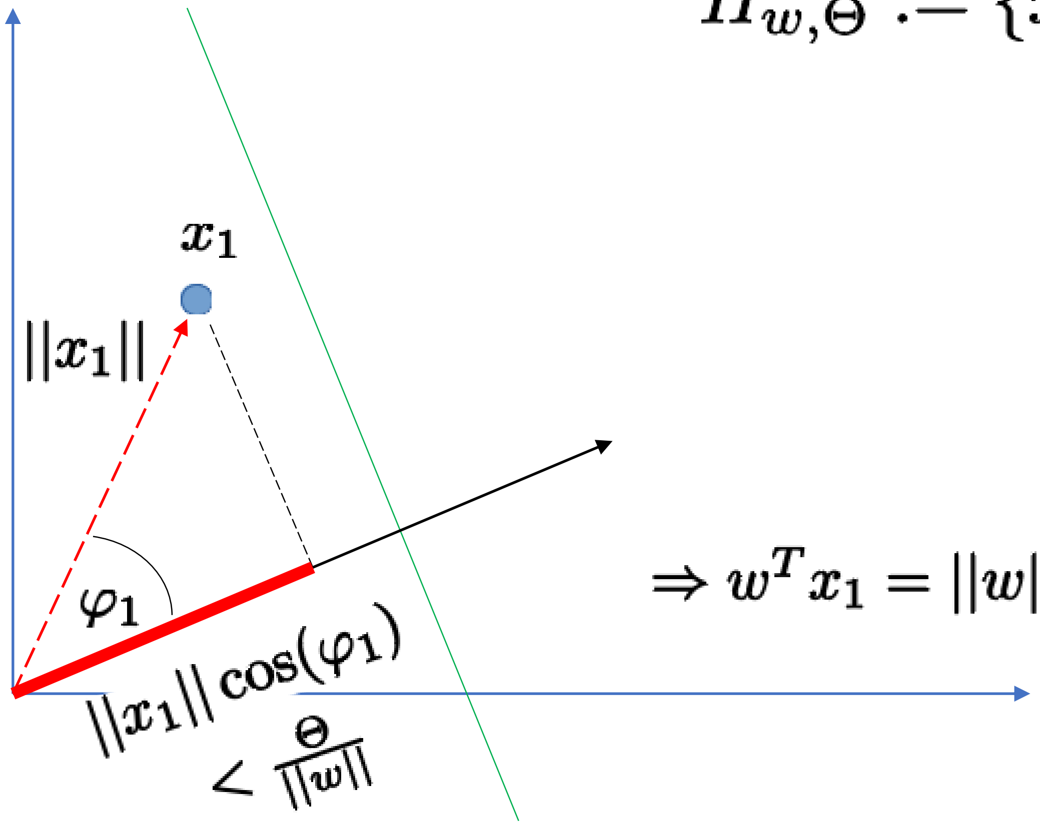

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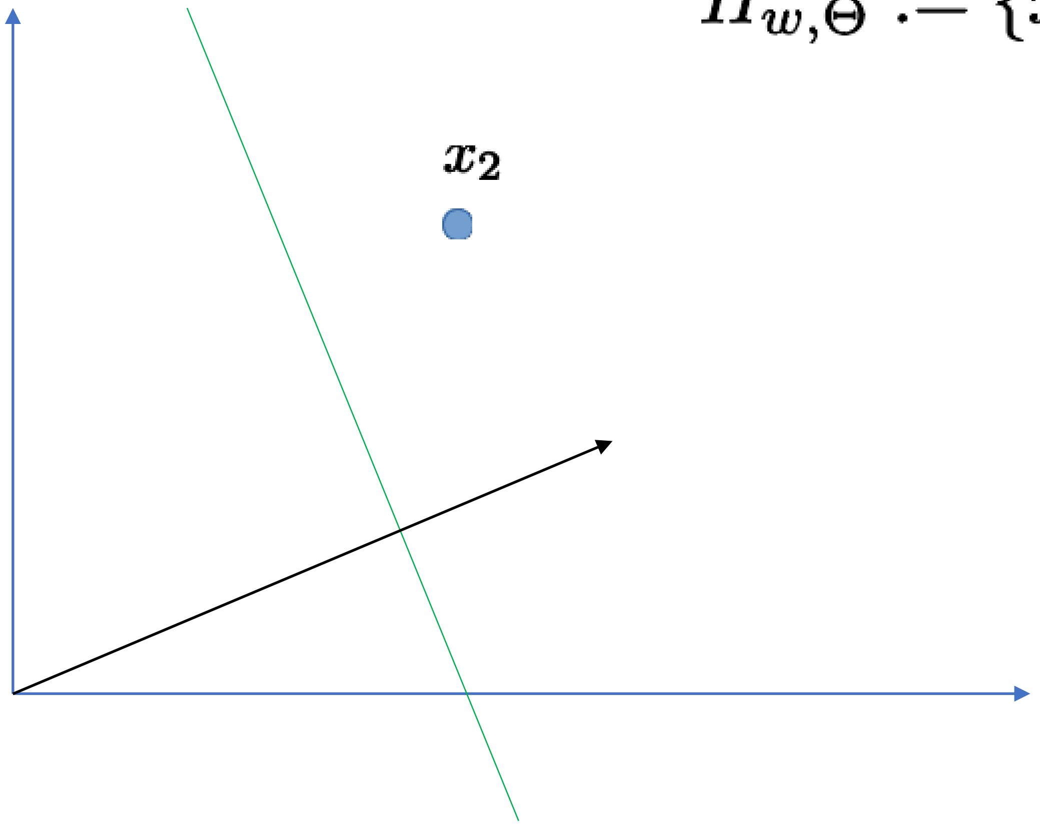

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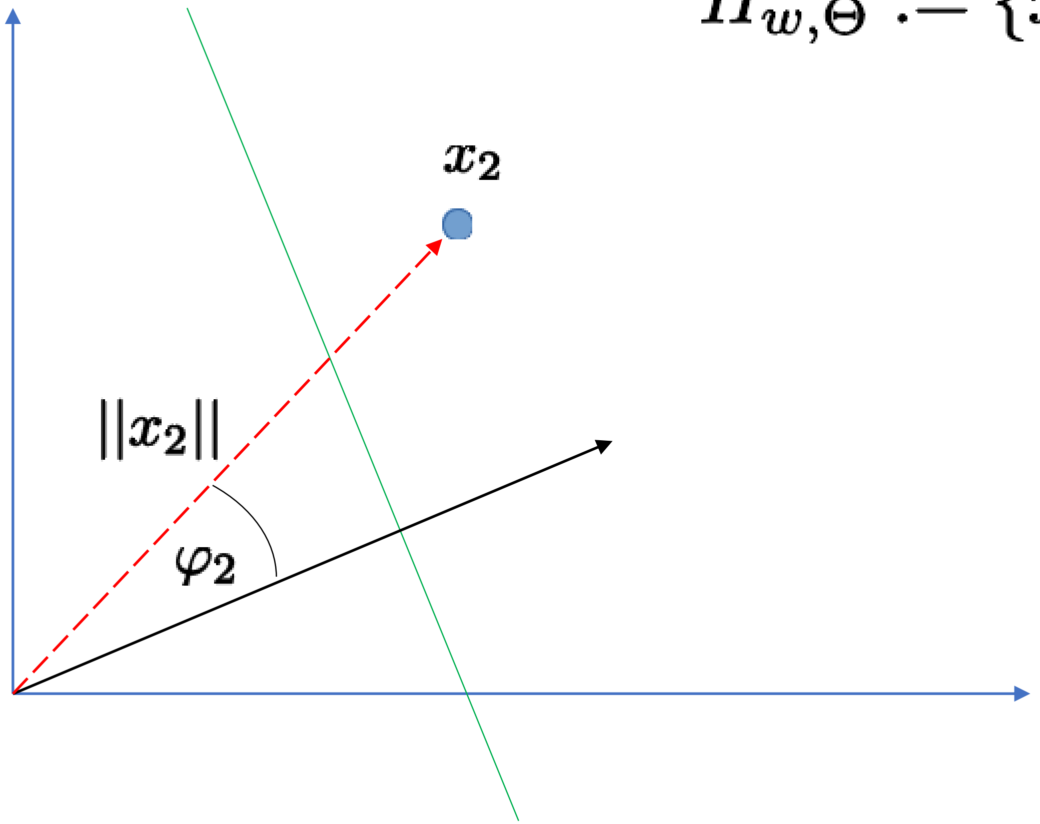

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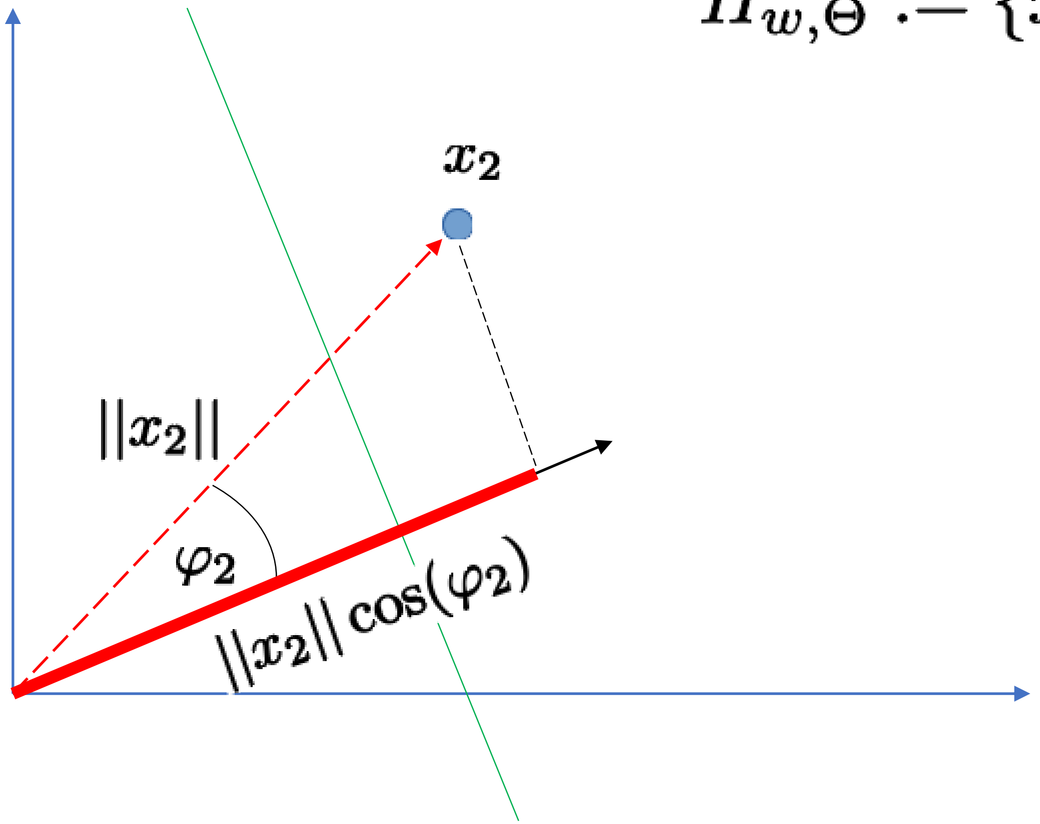

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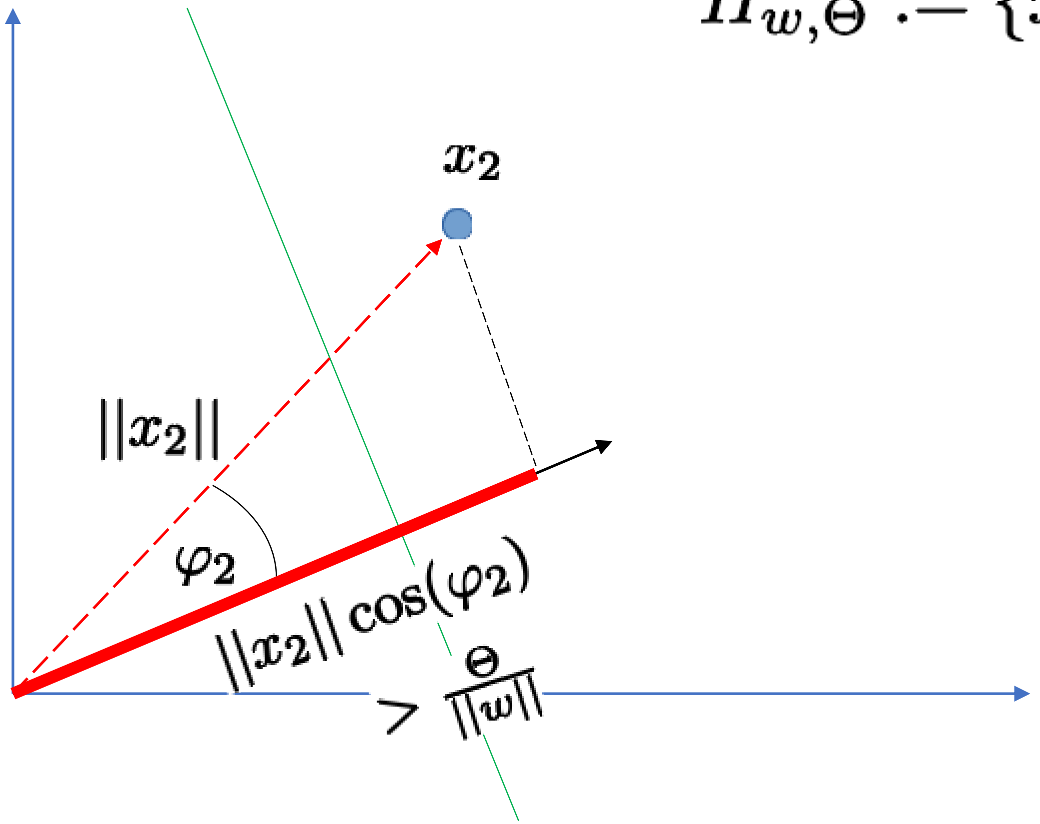

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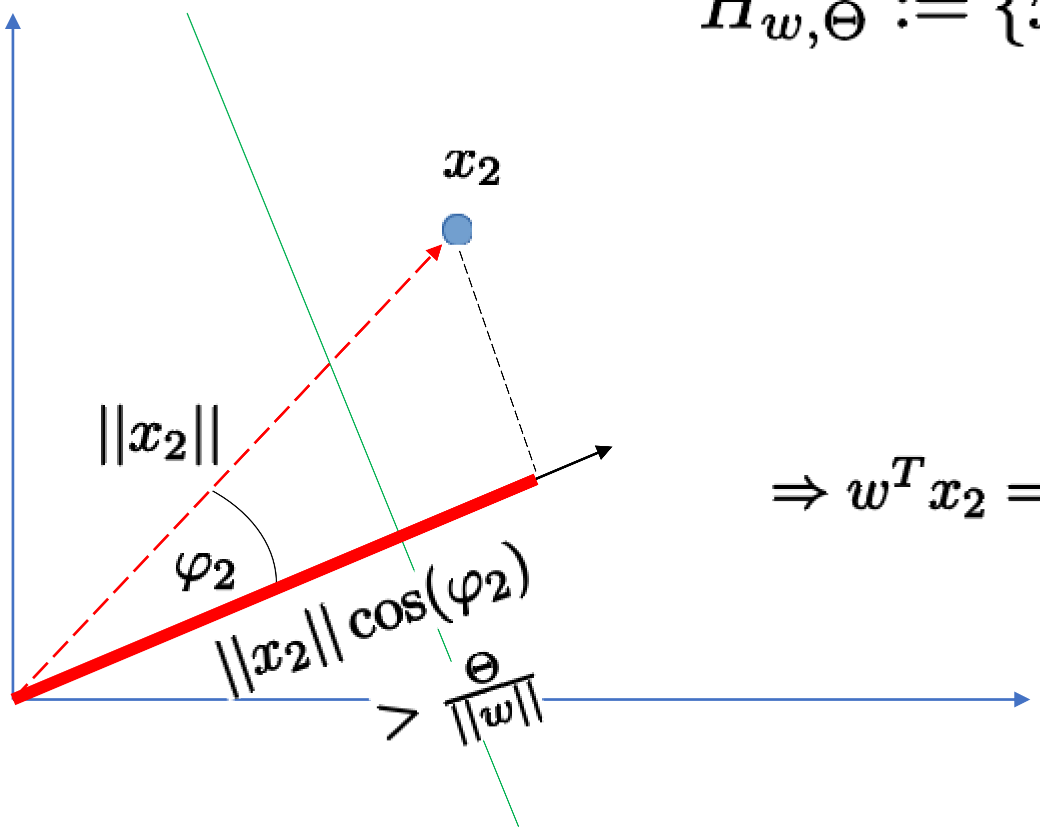

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5. Activation function is step function $y := f_a := \begin{cases} 1 & if \sum_{i=1}^{n} w_i x_i \geq \Theta \\ 0 & else \end{cases}$

Revision: Lecture

- Reformulation of step function

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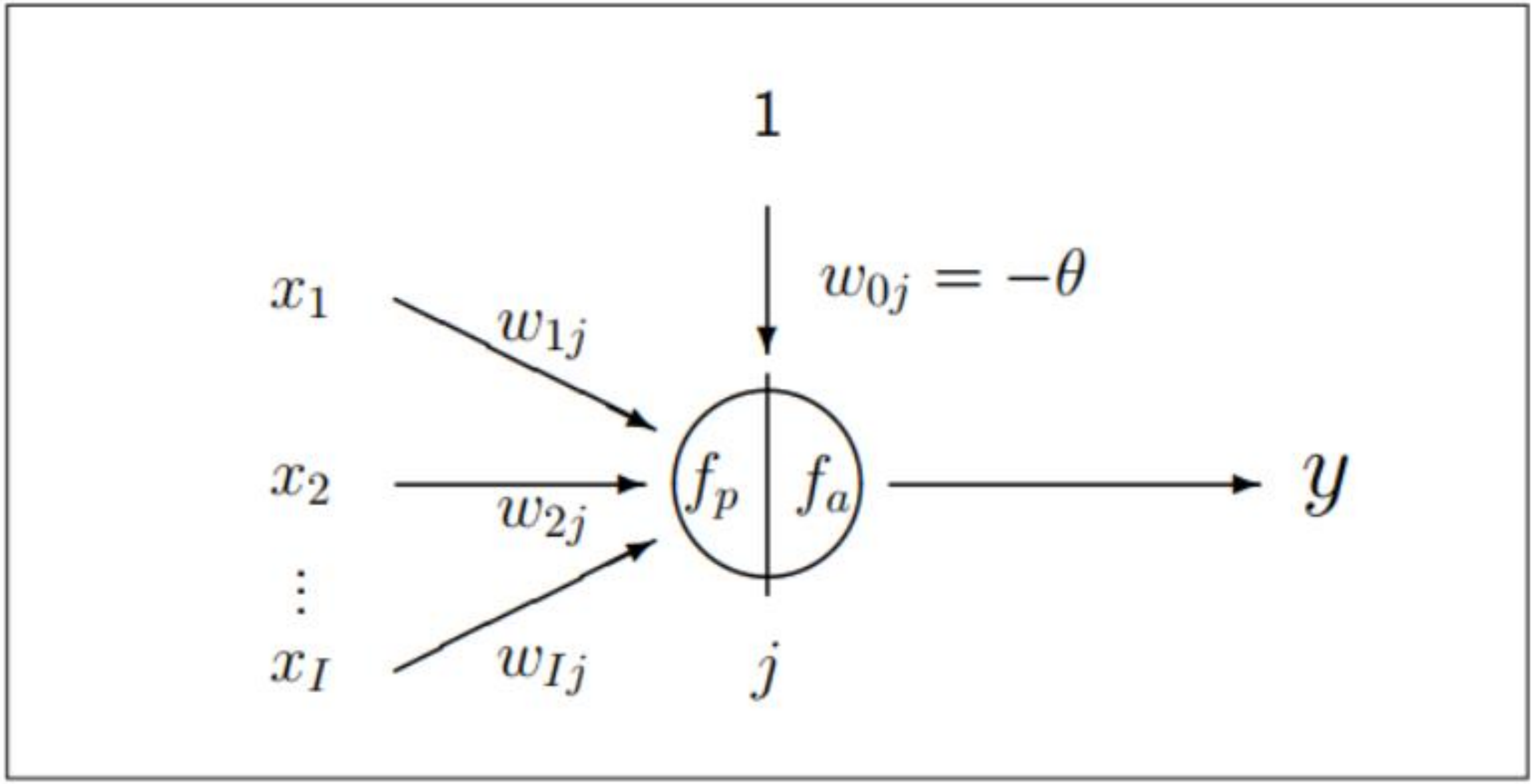
Scheme of Artificial Neuron



$f_p|f_a$ wird oft weggelassen, wenn aus dem Zusammenhang klar.

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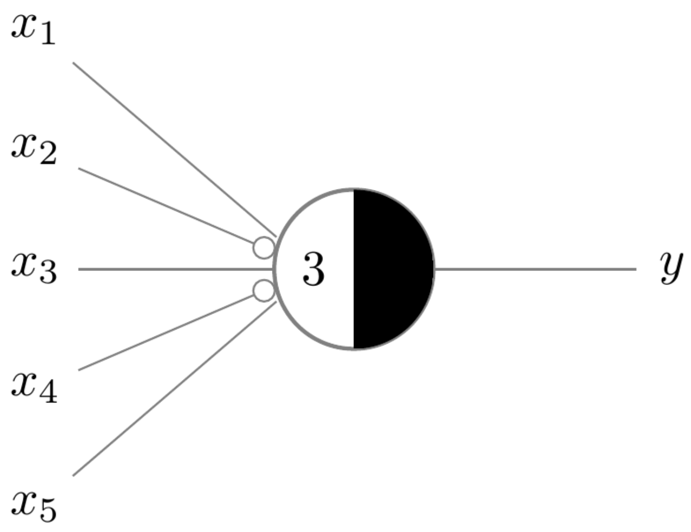

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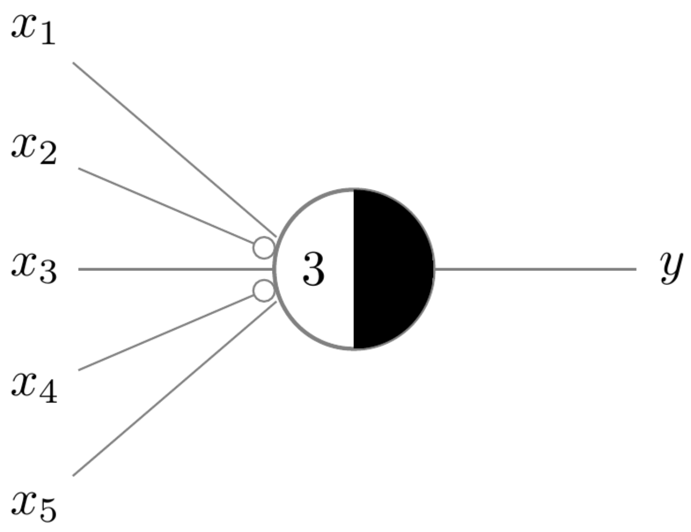

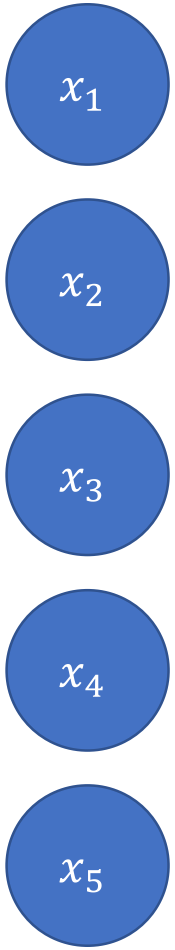

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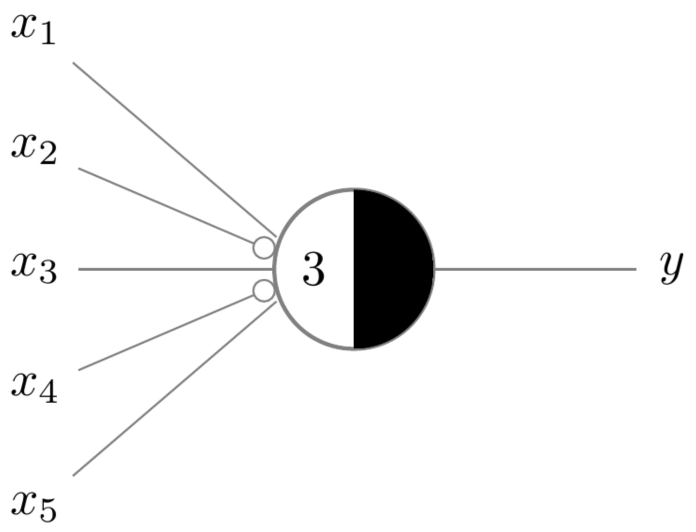

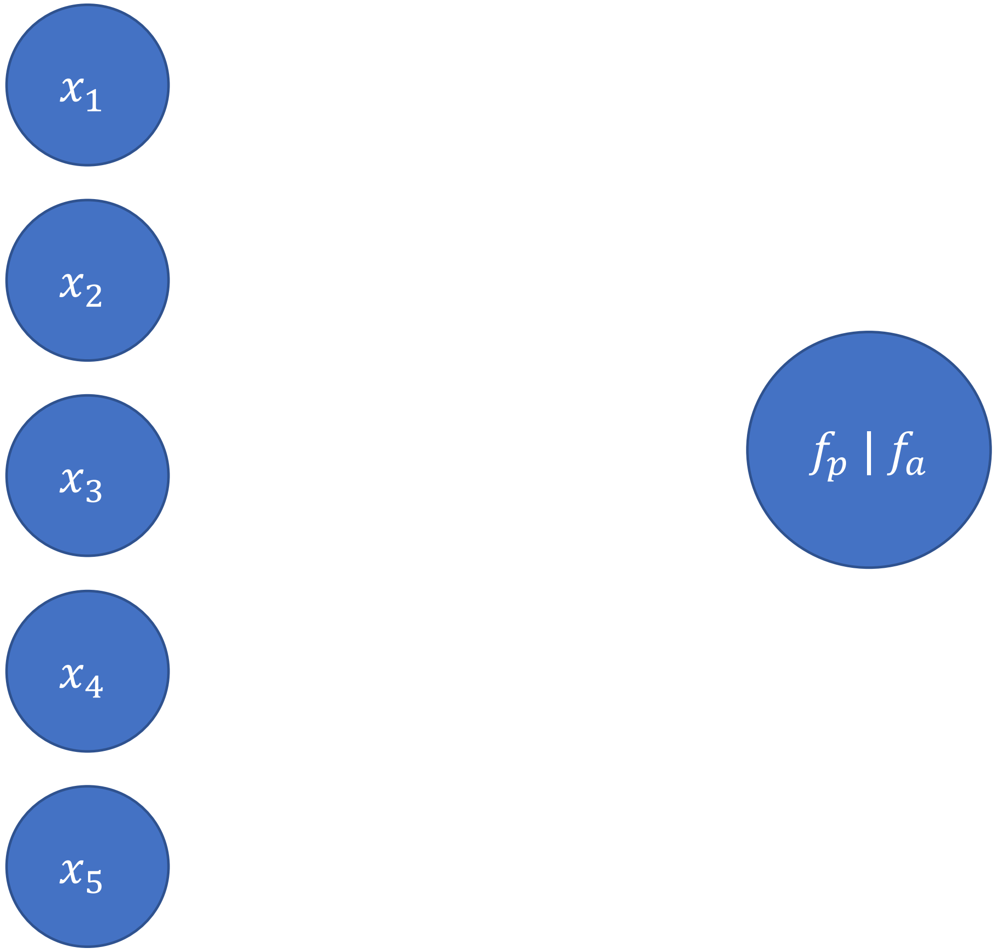

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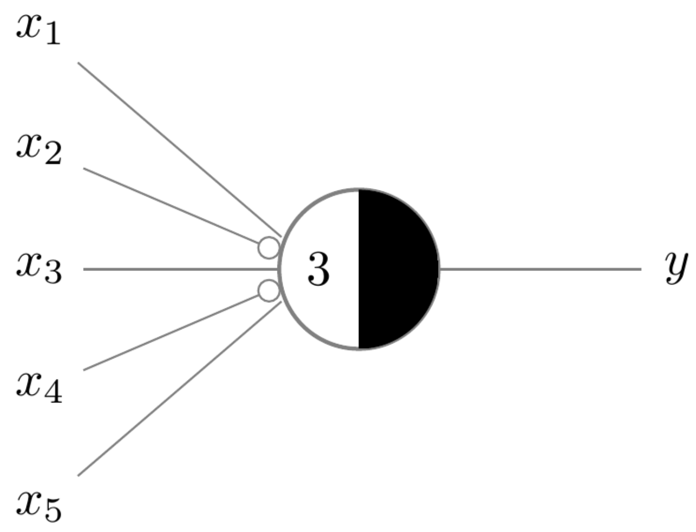

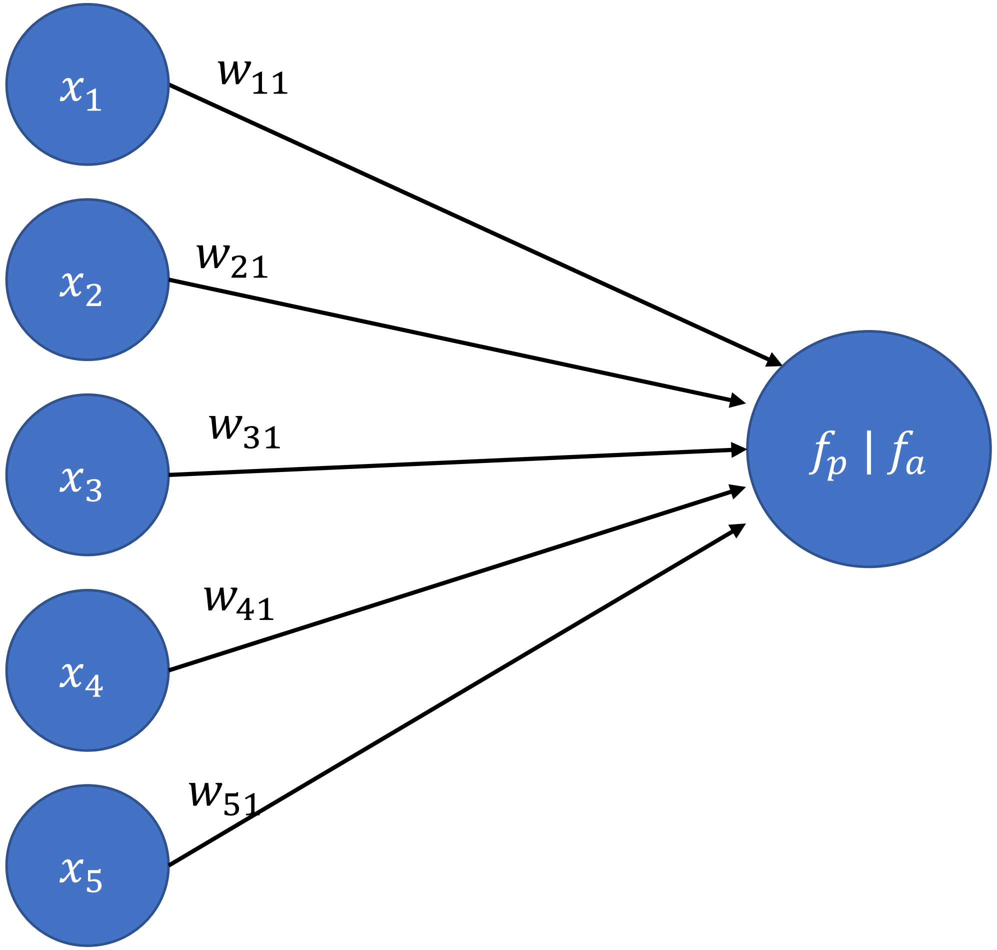

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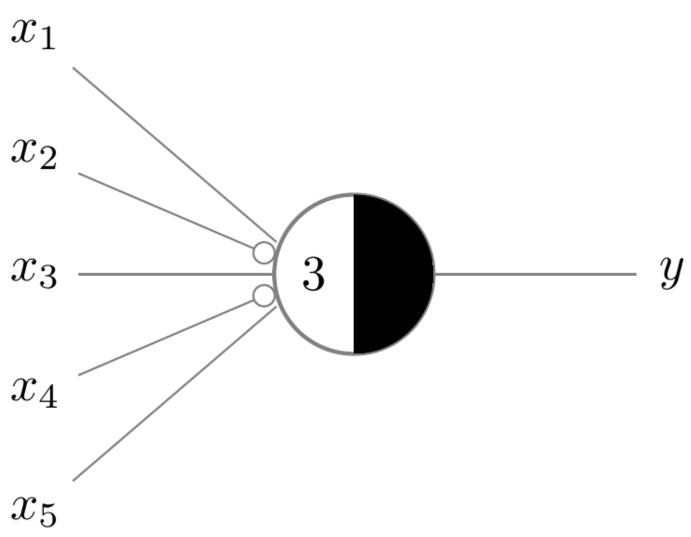

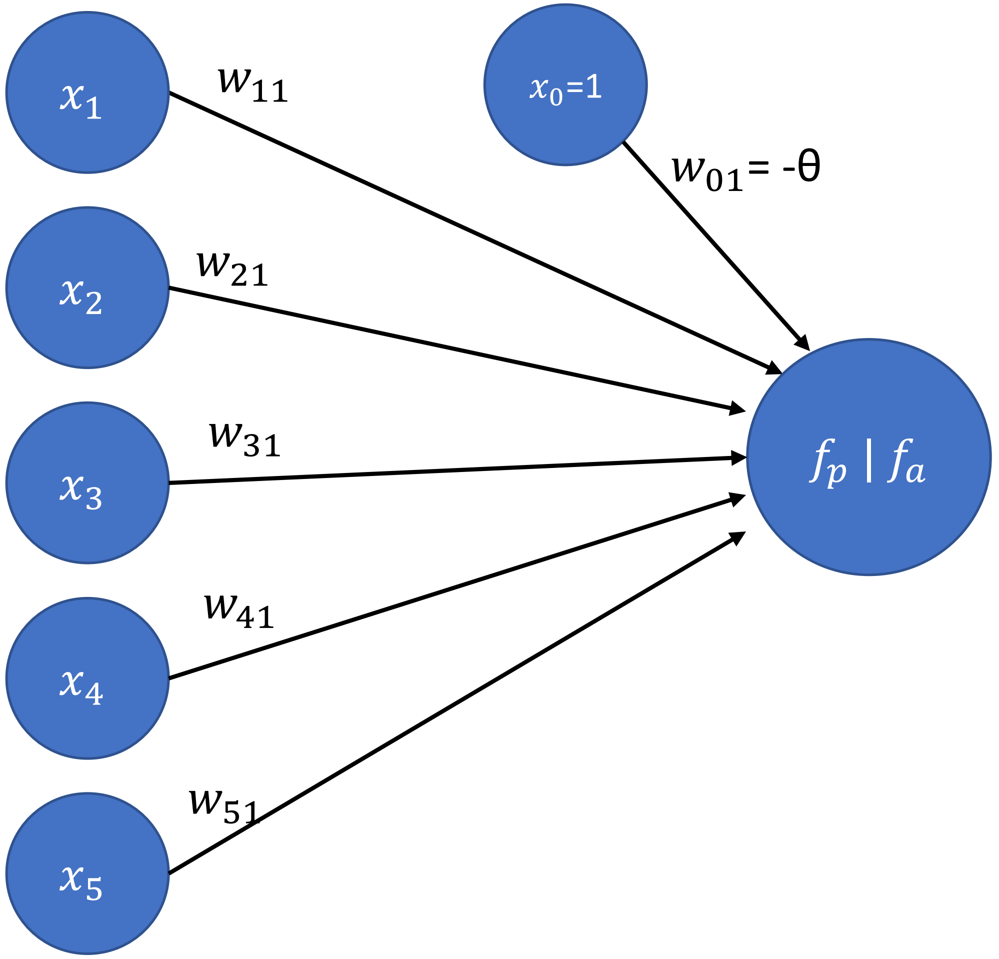

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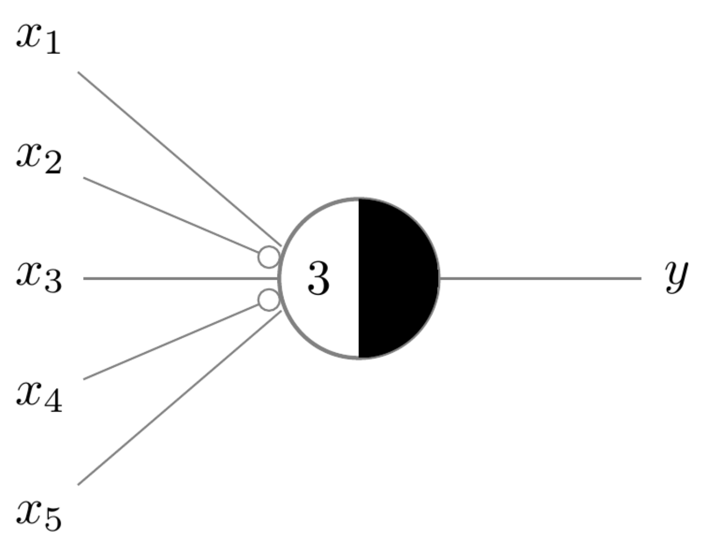

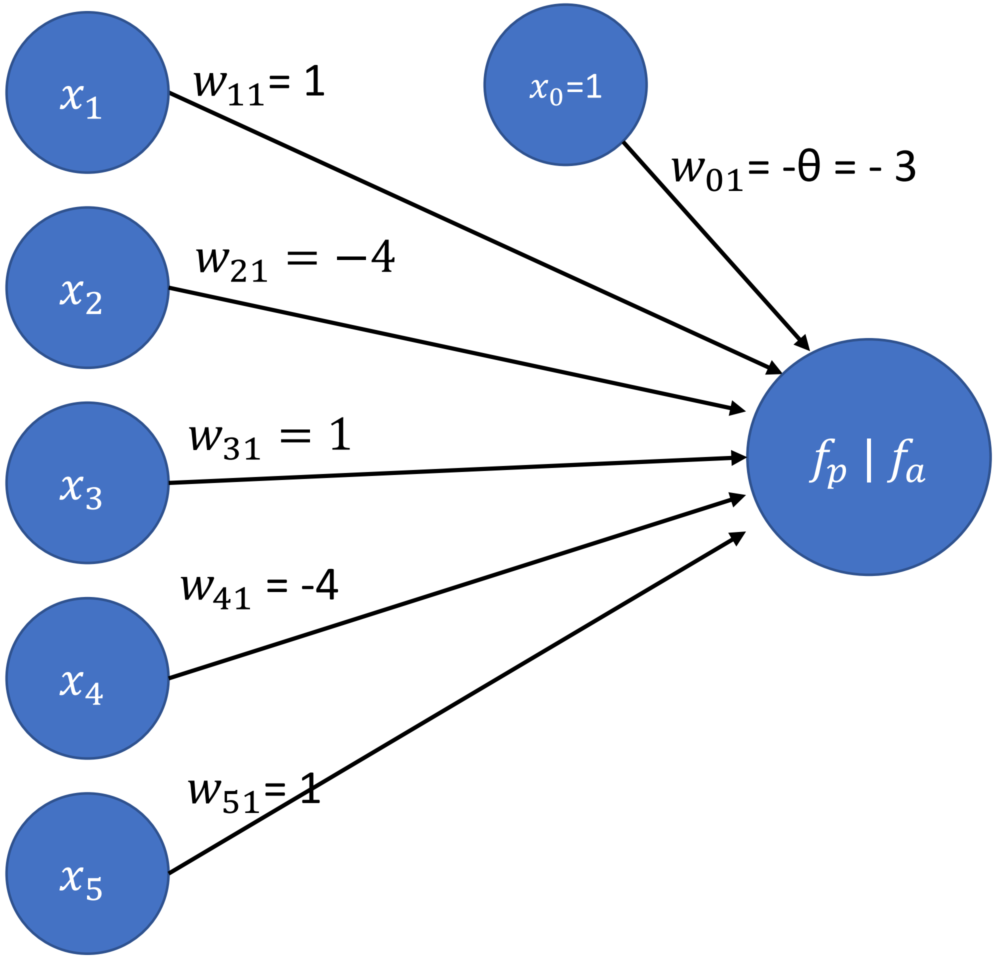

Content

- Revision: Mahalanobis Classifier
- Revision: Lecture
- Hebbian Learning

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- Postulated by Donald Olding Hebb in 1949
- Foundation of many learning rules

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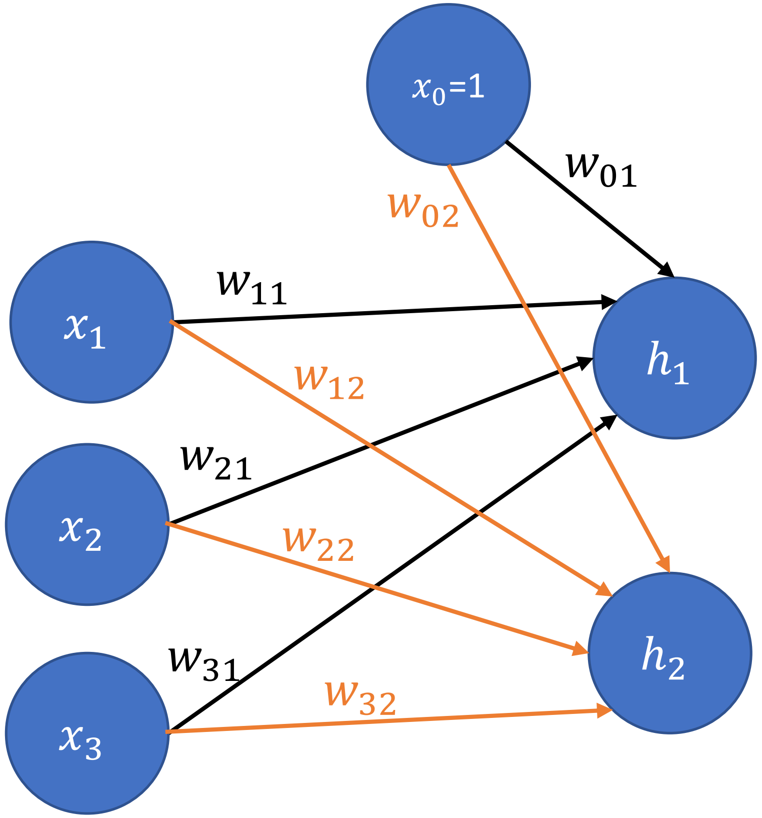
  - In artificial neural networks the strength og the connection is usually represented by the edge weight

Calculation of propagated value



$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^{3} w_{i2} x_i$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \qquad \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$

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