Rosenblatt Perceptron

Neuroinformatics Tutorial 6

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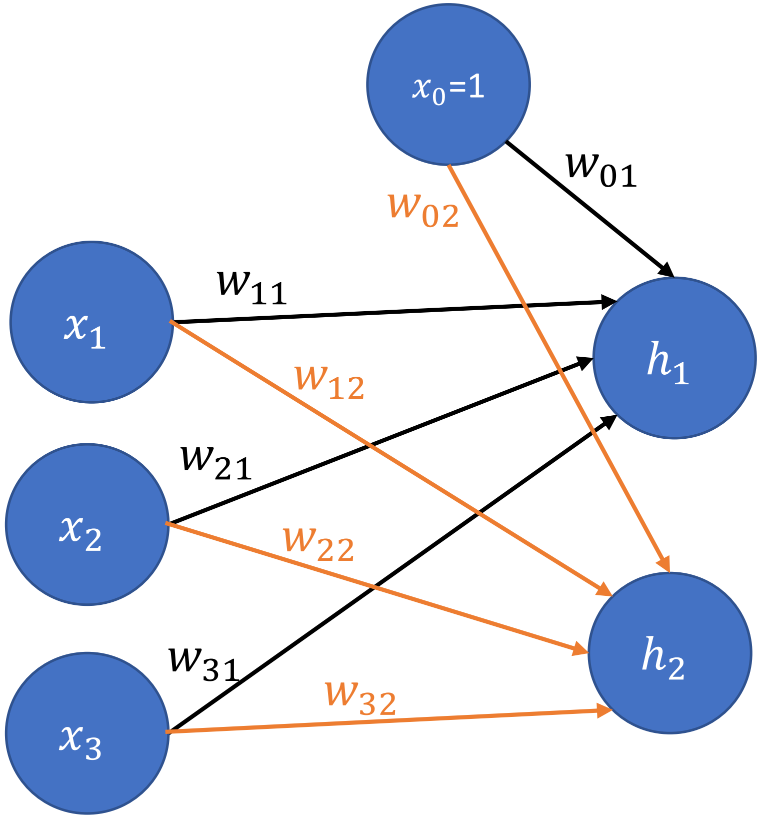
Content

- **Revision: Practical Task**
- Revision: Lecture
- New Practical Task

Calculation of propagated value



$$h_1 = \sum_{i=0}^{3} w_{i1} x_i$$

$$h_2 = \sum_{i=0}^{3} w_{i2} x_i$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad W = \begin{bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = W^T \cdot x$$

Hebbian Learning Rule

- Given:
  - Outputs of previous neurons (can also be input neurons)
  - In case of scheme : input neurons, i.e:

    $x_0, x_1, x_2, \ldots x_n \in \{-1, 1\}$

  - Output of current neurons
  - In case of scheme: $h_1, h_2 \in \{-1, 1\}$
  - Weights in between all neurons
  - Learning rate α
- Learning Rule:
  - Update weights by comparing similarity of outputs
  - $\Delta w_{i,j} \coloneqq \alpha \cdot x_i \cdot h_j$
  - $w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$

Hebbian Learning Rule

- Given:
  - Outputs of previous neurons (can also be input neurons)
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    $x_0, x_1, x_2, \dots x_n \in \{-1, 1\}$

  - Output of current neurons
  - In case of scheme: $h_1, h_2 \in \{-1, 1\}$
  - Weights in between all neurons
  - Learning rate α
- Learning Rule:
  - Update weights by cor
  - $\Delta w_{i,j} := \alpha \cdot x_i \cdot h_j$
  - $w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$

$$\begin{pmatrix} x_0 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} h_1 & h_2 \end{pmatrix}$$

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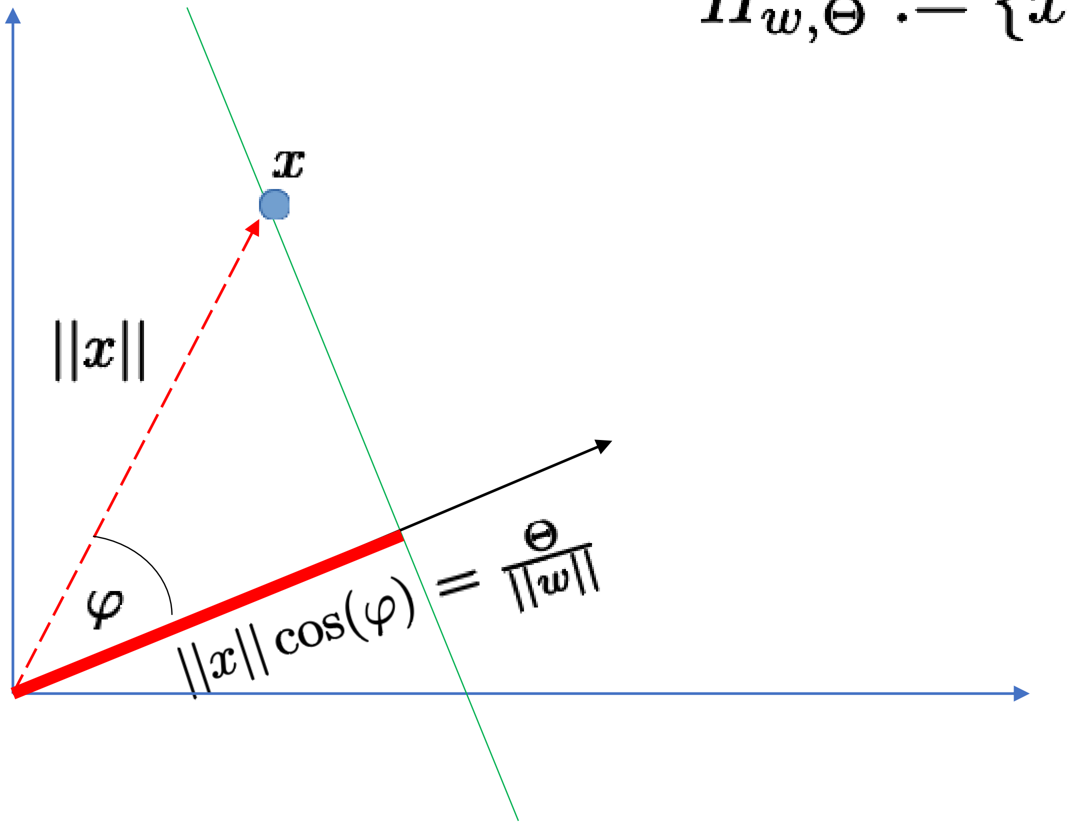
Revision: Lecture

- Which statements regarding Rosenblatt Perceptron are true?
  1. The weights and threshold of a RBP defines a hyperplane
  2. The extended version of the RBP lifts the input space into a higher dimension
  3. In the extended version of the RBP the hyperplane always goes through the origin
  4. The RBP can be viewed as a generalized McCulloch Pitts Neuron

Revision: Lecture

- Interpretation of formulas

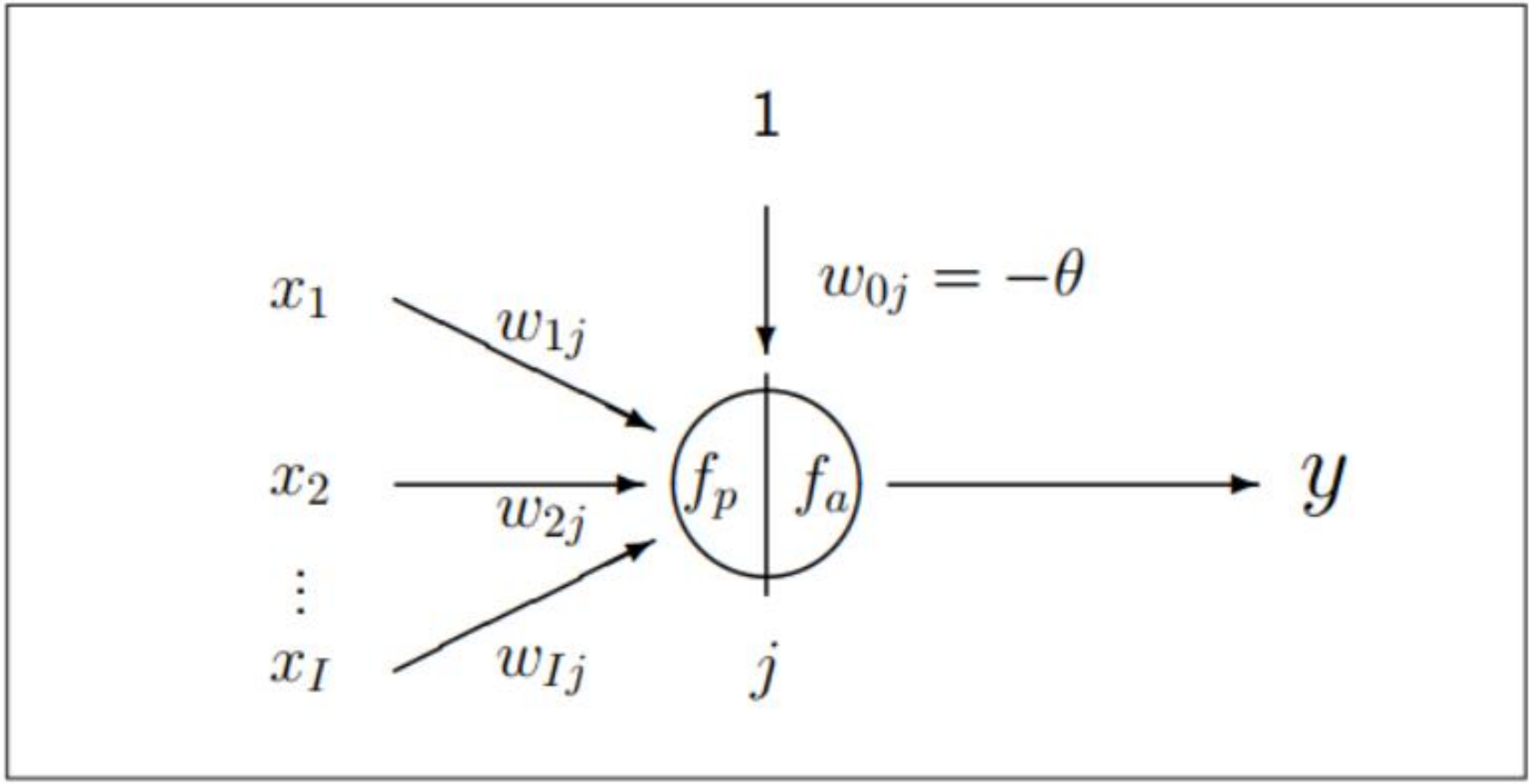


$$H_{w,\Theta} := \{x \in \mathbb{R}^n | w^T x = \Theta\}, w \in \mathbb{R}^n, \Theta \in \mathbb{R}$$

$$\Rightarrow \Theta = ||w|| \cdot ||x|| \cdot \cos(\varphi)$$

$$\Rightarrow ||x|| \cdot \cos(\varphi) = \frac{\Theta}{||w||}$$

Scheme of Artificial Neuron



$f_p|f_a$ wird oft weggelassen, wenn aus dem Zusammenhang klar.

Rosenblatt Learning Rule

- How does the Rosenblatt Learning Rule/Algorithm work? (must know!)

Rosenblatt Learning Rule

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  - Let $w := (-\Theta, w_1, \ldots, w_n)^T \in \mathbb{R}^{n+1}$ denote the extended weight vector including the bias
  - Let $w(i)$ denote the weight vector at iteration i
  - Let $x := (1, x_1, \ldots, x_n)^T \in \Omega := \mathcal{P} \cup \mathcal{N} \subset \mathbb{R}^{n+1}$ denote an arbitrary extended sample point from the trainind data set

Rosenblatt Learning Rule

- Let $\hat{y}(x) := \begin{cases} 1 & \text{if } x \in \mathcal{P} \\ -1 & \text{else} \end{cases}$ denote the desired target output
- Let $\tilde{y}_{w(i)}(x) := f_a(f_p(x))$ denote the actual output of the perceptron with weight vector $w(i)$

Rosenblatt Learning Rule

- Idea:
    - Draw a sample point $\boldsymbol{x}$ randomly
    - Check if perceptron output is target output
    - If not:
        - If should have been positive: add $\boldsymbol{x}$ to weight vector
        - If should have been negative: subtract $\boldsymbol{x}$ from weight vector

Rosenblatt Learning Rule

- If $\hat{y}(x) == \tilde{y}_{w(i)}(x)$

  Do nothing

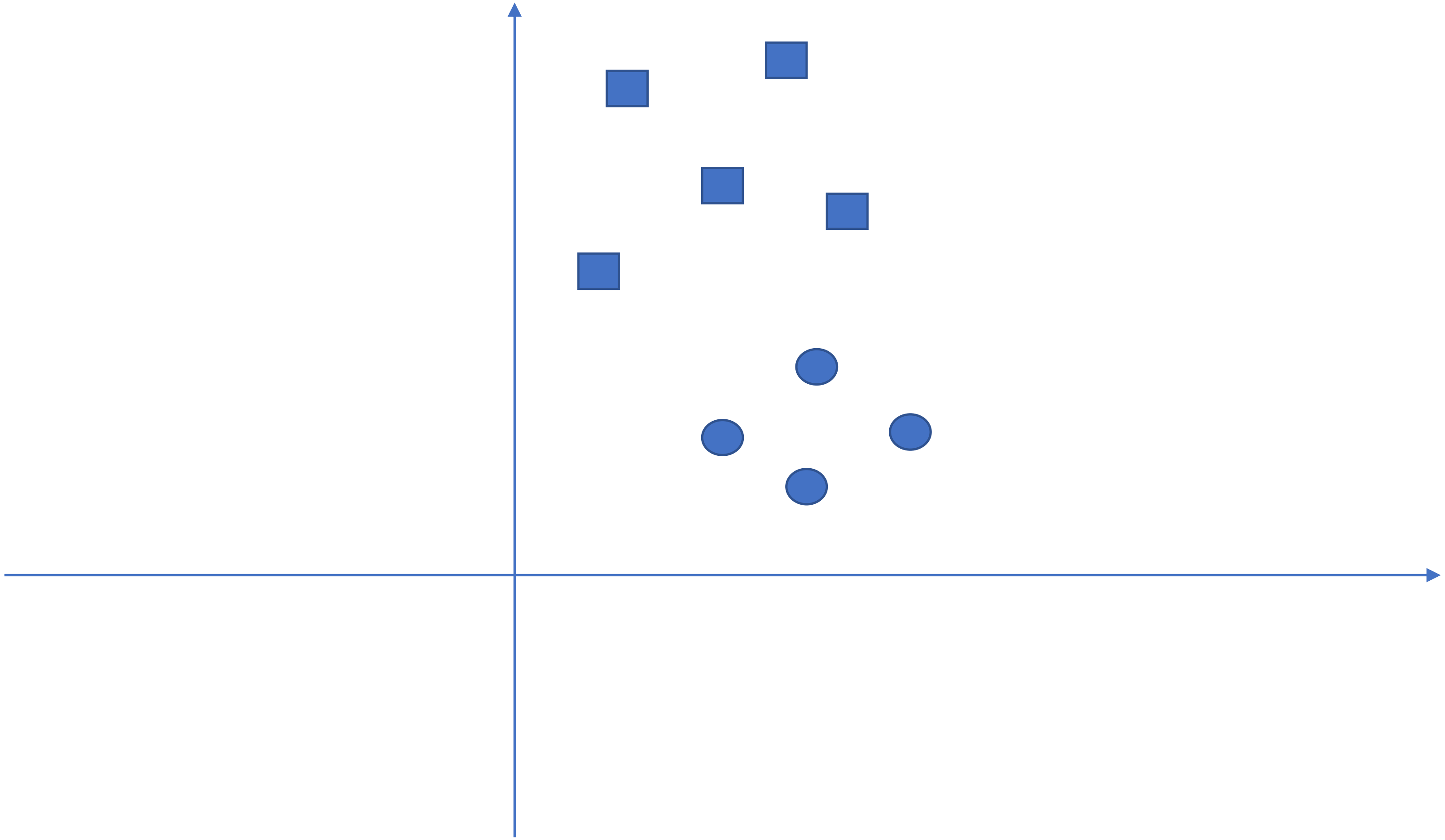
- If $\hat{y}(x) \neq \tilde{y}_{w(i)}(x)$
  - If $\hat{y}(x) == 1$

    $$w(i+1) \leftarrow w(i) + x$$

  - else

    $$w(i+1) \leftarrow w(i) - x$$

Rosenblatt Learning Rule



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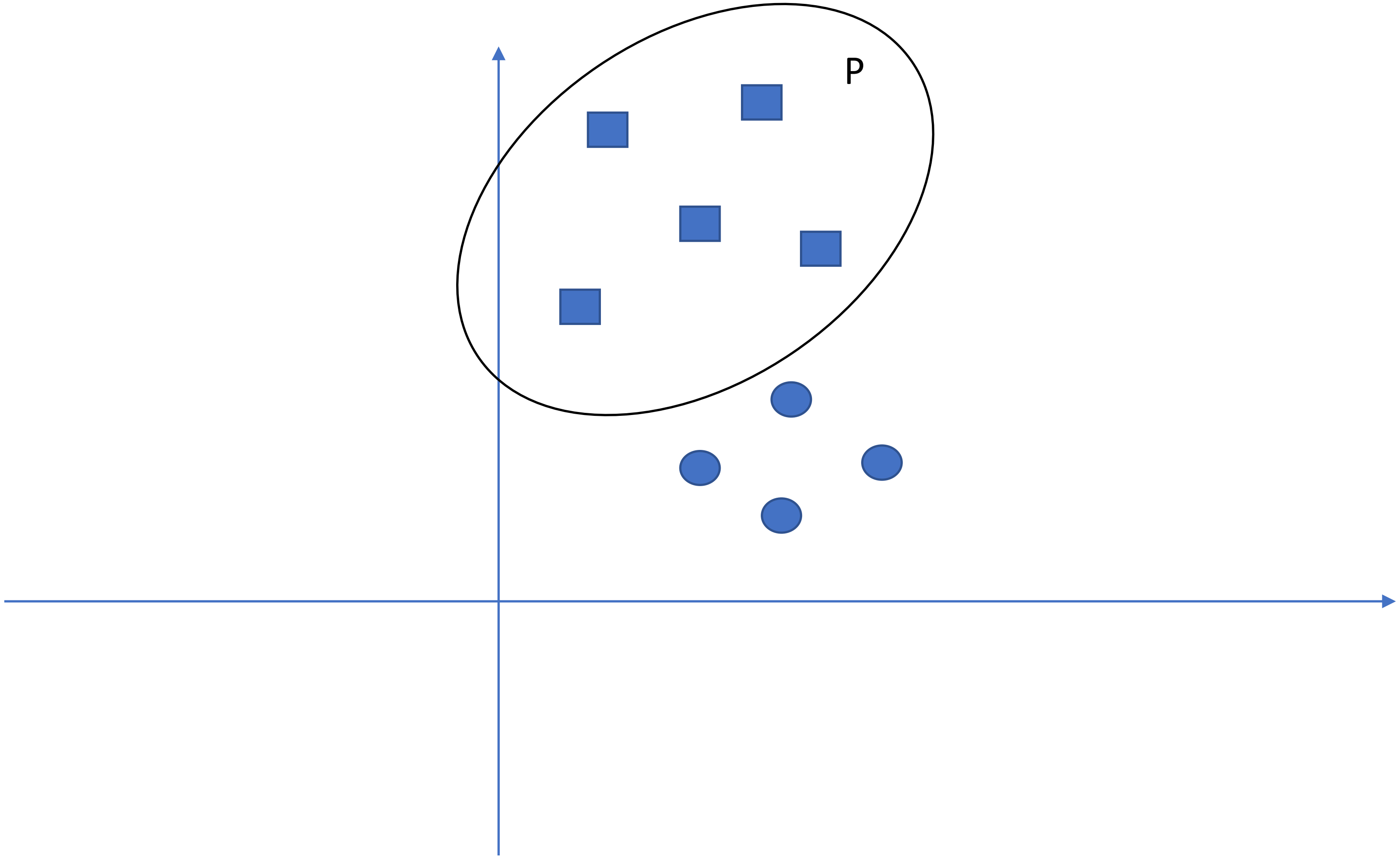

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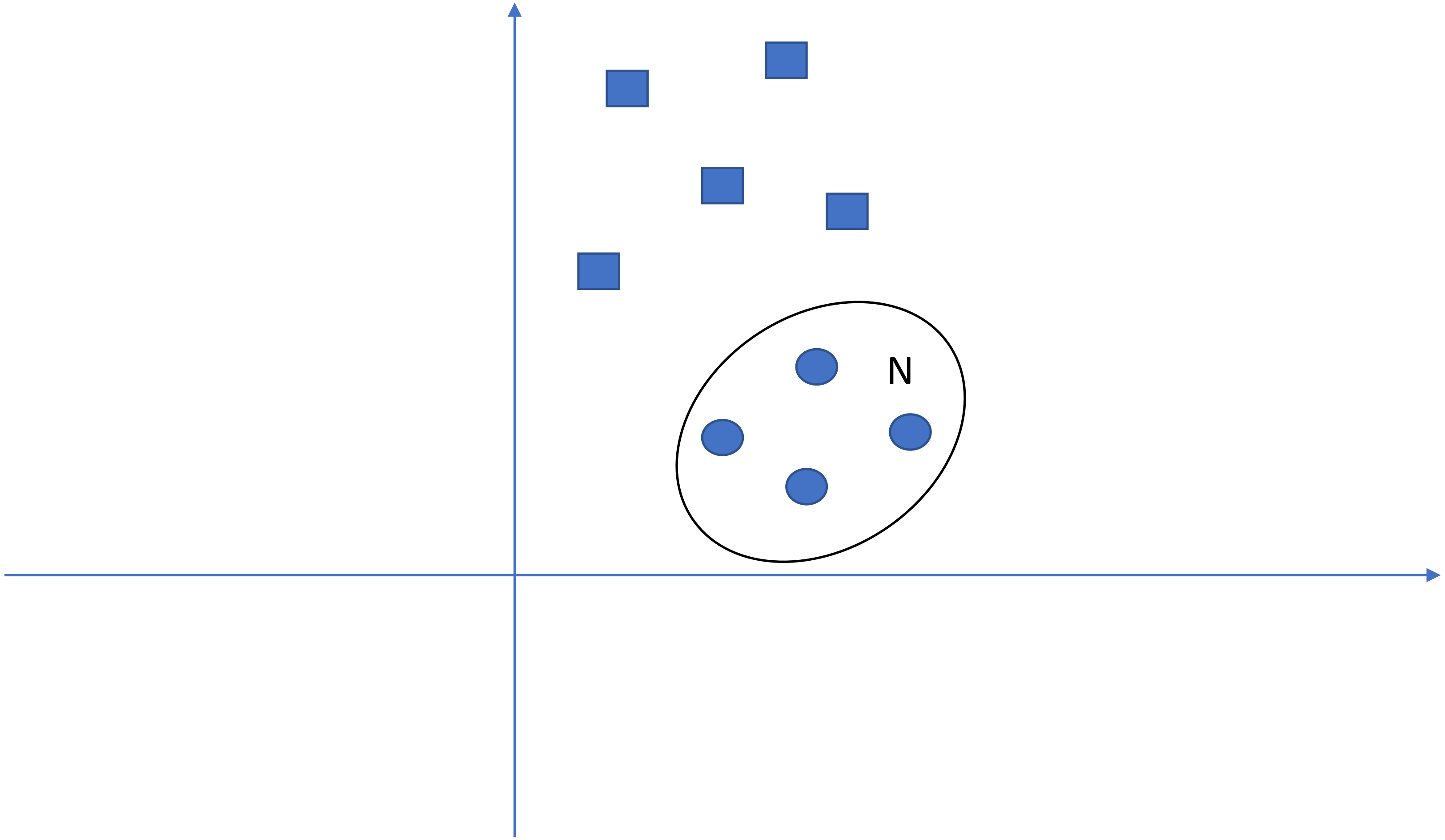

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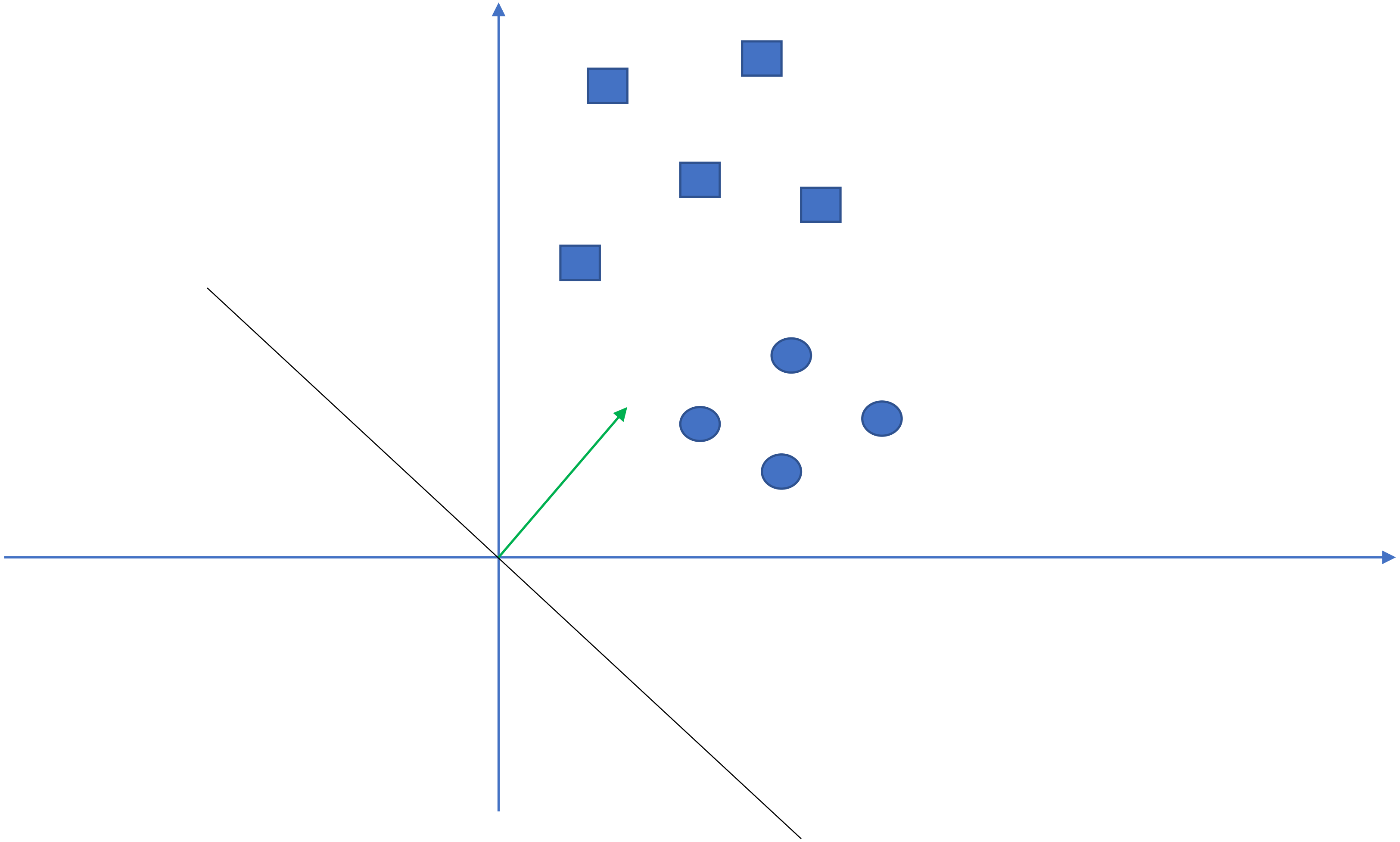

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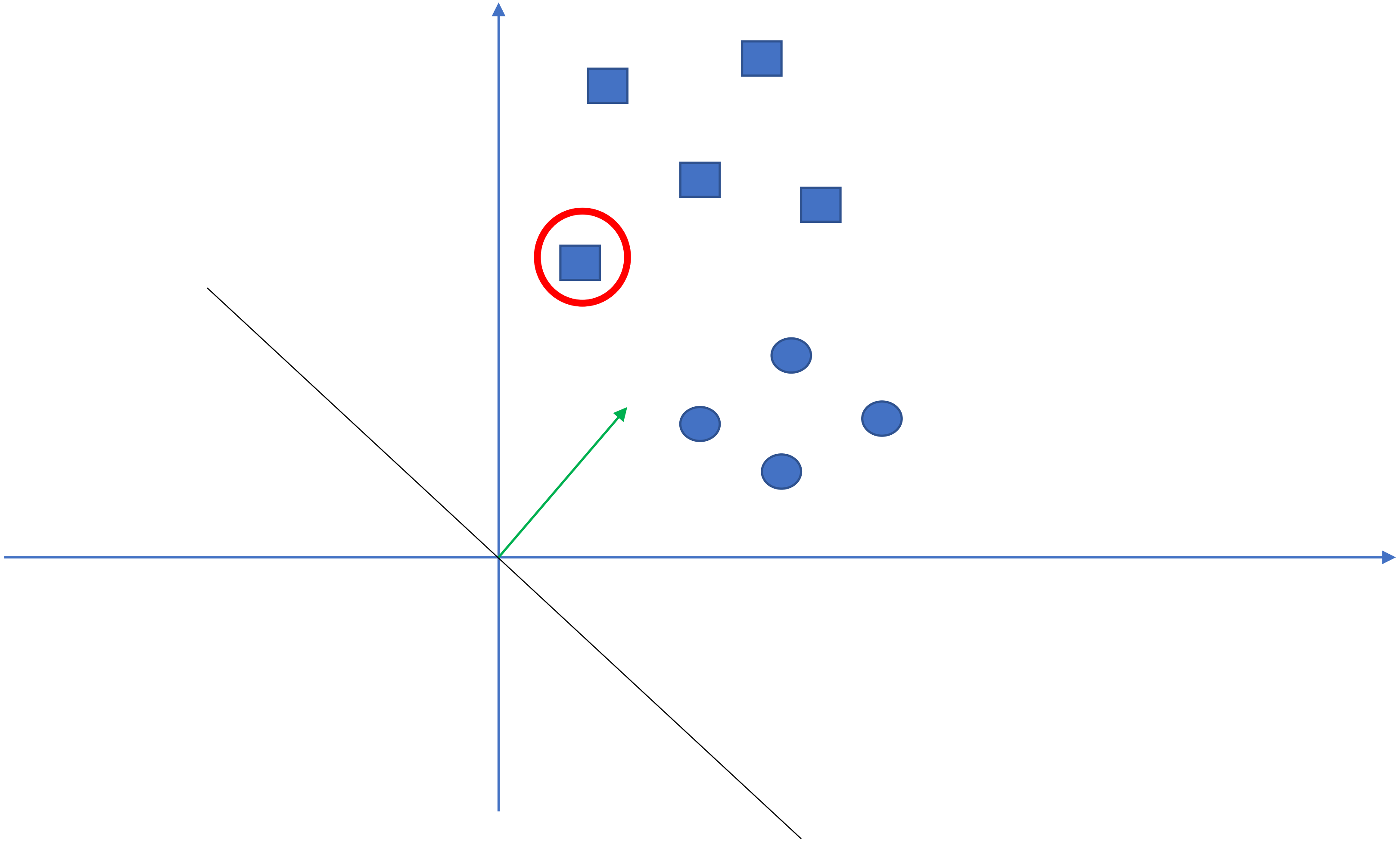

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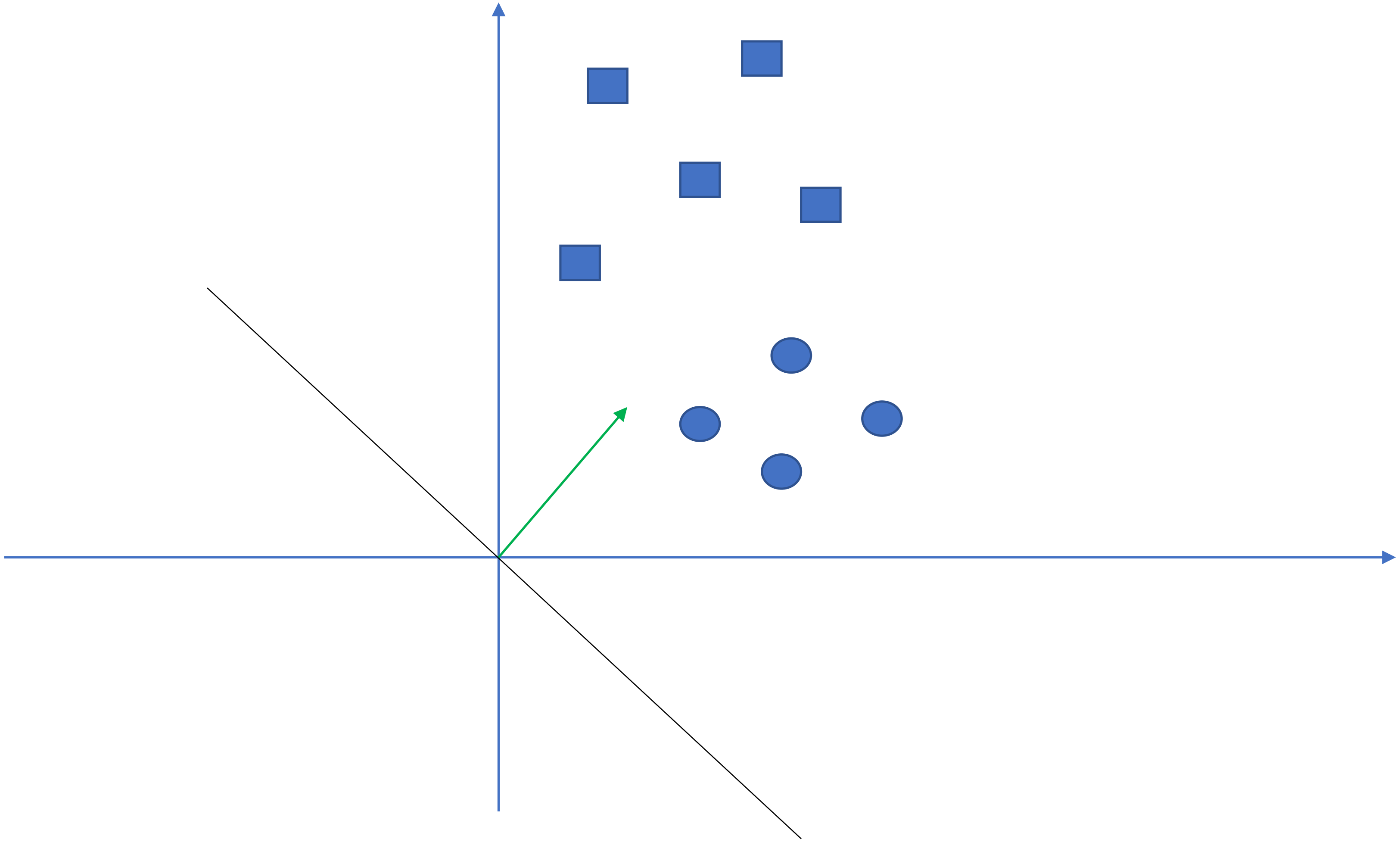

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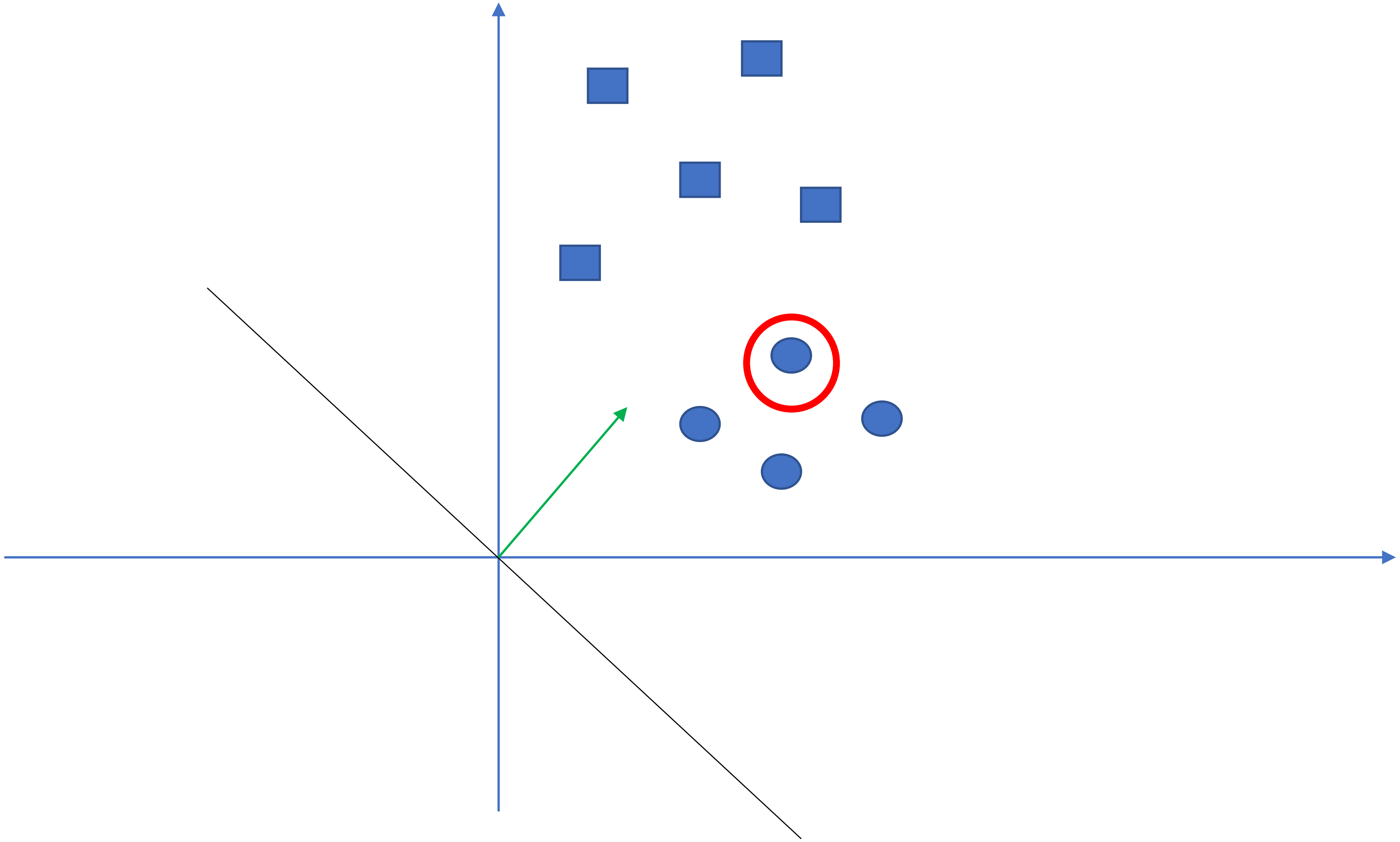

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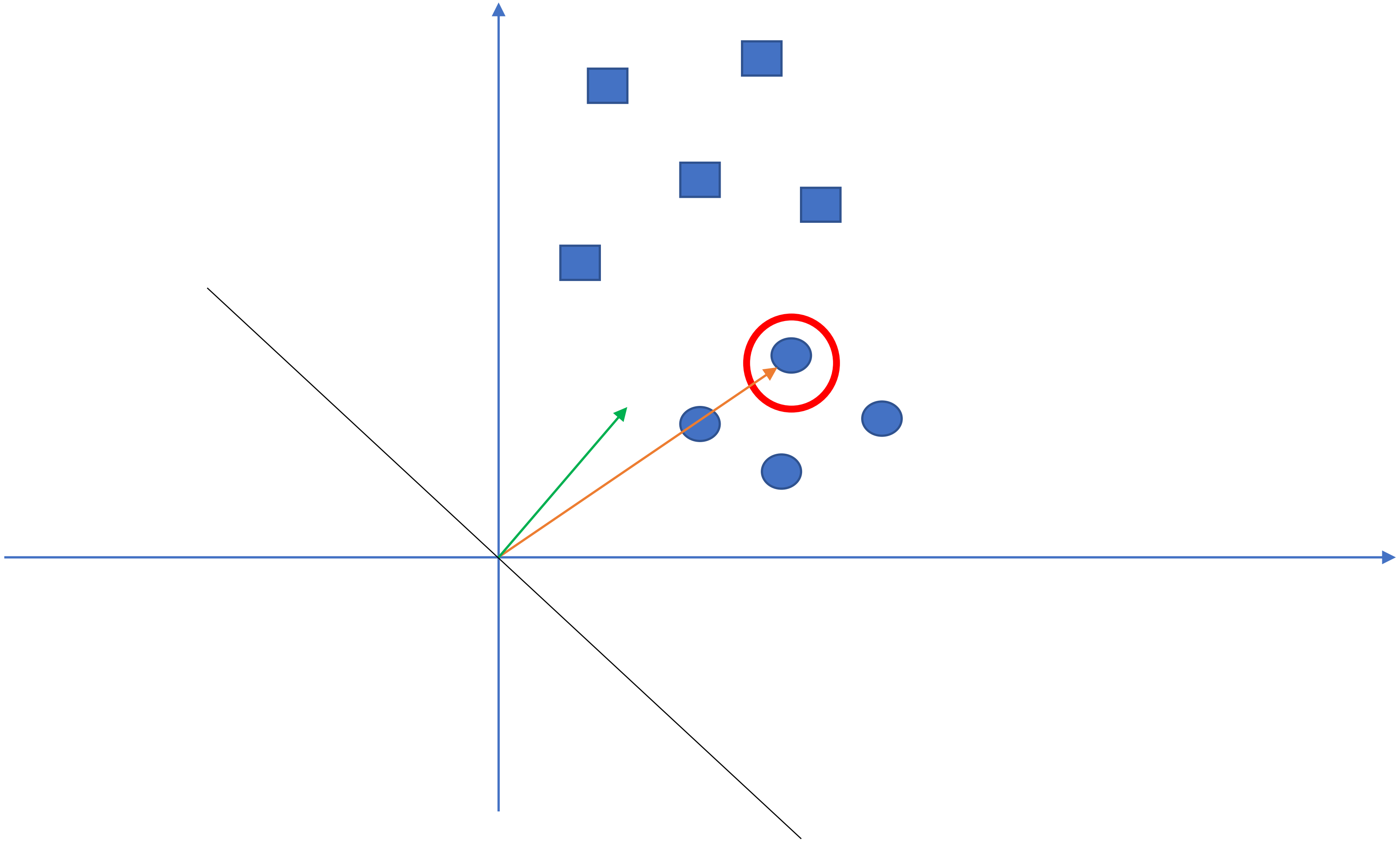

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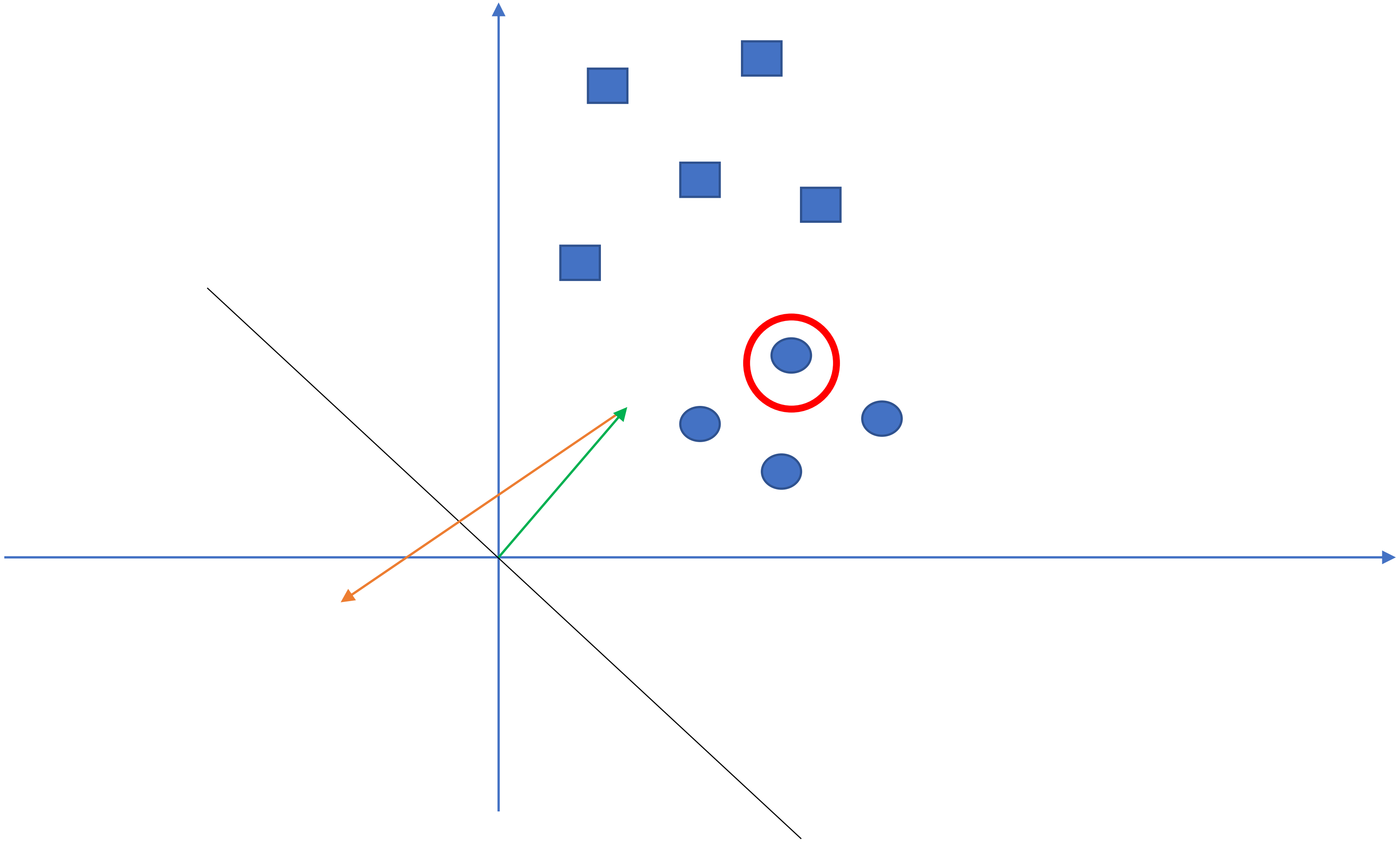

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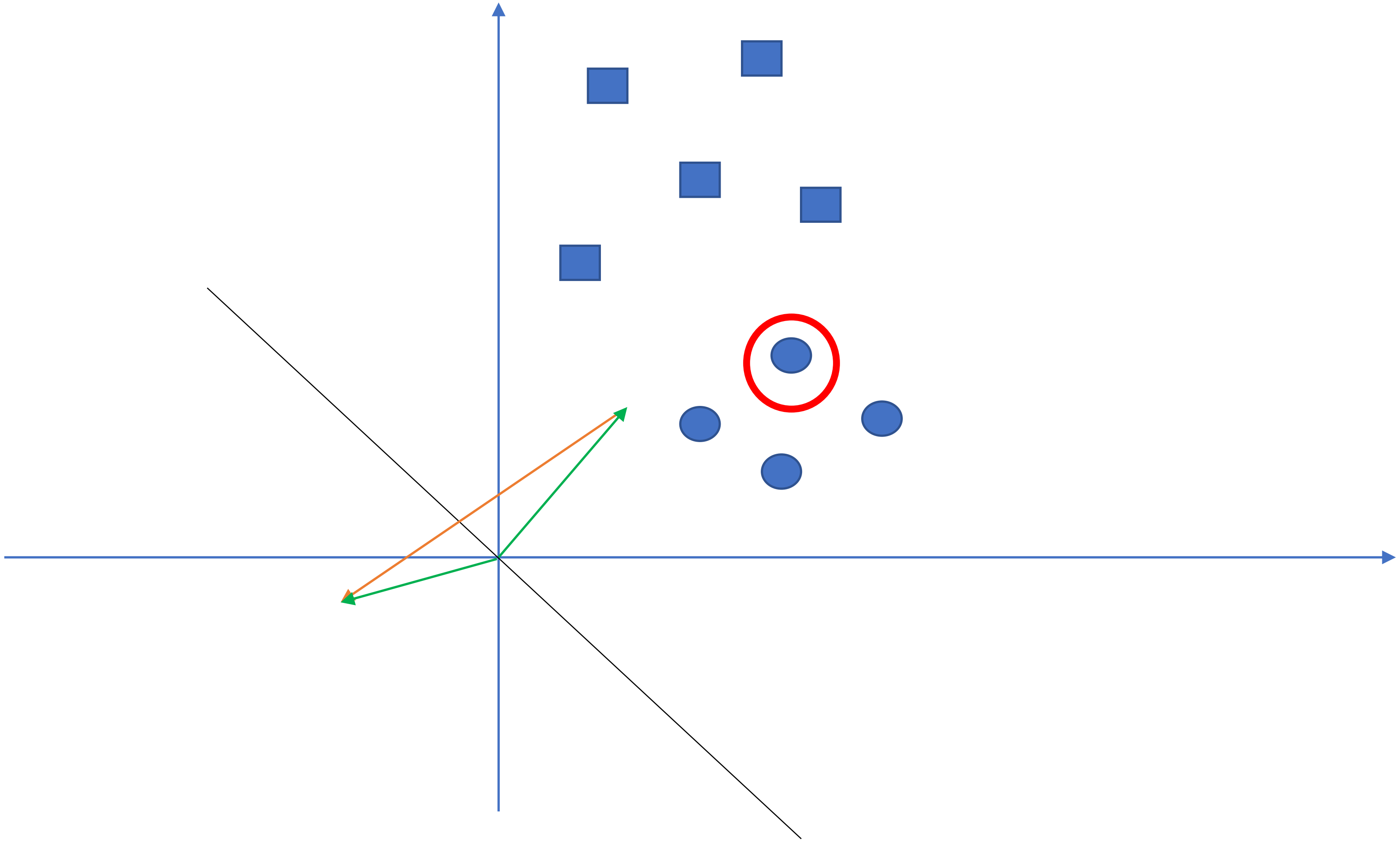

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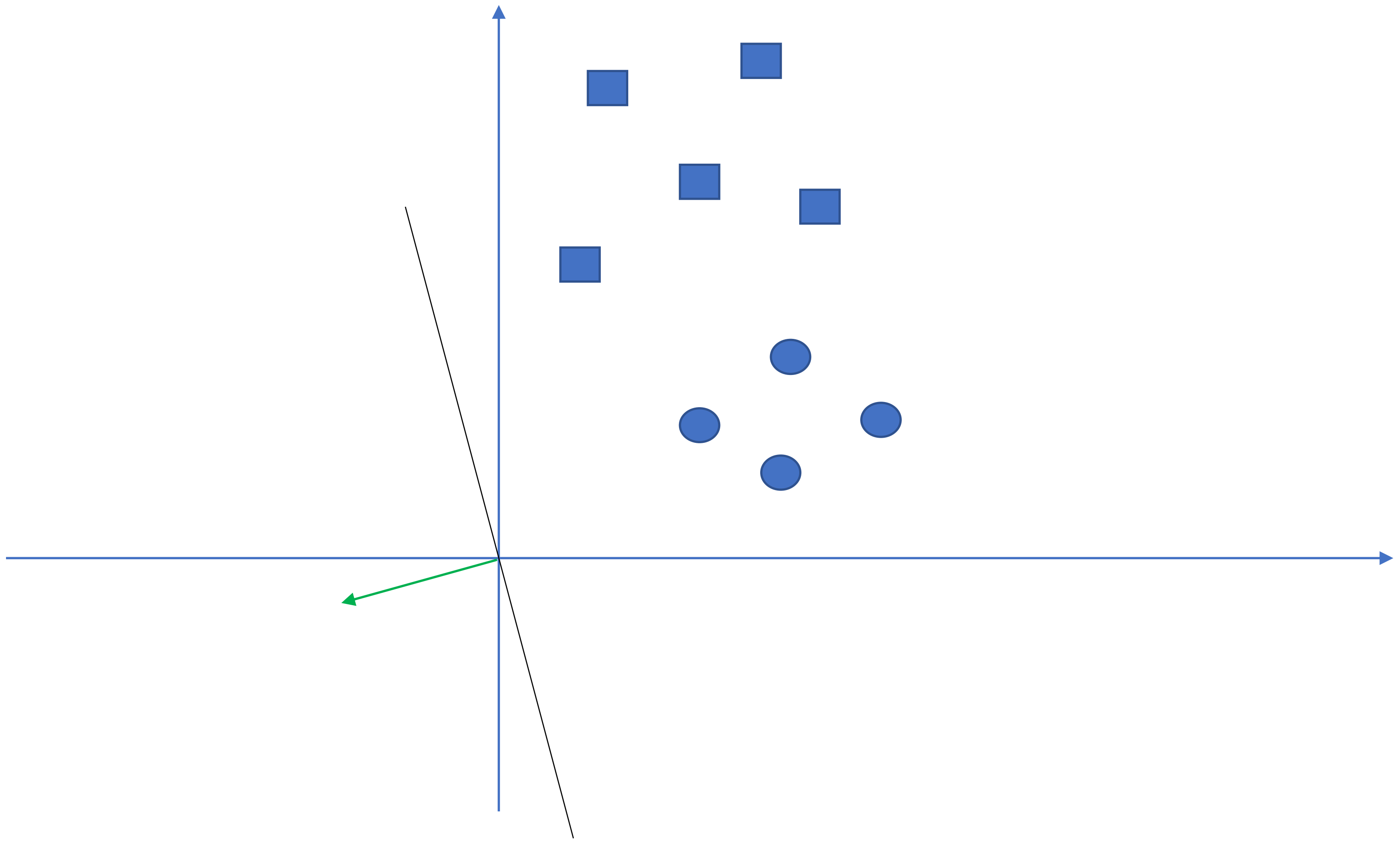

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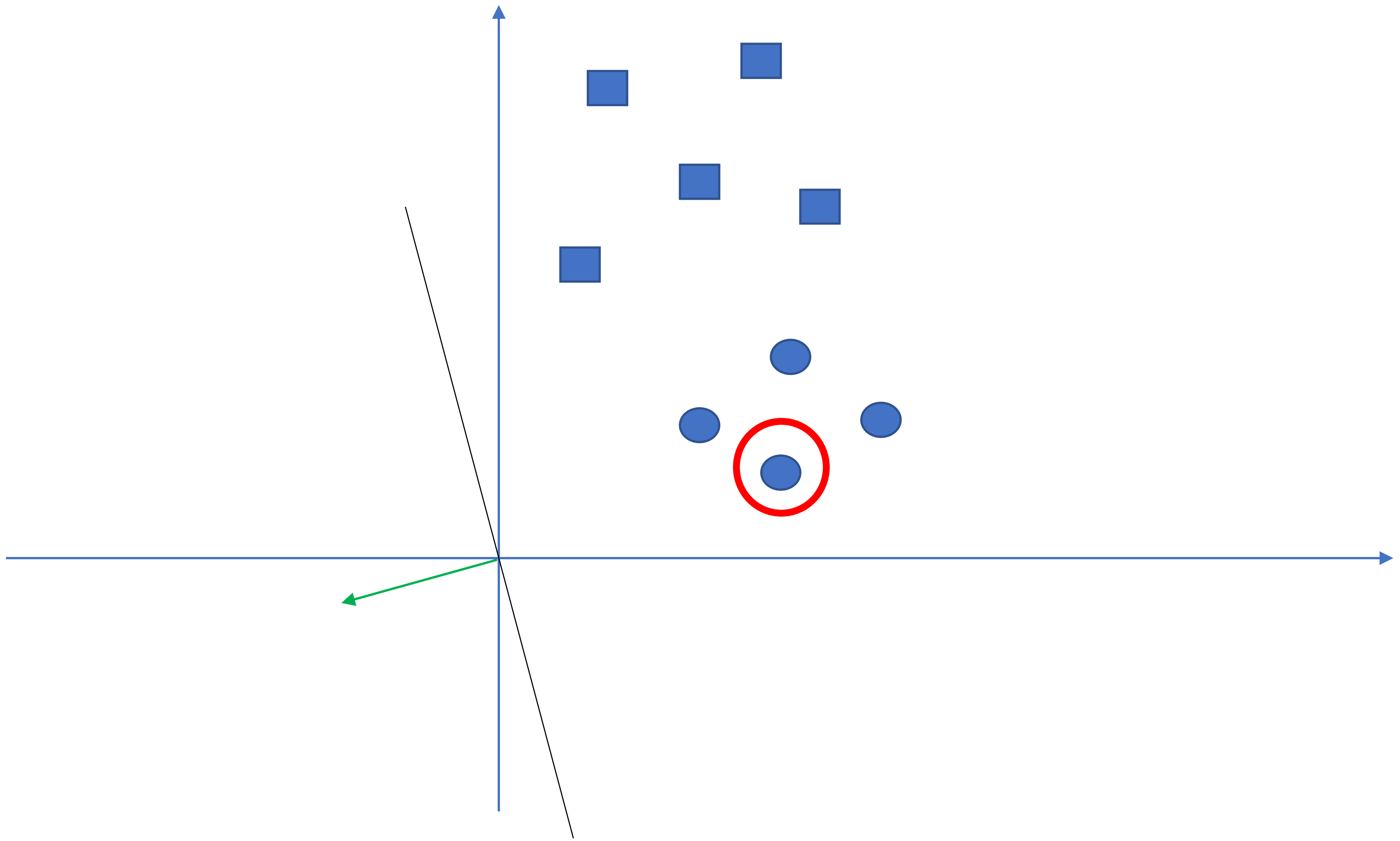

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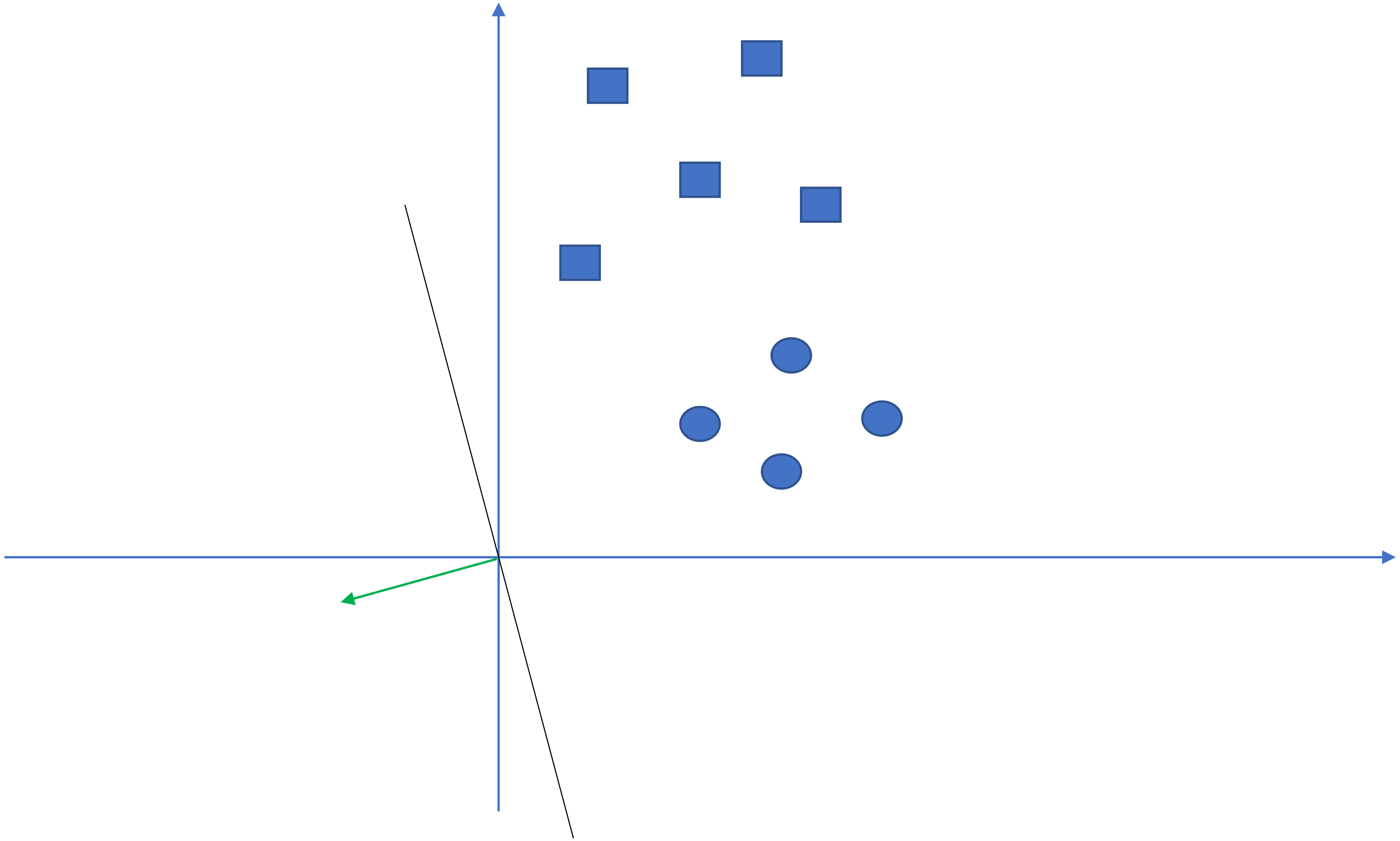

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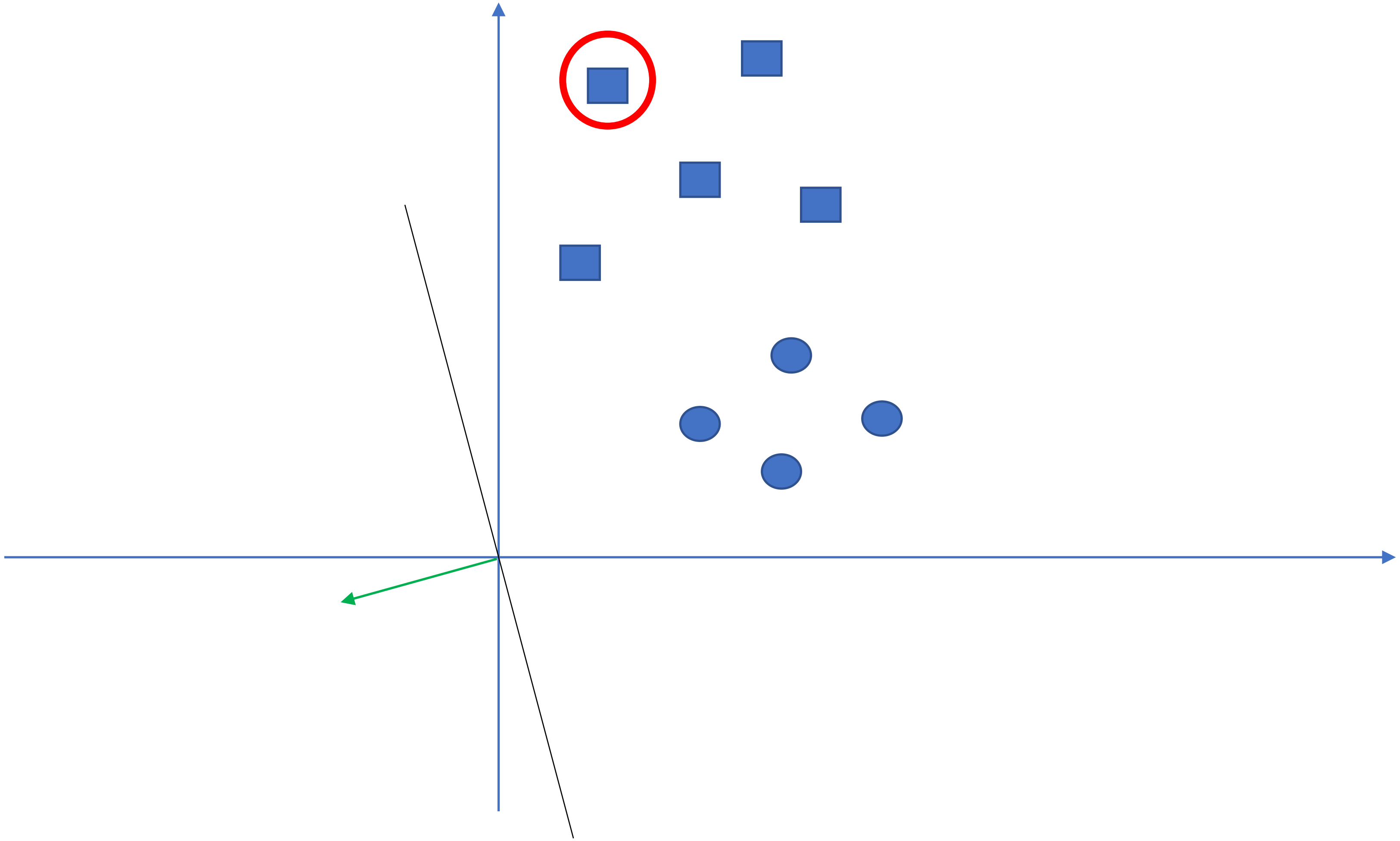

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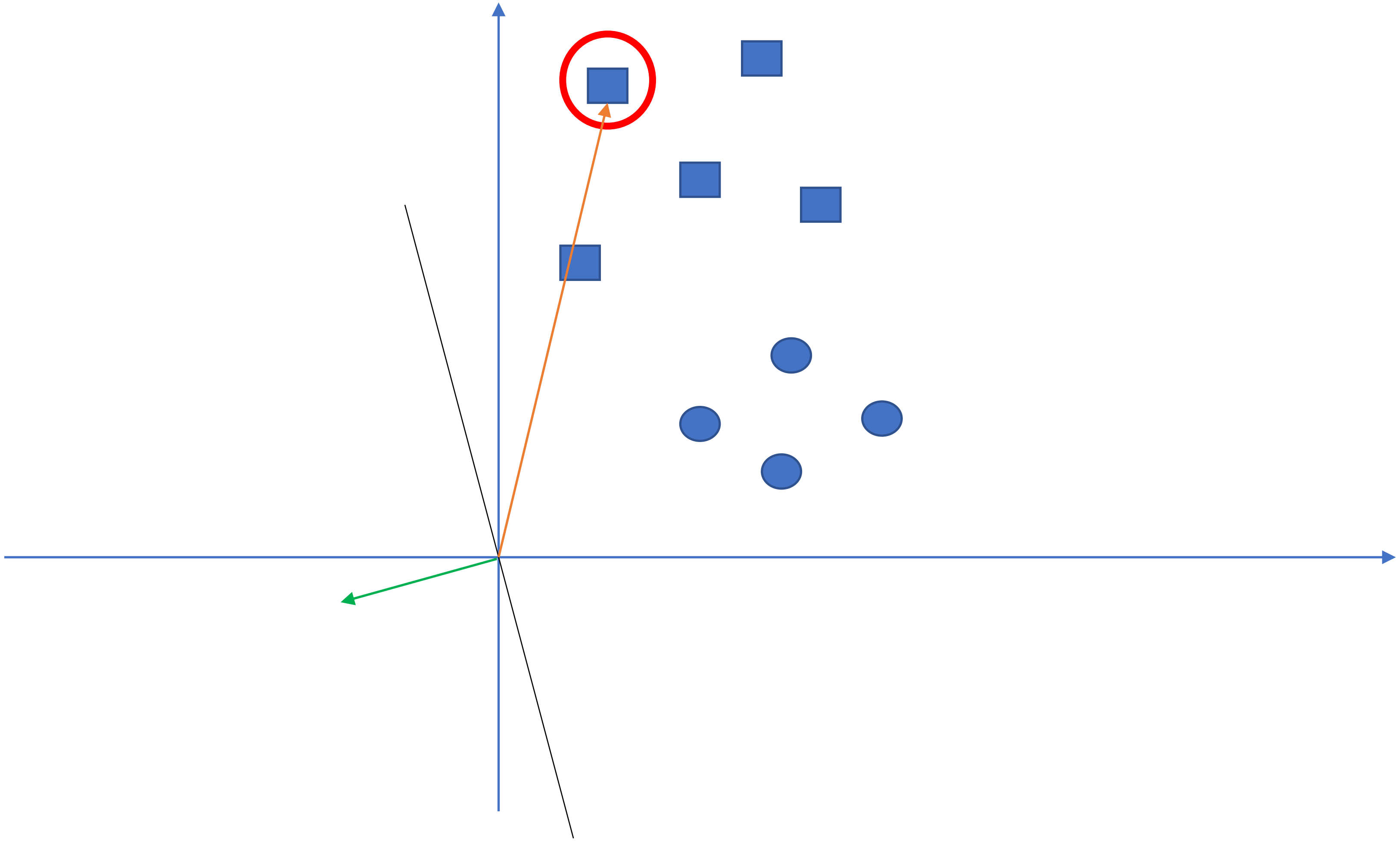

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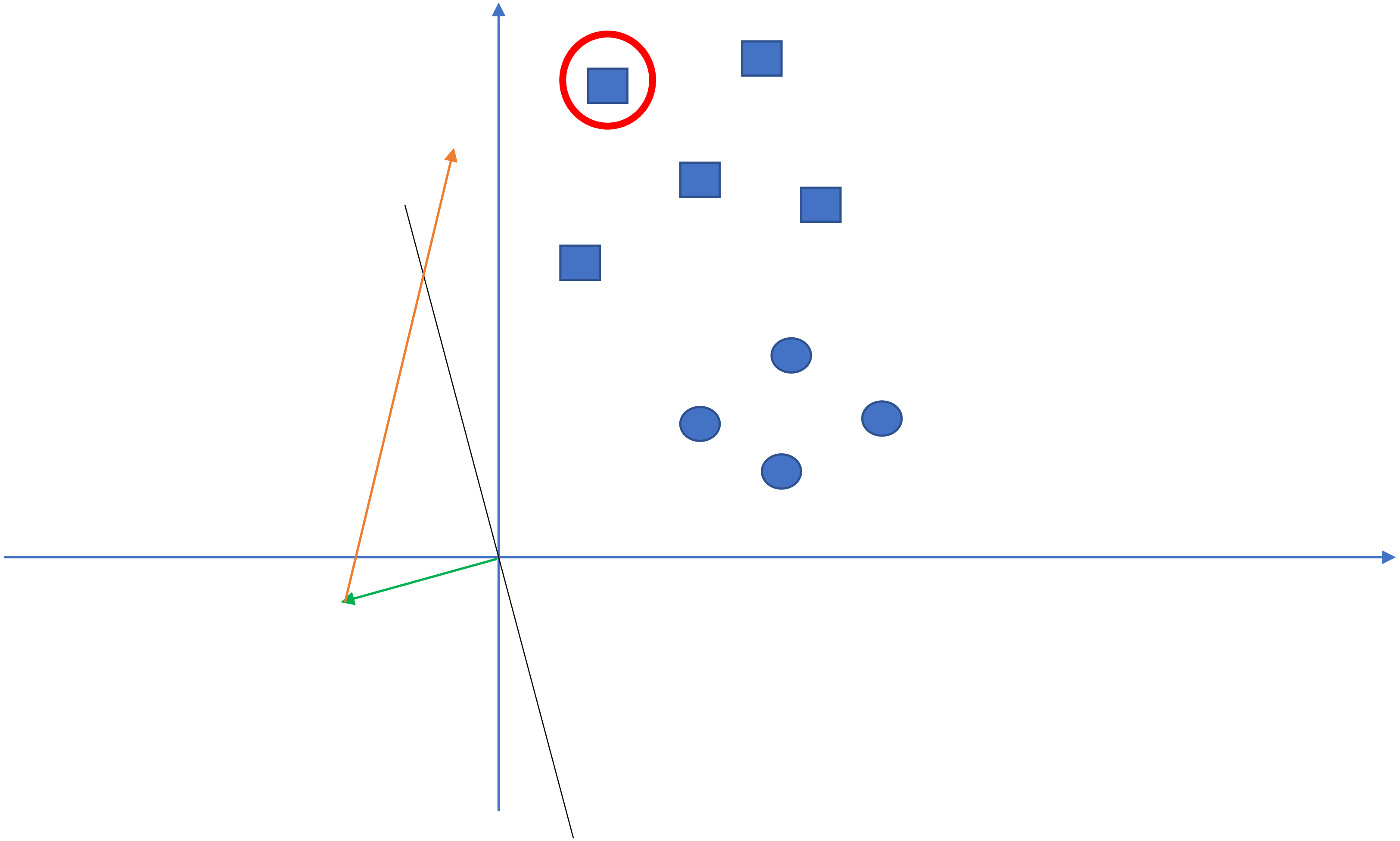

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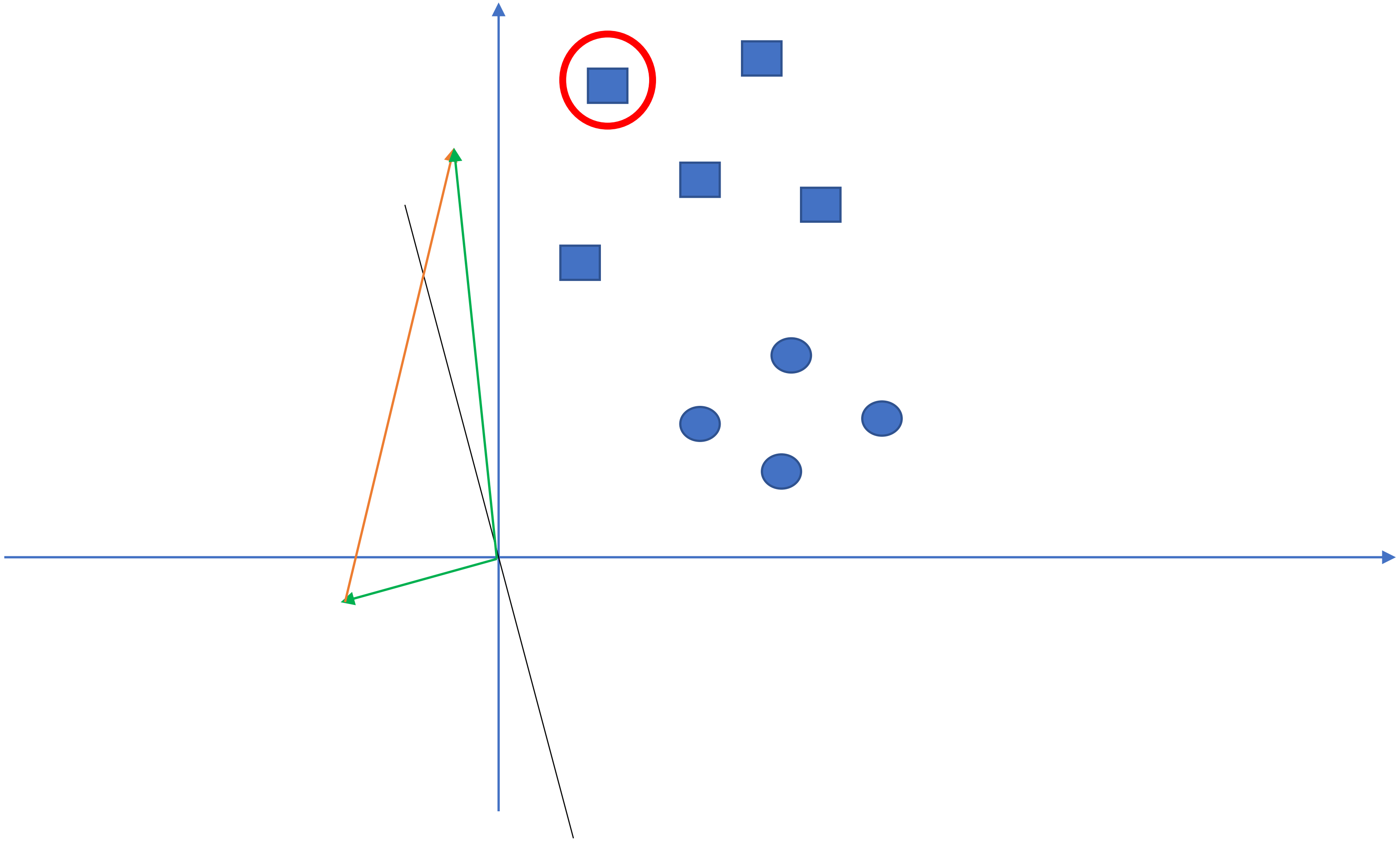

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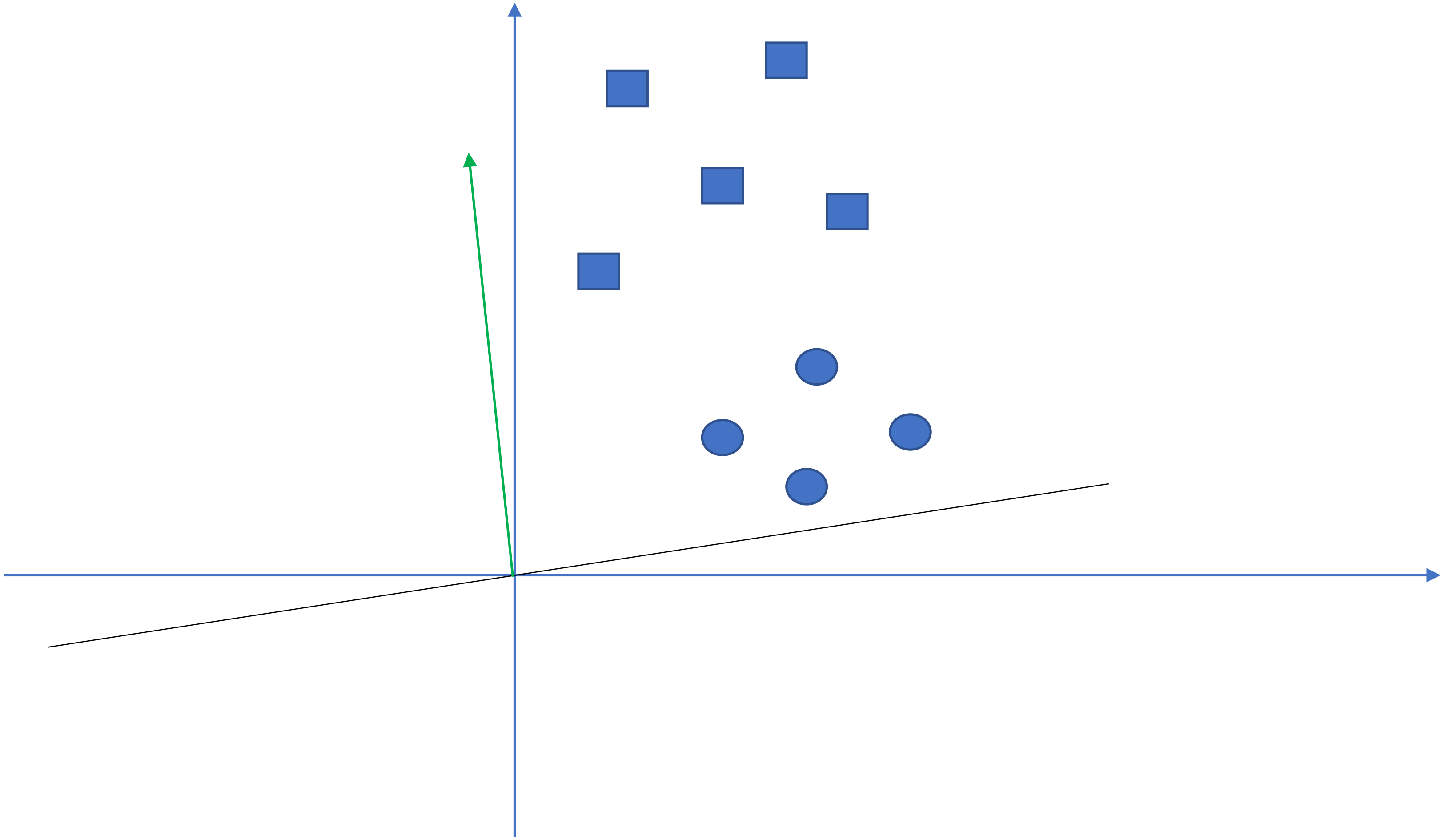

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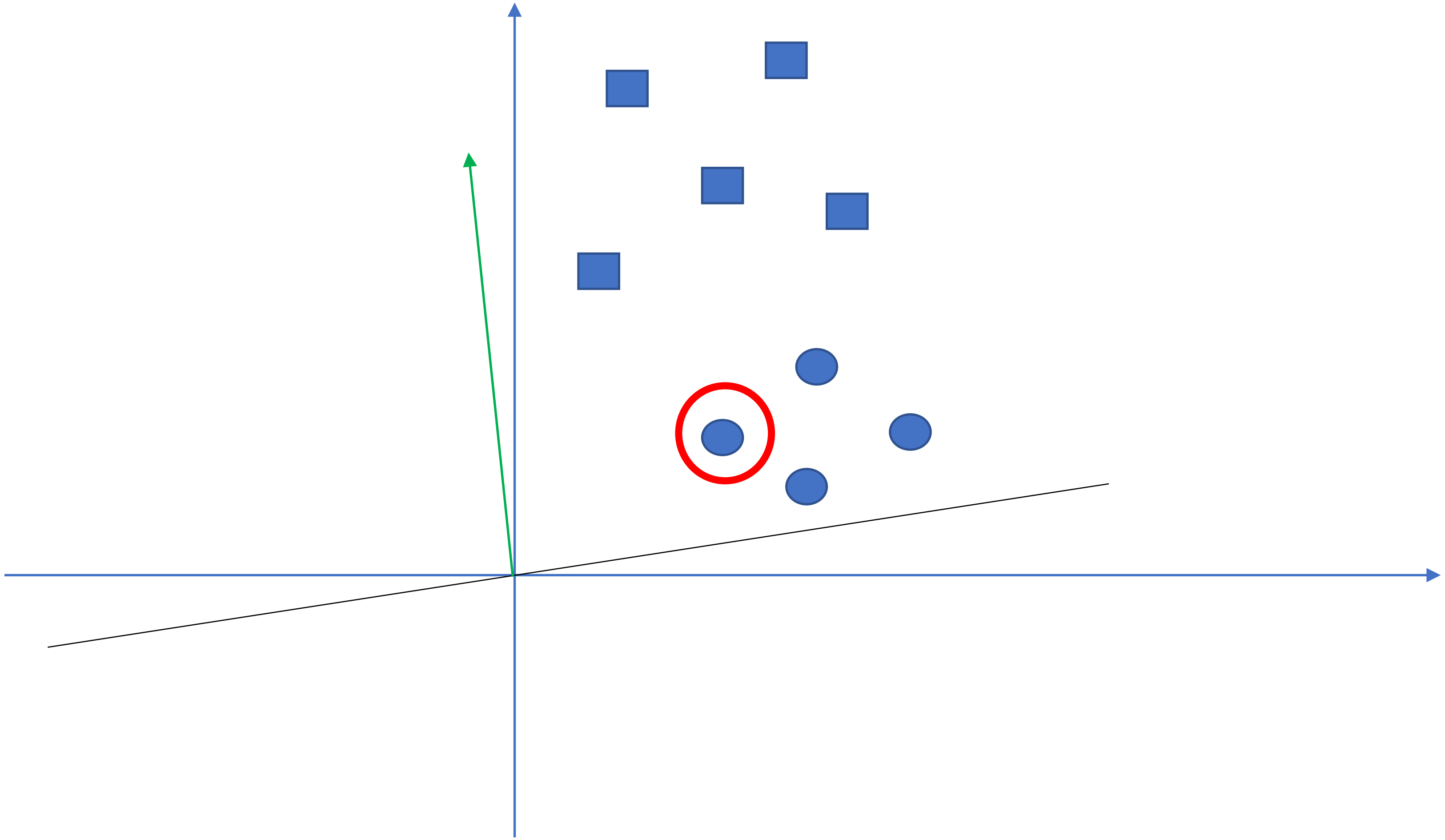

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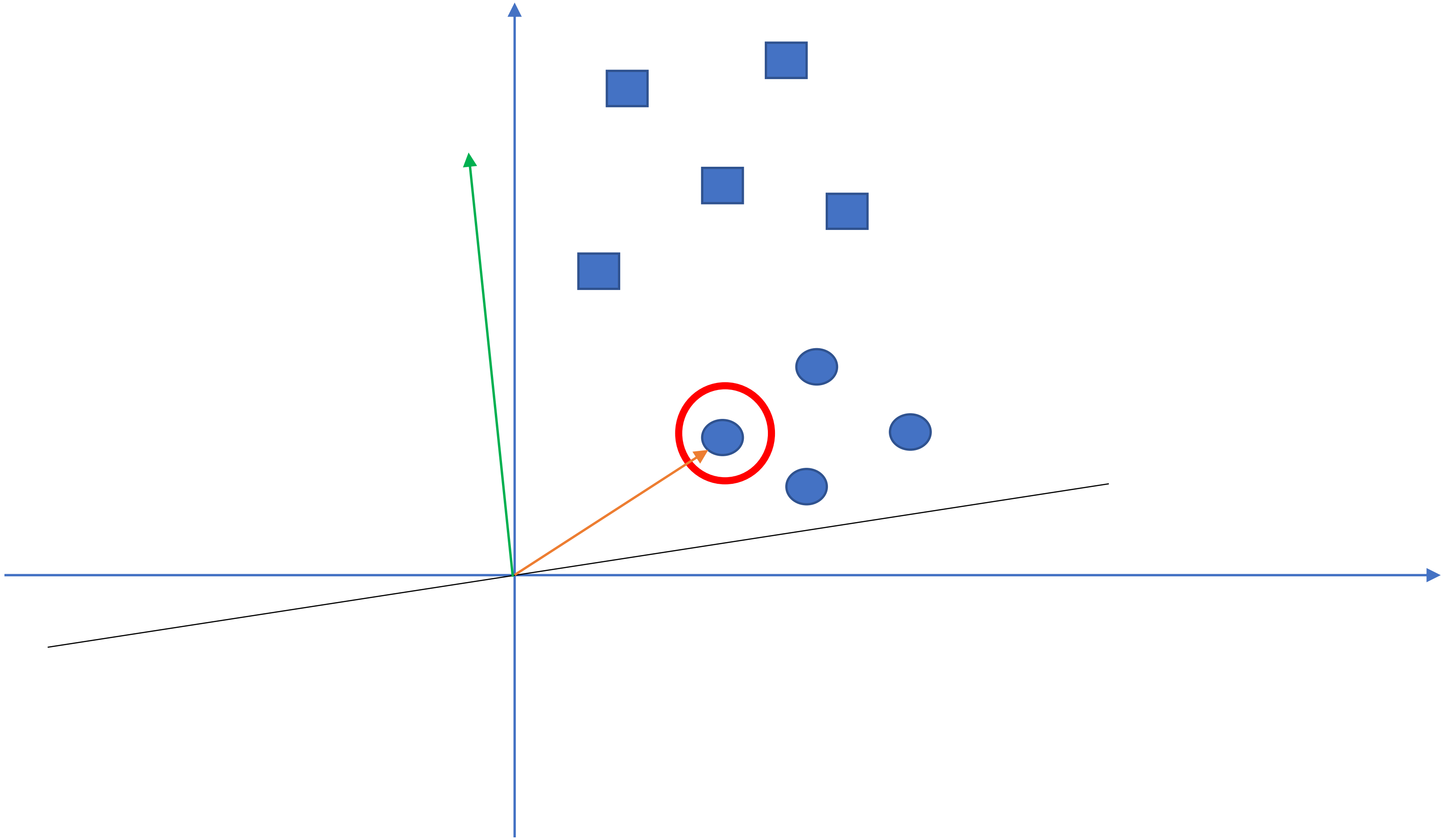

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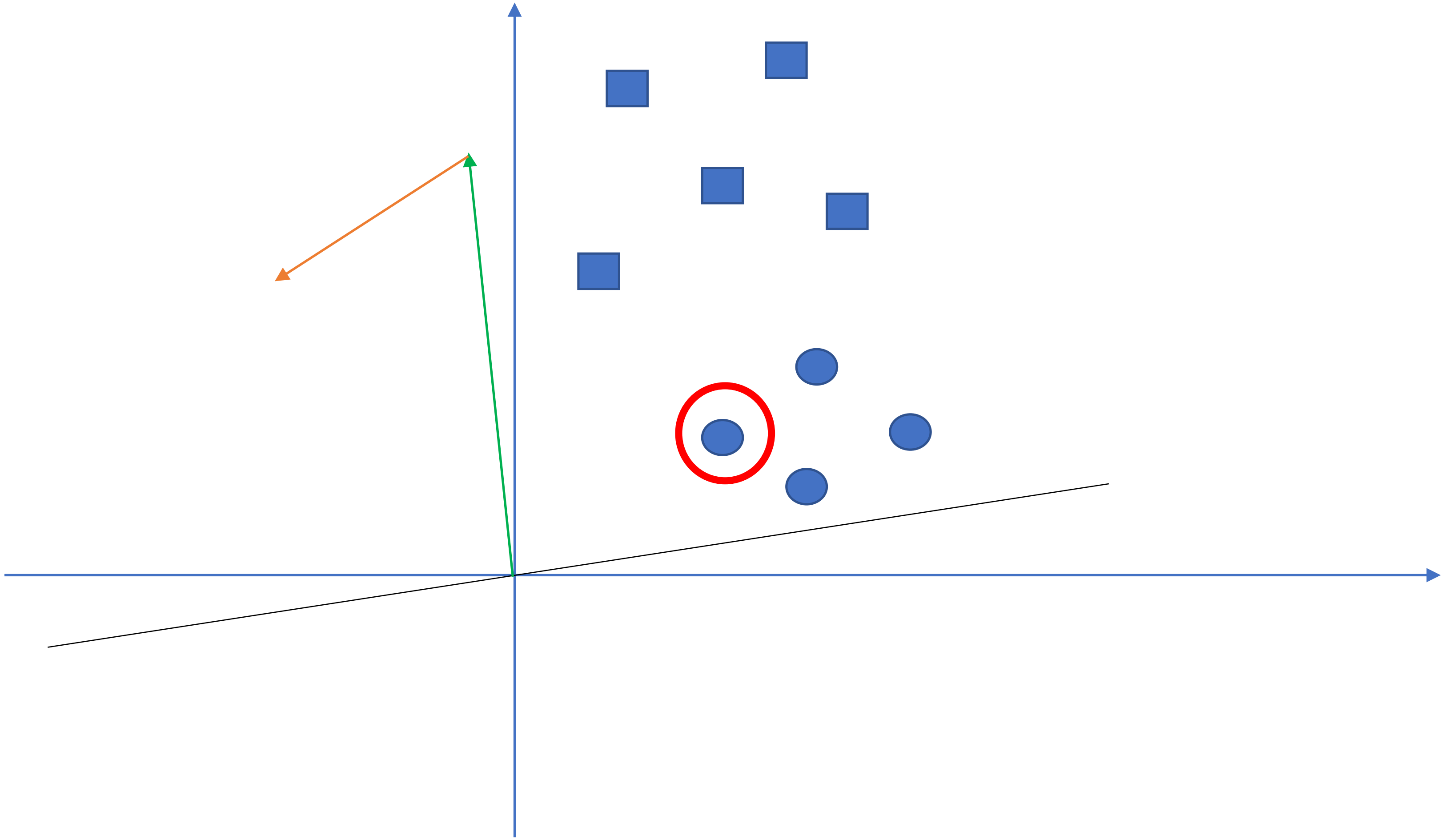

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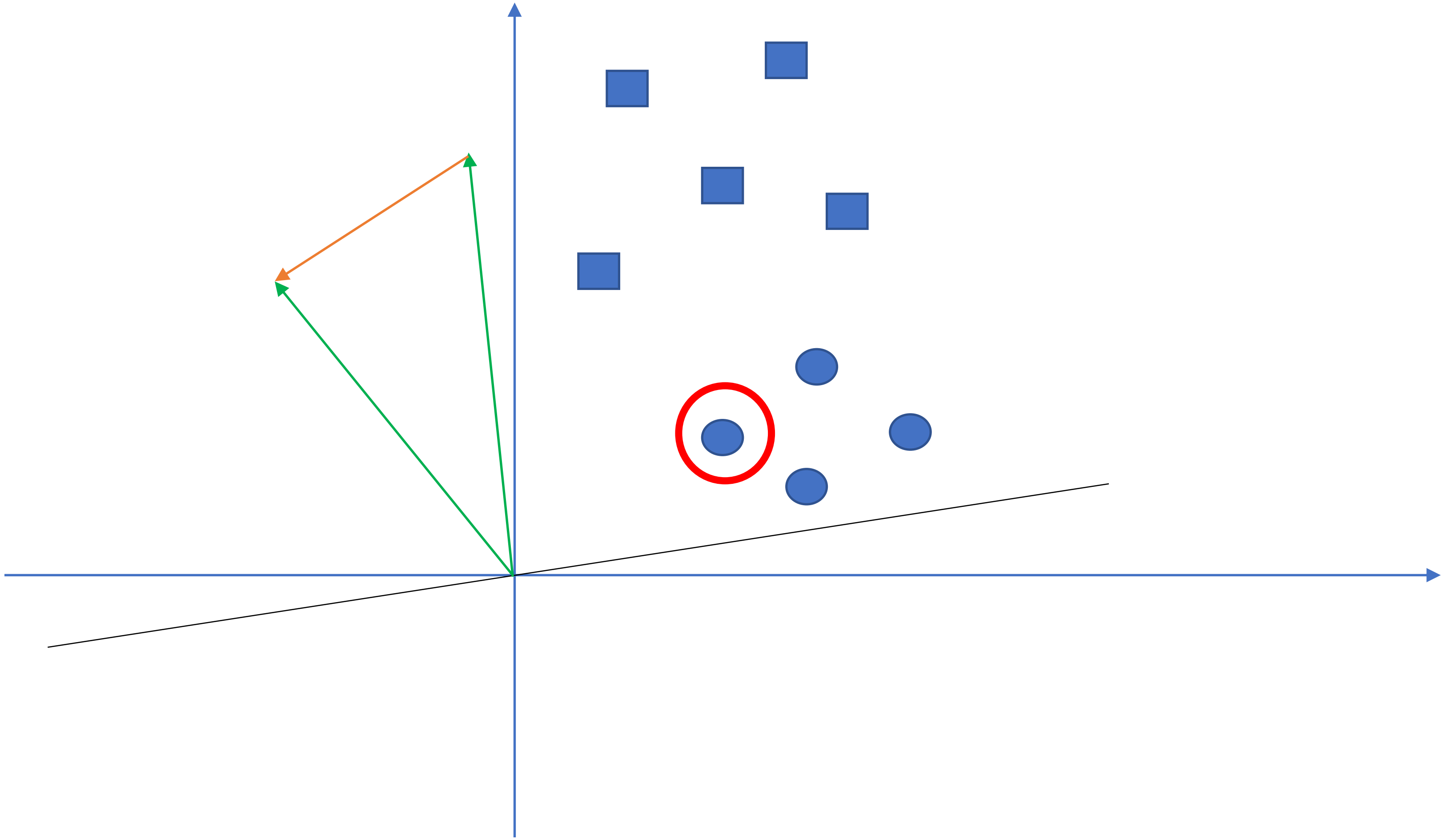

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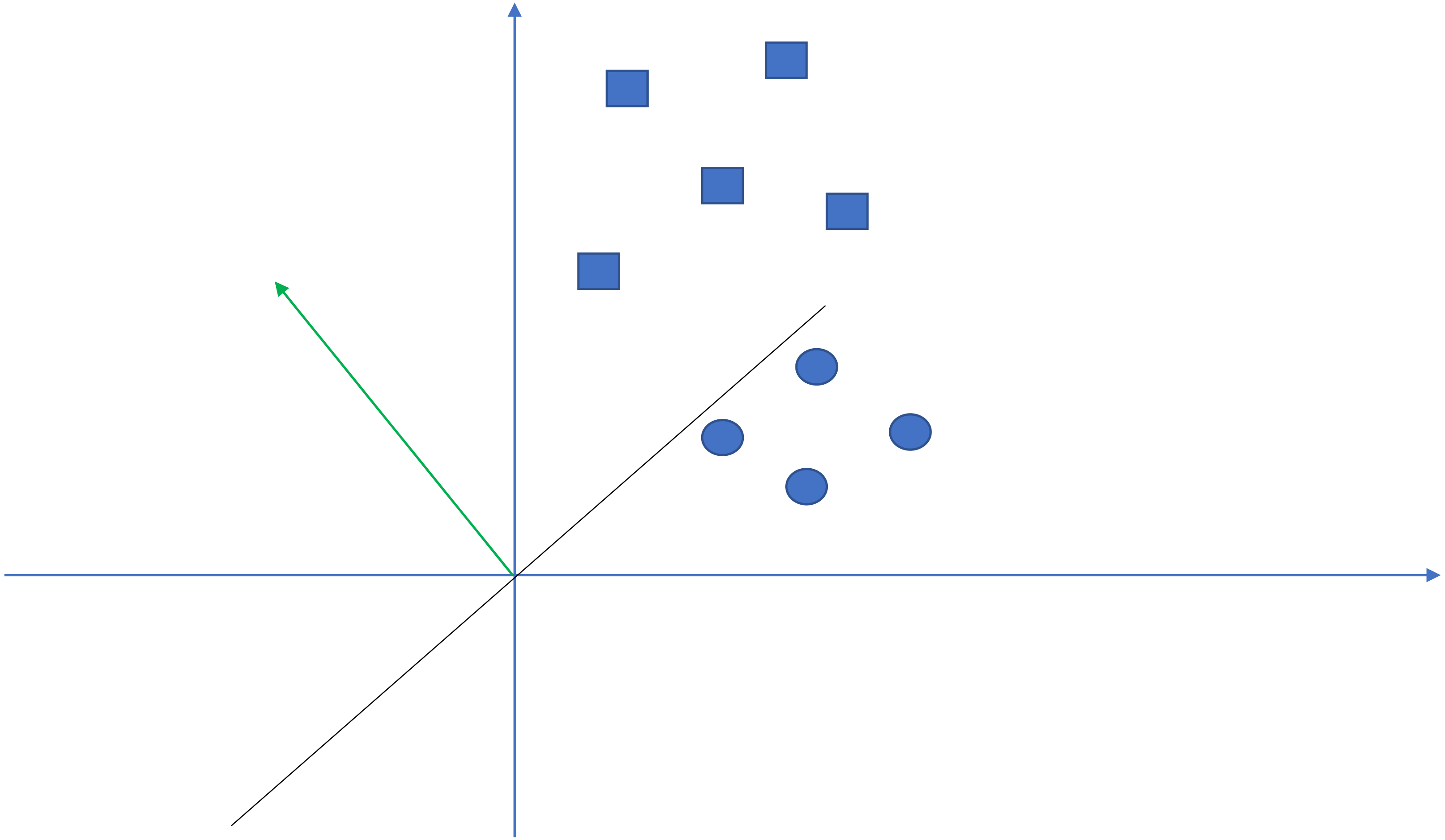


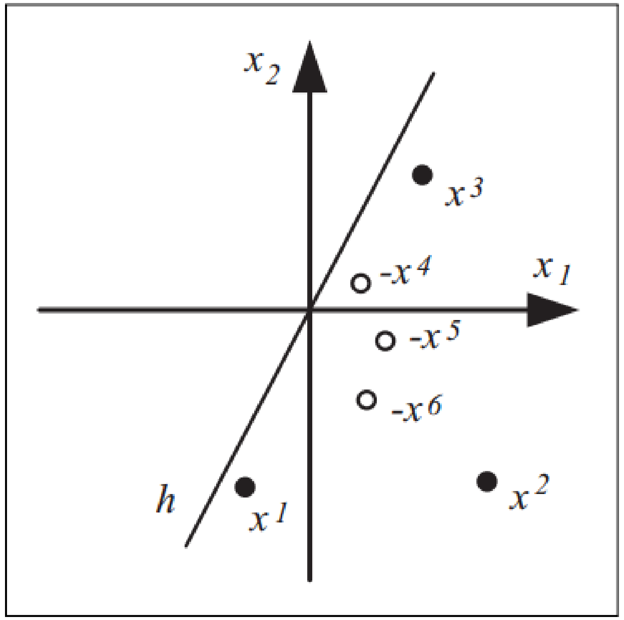
Revision: Lecture

- Under which condition(s) does the Perceptron Learning Rule terminate?

Revision: Lecture

- What is the *equivalent* Learning Problem?

Revision: Lecture



$\Omega' := \mathcal{P}' \cup \mathcal{N}'$ mit

$\mathcal{P}' :=$

$\{\zeta^m = \boldsymbol{x}^m | \boldsymbol{x}^m \in \mathcal{P}\}$

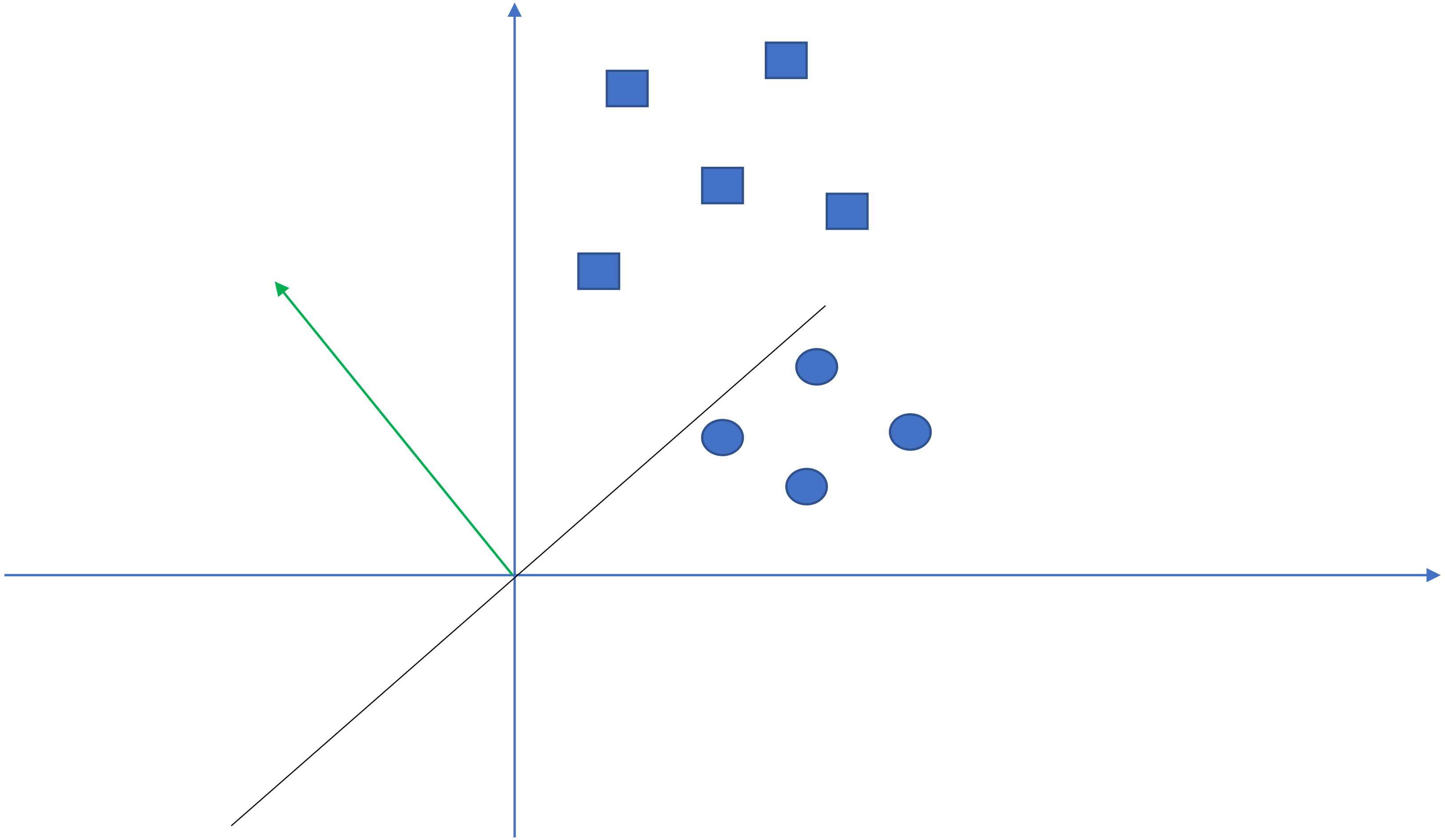
und

$\mathcal{N}' :=$

$\{\zeta^m = -\boldsymbol{x}^m | \boldsymbol{x}^m \in \mathcal{N}\}$

Äquivalentes Lernproblem: Finde Gewichtsvektor $\boldsymbol{w}$, so dass $\boldsymbol{w}^T\zeta > 0, \forall \zeta \in \Omega'$.

Revision: Lecture



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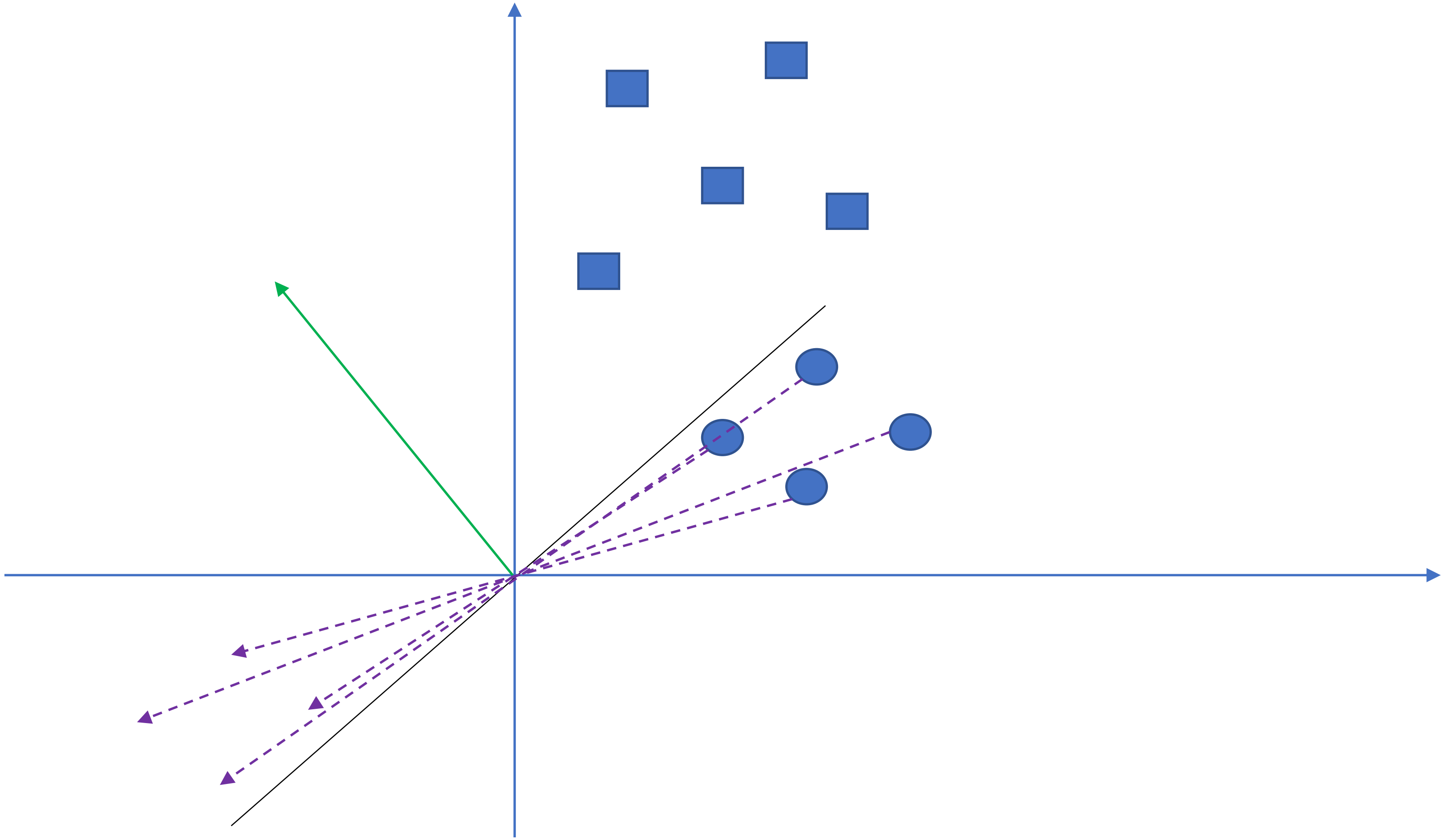

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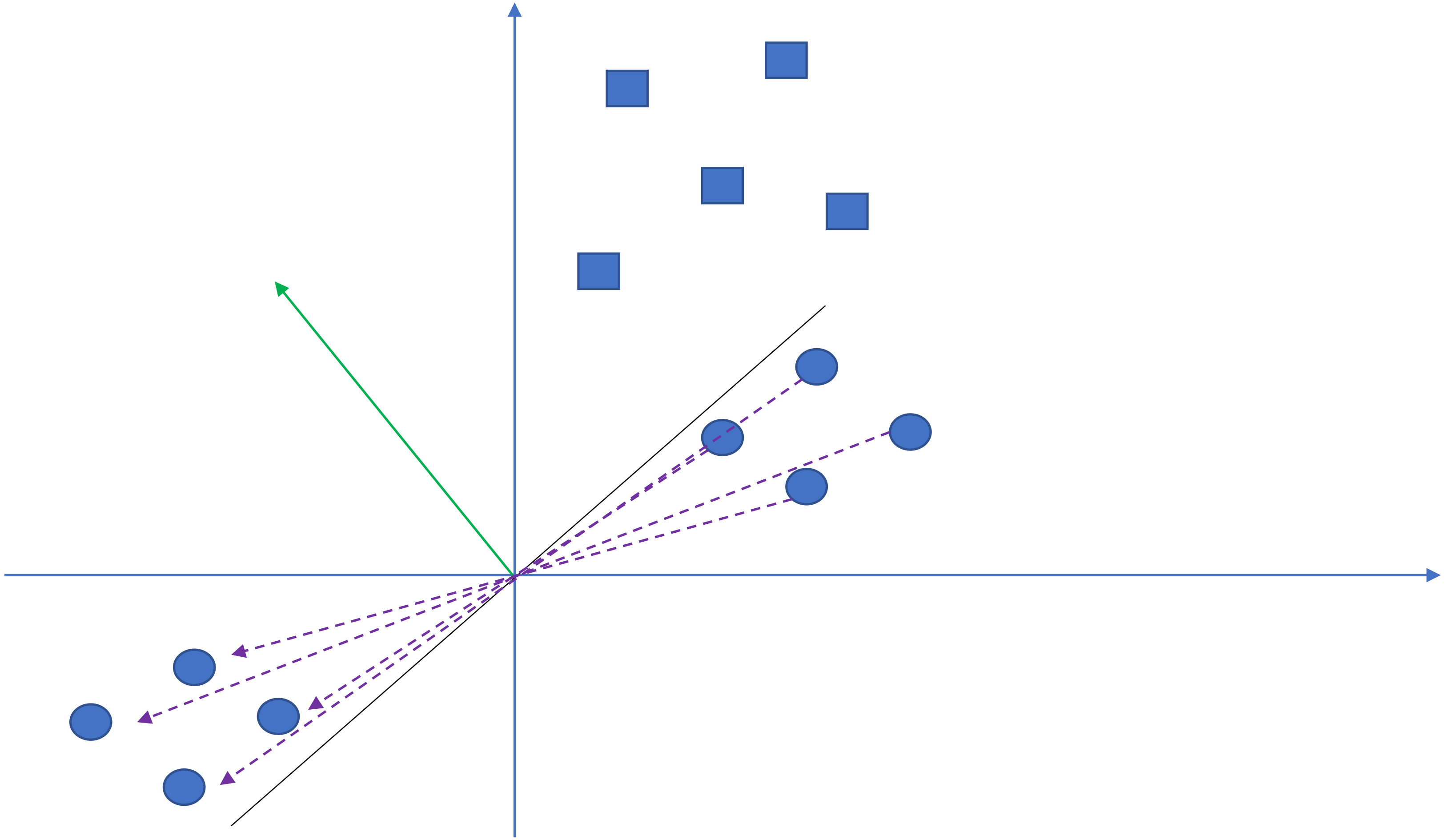

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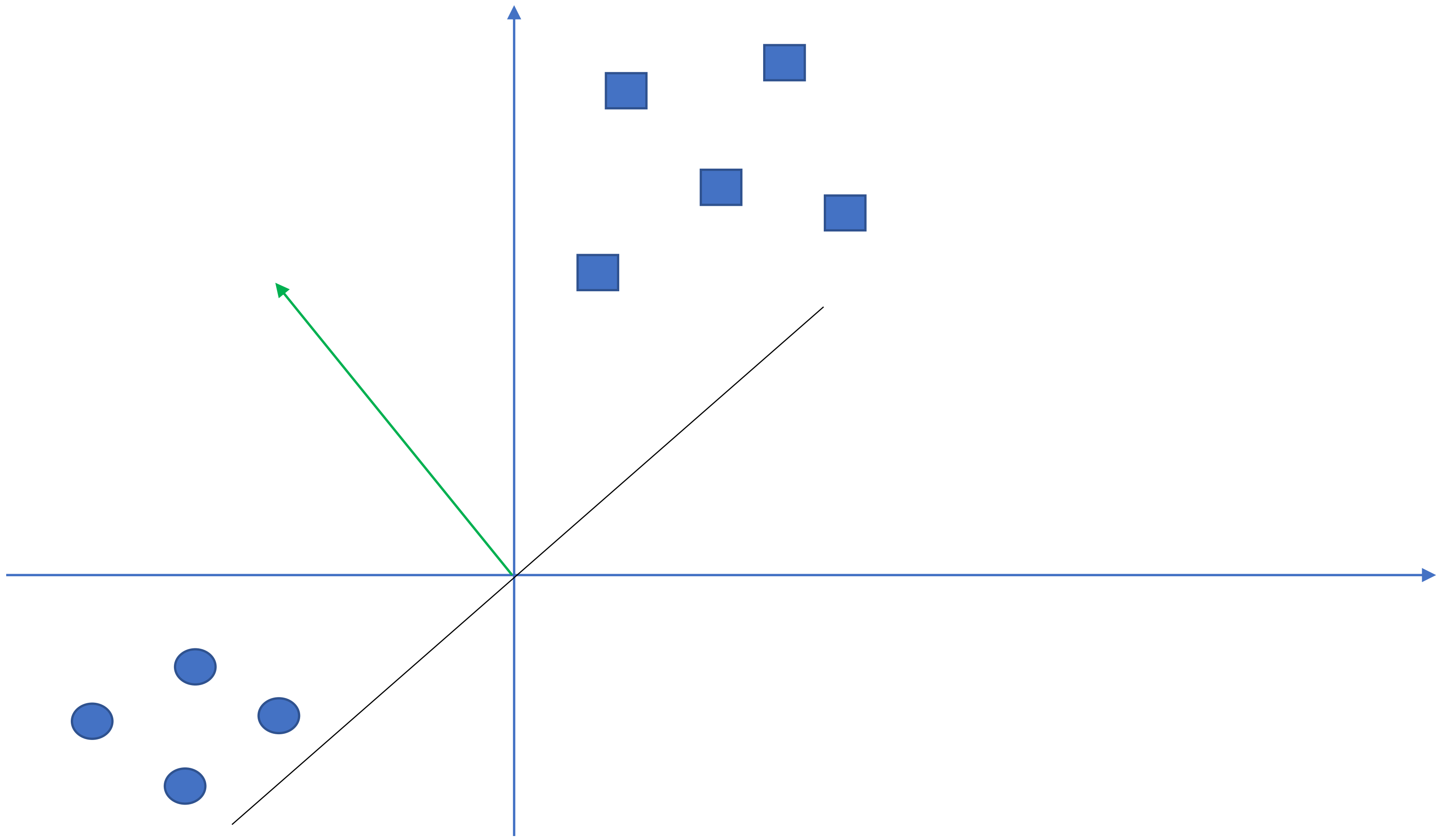

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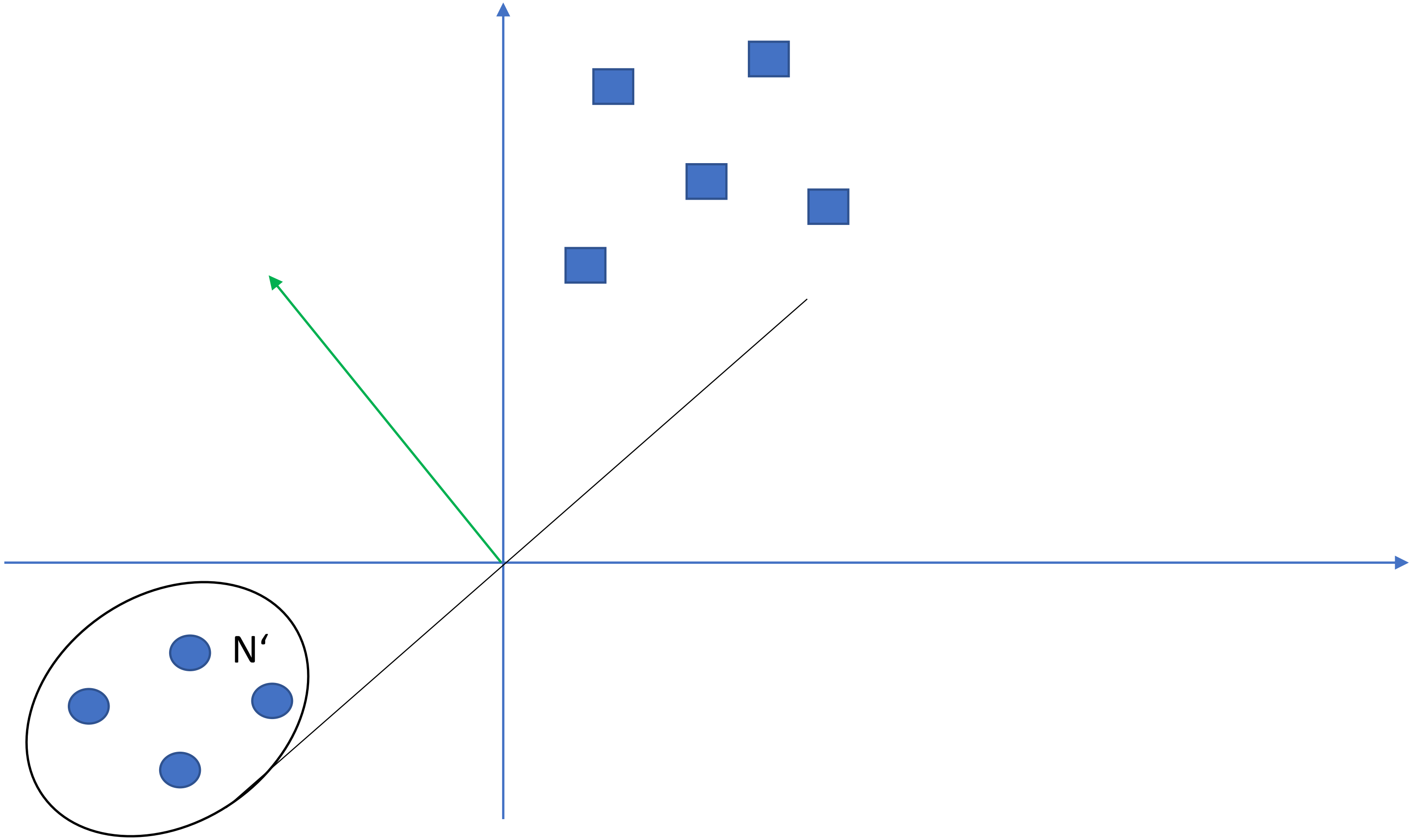

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