Lecture 6
Assignment

1. Let X and Y be two random variables such that Y has the binomial distribution with size N and probability π and,given $Y = y$,X has the binomial distribution with size y and probability p.
   (i)Suppose that $p \in (0, 1)$ and $\pi \in (0, 1)$ are unkonwn and N is known.Show that (X, Y) is minimal sufficient for (p, π).
   (ii) Suppose that π and N are konwn and $p \in (0, 1)$ is unkonwn.Show whether X is sufficient for p and whether y is sufficient for p.

2. Let $X_1, \cdots, X_n$ be independent and identically distributed random variables having the Lebesgue density

$$\exp\{-(\tfrac{x-\mu}{\sigma})^4 - \xi(\theta)\}$$

   where $\theta = (\mu, \sigma) \in \Theta = \mathcal{R} \times (0, \infty)$.Show that $\mathcal{P} = P_\theta : \theta \in \Theta$ is an exponential family,where P_Θ is the joint distribution of $X_1, \cdots, X_n$,and that the statistic $T = (\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2, \sum_{i=1}^{n} X_i^3, \sum_{i=1}^{n} X_i^4)$ is minimal sufficient for $\theta \in \Theta$.

3. Let X be a discrete random variable with

$$P_\theta(X = x) = \frac{\binom{\theta}{x}\binom{N-\theta}{n-x}}{\binom{N}{n}}, x = 0, 1, \cdots, \min\{\theta, n\},$$

   where $n - x \le N - \theta$,n and N are positive integers,$N \ge n$,and $\theta = 0, 1, \cdots, N$.Show that X is complete.