Lecture 2

Assignment

1. Let $X_1, \cdots, X_n, \cdots$ be a sequence of independent random variables such that

$$E[X_i] = \mu, \qquad Var(X_i) < \sigma^2, \qquad n = 1, 2, \cdots$$

   With Chebychev's inequality,prove that $\bar{X_n} \xrightarrow{P} \mu$.

2. Let $U_1, U_2, \cdots$ be independent random variables having the uniform distribution on [0,1] and $Y_n = (\prod_{i=1}^{n} U_i)^{-1/n})$. Show that $\sqrt{n}(Y_n - e) \xrightarrow{d} N(0, e^2)$.

3. Let $X_1, \cdots, X_n$ be i.i.d. random variables following Uniform[0,1].Let $Y_n = \min(X_1, \ldots, X_n)$.
   (i) Show that $Y_n \xrightarrow{a.s} 0$ as $n \to \infty$.
   (ii) Show that $nY_n \xrightarrow{d} \exp(1)$,where Exp(1) is the exponential distribution with density $f(x) = e^{-x}$ for $x > 0$.