Lecture 2

Assignment

1. Show that if $X_n \xrightarrow{d} X$ for a random variable X,then $X_n = O_p(1)$.

2. Let X_n and Y_n be two sequences of random variables such that X_n is bounded in probability and,for any real number t and $\epsilon > 0$, $\lim_n[P(X_n \leq t, Y_n \geq t+\epsilon) + P(X_n \geq t+\epsilon, Y_n \leq t)] = 0$.Show that $X_n - Y_n \xrightarrow{p} 0$.

3. Let $X, X_1, X_2, \cdots$ be a sequence of random variables.Show that $X_n \xrightarrow{p} X$ as $n \to \infty$ if and only if

$$E(\tfrac{|X_n - X|}{1+|X_n - X|}) \to 0, \qquad \text{as } n \to \infty.$$