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Partial symmetry detection of 3d shapes

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Abstract In this paper, we propose a new algorithm for detecting symmetries in shapes. Our algorithm computes generalized partial symmetries, i.e., subsets of a shape that reoccur multiple times within the model differing by combinations of translation, rotation and mirroring. Our algorithm is based on segmentation and matching locally coherent meaningful parts on the object surfaces. Working on relevant parts leads to a new algorithm that is able to detect partial symmetries more robust than the recent algorithms, which are based on the correlated correspondences among graphs of invariant features. We apply our algorithm to a number of 3D data sets, demonstrating high recognition rates for general partial symmetry.

Keywords symmetry detection · partial symmetry · segmentation

1 Introduction

Many objects, both man-made and natural, have some symmetries or self-similarities in them. Finding the symmetry from triangle meshes is a way to augment meshes with some structures thus helps various applications such as remeshing [1], mesh simplification [2], segmentation [3,4], repairing [5,6], and reconstruction [7]. Symmetry detection and analysis is a fundamental technique in computer graphics, computer vision and geometry processing. Whilst a lot of attention has been received in recent years, it is still challenging to robustly identify symmetries in general input meshes without resorting to user assistance.

Symmetries may have different definitions. In this work, we focus on generalized partial symmetry [3, 6] where some subsets of a shape reoccur multiple times within the model differing by combinations of translation, rotation and/or mirroring. Intrinsic symmetries are also considered in which case further isometric deformation is allowed between copies. This flexible definition allows symmetry detection to be more useful.

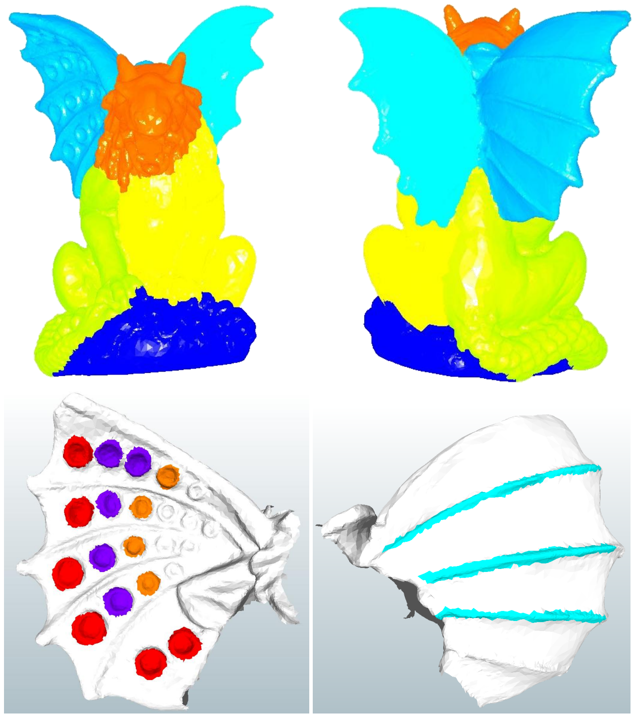

Fig. 1 Two-scale symmetries of the Gargoyle statue from two viewpoints. Top: At the coarse scale, two pairs of mirroring symmetry from six segmented parts are represented, with each pair being rendered by the same color. Bottom: At the fine scale, translation and rotation symmetries are detected on the left wing part for the detailed rings.

In this paper, we propose an effective method for generalized partial symmetry detection. The input mesh is first segmented into multiple meaningful pieces. The correspondence between every pair of pieces is established using robust matching. This is followed by a clustering stage where consistent matchings are recognized as a symmetric pair.

Compared to the recent graph matching algorithms [5, 6] based on salient lines, our algorithm could produce more robust partial symmetry detection results. As shown in Figure 1, the two-scale symmetries of the Gargoyle statue are correctly detected, which is a failure case in [6]. The top row shows the coarse-scale mirroring symmetry detected from six meaningfully segmented parts (head, left and right wing, left and right body parts, and status base). Two pairs of mirroring symmetries (shaded in the similar color) have been found. The bottom row shows the fine-scale symmetry detection on the left wing, both the translational and rotational symmetries have been found for the detailed rings.

The rest of the paper is structured as follows. After surveying related previous work in Section 2, Section 3 describes our symmetry detection algorithm. Section 4 demonstrates the effectiveness of our method with some results and comparisons with related works. Finally, section 5 draws conclusions and gives some discussions about future directions.

2 Related work

Symmetry detection and analysis have been thoroughly studied from theoretic, algorithmic, and applicative aspects during the last two decades. There exist a significant number of papers proposed for this purpose, which can mainly be classified into *global* and *partial* types. As we will aim to detect the intrinsic symmetry in 3D shapes in this paper, we concentrate on the most relevant 3D shape symmetry methods and recent advances in the intrinsic aspect. For the comprehensive review, please refer to papers [4, 6].

A common approach for symmetry detection is to identify the global symmetry in Euclidean space from the clusters in the transformed space [1–3]. Other approaches [5,6] formulate the symmetry detection as partial graph matching, and compute the reliable line features from scanned data. Extrinsic symmetry methods always employed symmetry of the Euclidean space for detecting symmetries [1, 3, 7], and patterns [2, 5, 6, 8].

Recent works have focused on analyzing partial and approximate symmetries, which is more difficult than the extrinsic, given the high dimensionality of the nonrigid transformations. Similar to extrinsic symmetry, intrinsic symmetry detection can also be categorized as global [9–17] and partial [4,18–21] cases. While from the viewpoint of transformation formulation, intrinsic symmetry algorithms are striving for the compact representation of parameters, e.g., translation, rotation, scaling, and reflection. Most algorithms considered discrete symmetries by sampling. An exception is the work [11] where global intrinsic symmetry is detected using the approximate killing vector field [11] from a continuous perspective. Ovsjanikov et al. perform global intrinsic symmetry detection using eigenanalysis of the surface Laplace-Beltrami operator [9], then resorting to the heat kernel maps [22]. By utilizing the Mobius transformation and conformal mapping, two robust symmetry detection algorithms were proposed by Kim and his coworkers [15, 17]. Meanwhile, the Global Multi-Dimensional Scaling (GMDS) computation was thoroughly studied by Bronstein and coworkers [10, 20, 21, 23] to detect the extrinsic and intrinsic symmetries, and mainly applied to the shape invariant retrieval applications. In addition, other researchers proposed variant symmetry methods by using different techniques, e.g., Markov random field model [18], symmetry axis voting [4], Eigen analysis [13, 14], Pseudo-polar Fourier Transform [24], etc.

It is more challenging to detect *partial* symmetries for 3D shapes. For the general partial symmetries, our algorithm would solve the reflectional, rotational and translational problems in a unified framework. Our approach is some similar to the partial symmetry detection [6], but going beyond it by robustly handling more challenging input data.

3 Algorithm

Given a shape $\mathbb{S}$, our algorithm mainly performs the following three steps (as illustrated in Figure 2). 1) Automatically decompose the shape into some pieces. 2) For each piece $\mathbb{S}_i$, match the remaining shape $\mathbb{S} - \mathbb{S}_i$ to $\mathbb{S}_i$ such that for any node in $\mathbb{S} - \mathbb{S}_i$, a unique match from $\mathbb{S}_i$ is selected. 3) Put together all the correspondences to form a many-to-many correspondence in $\mathbb{S}$, and cluster nodes in $\mathbb{S}$ based on local transformation consistency of the matching.

3.1 Automatic meaningful segmentation

For the input shape $\mathbb{S}$, in order to find the generalized intrinsic partial symmetry, we decompose the model into n pieces: $\mathbb{S}_1, \mathbb{S}_2, \ldots, \mathbb{S}_n$. The segmentation should ideally lead to each region representing a meaningful part of the object. We used random walks based segmentation [25] which involves distributing n seeds as far away as possible over the surface and segmenting

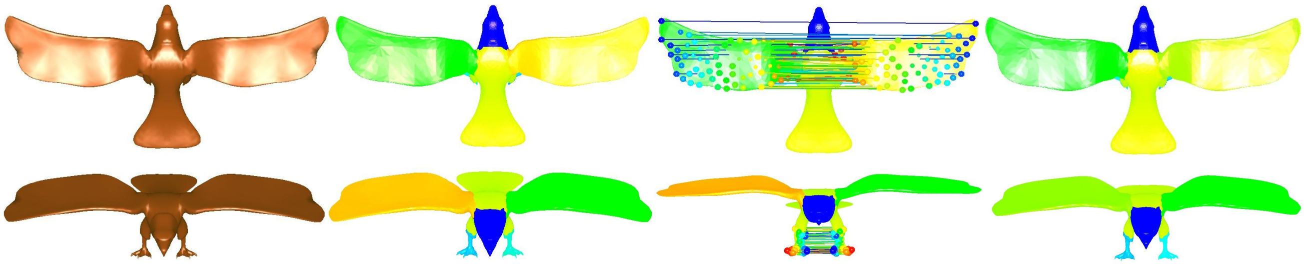

Fig. 2 Algorithm pipeline (demonstrated by the Eager example from two viewpoints). From left to right, for a given shape, our algorithm decomposes it into several parts, matches each part to the remaining shape, then detect the symmetry by clusternig the correspondences based on the consistency of the matching.

the shape by assigning a label to each face, indicating the seed with the maximum probability to be hit by a random walker starting from the face. The probability of movement is derived from the local geometry such that it is more difficult to move across sharp edges. Implied by the existing symmetry detection algorithms [4, 6], symmetric parts are always meaningful. This is the foundation of our algorithm and could explain why the meaningful segmentation is first performed to provide the parts for symmetry detection, although the other surface-based segmentation algorithm could also provide the initial segmentation.

3.2 Partial matching

In order to find the symmetries, we uniformly sample the shape $\mathbb{S}$ with m samples, and then build the partial matching between every pair of regions $\mathbb{S}_i$ and $\mathbb{S}_j$, $i \neq j$.

The partial matching is achieved by a third-order tensor matching algorithm [reference omitted for review], which is improved from the tensor-based algorithm for high-order graph matching [26]. Our algorithm formulates the matching using a supersymmetric tensor representing an affinity metric, which takes into account feature similarity and geometric constraints between features. Note that, although [26] claimed to use a supersymmetric affinity tensor, their approach did not make full use of supersymmetry when creating the supersymmetric affinity tensor, nor does it take advantage of supersymmetry to accelerate the power iteration process and sampling. Going beyond [26], our algorithm [reference omitted for review] really takes advantage of supersymmetry to devise an efficient sampling strategy to estimate the affinity tensor, as well as to store the estimated tensor compactly. Matching is performed by an efficient higher-order power iteration approach which takes advantage of the compact representation of the supersymmetric affinity tensor. efficient sampling strategy to estimate the affinity tensor, as well as to store the estimated tensor compactly.

The algorithm matches samples using a pair of three-tuples from both shapes and ensures consistency in geodesic distances. To speed up the geodesic distance computation, the approximate Dijkstra algorithm [27] is adopted and performed on the sampled points. Assume the number of matches between regions $\mathbb{S}_i$ and $\mathbb{S}_j$ is $match_{i,j}$, the regions are recognized as of candidate symmetry if $match_{i,j} > cm$, where c is a constant and typically assigned as $0.2/n$ and m equals to $100 \times n$ for all examples in the paper.

3.3 Correspondences clustering

In the following, we put together all the correspondences for each part $\mathbb{S}_i$, found by former partial matching. So, we form a many-to-many correspondence in the whole $\mathbb{S}$. Then, we cluster nodes in $\mathbb{S}$ based on local transformation consistency of the matching.

Benefiting from our partial matching, rigid transforms can be computed from each triple of compatible matching points. The transform which brings the most data points within a threshold distance of a point in the model is chosen as the optimal alignment transform [28]. As shown by [28], this transformation always exists for three non-collinear points, and is unique up to a reflective ambiguity. The solution method is closed-form and only involves second-order equations. The later paper [29] claimed that such a voting scheme is robust to the initial pose of the triples.

Our clustering method is similar to [3] based on the computed transformation that map local surface patches onto each other. Each matching pair provides evidence for a symmetry relation at the level of the local sample spacing. So, we could extract meaningful symmetries at larger scales by finding groups of pairs with a similar transformation that correspond to symmetric subsets of the shape. The suitable clustering method also is mean shift clustering [30], a non-parametric method based on gradient ascent on a density function.

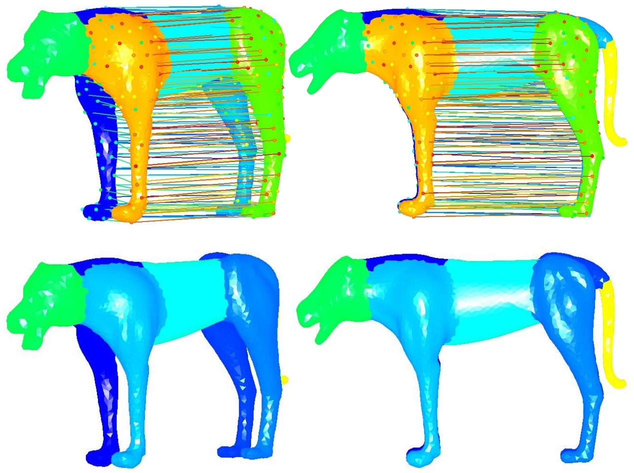

Fig. 3 The four legs of Chetah could be detected as one symmetric region at the current configuration $c = 0.2/n$ and $m = 100 \times n$. It is mainly resulting from the large similarity of four legs.

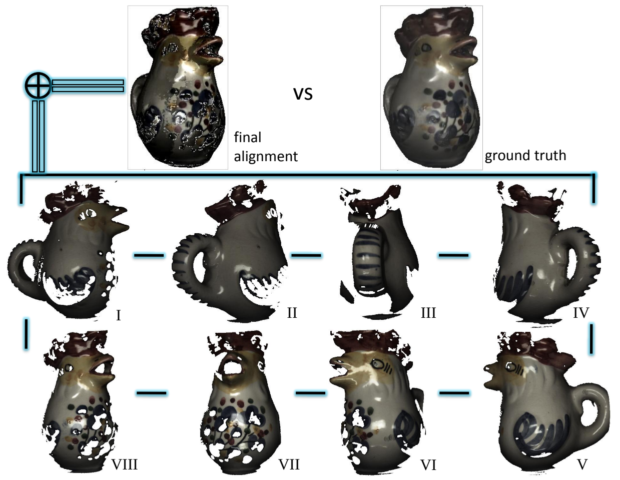

Fig. 4 The symmetry detection of point set Dinosaur with different resolutions in the left and right regions.

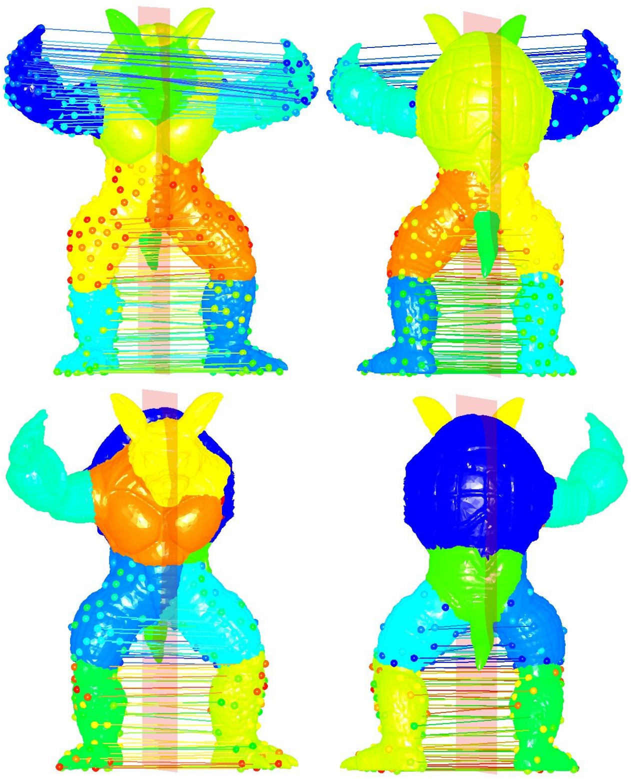

Fig. 5 The global reflectional planes are almost totally same, which are computed from both the complete and non-complete Armadillo shapes.

4 Results

Our algorithm is tested on a variety of 3D shapes. We visualize the discrete matches using the similar color as the corresponding symmetry parts.

We first show experimental results on clean manifold meshes, which are not corrupted with noise, and are complete. For such models, our algorithm provides good segmentation for the partial matching, and we reliably obtain perfect results (e.g. Figure 2 and 3).

As our algorithm works on the segmentation, uniform sampling, and clustering, the matching results do not depend on the representation and tessellation of the shapes. Figure 4 demonstrates that our algorithm performs very well on the shape in the point set form. Simultaneously, it proves that our algorithm is insensitive to the shape remeshing, as the left and right regions of Dinosaur are in large different resolutions.

Figure 1 shows an example using the three different symmetry group composed of reflection, rotation, and translation. The symmetry is detected in coarse-to-fine steps based on the hierarchical segmentation. In the coarse level, all major reflection symmetries are faithfully recovered. While in the fine level, most rotation and translation symmetries are detected. Note that, the example is clearly claimed as the failure case in the STAR algorithm [6].

We use another example on the complete and non-complete Armidillo (Figure 5) to demonstrate the robustness of our algorithm. Both on the complete and non-complete cases, the algorithm computes a global reflectional plane based on all final correspondences. Experiments show that the plane is almost totally same in the both cases.

The performance data of Table 1 indicates how the computation on the symmetries of the shapes. significantly more complex symmetry structure as compared to the dragon, hence computation times are substantially higher even though the models have roughly the same size.

Model	Vertices	Parts number	Segmentation	Partial matching	Correspondences clustering
Armadillo	172,974	9	982ms	2ms	7ms
Chetah	5,000	7	47ms	1ms	2ms
Dinosaur	69,215				
Eager	14,618	6	413ms	2ms	3ms
Gargoyl	250,003	6	218ms	3ms	3ms

Table 1 Timing for our algorithm: automatic meaningful segmentation, partial matching of each part, and clustering the correspondences, using a 1.6GHz Intel Core i7CPU laptop with 4GB of RAM.

5 Conclusions

We have proposed a symmetry detection algorithm for discovering and extracting partial symmetries of 3D geometric models. The algorithm could detects the general symmetry types, such as the rotation, translation, and mirroring symmetry. It is robust as relying on the shape segmentation, and shows the advantage over the graph matching using salient feature lines. Our algorithm is provably effective, easy to implement, and applicable to a wide range of 3D shapes.

Discussion. The main limitation of our symmetry detection algorithm stems from the foundation that we build it by using the segmentation algorithm. In presence of significant noise, this assumption breaks down for our choice of meaningful segmentation, thus forcing the algorithm to the unstable scenario. As the rapid development and maturity of meaningful segmentation [31], more segmentation algorithm would and could be employed to produce more reasonable decomposition. In addition, it is useful and necessary to develop more efficient algorithm by resorting to multi-resolution or parallelization techniques (e.g., using the GPU) to improve the speed of the algorithm.

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