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Segmentation-based partial symmetry detection for 3D shapes

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Abstract This paper proposes a new algorithm for detecting partial symmetries in 3D shapes. Our algorithm detects generalized partial approximate symmetries, i.e. subsets of a shape that approximately occur multiple times differing by combinations of translation, rotation and mirroring. Our algorithm is based on segmentation and matching locally coherent meaningful parts on the object's surfaces. A useful feature is that segmentation and symmetry detection can be performed iteratively to produce hierarchical better results. We apply our algorithm to a number of 3D data sets, demonstrating high detection rates for general partial symmetries.

Keywords symmetry detection · partial symmetry · segmentation

1 Introduction

Many objects, both man-made and natural, have approximate symmetries or self-similarities. Finding symmetries of meshes can be used to augment shapes with information relevant to applications such as finite element analysis [1], remeshing [2], shape simplification [3], segmentation [4–6], repair [7, 8], and reconstruction [9]. Symmetry detection and analysis is a fundamental tool in computer graphics, computer vision and geometry processing.

Symmetries may have different definitions. In this work, we focus on generalized partial symmetry [5, 8] where some subsets of a shape approximately occur multiple times within the model related by combinations of translation, rotation and mirroring. Intrinsic symmetries are also considered in which case further isometric deformation is allowed between copies. This flexible definition allows symmetry detection to be more useful.

Whilst a lot of attention has been paid to both segmentation and symmetry detection, their relationship has not been fully explored. Existing work mainly uses symmetry detection to guide shape segmentation, as symmetry has more intrinsic semantic information than a segmentation. However, the inverse problem, i.e. segmentation-based symmetry detection, has been little investigated. All symmetry detection algorithms explicitly or implicitly use the concept that a symmetrically-related subregion of a shape is a meaningful part. This is also the foundation of this paper, but here we employ segmentation results to benefit symmetry detection. We further note that segmentation and symmetry detection may be used iteratively to produce hierarchical symmetry relationships in objects.

In this paper, we propose an effective method for generalized partial approximate symmetry detection. The input shape is first segmented into multiple meaningful parts. Correspondences between each pair of parts are established using robust matching. This is followed by a clustering stage where consistent matchings are recognized as implying a symmetry relationship between a pair of subregions.

Compared to recent graph matching algorithms [7,8] based on salient lines our algorithm produces some robust partial symmetry detection results. For example, as shown in Figure 1, which [8] failed to handle as the paper claimed, the two-scale symmetries of the Gargoyle statue are correctly detected. The top row shows the coarse-scale mirror symmetry detected from first-time six meaningfully segmented parts (head, left and right wing, left and right body parts, and statue base). Two pairs of mirror symmetries (wings, body regions)

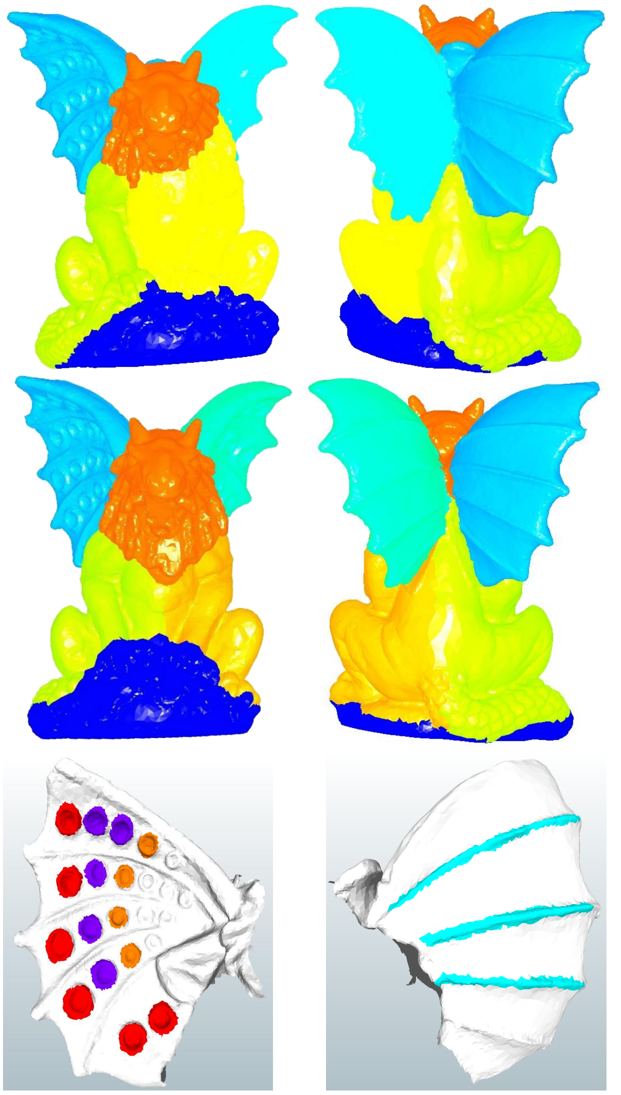

Fig. 1 Two-scale symmetries of the Gargoyle statue from two viewpoints. Top: At the coarse scale, two pairs of mirroring symmetry from six segmented parts are represented, with each pair being rendered by the similar color. Center: The results are produced by the segmentation-and-symmetry processing after 2 iterations. Bottom: At the fine scale, translation and rotation symmetries are detected on the left wing part for the detailed rings.

have been found. The center shows our segmentation-and-symmetry refinement results after 2 iteration, it is easy to notice that the both symmetry and segmentation are both better than the previous. The bottom row shows fine-scale symmetry further detected when performing an analysis just on the left wing, where both translational and rotational symmetries have been found for the detailed rings and ribs.

The rest of the paper is structured as follows. After surveying related previous work in Section 2, Section 3 describes our symmetry detection algorithm. Section 4 demonstrates the effectiveness of our method and provides comparisons with other approaches. Finally, Section 5 draws conclusions and discusses future work.

2 Related work

Symmetry detection and analysis have been thoroughly studied from theoretical, algorithmic, and applications aspects for many years. Papers on this topic can be classified according to whether they consider *global* or *partial* symmetries; most but not all recent papers consider approximate rather than exact symmetries. We aim to detect partial approximate symmetry in 3D shapes in this paper, and so we concentrate on the most closely related 3D symmetry detection methods, as well as ideas relating segmentation and symmetry. Comprehensive reviews can be found in [6, 8].

A common approach to symmetry detection is to identify the global symmetry in Euclidean space from clusters in the transformed space [2,3]. Other approaches [7, 10] formulate symmetry detection as graph matching, based on reliable line features extracted from scanned data. Extrinsic symmetry methods employ invariance under rigid transformations and possibly scaling for symmetry detection [2, 9, 11–14], and for finding patterns [3, 7, 15]. Ovsjanikov et al. perform global intrinsic symmetry detection for non-rigid isometric shapes using eigenanalysis of the surface Laplace-Beltrami operator [11], or by use of heat kernel maps [12]. Two robust symmetry detection algorithms proposed by Kim et al [16,17] utilize the Mobius transformation and conformal mapping. Most algorithms determine discrete symmetries by sampling. An exception is the work [18] where global intrinsic symmetry is detected using the approximate Killing vector field [18] from a continuous perspective.

Recent works have focused on analyzing partial and approximate symmetries, which is more difficult than finding global symmetries. As for global symmetry, partial symmetry detection can also be categorized as extrinsic [5, 8, 19] or intrinsic [6, 20–23]. A global multidimensional scaling (GMDS) approach devised by Bronstein and coworkers [19, 22–24] can detect extrinsic and intrinsic symmetries; it has been mainly applied to shape retrieval applications based on invariants. Other researchers have proposed methods based on a variety of techniques, e.g. Markov random field models [20], symmetry axis voting [6], eigenanalysis [25], pseudo-polar Fourier transforms [26], etc. For general partial symmetries, our algorithm can handle reflectional, rotational and translational problems in a unified framework.

The relationship between symmetry and segmentation can be viewed as a chicken-and-egg problem, as each provides clear guidance for the other. Symmetry detection has already been used to provide more plausible segmented parts, in both global [2, 27] and partial symmetry algorithms [6, 25]. The output of most symmetry detection algorithms is expressed in terms of segmented object parts. In this paper, we use segmentation to drive our symmetry detection algorithm.

3 Algorithm

Given a shape $\mathbb{S}$, our algorithm performs the following three steps (as illustrated in Figure 2)

1. Automatically decompose the shape into parts. Implied by the existing symmetry detection algorithms [6, 8], symmetric parts are always meaningful. This is the foundation of our algorithm and could explain why the meaningful segmentation is first performed to provide the parts for symmetry detection, although the other surface-based [28] segmentation algorithm could also provide the initial segmentation.
2. For each part $\mathbb{S}_i$, match the remaining shape $\mathbb{S} - \mathbb{S}_i$ to $\mathbb{S}_i$ such that for any node in $\mathbb{S} - \mathbb{S}_i$, a partial match from $\mathbb{S}_i$ is performed.
3. Put together all the correspondences to form a many-to-many correspondence in $\mathbb{S}$, and cluster nodes in $\mathbb{S}$ based on similarity of local matching transformation.

3.1 Automatic part-type segmentation

We start by decomposing the input shape $\mathbb{S}$ into n parts: $\mathbb{S}_1, \mathbb{S}_2, \ldots, \mathbb{S}_n$. The segmentation should ideally lead to each region representing a meaningful part of the object. We use a random-walk-based part-type segmentation approach [29] which starts by distributing n seeds as widely as possible over the surface. Each face of the shape is assigned the label belonging to the seed which has the maximum probability of being reached by a random walk starting from the face. The probability of stepping across face edges is derived from the local geometry such that it is more difficult to move across sharp edges.

3.2 Region matching

In order to find the symmetries, we uniformly sample the shape $\mathbb{S}$ with m sample regions, and then build the partial matching between every pair of regions $\mathbb{S}_i$ and $\mathbb{S}_j$, $i \neq j$. Typically we set $m = 100n$.

Partial matching is performed using a third-order tensor matching algorithm [reference omitted for review]. Our algorithm formulates the matching using a supersymmetric tensor representing an affinity metric, which takes into account feature similarity and geometric constraints between features. Matching is performed by an efficient higher-order power iteration approach which takes advantage of the compact representation of the supersymmetric affinity tensor; an efficient sampling strategy is used to compactly estimate the affinity tensor.

The algorithm matches point triples from each shape, taking into account geodesic distances between points. To compute geodesic distances, the approximate Dijkstra algorithm [30] is used. Suppose the number of matches between regions $\mathbb{S}_i$ and $\mathbb{S}_j$ is $M_{i,j}$. These regions are recognized as a candidate symmetry if $M_{i,j} > cm$, where c is a constant and set to $0.2/n$ for all examples in the paper.

3.3 Correspondence clustering

Next, we gather together all correspondences for each part $\mathbb{S}_i$, found by the partial matching, giving many-to-many correspondences for the whole of $\mathbb{S}$. Then, we cluster correspondences based on local transformation consistency of the matching.

For each partial matching output a rigid transform can be computed from each triple of compatible matching points, following [31]. As shown by [31], this transformation always exists for three non-collinear points, and is unique up to a reflective ambiguity. The solution is in closed-form and only involves second-order equations, so it is run in the fast speed.

For all sampled points, our clustering method is performed similar to the one used in [5], which determines transformations that map local surface patches onto each other. Each matching pair provides evidence for a symmetry relation at the level of the local sample spacing. From these, we extract meaningful symmetries at larger scales by finding groups of pairs with a similar transformation that correspond to symmetric subsets of the shape. We use mean shift clustering [32] for this purpose, a non-parametric method based on gradient ascent on a density function. The corresponding symmetry transformation is then defined by the clusters maximum. Please note that, as observed by Mitra et al. [5], the significant modes detected by the mean-shift clustering algorithm do not necessarily correspond to a meaningful symmetry. So, we also make use of the similar verification step [5] to compute the accurate symmetry regions. But our spatial consistency is also constrained by the former segmentation results. For more

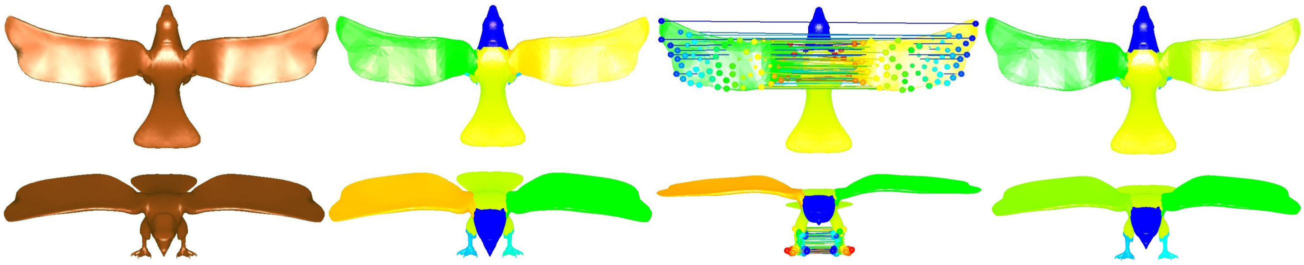

Fig. 2 Algorithm pipeline (using an Eagle example from two viewpoints). Left to right: the given shape, is decomposed into several parts, each part is matched to the remaining shape, and symmetry is detected by clustering the correspondences based on similarity of matching transform.

details on mean-shift clustering and verification we refer to [5].

3.4 Segmentation-and-symmetry iteration

Until now, one complete segmentation-based symmetry detection processing has been presented. As the initial segmentation does not often produce accurate meaningful part, the symmetry detection also is also not so perfect. So, the further refinement should be performed both on the segmentation and symmetry. Fortunately, as shown in Figure 1 top, the initial symmetry detection could detect accurate mirroring symmetry. The useful symmetry result could be further used to refine the segmentation, then improve the symmetry as a result. The symmetry information is used as the guide to place the seed points in the random-walk-based segmentation algorithm [29]. For the seeds at the Garoyl statues, two are placed at the symmetric left and right wings, same as those at the left and right bodies. Besides, one is placed at the head, and the last is place at status base. Beyond the seeds placement, the better segmentation could be produced as the center-left as 1. Consequently, based on the better segmentation, the better symmetry is further detected as the center-right.

4 Results

Our algorithm has been tested on a variety of 3D shapes. We visualize the discrete matches using similar colors to show symmetrically related parts.

We first show experimental results on clean, complete manifold meshes, without noise. For such models, our algorithm provides a good initial segmentation for the partial matching, and we obtain results which are in good agreement with those expected (e.g. Figures 2 and 3).

As our algorithm depends on segmentation, uniform sampling of partial matching, and transformation space clustering, the matching results do not depend on the representation or exact sampling (e.g. mesh) used. Figure 4 demonstrates that our algorithm can be applied to shapes in point set form. Simultaneously, it also demonstrates that our algorithm is insensitive to mesh variations, as the left and right sides of the dinosaur were sampled with quite different spatial resolutions.

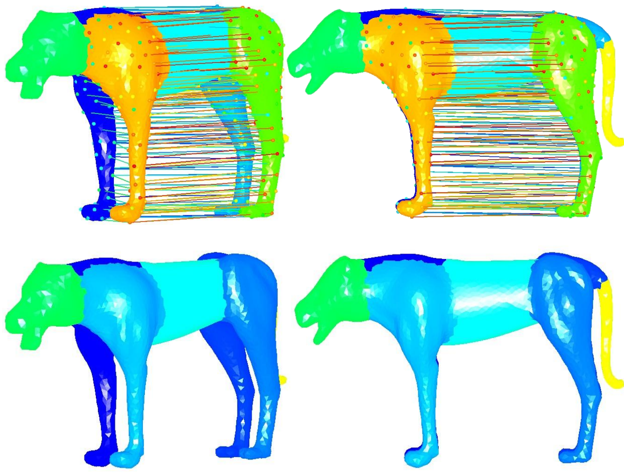

Fig. 3 The four legs of the cheetah are detected as symmetric copies when using configuration $c = 0.2/n$ and $m = 100 \times n$, where n is the user-defined segmentation number.

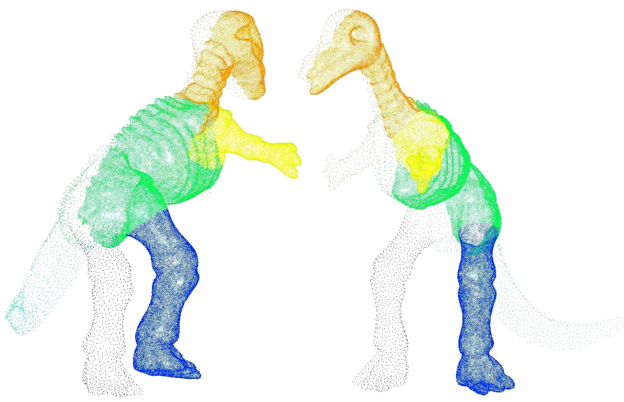

Fig. 4 Symmetry detection in a dinosaur model represented as a point set with different spatial resolutions on left and right.

Model	Vertices	Segments	Segmentation time	Partial matching time	Correspondence clustering time
Armadillo	172,974	9	1.6s	959.0s	7.8s
Chetah	5,000	7	0.2s	588.2s	5.2s
Dinosaur	69,215	7	3.2s	522.7s	5.1s
Eager	14,618	6	0.5s	403.1s	3.2s
Gargoyl	250,003	6	3.6s	448.9s	3.9s

Table 1 Times taken by our algorithm, using a 1.6GHz Intel Core i7 CPU laptop with 4GB of RAM.

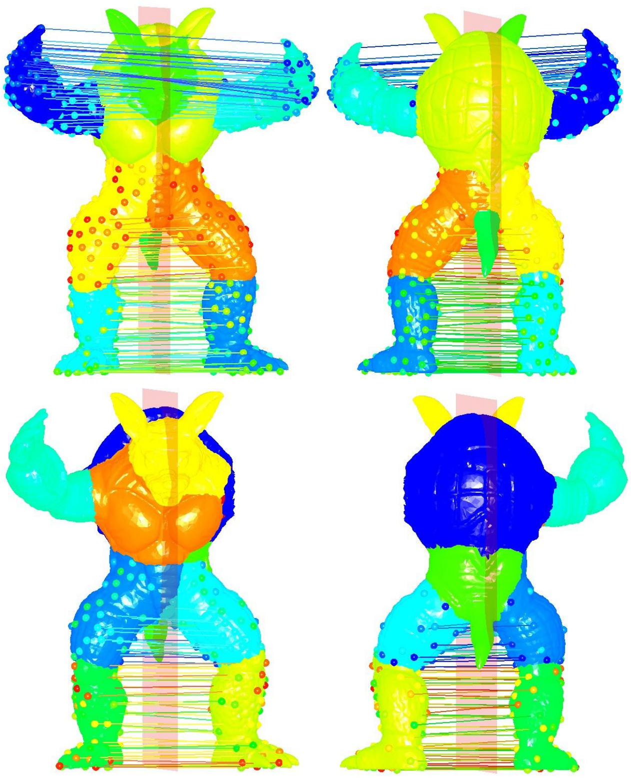

Fig. 5 The global reflection plane is almost identical, for both complete and incomplete armadillo models (note the missing arm).

Figure 1 shows an example containing three different symmetry groups composed of reflection, rotation, and translation. The symmetries are detected in coarse-to-fine steps based on hierarchical segmentation. At the coarse level, all major reflection symmetries are faithfully recovered, while at the fine level, many rotational and translational symmetries are also detected. Note that [8] cites this example as on on which its state of the art algorithm does not perform well.

Results of processing complete and incomplete armadillo models are shown in Figure 5, demonstrating our ability to handle incomplete shapes. In both cases, the algorithm computes a global reflection plane based on all final correspondences; the plane is almost identical in both cases.

Finally, we demonstrate iterative use of segmentation and symmetry detection steps to improve the output, both in terms of shape segmentation and symmetry detection. Initially, the segmentation of the shape is merely 'satisfactory', as the shape is complex. Following segmentation, symmetry is detected as before. Using the detected reflection symmetry information, we further improve the segmentation. Figure 1 shows the initial segmentation and symmetry detection, and the refined results after two iterations.

The performance data in Table 1 shows computation times for various 3D shapes. Partial matching takes more time than segmentation, but is less sensitive to the number of vertices in each shape: sampling is linearly dependent on the number of segmented parts.

5 Conclusions

We have proposed a symmetry detection algorithm for discovering and extracting partial approximate symmetries of 3D geometric models. It can detect rotational, translational, and mirror symmetries. It is based on first performing shape segmentation, and performs better than graph matching using salient feature lines. Our algorithm is effective, easy to implement, and applicable to a wide range of 3D shapes.

Our symmetry detection algorithm relies on helpful outputs from a segmentation algorithm which may be hard to ensure in the presence of significant noise. We have used a general purpose segmentation algorithm, but better results are likely to be achieved if the segmentation algorithm itself is symmetry aware. Further improvements both in performance and quality of results is likely to arise through use of multi-resolution approaches, as well as parallelization techniques.

References

1. M. Gellert and R. Harbord, "Symmetric forms for finite element analysis of the navier-stokes problem," *Computers and Fluids*, vol. 15, no. 4, pp. 379 – 389, 1987.
2. J. Podolak, P. Shilane, A. Golovinskiy, S. Rusinkiewicz, and T. Funkhouser, "A planar-reflective symmetry transform for 3D shapes," *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 549–559, 2006.
3. M. Pauly, N. J. Mitra, J. Wallner, H. Pottmann, and L. Guibas, "Discovering structural regularity in 3D geometry," *ACM Transactions on Graphics*, vol. 27, no. 3, pp. 43:1–11, 2008.

4. A. Shamir, "A survey on mesh segmentation techniques," *Computer Graphics Forum*, vol. 27, no. 6, pp. 1539–1556, 2008.
5. N. J. Mitra, L. J. Guibas, and M. Pauly, "Partial and approximate symmetry detection for 3D geometry," *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 560–568, 2006.
6. K. Xu, H. Zhang, A. Tagliasacchi, L. Liu, G. Li, M. Meng, and Y. Xiong, "Partial intrinsic reflectional symmetry of 3D shapes," *ACM Transactions on Graphics*, vol. 28, no. 5, 2009.
7. M. Bokeloh, A. Berner, M. Wand, H.-P. Seidel, and A. Schilling, "Symmetry detection using line features," *Computer Graphics Forum (Special Issue of Eurographics)*, vol. 28, no. 2, pp. 697–706, 2009.
8. A. Berner, M. Wand, N. J. Mitra, D. Mewes, and H.-P. Seidel, "Shape analysis with subspace symmetries," *Computer Graphics Forum*, vol. 30, no. 2, 2011.
9. H. Zabrodsky and D. Weinshall, "Using bilateral symmetry to improve 3D reconstruction from image sequences," *Computer Vision and Image Understanding*, vol. 67, pp. 48–57, 1997.
10. A. Berner, M. Bokeloh, M. Wand, A. Schilling, and H.-P. Seidel, "A graph-based approach to symmetry detection," in *Symposium on Volume and Point-Based Graphics*, pp. 1–8, Eurographics Association, 2008.
11. M. Ovsjanikov, J. Sun, and L. Guibas, "Global intrinsic symmetries of shapes," *Computer Graphics Forum (Symposium on Geometry Processing)*, vol. 27, no. 5, pp. 1341–1348, 2008.
12. M. Ovsjanikov, Q. Mérigot, F. Mémoli, and L. Guibas, "One point isometric matching with the heat kernel," in *Eurographics Symposium on Geometry Processing*, p. to appear, 2010.
13. M. Chertok and Y. Keller, "Spectral symmetry analysis," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 32, pp. 1227–1238, 2010.
14. A. Hooda, M. Bronstein, A. Bronstein, and R. P. Horaud, "Shape palindromes: Analysis of intrinsic symmetries in 2d articulated shapes," in *Proc. of the Third International Conference on Scale Space and Variational Methods in Computer Vision*, Springer, 2011.
15. Y.-T. Yeh and R. Mech, "Detecting symmetries and curvilinear arrangements in vector art," *Computer Graphics Forum (Special Issue of Eurographics)*, vol. 28, no. 2, pp. 707–716, 2009.
16. V. G. Kim, Y. Lipman, X. Chen, and T. Funkhouser, "Mobius transformations for global intrinsic symmetry analysis," *Computer Graphics Forum (Proc. of Symposium on Geometry Processing)*, vol. 29, no. 5, pp. 1689–1700, 2010.
17. V. G. Kim, Y. Lipman, and T. Funkhouser, "Blended intrinsic maps," *Transactions on Graphics (Proc. SIGGRAPH)*, 2011.
18. M. Ben-Chen, A. Butscher, J. Solomon, and L. Guibas, "On discrete killing vector fields and patterns on surfaces," *Computer Graphics Forum (Special Issue of SGP)*, vol. 29, no. 5, pp. 1701–1711, 2010.
19. A. M. Bronstein, M. M. Bronstein, A. M. Bruckstein, and R. Kimmel, "Partial similarity of objects, or how to compare a centaur to a horse," *International Journal of Computer Vision*, vol. 84, no. 2, pp. 163–183, 2009.
20. R. Lasowski, A. Tevs, H.-P. Seidel, and M. Wand, "A probabilistic framework for partial intrinsic symmetries in geometric data," in *IEEE International Conference on Computer Vision*, 2009.
21. N. J. Mitra, A. Bronstein, and M. Bronstein, "Intrinsic regularity detection in 3d geometry," in *Proceedings of the 11th European conference on computer vision conference on Computer vision: Part III*, pp. 398–410, 2010.
22. D. Raviv, A. M. Bronstein, M. M. Bronstein, and R. Kimmel, "Full and partial symmetries of non-rigid shapes," *International Journal of Computer Vision*, vol. 89, pp. 18–39, 2010.
23. A. M. Bronstein, M. M. Bronstein, L. J. Guibas, and M. Ovsjanikov, "Shape Google: Geometric words and expressions for invariant shape retrieval," *ACM Transactions on Graphics*, vol. 30, pp. 1:1–1:20, 2011.
24. A. M. Bronstein, M. M. Bronstein, and R. Kimmel, "Generalized multidimensional scaling: A framework for isometry-invariant partial surface matching," *Proc. National Academy of Sciences (PNAS)*, vol. 103, no. 5, pp. 1168–1172, 2006.
25. Y. Lipman, X. Chen, I. Daubechies, and T. Funkhouser, "Symmetry factored embedding and distance," *ACM Transactions on Graphics (Proc. SIGGRAPH)*, 2010.
26. A. Bermanis, A. Averbuch, and Y. Keller, "3D symmetry detection and analysis using the pseudo-polar fourier transform," *International Journal of Computer Vision*, vol. 90, pp. 166–182, 2010.
27. J. Podolak, A. Golovinskiy, and S. Rusinkiewicz, "Symmetry-enhanced remeshing of surfaces," *Symposium on Geometry Processing*, pp. 235–242, 2007.
28. A. Shamir, "Segmentation and shape extraction of 3D boundary meshes," *Eurographics STAR Report*, pp. 137–149, 2006.
29. Y.-K. Lai, S.-M. Hu, R. R. Martin, and P. L. Rosin, "Rapid and effective segmentation of 3d models using random walks," *Computer-Aided Geometric Design*, vol. 26, no. 6, pp. 665–679, 2009.
30. E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerical Mathematic*, vol. 1, no. 1, pp. 269–271, 1959.
31. D. P. Huttenlocher and S. Ullman, "Recognizing solid objects by alignment with an image," *International Journal of Computer Vision*, vol. 5, pp. 195–212, 1990.
32. D. Comaniciu, P. Meer, and S. Member, "Mean shift: A robust approach toward feature space analysis," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 24, pp. 603–619, 2002.