1.1

1. $R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} R_y(\delta) = \begin{bmatrix} cos\delta & 0 & sin\delta \\ 0 & 1 & 0 \\ -sin\delta & 0 & cos\delta \end{bmatrix} R_z(\phi) = \begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$
\begin{aligned}
R_1 &= R_z(\phi)R_y(\delta)R_x(\theta) \\
&= \begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\delta & 0 & sin\delta \\ 0 & 1 & 0 \\ -sin\delta & 0 & cos\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \\
&= \begin{bmatrix} cos\phi cos\delta & -sin\phi & cos\phi sin\delta \\ sin\phi cos\delta & cos\phi & sin\phi sin\delta \\ -sin\delta & 0 & cos\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \\
&= \begin{bmatrix} cos\phi cos\delta & -sin\phi cos\theta + cos\phi sin\delta sin\theta & sin\phi sin\theta + cos\phi sin\delta cos\theta \\ sin\phi cos\delta & cos\phi cos\theta + sin\phi sin\delta sin\theta & -cos\phi sin\theta + sin\phi sin\delta cos\theta \\ -sin\delta & cos\delta sin\theta & cos\delta cos\theta \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
R_2 &= R_z(\phi)R_x(\theta)R_y(\delta) \\
&= \begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} cos\delta & 0 & sin\delta \\ 0 & 1 & 0 \\ -sin\delta & 0 & cos\delta \end{bmatrix} \\
&= \begin{bmatrix} cos\phi & -sin\phi cos\theta & sin\phi sin\theta \\ sin\phi & cos\phi cos\theta & -cos\phi sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} cos\delta & 0 & sin\delta \\ 0 & 1 & 0 \\ -sin\delta & 0 & cos\delta \end{bmatrix} \\
&= \begin{bmatrix} cos\phi cos\delta - sin\phi sin\theta sin\delta & -sin\phi cos\theta & cos\phi sin\delta + sin\phi sin\theta cos\delta \\ sin\phi cos\delta - cos\phi cos\theta sin\delta & cos\phi cos\theta & sin\phi sin\delta - cos\phi sin\theta cos\delta \\ -cos\theta sin\delta & sin\theta & cos\theta cos\delta \end{bmatrix}
\end{aligned}
$$

2.

$$
\begin{aligned}
R_1R_1^T &= R_z(\phi)R_y(\delta)R_x(\theta)R_x^T(\theta)R_y^T(\delta)R_z^T(\phi) \\
&= R_z(\phi)R_y(\delta)R_x(\theta)R_x(-\theta)R_y(-\delta)R_z(-\phi) \\
&= I
\end{aligned}
$$

$$
\begin{aligned}
R_2R_2^T &= R_z(\phi)R_x(\theta)R_y(\delta)R_y^T(\delta)R_x^T(\theta)R_z^T(\phi) \\
&= R_z(\phi)R_x(\theta)R_y(\delta)R_y(-\delta)R_x(-\theta)R_z(-\phi) \\
&= I
\end{aligned}
$$

1.2

Apply SVD on matrixes A, B and C:

A = [3 3 13 8 9;1 6 10 2 4;9 7 6 9 5;14 15 3 4 10;11 3 7 6 4];

B = [13 9 13 7 7;15 2 4 5 2;11 14 9 3 9;6 14 1 3 8];

C = [11 5 11 10 23;11 8 15 12 21;10 10 8 7 8;1 7 5 2 4;2 13 2 4 13;1 24 3 9 11];

W1 = A.' * A;

[~,S,~] = svd(A)

W2 = B.' * B;

[~,S2,~] = svd(B)

W3 = C.' * C;

[~,S3,~] = svd(C)

1 and 2: According to lecture 3, the largest singular value is the largest amount by which a vector is scaled by A;

the smallest singular value is the smallest amount by which a vector is scaled by A(for low-rank or non-square matrices this can be 0). The results are shown in the following table.

3.According to the formula: $\dim\left(\mathrm{N(A)}\right) = \dim(V) - \dim(im(A))$, where $im(A)$ is the result vector of A transformation on x.

$$\dim\left(\mathrm{N(A)}\right) = \dim(V) - \mathrm{rank(A)} = 5 - 5 = 0$$
$$\dim\left(\mathrm{N(B)}\right) = \dim(V) - \mathrm{rank(B)} = 5 - 4 = 1$$
$$\dim\left(\mathrm{N(C)}\right) = \dim(V) - \mathrm{rank(C)} = 5 - 5 = 0$$
$$\dim\left(\mathrm{N}\left(A^T A\right)\right) = \dim(V) - rank\left(A^T A\right) = 5 - 5 = 0$$
$$\dim\left(\mathrm{N}\left(B^T B\right)\right) = \dim(V) - rank\left(B^T B\right) = 5 - 4 = 1$$
$$\dim\left(\mathrm{N}\left(C^T C\right)\right) = \dim(V) - rank\left(C^T C\right) = 5 - 5 = 0$$

	S	max projection length	min projection length
A	$\begin{matrix} 35.4271 & 0 & 0 & 0 & 0 \\ 0 & 14.0273 & 0 & 0 & 0 \\ 0 & 0 & 7.8850 & 0 & 0 \\ 0 & 0 & 0 & 3.7671 & 0 \\ 0 & 0 & 0 & 0 & 3.1270 \end{matrix}$	35.43	3.13

	S	max projection length	min projection length
B	$\begin{matrix} 37.0792 & 0 & 0 & 0 & 0 \\ 0 & 12.6429 & 0 & 0 & 0 \\ 0 & 0 & 6.7574 & 0 & 0 \\ 0 & 0 & 0 & 2.1503 & 0 \end{matrix}$	37.08	0
C	$\begin{matrix} 54.6186 & 0 & 0 & 0 & 0 \\ 0 & 21.0221 & 0 & 0 & 0 \\ 0 & 0 & 8.1179 & 0 & 0 \\ 0 & 0 & 0 & 3.9057 & 0 \\ 0 & 0 & 0 & 0 & 2.5932 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$	54.62	2.59

1.3

1. Run B = inv(eye(2017) - ones(2017)/2021) - eye(2017); in Matlab; observe B and get $\alpha = 0.25$

2. $(I - \frac{1}{n}A)e = \begin{bmatrix} 1-\frac{1}{n} & -\frac{1}{n} & \dots & -\frac{1}{n} \\ -\frac{1}{n} & 1-\frac{1}{n} & \dots & -\frac{1}{n} \\ \vdots & & & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \dots & 1-\frac{1}{n} \end{bmatrix}_{n\times n} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{1}{n}-\frac{1}{n}\times(n-1) \\ 1-\frac{1}{n}-\frac{1}{n}\times(n-1) \\ \vdots \\ 1-\frac{1}{n}-\frac{1}{n}(n-1) \end{bmatrix}_{1\times n} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Since Rank(e) is not zero while Rank($(I - \frac{1}{n}A)e$)=0, so Rank($I - \frac{1}{n}A$) = 0, so it is not invertible.

2.2

1.derivation of $\frac{dE}{dW}$

$$E = \frac{1}{DT}||M - NW||^2 = \frac{1}{DT}(M^T - W^T N^T)(M - NW) = \frac{1}{DT}(M^T M - W^T N^T M - M^T NW + W^T N^T NW)$$

$$\frac{dE}{dW} = \frac{2}{DT}N^T(NW - M)$$

2.3

Assume that there are two transformation matrixes that can be applied separately to the magnitude matrix and phase matrix of a piano music to convert it to guitar version. I calculated and applied two matrixes and got something like guitar music.