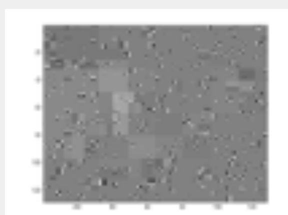
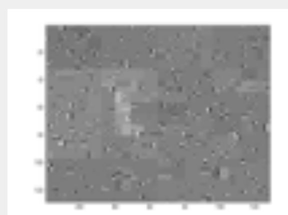
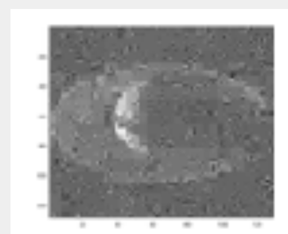
Problem 1

1a
reconstruction error

1a	1639	2048	4096
error	105.35	77.31	28.63
image reconstructed			

1b

1b	1639	2048	4096
error	4.941474457255049e+08	6.190870175281864e+08	1.261093037415351e+09
image reconstructed			

I use the function imagesc to observe the reconstructed images (to visualize data matrix Irec), since they are not as good as 1a.

IHT is not working as well as sparse recovery procedure in 1a, because it is a greedy algorithm, so it may not get a optimal result.

Problem 2

2.1
To get (X,Y), first Select any urn z at the probability P(Z); draw (X_1,Y) at the probability P(X_1,Y|Z); draw X_2 at the probability P(X_2|Z). Consider all combinations of X_2, (X_1,Y), Z.

$$P(X,Y) = \sum_Z P(Z) \sum_{X_2} P_{X_2}(X_2|Z) P(X - X_2, Y|Z)$$

2.2
Estimation:

$$P(Z|X,Y) = \frac{P(X,Y,Z)}{\sum_{Z'} P(X,Y,Z')} = \frac{P(Z)\sum_{X_2} P_{X_2}(X_2|Z) P_{X_1,Y_1}(X - X_2, Y|Z)}{\sum_{Z'} P(Z')\sum_{X_2} P_{X_2}(X_2|Z') P_{X_1,Y_1}(X - X_2, Y|Z')}$$

$$P(X_2|X,Y,Z) = \frac{P(X_2,X,Y|Z)}{P(X,Y|Z)} = \frac{P_{X_2}(X_2|Z) P_{X_1,Y_1}(X - X_2, Y|Z)}{\sum_{X_2'} P_{X_2}(X_2'|Z) P_{X_1,Y_1}(X - X_2', Y|Z)}$$

Maximization:

$$P_Z(Z) = \frac{\sum_X \sum_Y P(Z|X,Y) H(X,Y)}{\sum_{Z'} \sum_X \sum_Y P(Z'|X,Y) H(X,Y)}$$

$$P_{X_2}(X_2|Z) = \frac{\sum_X \sum_Y P(X_2|X,Y,Z) P(Z|X,Y) H(X,Y)}{\sum_{X_2'} \sum_X \sum_Y P(X_2'|X,Y,Z) P(Z|X,Y) H(X,Y)}$$

$$P_{X_1,Y_1}(X_1,Y|Z) = \frac{\sum_X P(X - X_1|X,Y,Z) P(Z|X,Y) H(X,Y)}{\sum_{X_1'} \sum_X P(X - X_1|X,Y,Z) P(Z|X,Y) H(X,Y)}$$

2.3
In this case, compare to 2.2, we don't have to pick the z urn, the problem is simplified to the following form.

Initialization

Normalize the blurred image matrix (of dimension 288 x 531) as $P_{X_1,Y_1}(X_1, Y_1)$

For N = 0-19, initialize $P_{X_2}(N) = e^{-0.1 \times N}$

Estimation

$$P(X_2|X,Y) = \frac{P_{X_2}(X_2) P_{X_1,Y_1}(X - X_2, Y)}{P(X,Y)} = \frac{P_{X_2}(X_2) P_{X_1,Y_1}(X - X_2, Y)}{\sum_{X_2'} P_{X_2}(X_2') P_{X_1,Y_1}(X - X_2', Y)}$$

Maximization

$$count_{X_1,Y_1} = \sum_X H(X,Y) P(X - X_1|X,Y)$$

$$count_{X_2} = \sum_X \sum_Y H(X, Y)P(X_2 | X, Y)$$

normalize the above two count values to get $P_{X_1}(X_1, Y),\ P_{X_2}(X_2)$

$$P_{X_1,Y_1}(X_1, Y) = \frac{\sum_X P(X - X_1 | X, Y)H(X, Y)}{\sum_{X_1'} \sum_X P(X - X_1 | X, Y)H(X, Y)}$$

$$P_{X_2}(X_2) = \frac{\sum_X \sum_Y P(X_2 | X, Y)H(X, Y)}{\sum_{X_2'} \sum_X \sum_Y P(X_2' | X, Y)H(X, Y)}$$

Got this result after 50 iterations. Scale the possibility matrix P_x1 by 1e5 to get an proper image.