Your original text:

step 5 Compute $\mu(\lambda, \theta^{(0)})$, $\mu(\lambda, \theta_i)$, $\mu(\lambda, \theta_j)$,

$$\mu(\lambda, \theta) = E_{H_1,\eta_0}\big(s(\theta, \eta_\theta\big) = \frac{1}{n}\big(l_{n,11}(n, 1, \hat{\eta}) + \theta l_{n,21}(n, 1\hat{\eta}) - (L_{13}^T(\hat{\eta}) + \theta L_{23}^T(\hat{\eta}))L_{33}^{-1}(\hat{\eta})l_{n,31}(1, 1, \hat{\eta})\big).$$

Your question: is the formula in step 5 correct?

Response. The likelihood is $l_n(\lambda, 1-\theta+\theta\lambda, \eta)$. The null hypothesis is $H_0 : \lambda = 1$ vs the alternative $H_1 : \lambda \neq 1$.

First, you have a typo: $l_{n,11}(n, 1, \hat{\eta})$ and $l_{n,21}(n, 1\hat{\eta})$, which should be $l_{n,11}(1, 1, \hat{\eta})$ and $l_{n,21}(1, 1\hat{\eta})$.

Also, $l_{n,31}(1, 1, \hat{\eta})$ should be $l_{n,13}(1, 1, \hat{\eta})$ (but I think they are the same under general condiitons).

For given (λ, θ) (with $\lambda \neq 1$) , I think $\mu(\lambda, \theta) = E_{H_1,\eta_0}\big(s(\theta, \eta_\theta\big)$ can be computed by the following Monte Carlo (method may not unique):

Note that for given (λ, θ), $(L_{13}^T(\hat{\eta}) + \theta L_{23}^T(\hat{\eta}))L_{33}^{-1}(\hat{\eta})$ is a constant, and $l_{1,11}(1, 1, \hat{\eta}) = l_{1,11}(1, 1, \hat{\eta}|y, x)$, $l_{1,21}(1, 1, \hat{\eta}) = l_{1,21}(1, 1, \hat{\eta}|y, x)$ and $l_{1,13}(1, 1, \hat{\eta}) = l_{1,13}(1, 1, \hat{\eta}|y, x)$ are random variables.

1). Given (λ, θ), for $i = 1, ..., n$ (typically $n \geq 10,000$), sample x_i from a given distribution (see ZLY for detail), then given this x_i, sample y_i from $f(y|\lambda, 1-\theta+\theta\lambda, \eta_0^T x_i)$ (see ZLY for how to choose the density $f(\cdot|\cdot, \cdot, \cdot)$). Then we get a sample $\{(y_i, x_i) : i = 1, ..., n\}$ under H_1.

2). Set

$$\mu(\lambda, \theta) = E_{H_1,\eta_0}\big(s(\theta, \eta_\theta\big)$$

$$\approx \frac{1}{n}\sum_{i=1}^{n}\Big(l_{1,11}(1, 1, \hat{\eta}|y_i, x_i) + \theta l_{1,21}(1, 1, \hat{\eta}|y_i, x_i) - (L_{13}^T(\hat{\eta}) + \theta L_{23}^T(\hat{\eta}))L_{33}^{-1}(\hat{\eta})l_{1,13}(1, 1, \hat{\eta}|y_i, x_i)\Big).$$