0.1 X：5，Y：4

0.1.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-1, 0, 1, 2)$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1)$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.4, -0.2, 0, 0, 2, 0, 4)$(linear effect)
3. $\boldsymbol{\eta} = (-0.30, -0.18, 0.20, 0.22, 0.24)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (-0.2, 0, 0.2, 0. - 0.2)$(nonmonotonic effect)

0.1.2 The result

NREPL=1000,Nemp=1000,N=500,Time=

Table 1. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.056	0.867	0.543	0.078
T2emp	0.059	0.87	0.555	0.08
T3emp	0.061	0.87	0.56	0.081
CobT1	0.054	0.856	0.582	0.182
T1s	0.052	0.864	0.538	0.076
T2s	0.057	0.874	0.565	0.078
T3s	0.056	0.874	0.565	0.078
X linear	0.055	0.881	0.499	0.067
X catego	0.054	0.706	0.537	0.281
iso	0.063	0.821	0.592	0.365
Spline	0.071	0.791	0.579	0.221

0.2 X：10，Y：4

0.2.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-4, -3, -2, -1, 0, 1, 2, 3, 4)$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1)$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1)$(linear effect)
3. $\boldsymbol{\eta} = (-0.65, -0.54, -0.3, 0.18, 0.2, 0.22, 0.24, 0.34, 0.36, 0.45)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (-0.2, 0, 0.2, 0, -0.2, -0.2, 0, 0.2, 0, -0.2)$(nonmonotonic effect)

0.2.2 The result

NREPL=1000,Nemp=1000,N=500,Time=

Table 1. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic

0.3 X：7，Y：4

0.3.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-2, -1, 0, 1, 2, 3)$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1)$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6)$(linear effect)
3. $\boldsymbol{\eta} = (-0.3, 0.18, 0.2, 0.22, 0.24, 0.34, 0.45)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (0, 0.2, 0, -0.2, -0.2, 0, 0.2)$(nonmonotonic effect)

0.3.2 The result

NREPL=1000,Nemp=1000,N=500,Time=

Table 1. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic