0.1 X：5，Y：4

0.1.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-1, 0, 1, 2), \beta^X = 1$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1), \beta^Y = 0.5$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.4, -0.2, 0, 0.2, 0.4)$(linear effect)
3. $\boldsymbol{\eta} = (-0.30, -0.18, 0.20, 0.22, 0.24)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (-0.2, 0, 0.2, 0, -0.2)$(nonmonotonic effect)

0.1.2 The result

- Parameter as above(The same as Li)

  NREPL=1000,Nemp=1000,N=500,Time 9h

Table 2. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.056	0.867	0.543	0.078
T2emp	0.059	0.87	0.555	0.08
T3emp	0.061	0.87	0.56	0.081
CobT1	0.054	0.856	0.582	0.182
T1s	0.052	0.864	0.538	0.076
T2s	0.057	0.874	0.565	0.078
T3s	0.056	0.874	0.565	0.078
X linear	0.055	0.881	0.499	0.067
X catego	0.054	0.706	0.537	0.281
iso	0.063	0.821	0.592	0.365
Spline	0.071	0.791	0.579	0.221

- $\beta^Y = 1$,NREPL=1000,Nemp=1000,N=500,Time=47354.37

Table 3. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.069	0.828	0.525	0.063
T2emp	0.05	0.832	0.552	0.06
T3emp	0.052	0.831	0.556	0.061
Cob2	0.06	0.826	0.555	0.168
Cob4	0.055	0.825	0.571	0.1
min2	0.056	0.795	0.515	0.212
Y X linear	0.059	0.857	0.483	0.058
Y X catego	0.057	0.655	0.483	0.256
X Y linear	0.056	0.823	0.55	0.068
X Y catego	0.056	0.685	0.387	0.064

- Set the η as

  1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0)$(the null)
  2. $\boldsymbol{\eta} = (-0.6, -0.3, 0, 0.3, 0.6)$(linear effect)
  3. $\boldsymbol{\eta} = (-0.30, -0.18, 0.20, 0.22, 0.5)$(monotonic nonlinear effect)

4. $\boldsymbol{\eta} = (-0.1, 0, 0.2, 0.1, -0.2)$(nonmonotonic effect)

NREPL=1000,Nemp=1000,N=500,Time=44460.03

Table 4. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.057	0.999	0.802	0.043
T2emp	0.054	0.999	0.8	0.055
T3emp	0.054	0.999	0.802	0.054
Cob2	0.063	0.996	0.793	0.092
Cob4	0.056	0.998	0.797	0.067
min2	0.06	0.997	0.764	0.118
min4	0.055	0.998	0.736	0.107
Y X linear	0.063	1	0.783	0.048
Y X catego	0.055	0.98	0.698	0.148
X Y linear	0.062	0.998	0.805	0.055
X Y catego	0.048	0.982	0.644	0.052

- Set the η as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.2, -0.1, 0, 0.1, 0.2)$(linear effect)
3. $\boldsymbol{\eta} = (-0.21, -0.18, 0.20, 0.22, 0.24)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (-0.25, 0, 0.2, 0, -0.15)$(nonmonotonic effect)

NREPL=1000,Nemp=1000,N=500,Time=40298.0

Table 5. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.057	0.343	0.576	0.113
T2emp	0.061	0.345	0.581	0.112
T3emp	0.062	0.345	0.582	0.11
Cob2	0.052	0.338	0.596	0.206
Cob4	0.054	0.345	0.598	0.146
min2	0.056	0.316	0.555	0.247
min4	0.055	0.279	0.528	0.217
Y X linear	0.053	0.362	0.543	0.088
Y X catego	0.045	0.205	0.531	0.305
X Y linear	0.061	0.338	0.583	0.108
X Y catego	0.051	0.226	0.427	0.091

- Set the η as

  1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0)$(the null)
  2. $\boldsymbol{\eta} = (-0.3, -0.15, 0, 0.15, 0.3)$(linear effect)
  3. $\boldsymbol{\eta} = (-0.21, -0.18, 0.10, 0.22, 0.24)$(monotonic nonlinear effect)
  4. $\boldsymbol{\eta} = (-0.35, 0.4, 0.2, 0, -0.15)$(nonmonotonic effect)

  NREPL=1000,Nemp=1000,N=500,Time=41262.45

Table 6. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T1emp	0.045	0.627	0.526	0.127
T2emp	0.046	0.643	0.507	0.135
T3emp	0.047	0.645	0.505	0.136
Cob2	0.05	0.611	0.507	0.599
Cob4	0.048	0.64	0.505	0.387
min2	0.047	0.582	0.477	0.728
min4	0.043	0.59	0.445	0.691
Y X linear	0.047	0.659	0.533	0.074
Y X catego	0.056	0.436	0.37	0.791
X Y linear	0.049	0.64	0.512	0.135
X Y catego	0.054	0.498	0.345	0.113

0.2 X：10，Y：4

0.2.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-4, -3, -2, -1, 0, 1, 2, 3, 4)$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1)$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1)$(linear effect)
3. $\boldsymbol{\eta} = (-0.65, -0.54, -0.3, 0.18, 0.2, 0.22, 0.24, 0.34, 0.36, 0.45)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (-0.2, 0, 0.2, 0, -0.2, -0.2, 0, 0.2, 0, -0.2)$(nonmonotonic effect)

0.2.2 The result

NREPL=1000,Nemp=1000,N=500,Time=

Table 1. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic
T2emp	0.045	0.977	0.559	0.056
T3emp	0.048	0.977	0.561	0.057
X conti	0.039	0.975	0.639	0.123
X catego	0.042	0.979	0.662	0.092
Cob2	0.041	0.983	0.626	0.06
min2	0.045	0.843	0.427	0.167
T1s	0.039	0.978	0.504	0.059
T2s	0.048	0.977	0.562	0.056
T3s	0.048	0.977	0.561	0.056
iso	0.046	0.969	0.647	0.082
Spline	0.06	0.923	0.626	0.145

0.3 X：7，Y：4

0.3.1 Generate ordinal data(same as the method in Li's paper)

Deriving from the (Li 2010 JASA),but don't generate the covariate Z.

The specifics of our four generating scenarios are as follows: we first generated X with five categories using th proportion odds model

$$P(X \leq i) = [1 + exp(-(\alpha_i^X + \beta^X Z))]^{-1}$$

with $\alpha^X = (\alpha_1^X, \alpha_2^X, \alpha_3^X, \alpha_4^X) = (-2, -1, 0, 1, 2, 3)$.The Y was generated with four levels using the proportional odds model

$$P(Y \leq j) = [1 + exp(-(\alpha_i^Y + \beta^Y Z + \eta_1 I_{\{X=1\}} + \eta_2 I_{\{X=2\}} + \cdots + \eta_5 I_{\{X=5\}}))]^{-1}$$

with $\alpha^Y = (\alpha_1^Y, \alpha_2^Y, \alpha_3^Y) = (-1, 0, 1)$,and $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_5)$ specified as

1. $\boldsymbol{\eta} = (0, 0, 0, 0, 0, 0, 0)$(the null)
2. $\boldsymbol{\eta} = (-0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6)$(linear effect)
3. $\boldsymbol{\eta} = (-0.3, 0.18, 0.2, 0.22, 0.24, 0.34, 0.45)$(monotonic nonlinear effect)
4. $\boldsymbol{\eta} = (0, 0.2, 0, -0.2, -0.2, 0, 0.2)$(nonmonotonic effect)

0.3.2 The result

NREPL=1000,Nemp=1000,N=500,Time=

Table 1. The result of the three statistics :Type I error and power

	Simulation scenarios			
Analysis method	Null	Linear	Nonlinear	Nonmonotonic