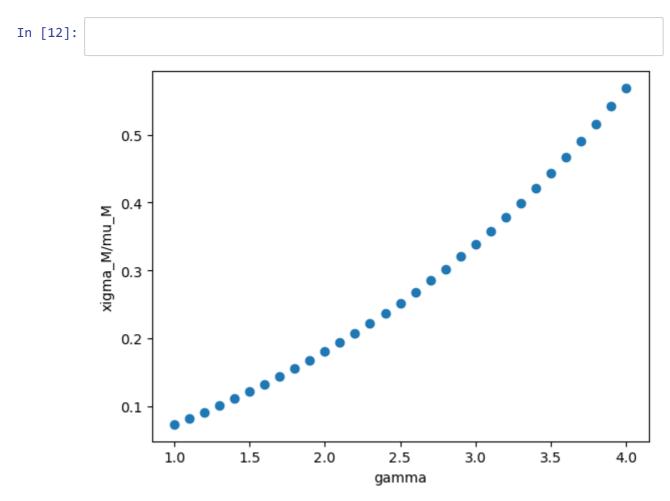
## Calculate $\mu M$ and $\sigma M$ for each value of $\gamma$ , and plot $\sigma M/\mu M$ (on the vertical axis) vs $\gamma$ (on the horizontal axis).



## Find the smallest value of $\gamma$ (in your data) for which $\sigma M/\mu M > 0.4$ .

In [13]:
Out[13]: 3.4

Explain (in words, without using any mathematical equations or formulas) the economic significance of this result.

- $1.\gamma$  comes from the power Utility functions we choose. In power Utility, investors will have a Constant Relative Risk Aversion. Higher  $\gamma$  means higher relative risk aversion, signifying that investors are more concerned about risk rather than potential returns.
- 2. $\sigma$ M/ $\mu$ M is the volatility ratio that measures the relative volatility of the pricing kernel in a Consumption Capital Asset Pricing Model (CAPM). The pricing kernel in Consumption CAPM reflects how investors discount future consumption. It quantifies the relative preferences of investors for consumption today versus consumption in the future and how much compensation they require for bearing the uncertainty (volatility) associated with their preferences for future consumption. A higher volatility ratio implies that investors demand a higher risk premium to exchange consumption today for uncertain consumption in the future.
- 3. The volatility ratio of the pricing kernel should have a lower bound determined by the highest Sharpe ratio among all possible portfolios. However, the pricing kernel has a lower limit of zero but no upper limit, indicating a significant rightward skew in its distribution.
- 4.Based on historical U.S. market data, the stock market has a Sharpe ratio of around 0.4. Therefore, the volatility ratio of the pricing kernel is at least 40%, indicating high volatility in the pricing kernel. As a preliminary estimation, a  $\gamma$  value greater than 20 is unreasonably high, leading to a 'L'-shaped power utility function.
- 5. Given this skewness characteristic, it is reasonable to assume that the investor's optimal consumption growth follows a lognormal distribution. After introducing the Rare Disasters component,  $\gamma$  is simulated to be in the range of 3.3-3.4, which aligns more closely with real-world scenarios.