Behavioural Finance

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Barberis, Huang and Santos (2001)

- Quasi-behavioral model with fully rational investors and non-standard preferences
- Preferences incorporate concepts from **prospect theory** of Kahneman and Tversky (1979):
  1. Gains and losses are measured relative to **reference level**
  2. Investors exhibit **loss aversion**: more sensitive to losses than gains (of same magnitude)
- Preferences can also incorporate **house money** effect: investors become more willing to gamble using prior gains, and less willing to gamble after prior losses
- Incorporate preferences into endowment economy to solve numerically for equilibrium prices and expected returns

Economic Environment

- Riskless bond provides (constant) risk-free rate of R_f
- Risky stock represents equity claim on perishable output, and provides (random) return of R_{t+1} over next time period
- In equilibrium, aggregate consumption and dividend growth both evolve as i.i.d. random walk with drift:

$$\ln\left(\frac{D_{t+1}}{D_t}\right) = \ln\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right) = \mu + \sigma\epsilon_{t+1}$$

- Here $\epsilon_t \sim N(0,1)$ represents effect of normal economic fluctuations on dividend and consumption growth
- Interpret investment in stock as investment in market portfolio

Investor Preferences

- Infinitely-lived investor receives time-separable utility from individual consumption as well as recent financial gain or loss:

$$E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_t \nu(X_{t+1})\right)\right]$$

- Here $\delta = e^{-\rho} \in (0,1)$ is subjective discount factor and $\gamma > 0$ is coefficient of relative risk aversion for consumption shocks
- Also X_t represents financial gain or loss, and $\nu(X_t)$ represents gain or loss of utility from financial gain or loss
- Then b_t is scale factor so that utility of consumption remains comparable in magnitude to utility of financial gain or loss

Prospect Theory – Part 1

- Let w_t be (dollar) value of investment in stock at time t, and assume that investor keeps track of year-to-year fluctuations
- Recent financial gain or loss is measured relative to reference level based on risk-free rate:

$$X_{t+1} = w_t\,(R_{t+1} - R_f)$$

- Loss aversion makes investor more sensitive to shortfall in financial gain (or outright financial loss), so $\lambda > 1$:

$$\nu(X_{t+1}) = \begin{cases} X_{t+1}, & X_{t+1} \geq 0 \\ \lambda X_{t+1}, & X_{t+1} < 0 \end{cases}$$

Prospect Theory – Part 2

- Utility from financial gain or loss is piecewise-linear, so define scale-invariant utility function for financial gain or loss:

$$\nu(X_{t+1}) = \nu(w_t\,(R_{t+1} - R_f)) = w_t\widehat{\nu}(R_{t+1})\,,$$
$$\widehat{\nu}(R_{t+1}) = \begin{cases} R_{t+1} - R_f\,, & R_{t+1} \geq R_f \\ \lambda\,(R_{t+1} - R_f)\,, & R_{t+1} < R_f \end{cases}$$

- Use marginal utility of aggregate consumption as scale factor:

$$b_t = b_0 \overline{C}_t^{-\gamma}$$

- Here $b_0 \geq 0$ determines amount of emphasis investor puts on utility from financial gain or loss, vs utility of consumption

Optimisation Problem

- Investor's consumption and asset allocation problem:

$$\max_{\{C_t, w_t\}} E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_0 \overline{C}_t^{-\gamma} w_t \widehat{\nu}(R_{t+1})\right)\right]$$

- Subject to investor's intertemporal budget constraint:

$$W_{t+1} = (W_t - C_t)\, R_f + w_t\,(R_{t+1} - R_f)$$

- Assume complete market, so there is representative investor who consumes real (per capita) aggregate consumption
- Solve for optimal choices using dynamic programming

Optimal Consumption

- First-order condition for optimal individual consumption:

$$\delta R_f E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} \right] = 1$$

- Optimality condition must also apply to representative investor (who consumes aggregate consumption), so use distribution of aggregate consumption growth to find risk-free rate:

$$\delta R_f E_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right] = 1 \quad \Longrightarrow \quad R_f = e^{\rho + \gamma\mu - \frac{1}{2}\gamma^2\sigma^2}$$

Optimal Asset Allocation

- First-order condition for optimal asset allocation:

$$\delta b_0 \left(\frac{\overline{C}_t}{C_t^*}\right)^{-\gamma} E_t[\widehat{\nu}(R_{t+1})] + \delta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} R_{t+1}\right] = 1$$

- Optimality condition must apply to representative investor (who consumes aggregate consumption):

$$\delta b_0 E_t[\widehat{\nu}(R_{t+1})] + \delta E_t\left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{t+1}\right] = 1$$

- In equilibrium, stock return must satisfy optimality condition

Stock Return – Part 1

- Assume that stock has constant price-dividend ratio, so that stock return has i.i.d. distribution:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \kappa \frac{D_{t+1}}{D_t} = \kappa \frac{\overline{C}_{t+1}}{\overline{C}_t}$$

- Here $\kappa = (1 + f)/f$, where f is constant price-dividend ratio
- Substitute into optimality condition for representative investor:

$$\delta b_0 E_t \left[\widehat{\nu} \left(\kappa \frac{\overline{C}_{t+1}}{\overline{C}_t} \right) \right] + \delta \kappa E_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{1-\gamma} \right] = 1$$

Stock Return – Part 2

- Now use distribution of aggregate consumption growth to get equilibrium condition for price-dividend ratio of stock:

$$\delta b_0 E_t\left[\widehat{\nu}\left(\kappa e^{\mu+\sigma\epsilon_{t+1}}\right)\right] + \delta\kappa E_t\left[e^{(1-\gamma)(\mu+\sigma\epsilon_{t+1})}\right] = 1$$
$$\implies \delta b_0 E_t\left[\widehat{\nu}\left(\kappa e^{\mu+\sigma\epsilon_{t+1}}\right)\right] + \delta\kappa e^{(1-\gamma)\mu+\frac{1}{2}(1-\gamma)^2\sigma^2} = 1$$

- No analytical solution, but can use numerical approach to find equilibrium price-dividend ratio of stock
- Use price-dividend ratio of stock and distribution of aggregate consumption growth to find expected stock return:

$$E_t[R_{t+1}] = \kappa E_t\left(e^{\mu+\sigma\epsilon_{t+1}}\right) = \kappa e^{\mu+\frac{1}{2}\sigma^2}$$

Empirical Results

- Set $\lambda = 2.25$, based on Tversky and Kahneman (1992)
- Set $\mu = 1.84\%$ and $\sigma = 3.79\%$ based on data from 1889–1995
- Set $\gamma = 0.9$ and $\delta = 0.98$, so $R_f \approx 3.5\%$ per year
- Equity premium will increase with b_0, as investor puts more emphasis on utility from financial gain or loss
- Annual equity premium is 0.06% for $b_0 = 0$ and 0.91% for $b_0 = 2$, and converges to 1.2% as $b_0 \to \infty$
- Annual equity premium of around 7% for U.S. stock market, so result is too small for reasonable levels of loss aversion

House Money Effect – Part 1

- Thaler and Johnson (1990) finds "house money" effect: investors are more willing to gamble using prior gains $\implies$ prior outcomes affect risk aversion and loss aversion
- Loss (of given magnitude) is less painful after prior gains and more painful after prior losses
- Hence subjects are more willing to gamble after prior gains and less willing to gamble after prior losses
- Prior financial gains provide "buffer" or "cushion" that reduces sensitivity to subsequent financial losses
- Conversely, prior financial losses make investors even more sensitive to subsequent financial losses

House Money Effect – Part 2

- In Barberis, Huang and Santos (2001), investor keeps track of **benchmark level** for value of investment in risky stock
- Benchmark level evolves over time, to reflect investor's updated view of accumulated financial gain or loss
- Investor becomes more loss averse with prior losses (i.e., if value of investment in risky stock falls short of benchmark) and less loss averse with prior gains
- Price-dividend ratio of market portfolio will be function of representative investor's (time-varying) benchmark level
- Equity premium of 4.1% per year for $b_0 = 2$, when λ rises by one for every 2% shortfall in value of investment in risky stock

References

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