Linear Factor Models
Performance Measurement

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August 18, 2022

Outline

1. Multi-Factor Models

2. Performance Measurement

Multi-Factor Pricing Model

- Multi-factor pricing model has two or more risk factors, each of which represents effect of different type of systematic risk
- Each risk factor comes with factor sensitivity (or factor loading) to show degree of exposure to that risk factor
- Often use factor-mimicking portfolios as risk factors, such as (excess) return on market portfolio for market risk
- Ideally, factor-mimicking portfolio should have no sensitivity to other risk factors, but difficult to achieve in practice
- Factor-mimicking portfolios are usually "long-short" portfolios, to reduce correlation amongst risk factors
- Multi-factor models can also include industry-specific, sector-specific, or country-specific risk factors

Fama–French 3-Factor Model – Part 1

- Eugene Fama and Kenneth French use three-factor model with risk factors for market risk, size risk, and value risk:

$$\tilde{R}_i - R_f = \alpha_i + \beta_i \left(\tilde{R}_m - R_f\right) + \gamma_i \left(\tilde{R}_s - \tilde{R}_b\right) + \delta_i \left(\tilde{R}_h - \tilde{R}_l\right) + \tilde{\epsilon}_i$$

- Risk factor for size risk ("SMB") is return on portfolio that is long on small-cap stocks and short on big-cap stocks
- Risk factor for value risk ("HML") is return on portfolio that is long on value stocks and short on growth stocks
- Provides motivation for Morningstar's "style box"

Fama–French 3-Factor Model – Part 2

- Use as linear regression model to determine exposure to market risk, size risk and value risk, for any asset
- Positive (or negative) coefficient for SMB means that portfolio behaves like small-cap (or big-cap) stock
- Positive (or negative) coefficient for HML means that portfolio behaves like value (or growth) stock
- If model is correct, then intercept coefficient should be zero for any individual asset or passive portfolio
- Hence interpret intercept coefficient as pricing error, relative to F–F three-factor model

Momentum Effect

- "Momentum effect" in short-term stock returns, over three- to six-month investment horizon
- Past winners will usually continue to outperform, while past losers will usually continue to underperform
- Fund managers often load up on momentum effect by buying past winners and selling past losers, either as deliberate investment strategy, or by coincidence
- "Reversal effect" (or "mean-reversion") in long-term stock returns, over three- to five-year investment horizon
- Past winners will usually underperform, while past losers will usually outperform

Carhart 4-Factor Model

- Mark Carhart augmented Fama–French three-factor model with additional risk factor for momentum effect:

$$\begin{aligned}\tilde{R}_i - R_f = \alpha_i + \beta_i \left(\tilde{R}_m - R_f\right) + \gamma_i \left(\tilde{R}_s - \tilde{R}_b\right) + \\ \delta_i \left(\tilde{R}_h - \tilde{R}_l\right) + \zeta_i \left(\tilde{R}_u - \tilde{R}_d\right) + \tilde{\epsilon}_i\end{aligned}$$

- Risk factor for momentum effect ("UMD") is return on portfolio that is long on past winners and short on past losers
- Often used to analyse performance and risk characteristics of actively-managed portfolios and mutual funds

Downside Beta

- Extension of market model to allow for different betas on upside and downside:

$$\tilde{R}_i - R_f = \alpha_i + \beta_i \left(\tilde{R}_m - R_f\right) + \beta_i^* \left(\tilde{R}_m - R_f\right) 1_{\left\{\tilde{R}_m \le R_f\right\}} + \tilde{\epsilon}_i$$

- Here $1_{\left\{\tilde{R}_m \le R_f\right\}}$ is "indicator variable" that has value of one when $\tilde{R}_m \le R_f$, and zero otherwise
- Hence beta of β_i on upside and beta of $\beta_i + \beta_i^*$ on downside
- If $\beta_i^* \ge 0$, then investor is more sensitive to downside risk
- Alternatively, if $\beta_i^* \ne 0$ for actively-managed portfolio, then interpret as (negative or positive) market-timing ability

Outline

Sharpe Ratio

- Sharpe ratio is risk premium per unit of standard deviation:

$$S_i = \frac{E\left(\tilde{R}_i - R_f\right)}{\sqrt{\mathrm{Var}\left(\tilde{R}_i - R_f\right)}}$$

- Denominator is designed to capture total risk, i.e., both systematic and idiosyncratic risk
- Hence not appropriate for comparing performance of individual investment to diversified portfolio
- Moreover, denominator ignores higher moments such as skewness and kurtosis, so may not fully reflect risk of investment if return distribution is not normal

Treynor Ratio

- Treynor ratio is risk premium per unit of market risk:

$$T_i = \frac{E\left(\tilde{R}_i - R_f\right)}{\beta_i}$$

- Denominator is designed to capture systematic (market) risk and ignore idiosyncratic risk
- In principle, can be used to compare performance of individual investment to diversified portfolio
- In practice, will fail to account for other types of systematic risk besides market risk

Jensen's Alpha

- Jensen's alpha is intercept coefficient from market model regression using excess returns:

$$\alpha_i = E\left(\tilde{R}_i - R_f\right) - \beta_i E\left(\tilde{R}_m - R_f\right)$$

- For passive portfolio, represents pricing error relative to CAPM
- For active portfolio, represents abnormal mean return after adjusting for exposure to market risk, due to fund manager's ability to identify underpriced or overpriced assets
- Can extend to multiple sources of systematic risk by using intercept coefficient from multi-factor pricing model

Information Ratio

- Information ratio (or appraisal ratio) is expected deviation from target (or benchmark) return, per unit of tracking error:

$$I_i = \frac{E\left(\tilde{R}_i - \tilde{R}_t\right)}{\sqrt{\mathrm{Var}\left(\tilde{R}_i - \tilde{R}_t\right)}}$$

- Measures ability of fund manager to exceed target return, relative to amount of tracking error
- Often used to evaluate fund managers who follow "enhanced indexing" strategy by overweighting or underweighting selected components of stock index

Downside Risk

- Downside risk of investment is risk that realised return on investment will fall below target return
- Below-target semi-variance is measure of downside risk:

$$SV\left(\tilde{R}_i; \tilde{R}_t\right) = E\left[\min\left\{\tilde{R}_i - \tilde{R}_t,\ \ 0\right\}^2\right]$$

- Below-target semi-deviation is square root of below-target semi-variance
- Can distinguish between asymmetric return distributions with same variance but different skewness

Sortino Ratio

- Sortino ratio is expected deviation from target (or benchmark) return, per unit of below-target semi-deviation:

$$St_i = \frac{E\left(\tilde{R}_i - \tilde{R}_t\right)}{\sqrt{\mathrm{SV}\left(\tilde{R}_i; \tilde{R}_t\right)}}$$

- Will produce rankings similar to information ratio when return distribution is close to symmetric, and expected asset return is close to expected target return
- More informative than information ratio or Sharpe ratio when return distribution is not normal