QF602 Homework

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Assignment 3: Due 2/17/24

Problem 1.

1.1.
$C_0 = e^{-rT}(F_0(T)\Phi(d_1) - K\Phi(d_2))$,where $d_1 = \frac{ln(F_0(T)/K)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$
According to put-call parity $P = C - e^{-rT}(F_0(T) - K)$
When $\sigma \to \infty$,

$C = S_0 e^{-qT}, P = e^{-rT}K$

1.2.
When $\sigma \to 0$, then there is no volatile in underlying price.
Which means $C = e^{-rT}(F_0(T) - K)^+, P = e^{-rT}(K - F_0(T))^+$

1.3.
According to arbitrage theory, easily we can induce that $0 \leq call \leq S_0 e^{-qT}$ since it's a non-dividend paying stock, $0 \leq call \leq S_0$.

Same logic to put option, $0 \leq put \leq Ke^{-rT}$.

Problem 2.

Since Vega equal $e^{-qT}S_0\phi(d_1)\sqrt{T}$, the only variable related to K is d_1, $max(\phi(d_1)) \to d_1 = 0$.

As $d_1 = \frac{ln(F_0(T)/K)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$,let $d_1 = 0 \to ln(F_0(T)/K) = -\frac{1}{2}\sigma^2 T \to$

$K = S_0 e^{(r-q)T+\frac{1}{2}\sigma^2 T}$

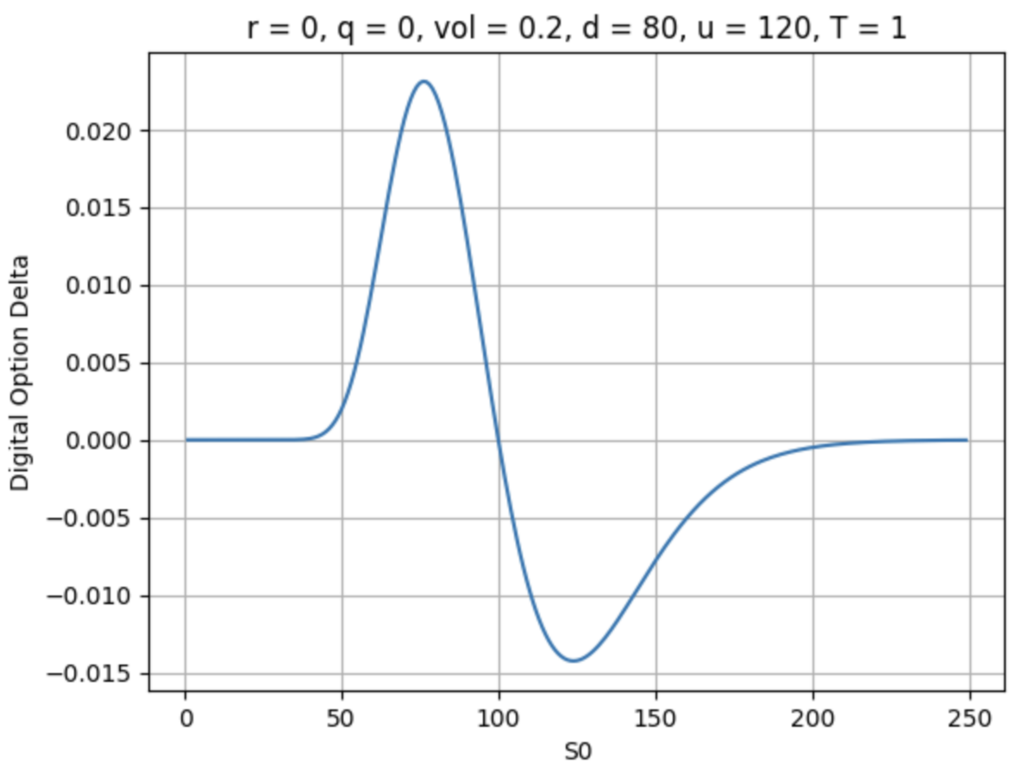
Problem 3.

1.1.
The logic is simple when showing on graph, basically just long call spread at L where spread is small enough, and short long call spread at U where spread is small enough.

$$S_T = S_0 e^{(r-q-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}x}$$
$$V_0 = e^{-rT}\int_{-\infty}^{\infty} 1_{d<S_T<u}\phi(x)dx$$
$$= e^{-rT}[\Phi(x_2)-\Phi(x_1)]$$
$$where\ x1 = \frac{ln\frac{d}{S_0}-(r-q-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, x2 = \frac{ln\frac{u}{S_0}-(r-q-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

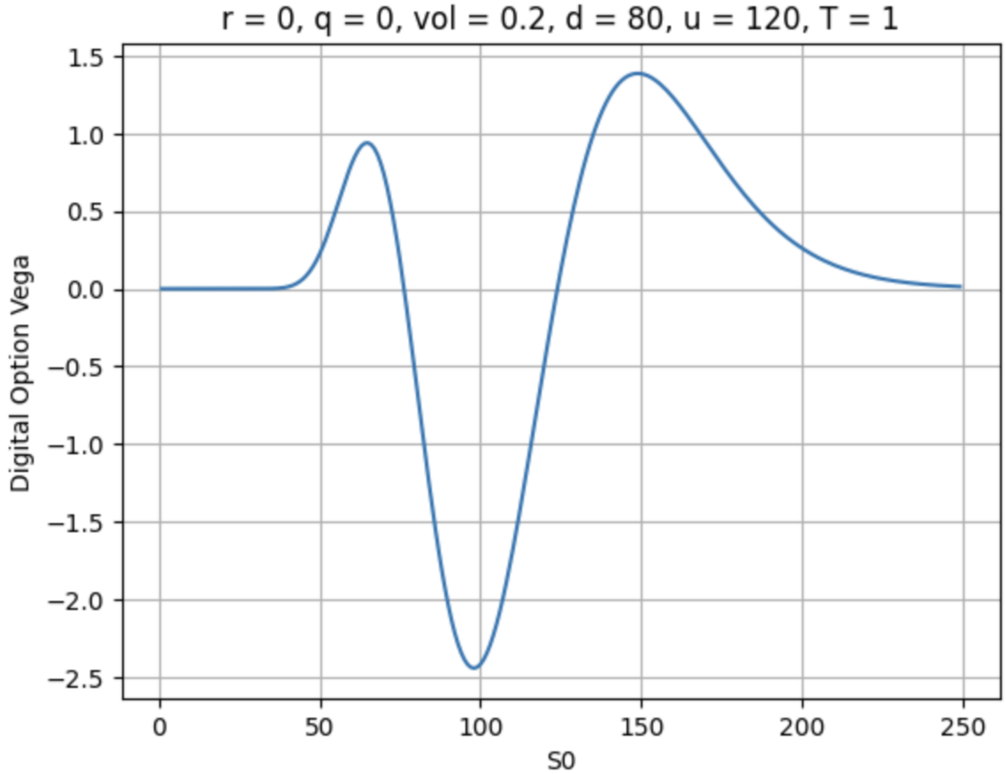
1.2.

$$\frac{\partial V}{\partial S_0} = e^{-rT}[\phi(x_1)\frac{1}{S_0\sigma\sqrt{T}}-\phi(x_2)\frac{1}{S_0\sigma\sqrt{T}}]$$



1.3.

$$\frac{\partial V}{\partial \sigma} = e^{-rT}\sqrt{T}(\phi(x_2) - \phi(x_1))(\frac{1}{2} + \frac{r-q}{\sigma^2}) + e^{-rT}(\phi(x_1)\frac{ln\frac{d}{S_0}}{\sigma^2\sqrt{T}} - \phi(x_2)\frac{ln\frac{u}{S_0}}{\sigma^2\sqrt{T}})$$



Problem 4.

$$C - P = e^{-rT}(F_0(T) - K)$$

1.1.

$$\frac{\partial Put_0}{\partial S_0} = x - e^{-qT}$$

1.2.
Put-call parity implies that Vega for European call and put options are the same.