QF602 Homework

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Assignment 2: Due 1/27/24

Problem 1.

When it comes to construct a derivative security the key point is "replicate".

The payoff is when Tesla first hits \$1000, pays \$100.

The answer should be straightforward, \$1000*0.1 = \$100, so it cost \$750*0.1 = \$75 to construct such derivative.

Problem 2.

For x:

We first use the butterfly constraint to find the range of x:

$$5 - 2x + 10 \ge 0 \rightarrow x \le 7.5$$

We can further narrow the range by using calendar spread. The put with strike 100 and 2y maturity is 9, this yields $x \le 9$

Finally, the put with strike 90(110) cannot worth more(less) than the put with strike 100 with the same maturity, this yields $5 \le x \le 10$

Combine them together, we get $5 \le x \le 7.5$

For y:

We first use the butterfly constraint to find the range of y:

$$y + 12 - 18 \ge 0 \rightarrow y \le 6$$

We can further narrow the range by using calendar spread. The put with strike 90 and 1y

maturity is 5, this yields $y \geq 5$

Finally, the put with strike 90 cannot worth more than the put with strike 100 with the same maturity, this yields $y \leq 9$

Combine them together, we get $6 \le y \le 9$

Problem 3.

Yes, there is calendar spread arbitrage opportunity. Call option price with strike 100 and maturity 1y is 11.923538474048499. Whereas, call option price with same strike and maturity 2y is 11.246291601828489.

In order to dismiss arbitrage, Call option with longer maturity should be at least has same price (i.e.11.923538474048499) Then $\sum(2) = 0.21213203435596242$.