

## QF602 - Homework 6

### Question

- Let's assume  $r = q = 0$ ,  $S_0 = 1$  and the implied volatility for  $T = 4$  are given by the following formula:

$$\Sigma(K) = 0.510 - 0.591K + 0.376K^2 - 0.105K^3 + 0.011K^4 \quad (1)$$

with an upper limit, which is given by  $\Sigma(K) = \Sigma(3)$  for  $K > 3$ .

- Any payoff that only depends on  $S_T$  can be priced with the following formula,

$$\begin{aligned} V_0 = & e^{-rT} V_T(F_0(T)) + \int_0^{F_0(T)} Put(K, T) \frac{\partial^2 V_T(K)}{\partial K^2} dK \\ & + \int_{F_0(T)}^{\infty} Call(K, T) \frac{\partial^2 V_T(K)}{\partial K^2} dK \end{aligned}$$

where  $Call(K, T)$  and  $Put(K, T)$  is computed by using the Black Scholes formula using the volatility  $\Sigma(K)$  obtained in (1).

- Using the B-L formula, compute numerically the option prices at time 0, for the following payoffs:
  - $V_T(S_T) = \sqrt{S_T}$
  - $V_T(S_T) = S_T^3$

You need to submit a Jupyter Note Book which contains executable Python code. You can modify the Python code for static replication for the square payoff in the lecture note 6.