QF602 - Homework 4

Question 1

- Assume $r = 0$, $q^X = q^Y = q^Z = 0$; X, Y and Z are driven by the following dynamics in the risk neutral measure:

  $$\frac{dX_t}{X_t} = \sigma^X dW_t^X, \ \ \frac{dY_t}{Y_t} = \sigma^Y dW_t^Y, \ \ \frac{dZ_t}{Z_t} = \sigma^Z dW_t^Z$$

  where $E[dW_t^X dW_t^Y] = \rho dt, E[dW_t^X dW_t^Z] = 0, E[dW_t^Y dW_t^Z] = 0.$

  Derive the pricing formula for the payoff which pays

  $$\max(X_T/Y_T - Z_T, 0)$$

  at time T.

Question 2

- Assume X and Y are driven by the following dynamics in the risk neutral measure:

  $$\frac{dX_t}{X_t} = (r - q^X)dt + \sigma^X dW_t^X$$
  $$\frac{dY_t}{Y_t} = (r - q^Y)dt + \sigma^Y dW_t^Y$$

  where $E[dW_t^X dW_t^Y] = \rho dt$.

  Derive the pricing formula for the payoff which pays

  $$\max(X_T, Y_T)$$

  at time T.

Question 3

- Using the same setup as Question 2. Derive the pricing formula for the payoff which pays

  $$\min(X_T, Y_T)$$

  at time T.

Question 4

- Assume you are a SGD investor. Let S be a SG stock, Y be a US stock, X be the USDSGD FX rate. The dynamics of the processes are given as:

$$\frac{dS_t}{S_t} = (r^d - q^S)dt + \sigma^S dW_t^S,$$

$$\frac{dY_t}{Y_t} = (r^f - q^Y)dt + \sigma^Y dW_t^Y,$$

$$\frac{dX_t}{X_t} = (r^d - r^f)dt + \sigma^Z dW_t^Z$$

- where dW^S and dW^X are BMs under the SGD risk neutral measure.
- dW^Y is a BM under the USD risk neutral measure.
- The correlations of the BMs are

$$E[dW_t^S dW_t^Y] = \rho_{SY} dt$$

$$E[dW_t^S dW_t^X] = \rho_{SX} dt$$

$$E[dW_t^Y dW_t^X] = \rho_{YX} dt$$

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Derive the pricing formula for the payoff which pays

$$\max(S_T/S_0 - Y_T/Y_0, 0)$$

at time T from your perspective.