QF602 - Homework 7

Question

- **Part A**. Consider the event that exactly one asset jump arrives in a interval $[0, T]$. Determine the probability of this event happening. The arrival intensity is λ.

- **Part B**. This event is the same as the event that there is exactly one asset jump in the interval $[0, \frac{T}{2})$ and no jumps in the interval $[\frac{T}{2}, T]$, or no jumps in $[0, \frac{T}{2})$ and exactly one in $[\frac{T}{2}, T]$. Confirm that this calculation gives rise to exactly the same probability.

① $P(K=1) = \dfrac{(\lambda T)^K e^{-\lambda T}}{K!} = \lambda T e^{-\lambda T}$

② $P($0 first half 1 second$) = \dfrac{(\lambda \frac{T}{2})^0 e^{-\lambda \frac{T}{2}}}{0!} \dfrac{(\lambda \frac{T}{2})^1 e^{-\lambda \frac{T}{2}}}{1!} = \dfrac{\lambda T}{2} e^{-\lambda T}$

③ $P($1 first half 0 second$) = \dfrac{(\frac{\lambda T}{2})^1 e^{-\lambda \frac{T}{2}}}{1!} \dfrac{(\lambda \frac{T}{2})^0 e^{-\lambda \frac{T}{2}}}{0!} = \dfrac{\lambda T}{2} e^{-\lambda T}$

① = ② + ③