QF602 Homework

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Assignment 1: Due 1/19/24

Problem 1.

(1)

$$100e^{1.5r_f} = 105$$

$$r_f = 3.25\%$$

(2)

$$100e^{1.5(r_f - 2\%)} = 105$$

$$r = 5.25\%$$

Problem 2.

$$105e^{r_{f12}} = 109$$

$$r_{f12} = 3.739\%$$

Problem 3.

(1)

$$100e^{2\% - 1\%} = 101.005 < 105$$

There is an arbitrage opportunity:
1.Borrow 100 at T = 0
2.buy with spot price
3.short forward contract
holding for one year
4.deliver to gain 105
5.pay interest with 101.005
6.earn 3.995 at T = 1

(2)

$$101.005$$

Problem 4.

$$F_0(1) = 100 \times 100e^{2\%} = 10202.0134$$

(1)

$$F_{\frac{1}{12}}(1) = 100 \times 120e^{\frac{11}{12}\times 2\%} = 12222.029$$

$$PNL = 2020.016e^{-\frac{11}{12}\times 2\%} = 1983.32$$

(2)

$$F_{\frac{1}{12}}(1) = 100 \times 120e^{\frac{11}{12}\times (2\%-5\%)} = 11674.496$$

$$PNL = 1472.483e^{-\frac{11}{12}\times 2\%} = 1445.73$$

Problem 5.

(1)
Exercise early requires paying the strike price early, hence loses the time value of money because if not exercise that money can generate interest till the expiry date on this cash amount.

On the other hand, exercise early holding the stock will generate cash flow too by receiving future dividends.

If dividend yield is higher than the interest rate then it may be optimal to exercise.

So the thumb of rule is to check if there is future dividend, if there is to compare the interest rate and the dividend yield, if the dividend yield is higher, it maybe optimal to exercise.

(2)
If the underlying that pays no dividend, then don't exercise early as exercise early require paying the strike price early, it will loses the time value of money.

(3)
Without dividends, it can be optimal to exercise an American put early.

For example:
An American put with K = 100 on a stock with S_t very close to 0.

S_t cannot go any lower than 0, and the put option pays $max(K - S_t, 0) = 100$ is the maximum that one can earn for holding this put option.

Exercise now, the option holder gets 100 today, with time value of money compared to exercise later.

Problem 6.

(1)
The main reason is that a margin account earns interest.

However, if and only if the risk-free interest vary, future price and forward price will be different.

Especially true, when risk-free interest positively(negatively) correlate with future price, as the long(short) party tends to benefit at expense of the short party.

To compensate, the futures price will be higher(lower) than the corresponding forward price.

(2)
Risk-free interest negatively correlate with future price, and margin account earns interest.