QF602 - Homework 3

Question 1

- What happens to the price of a vanilla call option as the volatility tends to infinity? How about put option?

  Answer. The Black Scholes Call option formula is:

  $$Z_0(T)(F_0(T)\Phi(d_1) - K\Phi(d_2))$$

  where $d_1 = \frac{\ln(F_0(T)/K)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, $d_2 = \frac{\ln(F_0(T)/K)-\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$. As $\sigma \to \infty$, $d_1 \to \infty$ and $d_2 \to -\infty$. This yields $\Phi(d_1) \to 1$ and $\Phi(d_2) \to 0$. Then the call price will tends to

  $$Z_0(T)F_0(T) = S_0 e^{-qT}$$

  By put call parity,

  $$Call - Put = Z_0(T)(F_0(T) - K)$$

  As $Call \to Z_0(T)F_0(T)$, the put option price tends to

  $$KZ_0(T)$$

- What happens to the price of a vanilla call option as the volatility tends to 0? How about put option?

  Answer. Volatility tends to 0 means the underlying is deterministic. It will move according to r and q. The call option will tend to

  $$Z_0(T)(F_0(T) - K)^+$$

  and put option will be

  $$Z_0(T)(K - F_0(T))^+$$

- What are the upper and lower bounds of the price of a call and put option on a non-dividend paying stock?

Answer. For call option, the bounds are

$$0 \le Call \le S_0 e^{-qT}$$

One can consider a call option is a delay purchase of stock. However, if the option is not exercised, the holder won't receive any dividend paid. For put option, the bound are

$$0 \le Put \le KZ_0(T)$$

One can consider a put option is a delay sale of stock. The maximum one can receive from exercising a put is K. However, this is a future cash flow and hence is discounted.

Question 2

- Black Scholes Vega is given as $e^{-qT} S_0 \phi(d_1)\sqrt{T}$, can you find the strike that gives the maximum vega for a given maturity T?

  Answer. Black Scholes vega is maximized when $d_1 = 0$, this yields

  $$K = F_0(T) e^{\frac{1}{2}\sigma^2 T}$$

Question 3

- Consider a digital option with a payoff at maturity $T = 1$

$$1_{L<S_T<U}$$

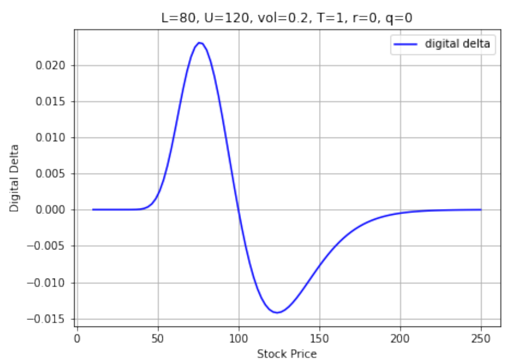
  where $L = 80$ and $U = 120$ are the lower and upper barriers.

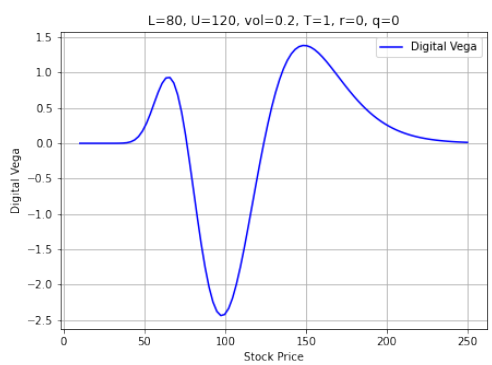
- Explain how to replicate the digital option using European options.

  Answer. The digital option can be replicated using long a call spread with strikes $L - \epsilon$ and $L + \epsilon$, and short a call spread with strikes $U - \epsilon$ and $U + \epsilon$.

- Draw the Black Scholes delta profile of the digital option. Assume the implied vol is 0.2, risk free rate and dividend yield are 0.

- Draw the Black Scholes vega profile of the digital option.





Question 4

- If the delta of a call with maturity T and strike K is x, what is the delta of a put with the same maturity and strike?

  Answer. By put call parity:

  $$Call - Put = Z_0(T)(F_0(T) - K)$$

  Differentiate the equation w.r.t S_0, we get

  $$CallDelta - PutDelta = e^{-qT}$$

  If $CallDelta = x$ then $PutDelta = x - e^{-qT}$

- If the vega of a call with maturity T and strike K is y, what is the vega of put with the same maturity and strike?

  Answer. The vega of a call is the same as the vega of a put.