QF 604
ECONOMETRICS OF FINANCIAL MARKETS

LECTURE 5

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SMU Master of Science in
Quantitative Finance 2023-2024 T2

LECTURE OUTLINE

01 Roles of Asset Pricing Models

- Expected Return of a stock = linear function of observable systematic risk (expected risk premium)

  Can construct profitable portfolios with high positive expected returns using long (short) high positive beta stocks when the factor is anticipated (expected) to be highly positive (negative)

- Even if next period factors cannot be anticipated, knowing the factors enables more accurate estimation of stock return covariance matrix that can help in portfolio risk-return optimization

- An asset pricing model provides a benchmark expected return for assessing if an asset is fetching superior (abnormally positive) returns, i.e., $\hat{\alpha} > 0$ or otherwise

- $\hat{\alpha}$ in event studies possibly also reveal presence of private information when $\hat{\alpha}$ changes significantly just prior to a public announcement

01 Arbitrage Pricing Theory (APT)

- Assume asset returns R_i's are generated by a K-factor model ($K < N$ where N is the total number of assets in the economy):

$$R_i = E(R_i) + \sum_{j=1}^{K} b_{ij}\delta_j + \varepsilon_i, \quad i = 1, 2, \ldots, N, \tag{7.1}$$

where $E(R_i) = E_i$; δ_j's are zero mean common risk factors (i.e. they affect asset i's return R_i via b_{ij}'s); b_{ij}'s are the factor loadings (or sensitivity coefficients to factors) for asset i, and ε_i is mean zero asset i's specific or unique risk.

- Also,

$$\begin{aligned} \mathrm{cov}(\varepsilon_i, \varepsilon_j) &= 0 \quad \text{for } i \neq j \\ \mathrm{cov}(\varepsilon_i, \delta_j) &= 0 \quad \text{for every } i, j. \end{aligned}$$

- Using matrix notations for Eq. (7.1):

$$\begin{aligned} R_{N\times 1} &= E_{N\times 1} + B_{N\times K}\delta_{K\times 1} + \varepsilon_{N\times 1} \\ E(\varepsilon\varepsilon^T) &= \sigma_\varepsilon^2 I_{N\times N} \\ E(\delta\varepsilon^T) &= 0_{K\times N}. \end{aligned}$$

01 Arbitrage Pricing Theory

- Suppose we can find a portfolio $x_{N\times 1}$ where the element are weights or fractions of investment outlay, such that

$$x^T l = 0; \quad l = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N\times 1} \tag{7.2}$$

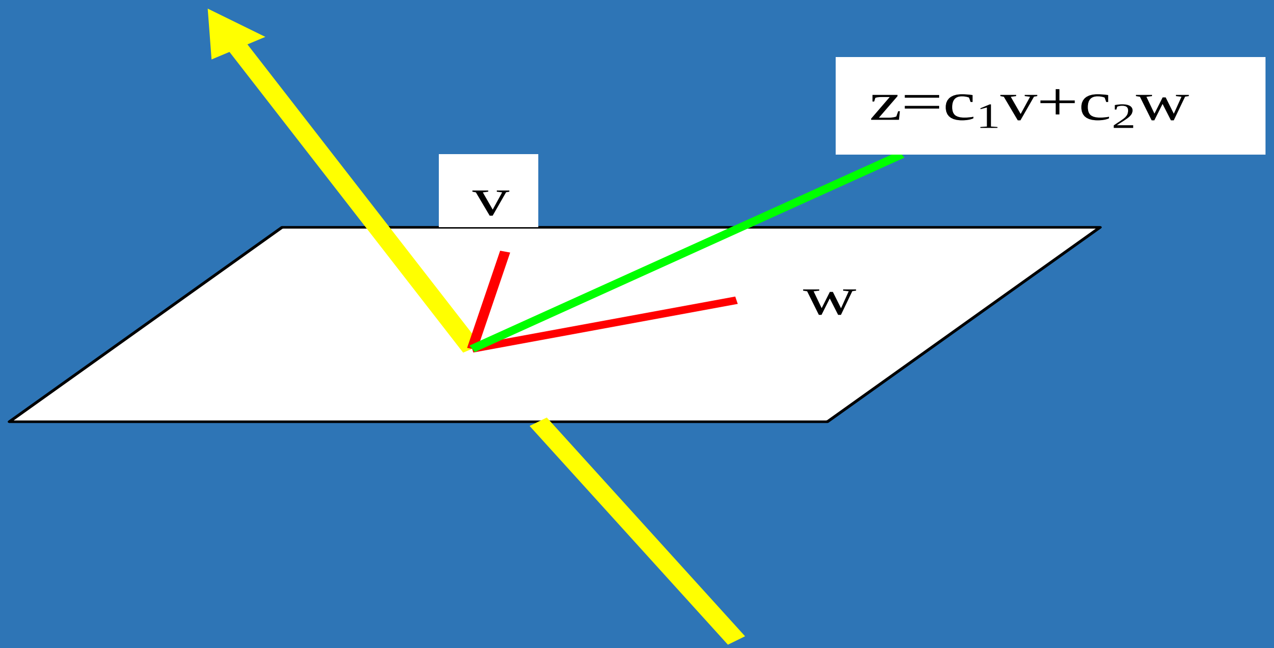
$$x^T B = 0_{1\times K}. \tag{7.3}$$

- Eq. (7.2) implies that x is an arbitrage portfolio, i.e., zero outlay, with zero systematic risk via Eq. (7.3).
- Suppose that x is a well-diversified portfolio, so $x^T \varepsilon \approx 0_{1\times 1}$. Hence, portfolio return is:

$$x^T R = x^T E + x^T B\delta + x^T \varepsilon \dot{\approx} \; x^T E \;\; (= x^T E \text{ as } N \uparrow \infty).$$

But since x is costless and riskless, then to prevent arbitrage profit, the return to x must be zero, i.e., $x^T E = 0$. (7.4)

01 Arbitrage Pricing Theory



Suppose vector u is orthogonal to v and also to w, then clearly u is orthogonal to any vector z that lies on the plane <u>spanned</u> by v and w, i.e. a linear combination of v and w.

01 Arbitrage Pricing Theory

- Since Eqs. (7.2) and (7.3) economically imply Eq. (7.4) always, then

$$E_{N\times 1} = \gamma_0 \underset{N\times 1}{l} + \underset{N\times K}{B} \gamma, \tag{7.5}$$

where γ_0 is a scalar constant and γ is a $K \times 1$ constant vector.

- Equation (7.5) is sometimes called the Arbitrage Pricing Model. Ex-ante unconditional expected return, E, is related to economy-wide constant γ_0 and to risk premia $\gamma_{K\times 1}$ for each of the K factor risks. If we put $B = 0$, then clearly γ_0 is the risk-free rate. Note that if B increases, then the systematic risks $B\gamma$ increase, and thus E also increases.
- For a single asset i, Eq. (7.5) implies:

$$E(R_i) = r_f + b_{i1}\gamma_1 + b_{i2}\gamma_2 + \cdots + b_{iK}\gamma_K. \tag{7.6}$$

risk premiums or compensation in the form of higher expected excess returns to exposures to the risk factors

01 Arbitrage Pricing Theory

- Putting Eq. (7.6) side by side the underlying process $R_i = E(R_i) + \sum_{j=1}^{K} b_{ij}\delta_i + \varepsilon_i$,

$$R_i = r_f + b_{i1}(\gamma_1 + \delta_1) + b_{i2}(\gamma_2 + \delta_2) + \cdots + b_{iK}(\gamma_K + \delta_K) + \varepsilon_i \qquad (7.7)$$

where γ_j's are constants, and δ_j's are zero mean r.v.'s.

- Equation (7.7) can be expressed in matrix form as:

$$R_{N\times1} = r_f l_{N\times1} + B_{N\times K}\theta_{K\times1} + \varepsilon_{N\times1} \qquad (7.8)$$

where $E(R_{N\times1}) = E_{N\times1}$

(1) $E(\theta) = E(\gamma + \delta) = \gamma_{K\times1}$, a $K \times 1$ vector of risk premia (7.9)

(2) $E(\varepsilon) = 0$

(3) $E(\theta\varepsilon^T)_{K\times N} = 0$

01 Arbitrage Pricing Theory

- Each equation in the system in Eq.(7.8) at time t for stocks i, where i= 1, 2,…, N is

$$R_i - r_f = b_{i1}\theta_1 + b_{i2}\theta_2 + \cdots + b_{iK}\theta_K + \varepsilon_i \qquad (7.10)$$

$\boldsymbol{b_{ij}}$**: the sensitivity** of stock i to the jth risk premium factor variable θ_j that is common to all stocks
$\boldsymbol{\theta_j}$**: the** ***j*****th risk factor**
$\boldsymbol{\gamma_j}$**: the** ***j*****th** factor **risk premium, = E(**$\boldsymbol{\theta_j}$**)**

- The APT model Eq. (7.6) is strictly speaking a single period asset pricing model.

  Eq.(7.6) holds for each period t. Over time t, however, we can put a time series structure onto each of the variables in Eq. (7.6) in order to be able to use time series data to estimate and test the model.

- Thus, over time t, $R_{it} - r_{ft} = b_{i1}\theta_{1t} + b_{i2}\theta_{2t} + \cdots + b_{iK}\theta_{Kt} + \varepsilon_{it}$ (7.11)

  where θ_{jt} is the jth risk (premium) random factor at t

- In Eq.(7.11), θ_{1t} can be defined as ones, i.e., allowing for a constant intercept. We add a bit more restrictions to enable nice econometric results, i.e., we assume $E(\varepsilon_{it}\theta_{jt}) = 0$, for every t, i, and j, var(δ_{jt}) is constant over time for each j, and var(ε_{it}) is constant over time.

01 Arbitrage Pricing Theory

- Empirical problems in estimating and testing the APT:

(1) The number of factors, *K*, is not known theoretically.
Setting different *K* will affect the estimation of the factor loadings *B*.

(2) Even if *K* is known, it is not known what the factors are.
One can only make guesses about economic variables that may have an impact on R_{it}.

(3) One may also make guesses about the factor loadings instead of the factors.

(4) This APT framework can be linked with the linear multi-factor asset pricing model that are multiple linear regression models using intuitive risk factors to explain stock returns.

However, in addition to just a statistical multi-factor model, APT informs that E_M(risk factor) = risk premium. If some "risk factors" are not priced or perfectly diversifiable, then their risk premium contribution to any stock is E_M("risk factor") = 0 where $E_M(\bullet)$ denotes expectation by the market

02 Merton's Intertemporal CAPM

- Another multi-factor model with a theory is Merton's ICAPM:

$$dP_{it}/\, P_{it} = \mu_i(x)dt + \sigma_i(x)\, dZ_i$$

for stock i = 1,2,…..,N and x is an exogenous state vector of dimension S. Hence S state variables and individual dZ_i (that may be correlated with other $dZ_j'\, s$) influence the motion of instantaneous return $dP_{it}/\, P_{it}$ of stock i.

- If an increase in x leads to a decrease in optimal consumption, i.e., $\frac{\partial C}{\partial x} < 0$, and if cov($dP_{it}/P_{it}$, x) $>$ 0, the representative agent will buy more of asset i to hedge against decline in consumption due to increase in x

- In equilibrium, just as there is a market portfolio that investor holds to hedge against variance risk, investor now also holds S number of optimal hedge portfolios θ_j, each with maximal correlation with a state variable indicated above. Hedge portfolio instantaneous expected return is μ_{θ_j}.

- Stock i expected return:

$$\mu_i - r = \beta_i^M(\mu_M - r) + \beta_i^{\theta_1}(\mu_{\theta_1} - r) + \beta_i^{\theta_2}(\mu_{\theta_2} - r) + \cdots + \beta_i^{\theta_S}(\mu_{\theta_S} - r)$$

02 Merton's Intertemporal CAPM

- $\beta_i^{\theta_j}$ typically > 0 when μ_{θ_j} > r, the instantaneous risk-free rate

  Hence more correlation of stock returns with hedge portfolio ⇒ higher systematic risk, hence higher expected return as risk-bearing compensation

  See **Proof** of ICAPM in pages 196 – 200 of book.

- Discretization gives, at each end of $t-1$ or start of t over period $(t, t+\triangle]$:

$$E(R_{it+\Delta} - r_{t+\Delta}) = \sum_{k=1}^{S+1} b_{ik}\gamma_k$$

  where $\gamma_1 = E(R_{Mt+\Delta} - r_{t+\Delta})$, $\gamma_k = E\left(R_{\theta_k t+\Delta} - r_{t+\Delta}\right)$ for k =2, 3,..., S+1 are factor risk premia. $b_{i1} = \beta_i^M$, $b_{ik} = \beta_i^{\theta_{k-1}}$ (for k =2, 3,..., S+1) are stock i's factor loadings on the unique state variables.

- This equation looks similar to the APT Eq. (7.6) for each t. Hence the ICAPM can also be employed to check out dynamic factors in the form of the market portfolio$(R_{Mt+\Delta} - r_{t+\Delta})$ and the hedge portfolios $\left(R_{\theta_k t+\Delta} - r_{t+\Delta}\right)$

02 Estimating and Testing Multi-Factor Models

- There are two specifications using APT theory for observations over time and cross-sectionally:

$$\boldsymbol{R_{it} - r_{ft} = b_{i1}\theta_{1t} + b_{i2}\theta_{2t} + \cdots + b_{iK}\theta_{Kt} + \varepsilon_{it}} \qquad \textbf{(7.11)}$$

where ε_{it} is independent of θ_{it}

$$\boldsymbol{R_{it} - r_{ft} = b_{i1}\gamma_{1t} + b_{i2}\gamma_{2t} + \cdots + b_{iK}\gamma_{Kt} + \xi_{it}} \qquad \textbf{(7.12)}$$

where $\xi_{it} = \sum_{j=1}^{K} b_{ij}\delta_{it} + \varepsilon_{it}$

- What are the estimation and testable specifications of APT in Eqs. (7.11) and (7.12) in a multi-period setting? $\boldsymbol{b_{ij}}$'s are risk sensitivities or risk factor loadings. $\boldsymbol{\theta_{jt}}$'s are the risk factors at t, and $\boldsymbol{\gamma_{jt}}$ are the risk premiums at t (assuming risk premiums or conditional expectations at each t can change over time). i is subscript for stock. j is subscript for the risk factor.

- Suppose: *K* is known. Identify the set of N stocks to investigate and this set forms observations for the dependent variable.

- Note that in (7.11) and (7.12), $\boldsymbol{b_{ij}}$'s for every i,j are constants across time t

02 Estimating and Testing Multi-Factor Models

- **Approach (i)**: **postulate what the risk factors $\boldsymbol{\theta_{jt}}$ are, $\forall_t$.** Given these, it is not possible to perform a cross-sectional regression at any *t*. This is because at *t*, there are only *K* number of observations for explanatory variable θ_j (left out *t* subscript) but a larger *NK* number of parameters b_{ij}'s to be estimated.
- However, **getting the loadings or $\boldsymbol{b_{ij}}$'s for each $\boldsymbol{t}$** is a necessary step to progress. To get them we can perform time series regressions based on Eq.(7.11) separately for each stock *i* using $T \times 1$ $R_{it} - r_{ft}$ as dependent variable and θ_{jt}, $j = 1, 2, \ldots, K$, and $t = 1, 2, \ldots, T$ as explanatory variables. The estimated $\hat{b}_{ij}$ ($i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, K$) **using sample period [1,*T*]** data are obtained.
- The next step: **perform a cross-sectional regression based on Eq.(7.12)** employing $N \times 1$ dependent variable $R_{it} - r_{ft}$ **at a time $\boldsymbol{T+1}$** and $\hat{b}_{ij}$ ($i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, K$) as explanatory variables. The coefficient estimates via OLS are $\boldsymbol{\hat{\gamma}_{j\ T+1}}$ for $j = 1, 2, \ldots, K$.
- Thus, by using different periods possibly on a rolling basis (allowing slightly different sets of $\hat{b}_{ij}$ for each cross-sectional regression), we would obtain time series of **estimates $\boldsymbol{\hat{\gamma}_{j\ T+n}}$** for each *j* and for $n = 1, 2, \ldots$ We can interpret each $\hat{\gamma}_{j\ T+n}$ as a conditional estimate. By taking sample average of these we can arrive at the unconditional estimates.

02 Estimating and Testing Multi-Factor Models

In **approach (i)**, Chen, Roll, and Ross (1986) specify macroeconomic and financial market variables that intuitively make economic sense in explaining co-movements with stock returns in a systematic fashion. The following five macroeconomic variables in their MLR are postulated as the risk factors:

(a) monthly industrial production growth, MP_t

(b) monthly change in expected inflation, DEI_t

(c) monthly unexpected inflation, UI_t

(d) unexpected monthly risk premium, URP_t, that is the difference between monthly yield on long-term Baa corporate bonds and yield on long-term government bonds. This proxied for default risk premium.

(e) Unexpected monthly term structure factor, UTS_t that is the difference in promised yields to maturity on long-term government bond and short-term Treasury bill, or approximately the slope of the government yield curve.

Chen, Roll, and Ross (1986), "Economic Forces and the Stock Market", The Journal of Business, 59(3).

02 Estimating and Testing Multi-Factor Models
Fama-MacBeth Method

- **Recap**: In the Sharpe CAPM, the testable implication is about whether given loadings or betas $\beta_i, \forall_i$, asset returns $R_i \ \forall_i$, at *t* are related cross-sectionally to risk premium at *t* via $R_i - r_f = \beta_i E(R_M - r_f) + e_i$.
- A simple verification would be to find cross-sectional regression of excess returns $R_i - r_f$ on estimated β_i; and test if the estimated coefficient of $\hat{b}$ in $R_i - r_f = a + b\hat{\beta}_i + e_i$ is indeed significantly positive. This estimated coefficient is the risk premium at *t* and could be interpreted as $E_t(R_M - r_f)$ at *t* that may change over time.
- If we perform one cross-sectional regression at each time *t*,we may have different risk premium estimates for different *t* since risk premium can conditionally change over time. After the cross-sectional estimates of $E_t(R_M - r_f)$ are obtained for each *t* in the sample space [1, *T*], we can average them over *T* and find the mean.
- Applying FM method in **Chen, Roll, and Ross (1986) study**:

$$R_{it} = r_f = a + b_{i,MP}MP_t + b_{i,DEI}DEI_t + b_{i,UI}UI_t + b_{i,URP}URP_t + b_{i,UTS}UTS_t + e_{i,t}$$

There are formally three steps:

(1) Grouping to find accurate betas or factor loadings
(2) Rolling window to form different sets of factor loadings at each t
(3) Month by month cross-sectional regressions

02 Estimating and Testing Multi-Factor Models

Chen, Roll, and Ross (1986)

Columns indicate the average slope of cross-sectional regressions on estimated beta loadings of stocks for each month.

TABLE 4 Economic Variables and Pricing (Percent per Month × 10), Multivariate Approach

C

	EWNY	MP	DEI	UI	UPR	UTS	Constant
1958–84	5.021	14.009	−.128	−.848	8.130	−5.017	6.409
	(1.218)	(3.774)	(−1.666)	(−2.541)	(2.855)	(−1.576)	(1.848)
1958–67	6.575	14.936	−.005	−.279	5.747	−.146	7.349
	(1.199)	(2.336)	(−.060)	(−.558)	(2.070)	(−.067)	(1.591)
1968–77	2.334	17.593	−.248	−1.501	12.512	−9.904	3.542
	(.283)	(2.715)	(−3.039)	(−3.366)	(2.758)	(−2.015)	(.558)
1978–84	6.638	7.563	−.132	−.729	5.273	−4.993	9.164
	(.906)	(1.253)	(−.529)	(−.847)	(.663)	(−.520)	(1.245)

Positive risk premium

Negative risk premium

Positive risk premium

Stock j's return correlates highly (lowly) with MP ⇒ high (low) beta_{ML} ⇒ high (low) expected return. Low beta_{ML} is low risk as stock cushions fall in MP affecting most other stocks.

Stock j's return correlates highly (lowly) with UI ⇒ high (low) beta_{UI} ⇒ low (high) expected return. High beta_{UI} is low risk as stock cushions inflation when other stocks are hurt.

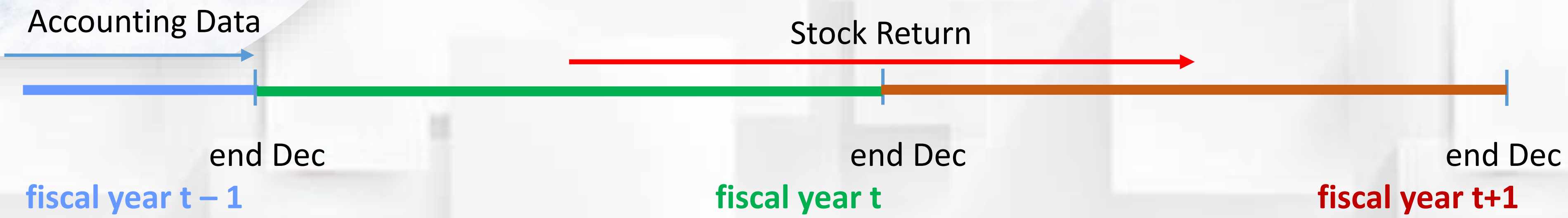
Stock j's return correlates highly (lowly) with UPR ⇒ high (low) beta_{UPR} ⇒ high (low) expected return. Low beta_{UPR} is low risk as steady stock return cushions defaults under high UPR affecting most other stocks (assuming low beta_{UPR} stocks are safer)

02 Estimating and Testing Multi-Factor Models

- **Approach (ii)** is to postulate instead what the risk factor loadings b_{ij} $(\forall i, j)$ are. In this case, we can directly perform a cross-sectional regression based on Eq. (7.12) employing $N \times 1$ dependent variable $R_{it} - r_{ft}$ $(\forall i = 1, 2, \ldots, N)$ at time t. The coefficient estimates via OLS are $\hat{\gamma}_{j\,t}$ for $j = 1, 2, \ldots, K$. For $t \in [1, T']$, we would obtain time series of estimates $\hat{\gamma}_{j\,t}$ for each j and for $t = 0, 1, 2, \ldots, T'$. We can interpret each $\hat{\gamma}_{j\,t}$ as a conditional estimate. By taking sample average of these we can arrive at the unconditional estimates.
- **Approach (iii),** like in approach (i), is to postulate what the risk factors θ_{jt} are, $\forall t$. Then directly perform time series regressions based on Eq. (7.11) but using portfolio stock returns as dependent variable instead of individual stock returns. θ_{jt}, $(\forall j, t)$ are explanatory variables. OLS estimates $\hat{b}_{ij}$ are then tested for significance where i is a portfolio. Grouping the stock returns into portfolio returns as dependent variable has the advantage of reducing the variance of the residual error ε_{it} in Eq. (7.11) and hence also the standard errors of the b_{ij} estimators.

03 Fama-French (1992)

Approach (ii): Employs Fama-MacBeth Method



Firm accounting data can explain stock returns due to investors acting on these accounting information/characteristics to build demand curves on the stocks. There is no mechanical relationship, so to make sense, the accounting data must precede return formation.

03 Fama-French (1992)

Eugene F. Fama and Kenneth R. French (1992), "The Cross-Section of Expected Stock Returns", The Journal of Finance, XLVII(2).

Table III

Average Slopes (*t*-Statistics) from Month-by-Month Regressions of Stock Returns on β, Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2.58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5.69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0.11 (−2.06)		0.35 (4.32)	−0.50 (−4.56)		
	−0.16 (−3.06)				0.06 (0.38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

- Finds weak or non-existence of relationship between average return on portfolios R_P and CAPM beta β_P during 1963-1990.
- Each month, cross-sectional regression of different portfolio returns on their betas yield a slope. (These are called Fama-MacBeth regressions.) Over many months, average of the estimated slopes has mean 0.15 with t-value (0.46) based on the monthly slope estimates.
- Average slopes of Size ln(ME) and Book-to-Market (Value due to under-valuation) in cross-sectional regression are significant at -0.11 (-1.99) and 0.35 (4.44).
- Hence these 2 factors explain the cross-section of stock returns.

03 Fama-French (1992)

Table V

Average Monthly Returns on Portfolios Formed on Size and Book-to-Market Equity; Stocks Sorted by ME (Down) and then BE/ME (Across): July 1963 to December 1990

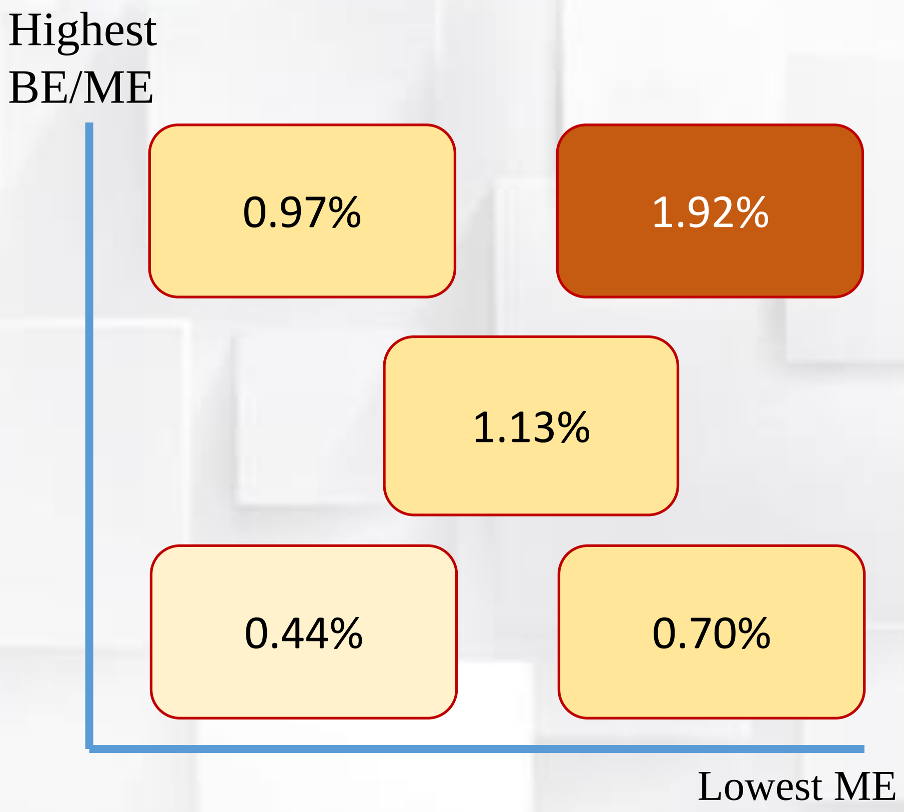
In June of each year t, the NYSE, AMEX, and NASDAQ stocks that meet the CRSP-COMPUSTAT data requirements are allocated to 10 size portfolios using the NYSE size (ME) breakpoints. The NYSE, AMEX, and NASDAQ stocks in each size decile are then sorted into 10 BE/ME portfolios using the book-to-market ratios for year $t-1$. BE/ME is the book value of common equity plus balance-sheet deferred taxes for fiscal year $t-1$, over market equity for December of year $t-1$. The equal-weighted monthly portfolio returns are then calculated for July of year t to June of year $t+1$.

Average monthly return is the time-series average of the monthly equal-weighted portfolio returns (in percent).

The All column shows average returns for equal-weighted size decile portfolios. The All row shows average returns for equal-weighted portfolios of the stocks in each BE/ME group.

		Book-to-Market Portfolios									
	All	Low	2	3	4	5	6	7	8	9	High
All	1.23	0.64	0.98	1.06	1.17	1.24	1.26	1.39	1.40	1.50	1.63
Small-ME	1.47	0.70	1.14	1.20	1.43	1.56	1.51	1.70	1.71	1.82	1.92
ME-2	1.22	0.43	1.05	0.96	1.19	1.33	1.19	1.58	1.28	1.43	1.79
ME-3	1.22	0.56	0.88	1.23	0.95	1.36	1.30	1.30	1.40	1.54	1.60
ME-4	1.19	0.39	0.72	1.06	1.36	1.13	1.21	1.34	1.59	1.51	1.47
ME-5	1.24	0.88	0.65	1.08	1.47	1.13	1.43	1.44	1.26	1.52	1.49
ME-6	1.15	0.70	0.98	1.14	1.23	0.94	1.27	1.19	1.19	1.24	1.50
ME-7	1.07	0.95	1.00	0.99	0.83	0.99	1.13	0.99	1.16	1.10	1.47
ME-8	1.08	0.66	1.13	0.91	0.95	0.99	1.01	1.15	1.05	1.29	1.55
ME-9	0.95	0.44	0.89	0.92	1.00	1.05	0.93	0.82	1.11	1.04	1.22
Large-ME	0.89	0.93	0.88	0.84	0.71	0.79	0.83	0.81	0.96	0.97	1.18

Average Monthly Return in Sorted Portfolio over 1 subsequent Year



03 Fama-French (JFE 1993)

Eugene F. Fama and Kenneth R. French (1993), "Common risk factors in the returns on stocks and bonds", Journal of Financial Economics, 33, 3-56.

Approach (iii)

This paper extends the asset-pricing tests in Fama and French (1992a) in three ways.

(a) "We expand the set of asset returns to be explained.The only assets considered in Fama and French (1992a) are common stocks. If markets are integrated, a single model should also explain bond returns. The tests here include U.S. government and corporate bonds as well as stocks."

(b) "We also expand the set of variables used to explain returns. The size and book-to-market variables in Fama and French (1992a) are directed at stocks. We extend the list to term-structure variables that are likely to play a role in bond returns. The goal is to examine whether variables that are important in bond returns help to explain stock returns, and vice versa.The notion is that if markets are integrated, there is probably some overlap between the return processes for bonds and stocks."

(c) "Perhaps most important, the approach to testing asset-pricing models is different. Fama and French (1992a) use the cross-section regressions of Fama and MacBeth (1973); the cross-section of stock returns is regressed on variables hypothesized to explain average returns."

03 Fama-French (JFE 1993)

(e) "This paper uses the time-series regression approach of Black, Jensen, and Scholes (1972). Monthly (excess) returns on stocks and bonds are regressed on the returns to a market portfolio of stocks and mimicking portfolios for size, book-to-market equity (BE / ME), and term-structure risk factors in returns. The time-series regression slopes are factor loadings that, unlike size or BE/ME, have a clear interpretation as risk-factor sensitivities for bonds as well as for stocks."

(f) "Our main results are easy to summarize. For stocks, portfolios constructed to mimic risk factors related to size and BE/ME capture strong common variation in returns, no matter what else is in the time-series regressions. This is evidence that size and book-to-market equity indeed proxy for sensitivity* to common risk factors in stock returns. Moreover, for the stock portfolios we examine, the intercepts from three-factor regressions that include the excess market return and the mimicking returns for size and BE/ME factors are close to 0. Thus, a market factor and our proxies for the risk factors related to size and book-to-market equity seem to do a good job explaining the cross-section of average."

* slope estimates of small-sized portfolio returns have larger slopes to the SMB risk factor than those of large-sized portfolio returns

03 Constructing Micmicking Portfolios

- Suppose:

  (1) $\text{Cov}(R_{it}, R_{jt}) \neq 0$ at t for any two stocks i, j.

  (2) There is a group of stocks j ∈ S with similar characteristics of small capitalizations, and another group of stocks k ∈ B with big capitalizations.

  (3) For all small cap stocks i, $\text{Cov}(R_{it}, \frac{1}{N}\sum_{j \in S} R_{jt})$ at t is high and $\text{Cov}(R_{it}, \frac{1}{N}\sum_{k \in B} R_{kt})$ at t is low.

  (4) For all large cap stocks i, $\text{Cov}(R_{it}, \frac{1}{N}\sum_{j \in S} R_{jt})$ at t is low and $\text{Cov}(R_{it}, \frac{1}{N}\sum_{k \in B} R_{kt})$ at t is high.

  If we form an index $\boldsymbol{F_t} = \frac{\boldsymbol{1}}{\boldsymbol{N}}\sum_{\boldsymbol{j \in S}} \boldsymbol{R_{jt}} - \frac{\boldsymbol{1}}{\boldsymbol{N}}\sum_{\boldsymbol{k \in B}} \boldsymbol{R_{kt}}$, then for small cap stocks $\text{Cov}(R_{it}, F_t)$ at t is high, while for large cap stocks $\text{Cov}(R_{it}, F_t)$ at t is low (possibly negative).

- Similarly suppose high value stocks' ($\in H$) returns have high positive correlations, low value stocks' ($\in L$) returns have high positive correlations, but high and low value stock returns have low correlations.

  If we form an index $\boldsymbol{G_t} = \frac{\boldsymbol{1}}{\boldsymbol{N}}\sum_{\boldsymbol{j \in H}} \boldsymbol{R_{jt}} - \frac{\boldsymbol{1}}{\boldsymbol{N}}\sum_{\boldsymbol{k \in L}} \boldsymbol{R_{kt}}$, then for high value stocks $\text{Cov}(R_{it}, G_t)$ at t is high, while for low value stocks, $\text{Cov}(R_{it}, G_t)$ at t is low (possibly negative).

03 Micmicking Portfolios (Stock Risk Factors)

- Size: The portfolio SMB (small minus big) is the difference, each month, between the simple average of the returns on the three small-stock portfolios (S/L, S/M, and S/H) and on the three big-stock portfolios (B/L, B/M and B/H).

  Note: Average of S/L, S/M, S/H is different from average in S when the sub-sample sizes in S/L, S/M, S/L are different.

  SMB is the difference between the returns on small- and big-stock portfolios with about the same weighted-average book-to-market equity.

  This difference should be largely free of the influence of BE/ME, focusing instead on the different return behaviors of small and big stocks.

- BE/ME: The portfolio HML(high minus low) is defined similarly.

  HML is the difference, each month, between the simple average of the returns on the two high-BE/ME portfolios(S/H and B/H) and on two low-BE/ME portfolios (S/L and B/L).

03 Fama-French (JFE 1993)

Table 7a

Regressions of excess stock returns on 25 stock portfolios formed on size and book-to-market equity (in percent) on the stock-market returns, $RM-RF$, SMB, and HML, and the bond-market returns, $TERM$ and DEF: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + mTERM(t) + dDEF(t) + e(t)$$

Size quintile	Low	2	3	4	High	Low	2	3	4	High
	Book-to-market equity (BE/ME) quintiles									
			b					$t(b)$		
Small	1.06	1.04	0.96	0.92	0.98	35.97	47.65	54.48	54.51	53.15
2	1.12	1.06	0.98	0.94	1.10	47.19	54.95	49.01	54.19	59.00
3	1.13	1.01	0.97	0.95	1.08	50.93	46.95	44.57	47.59	46.92
4	1.07	1.07	1.01	1.00	1.17	48.18	47.55	44.83	41.02	41.02
Big	0.96	1.02	0.98	1.00	1.10	53.87	51.01	41.35	48.29	35.96
			s					$t(s)$		
Small	1.45	1.26	1.20	1.15	1.21	37.02	43.42	50.89	51.36	49.55
2	1.01	0.98	0.89	0.74	0.89	32.06	38.10	33.68	32.12	35.79
3	0.76	0.66	0.60	0.49	0.68	25.82	22.97	20.83	18.54	22.32
4	0.38	0.34	0.30	0.26	0.42	12.71	11.36	9.99	8.05	11.07
Big	−0.17	−0.11	−0.23	−0.17	−0.06	−7.03	−4.07	−7.31	−6.07	−1.44
			h					$t(h)$		
Small	−0.27	0.10	0.27	0.40	0.63	−5.95	2.90	9.82	15.47	22.27
2	−0.51	0.02	0.25	0.44	0.71	−14.01	0.69	8.11	16.50	24.61
3	−0.37	−0.00	0.31	0.50	0.69	−10.81	−0.11	9.28	16.18	19.34
4	−0.42	0.04	0.29	0.53	0.75	−12.09	1.10	8.37	14.20	16.88
Big	−0.46	0.01	0.21	0.58	0.78	−16.85	0.38	5.70	18.16	16.59

- Regression results show that coefficient s increases for smaller sized portfolios and h increases for high value portfolios. T-stats show significance of estimates.

03 Forecasting Returns

J. Lewellen (2015), "The Cross-section of Expected Stock Returns", Critical Financial Reviews, 4, 1-44.

Lewellen (2015)'s paper on forecasting uses Fama and French (1992) approach.

Approach (ii):

- First ran cross-sectional regressions of stock returns on their pre-determined or lagged characteristics.
  - Model 1: used size, BE/ME, and past 12-month stock returns as characteristics
  - Model 2: added 3-year share issuance(log growth in split-adjusted outstanding shares) and one-year accruals, profitability, and asset growth as characteristics
  - Model 3: included 8 additional characteristics, such as beta, dividend yield, one-year share issuance, 3-year stock returns, 12-month volatility, 12-month turnover, market leverage, and sales-to-price ratio
- Obtain the time series of each monthly cross-sectional regression estimate of the risk premium associated with each firm characteristic.
- Compute the time series averages of the slope estimates (risk premium estimates each month) and their standard errors. This follows the Fama-MacBeth procedure. Hence perform a t-test if the slope average (unconditional risk premium) is significantly different from zero.

03 Forecasting Returns

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- Let the rolling average (or else cumulative average) of each estimated risk premium time series $\hat{\gamma}_{jt}$ over window $[d_1, d_2]$ be denoted as follows.

$$\bar{\hat{\gamma}}_j[d_1, d_2] = \frac{1}{d_2 - d_1 + 1} \sum_{t=1}^{d_2-d_1+1} \hat{\gamma}_{jt}$$

for each $j = 1, 2, \ldots, K$.

It is used as the expected premium for the future month $d_2 + 1$.

- Eq.(7.12) is then used to make a forecast or prediction of stock i excess return at $t = d_2 + 1$.

$$\mathbf{E}_{t\text{-}1}(\boldsymbol{R}_{it}) = \boldsymbol{r}_{ft} + \boldsymbol{b}_{i1}\boldsymbol{\gamma}_{1t} + \boldsymbol{b}_{i2}\boldsymbol{\gamma}_{2t} + \cdots + \boldsymbol{b}_{iK}\boldsymbol{\gamma}_{Kt}$$

$$\approx \bar{\hat{\boldsymbol{\gamma}}}_{0t} + \boldsymbol{b}_{i1}\bar{\hat{\boldsymbol{\gamma}}}_{1t} + \boldsymbol{b}_{i2}\bar{\hat{\boldsymbol{\gamma}}}_{2t} + \cdots + \boldsymbol{b}_{iK}\,\bar{\hat{\boldsymbol{\gamma}}}_{Kt}$$

where $\boldsymbol{b}_{ij}$ is firm i’s observed characteristic j at *t-1*.

04 Anomalies – Beyond Single Factor CAPM

There have been many applications of linear multiple regression to find drivers of stock returns

(1) Banz (1981) found low (high) market equity (number of shares outstanding $\times$ share price) correlates with high (low) $\hat{e}_i$ that is the residual error if market return is the only factor.

If we run OLS, $R_i - R_f = c_0 + c_1\hat{\beta}_i + c_2 ME_i + \eta_i$, then we would get a significantly negative estimate of c_2, i.e., $c_2 ME_i + \eta_i \approx \hat{e}_i$, so c_2 cov(ME_i, $\hat{e}_i$) $\approx$ var($\hat{e}_i$) or $c_2 \approx$ var($\hat{e}_i$)/cov(ME_i, $\hat{e}_i$) < 0

(2) Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) found that average returns on U.S. stocks are positively related to the ratio of a firm's book value of common equity, BE, to its market value. ME. [Time Series]

(3) Chan, Hamao, and Lakonishok (1991) found that book-to-market equity, BE/ME, also has a strong role in explaining the cross-section of average returns on Japanese stocks. If we run OLS $R_i - R_f = c_0 + c_1\hat{\beta}_i + c_2 ME_i + c_3 BE/ME_i + \eta_i$ across firms denoted i, we would get a significantly positive estimate of c_3.

04 Anomalies – Beyond Single Factor CAPM

(4) Basu (1983) showed that earnings-price ratios (E/P) help explain the cross-section of average returns on U.S. stocks in tests that also include size and market beta. E/P is likely to be higher (prices are lower relative to earnings) for stocks with higher risks and expected returns. If we run cross-sectional OLS $R_i - R_f = c_0 + c_1\hat{\beta}_i + c_2 ME_i + c_3 E/P_i + \eta_i$, we would get a significantly positive estimate of c_3.

(5) Bhandari (1988) reported positive relation between leverage and average return. It is plausible that leverage is associated with risk and expected return.

(6) Like Reinganum (1981) and Lakonishok and Shapiro (1986), Fama and French (1992) found that the relation between beta and average return disappears during the more recent 1963-1990 period, even when beta is used alone to explain average returns.

The simple relation between beta and average return is also weak in the 50-year 1941-1990 period. In short, FF (1992) did not support the most basic prediction of the SLB (Sharpe-Lintner-Black) model, that average stock returns are positively related to market betas.

04 Anomalies and Behavioral Finance

- Traditional finance asset pricing models or paradigms based on rationality, efficiency, and free market, i.e., optimization (of utility), rational expectations using all available information, and arbitraging away mispricings (no-arbitrage equilibrium), do not appear to explain all asset pricing aberrations
- Some of the anomalies (not currently explainable by rational equilibrium asset pricing models) may be explainable using behaviors. Behavioral finance is an alternative explanation of some systematic cases of apparent irrationality
- Price deviation (at least temporarily) from no-arbitrage equilibrium or evidence of behavioral finance could be due to:

(1) Irrationality or else bounded rationality (Herbert Simon)

(2) Limits to Arbitrage

(3) Psychological (behavioral) biases that affect how people make investment decisions despite the information. It is how they add their values/beliefs to the information

04 Anomalies and Behavioral Finance

- Limits to Arbitrage. Due to 3 sources of market frictions.

  (1) There is no perfect substitutes for mispriced security, e.g., LTCM -- Buying cheap Russian debt cannot be hedged by selling more expensive US debt. Most recently (Jan 2024), Asia Genesis's long China, short Japan strategy caused liquidation of fund.

  (2) There are transactions costs including short-selling constraints

  (3) Interruption by noisy traders (or uninformed liquidity traders) can de-rail convergence in mis-pricings. The increased mispricing gap due to noisy trades in the opposite direction means arbitrageur can suffer temporary loss. Arbitrageur may not want to take this risk if his/her trading capital is limited.

- One behavior is Loss Aversion tendency. Kahneman and Tversky– "Would you take a gamble with a 50% chance of losing $100 vs. a 50% chance of winning $101? Most people would say no. Despite the positive expected payoff, the possibility of losing $100 is enough to deter participation".

  This explains why investors hesitate to sell when there is a loss and sell too soon when there is a gain (as it "costs" more to take loss). Sometimes the tendency to hold on to loss stocks and realize gains too quickly in rising stocks is called Disposition Effect.

04 Anomalies and Behavioral Finance

- Another behavior is Ambiguity Aversion. People do not like situations where they are uncertain about the probability distribution of a gamble (ambiguity situations). Example: Ellsberg (1961) paradox :

Suppose that there are two urns, 1 and 2. Urn 1 contains 100 balls, an unknown mix of red and blue. Urn 2 contains a total of 100 balls, 50 red and 50 blue.

Subjects are asked to choose one of the following two gambles (a1 or a2), each of which involves a possible reward of $100.

a1 : a ball is drawn from Urn 1, reward $100 if red, $0 if blue
a2 : a ball is drawn from Urn 2, reward $100 if red, $0 if blue.

Subjects are then next also asked to choose between the following two gambles (b1 or b2):

b1 : a ball is drawn from Urn 1, reward $100 if blue, $0 if red,
b2 : a ball is drawn from Urn 2, reward $100 if blue, $0 if red.

If subject chooses a2 and then b2, he/she avoids uncertain distribution in Urn 1. The irrationality is as follows. He chooses a2 perhaps because he thinks there are more blue balls in Urn 1. But then he should choose b1 if this is the case. Hence ambiguity aversion can lead to suboptimal choices.

04 Anomalies and Behavioral Finance

- Investor sentiment is another behavioral factor affecting stock returns. Sentiment is the propensity to speculate and hence stocks that are more difficult to value (more speculation required) or stocks that have higher idiosyncratic volatility are more susceptible to sentiment biases (Daniel, Hirshleifer, and Subrahmanyam (1998, 2001), Hirshleifer (2001)) that are driven by emotions.

  Informed investor can exploit this sentiment of uninformed investor and sell more when the sentiment is to hold and buy more when the sentiment is to sell.

  On average strong sentiments would lead to underperformance, i.e. negative alpha. Thus, informed investors can profit more from trading in hard-to-value and high idiosyncratic volatility stocks.

- Overconfidence is another behavioral trait affecting full rationality. Overconfident investors assign high chances of obtaining higher returns than indicated by historical or objective data. Investors exhibit greater overconfidence when the market uncertainty is higher.

  Some individual investors commit larger investment mistakes and exhibit stronger behavioral biases in more uncertain environments. Thus, ex-post returns to investing in hard-to-value and also high idiosyncratic volatility stocks are lower for general market investors.

04 Anomalies and Behavioral Finance

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- Epstein & Schneider (2008) considered the effect of learning, with a focus on the role of signals with ambiguous precision.

  They show that such signals induce an asymmetric response to news — bad news (default risk is harder to calibrate or estimate objectively) is taken more seriously than good news — and contribute to **risk premia for idiosyncratic volatility** as well as negative skewness in returns

- Limited investor attention (bounded rationality) example – uninformed investors purchase or sell attention grabbing stocks that have bigger price changes and higher trading volumes or are linked with significant press events (Barber and Odean,2006). “Jump on the band wagon effect”

Practice Exercises (not graded)

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