QF 604
ECONOMETRICS OF FINANCIAL MARKETS

LECTURE 2

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LECTURE OUTLINE

01 EXPECTED UTILITY THEORY

02 SINGLE PERIOD CAPM

03 RANDOM WALK & MARKET EFFICIENCY

04 EVENT STUDIES

01 Expected Utility Theory

- X is a vector $(x_1, x_2, \ldots, x_n)$ where x_i is a scalar representing amount of a good or service for consumption. The corresponding price of one unit of x_i is p_i.
- A consumer chooses x_i's to max his/her objective function U called a direct utility function on consumption,

  $U : X \to [0,1] \in \mathcal{R}$.
- The optimal demands x_i are found in maximizing the " Lagrangian" function with Lagrange multiplier λ, where Y is wealth or income that forms a constraint:

$$\max_{X,\lambda} U(X) + \lambda\left(Y - \sum_i p_i x_i\right) \qquad s.t.\ \ X > 0$$

- A solution $x_1^*, x_2^*, \ldots, x_n^*$ can be found. Each x_i^* is a function of p_i's and Y, or $x_i^*(Y; p)$ where p is the vector of p_i's. When expressed in terms of given prices and income Y, the demand function $x_i^*(Y; p)$ is called a Marshallian demand function.
- Theorem (VM Expected Utility Representation): X, Y are bundles of goods. There is a utility function on space X, Y, $U : X \to [0,1] \in \mathcal{R}$, $U : Y \to [0,1] \in \mathcal{R}$, such that $U(p \odot X + (1-p) \odot Y) = pU(X) + (1-p)U(Y)$ where p is the probability of outcome X, and $1 - p$ is the probability of outcome Y.
- Likewise, $U(p \odot \$X + (1-p) \odot \$Y) = pU(\$X) + (1-p)U(\$Y)$

01 Risk Aversion

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- Let's define that a person/agent/investor) is risk neutral if he/she is indifferent between doing nothing or value 0 and an actuarially (probabilistically in the expectations sense) fair amount $E(X)$ = 0 where X is a RV or a lottery, and r > 0:

$$X = \begin{cases} \pi r, & with\ probability\ (1-\pi) \\ -(1-\pi)r, & with\ probability\ \pi \end{cases}$$

  - Risk-averse: prefer doing nothing to accepting the gamble, i.e., prefers certainty to an actuarially fair game.
  - Risk-loving: prefer the gamble to certainty

- Theorem: An agent is risk averse iff $U(W)$, where W is his or her \$ wealth, is a strictly concave function.

See **proof** on p. 67

01 Risk Aversion

- An insurance company charges a customer an insurance amount ***I*** to remove any uncertainty of a risky lottery *X*, with E(*X*) = 0, in his/her final wealth W + *X*. W is given original wealth. Then,

$$E[U(W+X)] = U(W-I)$$

Using Taylor series expansion, considering ***I*** is small relative to variance in *X*, then

$$E\left[U(W) + XU'(W) + \frac{1}{2}X^2U''(W) + o(U)\right] = U(W) - IU'(W) + o(U)$$

Thus, $\frac{1}{2}E[X^2]U''(W) \approx -IU'(W)$, or $I \approx -\frac{1}{2}\frac{U''(W)}{U'(W)}Var(X)$.

- Absolute risk aversion function: $A(W) = -\frac{U''(W)}{U'(W)}$, a positive number such that insurance premium I increases with this number *A*(*W*) for a given risk *X*. Insurance premium I also increases with *Var(X)* for a given *A(W)*.
- Risk tolerance function: $T(W) = \frac{1}{A(W)}$
- Relative risk aversion function: $R(W) = W \times A(W)$
- In some VM utility, when these functions become constants, we have the associated "risk aversion coefficients." Examples can be found in the book "Theory and Econometrics of Financial Asset Pricing" page 68.

01 St. Petersburg Paradox

- It was commonly accepted in 18th century that the price of a lottery would be its expected value. The lottery pays 2^{n-1} if the first head occurs in the nth toss of a coin.
- The probability of a head at the nth toss follows a geometric distribution (special case of negative binomial) $p(1-p)^{n-1}$ where p = 1/2 is the probability of a head. The expected payoff of the lottery is then,

$$\sum_{i=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{i-1} 2^{i-1} = \sum_{i=1}^{\infty}\frac{1}{2} = \infty$$

- The paradox is that no one would pay a large amount to buy this lottery with an expected payoff of ∞.
- Now suppose people are risk averse and not risk neutral. Suppose they have log utility ln(W). Then, a fair price π is ∋ (such that):

$$\ln(\pi) = \sum_{i=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{i-1} \ln(2^{i-1}) = \sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{i} (i-1)\ln 2 \; = \ln 2$$

- Hence, π = 2 which is a much smaller sum to pay given the risk aversion and can thus explain the paradox!

02 SINGLE PERIOD CAPM

- Assume:

  (1) N risky stocks and 1 risk-free bond with risk-free rate r_f

  (2) Utility function is strictly increasing and concave.

  (3) stock return rates r_i's are jointly normally distributed and/or investor has quadratic utility functions

- Investor k maximizes expected utility based on current wealth W_0 and investment decisions or portfolio weights (percentage investment) on the stocks, x_i:

$$\max_{\{x_i\}_{i=1,2,\dots,N}} E[U(W_1)]$$

where $W_1 = W_0(1 + r_P)$, r_P being the portfolio return rate, and

$$r_P = \sum_{i=1}^{N} x_i r_i + \left(1 - \sum_{i=1}^{N} x_i\right) r_f = r_f + x^T(r - r_f 1); \text{ x is N} \times 1$$

- Either the multivariate normal return distribution assumption or the quadratic utility assumption has the effect of ensuring that the investor's preference $E[U(W_1)] \equiv V(\mu_P, \sigma_P^2)$ ultimately depends only on the mean and variance of the return distribution.

02 SINGLE PERIOD CAPM

See equilibrium **proof** on pages 73 – 74.

- $\boldsymbol{\mu - r_f 1 = \frac{\sum x_M}{\sigma_M^2}(\mu_M - r_f)}$
- The above equation is the securities market line (SML) of the CAPM. Its *i*th element is

$$\boldsymbol{E(r_i) - r_f = \beta_i(E(r_M) - r_f)}$$

where $\boldsymbol{\beta_i} = \frac{\boldsymbol{Cov(r_i, r_M)}}{\boldsymbol{\sigma_M^2}}$.

- The SML says that the expected excess return of any security is equal to a risk premium (required compensation by a risk-averse investor for holding risky stock) which is proportional to its beta, β_i, a measure of its systematic risk.
- Since systematic risk cannot be diversified away, the CAPM shows importantly that only diversifiable risk does not cost, but non-diversifiable risk fetches a positive risk premium. The positive market risk premium is $\boldsymbol{(E(r_M) - r_f)}$.

02 SINGLE PERIOD CAPM

- **Curve A-B-M-V-C**: the hyperbola of μ_p versus σ_p.
- **Portfolio V**: the minimum variance (or minimum standard deviation) portfolio.
- **Segment of curve A-B-M-V**: the mean-variance portfolio efficient frontier
- **Segment VC:** the inefficient frontier
- When an investor invests in the risk-free asset with return rate r_f and the rest of the wealth in a frontier portfolio, the optimal risk-return would lie on the straight line that has intercept at r_f and has the point of tangency at *M* on the hyperbola.
- If all investors behave the same, *M* is the equilibrium market portfolio.
- Points on the tangent line **below *M***: **lending** at the risk-free rate and buying the risky market portfolio *M*.
- Points on the tangent line **above *M***: **borrowing** at the risk-free rate and buying the risky market portfolio M with leverage.

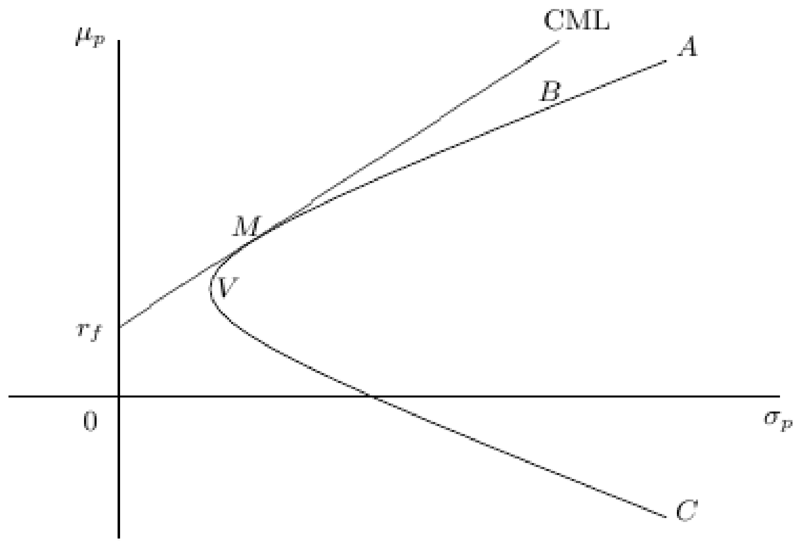

Figure 3.3: Mean-variance Risky Portfolio Efficient Frontier

- The tangent line is the Capital Market Line (CML):

$$\mu_p = r_f + \frac{E(r_M) - r_f}{\sigma_M}\sigma_p$$

where the mean and standard deviation of market portfolio *M*'s return are μ_M and σ_M.

02 SINGLE PERIOD CAPM

- The SML can be expressed as a regression equation that $\Rightarrow E(r_i) - r_f = \beta_i(E(r_M) - r_f)$. If returns are multivariate normal, then the regression specification viz.

$$r_{it} = r_{ft} + \beta_i(r_{mt} - r_{ft}) + e_{it}\,, \qquad \forall i, t$$

  where $E(e_{it}) = 0$, is plausible and consistent with SML. We have added time indexing for the risk-free rate r_{ft} and risky market portfolio return r_{mt} in order to allow the rates to vary over time.
- If we do not impose CAPM theory, statistical relationship is the market model, viz

$$r_{it} = a + br_{mt} + \xi_{it}\,, \qquad \forall i, t$$

  for constants a, b
- We can run the CAPM simple linear regression as:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + e_{it}\,, \qquad \forall i, t$$

  where $\alpha_i = 0$ and $Cov(e_{it}, r_{mt}) = 0$ for each i, t. $r_{it} - r_{ft} = \hat{\alpha}_i + \hat{\beta}_i(r_{mt} - r_{ft})$ is called the Security Characteristic Line
  - Hence it provides for OLS estimators that are BLUE under the classical conditions.
  - Estimated α_i: the alpha of stock i. It is theoretically 0 in equilibrium but could become positive or negative in actual regression. The interpretation of the latter then becomes one of financial performance: $\alpha_i > 0$: positive abnormal return and $\alpha_i < 0$: negative abnormal return

02 Application: CAPM Regression

- $\widehat{\alpha}_i = \overline{r_i - r_f} - \widehat{\beta}_i \overline{r_m - r_f}$. In the latter period (Table 3.2) $\hat{\alpha}$ is negative at -0.0143 with a 2-tail p-value of 0.0607. Thus, the stock appeared to underperform in the period after the global financial crisis of 2008-2009.
- Another possible interpretation is that the bench-mark CAPM model was not adequate, and the negative alpha could be explained by missing factors.
- The estimated betas of the Devon Energy stock in both periods are above 1. Thus, the stock is highly positively correlated with market movements.
- A good fit with reasonably high R^2's of 0.3189 and 0.4409.
- The estimate of the stock's systematic risk (2005-2009) is

$$\widehat{\beta}_i \sqrt{(1/T-1)\sum_{t=1}^{T}(X_t - \bar{X})^2} = 5.64\%.$$

where $X_t = r_{mt} - r_{ft}$. Unsystematic risk is estimated as

$$\hat{\sigma}_e = \sqrt{\frac{RSS}{(T-2)}} = 0.0832 \text{ or } 8.32\%.$$

Table 3.1: Regression of Monthly Excess Stock Return of Devon Energy on Monthly Excess Market Return, Jan 2005 - Dec 2009

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.0141	0.0108	1.307	0.196
Excess Market Return	1.1871	0.2278	5.211	0.0000***
R-squared	0.3189	F ($d.f.$ 1, 58)-statistic		27.16
Adjusted R-squared	0.3072	Prob(F-statistic)		0.0000
S.E. of regression	0.0832	Sum squared resid.		0.4013

Note: *** indicates significance at the 2-tail 0.1% level.

Table 3.2: Regression of Monthly Excess Stock Return of Devon Energy on Monthly Excess Market Return, Jan 2010 - Dec 2014

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.0143	0.0075	-1.913	0.0607
Excess Market Return	1.3144	0.1943	6.764	0.0000***
R-squared	0.4409	F ($d.f.$ 1, 58)-statistic		45.75
Adjusted R-squared	0.4313	Prob(F-statistic)		0.0000
S.E. of regression	0.0558	Sum squared resid.		0.1808

Note: *** indicates significance at the 2-tail 0.1% level.

02 CAPM Early Tests
["Feature Engineering"]

Black, Jensen, and Scholes (BJS) (1972) sorted 10 portfolios of stocks according to ranked estimated betas.

- Key idea is to reduce measurement or estimation errors of the betas
- To avoid selection bias due to chanced sample high beta or low beta, sorting is done on lagged 60 months data. Then next 60 months data are used to compute new beta of each stock.
- Portfolio betas are estimated by averaging the new betas of the individual stocks within each of the 10 portfolios.

- BJS tested the CAPM regression using both a time series approach and a cross-sectional approach.
- Time series regression: BJS ran regressions **on each portfolio *i*** based on

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + e_{it}$$

  and tested H_0: $\alpha_i = 0$. Found $\hat{\alpha}_i$ were consistently negative for the high $\hat{\beta}_i > 1$ portfolios and consistently positive for the low $\hat{\beta}_i < 1$ portfolios.
- Cross-sectional regression: BJS ran $r_i - r_f = a_i + b_i\hat{\beta}_i + \varepsilon_i$ across i=1,2,…,10 portfolios for different holding periods (different months). Some evidence of linearity between the average monthly portfolio excess returns $r_i - r_f$ and the portfolio betas $\hat{\beta}_i$, indicating significantly positive risk premium of $E(r_m - r_f)$ as in the CAPM SML.

02 CAPM Early Tests

Fama-MacBeth (1973) improved the cross-sectional regression procedures.

- For each month *t* of the test period, the betas of each stock are estimated using data from earlier months. At *t*, the sorted portfolios are used to obtain the average estimated beta for each portfolio *j* of stocks. This portfolio *j* beta, $\hat{\beta}_{jt}$, at t is then used in the cross-sectional regression, i.e. regression across portfolio returns *j* = 1,2,. . . ,*N* at *t*. Difference is in **rolling each month**.
- FM also used other cross-sectional explanatory variables at *t* besides $\hat{\beta}_{jt}$, including $(\hat{\beta}_{jt})^2$ and estimates of the standard deviation of the residual returns, $\hat{\sigma}_{jt}$.

$$r_{it} - r_{ft} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{jt} + \gamma_{2t}\left(\hat{\beta}_{jt}\right)^2 + \gamma_{3t}\hat{\sigma}_{jt} + \eta_{jt}$$

where η_{jt} is the residual error of the cross-sectional regression at *t*.

- OLS estimates of $\hat{\gamma}_{0t}$, $\hat{\gamma}_{1t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$ are **collected for each month *t*** in the test period. **These formed time series of *N* months each in the test period**. The **t-statistics were then computed to test if the mean of each of these time series was zero.** For each *t*, $\hat{\gamma}_{0t}$ should be zero according to CAPM. But since the estimated $\hat{\gamma}_{0t}$ is a RV with small sample errors, treating each time estimate as a stationary independent RV in the time series, the t-statistic testing the sample mean of this time series would indicate if indeed its expected value is zero according to CAPM.
- In general, there was support of the CAPM with data from the 40’s through the 70’s.

02 Performance Measures

- The Jensen measure: given by the alpha estimate in regressions where excess return is used as dependent variable, and the explanatory variables may include other risk factors besides the excess market return. Jensen alpha is also called risk-adjusted return or **abnormal return**.
- The Treynor measure: given by realized excess portfolio return rate per unit of estimated beta, over the sampling period, $\frac{\overline{r_{pt}-r_{ft}}}{\hat{\beta}_p}$, where subscript p denotes association with a portfolio. Compares with expected excess market portfolio return rate.
- Realized Sharpe measure or Sharpe ratio: $\frac{\overline{r_{pt}-r_{ft}}}{\hat{\sigma}_p}$, where $\overline{r_{pt} - r_{ft}}$ is realized excess portfolio return rate per unit of estimated standard deviation of the portfolio from the sampling period. Compares with portfolio on CML.

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02 Law of Iterated Expectations

$$E^{X,Y|Z}[Y|z] = \int_x \int_y y\, f(x,y|z) dy\, dx = \int_x \left(\int_y y \frac{f(x,y|z)}{f(x|z)} f(x|z) dy \right) dx$$
$$= \int_x \left(\int_y y\, f(y|x,z) dy \right) f(x|z)\, dx = \int_x \left(E^{Y|X,Z}(Y|x,z) \right) f(x|z)\, dx$$
$$= E^{X|Z}(E^{Y|X,Z}(Y|x,z))$$

- Think of random variables $\{X, Z\}$as the information set Φ_t, and values $\{x, z\}$ as the realised information ϕ_t at time t.
- Likewise $\{\mathrm{Z}\}$is an information set ς_t , and clearly $\varsigma_t \in \Phi_t$. We may rewrite the equation of the law of iterated expectations above as:

$$E[Y|\varsigma_t] = E(E(Y|\Phi_t)|\varsigma_t)$$

- Let $Y = P_{t+1}$, then $E[E(P_{t+1}|\Phi_t)|\varsigma_t] = E(P_{t+1}|\varsigma_t)$. In particular, if ς_t is null set (no information), then $E[E(P_{t+1}|\Phi_t)] = E(P_{t+1})$, unconditional expectation.

03 Random Walk

The mathematical construction of a random walk has an important application in finance - introducing the idea of market informational efficiency.

- Suppose a price process P_{t+1} follows an arithmetic random walk:

$$P_{t+1} = \mu + P_t + e_{t+1} \tag{1}$$

where μ is a constant, e_{t+1} is a mean zero white noise. Thus $E(e_{t+1}) = 0$, and e_{t+1}has zero serial correlations and needs not follow any specific probability distribution.

- A stronger version of white noise is typically used, i.e. e_{t+1} is i.i.d. with $E(e_{t+1}) = 0$.
- The property of e_{t+1} essentially implies that price changes at t+1, i.e. $P_{t+1} - P_t$ are unpredictable at t except for the mean μ.

03 Implication of Random Walk Model of Stock Prices

- In the arithmetic random walk (1), when drift $\mu = 0$, and e_{t+1} is mean zero i.i.d.,

$$P_{t+1} = P_t + e_{t+1} \tag{2}$$

Suppose Φ_t^Z represents observed information variables in the economy at time *t*. Hence, by time *t* and afterward, investors or agents in the economy would know about Φ_t^Z. At *t*, such a random variable (or variables) is taken as predetermined or given constants.

- If we take the conditional expectation on (2): $E(P_{t+1}|\Phi_t^Z) = E(P_t|\Phi_t^Z) + E(e_{t+1}|\Phi_t^Z) = P_t + 0$

Hence also, $E(P_{t+1} - P_t|\Phi_t^Z) = 0$ (3)

- Eq.(2) implies Eq.(3). But Eq.(3) does not necessarily imply Eq.(2). For example, $P_{t+1} = P_t\xi_{t+1}$, and $E(\xi_{t+1}|\Phi_t^Z) = 1$, can also lead to Eq.(3). Thus, the random walk process is a special case of Eq.(3). Nevertheless, the random walk process is a convenient workhorse to test a condition such as Eq.(3).

03 Implication of Random Walk Model of Stock Prices

- In (3): $E(P_{t+1} - P_t|\Phi_t^Z) = 0$. Eq. (3) ⇒ expected trading profit = 0 with latest information Φ_t^Z.
- (3) also implies $E(P_{t+1}|\Phi_t^Z) = P_t$.
  - This ⇒ whatever information Φ_t^Z at *t* do not add to expectation of next period price besides current price P_t.
  - Economically, this ⇒ that any information at t has already been **absorbed** into price P_t at *t*. P_t already **reflects** all such information Φ_t^Z including P_t itself.
  - P_t is as good a predictor as any about next period's price $E(P_{t+1})$.
  - Prices following a process with property such that the conditional expectation is the last period price is called a **martingale price process**.
  - The market is said to be **informationally efficient** with respect to Φ_t^Z if market information is Φ_t^Z
- Suppose there are smaller information sets $\Phi_t^X \subset \Phi_t^Y \subset \Phi_t^P$ where Φ_t^X is information set containing only lagged prices $\{P_t, P_{t-1}, ...\}$, and Φ_t^Y is information set containing only publicly available information at *t*. Private (including insider) information is contained in $\Phi_t^P \cap (\Phi_t^Y)^C$. Law of Iterated Expectations: $E(P_{t+1} - P_t|\Phi_t^P) = 0 \Rightarrow E(P_{t+1} - P_t|\Phi_t^Y) = 0 \Rightarrow E(P_{t+1} - P_t|\Phi_t^X) = 0$
- Hence strong form informational efficiency ⇒ semi-strong form ⇒ weak-form

03 Geometric Random Walk Model of Stock Prices

- Natural log form of random walk in prices (geometric random walk):

$$\ln P_{t+1} = \mu + \ln P_t + u_{t+1}$$

where u_{t+1} is i.i.d. Eq. can also be rewritten as: $r_{t+1} = \mu + u_{t+1}$ where the left-hand side $r_{t+1} \equiv \ln(P_{t+1}/P_t)$ is the continuously compounded return rate.

- Expected return r_{t+1} conditional on past information (including P_t) is μ. Shows why under random walk, there can be no past information that helps to produce higher returns. Random walk hypothesis is directly contrary to the idea of technical charting and to filtering rules.

- Suppose $u_{t+1} = \theta u_t + v_{t+1}$ (v_{t+1} is i. i. d.), so $\ln P_{t+1}$ is no longer a random walk because the residual noise are autocorrelated. Then past information/history can produce a return different from the population mean.

$$E(r_{t+1}|u_t \in \Phi_t^X) = \mu + \theta u_t \neq \mu$$

03 Example

- Suppose at time t = 0, there was information about whether the December 2008 GM, Ford, Chrysler bailout plan of $25 billion would pass through Senate. Suppose GM stock price at t = 0 was $3. If the bailout were successful, the stock price would increase to either $5 or $4 at t = 1. If the bailout were unsuccessful, the stock price would drop to either $2 or $1 at t = 1.
- All probabilities of the Bernoulli outcomes were 50%. Assume that the risk-adjusted discount rate from t = 0 to t = 1 was 0.
- If the market did not know about the outcome (i.e., did not process the information even when it was known), then at t = 0, its expected stock price at t = 1 was ¼ ($5+$4+ $2+$1) = $3.
- However， if the market was informationally efficient, then at $t = 0_{+}$, its conditional (upon the bailout out-come information) expectation of price at t = 1 was either $4.50 if the outcome were successful or $1.50 if the outcome were unsuccessful. Assume market is rational in computation.
- If at $t = 0_{+}$, the stock price did not quickly move away from $3, then the market was informationally inefficient as it did not capture the information immediately.

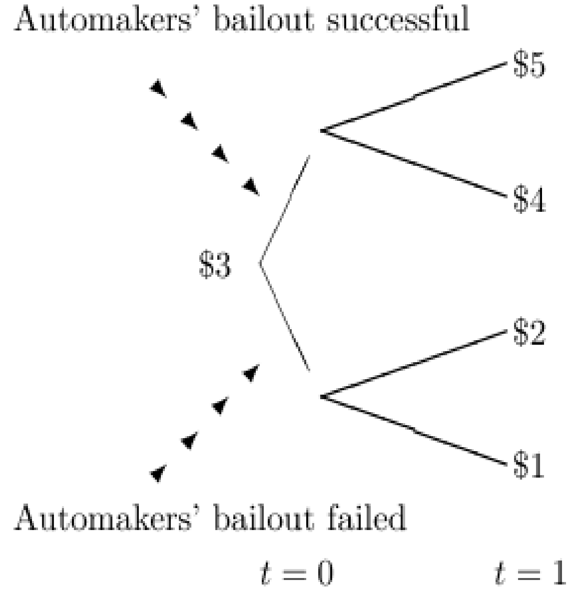

Figure 4.2: Stock Price Changes Contingent on Bailout News

04 Tests of Market Informational Efficiency

- An event is a news on a corporate happening whereby the firm or firms' traded stock prices may be affected.
- An asset benchmark pricing model is used to determine if the stock returns have changed significantly due to the event
- Event studies are typically not about tests of an asset pricing model
- Event studies are tests of

  (1) **whether the market is informationally efficient** given the public information of the event (assuming the benchmark model, also assuming we know or have a prior belief on how event has positive, negative, or neutral impact on returns)

  or (2) **whether the event has positive, negative, or neutral impact on returns** (assuming the benchmark model, also assuming the market is informationally efficient)
- In (1), it is also common to test for strong-form market efficiency, so that significant return deviations before the time of the publicly announced event may be interpreted as information leakage or inside information revelation. If the leakage occurs over several days before announcement, it may not be strong-form efficient as the price does not adjust immediately.

04 Tests of Market Informational Efficiency

- Three stages in an event study analysis:
  (1) define the event of interest and identify the period over which the event will be examined.
  (2) design a testing framework to define the way to measure impact and to test its significance.
  (3) collect appropriate data to perform the testing of the event's impact and draw conclusions in a model-theoretic and statistical sense.
- Event studies' various types:
  (a) Firm-specific event e.g. insider trading, announcement of board change, announcement of major strategic change, unusual rights issue announcement, announcement to file, executive stock option issues, employee stock option issues, etc.
  (b) Across firms system-wide event e.g. unanticipated better-than-forecast earnings announcement (good news), unanticipated worse-than-forecast earnings announcement (bad news), anticipated better-than or else worse-than forecast earnings announcement (no news). Others include announcement of bonus issues, stock splits, mergers and acquisitions, better than expected dividends or worse than expected dividends, new debt issues, seasoned equity issues, block sales, purchase of other companies' assets and stocks, etc.
  (c) Macro events e.g. increase in CPF employer contribution, GDP growth decline projection, regulatory changes, etc.

04 Testing Framework

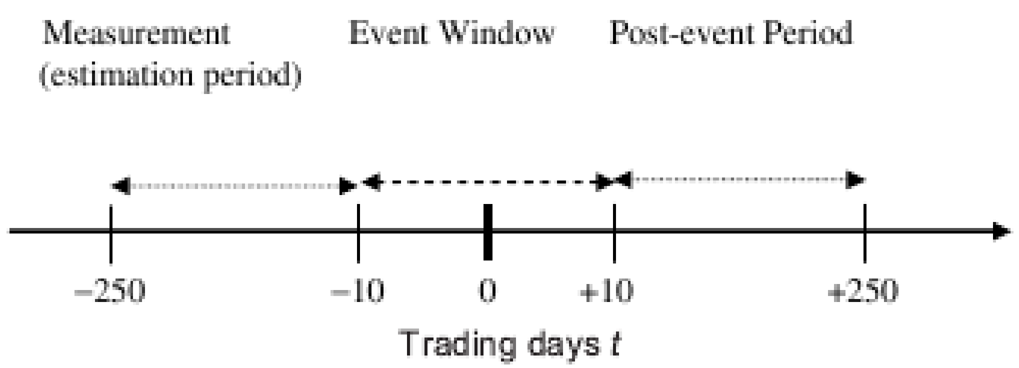

Figure 4.7: Event Sampling Frame

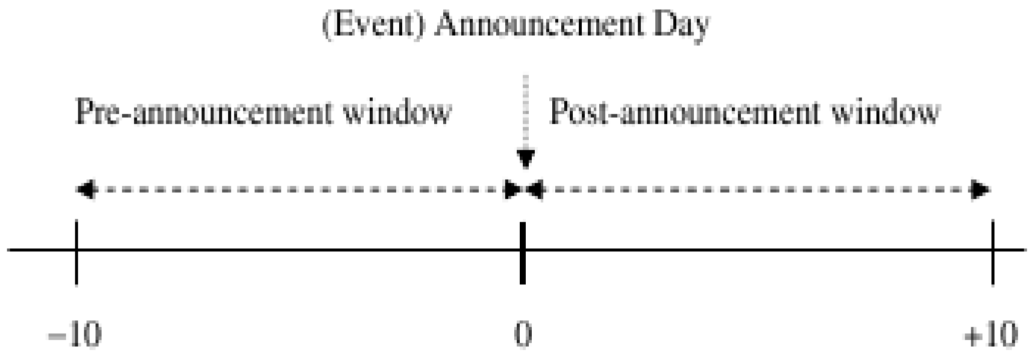

Figure 4.8: Announcement Windows

- Days are measured according to number of trading days before Event Day 0 (the event announcement day), or number of trading days after Event Day 0. In practical situation, announcement may be made during after-market close non-trading hours on calendar day t (in this case, event date is day t+1)
- The measurement/estimation period is used for estimating the parameters of the benchmark model employing historical data (typically daily continuously compounded returns) during the period.
- Estimation period: 240 sample points from t = –250 to t = –11. Sometimes, when we suspect that the market is disruptive and beta may change over more than one calendar year, then we can use a shorter time series e.g. 60 trading days (t = – 70 to – 11). During a stable measurement period, a longer or large sample is better in order to reduce sampling errors in the parameter estimators.
- The post-event period that goes up to one calendar year is less often used except for studies such as mergers and acquisitions, buyouts, IPOs, when a longer time is required before the effect of the event is to be seen.
- The Event Window typically includes two calendar weeks (or 10 trading days) before the announcement day, the announcement day itself, and 10 trading days after the announcement day. This window should be large enough to show up any possible changes to returns due to the event.

04 Benchmark Model

A normal return rate will have to be defined. Various benchmark models are used. In what follows, we shall take "returns" to mean return rates.

A. Market Model (MVN of stock returns leads to this specification)

$$r_{it} = \alpha_i + \beta_i r_{mt} + e_{it}$$

for return r_{it} stock *i* at time *t*, and conditions (a) $Cov(r_{mt}, e_{it}) = 0$; (b) $Var(e_{it}) = \sigma_i^2$, a constant; and (c) $Cov(e_{it}, e_{it-k}) = 0$ for $k \neq 0$, are assumed.

Then OLS regression for data set, $t = -250\ to\ t = -11$, will yield BLUE $\hat{\alpha}_i$and $\hat{\beta}_i$ that are also consistent. In addition, estimate of σ_i^2,

$$\hat{\sigma}_i^2 = \frac{1}{L-2} \sum_{t=-L-10}^{-11} \left(r_{it} - \hat{\alpha}_i - \hat{\beta}_i r_{mt}\right)^2 = \frac{1}{L-2} \sum_{t=-L-10}^{-11} (\hat{e}_{it})^2$$

is unbiased and consistent. L = 240 is number of sample points in measurement window.

- The benchmark or normal return during the Event Window is defined as $\hat{r}_{i\tau} = \hat{\alpha}_i + \hat{\beta}_i r_{m\tau}$ where the time subscript is now τ (to distinguish from *t*) outside the Event Window.
- $\tau = -10$ to +10 in the Event Window. The normal return on day τ is an expected return conditional on information available up to and including day τ. In the case of market model, this relevant information is just $r_{m\tau}$. The abnormal return to stock *i* at time $\tau \in [-10, +10]$

$$AR_{i\tau} = r_{i\tau} - \hat{r}_{i\tau} = r_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i r_{m\tau}$$

04 Benchmark Model

- **B. CAPM Model** $r_{it} = r_{ft} + \beta_i(r_{mt} - r_{ft}) + u_{it}$
  In (B), the normal return during the Event Window is defined as $\hat{r}_{i\tau} = r_{f\tau} + \hat{\beta}_i(r_{m\tau} - r_{f\tau})$. The abnormal return would then be $AR_{i\tau} = r_{i\tau} - \hat{r}_{i\tau} = r_{i\tau} - \left(r_{f\tau} + \hat{\beta}_i(r_{m\tau} - r_{f\tau})\right)$.

Supplementary measures of abnormal return include the following. They can be used to check the robustness of the results in case the benchmark model is not correct.

- **C. Market Adjusted Excess Return**, defined as $r_{i\tau} - r_{m\tau}$ ("abnormal return")
  Different abnormal return signs from (A) or (B) may warrant relook at those benchmark models. After relook, it could be benchmark models(A), (B) are correct but market adjusted excess return is too rough without an exact beta estimate.

- **D. Mean Adjusted Excess Return**, defined as $r_{i\tau} - \bar{r}_i$ ("abnormal return") where $\hat{r}_{i\tau} \equiv \bar{r}_i = \frac{1}{L}\sum_{t=-L-10}^{-11} r_{it}$. This is based on just the unconditional stationary mean of each stock return.

04 Event Studies

Some Testing Perspectives

- Suppose our event is a systematic one across many firms such as bonus issue information effect (whether bonus dividend announcement is good news, bad news, or no news).
  - We can test this over N firms each with a different bonus dividend announcement date scattered through time. However, the announcement date for each firm is denoted Event Day 0 though they are from different calendar dates.
    (Note: There could also be two events from the same firm if the firm happened to make two bonus issue announcements separated by time.)
- Avoiding clustering of the events in time prevents confounding events e.g. 9/11 when all stocks dived.
  - Spreading out also has the advantage of ensuring the various $AR_{i\tau}$'s of the various firms do not correlate so that it is easier to estimate the variance of a portfolio of the $AR_{i\tau}$'s across firms.

04 Event Studies

Some Testing Perspectives

- In an efficient and rational market, without significant information impact, the expected value of $AR_{i\tau}$ is zero, conditional on market information up to and including those at τ.
- Significant information in the event announcement is taken to be unanticipated news that causes the market to either (a) re-evaluate the stock's expected future earnings (thus also dividends), and, or (b) re-evaluate the stock's risk-adjusted discount rate, resulting in the immediate efficient adjustment of the stock price.
- With significant information impact on $\tau = 0$, the expected value of $AR_{i\tau}|_{\tau=0}$ is significantly non-zero (positive if good news on stock and negative if bad news on stock), conditional on market information up to and including those at τ. Immediate or fast reflection of news in prices is evidence of semi-strong form efficiency. If $AR_{i\tau}$ remains close to zero despite information, it may imply market inefficiency.

04 Event Studies

- Define the null hypothesis H_0: event has no impact on stock returns (or more specifically – no impact on stock's abnormal returns).

The market model abnormal return is $AR_{i\tau} = r_{i\tau} - \hat{\alpha}_i - \hat{\beta}_i r_{m\tau}$.

$$E(AR_{i\tau}|r_{m\tau}) = E(r_{i\tau}|r_{m\tau}) - E(\hat{\alpha}_i|r_{m\tau}) - r_{m\tau}E\left(\hat{\beta}_i\middle|r_{m\tau}\right) = E(r_{i\tau}|r_{m\tau}) - \alpha - \beta r_{m\tau} = 0$$

$$\begin{aligned} Var(AR_{i\tau}|r_{m\tau}) &= Var(r_{i\tau}|r_{m\tau}) + Var(\hat{\alpha}_i|r_{m\tau}) + r_{m\tau}^2 Var\left(\hat{\beta}_i\middle|r_{m\tau}\right) \\ &\quad +2r_{m\tau}Cov\left(\hat{\alpha}_i, \hat{\beta}_i\middle|r_{m\tau}\right) - 2r_{m\tau}Cov(r_{i\tau}, \hat{\alpha}_i|r_{m\tau}) \\ &\quad -2r_{m\tau}Cov\left(r_{i\tau}, \hat{\beta}_i\middle|r_{m\tau}\right) \\ &= \sigma_i^2\left(1 + \frac{1}{L} + \frac{(r_{m\tau} - \bar{r}_m)^2}{\sum_{t=-L-10}^{-11}(r_{mt} - \bar{r}_m)^2}\right) \end{aligned}$$

where $\bar{r}_m = \frac{1}{L}\sum_{t=-L-10}^{-11} r_{mt}$.

04 Event Studies Test Statistics

- If the estimation period sample size L is large, we can use the argument of asymptotic result to show $Var(AR_{i\tau}|r_{m\tau})$ converges to σ_i^2, or use the latter as an approximation in the case when L is fairly large, e.g. $L = 240$. So $AR_{i\tau}|r_{m\tau} \sim N(0, \sigma_i^2)$
- Test for each stock i using

$$\frac{AR_{i\tau}}{\hat{\sigma}_i} \sim N(0,1) \text{ or more accurately } \sim t_{L-2} \qquad (4.10)$$

- This is sometimes called the $SAR_{i\tau}$, the standardised abnormal return. $\hat{\sigma}_i^2$ is estimated via

$$\hat{\sigma}_i^2 = \frac{1}{L-2} \sum_{t=-L-10}^{-11} \left(r_{it} - \hat{\alpha}_i - \hat{\beta}_i r_{mt}\right)^2 \cong \frac{\sigma_i^2}{L-2} \chi_{L-2}^2$$

and $$\frac{AR_{i\tau}}{\hat{\sigma}_i} = \frac{AR_{i\tau}}{\sigma_i} \times \frac{\sigma_i}{\hat{\sigma}_i} = Z \Big/ \sqrt{\frac{\hat{\sigma}_i^2}{\sigma_i^2}} \cong t_{L-2}$$

where $Z \sim N(0,1)$

04 Event Studies Test Statistics

- If we have N firm-events，at any time τ within the Event Window, the average abnormal return (aggregated abnormal return), $AAR_\tau = \frac{1}{N}\sum_{i=1}^{N} AR_{i\tau}$.
- Assuming independence of disturbance across events (no clustering), and for large L,

$$Var(AAR_\tau | r_{m\tau}) \approx \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2$$

$$AAR_\tau | r_{m\tau} \approx N\left(0, \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2\right)$$

- Test for each Event Window day τ using

$$\frac{AAR_\tau}{\sqrt{\frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}_i^2}} \approx N(0,1) \qquad (4.11)$$

04 Event Studies Test Statistics

- In order to test for the persistence of the impact of the event during period $\tau_k - \tau_1$ (where τ_1 is start of Event Window, and τ_{21} is end of Event Window, and $\tau_1 \leq \tau_k \leq \tau_{21}$), the abnormal return can be added to obtain the cumulative abnormal return for each i, $CAR_i(\tau_1, \tau_K) = \sum_{\tau=\tau_1}^{\tau_k} AR_{i\tau}$
- Assuming independence of disturbance across time viz. $Cov(e_{it}, e_{it-k}) = 0$ for $k \neq 0$:

$$Var(CAR_i(\tau_1, \tau_k)|r_{m\tau k}, \dots) \approx \sum_{\tau=\tau_1}^{\tau_k} Var(AR_{i\tau}|r_{m\tau}) \approx (\tau_k - \tau_1 + 1)\sigma_i^2$$

- So, $CAR_i(\tau_1, \tau_k)|r_{m\tau k}, \dots \approx N(0, (\tau_k - \tau_1 + 1)\sigma_i^2)$. Test for each event window period (τ_1, τ_k) using

$$\frac{CAR_i(\tau_1, \tau_k)}{\sqrt{(\tau_k - \tau_1 + 1)\hat{\sigma}_i^2}} \approx N(0,1) \qquad (4.12)$$

04 Event Studies Test Statistics

The cumulative average abnormal return,

$$CAAR(\tau_1,\tau_K) = \frac{1}{N}\sum_{i=1}^{N}\sum_{\tau=\tau_1}^{\tau_k} AR_{i\tau} = \sum_{\tau=\tau_1}^{\tau_k}\frac{1}{N}\sum_{i=1}^{N} AR_{i\tau}$$

$$Var(CAAR_i(\tau_1,\tau_k)|r_{m\tau k},\dots) \approx \frac{1}{N^2}\sum_{i=1}^{N}(\tau_k-\tau_1+1)\sigma_i^2$$

$$CAAR_i(\tau_1,\tau_k)|r_{m\tau k},\dots \approx N(0,\frac{1}{N^2}\sum_{i=1}^{N}(\tau_k-\tau_1+1)\sigma_i^2)$$

CAAR may be construed as average of CAR or cumulation of AAR. Now test for each event window period (τ_1,τ_k) using

$$\frac{CAAR_i(\tau_1,\tau_k)}{\sqrt{(\tau_k-\tau_1+1)\frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}_i^2}} \approx N(0,1) \qquad (4.13)$$

04 Event Studies Test Statistics

- The z-statistics in Eqs.(4.10) to (4.13) can be used to test the H_0.
- The interpretation in each case will be slightly different.
- The tests are based on the null that the returns mean level and variance or volatility remain constant. A rejection could mean that the conditional mean had changed due to the event announcement. On the other hand, it is also possible that a rejection or non-rejection could be due to a change in volatility due to the event. If we want to test if there is a conditional mean change considering changed volatility as a result of the event， then we can use the sample variance of AAR_τ during the event window, $\tau \in (1,21)$, i.e. $\hat{\sigma}^2(ARR(1,21)) = \frac{1}{20}\sum_{\tau=1}^{21}(AAR_\tau - \overline{AAR})^2$

where $\overline{AAR} = \frac{1}{21}\sum_{\tau=1}^{21} AAR_\tau$, to construct the approximately t_{20} test statistic

$$\frac{AAR_\tau}{\sqrt{\hat{\sigma}^2 ARR(1,21)}} \approx t_{20}$$

for testing, where $\tau \in (1,21)$.

04 Event Studies

The three events in Figure 4.10 show different paths of CAR.

- In the dotted event path, CAR is significant only at event date due to a significantly positive AR. After that, AR is negative, and thus CAR falls back to zero.

This shows a price pressure (substitution) effect that could be due to excessive buying (or selling) not because of information but due to liquidity trading.

Due to temporary inelasticity, large volume buying will drive up price, hence return, temporarily.

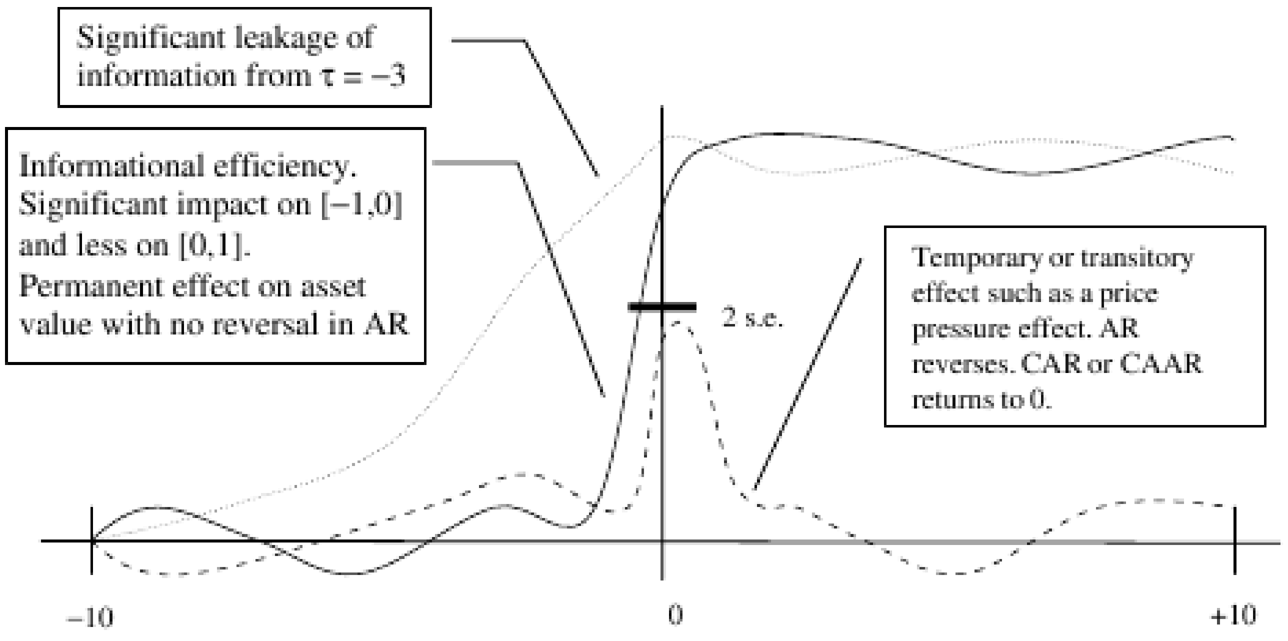

Figure 4.10: Graph of Cumulative Abnormal Returns Based on Events

As there is no information, the prices will revert, thus showing a reversal in AR over one or two days: CAR reverts back to zero after that. Such price pressure effects do not last and are said to be temporary or transitory.

04 Event Studies

- At times, even when a piece of information is known to produce asset price changes, but if this information is already known or anticipated, then its announcement at day 0 will not produce any significant AR or CAR.

  The solid line event illustrates significant event with a positive impact on price and returns, e.g., a positive earnings announcement. The impact is permanent. The news is also quickly absorbed, and price adjusted quickly so that after Day 1 of event, there is no more price adjustment, and AR is zero, CAR stays constant thereafter (semi-strong efficiency).

- In the small dotted event with positive CAR, the news appeared to hit before event date, at about $\tau = -3$. The AR and thus CAR are significantly positive.

  This may indicate information leakage. Inside information likely caused the significant price changes before the public news. Market appears to process strong-form information but at a slower speed. May be strong-form inefficiency (not immediate information reflection in price), or it could be information being released in bits.

Practice Exercises (not graded)

Betas.ipynb (data called from Yahoo Finance)
BOA_Event_Study.ipynb, BOA_Event_Study.csv
CSM_Event_Study.ipynb, CSM_Event_Study.cs