

QF605 Additional Examples

Session 2: Interest Rate and Swap Market

1 Questions

1. The spot LIBOR rates are as follow:

Tenor	Rate
1m	1.15%
2m	1.20%
3m	1.25%
6m	1.40%
9m	1.55%
12m	1.75%

Calculate:

- (a) The spot 3m discount factor $D(0, 3m)$.
- (b) The forward discount factor $D(3m, 6m)$.
- (c) The forward LIBOR rate $F(2m, 9m)$.
- (d) What rate would you show for a 2×12 FRA?
- (e) If your view is that one-month later the spot 1m rate would still remain at 1.15%, how should you trade?

2. Consider the following continuously compounded zero rates:

Maturity	Zero Rate
1y	4%
2y	4.5%
3y	4.75%

- (a) Calculate the continuously compounded forward rates $F(0y, 1y)$, $F(1y, 2y)$ and $F(2y, 3y)$.
 - (b) Show that the continuously compounded zero rate can be expressed as an arithmetic average of the corresponding forward rates.
3. Today's spot exchange rate for USD/SGD is $FX_0 = 1.42$. Furthermore, we observe the following zero-coupon bonds in the market:

$$D_{\text{SGD}}(0, T) = 0.98, \quad D_{\text{USD}}(0, T) = 0.964.$$

- (a) What should be the forward value of the exchange rate at time T ?
- (b) If we see that $FX_T = FX_0 = 1.42$, state an arbitrage.

4. Consider the following continuously compounded zero rates:

Maturity	Rate
3m	1.10%
6m	1.40%
12m	1.75%
18m	1.90%
24m	2.00%

Using 30/360 day count convention, calculate:

- (a) A 2y fixed leg pays 1.75% semi-annually. What is the PV of this fixed leg?
- (b) A 2y floating leg pays the 6m LIBOR rate semi-annually. What is the PV of this floating leg?
- (c) What is the par swap rate for a 2y interest rate swap with semi-annual payment?
- (d) Suppose you long a receiver swap at the par swap rate calculated in the previous question. Three months later, you observe the following zero rates in the market:

Maturity	Rate
3m	1.20%
6m	1.50%
12m	1.85%
18m	1.95%
24m	2.05%

What is the value of the receiver swap you have in your portfolio?

Note: Use linear interpolation on the zero rates if the maturity required is not provided.

5. We observe the following instruments in the swap market:

Instrument	Quote
6m LIBOR	2%
1y IRS	2.25%
2y IRS	2.40%
3y IRS	2.50%

All three interest rate swaps have semi-annual payment.

- (a) Determine the par swap rate for a 1.5y tenor interest rate swap with semi-annual payment.
 (b) A *forward starting* swap with a 2y tenor starting at $t = 1y$ has the following cashflows:

Time (y)	Pay	Rec
1.5	Par Swap Rate	6m LIBOR
2.0	Par Swap Rate	6m LIBOR
2.5	Par Swap Rate	6m LIBOR
3.0	Par Swap Rate	6m LIBOR

Calculate the par swap rate for this forward starting swap.

Note: Use 30/360 day count convention. Apply linear interpolation on the discount factors if necessary.

2 Suggested Solutions

1. (a) The spot $3m$ discount factor is

$$\frac{1}{1 + 0.25 \times 0.0125} \approx 0.996885. \quad \triangleleft$$

- (b) The forward discount factor $D(3m, 6m)$ is

$$D(3m, 6m) = \frac{D(0, 6m)}{D(0, 3m)} = \frac{\frac{1}{1+0.5 \times 0.014}}{\frac{1}{1+0.25 \times 0.0125}} \approx 0.996152. \quad \triangleleft$$

- (c) The forward LIBOR rate $F(2m, 9m)$ is

$$(1 + \Delta_{2m}L_{2m})(1 + \Delta_{7m}F(2m, 9m)) = 1 + \Delta_{9m}L_{9m}$$

$$F(2m, 9m) = \frac{1}{\frac{210}{360}} \left[\frac{1 + \frac{270}{360} \times 0.0155}{1 + \frac{60}{360} \times 0.012} - 1 \right] \approx 1.6467\%. \quad \triangleleft$$

- (d) We should show $F(2m, 12m)$ to prevent arbitrage

$$(1 + \Delta_{2m}L_{2m})(1 + \Delta_{10m}F(2m, 12m)) = 1 + \Delta_{12m}L_{12m}$$

$$F(2m, 12m) = \frac{1}{\frac{300}{360}} \left[\frac{1 + \frac{360}{360} \times 0.0175}{1 + \frac{60}{360} \times 0.012} - 1 \right] \approx 1.85629\%. \quad \triangleleft$$

- (e) If we think that the $1m$ spot rate will remain unchanged a month later, we should short the 1×2 FRA, since $F(1m, 2m) > 1.15\%$, we can borrow at 1.15% to deposit (lend) at $F(1m, 2m)$ if we were right. \triangleleft

2. Note that the zero rates provided are *continuously compounded*.

- (a) The first will be equal to the observed zero rate:

$$e^{F(0y, 1y) \times 1} = e^{4\% \times 1}$$

$$\Rightarrow F(0y, 1y) = 4\% \quad \triangleleft$$

By no-arbitrage, compounding in the forward rate should be identical to compounding in the zero rate for 2 years:

$$e^{F(0y, 1y) \cdot 1} \times e^{F(1y, 2y) \cdot 1} = e^{4.5\% \times 2}$$

$$\Rightarrow F(1y, 2y) = 5\% \quad \triangleleft$$

And similarly,

$$e^{F(0y, 1y) \cdot 1} \times e^{F(1y, 2y) \cdot 1} \times e^{F(2y, 3y) \cdot 1} = e^{4.75\% \times 3}$$

$$\Rightarrow F(2y, 3y) = 5.25\% \quad \triangleleft$$

- (b) Let r_n denote the continuously compounded zero rate for n years, and let F denote the continuously compounded forward rate. We have

$$e^{F(0,1)} \times e^{F(1,2)} \times \dots \times e^{F(n-1,n)} = e^{r_n \times n}$$

$$\Rightarrow r_n = \frac{F(0,1) + F(1,2) + \dots + F(n-1,n)}{n} \quad \triangleleft$$

3. (a) Let r^F and r^D denote the continuously compounded interest rate in the foreign and domestic economy, respective. Based on the definition of the exchange rate, we can arrive at

$$FX_T = FX_0 e^{(r^D - r^F)T}$$

using no-arbitrage argument. We can then write

$$FX_T = FX_0 \cdot \frac{e^{-r^F T}}{e^{-r^D T}} = FX_0 \cdot \frac{D_{\text{USD}}(0, T)}{D_{\text{SGD}}(0, T)} = 1.3968 \quad \triangleleft$$

- (b) If we see that $FX_T = FX_0 = 1.42$, enter into the FX forward to lock-in to this forward exchange rate. We long 1 unit of USD bond by shorting some SGD bond to generate a cash amount of \$1.36888 in SGD ($\because 0.964 \times 1.42 = 1.36888$). When the USD bond matures, we convert the \$1 USD back to SGD to get \$1.42 SGD. The short SGD bond position now becomes $-\$1.36888 \times \frac{1}{0.98} = \1.3968 . The difference is the arbitrage. \triangleleft

As long as your answer make use of the over-estimated FX forward (1.42 instead of 1.3968), and use long/short bond position to lock in to risk-free return, it is considered as correct.

4. (a) Given the zero rates, the discount factors are

$$D(0, 6m) = e^{-1.40\% \times 0.5} = 0.993$$

$$D(0, 12m) = e^{-1.75\% \times 1} = 0.9827$$

$$D(0, 18m) = e^{-1.90\% \times 1.5} = 0.9719$$

$$D(0, 24m) = e^{-2.00\% \times 2} = 0.9608$$

The PV of the fixed leg paying 1.75% semi-annually can be calculated as

$$\begin{aligned} PV_{\text{fix}} &= 0.5 \times [D(0, 6m) + D(0, 12m) + D(0, 18m) + D(0, 24m)] \times 1.75\% \\ &= 0.0342 \quad \triangleleft \end{aligned}$$

- (b) The PV of the floating leg is

$$PV_{\text{flt}} = 1 - D(0, 24m) = 0.03921 \quad \triangleleft$$

- (c) The par swap rate can be calculated as

$$S = \frac{1 - D(0, 24m)}{\sum_{i=1}^4 0.5 \times D(0, T_i)} = 2\% \quad \triangleleft$$

- (d) After three months, the swap's cashflows are 3m, 9m, 15m and 21m away. First we linearly interpolate for the necessary zero rates:

$$R_{9m} = \frac{1.50\% + 1.85\%}{2} = 1.675\%$$

$$R_{15m} = \frac{1.85\% + 1.95\%}{2} = 1.9\%$$

$$R_{21m} = \frac{1.95\% + 2.05\%}{2} = 2.0\%$$

Based on these zero rates, we can work out the forward LIBOR rates:

$$e^{1.20\% \times 0.25} \cdot (1 + 0.5L(3m, 9m)) = e^{1.675\% \times 0.75} \Rightarrow L(3m, 9m) = 1.92\%$$

$$e^{1.675\% \times 0.75} \cdot (1 + 0.5L(9m, 15m)) = e^{1.9\% \times 1.25} \Rightarrow L(9m, 15m) = 2.25\%$$

$$e^{1.9\% \times 1.25} \cdot (1 + 0.5L(15m, 21m)) = e^{2.0\% \times 1.75} \Rightarrow L(15m, 21m) = 2.263\%$$

The PV of the floating leg is

$$\begin{aligned} PV_{\text{flt}} &= 0.5 \times \left[D(0, 3m) \cdot 1.4\% + D(0, 9m) \cdot 1.92\% \right. \\ &\quad \left. + D(0, 15m) \cdot 2.25\% + D(0, 21m) \cdot 2.263\% \right] \\ &= 0.0383 \end{aligned}$$

The PV of the fixed leg is

$$\begin{aligned} PV_{\text{fix}} &= 0.5 \times \left[D(0, 3m) + D(0, 9m) + D(0, 15m) + D(0, 21m) \right] \times 2\% \\ &= 0.0393 \end{aligned}$$

So the value of the receiver swap is

$$V_{\text{rec}} = PV_{\text{fix}} - PV_{\text{flt}} \approx 0.001 \text{ per \$1 notional. } \triangleleft$$

5. (a) We need to know the discount factors $D(0, 6m)$, $D(0, 1y)$, and $D(0, 1.5y)$ to calculate the par swap rate for a 1.5y tenor swap. We have

$$D(0, 6m) = \frac{1}{1 + 0.5 \times 2.0\%} = 0.99$$

Using the 1y IRS quote, we can write

$$\begin{aligned} PV_{\text{fix}} &= 0.5 \cdot \left[D(0, 6m) + D(0, 1y) \right] \cdot 2.25\% \\ PV_{\text{flt}} &= D(0, 6m) \cdot 0.5 \cdot 2.0\% + D(0, 1y) \cdot 0.5 \cdot L(6m, 12m) \\ &= D(0, 6m) \cdot 0.5 \cdot 2.0\% + D(0, 1y) \cdot 0.5 \cdot \frac{1}{0.5} \frac{D(0, 6m) - D(0, 1y)}{D(0, 1y)} \\ &= D(0, 6m) \cdot 0.5 \cdot 2.0\% + D(0, 6m) - D(0, 1y) \\ &= D(0, 6m)(1 + 0.5 \cdot 2.0\%) - D(0, 1y) \\ &= 1 - D(0, 1y) \end{aligned}$$

Equating the PV of the fixed and floating leg, we can solve for $D(0, 1y) = 0.9779$.

Using the 2y IRS quote, we can write

$$\begin{aligned} PV_{\text{fix}} &= PV_{\text{flt}} \\ 0.5 \cdot \left[D(0, 6m) + D(0, 1y) + D(0, 1.5y) + D(0, 2y) \right] \cdot 2.4\% &= 1 - D(0, 2y) \end{aligned}$$

We linearly interpolate for $D(0, 1.5y)$ between $D(0, 1y)$ and $D(0, 2y)$, so that

$$D(0, 1.5y) = \frac{D(0, 1y) + D(0, 2y)}{2} = 0.48895 + 0.5D(0, 2y)$$

Substituting back, we obtain

$$D(0, 2y) = 0.9536, \quad D(0, 1.5y) = 0.96575.$$

The par swap rate for a 1.5y tenor swap with semi-annual payment is

$$S = \frac{1 - D(0, 1.5y)}{0.5 \cdot [D(0, 6m) + D(0, 1y) + D(0, 1.5y)]} = 2.335\% \triangleleft$$

- (b) For the forward starting swap with a $2y$ tenor starting at $t = 1y$, we need the discount factors: $D(0, 1y)$, $D(0, 1.5y)$, $D(0, 2y)$, $D(0, 2.5y)$, and $D(0, 3y)$. Again, we shall interpolate for the $D(0, 2.5y)$ discount factor linearly as follows:

$$D(0, 2.5y) = \frac{D(0, 2y) + D(0, 3y)}{2} = 0.4768 + 0.5D(0, 3y).$$

Using the $3y$ IRS quote, we write

$$\begin{aligned} PV_{\text{fix}} &= 0.5 \cdot \left[D(0, 6m) + D(0, 1y) + D(0, 1.5y) + D(0, 2y) + D(0, 2.5y) + D(0, 3y) \right] \cdot 2.5\% \\ PV_{\text{flt}} &= 1 - D(0, 3y) \end{aligned}$$

Setting the PV equal, we obtain

$$D(0, 3y) = 0.928, \quad D(0, 2.5y) = 0.941.$$

The par swap rate for this forward swap is

$$S = \frac{D(0, 1y) - D(0, 3y)}{0.5 \times [D(0, 1.5y) + D(0, 2y) + D(0, 2.5y) + D(0, 3y)]} = 2.63\% \quad \triangleleft$$