QF605 Fixed-Income Securities
Assignment 2, Due Date: 14-Feb-2024

1. Suppose the swap rate follows the stochastic differential equation

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(0) dW^{n+1,N},$$

derive a valuation formula for a payer swaption in the Black normal model

$$V^{pay}_{n,N}(0) = P_{n+1,N} \mathbb{E}^{n+1,N}[(S_{n,N}(T) - K)^+].$$

2. Consider a displaced-diffusion LIBOR model, defined as

$$dL_i(t) = \sigma_i \big[\beta L_i(t) + (1-\beta) L_i(0)\big] dW^{i+1}_t,$$

Evaluate the following expectations under the risk-neutral measure $\mathbb{Q}^{i+1}$, associated with the zero-coupon bond $D_{i+1}(t)$:

   (a) $\mathbb{E}^{i+1}[L_i(T_i)]$
   (b) $\mathbb{E}^{i+1}[(L_i(T_i) - K)^+]$

3. Write down the expectation of a receiver swaption payoff maturing at T and struck at K. Show that we cannot evaluate the expectation under $\mathbb{Q}^*$, the risk-neutral measure associated with the risk-free money market account numeraire $B_t = B_0 e^{\int_0^t r_u \, du}$, but by changing the measure to $\mathbb{Q}^{n+1,N}$, the risk-neutral measure associated with the present value of a basis point (PVBP) numeraire $P_{n+1,N}(t) = \sum_{i=n+1}^{N} \Delta_{i-1} D_i(t)$, we can derive an analytical expression for the receiver swaption.