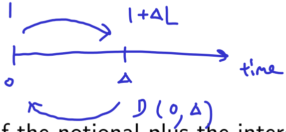
Term Rate: LIBOR & Its Replacement

If you put your money in a money-market account for a given period, the interest earned over this period is quoted as a **term rate** (e.g. LIBOR). At the end of a period of length Δ, one receives an interest equal to $\Delta \times L$, where L denotes the term rate and Δ denotes the day count fraction.

Since L is always quoted as annualized rate, Δ is often referred to as the **accrual fraction** or **day count fraction**.

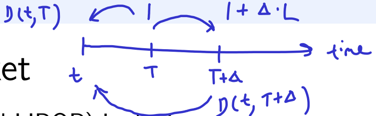
We have $1 \equiv (1 + \Delta \cdot L) \cdot D(0, \Delta)$.



This just states that the present value today of the notional plus the interest earned at the end of the Δ period is equal to the notional.

Note that in real markets Δ is not exactly equal to 0.25, 0.5, or 1, but is calculated according to a specific method for a given market, known as the **day count convention** (e.g. Act/365, Act/360 etc.)

Term Rate (LIBOR) Market



A **forward term rate** (e.g. forward LIBOR) is the interest rate one can contract for at time t to put money in a money-market account for the time period $[T, T+\Delta]$. Then we have

$$D(t,T) = (1+\Delta \cdot L) \cdot D(t, T+\Delta)$$

At the time when the **forward LIBOR** is fixed, it is then called a **spot LIBOR**. *Note that LIBOR is fixed at the beginning of each period T_i, and paid at the end of the period T_{i+1}. We call this* **"fixed in advance, paid in arrears."**

In most markets, only forward LIBOR rates of one specific tenor are actively traded, which is usually $3m$ (e.g. USD) or $6m$ (e.g. SGD, GBP, EUR).

Everyday we have a large number of forward LIBOR rates with this specific tenor, we can denote them as

$$i = 1, 2, \cdots, N: \; L(T_i, T_{i+1})$$

$$D(t,T)\cdot 1 = \big(1 + \Delta \cdot L(T,T+\Delta)\big)\cdot D(t,T+\Delta)$$

$$1 + \Delta L(T,T+\Delta) = \frac{D(t,T)}{D(t,T+\Delta)}$$

$$L(T,T+\Delta) = \frac{1}{\Delta}\cdot\frac{D(t,T) - D(t,T+\Delta)}{D(t,T+\Delta)}$$

Forward Rate Agreements (FRA)

Suppose you agree to an interest rate today, which you will obtain on an amount of money invested at later date, that is returned with interest at an even later date.

This kind of agreement is called a **forward rate agreement (FRA)**.

This level of the agreed rate that makes the value of entering into a forward rate agreement equal to zero is called the **forward rate**.

You have cashflow liability in the future (e.g need to borrow or deposit cash) but do not want to be exposed to interest rate risks.

Under a FRA contract:

- **Buyer** is obligated to **borrow** money at a pre-determined or fixed interest rate on the date when the FRA expires.
- **Seller** is obligated to **lend** at the fixed FRA rate. Thus both counterparties lock in a rate in advance and the actual lending and borrowing is done at the FRA expiration date.

Forward Rate Agreements (FRA)

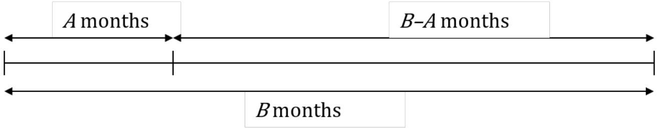
1x 7
2x 8
3 x 9

FRA allows you to **lock-in an interest rate today** at which you can borrow or deposit in the future.

4x10
5x11
6x12

⇒ For borrowers, if the market rate ends up higher, then it is good for you. The opposite is true for lenders.

⇒ For borrowers, if the market rate ends up lower, then it is bad for you. The opposite is true for lenders.



The FRA is quoted as $A \times B$ in terms of months. For example, a 3×9 FRA means the forward contract expires in 3 months, and the underlying interest rate is a 6-month LIBOR.

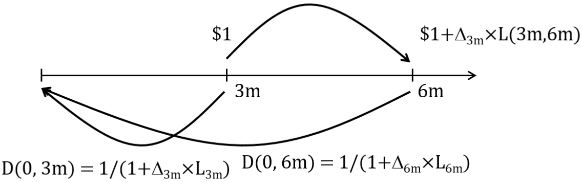
Forward Rate Agreements (FRA)

\$1 in $3m$'s time is worth $D(0, 0.25)$ today. If you invest that amount for $3m$ at the forward LIBOR $L(3m, 6m)$, then you earn an interest

$$D(0, 3m) = D(0, 6m)(1 + \Delta_{3m} L(3m, 6m))$$

$$\Rightarrow \quad L(3m, 6m) = \frac{1}{\Delta_{3m}} \frac{D(0, 3m) - D(0, 6m)}{D(0, 6m)}$$

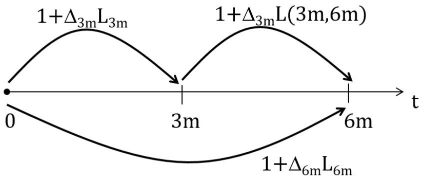
In words, we can work out the forward LIBOR rate based on spot LIBOR rates and discount factors.



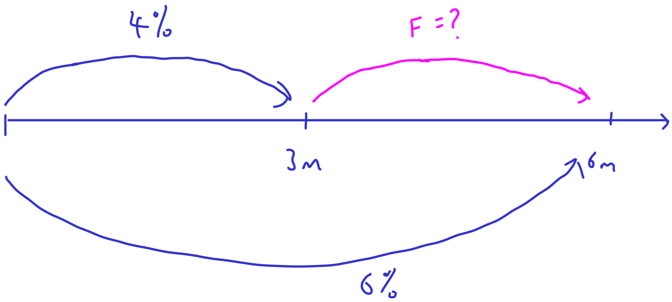
Forward Rate Agreements (FRA)

We can obtain the same result using either discount factors or spot LIBOR rates:

$$(1+\Delta_{3m}L_{3m})(1+\Delta_{3m}L(3m,6m)) = 1+\Delta_{6m}L_{6m}$$
$$\Rightarrow \quad L(3m,6m) = \frac{1}{\Delta_{3m}}\frac{(1+\Delta_{6m}L_{6m})-(1+\Delta_{3m}L_{3m})}{1+\Delta_{3m}L_{3m}}$$



Example If the $3m$ LIBOR rate is 4% and the $6m$ LIBOR rate is 6%, what is the in-$3m$-for-$3m$ forward rate $F(3m,6m)$?



$\sim 8\%$

$= 7.9\%$

Forward Rate Agreements (FRA)

Example Suppose we have the following discretely compounded zero rates:

Maturity	Zero Rate
$1y$	3.96%
$2y$	5.47%
$3y$	6.14%

Determine the forward rates $L(0, 1y)$, $L(1y, 2y)$, and $L(2y, 3y)$.

ans.: $L(1y, 2y) = 7\%$, $L(2y, 3y) = 7.5\%$.

$$D(0, 1y) = \frac{1}{(1 + 0.0396)^1}$$

$$D(0, 2y) = \frac{1}{(1 + 0.0547)^2}$$

$$D(0, 3y) = \frac{1}{(1 + 0.0614)^3}$$

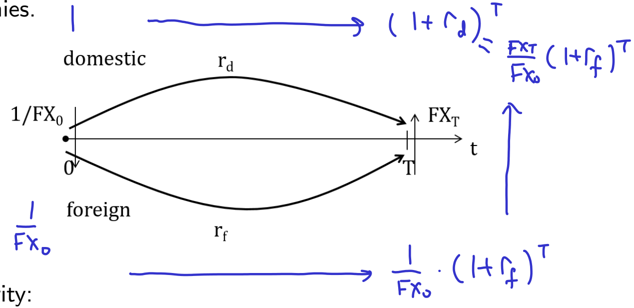
$$L(1y, 2y) = \frac{1}{1} \cdot \frac{D(0, 1y) - D(0, 2y)}{D(0, 2y)}$$

$$L(2y, 3y) = \frac{1}{1} \cdot \frac{D(0, 2y) - D(0, 3y)}{D(0, 3y)}$$

Interest Rate Parity

Interest rate parity is a no-arbitrage condition stating that it is impossible for investors to gain higher returns by borrowing and lending in the money market account of 2 economies.

This gives rise to a relationship between the interest rates and exchange rates of the 2 economies.



Interest rate parity:

$$FX_T = FX_0 \times \left(\frac{1+r_d}{1+r_f}\right)^T$$

SGD

$$1 \longrightarrow 1 + \Delta_{6m}^{SG} \cdot L_{6m}^{SG}$$

$$\|$$

$$\frac{FX_T}{FX_0}\left(1 + \Delta_{6m}^{US} L_{6m}^{US}\right)$$

time

$$\frac{1}{FX_0}$$

$$\longrightarrow \frac{1}{FX_0} \cdot \left(1 + \Delta_{6m}^{US} L_{6m}^{US}\right)$$

USD

$$1 + \Delta_{6m}^{SG} L_{6m}^{SG} = \frac{FX_T}{FX_0}\left(1 + \Delta_{6m}^{US} L_{6m}^{US}\right)$$

$$L_{6m}^{SG} = \left[\frac{FX_T}{F\cancel{X}_0}\left(1 + \Delta_{6m}^{US} L_{6m}^{US}\right) - 1\right] \cdot \frac{1}{\Delta_{6m}^{SG}}$$

Case Study: Swap Offer Rate (SOR) in Singapore

SOR is a **synthetic rate** in the Singapore market.

E. Calculation Methodology: Singapore Dollar Swap Offer Rate (SOR)

Benchmark	**Singapore Dollar Swap Offer Rate (SOR)**
Description	The synthetic rate for deposits in Singapore Dollars (SGD), which represents the effective cost of borrowing the Singapore Dollars synthetically by borrowing US Dollars (USD) for the same maturity, and swap out the US Dollars in return for the Singapore Dollars.
Calculation Methodology	The Administrator shall calculate and determine the Rate, for each maturity matching each Tenor specified below (each a "calculation period"), on each Business Day as follows: **SGD SOR** = $\boxed{\left\{\left[\left(\frac{\overbrace{\text{Spot Rate} + \text{Forward Points}}^{FX_T}}{\underbrace{\text{Spot Rate}}_{FX_0}}\right)\times\left(1+\frac{\text{USD Rate}\times\#\text{days}}{360}\right)\right]-1\right\}\times\frac{365}{\#\text{days}}\times 100}$ Where: **USD Rate** means the rate for deposits in USD for a period of the calculation period which appears on Thomson Reuters Screen LIBOR01 (or successor page displaying USD LIBOR) as of 11:00 am, London time, on the same Business Day.

Case Study: Swap Offer Rate (SOR) in Singapore

Case Study: MIFOR Rate in India

Mumbai Interbank Forward Offered Rate (MIFOR) in the India market.

Banks to provide a quotation of their offered side of INR/USD forward points for the forward sale of INR against USD for settlement on the last day of a period equivalent to the Designated Maturity and commencing on the Reset Date and the forward points so determined by the Calculation Agent shall be the "Forward Points" for purposes of the following formula. The Calculation Agent will then determine the rate for that Reset Date by applying the following formula:

Floating Rate =

$$\{[(\text{Spot Rate} + \text{Forward Points}) / \text{Spot Rate} * (1+ \text{LIBOR} * N1)] - 1\} * N2*100$$

where:

"Spot Rate" means the Reserve Bank of India's published USD/INR spot rate (expressed as a number of INR per one USD) which appears on Reuters Screen "RBIB" as of 13:00 p.m. India Standard Time on that Reset Date (if such rate is not available the Calculation Agent will ask each of the Reference Banks to provide a quotation of such rate);

"LIBOR" means USD-LIBOR-BBA for a period of the Designated Maturity commencing on the Reset Date;

"N1" means the number of days in the Calculation Period divided by 360;

"N2" means 365 divided by the number of days in the Calculation Period.

Case Study: THBFIX in Thailand

THBFIX—the synthetic rate for deposits in Thai Baht, representing the effective cost of borrowing the Thai Baht synthetically by borrowing USD and swapping out the USD in return for THB.

$$\boxed{THBFIX = \left\{\left[\left(\frac{Spot\ Rate + Forward\ Points}{Spot\ Rate}\right) \times \left(1 + \frac{USD\ Rate \times \#days}{360}\right)\right] - 1\right\} \times \frac{365}{\#days} \times 100}$$

Where:

"USD Rate" means US dollar interest rate (USD rate) in each tenor that is published on Reuters screen LIBOR01 (or other screens that shows USD LIBOR) at 11.55 a.m. London time during that business day.

"#days" means the actual number of days in each tenor (which may differ on each business day).

"Spot Rate" means USDTHB spot rate which is calculated via a volume weighted average method (excluding transactions with unusually large volume or unusually high or low spot rates), in relation to all USDTHB FX Spot Qualified Transactions on each business day.

"Forward Points" means Forward points which is calculated via a volume weighted average method (excluding transactions with unusually large volume or unusually high or low forward points), in relation to all USDTHB FX Swap Qualified Transactions on each business day.

Interest Rate Risk

- A standard company is usually financed by a combination of equity and debt. Consequently, **interest rate risk** is part and parcel of operating a company.
- A company might raise money from investors by issuing shares, and then borrows further to provide the rest of the funding.
- Clearly, the company will need to pay interest on the debt it issued, and eventually repay the debt in its entirety. If the interest rate fluctuates according to the prevailing rates in the market, it is said to be **floating**.
- The interest payments is typically a sizable burden, and could cripple a company if it rises too much. It is therefore better for a company to go for a **fixed-rate loan**.
- This obviously adds on to the complexity of the problem—if rates fall, it will be beneficial for the company to **refinance**.

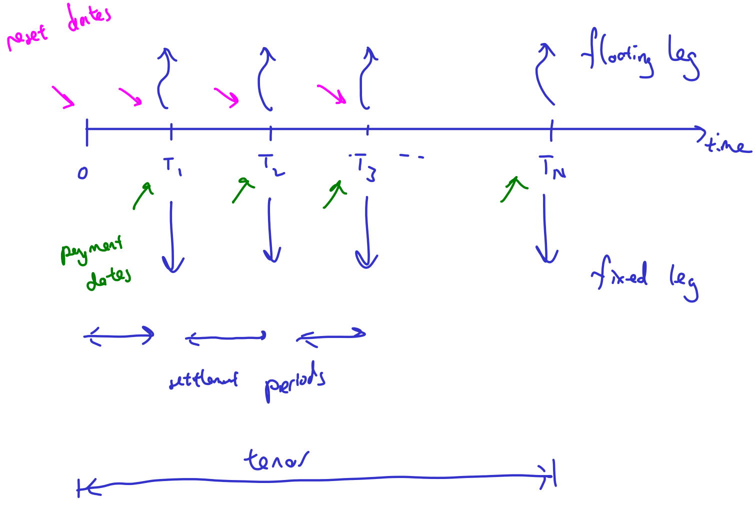
Interest Rate Swaps

Interest rate swap is a contract where two parties agree to exchange a set of floating interest rate payments for a set of fixed interest rate payments.

- The set of floating interest rate payments is based on LIBOR rates and is called the **floating leg**.
- The set of fixed payments is called the **fixed leg**.

The naming convention for swaps is based on the fixed side: **payer swap** means you pay the fixed leg, while **receiver swap** means you received the fixed leg

- The times at which the floating interest rates are set are called **reset dates**.
- The times at which the interest rate payments are made are called **payment dates**.
- The time between payment dates are called **settlement period**.
- The length of the swap is called the **tenor** of the swap (from start till maturity).
- Typically, the parties agree to exchange only the net amount owed from one party to the other party. This practice is called **netting**.
- If payments are not in the same currency, netting is not appropriate.



reset dates
floating leg
time
0
T_1
T_2
T_3
- -
T_N
payment dates
fixed leg
settlement periods
tenor

Interest Rate Swaps – Rationale

Interest rate swaps are useful because they allow you to turn a fixed-rate asset into a floating rate asset, and vice versa. For example:

- A bank funds itself via a mix of short-term deposits (floating rate) and fixed-rate bonds. It gives variable and fixed rate mortgages to its customers. When it raises financing, it does not know the quantities of variable and fixed rate mortgages that it will give.
- If it ends up giving more fixed rate mortgages than anticipated, it can enter into swaps to pay fixed and receive floating to hedge out interest rate risk.
- If it ends up giving more variable rate mortgages than anticipated, it can enter into swaps to receive fixed and pay floating to hedge out interest rate risk.

Interest rate swaps allow the bank to raise money first, then invest it, and fix any resulting interest rate risk exposure later.

Interest Rate Swaps – Fixed Leg

The process of **pricing the swap** involves finding the swap rate such that the value of the swap contract is **worth par** at trade initiation (i.e. NPV=0).

To this end, we proceed by determining the PV of the fixed and floating leg separately.

The PV of a fixed leg is straightforward. Consider a single fixed rate payment of K at time T, with an accrual fraction of Δ. The PV of this payment is

$$D(0,T) \times K \times \Delta$$

Therefore, for a series of payments starting at T_1 and ending at T_n, the leg of fixed rate payment is

$$PV_{fix} = K \sum_{i=1}^{n} D(0,T_i) \times \Delta_{i-1}$$

Interest Rate Swaps – Floating Leg

Recall that forward rate is related to discount factors:

$$L(3m, 6m) = \frac{1}{\Delta_{3m}} \frac{D(0,3m) - D(0,6m)}{D(0,6m)}.$$

Given a set of payment dates T_i, we can express the forward LIBOR rate as

$$L(T_{i-1}, T_i) = \frac{1}{\Delta_{i-1}} \frac{D(0,T_{i-1}) - D(0,T_i)}{D(0,T_i)}.$$

The PV of a single floating rate payment at T_i can therefore be expressed as:

$$D(0,T_i) \times \Delta_{i-1} \times L(T_{i-1}, T_i) = D(0,T_{i-1}) - D(0,T_i).$$

Generalizing, we see that if the floating leg comprises of n payments, then its PV is given by

$$\sum_{i=1}^{n} D(0,T_i) \times \Delta_{i-1} \times L(T_{i-1}, T_i) = 1 - D(0,T_n).$$

$$PV_{flt} = \sum_{i=1}^{n} \Delta_{i-1} \cdot D(0,T_i) \cdot L(T_{i-1}, T_i)$$

$$= \sum_{i=1}^{n} \Big(D(0,T_{i-1}) - D(0,T_i) \Big)$$

$$= \Big[D(0,T_0) - \cancel{D(0,T_1)} \Big] + \Big[\cancel{D(0,T_1)} - \cancel{D(0,T_2)} \Big]$$

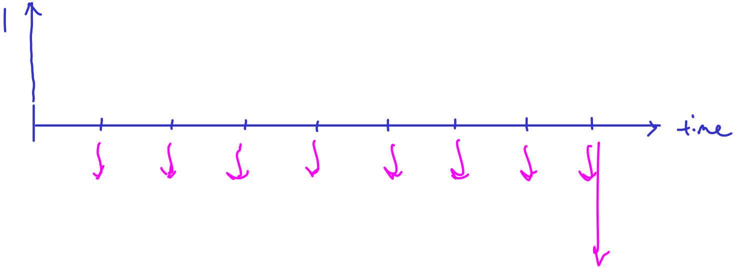
$$+ \cancel{\ldots} + \Big[\cancel{D(0,T_{n-1})} - D(0,T_n) \Big]$$

$$= D(0,T_0) - D(0,T_n)$$

$$= 1 - D(0,T_n)$$

$$PV_{flt} = 1 - D(0, T_n)$$

$$PV_{flt} + D(0, T_n) = 1$$



Par Swap Rate

In a swap multiple payments are exchanged:

- The value of a payer swap at time t that starts at T_0 and ends at T_n.
- At the start date T_0 the first LIBOR rate is fixed.
- Actual payments are exchanged at $T_1, \ldots, T_n$.
- The swap tenor is $T_n - T_0$.

The present value of a payer swap is given by

$$\sum_{i=1}^{n} V_i^{flt}(t) - \sum_{i=1}^{n} V_i^{fix}(t) = (D(0,T_0) - D(0,T_n)) - K \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i). \quad = 0$$

Similarly, the present value of a receiver swap is given by

$$K \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i) - (D(0,T_0) - D(0,T_n)). \quad = 0$$

In the market, swaps are not quoted as prices for different fixed rates K, but only the fixed rate K is quoted for each swap such that the present value of the swap is equal to 0.

Par Swap Rate

This particular rate is called the **par swap rate**. We denote the par swap rate for the $[T_0, T_n]$ swap with S. Solving, we obtain

$$K = S = \frac{D(0,T_0) - D(0,T_n)}{\sum_{i=1}^{n} \Delta_{i-1} D(0,T_i)}. = \frac{1 - D(0,T_n)}{P_{1,n}}$$

The term in the denominator is also called the **present value of a basis point** (PVBP) / PV01

$$P_{1,n} = \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i).$$

Pricing Interest Rate Swaps

Example Suppose we observe the following LIBOR discount factors in the market:

Discount Factor	Value
$D(0, 0.25)$	0.9876
$D(0, 0.5)$	0.9753
$D(0, 0.75)$	0.9632
$D(0, 1)$	0.9512

Determine the par swap rate for a $1y$ interest rate swap with quarterly payment under 30/360 day count convention.

ans.: 5.03%

$$S = \frac{1 - D(0,1)}{0.25\left[D(0,0.25) + D(0,0.5) + D(0,0.75) + D(0,1)\right]}$$

Pricing Interest Rate Swaps

Example Consider a $1y$ interest rate swap with semi-annual payments under 30/360 day count convention. The continuously compounded zero rates are as follows:

$L(6m,12m) = \frac{1}{\Delta_{6m}} \cdot \frac{D(0,6m) - D(0,12m)}{D(0,12m)}$

Maturity	Rate
$6m$	2.5%
$12m$	2.75%

$D(0,6m) = e^{-0.5 \times 0.025}$

$D(0,12m) = e^{-1 \times 0.0275}$

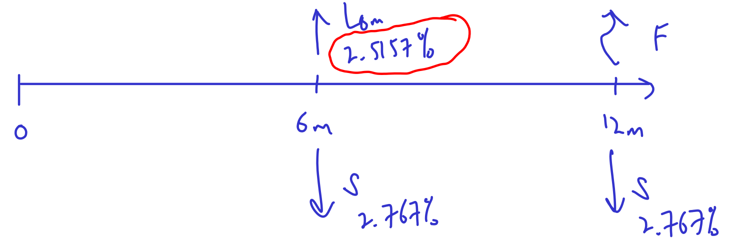
1. Calculate the forward LIBOR rate $L(6m, 12m)$.
2. Calculate the par swap rate.

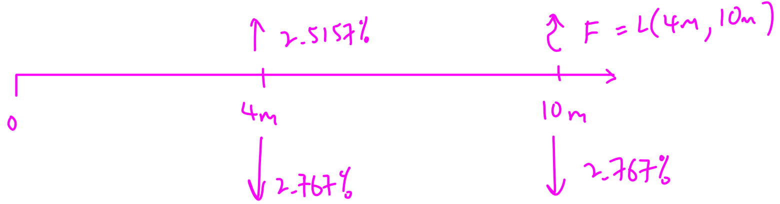
$$S = \frac{1 - D(0,12m)}{0.5\left[D(0,6m) + D(0,12m)\right]}$$

ans.: 3.023%. 2.767%

$$L(0,6m) = \frac{1}{0.5} \cdot \frac{1 - D(0,6m)}{D(0,6m)} = 2.5157\%$$

$$L(0,12m) = \frac{1}{1} \cdot \frac{1 - D(0,12m)}{D(0,12m)} = 2.789\%$$





$$PV_{fix} = D(0,4m)\cdot 0.5 \cdot 0.02767 + D(0,10m)\cdot 0.5 \cdot 0.02767$$

$$PV_{flt} = D(0,4m)\cdot 0.5 \; 0.025157 + D(0,10m)\cdot 0.5 \cdot L(4m,10m)$$

Valuation of Interest Rate Swaps

Example Continuing from the previous example, suppose we long a payer swap at the par swap rate. $2m$ later, the zero rate is as follows:

Maturity	Rate
$4m$	2.68%
$10m$	2.85%

$D(0,4m) = e^{-\frac{4}{12} \cdot 0.0268}$

$D(0,10m) = e^{-\frac{10}{12} \cdot 0.0285}$

What is the value of our swap position?

ans.: L(4m, 10m) = 2.985%. PVfix = 0.02722. PVflt = 0.02704

$$L(4m, 10m) = \frac{1}{0.5} \cdot \frac{D(0,4m) - D(0,10m)}{D(0,10m)}$$

Interest Rate Risk of IRS

Interest rate (IR) delta is the change in the NPV of the swap when interest rate increases:

$\Rightarrow$ Payer swap's PV <u>increases</u> when rates move <u>upward</u>

$\Rightarrow$ Payer swap's PV <u>decreases</u> when rates move <u>downward</u>

$\Rightarrow$ Receiver swap's PV <u>decreases</u> when rates move <u>upward</u>

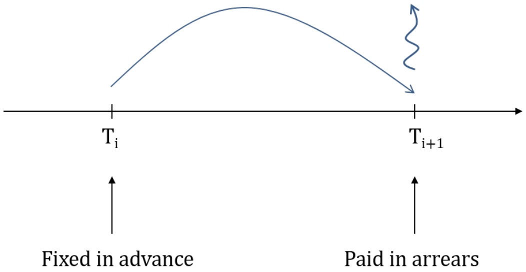
$\Rightarrow$ Receiver swap's PV <u>increases</u> when rates move <u>downward</u>

Understanding:

- Imagine you have a payer swap in your portfolio. You are paying fixed rate, and receiving floating rate.
- If LIBOR rates move up, it is good for you since you are paying fixed and receiving floating LIBOR.

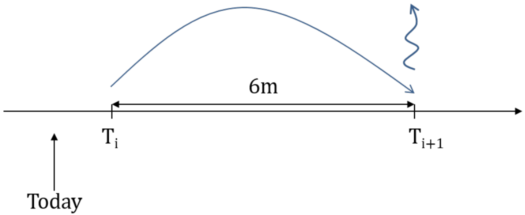
Floating Leg Convention

A forward LIBOR payment, accrued from T_i to T_{i+1}



Floating Leg Convention

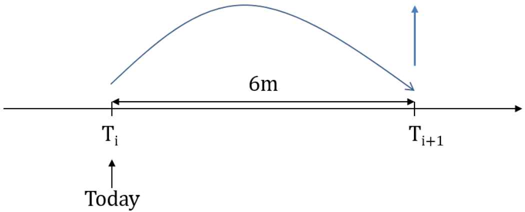
E.g. consider a 6m forward LIBOR payment



- We won't know that the LIBOR rate will be yet.
- But we can work out the no-arbitrage forward LIBOR rate if we know the spot LIBORs $L(0, T_i)$ and $L(0, T_{i+1})$
- $F(T_i, T_{i+1})$ needs to be calculated

Floating Leg Convention

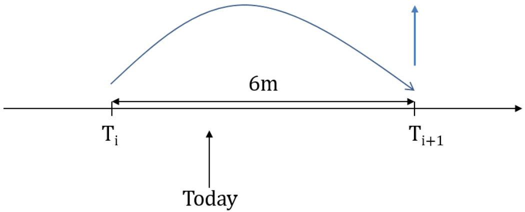
E.g. consider a 6m forward LIBOR payment



- The 6m LIBOR we will be paid is fixed today, it will be the spot 6m LIBOR rate in the market.

Floating Leg Convention

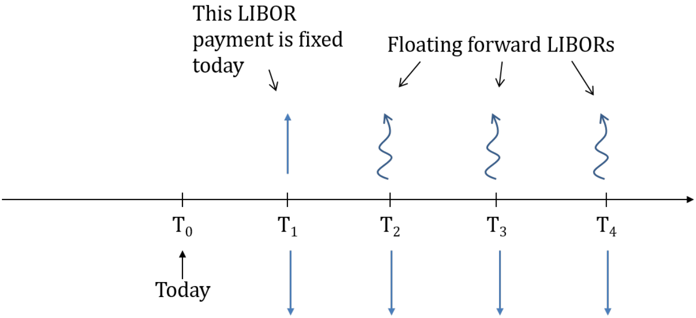
E.g. consider a 6m forward LIBOR payment



- The LIBOR payment at T_{i+1} will not change anymore since it's already been fixed
- Accrue period started at T_i and will end at T_{i+1}
- The day count fraction is always $T_{i+1} - T_i$

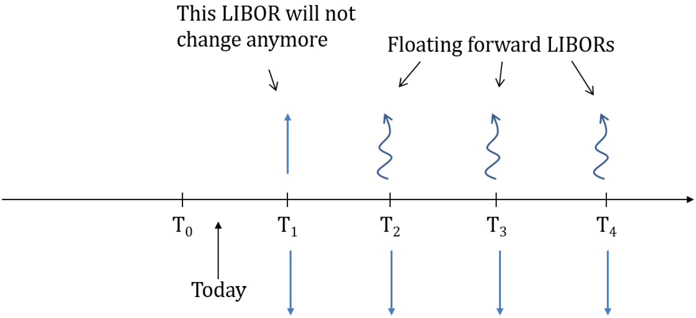
Floating Leg Convention

E.g. consider a spot starting interest rate swap



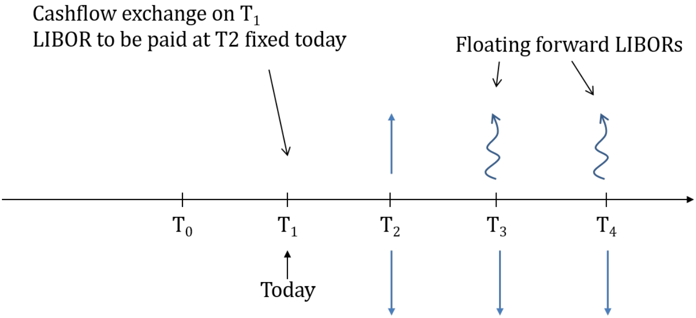
Floating Leg Convention

E.g. consider a spot starting interest rate swap



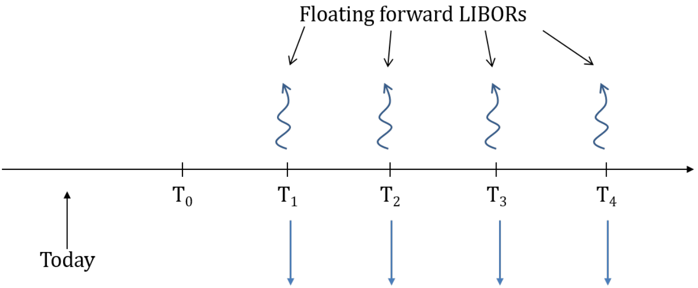
Floating Leg Convention

E.g. consider a spot starting interest rate swap



Floating Leg Convention

E.g. consider a forward starting interest rate swap



Session 3: Multicurve Framework and OIS Discounting

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QF605 Fixed Income Securities

$$1 + \frac{D_o(0,t_{i-1}) - D_o(0,t_i)}{D_o(0,t_i)} = \frac{\cancel{D_o(0,t_i)} + D_o(0,t_{i-1}) - \cancel{D_o(0,t_i)}}{D_o(0,t_i)}$$

Overnight Index Swaps

Overnight Index Swaps (OIS) pay a fixed swap rate *vs.* a floating leg whose rate is compounded according to the daily overnight rate.

The swaps are quoted like IRS, by their par swap rate. Notes these swaps do not depend on LIBOR – they capture a different segment of the rates market.

OIS with maturity of $1y$ or less have a single payment on each leg. The compounding leg accrues daily, this is typically equal to the rate length.

$$\prod_{i=1}^{N}(1+\Delta_{i-1} f_o(t_{i-1},t_i)) = (1+\Delta S_o)$$

$$\prod_{i=1}^{N}\left(1+\Delta_{i-1}\left(\frac{1}{\Delta_{i-1}}\cdot\frac{D_o(0,t_{i-1})-D_o(0,t_i)}{D_o(0,t_i)}\right)\right) = (1+\Delta S_o)$$

$$\prod_{i=1}^{N}\frac{D_o(0,t_{i-1})}{D_o(0,t_i)} = (1+\Delta S_o)$$

$$\frac{D_o(0,t_0)}{\cancel{D_o(0,t_1)}}\cdot\frac{\cancel{D_o(0,t_1)}}{\cancel{D_o(0,t_2)}}\cdot\frac{\cancel{D_o(0,t_2)}}{\cancel{D_o(0,t_3)}}\cdots\cdots\frac{\cancel{D(0,t_{n-1})}}{D(0,t_n)}$$

$$\frac{1}{D_o(0,t_N)} = (1+\Delta S_o)$$

$$\left(1+\frac{f_1}{360}\right)\left(1+\frac{f_2}{360}\right)\cdots\left(1+\frac{f_n}{360}\right)$$

Overnight Index Swaps

Example We see the following instruments in the market:

Instrument	Quote
$1y$ OIS	1.5%
$12m$ LIBOR	2.75%

S_0

Assuming 30/360 day count convention, calculate

1. The OIS discount factor for $1y$.
2. The $12m$-LIBOR discount factor for $1y$.

ans.: (1) $D_o(0,1y) = 0.9852$, $D(0,1y) = 0.9732$.

$$\frac{1}{1+1\cdot 0.015} \qquad \frac{1}{1+1\cdot 0.0275}$$

Types of Overnight Index

SOFR is an example of an overnight rate.

⇒ Based on transactions in the Treasury repurchase (**repo**) market where investors offer banks overnight loans backed by their bond assets.

Another important example is the **Effective Federal Funds Rate** (EFFR).

⇒ All US banks are required by law to hold reserves (a certain percentage of their deposits) with the banks that make up the Federal Reserve Bank system (US central bank).

⇒ There is incentive for these US banks to lend their excess money to other banks who might be short of reserves.

⇒ Every business day, US banks execute 1-day loan among themselves (hence the term "overnight"), and the interest rate charged is reported on the next day (by the New York Fed) as the weighted average called the **EFFR**.

⇒ These interbank transactions are known as the **Fed Funds market**.

Types of Overnight Index

The **Federal Open Market Committee** (FOMC) is responsible for implementing national monetary policy, and uses the EFFR to measure the success of their policy.

- The Federal Funds Rate is the target interest rate set by the Federal Open Market Committee (FOMC).
- FOMC cannot force banks to charge the exact Fed Fed Rate, but the Fed can adjust the money supply so that interest rates will move towards the target rate, either by increasing the amount of money so that interest rates fall or decreasing supply so interest rates rise.
- The FOMC meets eight times a year to set the target Fed Funds Rate based on prevailing economic conditions.

In summary, banks can borrow:

- **secured at SOFR** using Treasuries as collateral, or
- **unsecured at EFFR** without collateral.

Overnight Index Futures

SOFR and EFFR Futures are traded at CME. All contracts are cash settled, and for each contract month, trading ceases on the last day of the delivery month.

Used to hedge against unexpected shifts in short-term interest rates, or to speculate on interest rate movements.

- ⇒ Price Quote: $100 - F$, where F is the average daily overnight rate for the delivery month.
- ⇒ E.g. 98.5 means 1.5%

$$F = \sum_{i=1}^{30} \frac{f_i}{30}$$

For the **current month** the contract price is equal to a weighted average of the actual overnight rates realized to date and the expected overnight rates for the remainder of the month.

The pricing for contracts in **deferred (future) months** is based on the average expected overnight rates for the contract month—based only on expected rates.

Futures Settlement

The overnight rate implied by the futures contract is equal to 100 minus the contract price.

$100 - F$

$\parallel$

$$\text{Settlement Price} = 100 - \frac{\sum_{i=1}^{n} f_i}{n}$$

where n is the number of calendar days in that month.

For current month contract, before the final settlement, the pricing becomes a weighted average of the realized and expected rates:

$$\text{Futures Price} = 100 - \left(\frac{\sum_{i=1}^{k} f_i}{n} + \frac{\sum_{i=k+1}^{n} \mathbb{E}[f_i]}{n}\right)$$

where k is the number of days to date and n is the number of calendar days in that month.

If market participants anticipate a rate change by the Fed, the market price of futures contracts will adjust to reflect the anticipated rate change.

$100 - 97.75 = \underbrace{2.25}_{F}\%$

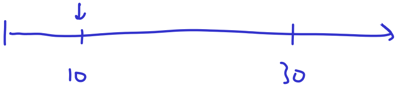
Final Settlement

Example If we are 10 days into a month, and suppose there are 30 calendar days in this month, and the averaged realized rate is 2.156%, and the futures quotes is 97.75 (=2.25%), then

$$2.25 = \frac{10}{30} \times 2.156 + \frac{20}{30} \times \mathbb{E}\left[\frac{\sum_{i=k+1}^{n} f_i}{20}\right]$$

$$= \frac{10}{30} \times 2.156 + \frac{20}{30} \times \mathbb{E}[\bar{f_i}]$$

$$\mathbb{E}[\bar{f_i}] = 2.297$$



Implied Probability

Example Suppose today is 16-Aug, the Sep futures is trading at 98.1 (=1.9%). The prevailing Fed rate is 2%, we wish to work out the implied probability of a rate cut to 1.75% on 10-Sep, when an FOMC meeting is held.

$\Rightarrow$ We can work out the probability of a rate cut based on the futures quotes:

$$1.9 = \frac{10}{30} \times 2 + \frac{20}{30} \times \Big[1.75 \times p + (1-p) \times 2\Big]$$