SMU
SINGAPORE MANAGEMENT UNIVERSITY

# Session 2: Interest Rate and Swap Market

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QF605 Fixed Income Securities

# Bond Portfolio Immunization

The process of obtaining a portfolio with zero dollar duration and dollar convexity is called **portfolio immunization**, and can be done by taking positions in other bonds available in the market.

Let $V$ be the value of a portfolio with dollar duration $D_\$(V)$ and dollar convexity $C_\$(V)$. Take positions of sizes $B_1$ and $B_2$, respectively, in two bonds with duration and convexity $D_i$ and $C_i$ for $i = 1, 2$.

The value of the new (hedged) portfolio is

$$\Pi = V + B_1 + B_2.$$

Note that $D_\$(B_1) = B_1 D_1$, $D_\$(B_2) = B_2 D_2$, $C_\$(B_1) = B_1 C_1$, $C_\$(B_2) = B_2 C_2$.

Choose $B_1$ and $B_2$ such that the dollar duration and dollar convexity of the portfolio $\Pi$ are equal to 0, i.e. such that

$$\begin{cases} B_1 D_1 + B_2 D_2 = -D_\$(V) \\ B_1 C_1 + B_2 C_2 = -C_\$(V) \end{cases}$$

# Bond Portfolio Immunization

**Example** Suppose we have just invested \$1 million in a bond with duration 3.2 and convexity 16, and \$2.5 million in a bond with duration 4 and convexity 24.

1. What are the dollar duration and the dollar convexity of your portfolio?
2. If the yield curves go up by 10 basis points, estimate the new value of the portfolio.
3. You can buy or sell two other bonds, one with duration 1.6 and convexity 12, and another one with duration 3.2 and convexity 20. What positions would you take in these bonds to immunize your portfolio, i.e. to obtain a portfolio with zero dollar duration and dollar convexity?

# Bond Portfolio Immunization

**Solution**

1. We have

$$D_\$(V) = B_1D_1 + B_2D_2 = \$13,200,000$$
$$C_\$(V) = B_1C_1 + B_2C_2 = \$76,000,000$$

2. We have

$$\begin{aligned}\Delta V &\approx -D_\$(V)\Delta r + \frac{C_\$(V)}{2}\cdot(\Delta r)^2\\ &\approx -\$13,162\end{aligned}$$

Hence $V' \approx V + \Delta V = \$3,500,000 - \$13,162 = \$3,486,838.$

3. Let $B_3$ and $B_4$ be the values of the positions taken in the two bonds. We have $\Pi = V + B_3 + B_4$, and

$$\begin{aligned}D_\$(\Pi) &= \$13.2mil + D_3B_3 + D_4B_4\\ &= \$13.2mil + 1.6B_3 + 3.2B_4\\ C_\$(\Pi) &= \$76mil + 12B_3 + 20B_4.\end{aligned}$$

Solving, we obtain $B_3 = \$3.25mil$ and $B_4 = -\$5.75mil$.

# Bonds with Negative Yields

Why did investors buy bonds with negative yields?

1. **Regulatory requirements**

   ⇒ Central banks, pension funds, insurance companies, banks are required to hold government bonds to meet liquidity requirements and also to pledge as collateral when they borrow.

2. **Potential capital gain**

   ⇒ Further interest rate cuts will cause bond prices to appreciate.

   ⇒ If a negative yielding currency might appreciate, investors buy bonds to benefit from currency appreciation.

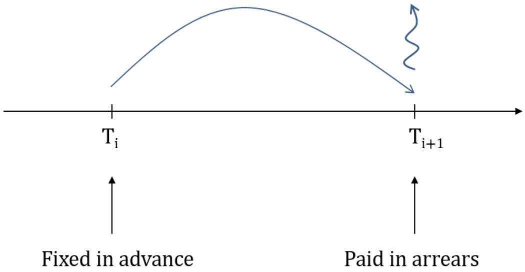
   ⇒ If deflation is likely, investors buy bonds to profit in "real" terms (buy more goods and services), even though there is nominal loss.

3. **Cash is not optimal**

   ⇒ If the government bond yields are negative, cash interest rates are likely to be negative too, so holding cash is not better.

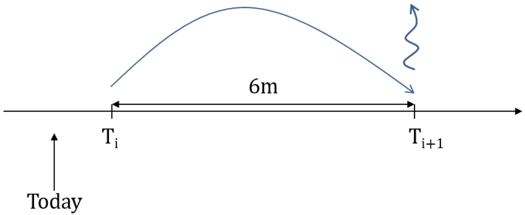
# Floating Leg Convention

A forward LIBOR payment, accrued from $T_i$ to $T_{i+1}$



# Floating Leg Convention

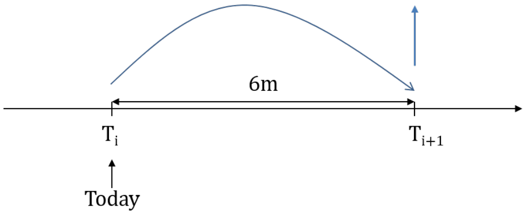
E.g. consider a 6m forward LIBOR payment




- We won't know that the LIBOR rate will be yet.
- But we can work out the no-arbitrage forward LIBOR rate if we know the spot LIBORs $L(0, T_i)$ and $L(0, T_{i+1})$
- $F(T_i, T_{i+1})$ needs to be calculated

# Floating Leg Convention

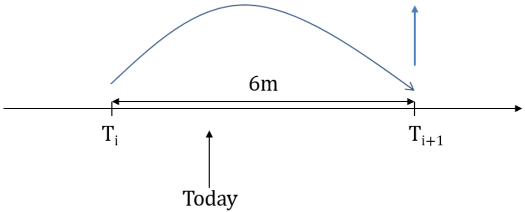
E.g. consider a 6m forward LIBOR payment




- The 6m LIBOR we will be paid is fixed today, it will be the spot 6m LIBOR rate in the market.

# Floating Leg Convention

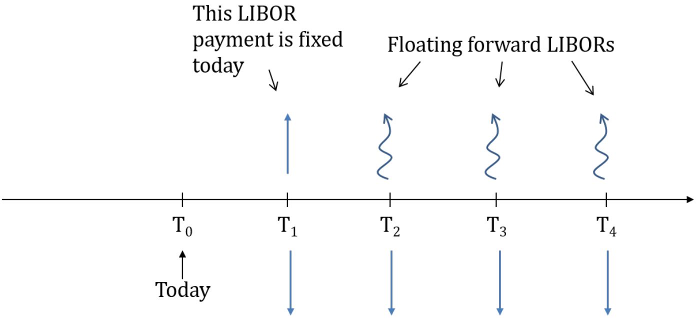
E.g. consider a 6m forward LIBOR payment




- The LIBOR payment at $T_{i+1}$ will not change anymore since it's already been fixed
- Accrue period started at $T_i$ and will end at $T_{i+1}$
- The day count fraction is always $T_{i+1} - T_i$

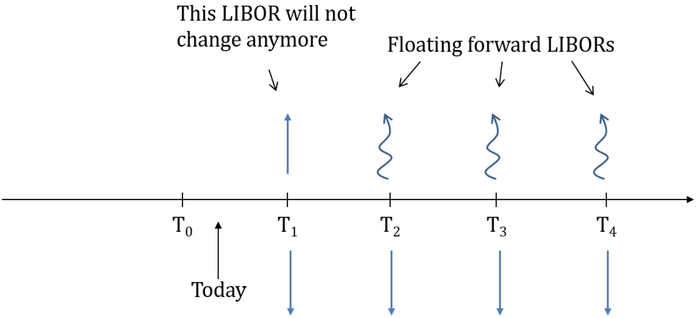
# Floating Leg Convention

E.g. consider a spot starting interest rate swap



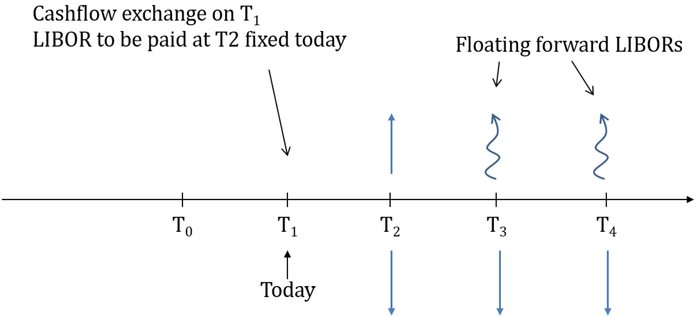
# Floating Leg Convention

E.g. consider a <u>spot starting</u> interest rate swap



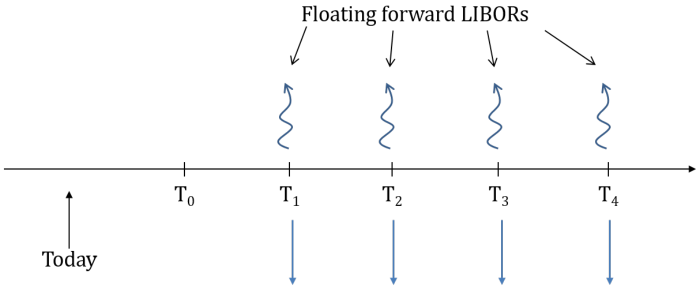
# Floating Leg Convention

E.g. consider a spot starting interest rate swap



# Floating Leg Convention

E.g. consider a forward starting interest rate swap



# Case Study: LIBOR Origin

Created in 1969 by Minos Zombanakis in London, who arranged an $80 million syndicated loan for the Shah of Iran.

$\Rightarrow$ The first loan to charge a floating rate, split among a group of banks.

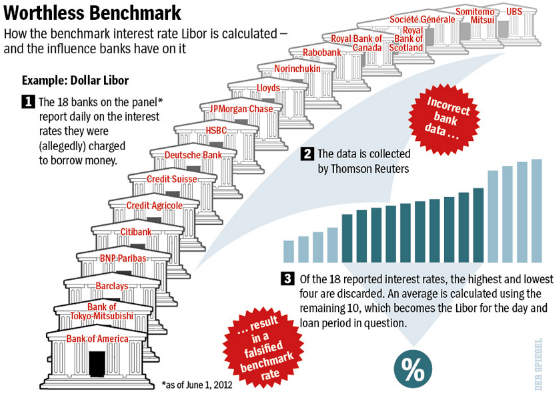
Subsequently became the benchmark for loans or bonds pricing.

Starting in 1980s, it became also the benchmark for complex derivatives.

Up until just before the 2008 crisis, LIBOR has become a rate that reflects the cost of unsecured borrowing for a specific period (generally $3m$ or $6m$).

# Case Study: LIBOR Scandal

Source: Der Spiegel



# Case Study: How was LIBOR calculated?

- Everyday, the Intercontinental Exchange (ICE) surveys a panel of banks asking the question
  *At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?*
- Throw away the highest and lowest portion of the responses (4 each for USD LIBOR), and averages the remaining middle (10 for USD LIBOR).
- The average is reported at 11:30 am and published worldwide by Thomson Reuters.
- This process is carried out for 10 currencies for maturity up to 1 year.

# Case Study: How was LIBOR rigged?

*Panel A: Proper Submission*

| Bank | Rate | Quartile |
|---|---|---|
| 1 | 5.00% | Upper 25% |
| 2 | 4.90% | Upper 25% |
| 3 | 4.80% | Upper 25% |
| 4 | 4.70% | Upper 25% |
| 5 | 4.60% | Included |
| 6 | 4.50% | Included |
| 7 | 4.40% | Included |
| 8 | 4.30% | Included |
| 9 | 4.20% | Included |
| 10 | 4.10% | Included |
| 11 | 4.00% | Included |
| 12 | 3.90% | Included |
| 13 | 3.80% | Included |
| 14 | 3.70% | Included |
| 15 | 3.60% | Lower 25% |
| 16 | 3.50% | Lower 25% |
| 17 | 3.40% | Lower 25% |
| 18 | 3.30% | Lower 25% |

Calculated LIBOR: 4.15%

*Panel B: Misrepresentation of Rate*

| Bank | Rate | Quartile |
|---|---|---|
| 1 | 5.00% | Upper 25% |
| 2 | 4.90% | Upper 25% |
| 3 | 4.80% | Upper 25% |
| 4 | 4.70% | Upper 25% |
| 5 | 4.60% | Included |
| 6 | 4.50% | Included |
| 7 | 4.40% | Included |
| 8 | 4.30% | Included |
| 9 | 4.20% | Included |
| 10 | 4.10% | Included |
| 11 | 4.00% | Included |
| 12 | 3.90% | Included |
| 13 | 3.80% | Included |
| 14 | 3.70% | Included |
| 15 | 3.60% | Lower 25% |
| 16 | 3.50% | Lower 25% |
| 17 | 3.40% | Lower 25% |
| 18 | 3.30% | Lower 25% |

Misrepresented as 3.35%

*Panel C: Understatement of LIBOR*

| Bank | Rate | Quartile | |
|---|---|---|---|
| 1 | 5.00% | Upper 25% | |
| 3 | 4.80% | Upper 25% | |
| 4 | 4.70% | Upper 25% | |
| 5 | 4.60% | Upper 25% | Improperly excluded |
| 6 | 4.50% | Included | |
| 7 | 4.40% | Included | |
| 8 | 4.30% | Included | |
| 9 | 4.20% | Included | |
| 10 | 4.10% | Included | |
| 11 | 4.00% | Included | |
| 12 | 3.90% | Included | |
| 13 | 3.80% | Included | |
| 14 | 3.70% | Included | |
| 15 | 3.60% | Included | Improperly included |
| 16 | 3.50% | Lower 25% | |
| 17 | 3.40% | Lower 25% | |
| 2 | 3.35% | Lower 25% | |
| 18 | 3.30% | Lower 25% | |

Calculated LIBOR: 4.05% Understated result

If your "fair" rate is in the middle, you can raise or lower LIBOR by submitting a rate on the desired direction.

If your "fair" rate is on the upper end, you can only lower LIBOR, by submitting a low rate, and vice versa.

# Case Study: Barclays' LIBOR Scandal

3 types of manipulation

1. Altering survey's response for the benefit of own derivative positions – requests from the trading desks.
2. Altering survey's reponse to protect Barclays' reputation – high submission rate is seen as a sign of weakness.
3. Attempting to induce other banks to alter their survey responses.

In short, the mechanism (trimmed mean) put in place to remove extreme values leaves survey participants with partial power to move rates in the direction they want.

# Case Study: Recent LIBOR

Every morning, a group of banks from around the world submits estimates of the lowest possible interest rate at which the institutions can borrow money from another bank on that day. There are five panels, each for a different currency, and every panel produces a rate for seven maturities, for a total of 35 rates a day. Below are the rates submitted on July 29 for U.S. dollar-denominated loans with a three-month maturity.

| | |
|---|---|
| RABOBANK | 0.70000 |
| JPMORGAN CHASE | 0.70000 |
| SOCIETE GENERALE | 0.72000 |
| ROYAL BANK OF CANADA | 0.72000 |
| CITIGROUP | 0.72500 |
| BANK OF AMERICA | 0.73000 |
| HSBC | 0.73000 |
| UBS | 0.73100 |
| BANK OF TOKYO-MITSUBISHI | 0.76000 |
| CREDIT SUISSE | 0.76000 |
| ROYAL BANK OF SCOTLAND | 0.77500 |
| BARCLAYS | 0.78000 |
| SUMITOMO MITSUI FINANCIAL GROUP | 0.79000 |
| NORINCHUKIN | 0.81000 |
| BNP PARIBAS | 0.81000 |
| LLOYDS | 0.84000 |
| CREDIT AGRICOLE | 0.86250 |
| DEUTSCHE BANK | 0.90000 |

The top and bottom quarter of submissions are discarded to avoid outliers. There were 18 contributors, so the highest and lowest four were cut.

| | |
|---|---|
| RABOBANK | 0.70000 |
| JPMORGAN CHASE | 0.70000 |
| SOCIETE GENERALE | 0.72000 |
| ROYAL BANK OF CANADA | 0.72000 |
| CITIGROUP | 0.72500 |
| BANK OF AMERICA | 0.73000 |
| HSBC | 0.73000 |
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Source: Bloomberg

# Case Study: LIBOR Replacement

After the crisis, the market for unsecured inter-bank lending failed to recover – most daily LIBOR submissions are based on “expert judgement”.


US central bank data shows that in the $2^{nd}$ quarter in 2018, the median number of unsecured borrowing was 7.

$\Rightarrow$ Many may not be aware of how truly thin these markets have become.

# Case Study: LIBOR Replacement

Central banks in different economies started to look for alternative rates to be used instead of LIBOR-style reference rates.

- USD: **S**ecured **O**vernight **F**inancing **R**ate (SOFR), the Treasury repo rate for overnight loan. Daily average of US Treasury-collateralized overnight repurchase agreement (repo) transaction data collected by the US Federal Reserve
- GBP: **S**terling **O**ver **N**ight **I**ndex **A**verage (SONIA), which tracks the actual (unsecured) overnight deals in money markets.
- JPY: **T**okyo **O**ver **N**ight **A**verage **R**ate (TONAR), unsecured overnight rate published by Bank of Japan.
- CHF: **S**wiss **A**verage **R**ate **O**ver**N**ight (SARON), based on CHF repo market.
- EUR: **E**uro **S**hort **T**erm **R**ate (€STER), based on actual money market transactions.
- SGD: **S**ingapore **O**vernight **R**ate **A**verage (SORA). Determined by the volume-weighted average rate of borrowing transactions in the unsecured overnight interbank SGD cash market in Singapore.

# Term Rate: LIBOR & Its Replacement

If you put your money in a money-market account for a given period, the interest earned over this period is quoted as a **term rate** (e.g. LIBOR). At the end of a period of length $\Delta$, one receives an interest equal to $\Delta \times L$, where $L$ denotes the term rate and $\Delta$ denotes the day count fraction.

Since $L$ is always quoted as annualized rate, $\Delta$ is often referred to as the **accrual fraction** or **day count fraction**.

We have $1 \equiv (1 + \Delta \cdot L) \cdot D(0, \Delta)$.

This just states that the present value today of the notional plus the interest earned at the end of the $\Delta$ period is equal to the notional.

Note that in real markets $\Delta$ is not exactly equal to 0.25, 0.5, or 1, but is calculated according to a specific method for a given market, known as the **day count convention** (e.g. Act/365, Act/360 etc.)

# Term Rate (LIBOR) Market

A **forward term rate** (e.g. forward LIBOR) is the interest rate one can contract for at time $t$ to put money in a money-market account for the time period $[T, T+\Delta]$. Then we have

$$D(t,T) = (1 + \Delta \cdot L) \cdot D(t, T+\Delta)$$

At the time when the **forward LIBOR** is fixed, it is then called a **spot LIBOR**. *Note that LIBOR is fixed at the beginning of each period* $T_i$*, and paid at the end of the period* $T_{i+1}$*. We call this* **"fixed in advance, paid in arrears."**

In most markets, only forward LIBOR rates of one specific tenor are actively traded, which is usually $3m$ (e.g. USD) or $6m$ (e.g. SGD, GBP, EUR).

Everyday we have a large number of forward LIBOR rates with this specific tenor, we can denote them as

$$i = 1, 2, \cdots, N: \; L(T_i, T_{i+1})$$

# Forward Rate Agreements (FRA)

Suppose you agree to an interest rate today, which you will obtain on an amount of money invested at later date, that is returned with interest at an even later date.

This kind of agreement is called a **forward rate agreement (FRA)**.

This level of the agreed rate that makes the value of entering into a forward rate agreement equal to zero is called the **forward rate**.

You have cashflow liability in the future (e.g need to borrow or deposit cash) but do not want to be exposed to interest rate risks.

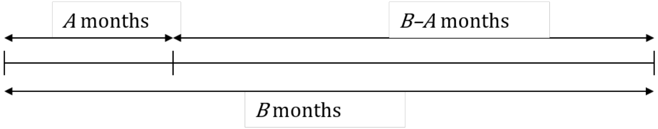
Under a FRA contract:

- **Buyer** is obligated to **borrow** money at a pre-determined or fixed interest rate on the date when the FRA expires.
- **Seller** is obligated to **lend** at the fixed FRA rate. Thus both counterparties lock in a rate in advance and the actual lending and borrowing is done at the FRA expiration date.

# Forward Rate Agreements (FRA)

FRA allows you to **lock-in an interest rate today** at which you can borrow or deposit in the future.

- ⇒ For borrowers, if the market rate ends up higher, then it is good for you. The opposite is true for lenders.
- ⇒ For borrowers, if the market rate ends up lower, then it is bad for you. The opposite is true for lenders.




The FRA is quoted as $A \times B$ in terms of months. For example, a $3 \times 9$ FRA means the forward contract expires in 3 months, and the underlying interest rate is a 6-month LIBOR.

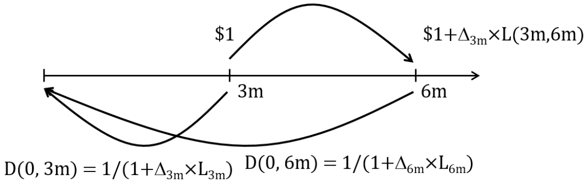
# Forward Rate Agreements (FRA)

\$1 in $3m$'s time is worth $D(0, 0.25)$ today. If you invest that amount for $3m$ at the forward LIBOR $L(3m, 6m)$, then you earn an interest

$$D(0,3m) = D(0,6m)(1 + \Delta_{3m} L(3m,6m))$$

$$\Rightarrow \quad L(3m,6m) = \frac{1}{\Delta_{3m}} \frac{D(0,3m) - D(0,6m)}{D(0,6m)}$$

In words, we can work out the forward LIBOR rate based on spot LIBOR rates and discount factors.

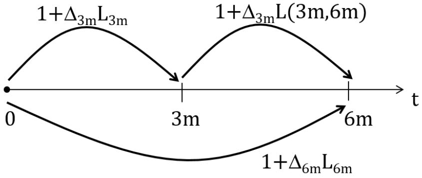


# Forward Rate Agreements (FRA)

We can obtain the same result using either discount factors or spot LIBOR rates:

$$(1+\Delta_{3m}L_{3m})(1+\Delta_{3m}L(3m,6m)) = 1+\Delta_{6m}L_{6m}$$

$$\Rightarrow \quad L(3m,6m) = \frac{1}{\Delta_{3m}}\frac{(1+\Delta_{6m}L_{6m})-(1+\Delta_{3m}L_{3m})}{1+\Delta_{3m}L_{3m}}$$




**Example** If the $3m$ LIBOR rate is 4% and the $6m$ LIBOR rate is 6%, what is the in-$3m$-for-$3m$ forward rate $F(3m,6m)$?

# Forward Rate Agreements (FRA)

**Example** Suppose we have the following discretely compounded zero rates:

| Maturity | Zero Rate |
|---|---|
| $1y$ | 3.96% |
| $2y$ | 5.47% |
| $3y$ | 6.14% |

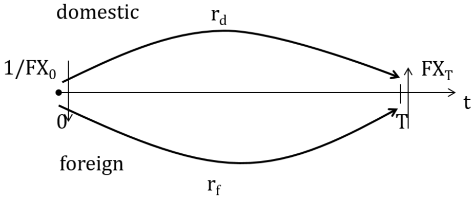
Determine the forward rates $L(0, 1y)$, $L(1y, 2y)$, and $L(2y, 3y)$.

ans.: $L(1y, 2y) = 7\%$, $L(2y, 3y) = 7.5\%$.

# Interest Rate Parity

**Interest rate parity** is a no-arbitrage condition stating that it is impossible for investors to gain higher returns by borrowing and lending in the money market account of 2 economies.

This gives rise to a relationship between the interest rates and exchange rates of the 2 economies.




Interest rate parity:

$$FX_T = FX_0 \times \left(\frac{1 + r_d}{1 + r_f}\right)^T$$

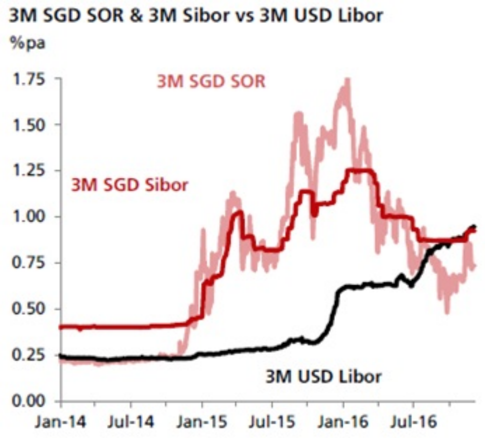
# Case Study: Swap Offer Rate (SOR) in Singapore

**SOR** is a **synthetic rate** in the Singapore market.

**E. Calculation Methodology: Singapore Dollar Swap Offer Rate (SOR)**

| | |
|---|---|
| **Benchmark** | **Singapore Dollar Swap Offer Rate (SOR)** |
| **Description** | The synthetic rate for deposits in Singapore Dollars (SGD), which represents the effective cost of borrowing the Singapore Dollars synthetically by borrowing US Dollars (USD) for the same maturity, and swap out the US Dollars in return for the Singapore Dollars. |
| **Calculation Methodology** | The Administrator shall calculate and determine the Rate, for each maturity matching each Tenor specified below (each a "calculation period"), on each Business Day as follows:<br><br>**SGD SOR** =<br>$\left\{\left[\left(\frac{\text{Spot Rate}+\text{Forward Points}}{\text{Spot Rate}}\right)\times\left(1+\frac{\text{USD Rate}\times\#\text{days}}{360}\right)\right]-1\right\}\times\frac{365}{\#\text{days}}\times 100$<br><br>Where:<br><br>**USD Rate** means the rate for deposits in USD for a period of the calculation period which appears on Thomson Reuters Screen LIBOR01 (or successor page displaying USD LIBOR) as of 11:00 am, London time, on the same Business Day. |

# Case Study: Swap Offer Rate (SOR) in Singapore



# Case Study: MIFOR Rate in India

**Mumbai Interbank Forward Offered Rate** (MIFOR) in the India market.

Banks to provide a quotation of their offered side of INR/USD forward points for the forward sale of INR against USD for settlement on the last day of a period equivalent to the Designated Maturity and commencing on the Reset Date and the forward points so determined by the Calculation Agent shall be the "Forward Points" for purposes of the following formula. The Calculation Agent will then determine the rate for that Reset Date by applying the following formula:

Floating Rate =

*{[(Spot Rate + Forward Points) / Spot Rate * (1+ LIBOR * N1)] – 1} * N2*100*

where:

"Spot Rate" means the Reserve Bank of India's published USD/INR spot rate (expressed as a number of INR per one USD) which appears on Reuters Screen "RBIB" as of 13:00 p.m. India Standard Time on that Reset Date (if such rate is not available the Calculation Agent will ask each of the Reference Banks to provide a quotation of such rate);

"LIBOR" means USD-LIBOR-BBA for a period of the Designated Maturity commencing on the Reset Date;

"N1" means the number of days in the Calculation Period divided by 360;

"N2" means 365 divided by the number of days in the Calculation Period.

# Case Study: THBFIX in Thailand

**THBFIX**—the synthetic rate for deposits in Thai Baht, representing the effective cost of borrowing the Thai Baht synthetically by borrowing USD and swapping out the USD in return for THB.

$$THBFIX = \left\{\left[\left(\frac{Spot\ Rate + Forward\ Points}{Spot\ Rate}\right) \times \left(1 + \frac{USD\ Rate \times \#days}{360}\right)\right] - 1\right\} \times \frac{365}{\#days} \times 100$$

Where:

"USD Rate" means US dollar interest rate (USD rate) in each tenor that is published on Reuters screen LIBOR01 (or other screens that shows USD LIBOR) at 11.55 a.m. London time during that business day.

"#days" means the actual number of days in each tenor (which may differ on each business day).

"Spot Rate" means USDTHB spot rate which is calculated via a volume weighted average method (excluding transactions with unusually large volume or unusually high or low spot rates), in relation to all USDTHB FX Spot Qualified Transactions on each business day.

"Forward Points" means Forward points which is calculated via a volume weighted average method (excluding transactions with unusually large volume or unusually high or low forward points), in relation to all USDTHB FX Swap Qualified Transactions on each business day.

# Case Study: LIBOR Replacement and Fallback

The USD LIBOR panel has ended on 30-June-2023. Some ($1m$, $3m$, $6m$) Synthetic LIBORs will continue to be published until 30-Sep-2024. The Alternative Reference Rates Committee (ARRC) recommended the US Secured Overnight Financing Rate (SOFR) to replace the USD LIBOR.

Main LIBOR transition issues to be addressed due the **term mismatch** and the **credit sensitivity**:

- ⇒ SOFR is an **overnight** rate while LIBOR is a **term** rate. SOFR must be compounded daily for a period to arrive at a simple interest rate.
- ⇒ LIBOR is paid in arrears but set in advance, so it involves expectation of future short rates.
- ⇒ Different **credit exposure**.

The International Swaps and Derivatives Association (ISDA) has provided **collateral agreement** interest rate definitions that incorporate these new **risk-free rates** (RFRs):

- ⇒ SOFR to replace Effective Federal Funds Rate (EFFR) for USD
- ⇒ €STR to replace the Euro Overnight Index Average (EONIA)

# Case Study: LIBOR Replacement and Fallback

The ARRC also indicated support for the use of the **CME Term SOFR Rates** in areas where use of overnight or averages of SOFR has proven to be difficult:

- The CME Term SOFR Rates aim to provide a robust measure of forward-looking SOFR term rates based on market expectations implied from transactions in the derivatives markets.
- The methodology for determining CME Term SOFR Rates uses a combination of SOFR **overnight indexed swaps** (OIS) and one-month and three-month SOFR **futures contracts**.
  - 1m SOFR futures (SR1): 13 consecutive months contracts
  - 3m SOFR futures (SR3): 5 consecutive quarterly contracts

  CME determines the path of overnight SOFR rates by assuming the overnight SOFR rates follow a **piecewise constant step function** and can only jump up or down the day after **FOMC policy rate announcement dates** and remains at those levels across all dates in between the FOMC policy rate announcement dates.
- No expert judgement is required.

# Case Study: LIBOR Replacement and Fallback

| | Term SOFR | Overnight SOFR |
|---|---|---|
| Tenor | 1m, 3m, 6m, 12m | Overnight compounded |
| Source | CME Group | New York Fed |
| Term | Forward-looking | Backward-looking rate |
| Fixing | Known before period starts | Determine after period ends |
| Pros | Similar to LIBOR | Consistent with other currencies |
| Cons | Missing for other currencies | Interest payable unknown when loan begins |

# Case Study: LIBOR Replacement and Fallback

**LIBOR fallback** To substitute the LIBOR fixing in the derivatives by a new quantity obtained as the sum of an **adjusted RFR** and an **adjustment spread**.

- The adjusted RFR plays the role of the floating term rate—it will depend on an overnight benchmark and be known around the date where LIBOR should have been fixed.
- The adjustment spread will be decided after the discontinuation and be seen as an adjustment to avoid **value transfer** between the original LIBOR fixing and the new fixing mechanism.

**Historical mean/median approach** The spread adjustment is based on the mean or median spot spread between the LIBOR and the adjusted RFR calculated over a signficant, static lookback period prior to the relevant announcement or publication triggering the fallback provisions.

# Interest Rate Risk

- A standard company is usually financed by a combination of equity and debt. Consequently, **interest rate risk** is part and parcel of operating a company.
- A company might raise money from investors by issuing shares, and then borrows further to provide the rest of the funding.
- Clearly, the company will need to pay interest on the debt it issued, and eventually repay the debt in its entirety. If the interest rate fluctuates according to the prevailing rates in the market, it is said to be **floating**.
- The interest payments is typically a sizable burden, and could cripple a company if it rises too much. It is therefore better for a company to go for a **fixed-rate loan**.
- This obviously adds on to the complexity of the problem—if rates fall, it will be beneficial for the company to **refinance**.

# Interest Rate Swaps

**Interest rate swap** is a contract where two parties agree to exchange a set of floating interest rate payments for a set of fixed interest rate payments.

- The set of floating interest rate payments is based on LIBOR rates and is called the **floating leg**.
- The set of fixed payments is called the **fixed leg**.

The naming convention for swaps is based on the fixed side: **payer swap** means you pay the fixed leg, while **receiver swap** means you received the fixed leg

- The times at which the floating interest rates are set are called **reset dates**.
- The times at which the interest rate payments are made are called **payment dates**.
- The time between payment dates are called **settlement period**.
- The length of the swap is called the **tenor** of the swap (from start till maturity).
- Typically, the parties agree to exchange only the net amount owed from one party to the other party. This practice is called **netting**.
- If payments are not in the same currency, netting is not appropriate.

# Interest Rate Swaps – Rationale

Interest rate swaps are useful because they allow you to turn a fixed-rate asset into a floating rate asset, and vice versa. For example:

- A bank funds itself via a mix of short-term deposits (floating rate) and fixed-rate bonds. It gives variable and fixed rate mortgages to its customers. When it raises financing, it does not know the quantities of variable and fixed rate mortgages that it will give.
- If it ends up giving more fixed rate mortgages than anticipated, it can enter into swaps to pay fixed and receive floating to hedge out interest rate risk.
- If it ends up giving more variable rate mortgages than anticipated, it can enter into swaps to receive fixed and pay floating to hedge out interest rate risk.

Interest rate swaps allow the bank to raise money first, then invest it, and fix any resulting interest rate risk exposure later.

# Interest Rate Swaps – Fixed Leg

The process of **pricing the swap** involves finding the swap rate such that the value of the swap contract is **worth par** at trade initiation (i.e. NPV=0).

To this end, we proceed by determining the PV of the fixed and floating leg separately.

The PV of a fixed leg is straightforward. Consider a single fixed rate payment of $K$ at time $T$, with an accrual fraction of $\Delta$. The PV of this payment is

$$D(0,T) \times K \times \Delta$$

Therefore, for a series of payments starting at $T_1$ and ending at $T_n$, the leg of fixed rate payment is

$$K \sum_{i=1}^{n} D(0,T_i) \times \Delta_{i-1}$$

# Interest Rate Swaps – Floating Leg

Recall that forward rate is related to discount factors:

$$L(3m, 6m) = \frac{1}{\Delta_{3m}} \frac{D(0, 3m) - D(0, 6m)}{D(0, 6m)}.$$

Given a set of payment dates $T_i$, we can express the forward LIBOR rate as

$$L(T_{i-1}, T_i) = \frac{1}{\Delta_{i-1}} \frac{D(0, T_{i-1}) - D(0, T_i)}{D(0, T_i)}.$$

The PV of a single floating rate payment at $T_i$ can therefore be expressed as:

$$D(0, T_i) \times \Delta_{i-1} \times L(T_{i-1}, T_i) = D(0, T_{i-1}) - D(0, T_i).$$

Generalizing, we see that if the floating leg comprises of $n$ payments, then its PV is given by

$$\sum_{i=1}^{n} D(0, T_i) \times \Delta_{i-1} \times L(T_{i-1}, T_i) = 1 - D(0, T_n).$$

# Par Swap Rate

In a swap multiple payments are exchanged:

- The value of a payer swap at time $t$ that starts at $T_0$ and ends at $T_n$.
- At the start date $T_0$ the first LIBOR rate is fixed.
- Actual payments are exchanged at $T_1, \ldots, T_n$.
- The swap tenor is $T_n - T_0$.

The present value of a payer swap is given by

$$\sum_{i=1}^{n} V_i^{flt}(t) - \sum_{i=1}^{n} V_i^{fix}(t) = (D(0,T_0) - D(0,T_n)) - K \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i).$$

Similarly, the present value of a receiver swap is given by

$$K \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i) - (D(0,T_0) - D(0,T_n)).$$

In the market, swaps are not quoted as prices for different fixed rates $K$, but only the fixed rate $K$ is quoted for each swap such that the present value of the swap is equal to 0.

# Par Swap Rate

This particular rate is called the **par swap rate**. We denote the par swap rate for the $[T_0, T_n]$ swap with $S$. Solving, we obtain

$$S = \frac{D(0,T_0) - D(0,T_n)}{\sum_{i=1}^{n} \Delta_{i-1} D(0,T_i)}.$$

The term in the denominator is also called the **present value of a basis point** (PVBP)

$$P_{1,n} = \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i).$$

# Pricing Interest Rate Swaps

**Example** Suppose we observe the following LIBOR discount factors in the market:

| Discount Factor | Value |
|---|---|
| $D(0, 0.25)$ | 0.9876 |
| $D(0, 0.5)$ | 0.9753 |
| $D(0, 0.75)$ | 0.9632 |
| $D(0, 1)$ | 0.9512 |

Determine the par swap rate for a $1y$ interest rate swap with quarterly payment under 30/360 day count convention.

ans.: 5.03%

# Pricing Interest Rate Swaps

**Example** Consider a $1y$ interest rate swap with semi-annual payments under 30/360 day count convention. The continuously compounded zero rates are as follows:

| Maturity | Rate |
|---|---|
| $6m$ | 2.5% |
| $12m$ | 2.75% |

1. Calculate the forward LIBOR rate $L(6m, 12m)$.
2. Calculate the par swap rate.

ans.: 3.023%. 2.767%

# Valuation of Interest Rate Swaps

**Example** Continuing from the previous example, suppose we long a payer swap at the par swap rate. $2m$ later, the zero rate is as follows:

| Maturity | Rate |
|---|---|
| $4m$ | 2.68% |
| $10m$ | 2.85% |

What is the value of our swap position?

ans.: L(4m, 10m) = 2.985%. PVfix = 0.02722. PVflt = 0.02704

# Interest Rate Risk of IRS

Interest rate (IR) delta is the change in the NPV of the swap when interest rate increases:

$\Rightarrow$ Payer swap's PV increases when rates move upward

$\Rightarrow$ Payer swap's PV decreases when rates move downward

$\Rightarrow$ Receiver swap's PV decreases when rates move upward

$\Rightarrow$ Receiver swap's PV increases when rates move downward

Understanding:

- Imagine you have a payer swap in your portfolio. You are paying fixed rate, and receiving floating rate.
- If LIBOR rates move up, it is good for you since you are paying fixed and receiving floating LIBOR.