Pricing a Caplet

floorlet : put on forward libor

$dL_i(t) = \sigma_i L_i(t)\, dW^{i+1}(t)$

The payoff of a **caplet** C_i at time T_{i+1} is given by

$dF_t = \sigma F_t \, dW_t$

call on forward libor

$$C_i(T_{i+1}) = \Delta_i (L_i(T_i) - K)^+.$$

Choosing D_{i+1} as a numeraire and working under the associated martingale measure $\mathbb{Q}^{i+1}$, we know that

$$\frac{C_i(0)}{D_{i+1}(0)} = \mathbb{E}^{i+1}\left[\frac{C_i(T_{i+1})}{D_{i+1}(T_{i+1})}\right]$$
$$\Rightarrow \quad C_i(0) = D_{i+1}(0)\Delta_i \mathbb{E}^{i+1}[(L_i(T_i) - K)^+].$$

The remaining steps required to derive a formula for a caplet price is identical to how we would handle a vanilla European option.

$$\mathbb{E}^{i+1}\left[L_i(T)\right] = \mathbb{E}^{i+1}\left[L_i(0)\, e^{-\frac{1}{2}\sigma_i^2 T + \sigma_i W^{i+1}(T)}\right]$$

Pricing a Caplet

The LIBOR rate follows the stochastic differential equation $= L_i(0)\, \cancel{e^{-\frac{1}{2}\sigma_i^2 T}}\ \cancel{e^{\frac{\sigma_i^2 T}{2}}}$

$$dL_i(t) = \sigma_i L_i(t) dW^{i+1}(t),$$

where $W^{i+1}(t)$ is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}^{i+1}$ associated with the numeraire $D_{i+1}(t)$. The solution is given by

$$L_i(T) = L_i(0) e^{-\frac{1}{2}\sigma_i^2 T + \sigma_i W^{i+1}(T)}.$$

Evaluating the expectation, we obtain

$$\begin{aligned} C_i(0) &= D_{i+1}(0)\Delta_i \mathbb{E}^{i+1}[(L_i(T_i) - K)^+] \\ &= D_{i+1}(0)\Delta_i [L_i(0)\Phi(d_1) - K\Phi(d_2)], \end{aligned}$$ $= \text{Black Call}(\quad)$

where

$$d_1 = \frac{\log \frac{L_i(0)}{K} + \frac{1}{2}\sigma_i^2 T}{\sigma_i \sqrt{T}}, \quad d_2 = d_1 - \sigma_i \sqrt{T}.$$

$\mathbb{Q}^*$: $\quad dr_t = k(\theta - r_t)dt + \sigma dW_t^*$

$$D(t,T) = e^{-\int_t^T r_u\,du} \qquad B(T) = B(t)\,e^{\int_t^T r_u\,du}$$

$)$

$$\frac{C_{i+1}(0)}{B_0} = \mathbb{E}^*\left[\frac{C_{i+1}(T_{i+1})}{B_{T_{i+1}}}\right]$$

$$\frac{1}{\Delta_i}\frac{D(t,T_i) - D(t,T_{i+1})}{D(t,T_{i+1})}$$

$$\Rightarrow \quad C_{i+1}(0) = \mathbb{E}^*\left[\frac{B_0 \cdot \Delta_i (L_i(T) - K)^+}{B_0\, e^{\int_0^T r_u\,du}}\right]$$

$$= \mathbb{E}^*\left[\Delta_i (L_i(T) - K)^+ \cdot \frac{D(T,T)/D(0,T)}{B_T/B_0}\right] \cdot D(0,T)$$

$$= D(0,T)\,\mathbb{E}^{\cancel{*}}\left[\Delta_i\left(L_i(T)-K\right)^+ \cdot \frac{dQ^T}{d\cancel{Q^*}}\right]$$

$$= D(0,T)\,\mathbb{E}^T\left[\Delta_i\left(L_i(T)-K\right)^+\right]$$

$$\frac{C_0}{B_0} = \mathbb{E}^*\left[\frac{C_T}{B_T}\right]$$

$$\frac{C_0}{D_0} = \mathbb{E}^T\left[\frac{C_T}{D_T}\right]$$

$$B_0\, \mathbb{E}^*\left[\frac{C_T}{B_T}\right] = D_0\, \mathbb{E}^T\left[\frac{C_T}{D_T}\right]$$

$$\mathbb{E}^T\left[\frac{C_T}{D_T}\right] = \mathbb{E}^*\left[\frac{C_T}{B_T/B_0} \cdot \frac{1/D_0}{1}\right]$$

$$\mathbb{E}^{T}\left[C_T \right] = \mathbb{E}^{*}\left[C_T \cdot \frac{D_T / D_0}{B_T / B_0} \right]$$

$$\mathbb{E}^{T}\left[C_T \right] = \mathbb{E}^{*}\left[C_T \cdot \frac{d\varphi^{T}}{d\varphi^{*}} \right]$$

Swap Market Model

$t \quad T_n \quad T_{n+1} \quad T_{n+2} \quad \cdots \quad T_N$

Let us denote the **par swap rate** for the $[T_n, T_N]$ swap as $S_{n,N}$: tenor

$$S_{n,N}(t) = \frac{D_n(t) - D_N(t)}{\sum_{i=n+1}^{N} \Delta_{i-1} D_i(t)}.$$

The term in the denominator is also called the **present value of a basis point** (PVBP)

$$P_{n+1,N}(t) = \sum_{i=n+1}^{N} \Delta_{i-1} D_i(t).$$

Note that a one-period swap rate $S_{i,i+1}$ is equal to the LIBOR rate. We can now write the value of a payer and receiver swap as

$$\text{Payer Swap} = P_{n+1,N}(t)(S_{n,N}(t) - K)$$
$$\text{Receiver Swap} = P_{n+1,N}(t)(K - S_{n,N}(t))$$

Payer swap $= PV_{flt} - PV_{fix}$

$$= \left[D_n(t) - D_N(t) \right] - \left[P_{n+1,N}(t) \cdot K \right]$$

$$= P_{n+1,N}(t) \left[\frac{D_n(t) - D_N(t)}{P_{n+1,N}(t)} - K \right]$$

$$= P_{n+1,N}(t) \left[S_{n,N}(t) - K \right]$$

$dF_t = \sigma F_t dW_t^*$

Pricing a Swaption

$dL_i(t) = \sigma_i L(t)\, dW^{i+1}(t)$

The PVBP is a portfolio of traded assets and has strictly positive value. It can therefore be used as a numeraire.

If we use $P_{n+1,N}(t)$ as a numeraire, then under the measure $\mathbb{Q}^{n+1,N}$ associated to the numeraire $P_{n+1,N}(t)$, all $P_{n+1,N}$ rebased values must be martingales in an arbitrage-free world.

In particular, the par swap rate $S_{n,N}$ must be a martingale under $\mathbb{Q}^{n+1,N}$. The swap market model makes the assumption that $S_{n,N}$ is a lognormal martingale under $\mathbb{Q}^{n+1,N}$. We write down the process

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t) dW^{n+1,N}(t),$$

where $W^{n+1,N}(t)$ is a Brownian motion under $\mathbb{Q}^{n+1,N}$.

A **swaption** (short for swap option) gives the right to enter at time T_n into a swap with fixed rate K. A **receiver swaption** gives the right to enter into a receiver swap, and a **payer swaption** gives the right to enter into a payer swap.

Pricing a Swaption

10x10 expiry x tenor

Swaptions are often denoted as $T_n \times (T_N - T_n)$, where T_n is the option expiry date (and also the start of the underlying swap), and $T_N - T_n$ is the tenor of the underlying swap.

The payoff of a payer swaption is given by

$$[P_{n+1,N}(T)(S_{n,N}(T) - K)]^+ . \quad = P_{n+1,N}(T)\left[S_{n,N}(T) - K\right]^+$$

Using $P_{n+1,N}$ as a numeraire, we can value the payer swaption under the measure $\mathbb{Q}^{n+1,N}$

$$\frac{V^{\text{payer}}_{n,N}(0)}{P_{n+1,N}(0)} = \mathbb{E}^{n+1,N}\left[\frac{V^{\text{payer}}_{n,N}(T_n)}{P_{n+1,N}(T_n)}\right] = \mathbb{E}^{n+1,N}\left[\frac{P_{n+1,N}(T_n)\cdot(S_{n,N}(T)-K)^+}{P_{n+1,N}(T_n)}\right]$$

$$\Rightarrow \quad V^{\text{payer}}_{n,N}(0) = P_{n+1,N}(0)\mathbb{E}^{n+1,N}[(S_{n,N}(T) - K)^+].$$

The remaining steps required to derive a formula for a swaption is identical to how we would handle a vanilla European option.

Pricing a Swaption

The swap rate follows the stochastic differential equation

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t) dW^{n+1,N}(t),$$

where $W^{n+1,N}(t)$ is a Brownian motion under $\mathbb{Q}^{n+1,N}$. The solution is given by

$$S_{n,N}(T) = S_{n,N}(0) e^{-\frac{1}{2}\sigma_{n,N}^2 T + \sigma_{n,N} W^{n+1,N}(T)}.$$

Evaluating the expectation, we obtain

$$\begin{aligned} V_{n,N}^{payer}(0) &= P_{n+1,N}(0)\mathbb{E}^{n+1,N}[(S_{n,N}(T) - K)^+] \\ &= P_{n+1,N}(0)[S_{n,N}(0)\Phi(d_1) - K\Phi(d_2)], \end{aligned}$$

where

$$d_1 = \frac{\log \frac{S_{n,N}(0)}{K} + \frac{1}{2}\sigma_{n,N}^2 T}{\sigma_{n,N}\sqrt{T}}, \quad d_2 = d_1 - \sigma_{n,N}\sqrt{T}. \quad \triangleleft$$

$$\mathbb{Q}^* : \quad \frac{V_{pay}(0)}{B_0} = \mathbb{E}^*\left[\frac{V_{pay}(T)}{B_T}\right]$$

$$V_{pay}(0) = \mathbb{E}^*\left[\frac{B_0 \, P_{1+1,N}(T)\,(S(T)-K)^+}{B_T}\right]$$

$$= \mathbb{E}^*\left[(S(T)-K)^+ \cdot \frac{P_{1+1,N}(T) / P_{1+1,N}(0)}{B_T / B_0}\right] \cdot P_{1+1,N}(0)$$

$$= P_{1+1,N}(0)\ \cancel{\mathbb{E}^*}\left[(S(T)-K)^+ \cdot \frac{d\mathbb{Q}^{1+1,N}}{d\cancel{\mathbb{Q}^*}}\right]$$

$$= P_{1+1,N}(0)\ \mathbb{E}^{1+1,N}\left[(S(T)-K)^+\right]$$

Swaption Vols – ATM Vols

$$dS(t) = \sigma S(t)\, dW(t)$$

91) Asset | 92) Actions | 93) Settings | Interest Rate Volatility Cube

USD | USD BVOL Cube (Default) | Bid | Date 10/29/15

1) Analyze Cube | 2) Market Data

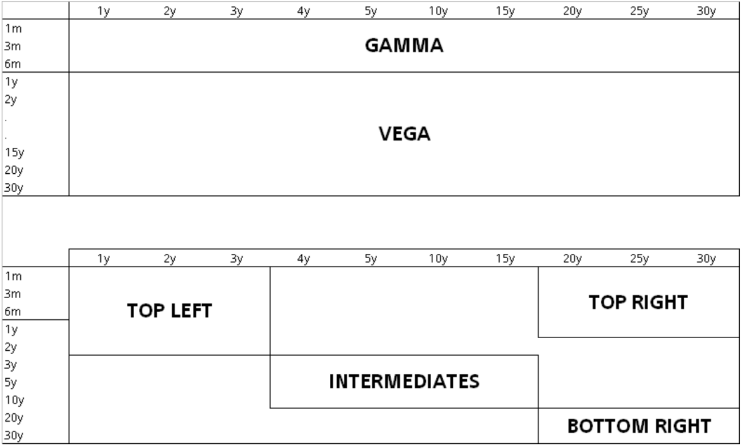
Swap Curve (23) US Dollar | Index Tenor 3M | Show Vol Black

View Strike ATM | Discounting IBOR | Show Strikes

Table | Charts

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr	12Yr	15Yr
1Mo	66.10	62.21	55.17	49.96	46.57	42.67	40.01	37.95	36.32	34.93	33.42	31.65
3Mo	70.23	63.36	57.30	52.53	48.96	44.96	42.20	40.23	38.56	37.12	35.62	33.86
6Mo	64.49	59.82	55.44	50.78	47.56	44.22	41.86	40.16	38.75	37.52	36.11	34.49
9Mo	61.24	56.74	52.60	48.62	45.77	43.00	40.94	39.50	38.25	37.09	35.75	34.21
1Yr	58.49	54.16	50.10	46.94	44.14	41.76	39.93	38.66	37.61	36.68	35.47	34.08
2Yr	52.60	48.17	44.87	42.30	40.20	38.71	37.53	36.65	35.84	35.19	34.21	32.99
3Yr	47.94	44.17	41.45	39.51	37.94	36.84	35.88	35.12	34.48	33.94	33.11	32.00
4Yr	43.43	40.52	38.55	37.09	35.81	34.97	34.30	33.72	33.27	32.92	32.17	31.12
5Yr	39.96	37.89	36.69	35.71	34.78	34.01	33.41	32.99	32.61	32.31	31.59	30.56
6Yr	37.44	36.00	35.03	34.21	33.40	32.80	32.32	31.97	31.66	31.40	30.73	29.77
7Yr	35.22	34.29	33.51	32.84	32.18	31.75	31.38	31.06	30.80	30.57	29.95	29.04
8Yr	33.81	32.87	32.23	31.74	31.25	30.91	30.60	30.33	30.09	29.90	29.32	28.48
9Yr	32.43	31.53	31.13	30.79	30.45	30.18	29.89	29.67	29.46	29.29	28.77	27.95
10Yr	31.21	30.41	30.19	29.98	29.75	29.52	29.26	29.07	28.88	28.72	28.24	27.45
12Yr	30.02	29.26	28.90	28.89	28.88	28.62	28.37	28.20	28.16	28.12	27.62	26.86
15Yr	28.25	27.56	27.43	27.33	27.36	27.26	27.19	27.20	27.23	27.23	26.76	25.99

Swaption ATM Vols



Swaption ATM Vols

$$dS(t) = \sigma \, dW(t)$$

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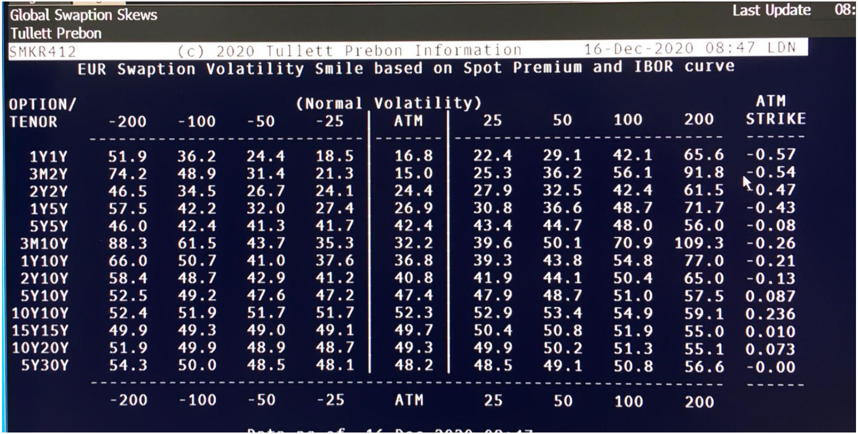
EUR Cash IRR Cal Day | EUR Phys (LCH) Cal Day | EUR Cash IRR Bus Day | EUR Phys (LCH) Bus Day | GBP Calendar D...
ICAP
EUR Swaption - BP Vol OIS Ph
60) MSG Contributor 10:56:45
ICAP Global Menu -> ICAP EMEA -> Interest Rate Options -> IR Options - Digital -> Swaption Normal Vols -> EUR Phys (LCH)...
ICAP - ATM Swaptions

Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M	12.70	13.60	15.90	18.40	20.60	22.60	24.80	26.50	28.00
2) 2M	13.30	14.20	16.70	19.40	21.90	24.20	26.40	28.30	29.90
3) 3M	14.10	15.00	17.20	20.00	22.50	25.10	27.60	29.50	31.10
4) 6M	14.60	16.10	18.90	22.20	24.70	27.20	29.40	31.60	33.50
5) 9M	15.80	17.70	20.50	23.60	[illegible]	[illegible]	30.90	32.80	34.70
6) 1Y	16.80	19.00	22.10	24.90	27.50	30.00	32.20	34.10	35.80
7) 18Y	18.80	21.90	24.90	27.40	29.90	32.40	34.40	36.30	38.00
8) 2Y	21.70	24.50	27.80	30.40	32.40	34.40	36.60	38.20	39.90
9) 3Y	28.00	30.50	33.10	35.00	36.80	38.60	40.30	41.80	43.10
10) 4Y	33.40	35.50	37.50	39.10	40.40	41.60	43.00	44.20	45.50
11) 5Y	37.80	39.70	41.10	42.20	43.30	44.40	45.40	46.40	47.40
12) 6Y	41.90	43.00	43.90	45.20	45.70	46.70	47.50	48.40	49.10
13) 7Y	44.70	45.60	46.30	47.10	47.70	48.40	49.00	49.60	50.20
14) 10Y	49.70	49.80	50.30	50.80	51.10	51.40	51.70	51.80	52.10
15) 12Y	51.30	50.90	51.00	51.40	51.60	52.00	52.30	52.50	52.50
16) 15Y	51.70	51.50	51.90	52.00	52.00	52.10	52.20	52.10	52.40
17) 20Y	51.20	51.10	51.40	51.30	51.30	51.50	51.30	51.20	51.20
18) 25Y	50.30	50.30	50.50	50.40	50.30	50.10	49.90	49.30	49.30
19) 30Y	49.30	49.40	49.60	49.60	49.70	49.30	48.60	48.00	47.40

Suggested Functions | FED See central bank info for the US | GOVY See a government's richest/cheapest bon...

Swaption Vols – Smile/Skew

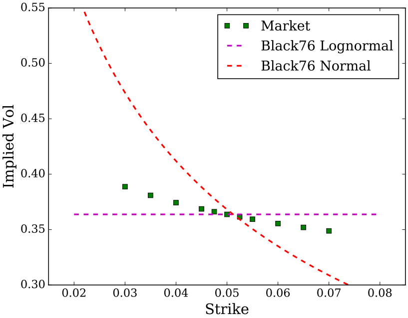


Global Swaption Skews
Tullett Prebon
Last Update 08:
SMKR412 (c) 2020 Tullett Prebon Information 16-Dec-2020 08:47 LDN
EUR Swaption Volatility Smile based on Spot Premium and IBOR curve

OPTION/ TENOR	-200	-100	-50	-25	ATM	25	50	100	200	ATM STRIKE
			(Normal	Volatility)						
1Y1Y	51.9	36.2	24.4	18.5	16.8	22.4	29.1	42.1	65.6	-0.57
3M2Y	74.2	48.9	31.4	21.3	15.0	25.3	36.2	56.1	91.8	-0.54
2Y2Y	46.5	34.5	26.7	24.1	24.4	27.9	32.5	42.4	61.5	-0.47
1Y5Y	57.5	42.2	32.0	27.4	26.9	30.8	36.6	48.7	71.7	-0.43
5Y5Y	46.0	42.4	41.3	41.7	42.4	43.4	44.7	48.0	56.0	-0.08
3M10Y	88.3	61.5	43.7	35.3	32.2	39.6	50.1	70.9	109.3	-0.26
1Y10Y	66.0	50.7	41.0	37.6	36.8	39.3	43.8	54.8	77.0	-0.21
2Y10Y	58.4	48.7	42.9	41.2	40.8	41.9	44.1	50.4	65.0	-0.13
5Y10Y	52.5	49.2	47.6	47.2	47.4	47.9	48.7	51.0	57.5	0.087
10Y10Y	52.4	51.9	51.7	51.7	52.3	52.9	53.4	54.9	59.1	0.236
15Y15Y	49.9	49.3	49.0	49.1	49.7	50.4	50.8	51.9	55.0	0.010
10Y20Y	51.9	49.9	48.9	48.7	49.3	49.9	50.2	51.3	55.1	0.073
5Y30Y	54.3	50.0	48.5	48.1	48.2	48.5	49.1	50.8	56.6	-0.00
	-200	-100	-50	-25	ATM	25	50	100	200	

Swaption Vol Calibration

Suppose the implied volatility across strike for a given swaption maturity and tenor is given by the green markers in the following figure:



The at-the-money volatility is 0.36, and the forward swap rate is 0.05.

Extension to the Black Model

An immediate and straightforward extension is the Black Normal model:

$$dS_{n,N}(t) = \sigma_{n,N} dW^{n+1,N}(t).$$

This is an arithmetic Brownian motion.

If the implied volatility skew we observed in the market is between normal and lognormal, then we can make use of the displaced-diffusion (shifted lognormal) model:

$dF_t = \sigma[\beta F_t + (1-\beta) F_0] dW_t$

$$dS_{n,N}(t) = \sigma_{n,N}[\beta S_{n,N}(t) + (1-\beta) S_{n,N}(0)] dW^{n+1,N}(t).$$

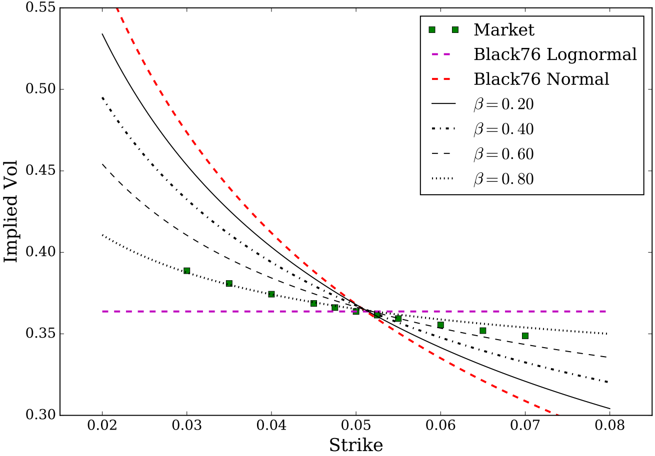
Recall that the solution is given by

$$S_{n,N}(T) = \frac{S_{n,N}(0)}{\beta} e^{\sigma_{n,N}\beta W^{n+1,N}(T) - \frac{\sigma_{n,N}^2 \beta^2 T}{2}} - \frac{1-\beta}{\beta} S_{n,N}(0)$$

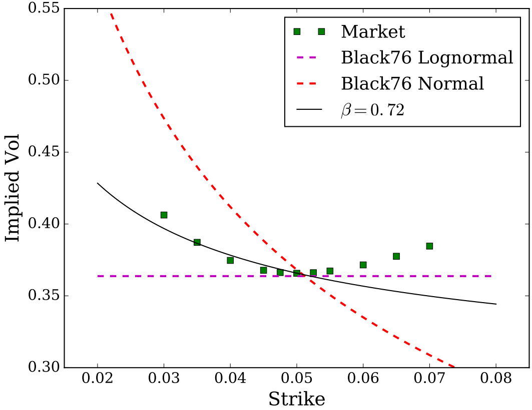
The swaption price under the displaced-diffusion model is

$$V_{n,N}(0) = P_{n+1,N}(0)\text{Black}\left(\frac{S_{n,N}(0)}{\beta},\ K + \frac{1-\beta}{\beta} S_{n,N}(0),\ \sigma\beta,\ T\right)$$

Swaption Vol Calibration – Displaced Diffusion

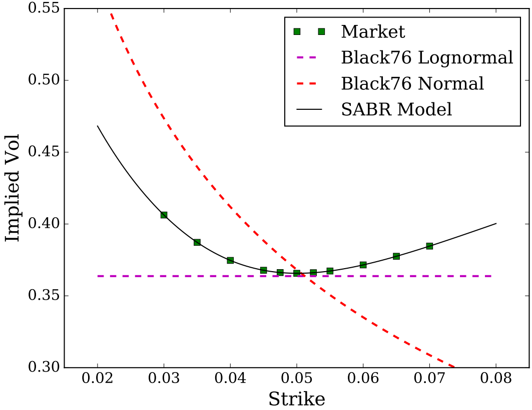


SABR Model



Displaced-diffusion model can only fit to implied volatility skew – there will be mismatch if the implied volatility surface also exhibit “smile” characteristic.

SABR Model



SABR model is able to fit both skew and smile in the implied volatility surface – this is the standard volatility model used in fixed-income market.

Session 5
Constant Maturity Swap Payoffs

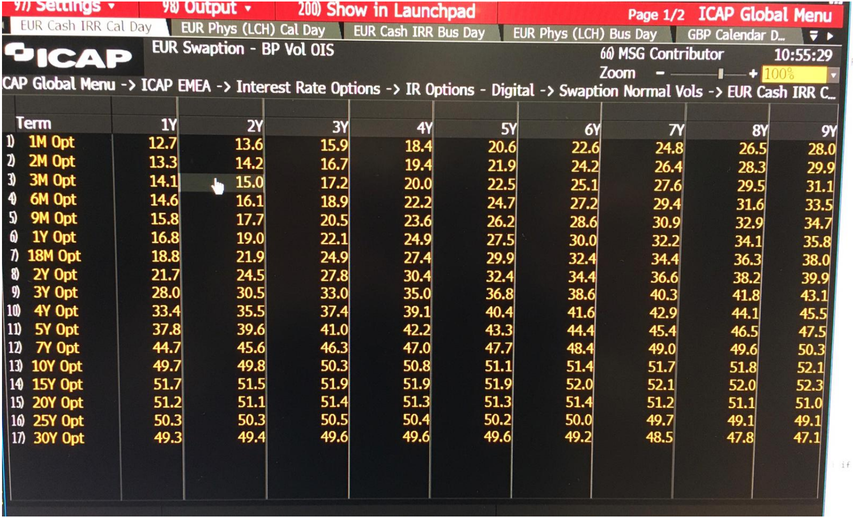
Tee Chyng Wen

QF605 Fixed Income Securities

Swap-Settled Swaptions

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M	12.70	13.60	15.90	18.40	20.60	22.60	24.80	26.50	28.00
2) 2M	13.30	14.20	16.70	19.40	21.90	24.20	26.40	28.30	29.90
3) 3M	14.10	15.00	17.20	20.00	22.50	25.10	27.60	29.50	31.10
4) 6M	14.60	16.10	18.90	22.20	24.70	27.20	29.40	31.60	33.50
5) 9M	15.80	17.70	20.50	23.60	[illegible]	[illegible]	30.90	32.80	34.70
6) 1Y	16.80	19.00	22.10	24.90	27.50	30.00	32.20	34.10	35.80
7) 18Y	18.80	21.90	24.90	27.40	29.90	32.40	34.40	36.30	38.00
8) 2Y	21.70	24.50	27.80	30.40	32.40	34.40	36.60	38.20	39.90
9) 3Y	28.00	30.50	33.10	35.00	36.80	38.60	40.30	41.80	43.10
10) 4Y	33.40	35.50	37.50	39.10	40.40	41.60	43.00	44.20	45.50
11) 5Y	37.80	39.70	41.10	42.20	43.30	44.40	45.40	46.40	47.40
12) 6Y	41.90	43.00	43.90	45.20	45.70	46.70	47.50	48.40	49.10
13) 7Y	44.70	45.60	46.30	47.10	47.70	48.40	49.00	49.60	50.20
14) 10Y	49.70	49.80	50.30	50.80	51.10	51.40	51.70	51.80	52.10
15) 12Y	51.30	50.90	51.00	51.40	51.60	52.00	52.30	52.50	52.50
16) 15Y	51.70	51.50	51.90	52.00	52.00	52.10	52.20	52.10	52.40
17) 20Y	51.20	51.10	51.40	51.30	51.30	51.50	51.30	51.20	51.20
18) 25Y	50.30	50.30	50.50	50.40	50.30	50.10	49.90	49.30	49.30
19) 30Y	49.30	49.40	49.60	49.60	49.70	49.30	48.60	48.00	47.40

IRR-Settled Swaptions


97) Settings ▾ 98) Output ▾ 200) Show in Launchpad Page 1/2 ICAP Global Menu
EUR Cash IRR Cal Day | EUR Phys (LCH) Cal Day | EUR Cash IRR Bus Day | EUR Phys (LCH) Bus Day | GBP Calendar D...
ICAP EUR Swaption - BP Vol OIS 60) MSG Contributor 10:55:29
Zoom 100%
CAP Global Menu -> ICAP EMEA -> Interest Rate Options -> IR Options - Digital -> Swaption Normal Vols -> EUR Cash IRR C...

Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M Opt	12.7	13.6	15.9	18.4	20.6	22.6	24.8	26.5	28.0
2) 2M Opt	13.3	14.2	16.7	19.4	21.9	24.2	26.4	28.3	29.9
3) 3M Opt	14.1	15.0	17.2	20.0	22.5	25.1	27.6	29.5	31.1
4) 6M Opt	14.6	16.1	18.9	22.2	24.7	27.2	29.4	31.6	33.5
5) 9M Opt	15.8	17.7	20.5	23.6	26.2	28.6	30.9	32.9	34.7
6) 1Y Opt	16.8	19.0	22.1	24.9	27.5	30.0	32.2	34.1	35.8
7) 18M Opt	18.8	21.9	24.9	27.4	29.9	32.4	34.4	36.3	38.0
8) 2Y Opt	21.7	24.5	27.8	30.4	32.4	34.4	36.6	38.2	39.9
9) 3Y Opt	28.0	30.5	33.0	35.0	36.8	38.6	40.3	41.8	43.1
10) 4Y Opt	33.4	35.5	37.4	39.1	40.4	41.6	42.9	44.1	45.5
11) 5Y Opt	37.8	39.6	41.0	42.2	43.3	44.4	45.4	46.5	47.5
12) 7Y Opt	44.7	45.6	46.3	47.0	47.7	48.4	49.0	49.6	50.3
13) 10Y Opt	49.7	49.8	50.3	50.8	51.1	51.4	51.7	51.8	52.1
14) 15Y Opt	51.7	51.5	51.9	51.9	51.9	52.0	52.1	52.0	52.3
15) 20Y Opt	51.2	51.1	51.4	51.3	51.3	51.4	51.2	51.1	51.0
16) 25Y Opt	50.3	50.3	50.5	50.4	50.2	50.0	49.7	49.1	49.1
17) 30Y Opt	49.3	49.4	49.6	49.6	49.6	49.2	48.5	47.8	47.1

Swap-Settled Swaptions

The swaptions we have covered so far in our Market Model discussion are **swap-settled swaptions** — when you exercise, you enter into a swap contract with your counterparty.

The payoff of the swaptions are

$$\text{Payer Swaption} = \Big[P_{n+1,N}(T)(S_{n,N}(T) - K)\Big]^+$$
$$\text{Receiver Swaption} = \Big[P_{n+1,N}(T)(K - S_{n,N}(T))\Big]^+$$

where

$$P_{n+1,N}(T) = \sum_{i=n+1}^{N} \Delta_{i-1} D_i(T).$$

Upon exercising, we get

$$\text{Payer Swaption} = V^{flt}(T) - V^{fix}(T)$$
$$\text{Receiver Swaption} = V^{fix}(T) - V^{flt}(T)$$

$\frac{1}{m} \simeq \Delta_i$ $D(T) \simeq \frac{1}{(1+S)^T}$

IRR-Settled Swaptions

An **Internal-Rate-of-Return (IRR)-settled swaption** has the following payoff:

$$\text{Payer Swaption} = \Big[\text{IRR}(S_{n,N}(T))(S_{n,N}(T) - K)\Big]^+$$

$$\text{Receiver Swaption} = \Big[\text{IRR}(S_{n,N}(T))(K - S_{n,N}(T))\Big]^+$$

where

$$\text{IRR}(S) = \sum_{i=1}^{(T_N - T_n)\times m} \frac{\frac{1}{m}}{\left(1+\frac{S}{m}\right)^i}$$

and $\frac{1}{m} = \Delta$ is the day count fraction corresponding to the payment frequency (m) of the swap.

IRR-settled swaptions are <u>settled in cash</u> based on the value of the payoff observed on the maturity date.

Swap-settled swaptions are common in the USD market, while IRR-settled swaptions are common in the European (EUR & GBP) markets.

IRR-Settled Swaptions

The Market Model used to value IRR-settled swaptions is:

$$V_{n,N}(0) \approx D(0,T) \cdot \mathsf{IRR}(S_{n,N}(0)) \cdot \mathsf{Black}(S_{n,N}(0), K, \sigma_{n,N}, T)$$

Historical Note:

- In the USD market, participants agree on the value of the PV01 $P_{n+1,N}$, i.e. there is no dispute on the discount factors.
- In the earlier days, market participants disagree on the PV01 value in the Euro and Sterling market.
- To avoid ambiguity, market participants agree to use the IRR formula to discount cashflows in the EUR and GBP market.
- The rational was that since $D(0,T) = \frac{1}{(1+r)^T}$, a good approximation would be to use the observed swap rate $S_{n,N}(T)$ for discounting.