QF605 Fixed-Income Securities
Assignment 1, Due Date: 31-Jan-2024

1. Suppose we observe the following spot LIBOR rates:

LIBOR	Rate
$1m$	1.25%
$2m$	1.50%
$3m$	1.65%
$6m$	1.75%
$12m$	1.80%

   The market is uncollateralized.

   (a) What forward rate would you show for a 9×12 FRA?

   (b) Calculate the $1y$ par swap rate for an interest rate swap with quarterly payment.

   (c) Calculate the continuously compounded zero rates $R(0, 3m)$, $R(0, 6m)$, and $R(0, 12m)$.

2. Suppose the spot exchange rate for USD/SGD is 1.42, and the USD $6m$ LIBOR rate is 1.5%. If the $6m$ forward exchange rate for USD/SGD is 1.39, calculate the implied $6m$ interest rate (SOR) in SGD using these instruments.

3. Suppose you observe the following instruments in the swap market:

Instrument	Quote
$6m$ LIBOR	1.5%
$1y$ IRS	1.8%
$2y$ IRS	2.0%
$3y$ IRS	2.05%

   The market is uncollateralized, and all interest rate swaps (IRS) have semi-annual payment.

   (a) What is the $1.5y$ tenor interest rate swap with semi-annual payment?

   (b) A forward starting swap with a $2y$ tenor starting at $t = 1y$ has the following cashflows:

Time (y)	Pay	Rec
1.5	Par Swap Rate	$6m$ LIBOR
2.0	Par Swap Rate	$6m$ LIBOR
2.5	Par Swap Rate	$6m$ LIBOR
3.0	Par Swap Rate	$6m$ LIBOR

   What is the par swap rate for this forward starting swap?

1.

(a)

$$D(0,6m) = \frac{1}{1+1.75\% \times 0.5} = 0.9913$$

$$D(0,12m) = \frac{1}{1+1.8\%} = 0.9823$$

$$D(0,9m) = \frac{D(0,6m)+D(0,12m)}{2} = 0.9868$$

$$L(9m,12m) = \frac{1}{\Delta 3m} \frac{D(0,9m)-D(0,12m)}{D(0,12m)} = 0.01834 = 1.83\%$$

(b)

$$D(0,3m) = \frac{1}{1+1.65\% \times 0.25} = 0.9959$$

$$S = \frac{1-D(0,12m)}{0.25[D(0,3m)+D(0,6m)+D(0,9m)+D(0,12m)]} = 0.01788 = 1.79\%$$

(c)

$$R(0,3m) = 4\ln(1+1.65\% \times 0.25) = 0.01647 = 1.65\%$$

$$R(0,6m) = 2\ln(1+1.75\% \times 0.5) = 0.01742 = 1.74\%$$

$$R(0,12m) = \ln(1+1.8\%) = 0.01784 = 1.78\%$$

2.

$$1+L^{SGD}(0,6m) \times 0.5 = \frac{FX_{6m}}{FX_0}(1+L^{USD}(0,6m) \times 0.5)$$

$$\Rightarrow L^{SGD}(0,6m) = -0.02757 = -2.76\%$$

3.

(a)

$$D(0,6m) = \frac{1}{1+1.5\% \times 0.5} = 0.9926$$

$$1.8\% \times 0.5[D(0,6m)+D(0,1y)] = 1-D(0,1y)$$

$$\Rightarrow D(0,1y) = 0.9822$$

$$\begin{cases} 2\% \times 0.5 \times [D(0,6m)+D(0,1y)+D(0,1.5y)+D(0,2y)] = 1-D(0,2y) \\ D(0,1.5y) = \dfrac{D(0,1y)+D(0,2y)}{2} \end{cases}$$

$$\Rightarrow \begin{cases} D(0,2y) = 0.9609 \\ D(0,1.5y) = 0.9716 \end{cases}$$

$$S = \frac{1-D(0,1.5y)}{0.5 \times [D(0,6m)+D(0,1y)+D(0,1.5y)]} = 1.93\%$$

b.

$$\begin{cases} 2.05\% \times 0.5 \times [D(0,6m) + D(0,1y) + D(0,1.5y) + D(0,2y) + D(0,2.5y) + D(0,3y)] = 1 - D(0,3y) \\ D(0,2.5y) = \dfrac{D(0,2y) + D(0,3y)}{2} \end{cases}$$

$$\Rightarrow \begin{cases} D(0,2.5y) = 0.9507 \\ D(0,3y) = 0.9406 \end{cases}$$

$$S = \frac{D(0,1y) - D(0,3y)}{0.5 \times [D(0,1.5y) + D(0,2y) + D(0,2.5y) + D(0,3y)]} = 0.02178 = 2.178\%$$