QF609 Risk Analysis
Lecture Notes 2

Dr. Tony Wong

Adjunct Professor of MQF
Head of Market Risk Model Validation, Maybank

tonywong@smu.edu.sg

Overview

Main topics:

- Recap
- IRRBB: Repricing Gap, ΔNII, and ΔEVE
- Interest Rate Notions
- Coupon Bond Pricing

Recap

Recap from the previous lecture:

- A Background Introduction of Financial Institutions
- What is a banking book? a trading book?
- Accrual Accounting (AA) Method (mainly used for banking books)
- Mark-to-Market (MTM) Method (mainly used for trading books)

Trading Book

A trading book mainly serves to keep the positions that are mainly used for short term trading activities (e.g. short term re-sale, profit taking from short term market movements, arbitrage, and hedging)

Banking Book

A banking book mainly serves to keep the positions on a bank's balance sheet that are expected to be held to maturity, typically consisting of customer loans to and deposits from retail and corporate customers, which are not meant for active trading purposes. A banking book can also include those derivatives that are used to hedge exposures arising from the banking book activities, including interest rate risk.

A Review Example: AA Accounting vs MTM Accounting

The initial BS of a bank is given as follows:

Liabilities		Assets	
Capital	20	FRB	20
Deposits	100	Loans	100
	120		120

- 3Y/6% loan and 1Y/3% deposit, interests paid annually
- discount rate for MTM accounting is 3% (effective annual rate)

Under AA, the BS will look like below in 6M and 1Y:

BS: 6M				BS: 1Y			
Liabilities		Assets		Liabilities		Assets	
Capital	20	FRB	20	Capital	20	FRB	20
Retained profit	1.5	Loans	100	Retained profit	3	Loans	100
Deposits	100	Interest receivable	1.5	Deposits	100	Interest receivable	3
	121.5		121.5	123		123	

For MTM, we have:

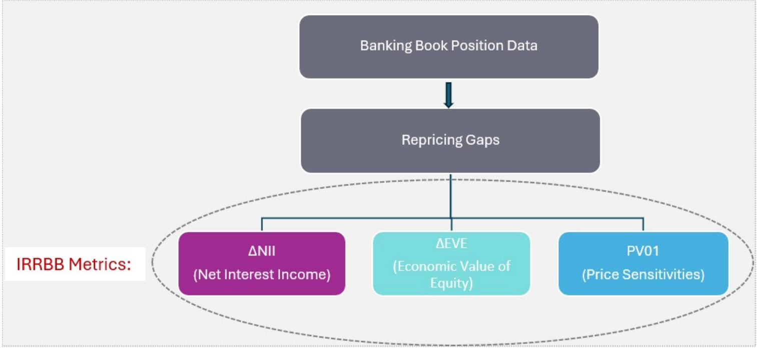
T	6M	1Y
Net Realized Cash	0	3
NPV of Loan	110.10	105.74
NPV of Deposit	-101.49	-100.00
MTM	8.61	8.74
Net Profit (MTM)	8.61	0.13
Net Profit (Accrual Accounting)	1.5	1.5

IRRBB: Background

IRRBB refers to the current or prospective risk to the bank's capital and earnings arising from adverse movements in interest rates that affect the bank's banking book positions:

- When interest rates change, the present value and timing of future cash flows change, and hence the economic value of a bank's assets and liabilities. There are different sub-types of IRRBB risks. Our focus is mainly on **repricing risk**.
- The Basel Committee proposed a Pillar 2 framework to address IRRBB for banks which sets out a set of principles for defining supervisory expectations and management of IRRBB.
- Two key reporting metrics for measuring IRRBB are the net interest income (NII) sensitivity and economic value of equity (EVE) sensitivity.
- The **repricing gaps**, extracted from the banking book positions, are the core inputs to the calculations of IRRBB metrics.

The general framework for calculating these IRRBB metrics in practice is illustrated by the chart below.



Repricing refers to the nearest future interest rate update/fixing event associated with a financial claim which can be a result of:

- a contractual interest rate fixing for an instrument payment
- a rollover/renewal of an expiring instrument or a partially retiring balance of an instrument

A repricing is defined by 2 attributes:

- **repricing gap**: notional exposure or book value of the contract subject to repricing
- **repricing date**: date on which the interest rate get "repriced" (effectively, fixed)

Interest rate repricing clearly has economic impacts on a bank's position, e.g. if rates were to increase and a bank's deposits **reprice** sooner than its loans, the bank may need to pay out more interest on deposits than the interest received from loans. This mismatch would subsequently bite into the bank's NII, as well as affecting the economic value of its EVE.

IRRBB: The Repricing Gap Model

Some conventions:

- The repricing gap for an asset (resp. liability) is expressed as a positive (resp. negative) number
- For some instruments such as mortgage, different portions of the principal amount may be repriced on different dates, the instrument in such cases is considered to have multiple repricing dates each of which is associated with a specific repricing amount
- If an instrument gets repriced periodically with each repricing applying to the full notional amount (e.g. an interest rate swap), only the nearest next repricing date will be considered as its repricing date (we shall see one example in a later section)

IRRBB: Repricing of Some Example Instruments

To get an intuitive sense of what repricing means, we shall look at how the repricing gap and date for the following representative instruments are determined (a detailed explanation is given on the notes uploaded to course site):

- (Interest-Only) Fixed Rate Deposit and Loan
- (Interest-Only) Floating Rate Deposit and Loan
- Fixed Rate Mortgage
- Interest Rate Swap (IRS)
- Forward Starting Fixed Rate Deposit and Loan

In practice, some instruments such as mortgage and current/saving account, either prepayment is allowed or the maturity is unknown. Behavioral assumptions need to be factored in for deriving their repricing gaps. In this course, we shall not concern such cases,

IRRBB: Exercise

Fill out the repricing date and gap:

Product	Interest Rate	Notional	Maturity	repricing date	repricing gap	notes
Interest-Only Loan	3%	$1 million	5Y			
Interest-Only Loan	USD 3M Libor	$1 million	5Y			
Deposit	3%	$1 million	1Y			
Deposit	USD 3M Libor	$1 million	1Y			
Treasury bill	4.5%	$1 million	3M			
Treasury notes	4%	$1 million	6M			
Treasury notes	3.8%	$1 million	10Y			
Mortgage	flat rate at 4% flat	$1 million	20Y			
IRS pay	3%	$1 Million	10Y			
IRS receive	USD 3M Libor	$1 Million	10Y			
1M-forward starting deposit	3%	$1 million	1Y			
1M-forward starting deposit	USD 3M Libor	$1 million	1Y			
1M-forward starting IRS pay	3%	$1 Million	10Y			
1M-forward starting IRS receive	USD 3M Libor	$1 Million	10Y			
1M-forward starting IRS pay	10Y par swap rate quoted in 1M	$1 Million	10Y			
1M-forward starting IRS receive	USD 3M Libor	$1 Million	10Y			

IRRBB: Interest Rate Gap Report

The core input to calculations of NII and EVE sensitivities is an interest rate gap report that capture the aggregated repricing gaps of all banking positions.

An interest gap report produced by banks typically is a two-dimensional table with one axis representing a set of standardized time-to-repricing buckets over a selected planning horizon and the other axis representing the product category.

A bank may have IRRBB exposures across different currencies. In that case, some aggregation logic should apply. We shall consider the case of one exposure currency only.

The next slide shows a simple example of a gap report.

IRRBB: Interest Rate Gap Report

time-to-repricing	loan	deposit	···	net gap
(0D,1M]	2	-10	···	-5
(1M, 2M]	3	-8	···	-15
(2M, 3M]	5	-20	···	5
(3M, 6M]	6	-25	···	15
(6M, 9M]	10	-10	···	-30
(9M, 1Y]	20	-5	···	-40
(1Y, 2Y]	···	···	···	···
(2Y, 3Y]	···	···	···	···
(3Y, 4Y]	···	···	···	···
(4Y, 5Y]	···	···	···	···
(5Y, 10Y]	···	···	···	···
Total	···	···	···	···

IRRBB: ΔNII

NII Sensitivity is an earning-based measure which focuses more on the sensitivities of a bank's short-term earnings to interest rate moves. To compute ΔNII, one needs to specify:

- a projection time horizon, say N year (N is typically chosen to be 1)
- a selected set of interest rate shock scenarios

For a given scenario s, we have

$$\Delta NII^s = \sum_{\{(T_i,A_i)\mid T\leq N\}} \Delta R_i^s \cdot A_i \cdot (N - T_i)$$

where T_i and A_i denote the time-to-repricing and the net repricing amount from the gap report. Once the sensitivities ΔNII^s under all specified scenarios are computed, a final reported number can be produced, e.g. the worst case ΔNII^s. **When using the formula above, one typically applies a mid-point rule such that T_i is set to the mid-point of a time-to-repricing bucket.**

IRRBB: ΔEVE

The EVE sensitivity under a given interest rate scenario is calculated as the difference in the economic value of aggregated assets and liabilities between the base case and the scenario. Here, economic value of an instrument is given by the NPV of the repricing gap of the position under a specified discount curve. Therefore, we have:

$$\Delta EVE^s = EV_s - EV_0$$

where

- EV_0: base EV of the banking book positions, given by the sum of all discounted interest rate gaps over all time buckets under a base discount curve.
- EV_s: scenario EV recalculated by applying the interest rate shock scenario s to the base discount curve.

Once the sensitivities ΔNII^s under all scenarios, a final reported number can be produced.

IRRBB: Example Calculation of ΔEVE

Gap Report

Bucket	Net Gap
[0M,1M]	-5
[1M,2M]	-15
[2M,3M]	5
[3M,6M]	15
[6M,9M]	-30
[9M,1Y]	-40
[1Y,2Y]	5
[2Y,3Y]	10
[3Y,4Y]	5
[4Y,5Y]	20
[5Y,6Y]	5
[6Y,7Y]	15
[7Y,8Y]	0
[8Y,9Y]	5
[9Y,10Y]	5
Total	**0**

EVE Sensitivity Calculation

	Discount Curve						EV	
		Zero Rate		Discount Factor				
Tenor	Tenor (in year)	base	parallel shift	base	scenario		base	scenario
1M	0.08	0.4330%	2.4330%	0.999639	0.997975		-5.00	-4.99
2M	0.17	0.5227%	2.5227%	0.999129	0.995804		-14.99	-14.94
3M	0.25	0.5985%	2.5985%	0.998505	0.993525		4.99	4.97
6M	0.50	0.7817%	2.7817%	0.996099	0.986188		14.94	14.79
9M	0.75	0.9289%	2.9289%	0.993058	0.978273		-29.79	-29.35
1Y	1.00	1.0554%	3.0554%	0.989501	0.969908		-39.58	-38.80
2Y	2.00	1.4548%	3.4548%	0.971323	0.933237		4.86	4.67
3Y	3.00	1.7419%	3.7419%	0.949084	0.893814		9.49	8.94
4Y	4.00	2.0088%	4.0088%	0.922792	0.851845		4.61	4.26
5Y	5.00	2.2626%	4.2626%	0.893036	0.808053		17.86	16.16
6Y	6.00	2.4738%	4.4738%	0.862063	0.764581		4.31	3.82
7Y	7.00	2.6683%	4.6683%	0.829624	0.721240		12.44	10.82
8Y	8.00	2.8496%	4.8496%	0.796149	0.678434		0.00	0.00
9Y	9.00	3.0200%	5.0200%	0.762006	0.636481		3.81	3.18
10Y	10.00	3.1682%	5.1682%	0.728463	0.596415		3.64	2.98

EV_0	-8.39
EV_s	-13.48
EVE sensitivity	**-5.09**

IRRBB: Example Calculation of ΔNII

The below uses the same gap report as in the ΔEVE calculation.

NII sensitivity: Horizon = 1Y, i.e. N=1

scenario +200bp

bucket	T = mid-point (in year)	N - T	gap	ΔNII_k
[0M,1M]	0.0417	0.9583	-5	-0.0958
[1M,2M]	0.1250	0.8750	-15	-0.2625
[2M,3M]	0.2083	0.7917	5	0.0792
[3M,6M]	0.3750	0.6250	15	0.1875
[6M,9M]	0.6250	0.3750	-30	-0.2250
[9M,1Y]	0.8750	0.1250	-40	-0.1000
			ΔNII (Total)	**-0.4167**

NII sensitivity allocation

bucket	ΔNII_k	ΔNII Allocation					
		[0M, 1M]	[1M, 2M]	[2M, 3M]	[3M, 6M]	[6M, 9M]	[9M, 1Y]
[0M,1M]	-0.0958	-0.0042	-0.0083	-0.0083	-0.0250	-0.0250	-0.0250
[1M,2M]	-0.2625		-0.0125	-0.0250	-0.0750	-0.0750	-0.0750
[2M,3M]	0.0792			0.0042	0.0250	0.0250	0.0250
[3M,6M]	0.1875				0.0375	0.0750	0.0750
[6M,9M]	-0.2250					-0.0750	-0.1500
[9M,1Y]	-0.1000						-0.1000
Total	**-0.4167**	**-0.0042**	**-0.0208**	**-0.0292**	**-0.0375**	**-0.0750**	**-0.2500**

IRRBB: Reconciliation of ΔEVE and ΔNI

While the two metrics ΔEVE and ΔNI focus on different perspectives of looking at IRRBB, is there a way to reconcile the two?

We explore the answer of this by looking at a very simple gap report:

time-to-reprice bucket	deposit	loan	net gap
$[0Y, 1Y]$	-100	0	-100
$[1Y, 2Y]$	0	0	0
$[2Y, 3Y]$	0	0	0
$[3Y, 4Y]$	0	0	0
$[4Y, 5Y]$	0	100	100
Total	-100	100	0

This is the gap report corresponding to the following simple positions:

- a \$100 5-year fixed rate loan paying 5% annually
- a \$100 1-year fixed rate deposit to fund the loan which pays 5% annually

IRRBB: Reconciliation of ΔEVE and ΔNI

Furthermore, we assume:

- The base discount curve is 5% flat for all maturities. Here, we assume the 5% is an effective annually compounded rate, i.e. for any maturity t (expressed in years), the discount factor is given by $(1+5\%)^{-t}$.
- The scenario is a $+200$bps parallel shift to the base curve.
- For ΔNII, the planning horizon is $N = 5$. For this example, we use the exact time-to-repricing for T (instead of applying the mid-point rule).

Following the above, we should obtain:

$$\begin{aligned}
\Delta EVE &= \left[\frac{100}{1.07^5} + \frac{-100}{1.07}\right] - \left[\frac{100}{1.05^5} + \frac{-100}{1.05}\right] = -5.27, \\
\Delta NII &= -100 \cdot 4 \cdot 0.02 = -8.
\end{aligned}$$

Hence, the two measures agree in sign but not in value.

IRRBB: Reconciliation of ΔEVE and ΔNI

To reconcile the two, we further take into account the followings in the ΔEVE:

- repricing risk of the interest payments associated with the deposit and the loan
- impacts of the interest rate shock on deposit interest payments occurring after its repricing date

Then, we obtain the base and scenario gap reports for ΔEVE:

	Base: 5% flat			Scenario: parallel shift +200bps		
bucket	deposit	loan	net gap	deposit	loan	net gap
$[0Y, 1Y]$	-5	5	0	-5	5	0
$[1Y, 2Y]$	-5	5	0	-7	5	-2
$[2Y, 3Y]$	-5	5	0	-7	5	-2
$[3Y, 4Y]$	-5	5	0	-7	5	-2
$[4Y, 5Y]$	-105	105	0	-107	105	-2
Total	-125	125	0	-133	125	-8

We then have:

$$\Delta EVE = \sum_{i=2}^{5} \frac{-2}{1.07^i} = -6.33.$$

If we further take into account discounting effect on ΔNII, we have:

$$\Delta NII = \sum_{i=2}^{5} \frac{-100 * 0.02}{1.07^i} = -6.33.$$

In this case, the ΔNII can be interpreted as the additional funding cost/surplus as of today if the scenario becomes realized today and holds valid until the relevant repricing dates.

The above example only serves to provide some intuitions on the connection between ΔNII and ΔEVE. In practical IRRBB reporting, ΔNII focuses on earning impacts over the short term (N=1), while ΔEVE looks at economic value impacts typically over a longer planning period, e.g. 10Y.

IRRBB: Limitations of Repricing Gap Model

The repricing gap model has some limitations:

- It does not take into account repricing risks of interest payments. Strictly speaking, interest payments related to deposits/loans are subject to repricing risk due to reinvestment/funding. However, the model considers this as a negligible second order risk.
- Using standardized repricing buckets for a gap report can either understate or overstate the actual repricing risk.
- A gap report gives no indications of basis risk, i.e. items that appear to offset by virtue of having the same repricing date may not actually reprice by the same amount.
- behavioral assumptions need to be embedded into the repricing model to deal with contracts with balance amortization, prepayment, or early closure.
- ΔNII assumes that the total balance sheet size and shape are maintained by having like-for-like replacement of assets and liabilities as they run off.

IRRBB: Remarks

This ends our coverage on the IRRBB topic. The goal here is to provide a basic understanding the IRRBB framework. In practice, there are a lot of subtleties in IRRBB implementation in a bank (*devils are in the details*, e.g. data and infrastructure issues, behavioral models, etc.).

IRRBB is of practical importance given that there is increasing regulatory attention towards IRRBB and it has become one of the hot topics for regulators in recent years.

For those who have further interests on this topic, there is new book written by a practitioners which should be of great help:

- *Interest Rate Risk in the Banking Book: A Best Practice Guide to Management and Hedging*, Beata Lubinska, Wiley Finance.

Let us introduce some common notions of interest rates that are useful for our later discussions.

Notional Interest Rate

A nominal interest rate is defined by its value as well as a specified annual compounding frequency m. Let us denote the rate by $y^{(m)}$. The discounting factor for a maturity t (in years) is given by:

$$DF(t) \;=\; \left(1 + \frac{y^{(m)}}{m}\right)^{-m\cdot t}.$$

Effective Interest Rate

This is simply the case with $m = 1$. So it refers to the effective annual interest rate.

If we are given a nominal rate $y^{(m)}$, how can we convert it to an equivalent nominal rate with a different compounding frequency, say n?

For this, we can obtain $y^{(n)}$ by enforcing an condition that both rates should result in the same discount factor for all t, i.e.:

$$DF(t) = \left(1 + \frac{y^{(m)}}{m}\right)^{-m \cdot t} = \left(1 + \frac{y^{(n)}}{n}\right)^{-n \cdot t}.$$

This gives:

$$y^{(n)} = \left[\left(1 + \frac{y^{(m)}}{m}\right)^{\frac{m}{n}} - 1\right] \cdot n$$

Once can show that $y^{(m)} < y^{(n)}$ if $m > n$.

Zero Rate

This is just the limiting case with continuous compounding. Denoting the rate by r, we have:

$$DF(t) \quad = \quad \exp\left(-r \cdot t\right)$$

It is easy to show (exercise?) that:

$$r = \lim_{m \to \infty} y^{(m)}.$$

All rates discussed above assumes no term structure - so the rate is applied to obtain the discount factor for all maturities. In practice, interest rates have a term structure,e.g. if you look at the zero rate curve for USD SOFR or SGD SORA discounting, it is far from being flat. Another way of saying this is that we need to index the rate by maturity, e.g. r(t), and $r(t)$ is not a constant function.

Coupon Bond Pricing

A coupon bond is a debt instrument that pays periodic coupons and a principal at maturity. Consider a coupon bond with:

- coupon payment frequency $= m$ times per year
- coupon rate $= C$
- notional $= \$100$
- maturity $=$ N years
- effective yield $= y$

The price of the bond is given by:

$$P = \sum_{i=1}^{N \cdot m} \frac{\frac{100 \cdot C}{m}}{(1+y)^{\frac{i}{m}}} + \frac{100}{(1+y)^N}$$

Coupon Bond Pricing

The effective bond yield can be converted to a nominal yield, e.g. with compounding frequency = m. In that case, we simply have:

$$\begin{aligned} y^{(m)} &= \left[(1+y)^{\frac{1}{m}} - 1\right] \cdot m, \\ P &= \sum_{i=1}^{N \cdot m} \frac{\frac{100 \cdot C}{m}}{(1 + \frac{y^{(m)}}{m})^i} + \frac{100}{(1+y)^{N \cdot m}} \end{aligned}$$