Risk Analysis (QF609, AY2022-2023)

Practice Problem Set #2

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1. Consider a 10-year coupon bond with notional = \$1,000,000 and semi-annual coupon payments at an annualized coupon rate of 5%. The current annually compounding yield of the coupon bond is 6%.

   (a) Calculate the PV of the bond.

   (b) Calculate the Macaulay duration, modified duration, and convexity of the bond.

   (c) Repeat (a)-(b) if the yield is 3% and 8%, respectively.

   (d) Using the modified duration and convexity obtained in (b), enstimate the PV impacts when the yield changes from 6% to 3% and 8%, respectively (compare them to the exact impacts calculated using (c)).

2. Assume that the current values of the assets (A) and liabilities (L) on a bank's balance sheet are $A = 100 + 30$ and $L = 140$, respectively. The Macaulay duration of the liabilities is equal to 6 years. The assets consist of two items:

   - Asset 1: PV = 100, Macaulay duration = 5 years
   - Asset 2: PV = 30, Macaulay duration = 4.5 years

   (a) Calculate the combined Macaulay duration of the two assets.

   (b) Calculate the leverage-adjusted duration gap (D-Gap) of the balance sheet.

   (c) Consider the 10-year bond in question 1 with a 5% coupon rate and a 6% yield. What notional amount of the bond should the bank purchase in order to immunize its equity value change?

3. Consider an asset A with a current value of \$100. The relative 1-day return of the asset is denoted by R (hence, the asset value in one day is $100 \cdot (1 + R)$). Calculate the 99% (i.e. confidence level = 99% and significance level =1%) **1-day VaR** for the asset in each of the following 3 cases:

   (a) R has the following discrete distribution (you may apply interpolation as needed for percentile calculation in this case):

R	probability
-20%	0.005
-10%	0.12
-3%	0.15
0%	0.25
2%	0.30
10%	0.10
20%	0.05
30%	0.015
50%	0.01

   (b) R has a normal distribution with mean 1.8% and variance 0.8181%

4. A portfolio consists of \$1 million in each of three stocks (stock A, B and C). The daily returns are i.i.d and normally distributed with the estimated mean and covariance matrix below:

$$\mu = \begin{pmatrix} 0.0356\% \\ 0.0267\% \\ 0.0133\% \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 0.00007 & 0.0001 & -0.000045 \\ 0.0001 & 0.0004 & -0.00008 \\ -0.000045 & -0.00008 & 0.000178 \end{pmatrix}$$

   What are the expected value and variance of the portfolio PNL in 1 day? What is the 1-day 99% VaR?

5. Calcute the Cholesky Decomposition of the following correlation matrix:

$$\begin{pmatrix} 1 & 0.4 & 0.2 \\ 0.4 & 1 & 0.35 \\ 0.2 & 0.35 & 1 \end{pmatrix}$$

6. Consider the full-rank correlation matrix A given below:

$$A = \begin{pmatrix} 1 & 0.98 & 0.95 & 0.93 & 0.91 \\ 0.98 & 1 & 0.98 & 0.95 & 0.93 \\ 0.95 & 0.98 & 1 & 0.98 & 0.95 \\ 0.93 & 0.95 & 0.98 & 1 & 0.98 \\ 0.91 & 0.93 & 0.95 & 0.98 & 1 \end{pmatrix}$$

The eigenvectors and their associated eigenvalues are given below:

Eigenvalues				
4.8163	0.1239	0.0397	0.0161	0.0040
Eigenvectors				
e1	e2	e3	e4	e5
0.4429	0.5855	0.5030	0.3964	0.2254
0.4494	0.3964	-0.1622	-0.5855	-0.5212
0.4513	0.0000	-0.6643	0.0000	0.5958
0.4494	-0.3964	-0.1622	0.5855	-0.5212
0.4429	-0.5855	0.5030	-0.3964	0.2254

(a) Calculate the full-rank factor loading matrix.

(b) Calculate the rank-reduced factor loading matrix (target rank = 3).

(c) Calcualte the model correlation matrix resulted from rank reduction.

7. Assume that the correlation matrix A given in the previous question is a sample correlation matrix calculated from a certain standardized dataset.

(a) What are the first 3 principle components (PCs) if we were to run a PCA on the standardized dataset?

(b) Let $(0.3 \;\; 0.2 \;\; 0.15 \;\; 0.15 \;\; 0.1)$ be a particular data point from the standardized dataset. Calculate the projected coordinates of this data point onto the first 3 PCs?

8. Review the method to simulate correlated standard normal numbers from independent uniform[0,1] numbers and vice versa, and try it out yourself on Excel as an exercise.