QF609: Supplementary Notes on Repricing Date of a Floating Rate Bond/Loan Valuation

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Let us first consider a one-period ($[T_{n-1}, T_n]$) floating rate loan with notional amount N. The cash flows simply consists of:

- an floating interest rate payment at T_n given by $N \cdot (T_n - T_{n-1}) \cdot L(T_{n-1}, T_n)$, where $L(T_{n-1}, T_n)$ is the floating rate fixed at T_{n-1}
- notional repayment N at T_n

From your other courses, you probably learn that the value of the loan at T_{n-1} should be given by:

$$V(T_{n-1}) = P(T_{n-1}, T_n)\left[N + N \cdot (T_n - T_{n-1}) \cdot L(T_{n-1}, T_n)\right]$$

where $P(T_{n-1}, T_n)$ is the discount factor[1] at time T_{n-1} for the maturity T_n. If we assume the floating rate and discount rates are coming from the same curve, the following will hold:

$$L(T_{n-1}, T_n) = \frac{1}{T_n - T_{n-1}}\left[\frac{1}{P(T_{n-1}, T_n)} - 1\right].$$

Substitute this into the formula above for $V(T_{n-1})$, we have:

$$\begin{aligned}
V(T_{n-1}) &= P(T_{n-1}, T_n)\left[N + N \cdot (T_n - T_{n-1}) \cdot \frac{1}{T_n - T_{n-1}}\left[\frac{1}{P(T_{n-1}, T_n)} - 1\right]\right] \\
&= N \cdot P(T_{n-1}, T_n) + N \cdot [1 - P(T_{n-1}, T_n)] \\
&= N.
\end{aligned}$$

Hence, $V(T_{n-1})$ is simply equal to the notional amount.

Next, consider a two-period floating rate loan with the following cash flows:

Payment Date	Payment
T_{n-1}	$N \cdot (T_{n-1} - T_{n-2}) \cdot L(T_{n-2}, T_{n-1})$
T_n	$N + N \cdot (T_n - T_{n-1}) \cdot L(T_{n-1}, T_n)$

We already know that the value of the time-T_n cash flow at T_{n-1} is equal to N. So the value of the two period loan can be obtained by discounting the following cash flow at T_{n-1} back to time T_{n-2}:

$$P(T_{n-2}, T_{n-1})\left[N + N \cdot (T_{n-1} - T_{n-2}) \cdot L(T_{n-2}, T_{n-1})\right]$$

[1] Imagine that we are now at time T_{n-1}, and $P(T_{n-1}, T_n)$ is the discount factor used for discounting cash flow at T_n back to T_{n-1}.

This again will give a value of N for the two-period floating rate loan at T_{n-2}. Repeating the above for an (n-1)-period loan, we see its value at T_1 is also equal to the notional amount N.

Finally, we consider the n-period loan. Following the same calculations, the value of the loan at time $T_0 = 0$ is simply the discounted value of a T_1 cash flow of amount $N + N \cdot (T_1 - T_0) \cdot L(T_0, T_1)$ back to T_0.

Let us imagine now that we stand at T_0 and assume the floating rate $L(T_0, T_1)$ has already been fixed. Then, the amount $N + N \cdot (T_1 - T_0) \cdot L(T_0, T_1)$ is simply a fixed amount. Hence, its discounted value back to time $T_0 = 0$ is only exposed to interest rate through discounting, i.e. $P(0, T_1)$. Let's express the discounting factor in terms of zero rate:

$$P(0, T_1) \;\; = \;\; e^{-r(T_0, T_1) \cdot T_1}.$$

Then, the value of the floating rate loan is only exposed to a risk that the zero rate $r(T_0, T_1)$ changes that will impact the disocunted value. For example, if $r(T_0, T_1)$ suddently increases by 1%, the value of the float rate loan will change by:

$$\begin{aligned}
\Delta &= [N + N \cdot (T_1 - T_0) \cdot L(T_0, T_1)] \cdot \left[e^{-[r(T_0,T_1)+1\%] \cdot T_1} - e^{-r(T_0,T_1) \cdot T_1} \right]] \\
&\approx [N + N \cdot (T_1 - T_0) \cdot L(T_0, T_1)] \cdot (-1\%) \cdot T_1 \\
&\approx N \cdot (-1\%) \cdot T_1
\end{aligned}$$

This is the case when $L(T_0, T_1)$ has been fixed. If $L(T_0, T_1)$ is not fixed yet (e.g. it will only get fixed in late evening on the date T_0), a rate changes will have even less (effectively zero) impact on the value of the floating rate loan, as when the rates changes, both the discount rate and the $L(T_0, T_1)$ will change and their effects will be offset.

The Δ above, calculated from first principle, is essentially what ΔEVE attempts to capture (under pre-defined scenarios). If the repricing gap model along with the ΔEVE calculation approach presented in class were to capture this properly, the repricing date for this floating rate loan needs to be defined as the nearest future interest rate fixing date, i.e. T_1. By defining it so, the ΔEVE for a floating rate loan, presented in class, will be largely consistent with our first-principle estimate Δ above.

Next, consider what NII attempts to measure. It essentially tries to estimate the impact of an interest rate shock on the interest rate earnings/expenses over a pre-defined time window, e.g. 1Y. Assume the loan has a notional of 100 dollars and pays quarterly floating rate interest. Since the the first floating interest rate is already fixed, there remains 3 floating rate interest payments over the 1Y window that are subject to the risk of rates changes. If now the rates environment changes suddently with a parallel shift of 1%, we would have to revise our expected interest income over the 1 year period by an amount of $100 \cdot 0.25 \cdot 1\% =$ for each of the 3 unfixed interest rate payment. This sums to a total NII impact of 0.75 over the 1Y window. If the repricing gap model along with the ΔNII calculation approach presented in class were to capture this expected NII impact, what should be repricing date for the loan? Clearly, it should be 3M as following the the ΔNII the formula presented in class, we would also obtain:

$$\Delta R \cdot A \cdot (N - T) = 1\% \cdot 100 \cdot (1 - 0.25) \;\; = \;\; 0.75.$$

In summary, there is a reason why the repricing date for a floating rate instrument is defined as the nearest interest rate fixing date. Following the above, one possible explanation is that it enables the ΔEVE and ΔNII approaches to return sensible EVE and NII sensitivity measures that are in line with expectations.