Risk Analysis (QF609, AY2023-2024)

Practice Problem Set #1

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1. Consider a newly established bank with an intial balance sheet:

Liabilities		Assets	
Capital	20	FRB	20
Deposits	100	Loans	100
	120		120

Assume:

- both the loans and deposits have a fixed maturity of 5Y
- 3% interest on the deposits and 5% interest on the loans, both paid annually
- the bank will spend $1 billion per year on operating expenses
- no interests earned on the capital deposited in FRB

(a) With accrual accounting, how will the balance sheet look like after 3M and 6M, respectively?

(b) With MTM accounting, how will the balance sheet look like after 3M and 6M, respectively, if the benchmark discount rate (quoted as an annually compounding rate) in 3M and 6M becomes 2% and 8%, respectively?

In above, simply provide the values of the balance sheet items (no need to provide profit and loss numbers).

2. What is the key difference between a banking book and a trading book?

3. (*A small challenge*) Consider an 20-year fixed-rate mortgage with a notional amount of $1 million. The nominal mortgage rate is 5% (monthly compounding, i.e. the monthly compounding factor is given by $1 + \frac{1}{12} \cdot 5\%$). The mortage payment for the k^{th} month is defined by $A \cdot B^{k-1}$.

(a) Find A given $B = 1.001$ (you do not need to provide the calculation details, but do mention briefly what's the criteria you follow to obtain your answer).

(b) With the A found in (a) and $B = 1.001$, generate the amortization table for this mortgage in EXCEL and provide the entries for the 10-th, 100-th, 200-th, and 240-th month, respectively (you only need to provide the entries below).

Month	Opening Balance	Payment	Interest	Principal Repaid	Closing Balance
10	?	?	?	?	?
100	?	?	?	?	?
200	?	?	?	?	?
240	?	?	?	?	?

1. (a)

BS : 3M

Liabilities		Assets	
Capital	20	FRB	20
Retained profit	0.25	Loans	100
Deposit	100	Interest receivable	0.25
	120.25		120.25

BS : 6M

Liabilities		Assets	
Capital	20	FRB	20
Retained profit	0.5	Loans	100
Deposit	100	Interest receivable	0.5
	120.5		120.5

(b)

T	3M	6M
Net Realized cash	0	0
NPV of Loan	114.7	91.5
NPV of deposit	-105.2	-83.2
MTM	9.5	8.3
operating expense	-0.25	-0.5
Net Profit (MTM)	9.25	-1.95

2. Trading Book used for short term trading activities
Banking Book are expected to be held to maturity

3.

$$\sum_{k=1}^{20\times 12} \left(1+\tfrac{1}{12}\times 5\%\right)^{-k} A\cdot B^{k-1}$$

$$\sum_{k=1}^{240} \left(\frac{B}{1+\frac{1}{12}\times 5\%}\right)^{k} \frac{A}{B}$$

$$\frac{a_1(1-q^n)}{1-q}$$

$$q = \frac{B}{1+\frac{1}{12}\times 5\%}$$

$$a_1 = \frac{A}{1+\frac{1}{12}\times 5\%}$$

$$\frac{\frac{A}{1+\frac{1}{12}\times 5\%}\left(1-q^{240}\right)}{1-q} = 1\text{m}$$

$$\Rightarrow A = 5958.90$$

Month	Opening Balance	Payment	Interest	Principal Repaid	Closing Balance
10	? 883387.2749	? 6012.734867	? 4097.492177	? 1915	? 981466
100	? 746648	? 6578	? 3111	? 3468	? 743180
200	? 278985	? 7270	? 1161	? 6108	? 272757
240	? 9137	? 7566	? 31	? 7535	? 0

4. Provide the repricing amount(s) and date(s) for each of the transaction below.

Product	Interest Rate	Notional	Maturity	repricing date	repricing gap
Interest-Only Loan	1%	\$100k	2Y	2Y	+\$100k
Interest-Only Loan	USD 3M Libor	\$100k	5Y	3M	+\$100k
Deposit	3%	\$100k	2Y	2Y	-\$100k
Deposit	USD 3M Libor	\$100k	5Y	3M	-\$100k
Treasury notes	4%	\$100k	5Y	5Y	+\$100k
IRS pay	3%	\$100k	5Y	5Y	-
IRS receive	USD 3M Libor	\$100k	5Y	3M	+
1M-forward starting deposit	3%	\$100k	2Y	1M 2Y1M	+ -
1M-forward starting deposit	USD 3M Libor	\$100k	2Y	1M 1M	+ -
1M-forward starting IRS pay	3%	\$100k	5Y	5Y1M	-
1M-forward starting IRS receive	USD 3M Libor	\$100k	5Y	1M	+

5. A bank has only two products on its balance sheet, loans and deposits, respectively. The repricing time buckets and amounts for the two products are provided in the gap report below. Complete the Net Gap and CGAP (i.e. the accumulated net gap along the buckets) columns, respectively.

Repricing Time Bucket	Loans	Deposits	Net Gap	CGAP
1D	5	-30	-25	-25
1M	5	-20	-15	-40
2M	5	-20	-15	-55
3M	10	-20	-10	-65
6M	10	-30	-20	-85
9M	15	-15	0	-85
1Y	20	-15	5	-80
2Y	30	0	30	-50
3Y	15	0	15	-35
4Y	20	0	20	-15
5Y	10	0	10	-5
7Y	5	0	5	0
10Y	0	0	0	0

6. Take the repricing gap report from the previous question and assume that the base discount curve is given by a flat zero rate curve of 3% for all tenors.
   (a) Calcualte the econmic value sensitivity (ΔEV) under a +100bp parallel shift of the zero rates.
   (b) Calculate the net interest income sensitivity (ΔNII) over a 2-year horizon with the mid-point approximation under a +100bp parallel shift of the zero rates.

7. Assume r_1 and r_2 are two positive (annually) compounding rates, where $0 < r_1 < r_2$, and $n \geq 1$ is a positive integer. When is C defined below positive (or negative, respectively)? [**A Small Challenge**]

$$\frac{1}{(1+r_1)^n} - \frac{1}{(1+r_2)^n} > \frac{r_1}{1+r_1} - \frac{r_2}{1+r_2} C = \sum_{k=2}^{n} \frac{-r_2}{(1+r_2)^k} - \sum_{k=2}^{n} \frac{-r_1}{(1+r_1)^k}$$