Risk Analysis (QF609, AY2023-2024)

Practice Problem Set #1 Answers

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1. AA

(a)

3M			
Liabilities		Assets	
Capital	20	FRB	20
Retained profit	0.25	Loans	100
Deposits	100	Net accrued income/expense	0.25
Total	**120.25**	**Total**	**120.25**

6M			
Liabilities		Assets	
Capital	20	FRB	20
Retained profit	0.5	Loans	100
Deposits	100	Net accrued income/expense	0.5
Total	**120.5**	**Total**	**120.5**

For MTM in 3M:

$$\begin{aligned} Deposit - PV &= -\left[\sum_{i=1}^{5} \frac{3}{1.02^{i-1}} + \frac{100}{1.02^4}\right] \cdot \frac{1}{1.02^{0.75}} = -105.2331 \\ Loan - PV &= \left[\sum_{i=1}^{5} \frac{5}{1.02^{i-1}} + \frac{100}{1.02^4}\right] \cdot \frac{1}{1.02^{0.75}} = 114.7068 \end{aligned}$$

For MTM in 6M:

$$\begin{aligned} Deposit - PV &= -\left[\sum_{i=1}^{5} \frac{3}{1.08^{i-1}} + \frac{100}{1.08^4}\right] \cdot \frac{1}{1.08^{0.5}} = -83.1763 \\ Loan - PV &= \left[\sum_{i=1}^{5} \frac{5}{1.08^{i-1}} + \frac{100}{1.08^4}\right] \cdot \frac{1}{1.08^{0.5}} = 91.4750 \end{aligned}$$

2. One key difference between the two is that banking book is mainly meant for holding positions until maturity while trading book is more for active short term position taking.

3. (a) The PV of the mortgage cash flows are given by:

$$PV = \sum_{k=1}^{240} A(1.001)^{k-1} \cdot (1 + 0.05/12)^{-k}$$

This PV should be equal to the mortgag principle 1,000,000. Hence,

$$1000000 = \sum_{k=1}^{240} A(1.001)^{k-1} \cdot (1 + 0.05/12)^{-k} = \frac{A}{1 + 0.05/12} \cdot \sum_{k=1}^{240} \left(\frac{1.001}{1 + 0.05/12}\right)^{k-1}$$

Thus, we have

$$
\begin{aligned}
A &= \frac{1000000 \cdot (1+0.05/12)}{\sum_{k=1}^{240}(0.996846473)^{k-1}} \\
&= \frac{1000000 \cdot (1+0.05/12)}{\frac{1-0.996846473^{240}}{1-0.996846473}} \\
&= 5959
\end{aligned}
$$

(b) The required amortization entries are as below:

Month	Opening Balance	Payment	Interest	Principal Repaid	Closing Balance
10	983381	6013	4097	1915	981466
100	746648	6579	3111	3468	743181
200	278866	7270	1162	6108	272757
240	7535	7567	31	7535	0

4. Repricing date/amount:

Product	Interest Rate	Notional	Maturity	repricing date	repricing gap
Interest-Only Loan	1%	\$100k	2Y	2Y	\$100k
Interest-Only Loan	USD 3M Libor	\$100k	5Y	3M	\$100k
Deposit	3%	\$100k	2Y	2Y	-\$100k
Deposit	USD 3M Libor	\$100k	5Y	3M	-\$100k
Treasury notes	4%	\$100k	5Y	5Y	\$100k
IRS pay	3%	\$100k	5Y	5Y	-\$100k
IRS receive	USD 3M Libor	\$100k	5Y	3M	\$100k
1M-forward starting deposit	3%	\$100k	2Y	2Y1M 1M	-\$100k \$100k
1M-forward starting deposit	USD 3M Libor	\$100k	2Y	1M 1M	-\$100k \$100k
1M-forward starting IRS pay	3%	\$100k	5Y	5Y1M	-\$100k
1M-forward starting IRS receive	USD 3M Libor	\$100k	5Y	1M	\$100k

5. Net gap and cumulative gap:

Repricing Time Bucket	Loans	Deposits	Net Gap	CGAP
1D	5	-30	-25	-25
1M	5	-20	-15	-40
2M	5	-20	-15	-55
3M	10	-20	-10	-65
6M	10	-30	-20	-85
9M	15	-15	0	-85
1Y	20	-15	5	-80
2Y	30	0	30	-50
3Y	15	0	15	-35
4Y	20	0	20	-15
5Y	10	0	10	-5
7Y	5	0	5	0
10Y	0	0	0	0

6. ΔEVE and ΔNII:

T	mid-point	T	DF (at 3%)	DF (at 3%+100bp)	ΔEVE	ΔNII (2Y horizon)
0.0028	0.0014	0	0.9999	0.9999	0.0007	-0.4997
0.0833	0.0431	0	0.9975	0.9967	0.0125	-0.2935
0.1667	0.1250	0	0.9950	0.9934	0.0249	-0.2813
0.25	0.2083	0	0.9925	0.9900	0.0248	-0.1792
0.50	0.3750	1	0.9851	0.9802	0.0983	-0.3250
0.75	0.6250	1	0.9778	0.9704	0.0000	0.0000
1	0.8750	1	0.9704	0.9608	-0.0483	0.0563
2	1.5000	2	0.9418	0.9231	-0.5594	0.1500
3	2.5000	3	0.9139	0.8869	-0.4052	-
4	3.5000	4	0.8869	0.8521	-0.6955	-
5	4.5000	5	0.8607	0.8187	-0.4198	-
7	6.0000	7	0.8106	0.7558	-0.2740	-
10	8.5000	10	0.7408	0.6703	0.0000	-
Total					-2.2411	-1.3724

7. Let us define:

$$f(r) \quad = \quad \sum_{k=2}^{n} \frac{-r}{(1+r)^k} \tag{1}$$

Following the hint in the lecture, we have:

$$f(r) \quad = \quad \frac{1}{(1+r)^n} - \frac{1}{1+r} \tag{2}$$

Therefore,

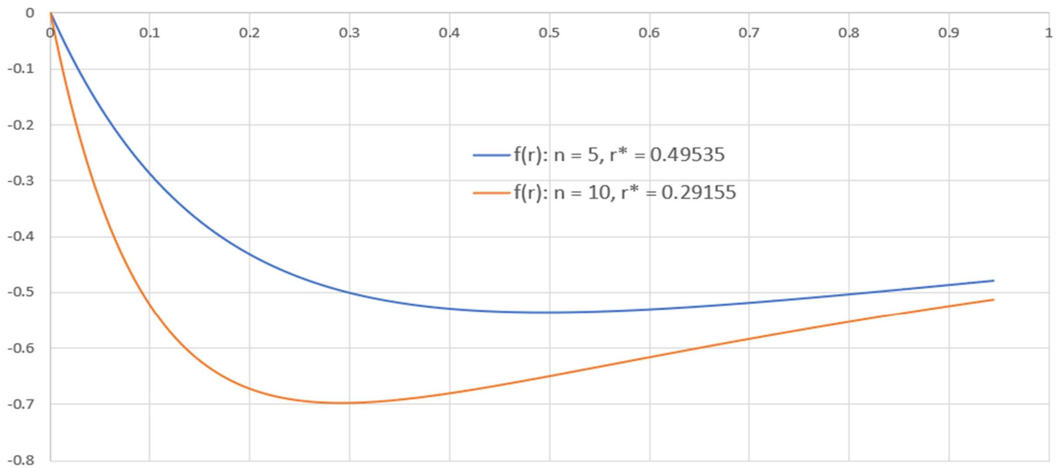
$$\frac{df(r)}{dr} \quad = \quad \frac{1}{(1+r)^2} - \frac{n}{(1+r)^{n+1}} \tag{3}$$

Setting $\left.\frac{df(r)}{dr}\right|_{r=r^*} = 0$ gives:

$$r^* = n^{\frac{1}{n-1}} - 1 \tag{4}$$

Since $f(r)$ starts off at 0 for $r = 0$ and is negative for all $r > 0$, it is easy to imagine that $(r^*, f(r^*))$ is a local minimum. This means $f(r)$ is decreasing on $(0, r^*)$ and increasing on (r^*, ∞). This means that C below is negative when $0 < r_1 < r_2 \leq r^*$ and positive for $r^* < r_1 < r_2$. However, the sign of C will not be clearly determined in the case $r_1 < r^* < r_2$.

$$C \quad = \quad \sum_{k=2}^{n} \frac{-r_2}{(1+r_2)^k} - \sum_{k=2}^{n} \frac{-r_1}{(1+r_1)^k} \tag{5}$$


f(r): n = 5, r* = 0.49535
f(r): n = 10, r* = 0.29155
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1