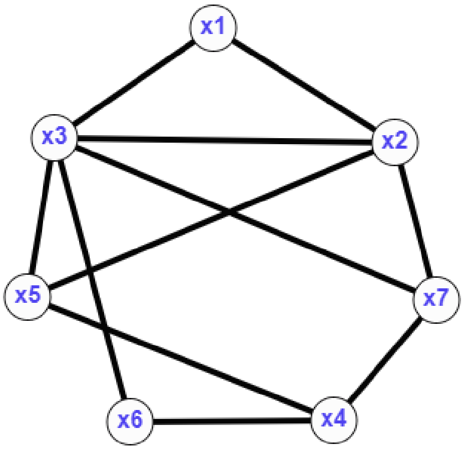
The Computation of All Maximal Independent Sets

Bron-Kerbosch Algorithm. Example



1. $S_0 = Q_0^- = \emptyset$, $Q_0^+ = X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.
2. $S_1 = \{x_1\}$, $Q_1^- = \emptyset$, $Q_1^+ = \{x_4, x_5, x_6, x_7\}$.
3. $S_2 = \{x_1, x_4\}$, $Q_2^- = \emptyset$, $Q_2^+ = \emptyset$. $\{x_1, x_4\}$ – maximal independent set.
4. (a backtracking step) $S_1 = \{x_1\}$, $Q_1^- = \{x_4\}$, $Q_1^+ = \{x_5, x_6, x_7\}$ (we retrieve Q_1^- and Q_1^+ from the step 2 and change them).
5. $S_2 = \{x_1, x_5\}$, $Q_2^- = \emptyset$, $Q_2^+ = \{x_6, x_7\}$.
6. $S_3 = \{x_1, x_5, x_6\}$, $Q_3^- = \emptyset$, $Q_3^+ = \{x_7\}$.
7. $S_4 = \{x_1, x_5, x_6, x_7\}$, $Q_4^- = \emptyset$, $Q_4^+ = \emptyset$. $\{x_1, x_5, x_6, x_7\}$ – maximal independent set.
8. $S_3 = \{x_1, x_5, x_6\}$, $Q_3^- = \{x_7\}$, $Q_3^+ = \emptyset$ (we retrieve Q_3^- and Q_3^+ from the step 6 and change them) (the condition (8) is satisfied$\Rightarrow$ a backtracking step).
9. $S_2 = \{x_1, x_5\}$, $Q_2^- = \{x_6\}$, $Q_2^+ = \{x_7\}$ (the condition (8) is satisfied$\Rightarrow$ a backtracking step).
10. $S_1 = \{x_1\}$, $Q_1^- = \{x_4, x_5\}$, $Q_1^+ = \{x_6, x_7\}$ (we retrieve Q_1^- and Q_1^+ from the step 4 and change them; we will do so at all backtracking steps) (the condition (3.8) is satisfied$\Rightarrow$ a backtracking step).

11.

12. $S_0 = \emptyset$, $Q_0^- = \{x_1\}$ (this means that we have already considered all maximal independent sets with x_1 and prohibit its use), $Q_0^+ = \{x_2, x_3, x_4, x_5, x_6, x_7\}$.

13. $S_1 = \{x_2\}$, $Q_1^- = \emptyset$, $Q_1^+ = \{x_4, x_6\}$.

14. $S_2 = \{x_2, x_4\}$, $Q_2^- = \emptyset$, $Q_2^+ = \emptyset$ $\{x_2, x_4\}$ – maximal independent set.

15. $S_1 = \{x_2\}$, $Q_1^- = \{x_4\}$, $Q_1^+ = \{x_6\}$.

16. $S_2 = \{x_2, x_6\}$, $Q_2^- = \emptyset$, $Q_2^+ = \emptyset$ $\{x_2, x_6\}$ – maximal independent set.

17. $S_1 = \{x_2\}$, $Q_1^- = \{x_4, x_6\}$, $Q_1^+ = \emptyset$ (the condition (8) is satisfied$\Rightarrow$ a backtracking step).

18. $S_0 = \emptyset$, $Q_0^- = \{x_1, x_2\}$ (this means that we have already considered all maximal independent sets with x_1, x_2 and prohibit their use), $Q_0^+ = \{x_3, x_4, x_5, x_6, x_7\}$.

19. $S_1 = \{x_3\}$, $Q_1^- = \emptyset$, $Q_1^+ = \{x_4\}$

20. $S_2 = \{x_3, x_4\}$, $Q_2^- = \emptyset$, $Q_2^+ = \emptyset$ $\{x_3, x_4\}$ – maximal independent set.

21. $S_1 = \{x_3\}$, $Q_1^- = x_4$, $Q_1^+ = \emptyset$ (the condition (8) is satisfied$\Rightarrow$ a backtracking step)

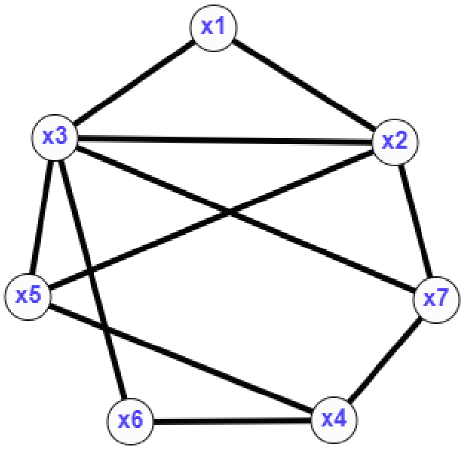
22. $S_0 = \emptyset$, $Q_0^- = \{x_1, x_2, x_3\}$, $Q_0^+ = \{x_4, x_5, x_6, x_7\}$. The condition (8) is satisfied, but we could not do a backtracking step$\Rightarrow$ Stop (all maximal independent sets have been found)

    $\{x_1, x_4\}$, $\{x_1, x_5, x_6, x_7\}$, $\{x_3, x_4\}$, $\{x_2, x_6\}$, $\{x_2, x_4\}$ – maximal independent sets

    $\{x_1, x_5, x_6, x_7\}$, – maximum independent set, $\alpha[G] = 4$ – the independence number of the graph G.

Boolean arithmetic. Example

Consider the same example.



$$\varphi' = (x_1' + x_2')(x_1' + x_3')(x_2' + x_3')(x_2' + x_5')(x_2' + x_7')(x_3' + x_5')*$$

$$*(x_3' + x_6')(x_3' + x_7')(x_4' + x_5')(x_4' + x_6')(x_4' + x_7') =$$

$$= (x_1' + x_2'x_3')(x_2' + x_3'x_7')(x_5' + x_2'x_3')(x_3' + x_6'x_7')(x_4' + x_5'x_6'x_7') =$$

$$= (x_1'x_2' + x_1'x_3'x_7' + x_2'x_3')(x_5' + x_2'x_3')(x_3'x_4' + x_6'x_7'x_4' + x_5'x_6'x_7') =$$

$$= (x_1'x_2'x_5' + x_1'x_3'x_7'x_5' + x_2'x_3')(x_3'x_4' + x_6'x_7'x_4' + x_5'x_6'x_7') =$$

$$= (x_1'x_2'x_5'x_6'x_7' + x_1'x_3'x_7'x_5'x_4' + x_1'x_3'x_7'x_5'x_6' + x_2'x_3'x_4' + x_2'x_3'x_5'x_6'x_7').$$

So, we have 5 maximal independent sets:

$\{x_3, x_4\} = X \backslash \{x_1, x_2, x_5, x_6, x_7\}$
$\{x_2, x_6\} = X \backslash \{x_1, x_3, x_7, x_5, x_4\}$
$\{x_2, x_4\} = X \backslash \{x_1, x_3, x_7, x_5, x_6\}$
$\{x_1, x_5, x_6, x_7\} = X \backslash \{x_2, x_3, x_4\}$
$\{x_1, x_4\} = X \backslash \{x_2, x_3, x_5, x_6, x_7'\}$