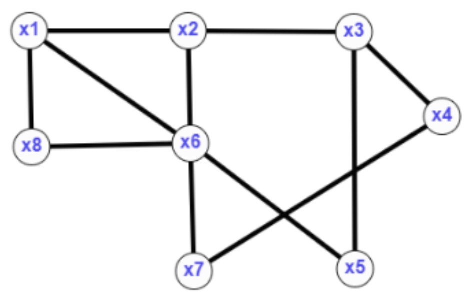
Homework_28022022_XuFeiran

Task 3



Graph G

Find all maximal independent sets and the independence number of graph G.

1. $S_0 = Q_0^- = \emptyset$, $Q_0^+ = X = \{x1, x2, x3, x4, x5, x6, x7, x8\}$.
2. $S_1 = \{x1\}, Q_1^- = \emptyset, Q_1^+ = \{x3, x4, x5, x7\}$
3. $S_2 = \{x1, x3\}$. $Q_2^- = \emptyset$, $Q_2^+ = \{x7\}$
4. $S_3 = \{x1, x3, x7\}, Q_3^- = \emptyset, Q_3^+ = \emptyset$
5. $S_2 = \{x1, x3\}, Q_2^- = \{x7\}, Q_2^+ = \emptyset$ (We retrieve Q_2^- and Q_2^+ from the step 3 and change them) The condition (8) is satisfied → a backtracking step.

the condition:

$$\exists x \in Q_k^- \text{ so that } \Gamma(x) \cap Q_k^+ = \emptyset \tag{8}$$

6. $S_1 = \{x1\}, Q_1^- = \{x3\}, Q_1^+ = \{x4, x5, x7\}$ (the condition (8) is not satisfied

7. $S_2 = \{x1, x4\}, Q_2^- = \emptyset, Q_2^+ = \{x5\}$
8. $S_3 = \{x1, x4, x5\}. Q_3^- = \emptyset, Q_3^+ = \emptyset$
9. $S_2 = \{x1, x4\}, Q_2^- = \{x5\}, Q_2^+ = \emptyset$ (Condition (8) is satisfied → backtracking step)
10. $S_1 = \{x1\}, Q_1^- = \{x3, x4\}, Q_1^+ = \{x5, x7\}$ (Condition (8) is not satisfied
11. $S_2 = \{x1, x5\}, Q_2^- = \{x4\}, Q_2^+ = \{x7\}$
12. $S_3 = \{x1, x5, x7\}, Q_3^- = \emptyset, Q_3^+ = \emptyset$
13. $S_2 = \{x1, x5\}, Q_2^- = \{x7\}, Q_2^+ = \emptyset$ (Condition (8) is satisfied → backtracking step)
14. $S_1 = \{x1\}, Q_1^- = \{x3, x4, x5\}, Q_1^+ = \{x7\}$ (Condition (8) is satisfied → backtracking step)
15. $S_0 = \emptyset$, $Q_0^- = \{x1\}$ (This means that we have already considered all maximal independent sets with x1 and prohibit its use) ,$Q_0^+ = \{x2, x3, x4, x5, x6, x7, x8\}$
16. $S_1 = \{x2\}, Q_1^- = \emptyset$, $Q_1^+ = \{x4, x5, x7, x8\}$
17. $S_2 = \{x2, x4\}, Q_2^- = \emptyset, Q_2^+ = \{x5, x8\}$
18. $S_3 = \{x2, x4, x5\}, Q_3^- = \emptyset, Q_3^+ = \{x8\}$
19. $S_4 = \{x2, x4, x5, x8\}, Q_4^- = \emptyset, Q_4^+ = \emptyset$,
20. $S_3 = \{x2, x4, x5\}, Q_3^- = \{x8\}, Q_3^+ = \emptyset$ (Condition (8) is satisfied → backtracking step)
21. $S_2 = \{x2, x4\}, Q_2^- = \{x5\}, Q_2^+ = \{x8\}$(Condition (8) is satisfied → backtracking step)
22. $S_1 = \{x2\}, Q_1^- = \{x4\}, Q_1^+ = \{x5, x7, x8\}$ (Condition (8) is not satisfied)
23. $S_2 = \{x2, x5\}, Q_2^- = \{x4\}, Q_2^+ = \{x7, x8\}$
24. $S_3 = \{x2, x5, x7\}, Q_3^- = \emptyset, Q_3^+ = \{x8\}$
25. $S_4 = \{x2, x5, x7, x8\}, Q_4^- = \emptyset, Q_4^+ = \emptyset$
26. $S_3 = \{x2, x5, x7\}, Q_3^- = \{x8\}, Q_3^+ = \emptyset$ (Condition (8) is satisfied → backtracking step)

27. $S_2 = \{x2, x5\}, Q_2^- = \{x4, x7\}, Q_2^+ = \{x8\}$ (Condition (8) is satisfied → backtracking step)

28. $S_1 = \{x2\}, Q_1^- = \{x4, x5\}, Q_1^+ = \{x7.x8\}$ (Condition (8) is satisfied → backtracking step)

29. $S_0 = \emptyset, Q_0^- = \{x1, x2\}, Q_0^+ = \{x3, x4, x5, x6, x7, x8\}$ (This means that we have already considered all maximal independent sets with x1,x2 and prohibit its use)

30. $S_1 = \{x3\}, Q_1^- = \{x1\}, Q_1^+ = \{x6, x7, x8\}$

31. $S_2 = \{x3, x6\}, Q_2^- = \emptyset, Q_2^+ = \emptyset$

32. $S_1 = \{x3\}, Q_1^- = \{x1, x6\}, Q_1^+ = \{x7, x8\}$ (Condition (8) is not satisfied)

33. $S_2 = \{x3, x7\}, Q_2^- = \{x1\}, Q_2^+ = \{x8\}$

34. $S_3 = \{x3, x7, x8\}, Q_3^- = \emptyset, Q_3^+ = \emptyset$

35. $S_2 = \{x3, x7\}, Q_2^- = \{x1, x8\}, Q_2^+ = \emptyset$ (Condition (8) is satisfied → backtracking step)

36. $S_1 = \{x3\}, Q_1^- = \{x1, x6, x7\}, Q_1^+ = \{x8\}$ (Condition (8) is satisfied)

37. $S_0 = \emptyset, Q_0^- = \{x1, x2, x3\}, Q_0^+ = \{x4, x5, x6, x7, x8\}$ (This means that we have already considered all maximal independent sets with x1,x2,x3 and prohibit its use)

38. $S_1 = \{x4\}, Q_1^- = \{x1, x2\}, Q_1^+ = \{x5, x6, x8\}$

39. $S_2 = \{x4, x6\}, Q_2^- = \emptyset, Q_2^+ = \emptyset$

40. $S_1 = \{x4\}, Q_1^- = \{x1, x2, x6\}, Q_1^+ = \emptyset$

41. $S_0 = \emptyset, Q_0^- = \{x1, x2, x3, x4\}, Q_0^+ = \{x5, x6, x7, x8\}$

42. $S_1 = \{x5\}, Q_1^- = \{x1, x2, x4\}, Q_1^+ = \{x7, x8\}$ (Condition (8) is satisfied → backtracking step)

43. $S_0 = \emptyset, Q_0^- = \{x1, x2, x3, x4, x5\}, Q_0^+ = \{x6, x7, x8\}$ (Condition (8) is satisfiedn ,, but we cant do a backtracking step → stop)

$\{x1, x3, x7\}, \{x1, x4, x5\}, \{x1, x5, x7\}, \{x2, x4, x5, x8\},$

$\{x2, x5, x7, x8\}, \{x3, x6\}, \{x3, x7, x8\}, \{x4, x6\}, \alpha[G] = 4$