Written Exam in Intelligent Vehicles – MK8005

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Assistant aids:	**Writing tools, calculator and an arbitrary book on formulas (e.g. Beta).**
Date:	**Halmstad, 2011-03-17**
Time limit:	**4 hours**
Answers:	**All answers should be motivated. The answers should be kept as <u>short</u> as possible.**
Language:	**Write your answers in either Swedish or English language.**
Contact:	**Björn Åstrand, 0733 – 121285**
Points and grades:	**Maximum points = 50** **[20 – 29.5]p gives grade = 3** **[30 – 39.5]p gives grade = 4** **[40 – 50.0]p gives grade = 5**
Passing the exam / Final grade:	**You should, to pass the exam, achieve at least the grade 3**

Good luck,

/Björn

1. **In Exercise 1 we found that speed estimates using GPS was much better than the position estimate.**
   a. Explain why? (4p)
   b. How many satellites are needed for estimating the position and why? (2p)

2. **Triangulation can be used for calculating robot position. Briefly explain how triangulation works?** (3p)

3. **Your robot is equipped with a rate-gyro. Explain:**
   a. What information does it give? (2p)
   b. How can this be used to reduce the uncertainty in the pose of a mobile robot? (2p)
   c. What are the main limitations of this sensor in this situation? (2p)

4. **The differential drive robot, which was used in exercise 2 and Wang paper, the vehicle's relative movement in between time steps k and k+1 ($\Delta x, \Delta y, \Delta\theta$) can be seen in Equation 2.**

$X_{k+1} = X_k + \Delta X$ there

$$\Delta X = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} \Delta s \cos\left(\theta + \frac{\Delta\theta}{2}\right) \\ \Delta s \sin\left(\theta + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{pmatrix} \quad \text{and} \quad \begin{aligned} \Delta s &= \frac{\Delta s_r + \Delta s_l}{2} \\ \Delta\theta &= \frac{\Delta s_r - \Delta s_l}{L} \end{aligned} \qquad \text{(Equation 2)}$$

there $\Delta s_r, \Delta s_l$ are the incremental distance traveled by the robots left and right wheel, $\Delta\theta$ the change of heading, and L the distance between the wheels.

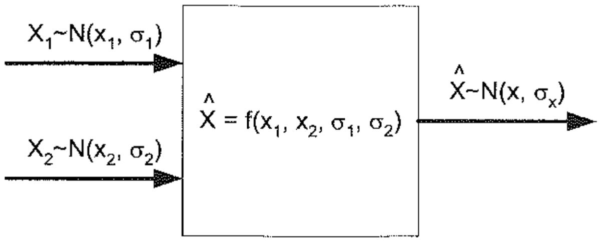
   a. The input signals, Δs and $\Delta\theta$, are not known with absolute certainty. In the Wang paper they assumed random error of odometry and this can be modeled by a parametric distribution. They derived following expression for the variance of change in robots heading, $\sigma_{\Delta\theta}^2 = \left(\sigma_r^2 + \sigma_l^2\right)/L^2$, and the variance of robot incremental distance traveled, $\sigma_{\Delta s}^2 = \left(\sigma_r^2 + \sigma_l^2\right)/4$. Calculate the co-variance matrix of X_{k+1}, i.e. the position uncertainty $\Sigma_{X_{k+1}}$, with respect to the uncertainty in the input signals. (You should do all necessary calculations.) (7p)
   b. Explain the term $\Delta\theta/2$ in Equation 2? (3p)

5. **The Snowhite robot, which was used in exercise 2-4, the vehicle's relative movement in between time steps k and k+1 (Δx, Δy, Δθ) can be seen in below.**

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} v(k)\cos(\alpha(k))T\cos(\theta(k) + \frac{v(k)\sin(\alpha(k))T}{2L}) \\ v(k)\cos(\alpha(k))T\sin(\theta(k) + \frac{v(k)\sin(\alpha(k))T}{2L}) \\ \frac{v(k)\sin(\alpha(k))T}{L} \end{pmatrix}$$

   a. Under the assumption that all input signals but the speed, v(k), are known with absolute certainty, calculate the co-variance matrix of (Δx, Δy, Δθ). (You should do all necessary calculations.) (6p)
   b. Explain the difference between a systematic and a random error. Give two examples of something that might cause a systematic error in odometry. (2p)

6. **Assume you have two independent (both having errors that are zero mean and Gaussian distributed with variances σ_1^2 and σ_2^2 respectively) measurement systems, both measuring X . See the below figure for an illustration.**



a. Derive the expression for the linear combination $\hat{X} = f(X_1, X_2, \sigma_1^2, \sigma_2^2)$ that gives you the smallest variance of the error in the estimated $\hat{X}$. (4p)

b. Derive an expression for the variance of $\hat{X}$. (2p)

7. **If you want to move a long distance and arrive at a point with little position uncertainty, would you prefer to start with low angular uncertainty or low position uncertainty? Motivate you answer!**

(2p)

8. **Explain what an occupancy grid is and can be used for?**

(5p)

9. **In exercise 3 we used Cox algorithm for scan matching range scans. Explain how the algorithm works? You don't have to write any equations!**

(4p)