Written Exam in Intelligent Vehicles – MK8005

Björn Åstrand
Halmstad University, 2010.03.18

Assistant aids:	Writing tools, calculator and an arbitrary book on formulas (e.g. Beta).
Date:	Halmstad, 2010-03-18
Time limit:	4 hours
Answers:	All answers should be motivated. The answers should be kept as <u>short</u> as possible.
Language:	Write your answers in either **Swedish** or **English** language.
Contact:	Björn Åstrand, 035 – 167143 or 0733 – 121285
Points and grades:	Maximum points = 50 [20 – 29.5]p gives grade = 3 [30 – 39.5]p gives grade = 4 [40 – 50.0]p gives grade = 5
Passing the exam / Final grade:	You should, to pass the exam, achieve at least the grade 3

Good luck,

/Björn

prediction
observation
matching
position update

1. **In Exercise 3 the Cox algorithm was used for matching range scans. Explain how the algorithm works? You don't have to write any equations.** (4p)

2. **Two approaches for robot localization are: *relative* localization and *absolute* localization. Name two localizations methods for each approach.** (4p)

Kalman Localization — Markov Localization

3. **What is triangulation and explain how it works?** (2p)

4. **Assume we measure the relative movement of an airplane (see Figure) by a distance, $\hat{d}_i = d_i + \varepsilon_d$, which is the true distance plus some noise, which is normally distributed with zero mean and known variance σ_d^2. We also measures (by a gyro or something similar) the change in heading angle of the airplane, $\hat{\alpha}_i = \alpha_i + \varepsilon_\alpha$, i.e. the change in the airplane's heading relative ground. Also the error in the angle measurement is normally distributed with zero mean and known variance σ_α^2. As the two parameters are given by different measuring systems we can further assume that the errors are uncorrelated, i.e. the co-variance between ε_d and ε_α is zero.**

$d_i \cos(\alpha + \frac{\alpha}{2}$

a. Derive an expression for the predicted position at time k+1, i.e. so that X(k+1) = [x(k+1) y(k+1) θ(k+1)]^T becomes a function of the position at time k, the relative change in movement d(k) and the relative change in heading α(k). (3p)

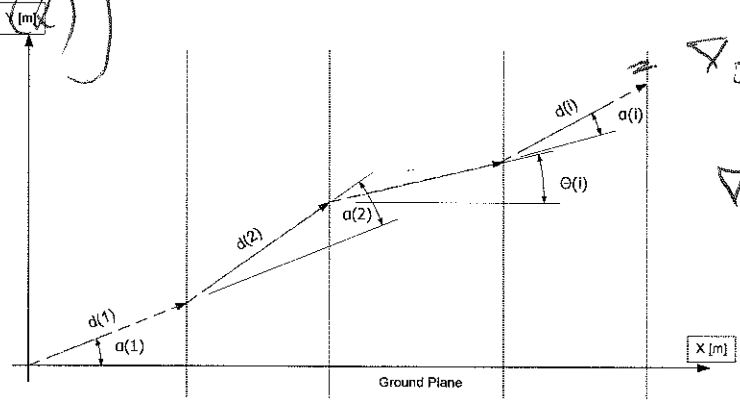
b. Derive an expression for the position uncertainty Σ_{k+1}, i.e. the covariance matrix of at time step k+1. Use the error propagation law. (6p)

$P_e + P_\Delta$

$\begin{bmatrix} x(k-1) \\ y(k-1) \\ \theta(k-1) \end{bmatrix} + \begin{bmatrix} d_i \cos(\frac{\alpha}{2}) \\ d_i \sin(\frac{\alpha}{2}) \\ \alpha_i \end{bmatrix}$

$\partial\theta$

$\nabla f \cdot \Sigma_k \nabla f^T + \nabla f_{dr} \cdot \Sigma \nabla f_{dr}^T$



5. **The Snowhite robot, which was used in exercise 2-4, the vehicle's relative movement in between time steps k and k+1 (Δx, Δy, Δθ) can be seen in below.**

$P(k+1) \sim P(k) + \Delta P$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} v(k)\cos(\alpha(k))T\cos(\theta(k) + \frac{v(k)\sin(\alpha(k))T}{2L}) \\ v(k)\cos(\alpha(k))T\sin(\theta(k) + \frac{v(k)\sin(\alpha(k))T}{2L}) \\ \frac{v(k)\sin(\alpha(k))T}{L} \end{pmatrix}$$

a. Under the assumption that all input signals but the speed, *v(k)*, are known with absolute certainty, calculate the co-variance matrix of (Δx, Δy, Δθ). (You should do all necessary calculations.) (6p)

6. **Assume you have a robot equipped with a dead reckoning system, which provides you with a estimate of its position, (x(k), y(k), θ(k)), and its co-variance matrix, Σ_{dr}(k), at time step k. Also assume that you have implemented the Cox scan matching algorithm which provides you with position fixes, (Δx(k), Δy(k), Δθ(k)), and its corresponding co-variance matrix, Σ_m(k), at the same time intervals, i.e. at each time step k it provides you with a position fix.**

a. Briefly explains what happens to the position estimate (and its co-variance matrix) if we decide not to use any of the position fixes. (2p)

b. Briefly explains what happens to the position estimate (and its co-variance matrix) if we use every second position fix. (It is assumed that we are using a Kalman filter to fuse the position estimates and the position fixes.) (2p)

c. To decide whether the position fixes are correct or not, we can use a validation gate and only use position fixes that pass the gate. As a validation gate, one might think of the following three possibilities:

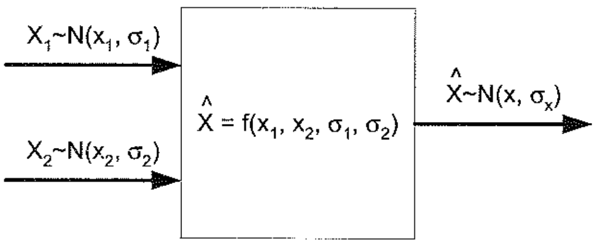
$$(\Delta x \quad \Delta y \quad \Delta\theta)(\Sigma_m)^{-1}(\Delta x \quad \Delta y \quad \Delta\theta)^T \leq \mathit{Threshold}$$
$$(\Delta x \quad \Delta y \quad \Delta\theta)(\Sigma_{dr})^{-1}(\Delta x \quad \Delta y \quad \Delta\theta)^T \leq \mathit{Threshold}$$
$$(\Delta x \quad \Delta y \quad \Delta\theta)(\Sigma_m + \Sigma_{dr})^{-1}(\Delta x \quad \Delta y \quad \Delta\theta)^T \leq \mathit{Threshold}$$

Which of the above would you use and why? Why would you use one of the above over the other? (3p)

7. **What does it mean that a mobile robot has a non-holonomic constraint.** (2p)

8. **Two papers were about the Vector Field Histogram (VFH) method.**
   a. Describe how the method works? (3p)
   b. What is the method mainly used for? (2p)

9. **Assume you have two independent (both having errors that are zero mean and Gaussian distributed with variances σ_1^2 and σ_2^2 respectively) measurement systems, both measuring X. See the below figure for an illustration.**



   a. Derive the expression for the linear combination $\hat{X} = f(X_1, X_2, \sigma_1^2, \sigma_2^2)$ that gives you the smallest variance of the error in the estimated $\hat{X}$. (4p)
   b. Derive an expression for the variance of $\hat{X}$. (2p)

10. **The exercise on GPS is about investigating a well used sensor in terms of e.g. *accuracy*, *repeatability* and *bias*.**
   a. In general, how can you improve the repeatability of a sensor? Why is this problematic in the case of the GPS receiver? (2p)
   b. In the exercise we found that the speed estimates using GPS was much better that the position estimate. Explain why (3p)

$y = a_1 x_1 + a_2 x_2$

$a_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$

σ