Digital Control: Exercise 5

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1. Basic theory

a) Tank dynamics

A tank process is there animated and can be controlled manually or by feedback control. The dynamics of the process can be described by the mass balance equation:

$$A\frac{dh}{dt} = q_{in} - q_{out}$$

Where A is the tank area, h is the height of water, q_{in} and q_{out} are the inflow and outflow of water, respectively. Energy balance (Bernoulli) gives:

$$\rho gh = \frac{\rho v^2}{2} \Rightarrow v = \sqrt{2gh}$$

Where v is the outlet water velocity. With outlet area a, the outflow then becomes:

$$q_{out} = av = a\sqrt{2gh}$$

The pump flow is proportional to the pump voltage V, according to:

$$q_{in} = kV$$

The nonlinear tank dynamics are therefore:

$$A\frac{dh}{dt} = kV - a\sqrt{2gh} \ \rightarrow \frac{dh}{dt} = -\alpha\sqrt{h} + \beta V = f(h,V)$$

Where $\alpha = \frac{\alpha\sqrt{g}}{A}$ and $\beta = \frac{k}{A}$. For simplicity, the values are chosen $\alpha = \beta = 1$ and when the valve is opened $\alpha = 4$.

b) Linearization

Balance level $(\dot{h} = 0)$ for a constant pump voltage v_0 corresponds to the level $h_0 = {v_0}^2$. Linearization around the balance level gives:

$$\frac{d\Delta h}{dt} = f(h,V) \approx \frac{df}{dh}(h_0,V_0)\Delta h + \frac{df}{dV}(h_0,V_0)\Delta V$$

Where with notation $y = \Delta h = h - h_0$ and $u = \Delta V = V - V_0$

$$\frac{dy}{dt} = py + du, \quad \begin{cases} p = -\frac{\alpha}{2\sqrt{h_0}} \\ d = 1 \end{cases}$$

c) Sampling

Zero-order-hold sampling with sampling period $h_s = 1$ gives

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^p \\ b = (\lambda - 1)/p \end{cases}$$

In polynomial form $A(q^{-1}) = 1 - \lambda q^{-1}$ and $B(q^{-1}) = bq^{-1}$.

d) Identification

The parameters λ and b can be estimated experimentally by the least-squares method. This is performed as follows: Excite the system around the balance level h_0, collect input and output data.

2. Tank models at different operating points

Now do the following experiment with the tank process. To access the samples, first write in the command window

> global samples

The variable samples will now continuously change with the data from the simulation. It contains 100 of the latest data samples, as $samples = [t, u, y]$, where is the time (in seconds), u the input samples and y the output samples (height level h).

a) Excite the tank system manually using $u_0(K = 0)$ such that y varies with mean close to $h_0 = 20$.

As we already know from part1,

$$\begin{cases} p = -\frac{\alpha}{2\sqrt{h_0}} = -\frac{1}{2\sqrt{20}} = -\frac{\sqrt{5}}{20} \\ d = 1 \end{cases}$$

Zero-order-hold sampling with sampling period $h_s = 1$ gives

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^p = e^{-\frac{\sqrt{5}}{20}} \approx 0.8942 \\ b = \frac{\lambda - 1}{p} = \frac{e^{-\frac{\sqrt{5}}{20}} - 1}{-\frac{\sqrt{5}}{20}} \approx 0.9461 \end{cases}$$

Then I manually set $u_0(K = 0)$ in P-control mode to make the y varies with mean close to $h_0 = 20$. I got the samples as following table:

Table 2.1 Samples when $h_0 = 20$

21587.7574	4.3698	19.8371	20

21588.7779	4.3698	19.7561	20
21589.798	4.3698	19.6838	20
21590.8787	4.3698	19.6159	20
21591.9987	4.3698	19.5537	20
21593.018	4.3698	19.5035	20
21594.0379	4.3698	19.4588	20
21595.1378	4.3698	19.416	20
21596.1782	4.3698	19.3802	20
21597.2176	4.3698	19.3484	20
21598.2578	4.3698	19.3201	20
21599.259	4.3698	19.2959	20
21600.3392	4.3698	19.2726	20
21601.3987	4.3698	19.2525	20
21602.4792	4.3698	19.2342	20
21603.5392	4.3698	19.2184	20
21604.5987	4.3698	19.2044	20
21605.6386	4.3698	19.1922	20
21606.6588	4.3698	19.1816	20
21607.6837	4.3698	19.1721	20
21608.6993	4.3698	19.1637	20
21609.7797	4.3698	19.1558	20
21610.8001	4.3698	19.1492	20
21611.84	4.3698	19.1431	20
21612.9	4.3698	19.1377	20
21614.0196	4.3698	19.1327	20
21615.1196	4.3698	19.1283	20
21616.1205	4.3698	19.1247	20
21617.1599	4.3698	19.1215	20
21618.1605	4.3698	19.1187	20
21619.241	4.3698	19.116	20
21620.3408	4.3698	19.1136	20
21621.4411	5.0703	19.1114	20
21622.461	4.691	19.7842	20
21623.4611	4.691	20.0141	20
21624.5411	4.3755	20.2352	20
21625.6409	4.557	20.108	20
21626.7016	4.557	20.1809	20
21627.7217	4.557	20.2433	20
21628.7613	4.557	20.3	20
21629.7827	4.557	20.3497	20
21630.8822	4.557	20.3972	20
21631.9216	4.557	20.4372	20
21632.9622	4.557	20.4728	20
21633.9627	4.557	20.5034	20

21634.9822	4.557	20.5313	20
21636.0227	4.557	20.5567	20
21637.0827	4.557	20.5798	20
21638.1426	4.557	20.6003	20
21639.2226	4.557	20.6189	20
21640.2426	4.557	20.6345	20
21641.2431	4.557	20.6482	20
21642.2826	4.557	20.661	20
21643.283	4.557	20.672	20
21644.383	4.557	20.6827	20
21645.4625	4.557	20.692	20
21646.4625	4.557	20.6998	20
21647.563	4.557	20.7073	20
21648.6626	4.557	20.714	20
21649.744	4.557	20.7199	20
21650.7841	4.557	20.7249	20
21651.8835	4.557	20.7296	20
21652.884	4.557	20.7334	20
21653.9836	4.557	20.7372	20
21655.0046	4.557	20.7403	20
21656.0243	4.557	20.743	20
21657.1443	4.557	20.7457	20
21658.2638	4.557	20.7481	20
21659.3048	4.557	20.7501	20
21660.3451	4.557	20.7518	20
21661.4047	4.557	20.7534	20
21662.4849	4.557	20.7548	20
21663.5645	4.557	20.7561	20
21664.5646	4.557	20.7572	20
21665.565	4.557	20.7581	20
21666.5848	4.557	20.759	20
21667.625	4.557	20.7598	20
21668.725	4.557	20.7605	20
21669.7861	4.557	20.7612	20
21670.8258	4.557	20.7617	20
21671.8664	4.557	20.7622	20
21672.9664	4.272	20.7627	20
21674.0862	4.272	20.4628	20
21675.206	4.272	20.1979	20
21676.2062	4.4535	19.9876	20
21677.2264	4.4535	19.971	20
21678.3463	4.4535	19.9548	20
21679.4074	4.4535	19.9412	20
21680.5069	4.4535	19.9287	20

21681.6066	4.4535	19.9177	20
21682.707	4.4535	19.908	20
21683.7071	4.4535	19.9001	20
21684.787	4.4535	19.8925	20
21685.8076	4.4535	19.8861	20
21686.9264	4.4535	19.8799	20
21687.987	4.4535	19.8748	20
21688.9875	4.4535	19.8704	20
21690.008	4.4535	19.8664	20
21691.108	4.4535	19.8626	20
21692.1275	4.4535	19.8595	20

And I use these data to calculate θ, then we can get:

$$\theta = \begin{pmatrix} \lambda \\ b \end{pmatrix} = \begin{pmatrix} 0.89 \\ 0.9822 \end{pmatrix} \approx \begin{pmatrix} 0.8942 \\ 0.9461 \end{pmatrix} (theory\ value)$$

From the result, we can found it's quite close to the theory value.

b) Calculate the response of your estimated model and compare it to the real response.

The result is shown in figure 2.1.

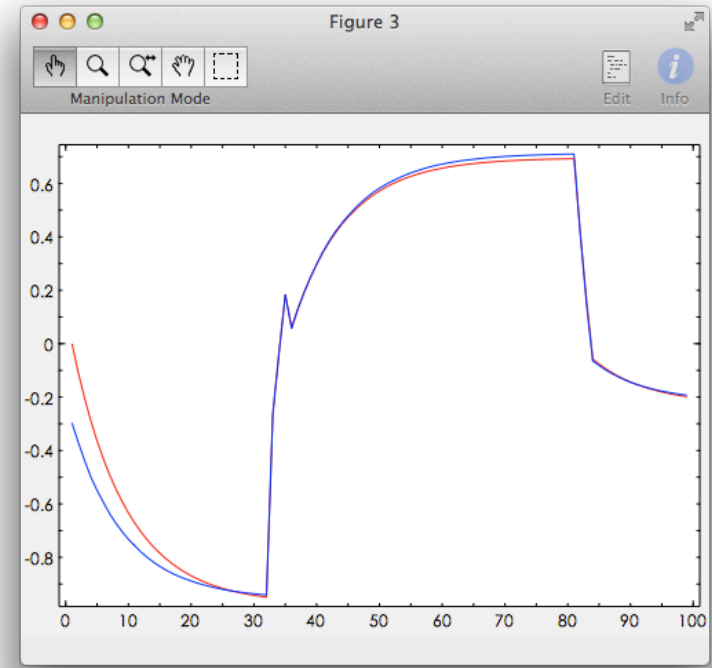

Figure 2.1 Comparison between the responses of estimated model and real model

In the figure, the red curve is the response of estimated model and the blue curve is the response of real model, we can see, at first these two curves have a bigger error, and then they got almost the same, the error is very small.

c) Repeat the above experiment with excitation around $h_0 = 350$.

$$\begin{cases} p = -\frac{\alpha}{2\sqrt{h_0}} = -\frac{1}{2\sqrt{350}} = -\frac{\sqrt{14}}{140} \approx -0.0267 \\ d = 1 \end{cases}$$

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^{p} = e^{-\frac{\sqrt{14}}{140}} \approx 0.9737 \\ b = \frac{\lambda - 1}{p} = \frac{e^{-\frac{\sqrt{14}}{140}} - 1}{-\frac{\sqrt{14}}{140}} \approx 0.9868 \end{cases}$$

Then I manually set $u_0(K=0)$ in P-control mode to make the y varies with mean close to $h_0 = 350$. I got the samples as following table:

Table 2.2 Samples when $h_0 = 350$

22412.5586	18.828	347.5852	350
22413.6787	18.828	347.7887	350
22414.7986	18.828	347.9861	350
22415.8386	18.828	348.1643	350
22416.9386	18.828	348.3473	350
22417.9583	18.828	348.5123	350
22418.9586	18.828	348.6698	350
22419.9596	18.828	348.8232	350
22420.9996	18.828	348.9783	350
22422.0395	18.828	349.1291	350
22423.0593	18.828	349.273	350
22424.1589	18.828	349.4239	350
22425.1592	19.119	349.5573	350
22426.1792	18.6172	349.9825	350
22427.1992	18.6172	349.8912	350
22428.2991	18.6172	349.7956	350
22429.4001	18.6172	349.7027	350
22430.5002	18.6172	349.6125	350
22431.5199	18.6172	349.5312	350
22432.6201	18.6172	349.446	350
22433.6201	18.6172	349.3707	350
22434.7012	18.6172	349.2915	350
22435.8012	18.6172	349.2133	350
22436.9012	18.6172	349.1373	350
22437.9211	18.6172	349.0688	350
22439.0405	18.6172	348.9957	350
22440.0807	18.6172	348.9298	350
22441.081	18.6172	348.8681	350
22442.1807	18.6172	348.8022	350
22443.2009	18.6172	348.7427	350
22444.221	18.6172	348.6848	350

22445.3212	18.6172	348.6242	350
22446.4414	18.6172	348.5642	350
22447.4812	18.6172	348.5101	350
22448.4812	18.6172	348.4595	350
22449.5621	18.6172	348.4064	350
22450.5621	18.6172	348.3585	350
22451.6615	18.6172	348.3074	350
22452.7415	18.6172	348.2586	350
22453.7814	18.6172	348.2129	350
22454.7822	18.6172	348.1701	350
22455.8821	18.6172	348.1244	350
22456.9819	18.6172	348.08	350
22458.0622	18.6172	348.0377	350
22459.1026	18.6172	347.9981	350
22460.1228	18.6172	347.9603	350
22461.2063	18.6172	347.9213	350
22462.2832	18.6172	347.8836	350
22463.3029	18.6172	347.8489	350
22464.3836	18.6172	347.8131	350
22465.3837	18.6172	347.781	350
22466.3837	18.6172	347.7497	350
22467.5033	19.3085	347.7156	350
22468.5224	18.819	348.3804	350
22469.6237	18.819	348.5476	350
22470.6244	18.819	348.6953	350
22471.6444	18.819	348.8419	350
22472.6644	18.819	348.9845	350
22473.7245	18.819	349.1286	350
22474.8056	18.819	349.2714	350
22475.8255	18.819	349.4024	350
22476.9257	18.819	349.5398	350
22477.9846	18.819	349.6683	350
22479.0253	18.819	349.791	350
22480.1052	18.819	349.9148	350
22481.1055	18.819	350.0264	350
22482.1254	18.819	350.1371	350
22483.2055	18.819	350.2511	350
22484.2455	18.819	350.3578	350
22485.2458	18.819	350.4576	350
22486.2655	18.819	350.5567	350
22487.3665	18.819	350.6607	350
22488.4064	18.819	350.7562	350
22489.4065	18.819	350.8455	350
22490.4257	18.819	350.9342	350

22491.5257	18.819	351.0272	350
22492.5465	18.819	351.1111	350
22493.5662	18.819	351.1926	350
22494.6061	18.8935	351.2735	350
22495.6857	18.8935	351.4344	350
22496.7261	18.6047	351.5851	350
22497.7262	18.6047	351.4411	350
22498.7461	18.6047	351.2982	350
22499.787	18.6047	351.1564	350
22500.8866	18.6047	351.0107	350
22501.9264	18.6047	350.8768	350
22503.027	18.6047	350.7391	350
22504.128	18.6047	350.6054	350
22505.128	18.6047	350.4872	350
22506.1878	18.6047	350.3654	350
22507.1879	18.6047	350.2536	350
22508.1879	18.6047	350.1448	350
22509.1882	18.6047	350.0388	350
22510.3017	18.6047	349.924	350
22511.3083	18.6047	349.8232	350
22512.4077	18.6047	349.7161	350
22513.4079	18.6047	349.6214	350
22514.428	18.6047	349.5274	350
22515.4681	18.6047	349.4342	350
22516.4682	18.6047	349.3469	350

And I use these data to calculate θ, then we can get:

$$\theta = \begin{pmatrix} \lambda \\ b \end{pmatrix} = \begin{pmatrix} 0.9725 \\ 1.0244 \end{pmatrix} \approx \begin{pmatrix} 0.9737 \\ 0.9868 \end{pmatrix} (theory\ value)$$

Calculate the response of your estimated model and compare it to the real response. The result is shown in figure 2.2.

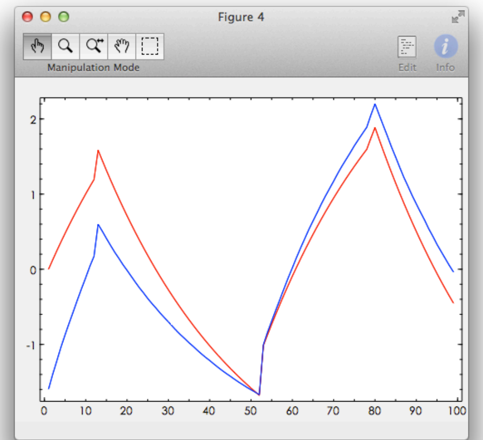

Figure 2.2 Comparison between the responses of estimated model and real model

3. Changed model when opening the valve

Click on the valve to opening it. Then the outflow is increased and $\alpha = 4$. Estimate how the model is changed by repeating Problem 1a) and b), now with the valve fully opened.

$$\begin{cases} p = -\dfrac{\alpha}{2\sqrt{h_0}} = -\dfrac{4}{2\sqrt{20}} = -\dfrac{\sqrt{5}}{5} \\ d = 1 \end{cases}$$

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^{p} = e^{-\frac{\sqrt{5}}{5}} \approx 0.6394 \\ b = \dfrac{\lambda - 1}{p} = \dfrac{-\frac{\sqrt{5}}{5} - 1}{-\frac{\sqrt{5}}{5}} \approx 0.8063 \end{cases}$$

Then I manually set $u_0(K = 0)$ in P-control mode to make the y varies with mean close to $h_0 = 20$ and also open the valve. I got the samples as following table:

Table 3.1 Samples when $h_0 = 20$ and $\alpha = 4$

23856.6389	17.7375	19.6637	20
23857.679	17.7375	19.6637	20
23858.7789	17.7375	19.6637	20
23859.7796	17.7375	19.6637	20
23860.88	17.7375	19.6637	20
23862.0001	17.7375	19.6637	20
23863.0599	17.7375	19.6637	20
23864.1	17.7375	19.6637	20
23865.22	17.7375	19.6637	20
23866.3	17.7375	19.6637	20
23867.36	17.7375	19.6637	20
23868.4001	17.7375	19.6637	20
23869.4219	17.7375	19.6637	20
23870.4616	17.7375	19.6637	20
23871.5806	17.7375	19.6637	20
23872.6813	17.7375	19.6637	20
23873.6817	17.7375	19.6637	20
23874.7009	17.8277	19.6637	20
23875.7009	17.8277	19.7363	20
23876.7409	17.8277	19.784	20
23877.761	17.539	19.8135	20
23878.841	17.539	19.5861	20
23879.8419	17.539	19.4545	20
23880.8819	16.9357	19.3684	20

23881.9619	16.9357	18.7988	20
23882.9706	17.2379	18.4705	20
23884.0218	17.7522	18.5095	20
23885.1219	17.7522	18.9777	20
23886.2019	17.4634	19.2561	20
23887.2219	18.1548	19.1833	20
23888.2419	18.1548	19.7018	20
23889.3229	18.1548	20.0444	20
23890.3629	18.1548	20.2494	20
23891.4229	18.1548	20.3805	20
23892.5028	18.1548	20.4637	20
23893.5293	17.8661	20.5132	20
23894.5427	18.0533	20.3086	20
23895.5629	17.7646	20.3309	20
23896.6632	17.7646	20.0948	20
23897.6638	17.7646	19.9606	20
23898.7029	17.7646	19.8722	20
23899.7638	17.7646	19.8159	20
23900.7639	17.7646	19.7825	20
23901.7839	17.7646	19.7609	20
23902.8439	17.7646	19.7468	20
23903.9243	18.0533	19.7379	20
23905.0242	18.0533	19.9829	20
23906.0243	18.0533	20.1219	20
23907.0643	18.0533	20.2137	20
23908.1643	18.0533	20.2741	20
23909.1647	17.8401	20.3085	20
23910.2452	17.8401	20.149	20
23911.3252	17.8401	20.0505	20
23912.4245	17.8401	19.9888	20
23913.4246	17.8401	19.9538	20
23914.5054	17.8401	19.93	20
23915.6055	17.8401	19.9152	20
23916.6249	17.8401	19.9066	20
23917.6648	17.8401	19.9011	20
23918.6653	17.8401	19.8978	20
23919.6662	17.8401	19.8956	20
23920.7465	17.8401	19.8942	20
23921.7863	17.8401	19.8933	20
23922.8263	17.8401	19.8928	20
23923.8266	17.8401	19.8925	20
23924.9454	17.8401	19.8922	20
23926.0458	18.0533	19.8921	20
23927.0458	18.0533	20.0638	20

23928.0854	18.0533	20.1771	20
23929.1466	17.7646	20.2496	20
23930.1468	18.254	20.0597	20
23931.2068	17.9653	20.3459	20
23932.2276	17.9653	20.2825	20
23933.3265	17.9653	20.2398	20
23934.3468	17.9653	20.215	20
23935.3468	17.9653	20.1996	20
23936.4468	17.9653	20.1889	20
23937.4668	17.9653	20.1827	20
23938.5268	17.9653	20.1787	20
23939.6078	17.9653	20.1761	20
23940.6279	17.9653	20.1746	20
23941.6478	17.9653	20.1737	20
23942.6678	17.9653	20.1731	20
23943.7677	17.9653	20.1727	20
23944.8678	17.9653	20.1724	20
23945.9078	17.9653	20.1723	20
23946.9678	17.9653	20.1722	20
23948.007	17.9653	20.1721	20
23949.0077	17.9653	20.1721	20
23950.0888	17.9653	20.172	20
23951.1288	17.9653	20.172	20
23952.1688	17.9653	20.172	20
23953.1888	17.9653	20.172	20
23954.289	17.9653	20.172	20
23955.3288	17.9653	20.172	20
23956.4285	17.9653	20.172	20
23957.5287	17.9653	20.172	20
23958.5288	17.9653	20.172	20
23959.5898	17.9653	20.172	20
23960.6697	17.9653	20.172	20

And I use these data to calculate θ, then we can get:

$$\theta = \begin{pmatrix} \lambda \\ b \end{pmatrix} = \begin{pmatrix} 0.6193 \\ 0.8392 \end{pmatrix} \approx \begin{pmatrix} 0.6394 \\ 0.8063 \end{pmatrix} (theory\ value)$$

Calculate the response of your estimated model and compare it to the real response. The result is shown in figure 3.1.

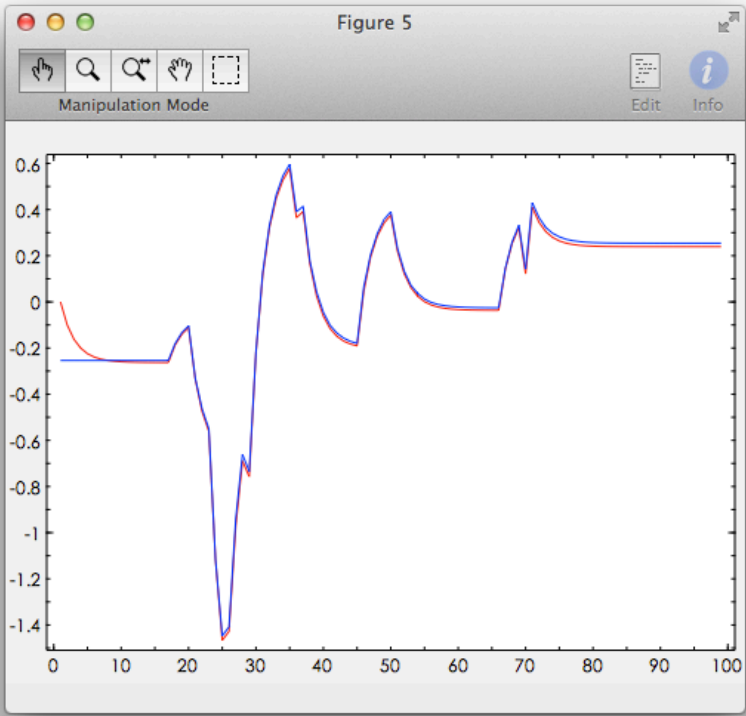

Figure 3.1 Comparison between the responses of estimated model and real model

We can see from the result, at first these two curves have a bigger error, and then they got almost the same, the error is very small.