Fourier Descriptors.

In this exercise you will practice using Fourier Descriptors (FDs) to describe the contour of an object (here: a character). The description of the contour should be translation, scale, rotation and parametrization (starting point) invariant but also insensitive to small distortions of the contour.

You can make FDs invariant to position, scale, and rotation by normalizing them as follows:

$$C(k) = \frac{F(k)}{F(1)} \quad \text{for k= -N/2, ..., -2, -1, 2, 3, ..., N/2}$$

where F(k) are the FDs of the complex contour f=x+i*y (x and y are the N coordinate points of the closed contour of the object). However, this assumes that the original sequence $f(m)$ starts at the same point after distorsions. Additional invariance to starting position, can be obtained as follows:

$$C(k) = \frac{|F(k)|}{|F(1)|} \quad \text{for k= -N/2, ..., -2, -1, 2, 3, ..., N/2}$$

You can make the C(k) insensitive to small distortions (=high frequencies) by using only the lower values of k (in this exercise: k=−4, −3, −2, −1, 2, 3, 4).
Note that the values k=0, 1 are left out. Why? (Hint: Ch. 17.5 in course book, Bigun 2006)

The C(k) can be used as a feature vector to describe the contour of an object. By calculating a "contour difference measure" between the feature vector C(k) and a reference vector R(k) a classification can be done.
To do this all the contours must be resampled to the same length of N points (in this exercise: N=128) before calculating the FDs.

A "contour difference measure" can be calculated as the Euclidean distance

$$ED = \sqrt{\sum_k |C(k) - R(k)|^2} = \text{norm}(C\text{-}R)$$

between the feature vector C(k) and the reference vector R (k) representing the object class j.
The object C(k) is classified according to minimum distance ED_j.

1. Computing the reference feature vectors $R_j(k)$
Load the binary image *char_e* and display it.
Compute the *reference feature vector Re* for character E as follows: (check the help for: perim_sort, resample_periodic, normFD)

%%Be careful! Cut-paste applied to big chunks of text of pdf may loose text.

load char_e; %load the character E
b_cell=bwboundaries(char_e); % extract the contour points. see help bwboundaries for output.
xe= b_cell{1}(:,2); %there is one letter, hence one cell, which is a matrix. Pick col 2 for x coords.
ye= -b_cell{1}(:,1);%... Pick col 1 for y coords and change sign to make it grow upwards.
xer=resample_periodic(xe,128); %and resample to 128 points %%%Question: Why resample?
yer=resample_periodic(ye,128);
figure(1); subplot(2,2,1); imshow(char_e); axis on; %display E
figure(1); subplot(2,2,2); plot(xe,ye); %and its contour images
figure(1); subplot(2,2,4); plot(xer,yer);

fe=xer+i*yer; %make a 1D complex signal out of the contour points
Fe=fft(fe,128); %and compute its FDs
% compute the reference feature vector
Re=normFD(Fe,[-4,-3,-2,-1, 2, 3, 4]); % use only the lower values of k
Re %print the reference vector on screen

Make a function FD_contour out of the previous code and save it as FD_contour.m such that you run the function by the command [Fv]=FD_contour(im,fig); where im is the input binary image and fig is the fig number to display the original and the contour images.
The output parameter Fv is the feature vector to "recognize" a letter or rather to differentiate it from another letter. First test the function FD_contour for character E.

load char_e; %load character E
img= char_e; %assign it to a new image matrix, img

[Re]=FD_contour(img,1); %compute ref vector and display contour images in Fig. 1
Rw %print the reference vector on screen

Then we use this function to calculate the *reference feature vector Rw for the character W*

load char_w; %load character W
[Rw]=FD_contour(char_w,2); %compute ref vector and contour images , and display
Rw %print the reference vector on screen

2. Matching with a scaled object
Change the size of the character E.
Compute its feature vector Se and its contour points sxe and sye.

img=imresize(img,1.6,'bicubic'); %Enlarge E 1.6 times
[Se]=FD_contour(img,3); %compute ref vector and contour points, and display
Se %print the feature vector on screen

Calculate the value of the "contour difference measure" between the scaled character E and the references for characters E and W represented by Se, Re, and Rw respectively.

dist_e=norm(Se-Re); %distance between scaled character E and reference E
dist_w=norm(Se-Rw); %distance between scaled character E and reference W
L=dist_w/dist_e %Likelihood of img being E compared to being W

Write down the values values of dist_e and dist_w and the feature vectors Se, Re, and Rw and likelihood L. What is your decision regarding the character described by the feature vector Se (is the character an "E" or a "W")?

3. Matching with a scaled and rotated object
Create a scaled and rotated version of the contour of the character E and compute its feature vector SRe and its contour points (x3r and y3r in the code below).

img2=imrotate(img,60,'bicubic'); %rotate img (scaled E) with 60 degrees
[SRe]=FD_contour(img2,4); %compute ref vector and contour points, and display
Se %print the feature vector on screen

Find the value of the "contour difference measure" between the rotated and scaled character E and the reference characters "E" and "W" (see above code: dist_e=.... and dist_w=....), and its likelihood of being "E" as commpared to "W" .

Write down the distances and the feature vectors SRe, Re and Rw.
What is your decision regarding the character described by vector SRe (is the character an "E" or a "W")?

4. Invariance with respect to translation

Finally, let us also translate the scaled and rotated contour of the character E. You can perform the translation with Δx (respectively a Δy) by copying img to a an appropriate position in a larger image.

zs=zeros(round(1.5*size(img2))); %create a larger image
zs(32:(size(img2,1)+31),15:(size(img2,2)+14))=img2; %Translate the E character with Δx=15 and Δy=32
img3=zs;
[SRTe]=FD_contour(img3,5); %compute ref vector and contour points, and display

Write down the distances and the feature vectors SRTe, Re, and Rw.
What is your decision regarding the character described by the feature vector SRTe (is it closer to an "E" or to a "W")?

Write down the reference vector for character E (Re) and the feature vectors for the scaled "E" (Se), the scaled and rotated "E" (SRe), and the scaled, rotated, translated "E" (SRTe).
Have you achieved a scale, rotation, and translation invariant description of the contour of E?

[Re Se SRe SRTe]