Digital Control: Exercise 1

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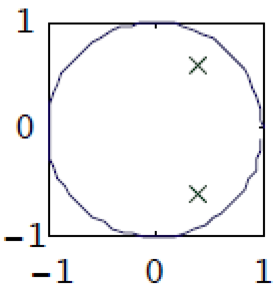
1. $\boldsymbol{y(k) = K\frac{(1-z_1q^{-1})(1-z_2q^{-1})\cdots(1-z_mq^{-1})}{(1-\lambda_1q^{-1})(1-\lambda_2q^{-1})\cdots(1-\lambda_mq^{-1})}u(k)}$, step response.

A 

From this zero-pole graph, we can see there are two poles in this unit circle, so we can regard the poles as $\lambda_1 = \frac{1}{2}+\frac{1}{2}j$ and $\lambda_2 = \frac{1}{2}-\frac{1}{2}j$, and then we can regard the constant K as $K = 1$.
So, we can get the equation as:

$$y(k) = \frac{1}{\left(1-(\frac{1}{2}+\frac{1}{2}j)q^{-1}\right)\left(1-(\frac{1}{2}-\frac{1}{2}j)q^{-1}\right)}u(k)$$

$$y(k) = \frac{1}{1-q^{-1}+\frac{1}{2}q^{-2}}u(k)$$

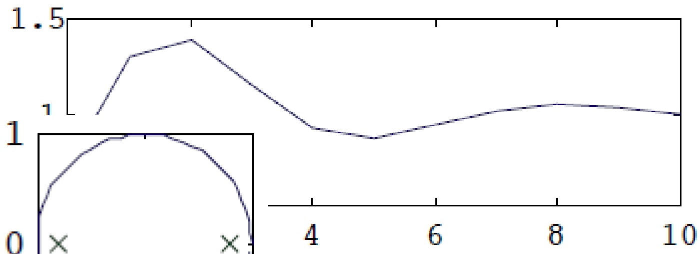
So, we can transfer it into recursive form:

$$y(k) = y(k-1) - \frac{1}{2}y(k-2) + u(k)$$

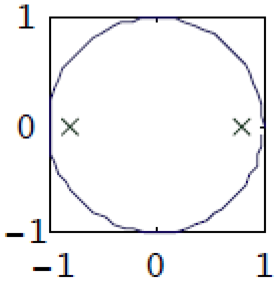
$y(0) = u(0) = 1$
$y(1) = y(0) + u(1) = 2$
$y(2) = y(1) - \frac{1}{2}y(0) + u(2) = 2.5$
$y(3) = y(2) - \frac{1}{2}y(1) + u(3) = 2.5$
$y(4) = y(3) - \frac{1}{2}y(2) + u(4) = 2.25$
$y(5) = y(4) - \frac{1}{2}y(3) + u(5) = 2$
…
We can see all the poles is inside the circle, so the system is stable.

$$y_\infty = G(1) = \frac{1}{1-1+\frac{1}{2}} = 2$$

2.



B



This unit step response is similar with number 2 picture.

From this zero-pole graph, we can see there are two poles in this unit circle, so we can regard the poles as $\lambda_1 = \frac{3}{4}$ and $\lambda_2 = -\frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 - \frac{3}{4}q^{-1}\right)\left(1 + \frac{3}{4}q^{-1}\right)} u(k)$$

$$y(k) = \frac{1}{1 - \frac{9}{16}q^{-2}} u(k)$$

So, we can transfer it into recursive form:

$$y(k) = \frac{9}{16} y(k-2) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = u(1) = 1$

$y(2) = \frac{9}{16} y(0) + u(2) = 1.5625$

$y(3) = \frac{9}{16} y(1) + u(3) = 1.5625$

$y(4) = \frac{9}{16} y(2) + u(4) = 1.8789$

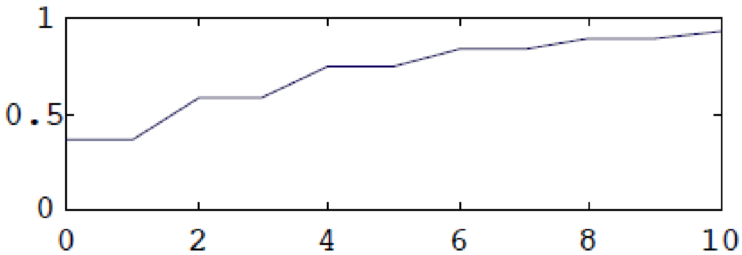
$y(5) = \frac{9}{16} y(3) + u(5) = 1.8789$

…

We can see all the poles is inside the circle, so the system is stable.

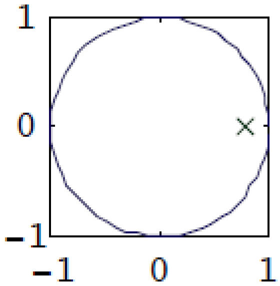
$y_\infty = G(1) = \frac{1}{1 - \frac{9}{16}} = \frac{16}{7}$

4.



This unit step response is similar with number 4 picture.

C



From this zero-pole graph, we can see there is one pole in this unit circle, so we can regard the pole as $\lambda = \frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 - \frac{3}{4}q^{-1}\right)} u(k)$$

So, we can transfer it into recursive form:

$$y(k) = \frac{3}{4}y(k-1) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = \frac{3}{4}y(0) + u(1) = 1.75$

$y(2) = \frac{3}{4}y(1) + u(2) = 2.3125$

$y(3) = \frac{3}{4}y(2) + u(3) = 2.7344$

$y(4) = \frac{3}{4}y(3) + u(4) = 3.0508$

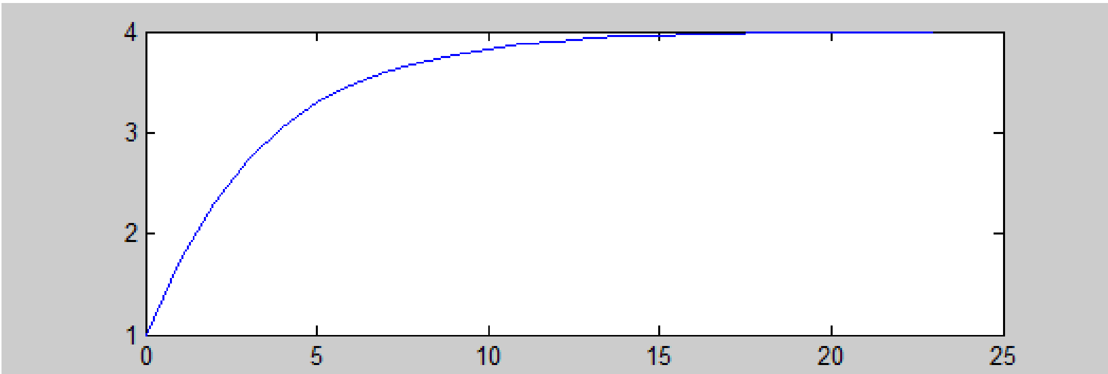
$y(5) = \frac{3}{4}y(4) + u(5) = 3.2881$

…

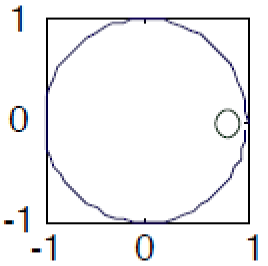
We can see the pole is inside the circle, so the system is stable.

$y_\infty = G(1) = \frac{1}{1 - \frac{3}{4}} = 4$

So, the graph for this is not match. The correct graph for this unit step response should be like:



D



From this zero-pole graph, we can see there is one zero in this unit circle, so we can regard the zero as $z = \frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = (1 - \frac{3}{4}q^{-1})u(k)$$

So, we can transfer it into recursive form:

$$y(k) = u(k) - \frac{3}{4}u(k-1)$$

$y(0) = u(0) - \frac{3}{4}u(-1) = 1 - 0 = 1$

$y(1) = u(1) - \frac{3}{4}u(0) = 0.25$

$y(2) = u(2) - \frac{3}{4}u(1) = 0.25$

$y(3) = u(3) - \frac{3}{4}u(2) = 0.25$

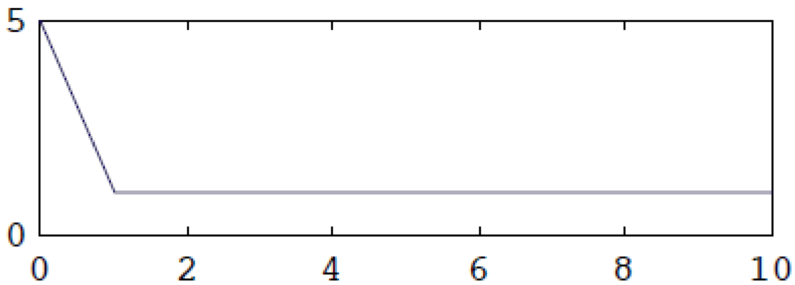
$y(4) = u(4) - \frac{3}{4}u(3) = 0.25$

$y(5) = u(5) - \frac{3}{4}u(4) = 0.25$

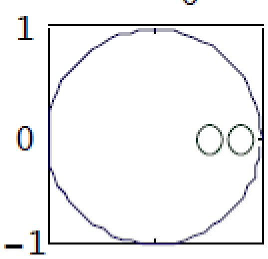
…

So, $y(k) = \begin{cases} 1 & k = 0 \\ 0.25 & k > 0 \end{cases}$. The unit step response of this system is like number 3.

3.



E



From this zero-pole graph, we can see there are two zeros in this unit circle, so we can regard the poles as $z_1 = \frac{1}{2}$, $z_2 = \frac{3}{4}$, then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = (1 - \frac{3}{4}q^{-1})(1 - \frac{1}{2}q^{-1})u(k)$$

$$y(k) = (1 + \frac{3}{8}q^{-2} - \frac{5}{4}q^{-1})u(k)$$

So, we can transfer it into recursive form:

$$y(k) = u(k) + \frac{3}{8}u(k-2) - \frac{5}{4}u(k-1)$$

$y(0) = u(0) = 1$

$y(1) = u(1) - \frac{5}{4}u(0) = -0.25$

$y(2) = u(2) + \frac{3}{8}u(0) - \frac{5}{4}u(1) = 0.125$

$y(3) = u(3) + \frac{3}{8}u(1) - \frac{5}{4}u(2) = 0.125$

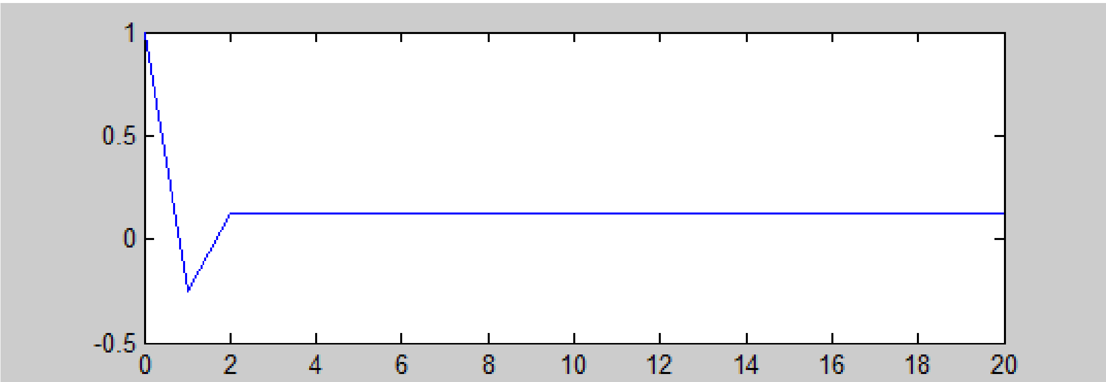
$y(4) = u(4) + \frac{3}{8}u(2) - \frac{5}{4}u(3) = 0.125$

$y(5) = u(5) + \frac{3}{8}u(3) - \frac{5}{4}u(4) = 0.125$

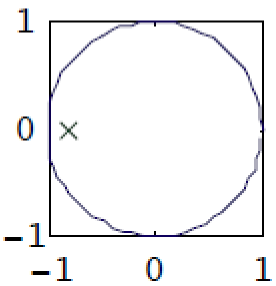
…

So, $y(k) = \begin{cases} 1 & k = 0 \\ -0.25 & k = 1 \\ 0.125 & k > 1 \end{cases}$. The unit step response of this system is like number 3.

So, the graph for this is not match. The correct graph for this unit step response should be like:



F



From this zero-pole graph, we can see there is one pole in this unit circle, so we can regard the pole as $\lambda = -\frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 + \frac{3}{4}q^{-1}\right)}u(k)$$

So, we can transfer it into recursive form:

$$y(k) = -\frac{3}{4}y(k-1) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = -\frac{3}{4}y(0) + u(1) = 0.25$

$y(2) = -\frac{3}{4}y(1) + u(2) = 0.8125$

$y(3) = -\frac{3}{4}y(2) + u(3) = 0.3906$

$y(4) = -\frac{3}{4}y(3) + u(4) = 0.7070$

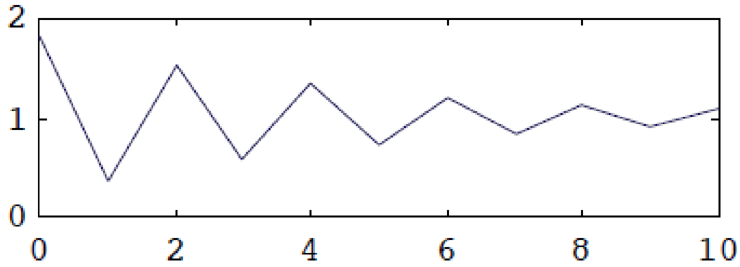
$y(5) = -\frac{3}{4}y(4) + u(5) = 0.4697$

…

We can see the pole is inside the circle, so the system is stable.

$y_\infty = G(1) = \frac{1}{1+\frac{3}{4}} = \frac{4}{7}$

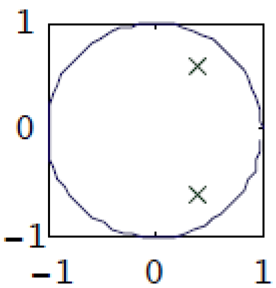
1.



The unit step response of this system is like number 1.

2. $\boldsymbol{y(k) = K\frac{(1-z_1q^{-1})(1-z_2q^{-1})\cdots(1-z_mq^{-1})}{(1-\lambda_1q^{-1})(1-\lambda_2q^{-1})\cdots(1-\lambda_mq^{-1})}u(k)}$, pulse response.

A



From this zero-pole graph, we can see there are two poles in this unit circle, so we can regard the poles as $\lambda_1 = \frac{1}{2} + \frac{1}{2}j$ and $\lambda_2 = \frac{1}{2} - \frac{1}{2}j$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1-(\frac{1}{2}+\frac{1}{2}j)q^{-1}\right)\left(1-(\frac{1}{2}-\frac{1}{2}j)q^{-1}\right)}u(k)$$

$$y(k) = \frac{1}{1-q^{-1}+\frac{1}{2}q^{-2}}u(k)$$

So, we can transfer it into recursive form:

$$y(k) = y(k-1) - \frac{1}{2}y(k-2) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = y(0) + u(1) = 1$

$y(2) = y(1) - \frac{1}{2}y(0) + u(2) = 0.5$

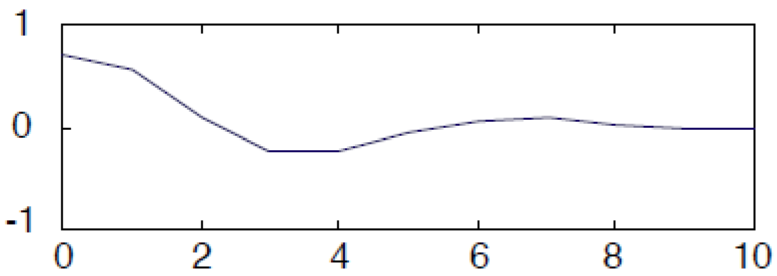
$$y(3) = y(2) - \frac{1}{2}y(1) + u(3) = 0$$

$$y(4) = y(3) - \frac{1}{2}y(2) + u(4) = -0.25$$

$$y(5) = y(4) - \frac{1}{2}y(3) + u(5) = -0.25$$

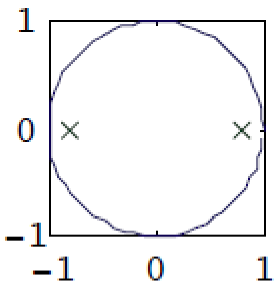
…

2



The impulse response of this system is like number 2.

B



From this zero-pole graph, we can see there are two poles in this unit circle, so we can regard the poles as $\lambda_1 = \frac{3}{4}$ and $\lambda_2 = -\frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 - \frac{3}{4}q^{-1}\right)\left(1 + \frac{3}{4}q^{-1}\right)}u(k)$$

$$y(k) = \frac{1}{1 - \frac{9}{16}q^{-2}}u(k)$$

So, we can transfer it into recursive form:

$$y(k) = \frac{9}{16}y(k-2) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = u(1) = 0$

$$y(2) = \frac{9}{16}y(0) + u(2) = 0.5625$$

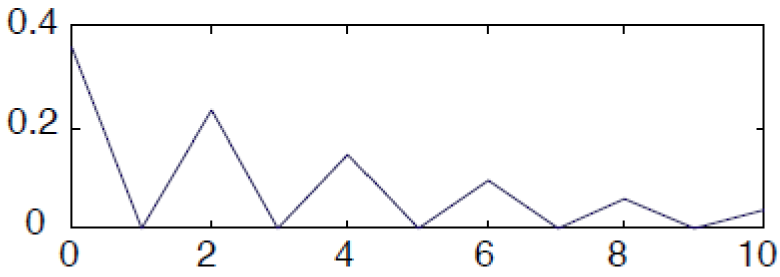
$$y(3) = \frac{9}{16}y(1) + u(3) = 0$$

$$y(4) = \frac{9}{16}y(2) + u(4) = 0.3164$$

$$y(5) = \frac{9}{16}y(3) + u(5) = 0$$

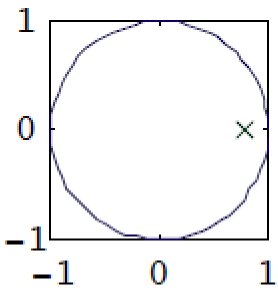
…

1



The impulse response of this system is like number 1.

C



From this zero-pole graph, we can see there is one pole in this unit circle, so we can regard the pole as $\lambda = \frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 - \frac{3}{4}q^{-1}\right)}u(k)$$

So, we can transfer it into recursive form:

$$y(k) = \frac{3}{4}y(k-1) + u(k)$$

$y(0) = u(0) = 1$

$y(1) = \frac{3}{4}y(0) + u(1) = 0.75$

$y(2) = \frac{3}{4}y(1) + u(2) = 0.5625$

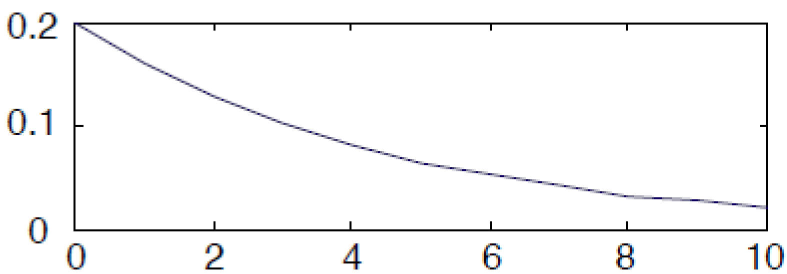
$y(3) = \frac{3}{4}y(2) + u(3) = 0.4219$

$y(4) = \frac{3}{4}y(3) + u(4) = 0.3164$

$y(5) = \frac{3}{4}y(4) + u(5) = 0.2373$

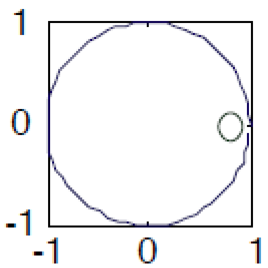
…

3



The impulse response of this system is like number 3.

D



From this zero-pole graph, we can see there is one zero in this unit circle, so we can regard the zero as $z = \frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = (1 - \frac{3}{4}q^{-1})u(k)$$

So, we can transfer it into recursive form:

$$y(k) = u(k) - \frac{3}{4}u(k-1)$$

$y(0) = u(0) - \frac{3}{4}u(-1) = 1 - 0 = 1$

$y(1) = u(1) - \frac{3}{4}u(0) = -0.75$

$y(2) = u(2) - \frac{3}{4}u(1) = 0$

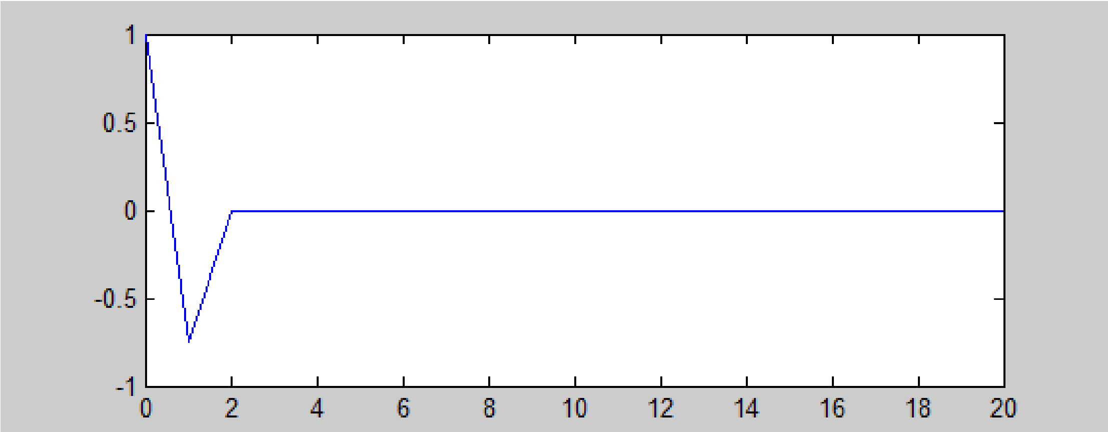
$y(3) = u(3) - \frac{3}{4}u(2) = 0$

$y(4) = u(4) - \frac{3}{4}u(3) = 0$

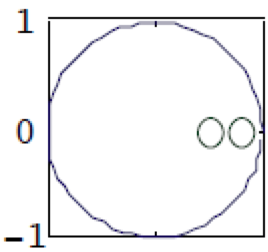
$y(5) = u(5) - \frac{3}{4}u(4) = 0$

…

So, the graph for this is not match. The correct graph for this impulse response should be like:



E



From this zero-pole graph, we can see there are two zeros in this unit circle, so we can regard the poles as $z_1 = \frac{1}{2}$, $z_2 = \frac{3}{4}$, then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = (1 - \frac{3}{4}q^{-1})(1 - \frac{1}{2}q^{-1})u(k)$$

$$y(k) = (1 + \frac{3}{8}q^{-2} - \frac{5}{4}q^{-1})u(k)$$

So, we can transfer it into recursive form:

$$y(k) = u(k) + \frac{3}{8}u(k-2) - \frac{5}{4}u(k-1)$$

$y(0) = u(0) = 1$

$$y(1) = u(1) - \frac{5}{4}u(0) = -1.25$$

$$y(2) = u(2) + \frac{3}{8}u(0) - \frac{5}{4}u(1) = 0.375$$

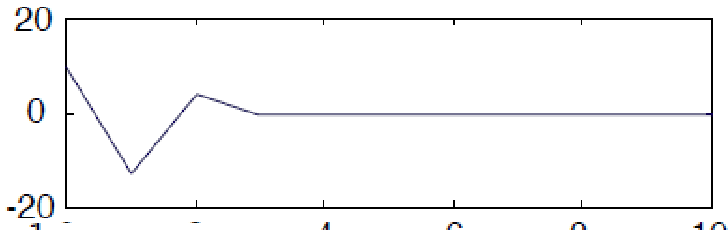
$$y(3) = u(3) + \frac{3}{8}u(1) - \frac{5}{4}u(2) = 0$$

$$y(4) = u(4) + \frac{3}{8}u(2) - \frac{5}{4}u(3) = 0$$

$$y(5) = u(5) + \frac{3}{8}u(3) - \frac{5}{4}u(4) = 0$$

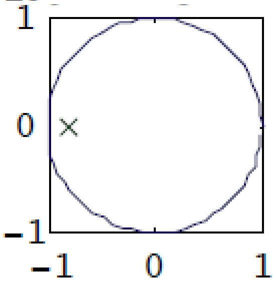
…

4



The impulse response of this system is like number 4.

F



From this zero-pole graph, we can see there is one pole in this unit circle, so we can regard the pole as $\lambda = -\frac{3}{4}$, and then we can regard the constant K as $K = 1$.

So, we can get the equation as:

$$y(k) = \frac{1}{\left(1 + \frac{3}{4}q^{-1}\right)}u(k)$$

So, we can transfer it into recursive form:

$$y(k) = -\frac{3}{4}y(k-1) + u(k)$$

$$y(0) = u(0) = 1$$

$$y(1) = -\frac{3}{4}y(0) + u(1) = -0.75$$

$$y(2) = -\frac{3}{4}y(1) + u(2) = 0.5625$$

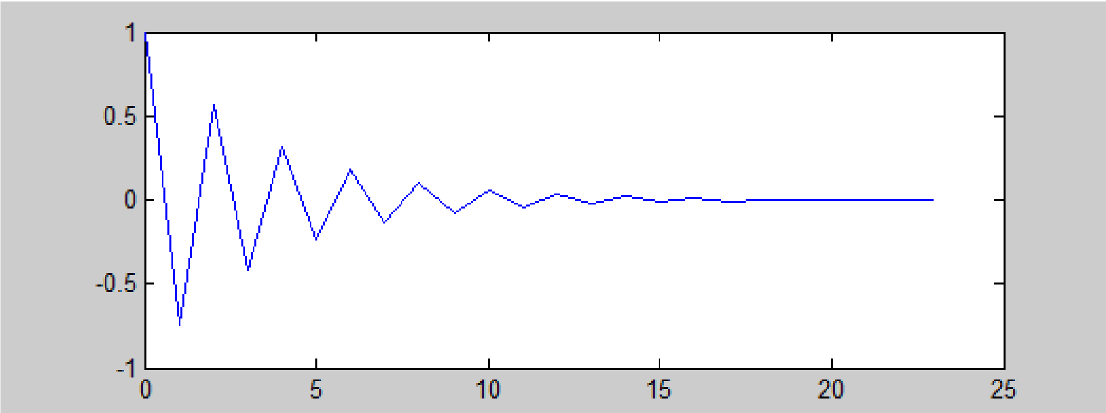
$$y(3) = -\frac{3}{4}y(2) + u(3) = -0.4219$$

$$y(4) = -\frac{3}{4}y(3) + u(4) = 0.3164$$

$$y(5) = -\frac{3}{4}y(4) + u(5) = -0.2373$$

…

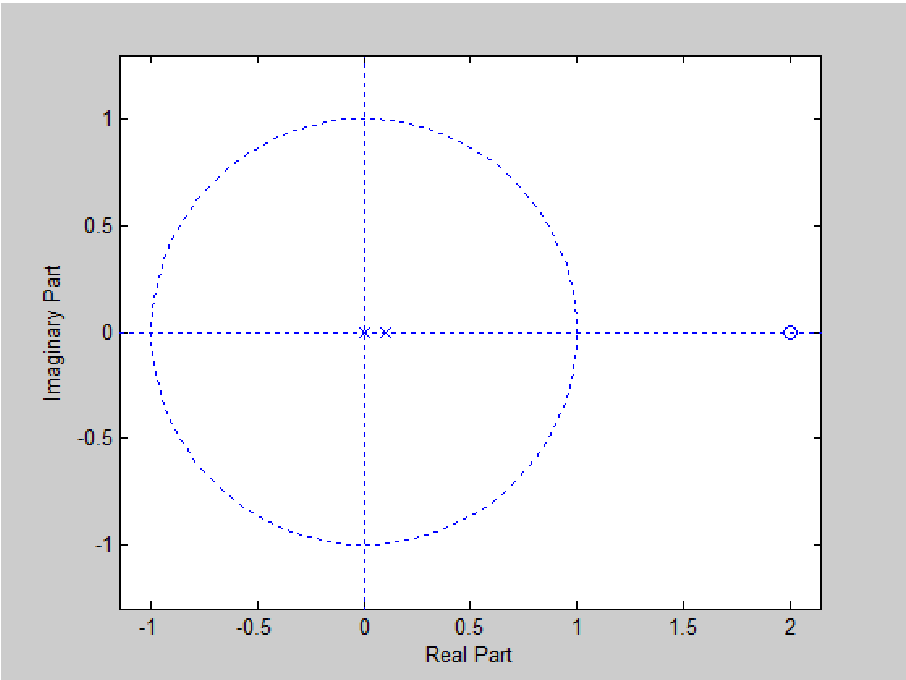
So, the graph for this is not match. The correct graph for this impulse response should be like:



3. What are the steady-state (stationary) gain of the following systems?

a) $y(k) = \frac{0.1q^{-1} - 0.2q^{-2}}{1 - 0.1q^{-1}} u(k)$

We can make the zero-pole graph like this:



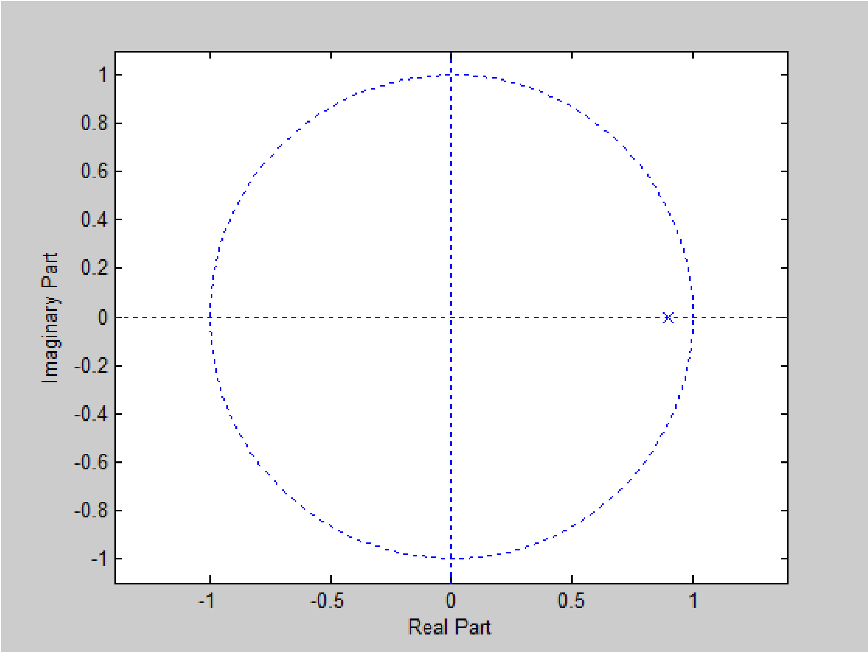
We can see clearly from the graph, that there are two poles, $\lambda_1 = 0$ and $\lambda_2 = 1/9$, and the only zero is $z = 2$, we can see all the poles are in the unit circle, so the system is stable, $y_\infty = G(1) = \frac{0.1 - 0.2}{1 - 0.1} = -\frac{1}{9}$

b) $y(k) = 0.9y(k-1) + 0.1u(k-1)$

We can transfer to differential equation as:

$$y(k) = \frac{0.1q^{-1}}{1 - 0.9q^{-1}} u(k)$$

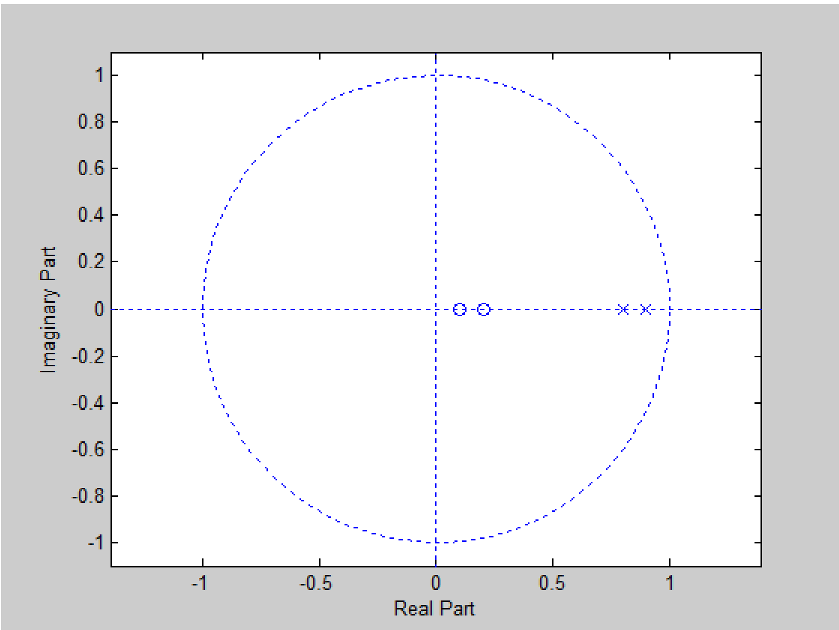
We can make the zero-pole graph like this:



We can see clearly from the graph, that there is one poles, $\lambda = 0.9$, we can see the pole is in the unit circle, so the system is stable, $y_\infty = G(1) = \frac{0.1}{1-0.9} = 1$

c) $y(k) = \frac{-10(1-0.1q^{-1})(1-0.2q^{-1})}{(1-0.9q^{-1})(1-0.8q^{-1})}u(k)$

We can make the zero-pole graph like this:



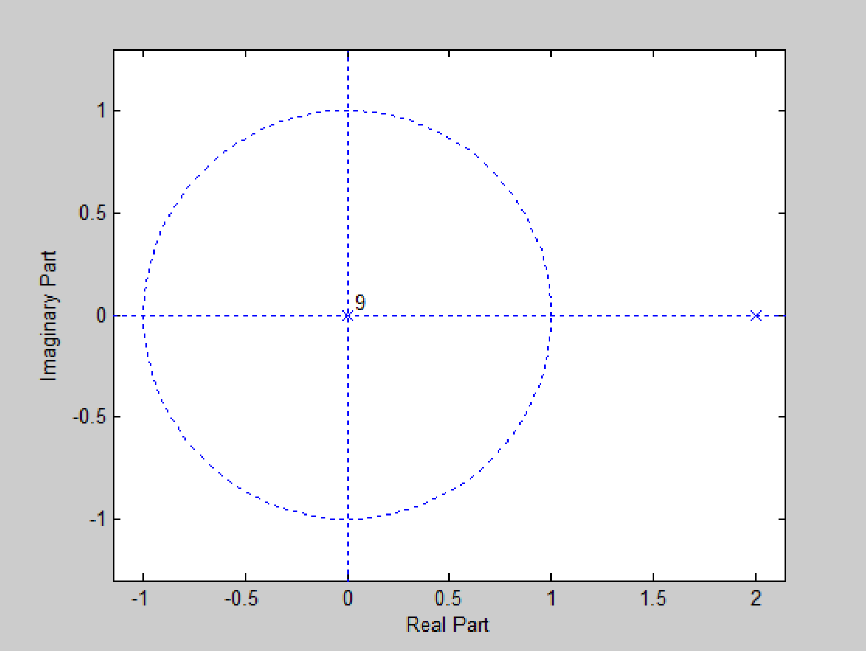
We can see clearly from the graph, that there are two poles, $\lambda_1 = 0.8$ and $\lambda_2 = 0.9$, and the only zero is $z_1 = 0.2$ and $z_2 = 0.1$, we can see all the poles are in the unit circle, so the system is stable, $y_\infty = G(1) = \frac{-10+3-0.2}{1-1.7+0.72} = -\frac{7.2}{0.02}$=-360

d) $y(k) - 2y(k-1) = 2u(k-10)$

We can transfer to differential equation as:

$$y(k) = \frac{2q^{-10}}{1-2q^{-1}}u(k)$$

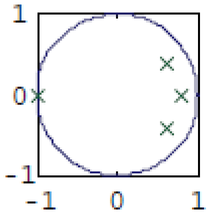
We can make the zero-pole graph like this:



We can see clearly from the graph, that there is one poles, $\lambda = 2$, we can see the pole is not in the unit circle, so the system is not stable, we can't get any steady-state gain from this system.

4. $G(q^{-1}) = K\frac{(1-z_1q^{-1})\cdots(1-z_mq^{-1})}{(1-\lambda_1q^{-1})\cdots(1-\lambda_nq^{-1})}q^{-2}$, step response.

a



From the zero-pole graph, we can see all the four poles are in this unit circle, so the system is stable, and we can assume these four poles as $\lambda_1 = -1,\ \lambda_2 = \frac{2}{3}+\frac{1}{3}i,\ \lambda_3 = \frac{2}{3}-\frac{1}{3}i$ and $\lambda_4 = \frac{3}{4}$, and then we regard K as $K = 1$.

So, we can get the equation as:

$$G(q^{-1}) = \frac{1}{(1+q^{-1})(1-(\frac{2}{3}+\frac{1}{3}i)q^{-1})(1-(\frac{2}{3}-\frac{1}{3}i)q^{-1})(1-\frac{3}{4}q^{-1})}q^{-2}$$

$$=> G(q^{-1}) = \frac{1}{1-\frac{13}{12}q^{-1}-\frac{19}{36}q^{-2}+\frac{41}{36}q^{-3}-\frac{5}{12}q^{-4}}$$

Then, we can transfer it to differential equation:

$$y(k)(1-\frac{13}{12}q^{-1}-\frac{19}{36}q^{-2}+\frac{41}{36}q^{-3}-\frac{5}{12}q^{-4}) = u(k)q^{-2}$$

$$y(k)-\frac{13}{12}y(k-1)-\frac{19}{36}y(k-2)+\frac{41}{36}y(k-3)-\frac{5}{12}y(k-4) = u(k-2)$$

$y(0) = u(-2) = 0$

$y(1) = \frac{13}{12}y(0) + u(-1) = 0$

$y(2) = \frac{13}{12}y(1) + \frac{19}{36}y(0) + u(0) = 1$

$y(3) = \frac{13}{12}y(2) + \frac{19}{36}y(1) - \frac{41}{36}y(0) + u(1) = 2.0833$

$y(4) = \frac{13}{12}y(3) + \frac{19}{36}y(2) - \frac{41}{36}y(1) + \frac{5}{12}y(0) + u(2) = 3.7847$

$y(5) = \frac{13}{12}y(4) + \frac{19}{36}y(3) - \frac{41}{36}y(2) + \frac{5}{12}y(1) + u(3) = 5.0608$

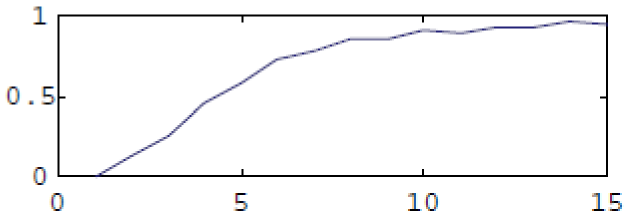
…

We can see the pole is inside the circle, so the system is stable.

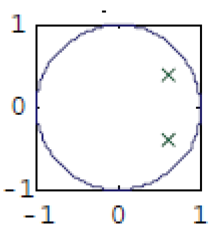
$$y_\infty = G(1) = \frac{1}{1 - \frac{13}{12} - \frac{19}{36} + \frac{41}{36} - \frac{5}{12}} = 9$$

The step response of this system is like number 5.

5



b



From the zero-pole graph, we can see all the two poles are in this unit circle, so the system is stable, and we can assume these two poles as $\lambda_1 = \frac{2}{3} - \frac{1}{3}i$, $\lambda_2 = \frac{2}{3} + \frac{1}{3}i$, and then we regard K as $K = 1$.

So, we can get the equation as:

$$G(q^{-1}) = \frac{1}{(1 - (\frac{2}{3} + \frac{1}{3}i)q^{-1})(1 - (\frac{2}{3} - \frac{1}{3}i)q^{-1})}q^{-2}$$

$$=> G(q^{-1}) = \frac{1}{1 - \frac{4}{3}q^{-1} + \frac{5}{9}q^{-2}}q^{-2}$$

Then, we can transfer it to differential equation:

$$y(k)(1 - \frac{4}{3}q^{-1} + \frac{5}{9}q^{-2}) = u(k)q^{-2}$$

$$y(k) - \frac{4}{3}y(k-1) + \frac{5}{9}y(k-2) = u(k-2)$$

$y(0) = u(-2) = 0$

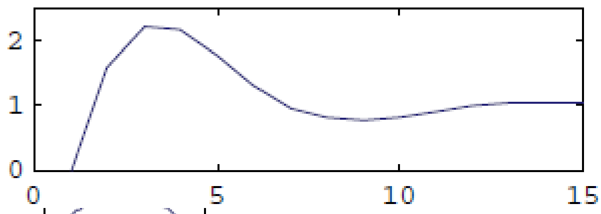
$y(1) = \frac{4}{3}y(0) + u(-1) = 0$

$$y(2)=\frac{4}{3}y(1)-\frac{5}{9}y(0)+u(0)=1$$
$$y(3)=\frac{4}{3}y(2)-\frac{5}{9}y(1)+u(1)=2.3333$$
$$y(4)=\frac{4}{3}y(3)-\frac{5}{9}y(2)+u(2)=3.5556$$
$$y(5)=\frac{4}{3}y(4)-\frac{5}{9}y(3)+u(3)=4.4444$$
…

We can see the pole is inside the circle, so the system is stable.

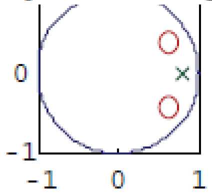
$$y_\infty=G(1)=\frac{1}{1-\frac{4}{3}+\frac{5}{9}}=4.5$$

1



The step response of this system is like number 1.

c



From the zero-pole graph, we can see the pole is in this unit circle, so the system is stable, and we can assume this pole as $\lambda=\frac{3}{4}$, two zeros as $z_1=\frac{2}{3}+\frac{1}{3}i$, $z_2=\frac{2}{3}-\frac{1}{3}i$, and then we regard K as $K=1$.

So, we can get the equation as:

$$G(q^{-1})=\frac{(1-(\frac{2}{3}+\frac{1}{3}i)q^{-1})(1-(\frac{2}{3}-\frac{1}{3}i)q^{-1})}{1-\frac{3}{4}q^{-1}}q^{-2}$$

$$=>G(q^{-1})=\frac{1-\frac{4}{3}q^{-1}+\frac{5}{9}q^{-2}}{1-\frac{3}{4}q^{-1}}q^{-2}$$

Then, we can transfer it to differential equation:

$$y(k)-\frac{3}{4}y(k-1)=u(k-2)-\frac{4}{3}u(k-3)+\frac{5}{9}u(k-4)$$

$$y(0)=u(-2)-\frac{4}{3}u(-3)+\frac{5}{9}u(-4)=0$$
$$y(1)=\frac{4}{3}y(0)+u(-1)-\frac{4}{3}u(-2)+\frac{5}{9}u(-3)=0$$
$$y(2)=\frac{4}{3}y(1)+u(0)-\frac{4}{3}u(-1)+\frac{5}{9}u(-2)=1$$
$$y(3)=\frac{4}{3}y(2)+u(1)-\frac{4}{3}u(0)+\frac{5}{9}u(-1)=0.4167$$

$y(4) = \frac{4}{3}y(3) + u(2) - \frac{4}{3}u(1) + \frac{5}{9}u(0) = 0.5347$

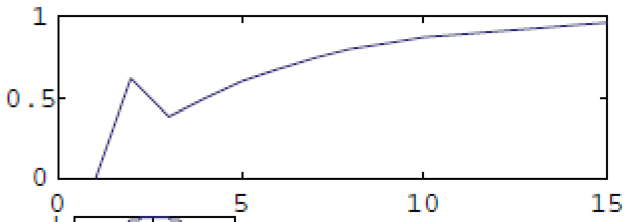
$y(5) = \frac{4}{3}y(4) + u(3) - \frac{4}{3}u(2) + \frac{5}{9}u(1) = 0.6233$

…

We can see the pole is inside the circle, so the system is stable.

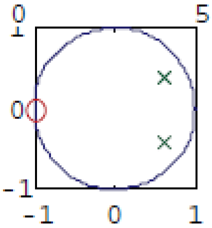
$$y_\infty = G(1) = \frac{1 - \frac{4}{3} + \frac{5}{9}}{1 - \frac{3}{4}} = \frac{8}{9}$$

2



The step response of this system is like number 2.

d



From the zero-pole graph, we can see all the poles are in this unit circle, so the system is stable, and we can assume these two poles as $\lambda_1 = \frac{2}{3} + \frac{1}{3}i$, $\lambda_2 = \frac{2}{3} - \frac{1}{3}i$, the zeros as $z = -1$, and then we regard K as $K = 1$.

So, we can get the equation as:

$$G(q^{-1}) = \frac{1 + q^{-1}}{(1 - (\frac{2}{3} + \frac{1}{3}i)q^{-1})(1 - (\frac{2}{3} - \frac{1}{3}i)q^{-1})}q^{-2}$$

$$=> G(q^{-1}) = \frac{1 + q^{-1}}{1 - \frac{4}{3}q^{-1} + \frac{5}{9}q^{-2}}q^{-2}$$

Then, we can transfer it to differential equation:

$$y(k) - \frac{4}{3}y(k-1) + \frac{5}{9}y(k-2) = u(k-2) + u(k-3)$$

$y(0) = u(-2) + u(-3) = 0$

$y(1) = \frac{4}{3}y(0) + u(-1) + u(-2) = 0$

$y(2) = \frac{4}{3}y(1) - \frac{5}{9}y(0) + u(0) + u(-1) = 1$

$y(3) = \frac{4}{3}y(2) - \frac{5}{9}y(1) + u(1) + u(0) = 3.3333$

$y(4) = \frac{4}{3}y(3) - \frac{5}{9}y(2) + u(2) + u(1) = 5.8889$

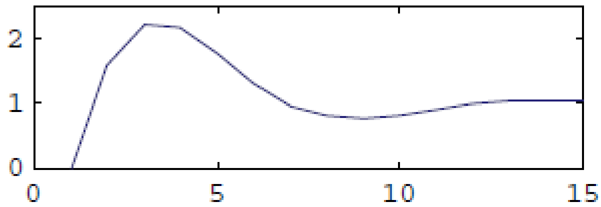
$y(5) = \frac{4}{3}y(4) - \frac{5}{9}y(3) + u(3) + u(2) = 8$

…

We can see the pole is inside the circle, so the system is stable.

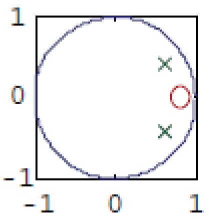
$$y_\infty = G(1) = \frac{1+1}{1-\frac{4}{3}+\frac{5}{9}} = 9$$

1



The step response of this system is like number 1.

e



From the zero-pole graph, we can see all the poles are in this unit circle, so the system is stable, and we can assume these two poles as $\lambda_1 = \frac{2}{3}+\frac{1}{3}i,\ \lambda_2 = \frac{2}{3}-\frac{1}{3}i$, the zeros as $z = -\frac{3}{4}$, and then we regard K as $K = 1$.
So, we can get the equation as:

$$G(q^{-1}) = \frac{1-\frac{3}{4}q^{-1}}{(1-(\frac{2}{3}+\frac{1}{3}i)q^{-1})(1-(\frac{2}{3}-\frac{1}{3}i)q^{-1})}q^{-2}$$

$$=> G(q^{-1}) = \frac{1-\frac{3}{4}q^{-1}}{1-\frac{4}{3}q^{-1}+\frac{5}{9}q^{-2}}q^{-2}$$

Then, we can transfer it to differential equation:

$$y(k) - \frac{4}{3}y(k-1) + \frac{5}{9}y(k-2) = u(k-2) - \frac{3}{4}u(k-3)$$

$$y(0) = u(-2) - \frac{3}{4}u(-3) = 0$$

$$y(1) = \frac{4}{3}y(0) + u(-1) - \frac{3}{4}u(-2) = 0$$

$$y(2) = \frac{4}{3}y(1) - \frac{5}{9}y(0) + u(0) - \frac{3}{4}u(-1) = 1$$

$$y(3) = \frac{4}{3}y(2) - \frac{5}{9}y(1) + u(1) - \frac{3}{4}u(0) = 1.5833$$

$$y(4) = \frac{4}{3}y(3) - \frac{5}{9}y(2) + u(2) - \frac{3}{4}u(1) = 1.8056$$

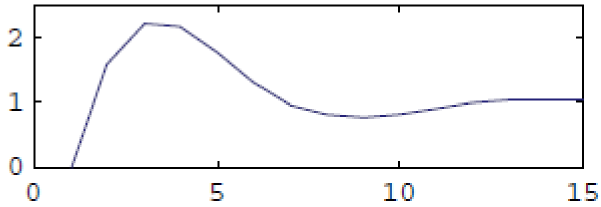
$$y(5) = \frac{4}{3}y(4) - \frac{5}{9}y(3) + u(3) - \frac{3}{4}u(2) = 1.7778$$

…

We can see the pole is inside the circle, so the system is stable.

$$y_\infty = G(1) = \frac{1-\frac{3}{4}}{1-\frac{4}{3}+\frac{5}{9}} = \frac{9}{8}$$

1



The step response of this system is like number 1.

5. Group Problems.

For fulfill the requirements, we can build the system function like this:

$$G(q^{-1}) = K\frac{1+q^{-1}}{(1-\lambda_1 q^{-1})(1-\lambda_2 q^{-1})}q^{-1}$$

We choose the group 3 which $\lambda_1 = 0.8+0.3i$, $\lambda_2 = 0.8-0.3i$, so we can the equation like this:

$$G(q^{-1}) = K\frac{1+q^{-1}}{(1-(0.8+0.3i)q^{-1})(1-(0.8-0.3i)q^{-1})}q^{-1}$$

$$=> G(q^{-1}) = K\frac{1+q^{-1}}{1-1.6q^{-1}+0.73q^{-2}}q^{-1}$$

We can see $|\lambda_1| = |\lambda_2| = 0.7616 < 1$, so the system is stable, from the requirement, we know the steady-state gain is 2.

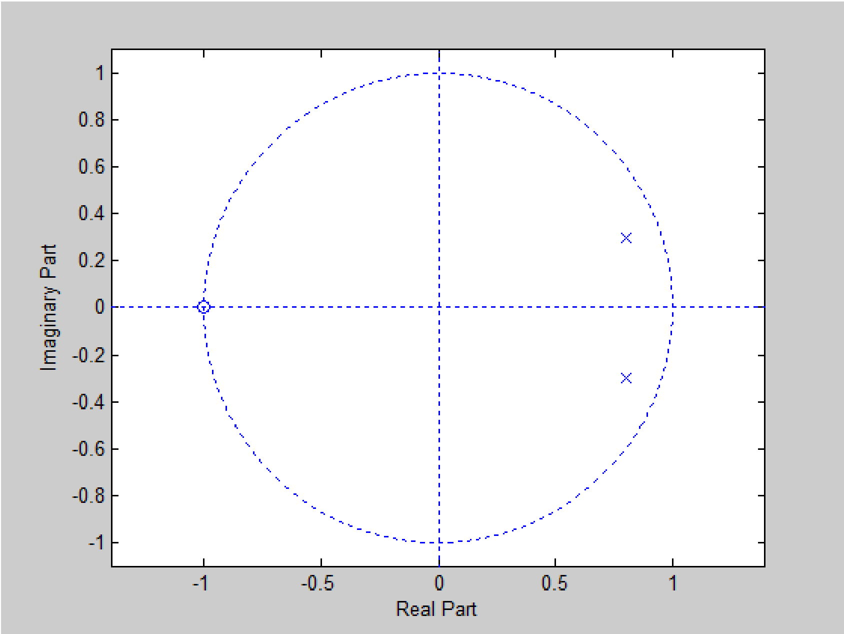
$$G(1) = K\frac{1+1}{1-1.6+0.73} = 2$$

$$=> K = 0.13$$

So, the system is:

$$G(q^{-1}) = \frac{0.13q^{-1}+0.13q^{-2}}{1-1.6q^{-1}+0.73q^{-2}}$$

We can draw the zero-pole graph as:



We can transfer the system to the differential equation:

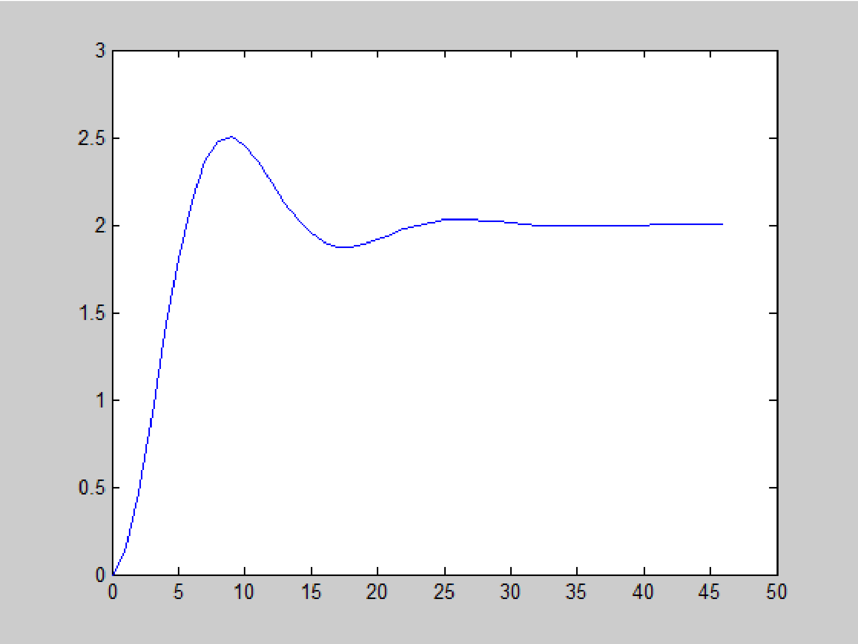
$$y(k)(1 - 1.6q^{-1} + 0.73q^{-2}) = u(k)(0.13q^{-1} + 0.13q^{-2})$$

$$=> y(k) - 1.6y(k-1) + 0.73y(k-2) = 0.13u(k-1) + 0.13u(k-2)$$

Here, we use step response, $u(k)\begin{cases}1 & k \geq 0 \\ 0 & k < 0\end{cases}$

$y(0) = 0.13u(-1) + 0.13u(-2) = 0$
$y(1) = 0.13u(0) + 0.13u(-1) + 1.6y(0) = 0.13$
$y(2) = 0.13u(1) + 0.13u(0) + 1.6y(1) - 0.73y(0) = 0.468$
$y(3) = 0.13u(2) + 0.13u(1) + 1.6y(2) - 0.73y(1) = 0.9139$
$y(4) = 0.13u(3) + 0.13u(2) + 1.6y(3) - 0.73y(2) = 1.3806$
$y(5) = 0.13u(4) + 0.13u(3) + 1.6y(4) - 0.73y(3) = 1.8018$
$y(6) = 0.13u(5) + 0.13u(4) + 1.6y(5) - 0.73y(4) = 2.1351$
$y(7) = 0.13u(6) + 0.13u(5) + 1.6y(6) - 0.73y(5) = 2.3608$
$y(8) = 0.13u(7) + 0.13u(6) + 1.6y(7) - 0.73y(6) = 2.4786$
$y(9) = 0.13u(8) + 0.13u(7) + 1.6y(8) - 0.73y(7) = 2.5025$
$y(10) = 0.13u(9) + 0.13u(8) + 1.6y(9) - 0.73y(8) = 2.4545$
…

Here's the step response:



3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
30
35
40
45
50