Digital Control – Exercise 2

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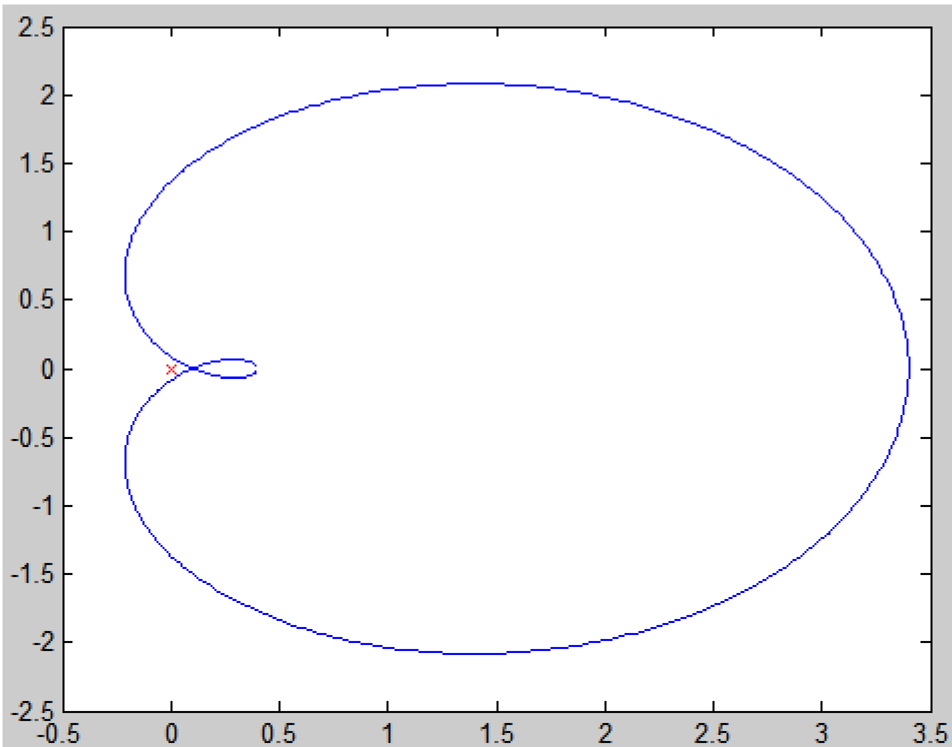
1. Determine stability/instability

a. $\mathrm{A}(q^{-1}) = 1 - 1.5q^{-1} + 0.9q^{-2}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:

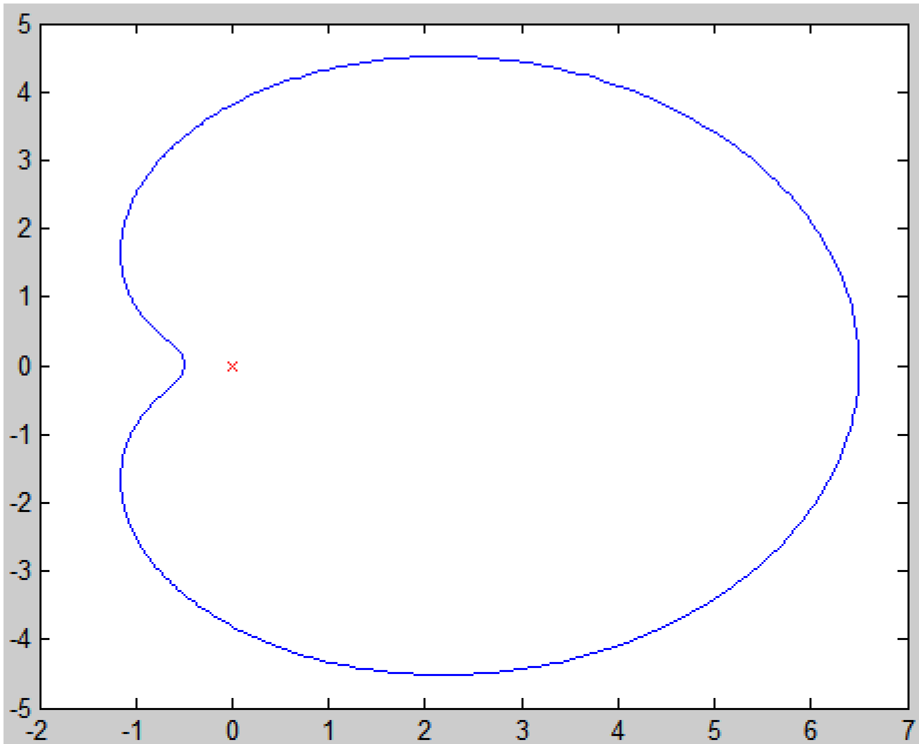


So the curve does not encircle the origin, i.e. there is no unstable pole. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $0.7500 \pm 0.5809\mathrm{i}$, which are all inside the unit circle.

This system is stable.

b. $A(q^{-1}) = 1 - 3q^{-1} + 2q^{-2} - 0.5q^{-3}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:

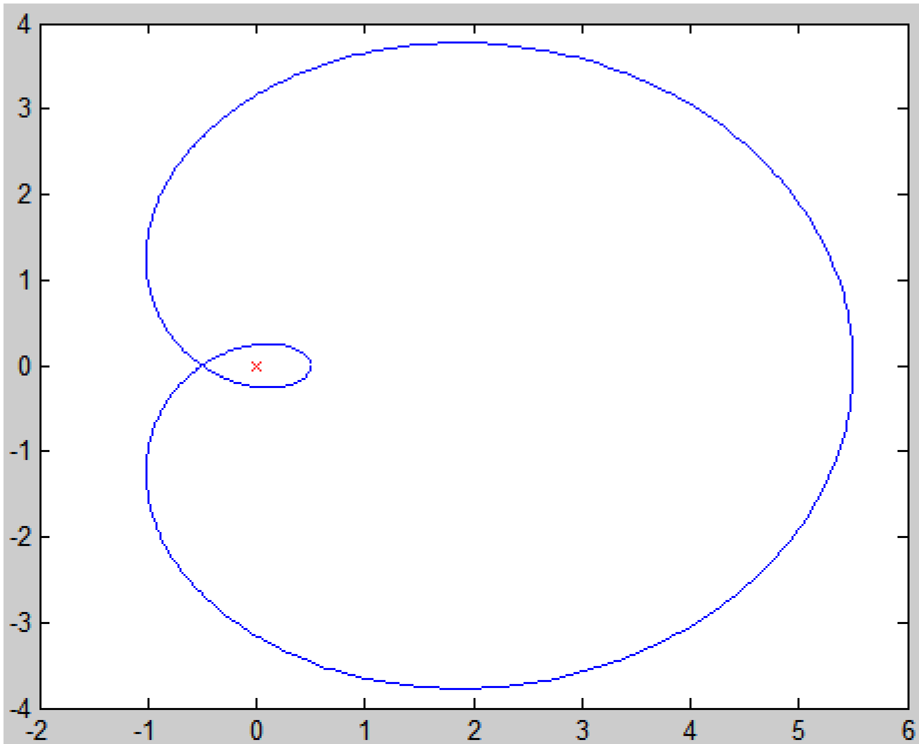


So the curve makes one revolution around the origin, i.e. there is an unstable pole. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $2.1915, 0.4043 \pm 0.2544i$. Absolutely, the pole 2.1915 is outside of the unit circle.

This system is unstable.

c. $A(q^{-1}) = 1 - 2q^{-1} + 2q^{-2} - 0.5q^{-3}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:

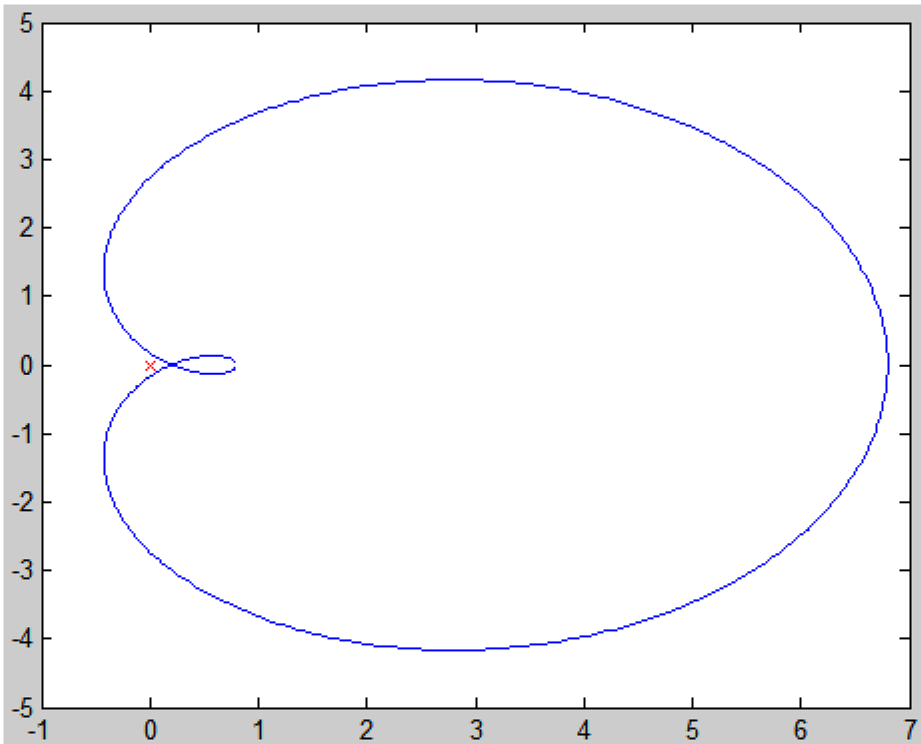


So the curve makes two revolutions around the origin, i.e. there are two unstable poles. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $0.3522, 0.8239 \pm 0.8607\mathrm{i}$. Absolutely, the poles $0.8239 \pm 0.8607\mathrm{i}$ are both outside of the unit circle.

This system is unstable.

d. $A(q^{-1}) = 2 - 3q^{-1} + 1.8q^{-2}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:

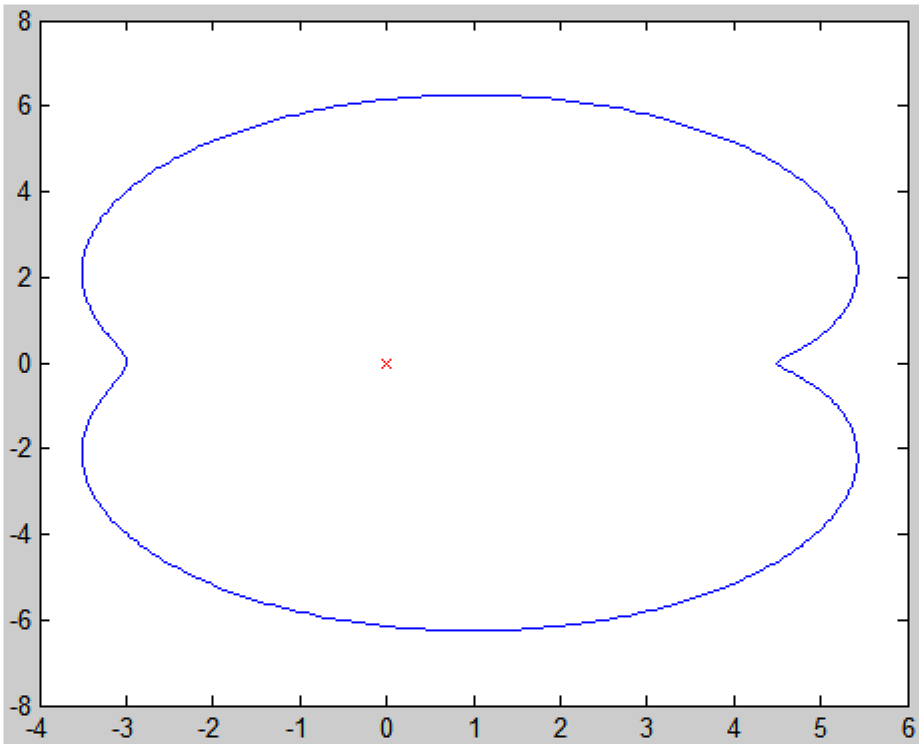


So the curve does not encircle the origin, i.e. there is no unstable pole. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $0.7500 \pm 0.5809\mathrm{i}$, which are all inside the unit circle.

This system is stable.

e. $A(q^{-1}) = 1 + 5q^{-1} - 0.25q^{-2} - 1.25q^{-3}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:

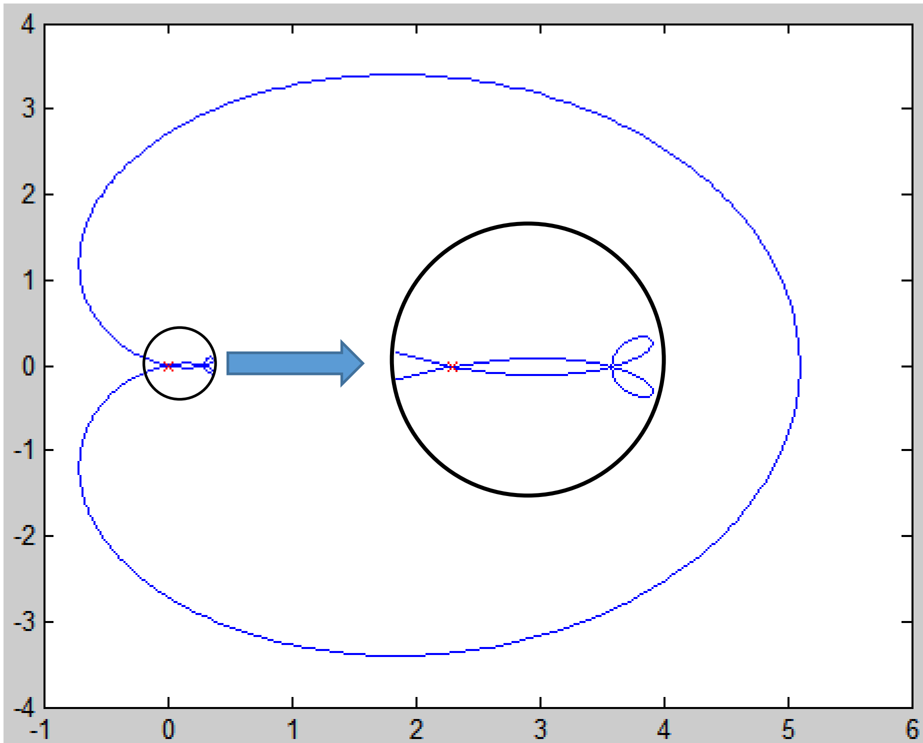


So the curve makes one revolution around the origin, i.e. there is an unstable pole. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $5, \pm 0.5$. Absolutely, the pole 5 is outside of the unit circle.

This system is unstable.

f. $A(q^{-1}) = 1 - 1.7q^{-1} + 1.7q^{-2} - 0.7q^{-3}$

We can plot the figure of Nyquist Curve, and the red cross marks the origin point:



As we can see, the curve makes cross right at the origin point, i.e. there are two poles which both are 1. To verify this, we can calculate the roots of this polynomial. Then we get the roots of $0.7, 0.5 \pm 0.866i$. Absolutely, the poles $0.5 \pm 0.866i$ both are just on the unit circle.

This system is unstable.

2. Consider the process:

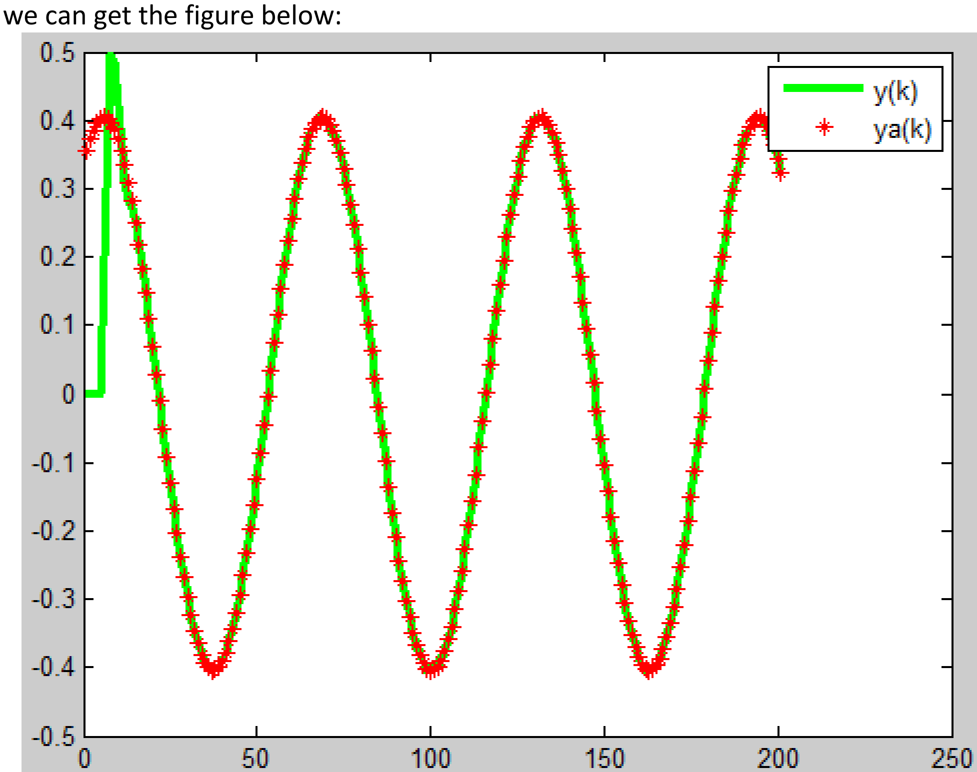
$$G(q^{-1}) = \frac{0.2q^{-5}}{1 - q^{-1} + 0.5q^{-2}}$$

a) Plot 2 figures together to compare:

Code in Matlab:

```
k=0:200;
u=[zeros(1,5),cos(0.1*k)];
y=filter(0.2,[1,-1,0.5,0,0,0],u);
figure;plot(y(1:201),'g','LineWidth',3);hold on;
z=exp(-i*0.1);
G=0.2*z^5/(1-z+0.5*z^2);
ya=abs(G)*cos(0.1*k+angle(G));
plot(ya,'r*');hold off;
legend('y(k)','ya(k)');
```

Then we can get the figure below:



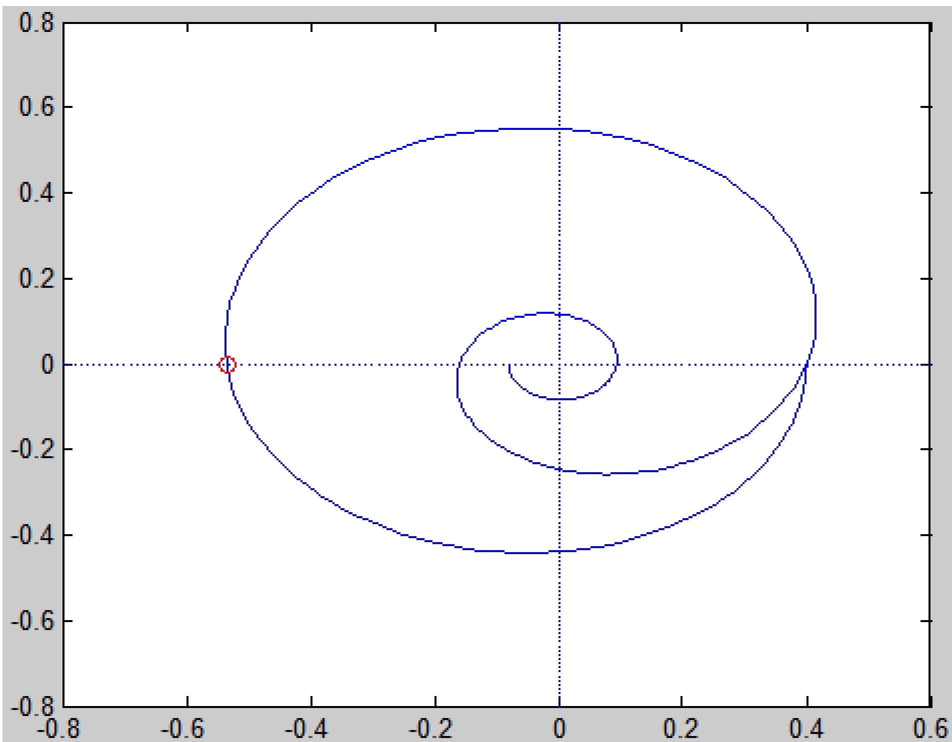
From the graph above, it is obvious that $y(k) \to y_a(k)$.

b)

Plot the Nyquist curve of $G(e^{-i\omega})$, $\omega = 0 \to \pi$:
Code in Matlab:

```
w=0:0.01:pi;
z=exp(-i*w);
G=0.2*z.^5./(1-z+0.5*z.^2);
plot(real(G),imag(G));
```

Then we can get the figure below:



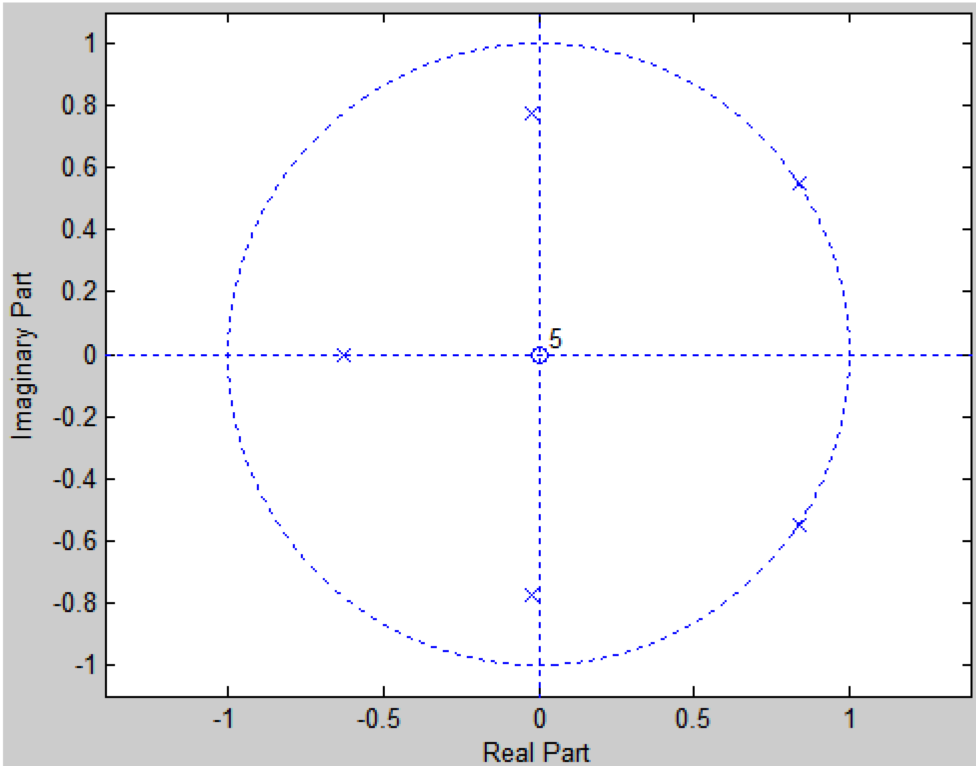
As we can see from the figure above, the curve crosses the negative real axis at about -0.534, so we can find the gain margin is approximately $\frac{1}{0.534} \approx 1.87$, which is the K_{max} for which the closed-loop system is stable.

To verify this, we can use this K_{max} to calculate the function for the closed-loop system which is:

$$G_{closed} = \frac{0.2q^{-5}}{1 - q^{-1} + 0.5q^{-2} + K * 0.2q^{-5}} = \frac{0.2q^{-5}}{1 - q^{-1} + 0.5q^{-2} + 0.374q^{-5}}$$

Then we can use the Matlab to plot the zero-pole figure on the z-plane:

```
num=[0.2]
den=[1 -1 0.5 0 0 0.374]
zplane(num,den)
```



As we can see, the poles which are in the first ant fourth quadrant are just on the unit circle, which means that they are on the stability margin.

To calculate the argument for this pole pair, we can use the Matlab to calculate the root of the denominator of the G_{closed}. Then we can get the roots are:

$$0.8364 \pm 0.5480\mathrm{i}, -0.0231 \pm 0.7722\mathrm{i}, -0.6267$$

Absolutely, the two poles which are on the stability margin are $0.8364 \pm 0.5480\mathrm{i}$. The argument for this pole pair is $\pm arctan \frac{0.5480}{0.8364} = \pm 0.58\, rad$.

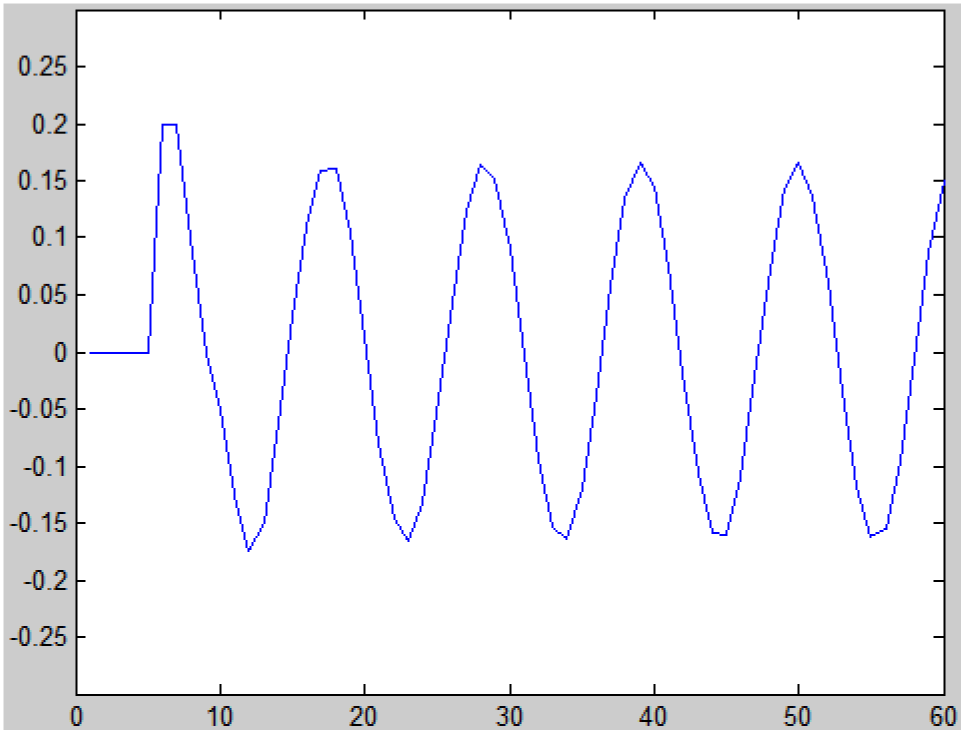
Next, we should calculate the ω for which the Nyquist curve crosses the negative axis. We can use the `fzero()` function to find it on the condition that we know it is near 0.5 . The Matlab code is shown below:

```
y=inline('imag(0.2*exp(-5*i*w)/(1-exp(-i*w)+0.5*exp(-2*i*w)))','w');
w=fzero(y,0.5);
```

Then we can get the result of 0.5799. This ω refers to the oscillation frequency when the closed-loop system uses the maximum gain $K_{max} = 1.87$. If this is true, the oscillation period should be $\frac{2\pi}{0.5799} \approx 10.835$. To verify this, we can plot the impulse response of the system:

```
num=[0.2];
den=[1 -1 0.5 0 0 0.374];
sys=tf(num,den,-1);
plot(impulse(sys));
axis([0 60 -0.3 0.3]);
```

Then we get the figure of the impulse response of this system:



As we see from the figure, the oscillation period is just what we expect, approximately 10.835. So we can say that the ω for which the Nyquist curve crosses the negative axis is the oscillation frequency when the closed-loop system uses the maximum gain.

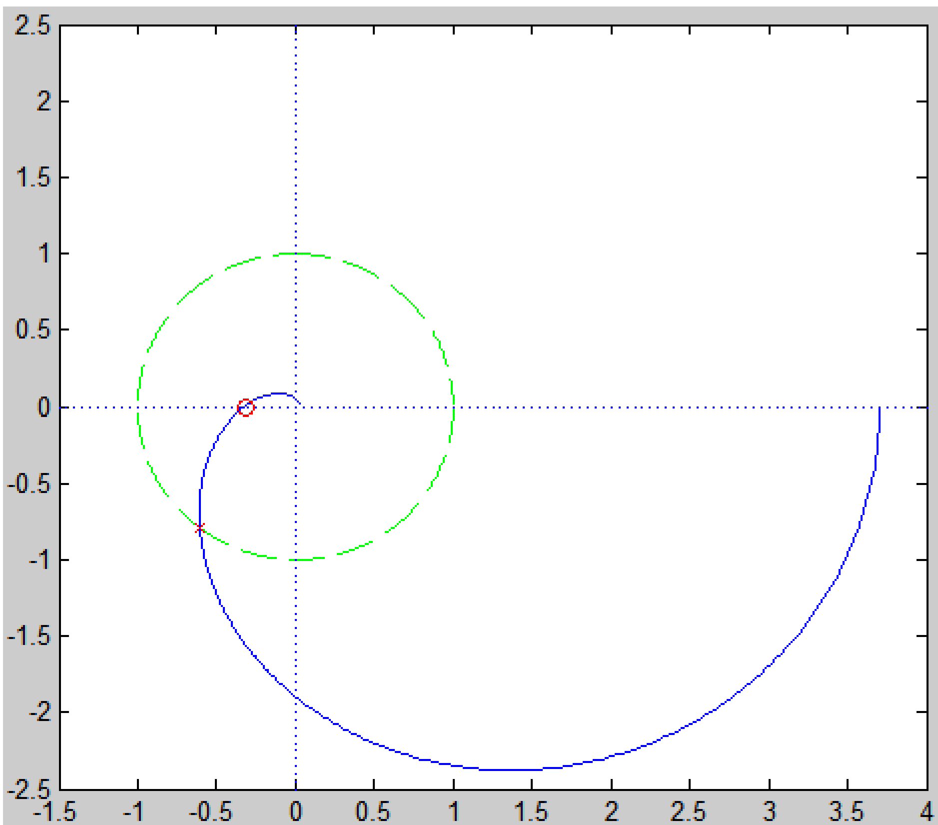
3. Consider the process:

$$G(q^{-1}) = \frac{0.1q^{-2}}{(1-0.1q^{-1})(1-0.7q^{-1})(1-0.9q^{-1})}$$

a)

Code in Matlab:

```
w=0:0.001:pi;
z=exp(-1i*w);
G=0.1*z.^2./((1-0.1*z).*(1-0.7*z).*(1-0.9*z));
plot(real(G),imag(G));
```



The figure above plots the Nyquist curve of $G(e^{-i\omega})$, $\omega = 0 \to \pi$.

The point with the red circle is the position which we get the gain margin, which is about -0.3194 when $\omega_o = 0.6120$. So the gain margin $A_m = \frac{1}{0.3194} \approx 3.13$.

The point with the red cross is the position which we get the phase margin, which is about $-0.6065 - 0.7951\mathrm{i}$ when $\omega_x = 0.2861$. So the phase margin $\phi_m = arctan\frac{0.7951}{0.6065} \approx 0.919\ rad$.

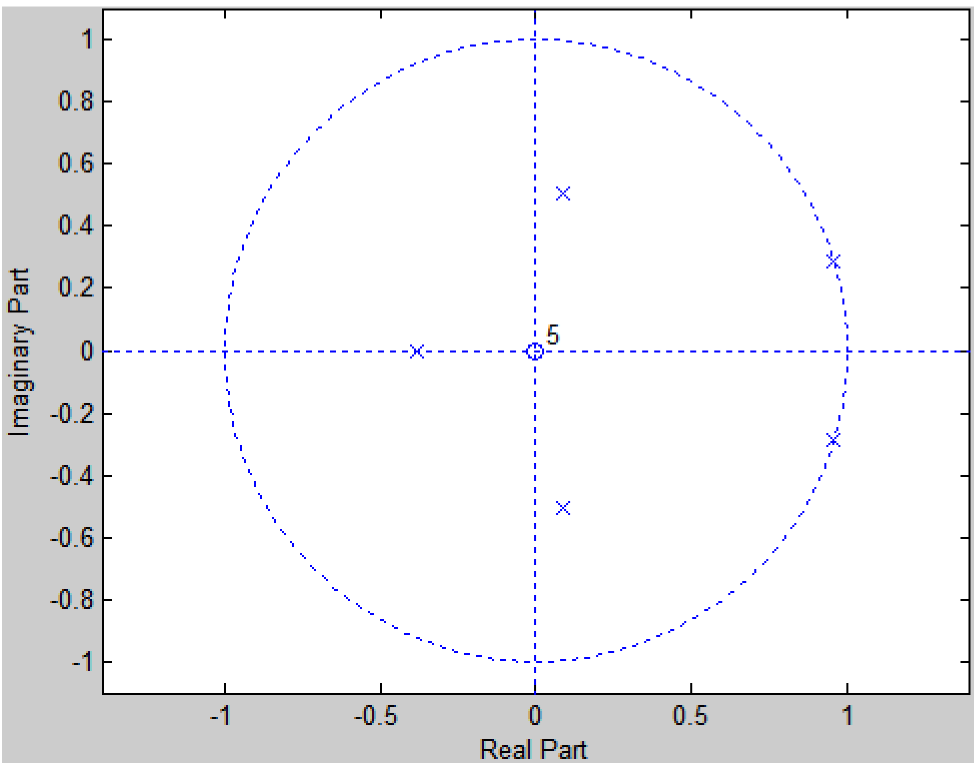
b)

Delay margin $\tau_m = \frac{\phi_m}{\omega_x} = \frac{0.9196}{0.2861} \approx 3.2127$. So 3 sample delays can be introduced before the closed-loop gets unstable.

To verify this, we can use $u(k) = -y(k-3) = -q^{-3}y(k)$ as the feedback function to calculate the function for the closed-loop system which is:

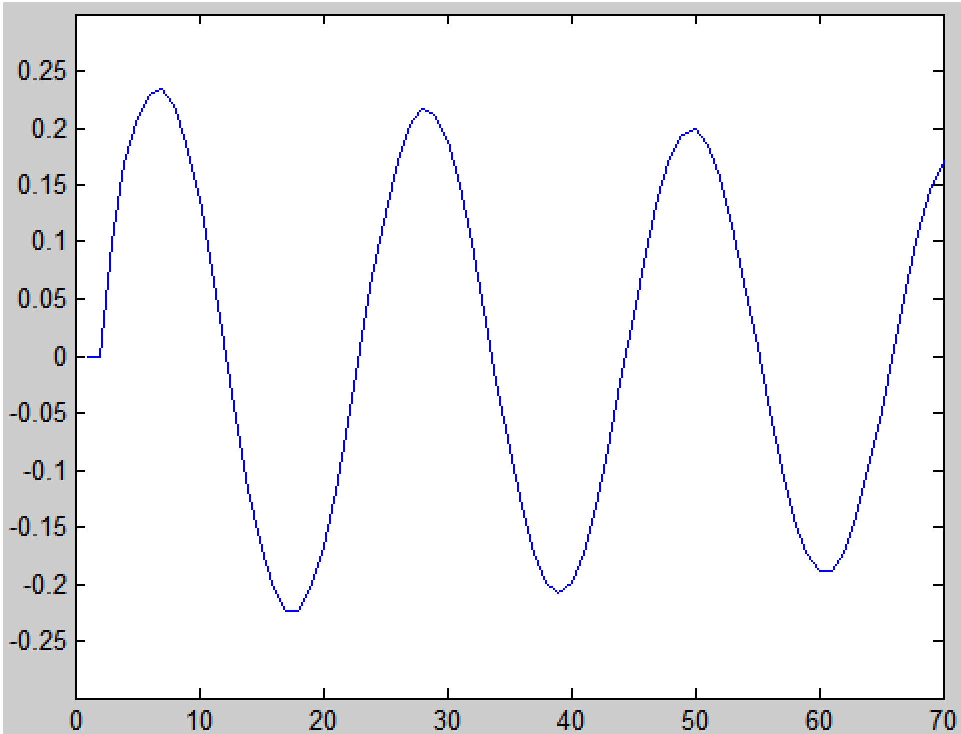
$$G_{closed} = \frac{0.1q^{-2}}{(1-0.1q^{-1})(1-0.7q^{-1})(1-0.9q^{-1}) + q^{-3} * 0.1q^{-2}}$$
$$= \frac{0.1q^{-2}}{1-1.7q^{-1}+0.79q^{-2}-0.063q^{-3}+0.1q^{-5}}$$

Then we can use the Matlab to plot the zero-pole figure on the z-plane:



As we can see, there two poles are just on the left part of the unit circle, which means that they are on the stability margin. After using Matlab to calculate the root of the denominator of the G_{closed}, we can get the argument of the two poles are $0.9538 \pm 0.2871\text{i}$. The argument for this pole pair is $\pm arctan\frac{0.2871}{0.9538} = \pm 0.29\, rad$.

The $\omega = 0.2861$ for which the Nyquist curve crosses the negative axis is the oscillation frequency when the closed-loop system uses the delay margin $\tau_m = 3$ as the feedback function. Same as the problem in 2.b), we can verify this by calculating the impulse response:



As we see from the figure, the oscillation period is just what we expect, approximately 21.96, corresponding with the $\omega = 0.2861$. So we can say that the ω for which the Nyquist curve crosses the negative axis is the oscillation frequency when the closed-loop system uses the maximum delay.