Comments on exercise 2

This is some additional comment on exercise 2

Task 2.1-2.4

In this part of the exercise the uncertainty parameters are, beside of the state parameters, distance driven, Δd , and heading change, Δθ. The model for position update will look like:

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ y_{k-1} + \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \theta_{k-1} + \Delta\theta \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{\mathrm{X_k}} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{\Delta d\Delta\theta} \Sigma_{\Delta d\Delta\theta} J_{\Delta d\Delta\theta}^T$$

There

$$J_{X_{k-1}} = \begin{bmatrix} 1 & 0 & -\Delta d \sin(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ 0 & 1 & \Delta d \cos(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{\Delta d\Delta\theta} = \begin{bmatrix} \cos(\theta_{k-1} + \frac{\Delta\theta}{2}) & -\frac{\Delta d}{2} \sin(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ \sin(\theta_{k-1} + \frac{\Delta\theta}{2}) & \frac{\Delta d}{2} \cos(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ 0 & 1 \end{bmatrix}$$

$$\Sigma_{\Delta d\Delta\theta} = \begin{bmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_{\Delta\theta}^2 \end{bmatrix}$$

From Wang paper (section 4) we got (assume no correlation between wheels) that :

$$\sigma_d^2 = \frac{\sigma_r^2 + \sigma_l^2}{4}$$

$$\sigma_{\Delta\theta}^2 = \frac{\sigma_r^2 - \sigma_l^2}{L^2}$$

Task 2.5-2.6

Here we got new uncertainty parameters, distance driven by the left wheel, Δl, and distance driven by the right wheel, Δr, and the wheel base. This change our model for position update to:

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + \frac{\Delta R + \Delta L}{2} \cos\left(\theta_{k-1} + \frac{\Delta R - \Delta L}{2b}\right) \\ y_{k-1} + \frac{\Delta R + \Delta L}{2} \sin\left(\theta_{k-1} + \frac{\Delta R - \Delta L}{2b}\right) \\ \theta_{k-1} + \frac{\Delta R - \Delta L}{b} \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{\Delta r \Delta l} \Sigma_{\Delta r \Delta l} J_{\Delta r \Delta}^T + + J_b \Sigma_b J_b^T$$

There

$$J_{X_{k-1}} = \begin{bmatrix} 1 & 0 & -\Delta d \sin(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ 0 & 1 & \Delta d \cos(\theta_{k-1} + \frac{\Delta\theta}{2}) \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{\Delta r \Delta l} = \mathit{See\ textbook\ page}\ 189\ (\mathit{equ}\ 5.11) \mathit{or\ the\ slides\ from\ lecture}\ 5$$

$$\Sigma_{\Delta r \Delta l} = \mathit{See\ textbook\ page}\ 189\ (\mathit{equ}\ 5.8) \mathit{or\ the\ slides\ from\ lecture}\ 5$$

$$J_b = \mathit{See\ Exercise}\ 3\ \mathit{and}\ 4\ \mathit{in\ the}\ \text{Error propagation}\ \mathit{material\ sent\ out}$$

$$\Sigma_b = \mathit{See\ Exercise}\ 3\ \mathit{and}\ 4\ \mathit{in\ the}\ \text{Error propagation}\ \mathit{material\ sent\ out}$$

Task 3

In this part of the exercise we use a steer-drive robot instead of a differential drive robot. This means that our model change for position update. The new uncertainty parameters are the steering angle, α, and the speed, V, and sampling time T.

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + v\cos(\alpha)\,\mathrm{T} \cos\left(\theta_{k-1} + \frac{v\sin(\alpha)\,T}{2L}\right) \\ y_{k-1} + v\cos(\alpha)\,T\ \sin\left(\theta_{k-1} + \frac{v\sin(\alpha)\,T}{2L}\right) \\ \theta_{k-1} + \frac{v\sin(\alpha)\,T}{L} \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{v\alpha T} \Sigma_{v\alpha T} J_{v\alpha T}^T$$

For guidance, see Exercise 4 in *Calculate Jacobian matrixes* material sent out.