a) ① $x(n) = 1 + \cos\left(\frac{\pi}{4} n\right) + 0.5\cos\left(\frac{3\pi}{4}(n-1)\right)$ periodic, infinite dur. signal

DTFS: $x(n) = \sum_{k=0}^{N-1} c_k \cdot e^{j\frac{2\pi}{N} k \cdot n}$ $\quad k = 0, 1, \dots, N-1$

Find the period $= N$ of $x(n)$.

$\cos\left(\frac{\pi}{4} \cdot n\right) = \cos\left(2\pi \frac{1}{8} \cdot n\right) \rightarrow N = 8$

$\cos\left(\frac{3\pi}{4} \cdot n\right) = \cos\left(2\pi \frac{3}{8} \cdot n\right) \rightarrow N = 8.$

So $x(n)$ has the period of $N = 8$.

DTFS: $x(n) = \sum_{k=0}^{7} c_k \cdot e^{j\frac{2\pi}{8} k \cdot n} = \sum_{k=0}^{7} c_k e^{j\frac{\pi}{4} k \cdot n}$ $\quad k = 0, 1, \dots, 7$

Compare!

$= c_0 + c_1 e^{j\frac{\pi}{4} \cdot n} + \dots + c_3 e^{j\frac{\pi}{4} \cdot 3 \cdot n} + \dots + c_7 e^{j\frac{\pi}{4} \cdot 7 n}$

Euler id:

$x(n) = 1 + \frac{1}{2}\left[e^{j\frac{\pi}{4} n} + e^{-j\frac{\pi}{4} \cdot n}\right] + 0.5 \frac{1}{2}\left[e^{j\frac{3\pi}{4}(n-1)} + e^{-j\frac{3\pi}{4}(n-1)}\right]$

$= 1 + \frac{1}{2} e^{j\frac{\pi}{4} n} + \frac{1}{2} e^{-j\frac{\pi}{4} n} + \frac{1}{4} \cdot e^{-j\frac{3\pi}{4}} \cdot e^{j\frac{\pi}{4} \cdot 3 \cdot n} + \frac{1}{4} e^{j\frac{3\pi}{4}} e^{-j\frac{\pi}{4} \cdot 3 \cdot n}$

Compare! $\Rightarrow$

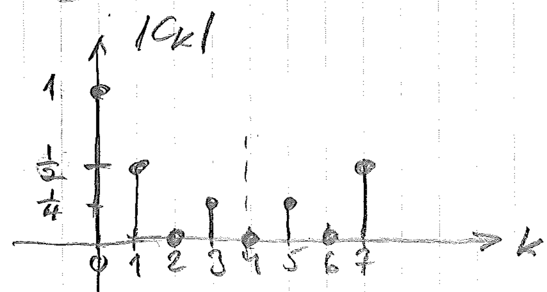
$\boxed{c_{k+N} = c_k}$ periodic spectra

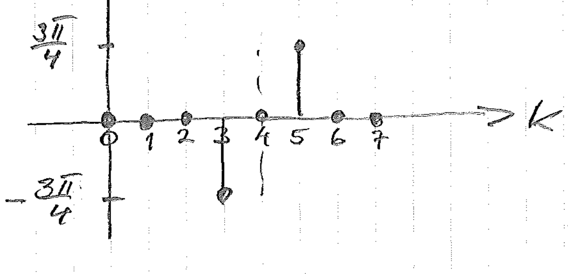
$c_0 = 1$

$c_1 = c_{-1} = \frac{1}{2} = c_7$

$c_3 = \frac{1}{4} \cdot e^{-j\frac{3\pi}{4}}$; $c_{-3} = \frac{1}{4} e^{j\frac{3\pi}{4}} = c_5$

$c_2 = c_4 = c_6 = 0$





① b) $y(n) = \frac{1}{2}[x(n) - x(n-8)]$

$x(n) = 1 + \cos(\frac{\pi}{4} \cdot n) + 0.5\cos(\frac{3\pi}{4}(n-1))$; $\underline{-\infty < n < \infty}$

steady state output.

Steady state output:

$$y(n) = H(0)\cdot 1 + |H(\tfrac{\pi}{4})| \cos(\tfrac{\pi}{4} \cdot n + \mathrm{Arg}\{H(\tfrac{\pi}{4})\})$$
$$+ |H(\tfrac{3\pi}{4})| \cdot 0.5 \cos(\tfrac{3\pi}{4}(n-1) + \mathrm{Arg}\{H(\tfrac{3\pi}{4})\})$$

So find $H(\omega) = H(z)\big|_{z=e^{j\omega}}$ $\qquad h(n) = \{\underset{\uparrow}{\tfrac{1}{2}}, 0, 0, 0, 0, 0, 0, 0, -\tfrac{1}{2}\}$

$\boxed{y(n) = \frac{1}{2}x(n) - \frac{1}{2}x(n-8)}$ diff. eq. FIR-system.

$\downarrow z$

$$Y(z) = \tfrac{1}{2}X(z) - \tfrac{1}{2}z^{-8}X(z) = X(z)\cdot\tfrac{1}{2}(1 - z^{-8})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 - z^{-8})$$ ROC: entire z-plane except $z=0$

$$H(\omega) = H(z)\big|_{z=e^{j\omega}} = \frac{1}{2}(1 - e^{-j\omega 8})$$

$$H(0) = \tfrac{1}{2}(1-1) = 0 \qquad (\Sigma h(n) = 0)$$

$$H(\tfrac{\pi}{4}) = \tfrac{1}{2}(1 - e^{-j\frac{\pi}{4}\cdot 8}) = \tfrac{1}{2}(1 - \underbrace{e^{-j2\pi}}_{=1}) = 0$$

$$H(\tfrac{3\pi}{4}) = \tfrac{1}{2}(1 - e^{-j\frac{3\pi}{4}8}) = \tfrac{1}{2}(1 - \underbrace{e^{-j6\pi}}_{=1}) = 0$$

$\Rightarrow$ $\underline{\underline{y(n) = 0}}$

(2)

$$H(z) = \frac{1-z^{-1}}{1-0{,}25z^{-2}} \qquad \text{causal LTI-system}$$

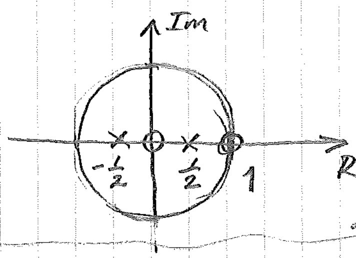
a)

$$H(z) = \frac{1-z^{-1}}{1-0{,}25z^{-2}} \begin{matrix} \times z^2 \\ \times z^2 \end{matrix} = \frac{z(z-1)}{z^2-0{,}25}$$

zeros: $z_1 = 0$, $z_2 = 1$

poles: $z^2 - 0.25 = 0$

$z^2 = \frac{1}{4} \rightarrow z_{1,2} = \pm\frac{1}{2}$; $p_1 = \frac{1}{2}$; $p_2 = -\frac{1}{2}$



ROC: $|z| > \frac{1}{2}$

b)

$x(n) = u(n)$

$Y(z) = H(z) \cdot X(z)$

$$= \frac{(1-z^{-1})}{(1-0{,}25z^{-2})} \cdot \frac{1}{(1-z^{-1})} \begin{matrix} \times z^3 \\ \times z^3 \end{matrix} \quad \frac{z^2(z-1)}{\underbrace{(z^2-0{,}25)}_{(z-\frac{1}{2})(z+\frac{1}{2})}(z-1)}$$

$$\frac{Y(z)}{z} = \frac{z(z-1)}{(z-\frac{1}{2})(z+\frac{1}{2})(z-1)} \overset{\text{p.f.e}}{=} \frac{A}{(z-\frac{1}{2})} + \frac{B}{(z+\frac{1}{2})} + \frac{C}{(z-1)}$$

$$A = (z-\tfrac{1}{2})\frac{Y(z)}{z}\Big|_{z=\frac{1}{2}} = \frac{\frac{1}{2}(\frac{1}{2}-1)}{(\frac{1}{2}+\frac{1}{2})(\frac{1}{2}-1)} = \frac{-\frac{1}{4}}{-\frac{1}{2}} = \frac{1}{2}$$

$$B = (z+\tfrac{1}{2})\frac{Y(z)}{z}\Big|_{z=-\frac{1}{2}} = \frac{-\frac{1}{2}(-\frac{1}{2}-1)}{(-\frac{1}{2}-\frac{1}{2})(-\frac{1}{2}-1)} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2}$$

$$C = (z-1)\frac{Y(z)}{z}\Big|_{z=1} \quad \frac{1(1-1)}{(1-\frac{1}{2})(1+\frac{1}{2})} = 0$$

(see the pole-zero plot!)

$$Y(z) = \frac{\frac{1}{2}}{(1-\frac{1}{2}z^{-1})} + \frac{\frac{1}{2}}{(1+\frac{1}{2}z^{-1})}$$

z^{-1}

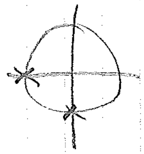
$$y(n) = \left[\frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{1}{2}\left(-\frac{1}{2}\right)^n\right]u(n)$$

(2)

c) $x(n) = \cos(0.5\pi \cdot n)$ $\quad -\infty < n < \infty$

steady state output!

$$y(n) = |H(0.5\pi)| \cos\left(0.5\pi \cdot n + \mathrm{Arg}\{H(0.5\pi)\}\right)$$



$$H(\tfrac{\pi}{2}) = H(z)\Big|_{z=e^{j\frac{\pi}{2}}}$$

$$= \frac{1 - e^{-j\frac{\pi}{2}}}{1 - 0.25\, e^{-j\pi}} = \frac{1-(-j)}{1-0.25(-1)}$$

$$= \frac{1+j}{5/4} = \frac{4}{5}(1+j)$$

$$|H(\tfrac{\pi}{2})| = \frac{4}{5} \cdot |1+j| = \frac{4\cdot\sqrt{2}}{5} \approx 1{,}13$$

$$\mathrm{Arg}\{H(\tfrac{\pi}{2})\} = \mathrm{Arg}\{\tfrac{4}{5}\} + \mathrm{Arg}\{1+j\} = 0 + 45^\circ$$

$$y(n) = \underbrace{\frac{4\cdot\sqrt{2}}{5}}_{\approx 1{,}13} \cos\left(0.5\pi \cdot n + 45^\circ\right)$$

③ $h(n) = 0.5\,\delta(n) + 0.7\,\delta(n-1) + 0.5\,\delta(n-2) = \{\underset{\uparrow}{0.5}, 0.7, 0.5\}$

a) $$H(\omega) = \sum_{n=0}^{2} h(n)\, e^{-j\omega n}$$

$$= 0.5 + 0.7 e^{-j\omega} + 0.5 e^{-j2\omega}$$

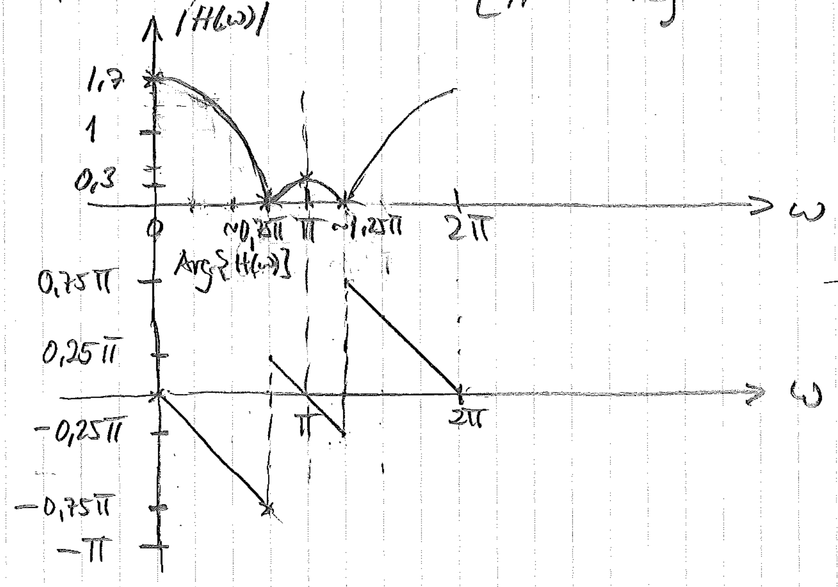
$$= 0.5\left[1 + e^{-j2\omega}\right] + 0.7 e^{-j\omega}$$

$$= 0.5 e^{-j\omega}\underbrace{\left[e^{j\omega} + e^{-j\omega}\right]}_{2\cdot\cos(\omega)} + 0.7 e^{-j\omega}$$

$$= e^{-j\omega}\left(0.7 + \cos(\omega)\right)$$

× $|H(\omega)| = |0.7 + \cos(\omega)|$

× $\mathrm{Arg}\{H(\omega)\} = -\omega + \begin{cases} 0 & \text{pos} \\ \pi & \text{neg} \end{cases}$



× $\omega = 0 \rightarrow 1.7$

× zeros ?

$\cos(\omega) = -0.7$

$\omega = \arccos(-0.7)$

$\Rightarrow \omega \approx \underset{(135°)}{0.75\pi}\ ;\ \underset{(225°)}{1.25\pi}$

× $\omega = \pi$

$\rightarrow |0.7 + \underbrace{\cos(\pi)}_{-1}| = 0.3$

b) $$|H(\omega_c)| = \frac{1.7}{\sqrt{2}} \approx 1.2$$

$\Rightarrow 1.2 = 0.7 + \cos(\omega_c)$

$\cos(\omega_c) = 1.2 - 0.7 \approx 0.5 \Rightarrow \omega_c \approx \frac{\pi}{3}$

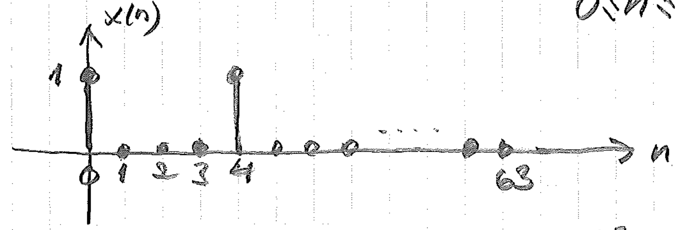
c) $x(n) = \cos(\omega_0 n)$; $-\infty < n < \infty$

$y(n) = 0$ where $|H(\omega_0)| = 0$

$\Rightarrow \omega_0 \approx 0.75\pi$ (see ampl. graf.)

④ $x(n) = [\delta(n) + \delta(n-4)]\,\underbrace{[u(n) - u(n-N)]}_{0 \le n \le 63}$; $N = 64$

a)



$x(n) = \{\overset{0}{\underset{\uparrow}{1}}, \overset{1}{0}, \overset{2}{0}, \overset{3}{0}, \overset{4}{1}, \overset{5}{0}, 0, \dots, \overset{63}{0}\} \longrightarrow n$ Time limited signal

$\underset{(N)}{\text{DFT}}$: $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot k \cdot n}$ $k = 0, 1, 2, \dots, N-1$

$N = 64 \Rightarrow X(k) = \sum_{n=0}^{63} (\delta(n) + \delta(n-4))\, e^{-j\frac{2\pi}{64}\cdot k \cdot n}$

$= 1 + 1 \cdot e^{-j\frac{2\pi}{64}k \cdot 4} = \left(1 + e^{-j\frac{\pi}{8}\cdot k}\right)$

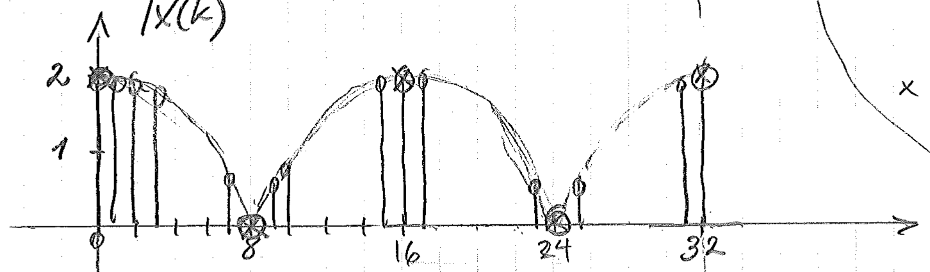
$k = 0, 1, 2, \dots, 63$

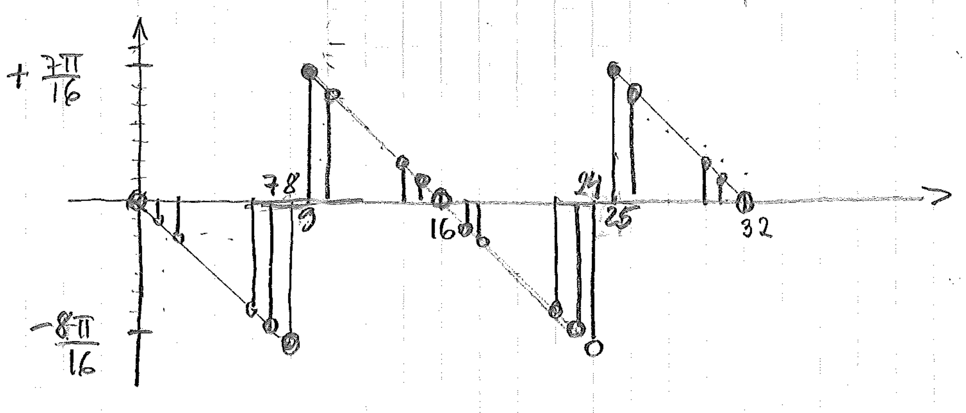
b) $X(k) = \left(1 + e^{-j\frac{\pi}{8}\cdot k}\right) = e^{-j\frac{\pi}{16}\cdot k}\,\underbrace{\left(e^{j\frac{\pi}{16}\cdot k} + e^{-j\frac{\pi}{16}\cdot k}\right)}_{2\cdot\cos(\frac{\pi}{16}\cdot k)}$

$= e^{-j\frac{\pi}{16}\cdot k} \cdot 2 \cdot \cos\left(\frac{\pi}{16}\cdot k\right)$

× $|X(k)| = 2\left|\cos\left(\frac{\pi}{16}\cdot k\right)\right|$

× $\operatorname{Arg}\{X(k)\} = -\frac{\pi}{16}\cdot k + \operatorname{Arg}\left\{\cos\left(\frac{\pi}{16}\cdot k\right)\right\}$





× Zeros?
$\frac{\pi}{16}\cdot k = m\cdot\frac{\pi}{2}$
$m = 1, 3, 5, 7,$
$k = 8 \cdot m$

× Max?
$\frac{\pi}{16}\cdot k = m\cdot\pi$
$m = 0, 1, 2, 3$
$k = 16 \cdot m$

× cosine signal