

Exercise 1 – Global Positioning Systems (GPS)

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Abstract

This exercise is mainly concern on the sensor applications of Global Positioning Systems(GPS). In this exercise, two kind of GPS receiver data are follow the details of NMEA-0183. For first data file, it's a series of stationary position data, Longitude and latitude conversion method^[1] is implemented on the original position data received by GPS. The error in position is plotted in histogram which shows the error distribution follows Guassian distribution. The error of position also plot with Co-variance matrix which centred around (0, 0) and more than 60% of the points located in the area of covariance matrix. Auto-correlation is calculated for errors, compare with the auto-correlation on random signal, the difference is obvious for the position data has more similarity than random signal. For the second series of data, which is a trace of an vehicle driving in the east part of halmstad. The trace is plotted using the data conversion method. Speed and heading of vehicle is calculated and plot against time which shows the measurement of speed is quite accurate and huge difference take place in variance of headings between while the vehicle is driving in low or high speed.

1.Introduction

Global navigate system can provide location and time information anywhere on earth or near earth where there is an GPS receiver. The main task of this report is to analyse the original data from an GPS receiver and implement Longitude and Latitude Conversion, Error analysis, calculation of co-variance matrix, stationary and mobile data analysis. This report take two kinds of data series into analysis, one is from a stationary position and another is from a mobile GPS receiver. In this exercise, all data follows the details of NMEA-0183, a common output of GPS receiver defined by National Marine Electronic Association interface standard and all calculation is operated by using Matlab. Error is estimated by calculate and plot the co-variance matrix and Error of position. Path retracing and speed,heading Analysis of mobile GPS receiver is presented. All exercises is performed in Matlab.

2. Theory and Method

In this section, Method of Longitude and latitude conversion, usage of co-variance matrix and auto-correlation is presented in these section.

2.1 Longitude and latitude conversion

In this report, the GPS data follows a group of comma delimited lines defined by National Marine Electronic Association interface standard, so called NMEA-0183. Both the latitude and longitude values are presented in degrees, minutes, and decimal minutes in an NMEA-0183 sentence^[1]. The conversion from latitude or longitude angles into degrees can be done by using the formula below:

$$dd.ddddddd = dd + \frac{mm.mmmmm}{60} = \left[\frac{ddmm.mmmmm}{100} \right] + \frac{ddmm.mmmmm - \left[\frac{ddmm.mmmmm}{100} \right] * 100}{60} ;$$

The conversion from degrees into meters can use the table in ^[2] to approximate factors Flon and Flat for a given latitude and height over the ellipsoid. This conversion method will be implemented in section 3 for transfer original GPS receiver data to meter form.

2.2 Co-variance matrix

Covariance matrix is calculated in this report for represent the variance of a set of point. In statistics, the element in covariance matrix specify the joint probability distribution of random variables^[3].

Think of a two order covariance matrix using for describe the variance of a position measurement or estimation process. The elements on principal diagonal donate the variance of x,y position, the elements on subordinate diagonal donate the covariance of x,y direction. Covariance is a measure of how much two random variables change together^[4]. If the variables tend to show similar behaviour, the covariance is positive, if it's on the opposite side, not similar changing behaviour, then the covariance will be negative. So the covariance here on the subordinate diagonal can be consider as the scale of rotate on position of x,y.

In section 3, covariance matrix will be plot with the error of x,y position and show the uncertainty of x,y.

2.3 auto-correlation

Autocorrelation is the cross-correlation of a signal with itself. It is the similarity between observations as a function of the time separation between them^[5]. In section 3, autocorrelation is calculated as an mathematical tool for finding repeating patterns, for the error of GPS receiver, auto-correlation will be plot compare with the auto-correlation of random signal, which totally not repeat itself at all.

3. Experiments and results

In this section, two kind of GPS receiver's data is used for analysis, first one is a stationary position's data, the second one is a mobile GPS receiver's data. All calculation and figure plotting is operated by using Matlab.

3.1 Transform NMEA-1083 's longitude and latitude data from angles to meters

First step, transform longitude and latitude data from angles to degrees. Use the formula:

$$dd.ddddddd = dd + \frac{mm.mmmm}{60} = \left[\frac{ddmm.mmmm}{100} \right] + \frac{ddmm.mmmm - \left[\frac{ddmm.mmmm}{100} \right] * 100}{60};$$

Save data of meter form in two arrays. The code in .m file can be:

```
LongDeg=floor(Longitude/100)+(Longitude-floor(Longitude/100)*100)/60;
LatDeg=floor(Latitude/100)+(Latitude-floor(Latitude/100)*100)/60;
```

Second step, data of degrees transform into meters using a coordinate system that set the Equator as x-axis and set Greenwich as y-axis. Refer to the longitude and latitude conversion table^[2], assume the height is 0 and latitude is 56 degree. The code can be:

```
X = F_lon * LongDeg;
Y = F_lat * LatDeg;
```

Figure 3.1 shows the X,Y position in the coordinate system.

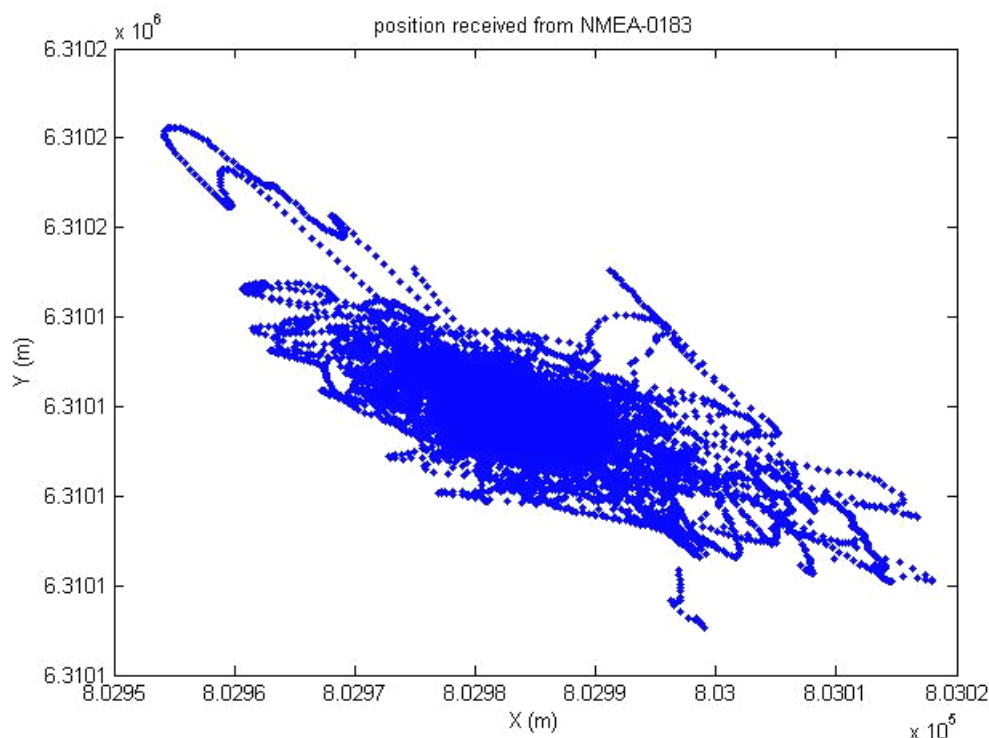


Figure 3.1 X,Y position

3.2 Estimation of X,Y position's mean and variance

(1) Error

Calculate the mean value of X,Y, and subtract from original X,Y position

data. Then plot the Error of two position in histogram form. Use `histfit()` function compare the Error histogram with normal distribution. Figure 3.2 shows the result. Red curve donate as the normal distribution.

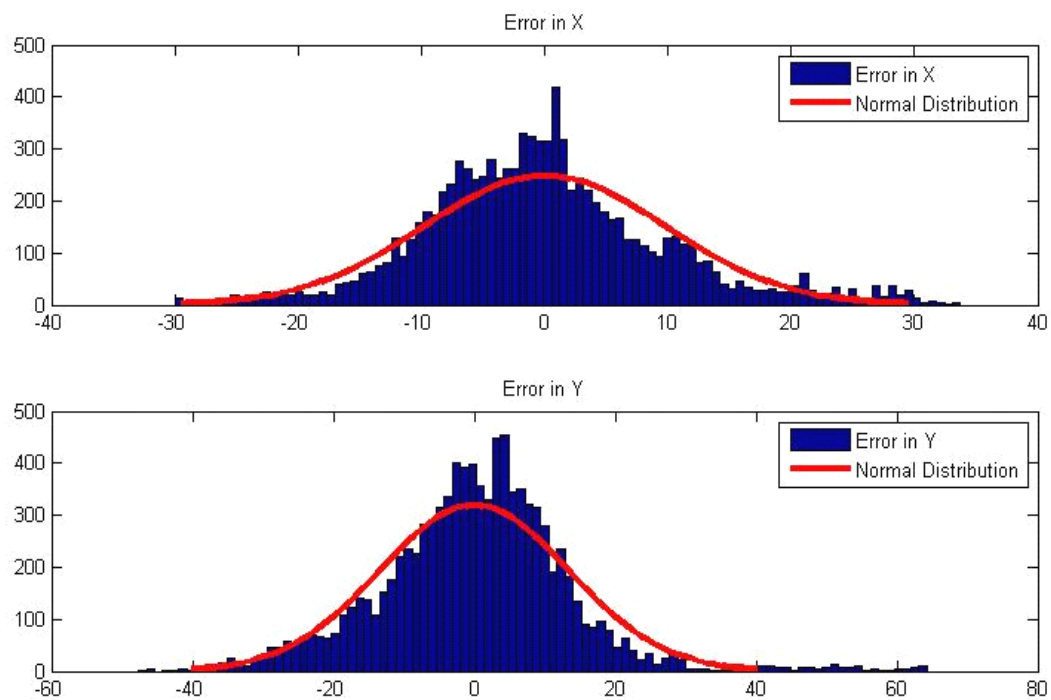


Figure 3.2 Error histogram of X and Y position

As shown on the figure 3.2, blue pillar represent as the error of X,Y position almost fit the shape of red curve which donate as the Guassian distribution. Therefore, the Error of X,Y position roughly follow the Guassian distribution.

Calculate the mean value of X,Y position, subtract them from X,Y, find the maxim value of error, the maxim error in x is 33.7343, maxim error in y is 64.2594.

(2) Co-variance Matrix

Calculate Co-variance matrix using command `'plot_uncertainty([0 0]', cov([X Y]), 1, 2)'`, where the centre of co-variance matrix is plotted around (0,0). Figure 3.3 shows the result. Blue point donate as the error and red circle donate the co-variance matrix, approximately 63% of the points should be inside the circle.

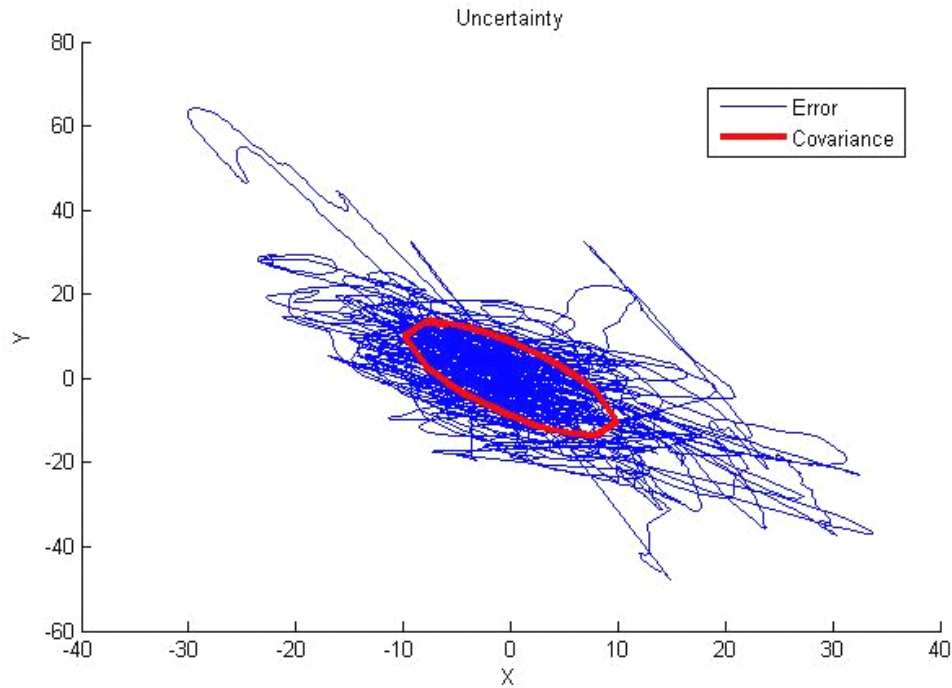


Figure 3.3 Error and co-variance matrix

3.3 Plot the error and auto-correlation in x and y separately

(1) Plot and compare X and Y

Figure 3.4 shows the plot of X and Y, the error of x and y in some case opposite to each other, for example, at point 7000 of y-axis, the value of x below the mean and y above the mean, which some how opposite or compensate for each other, same situation happens at point around 3500, 4200, 7900.

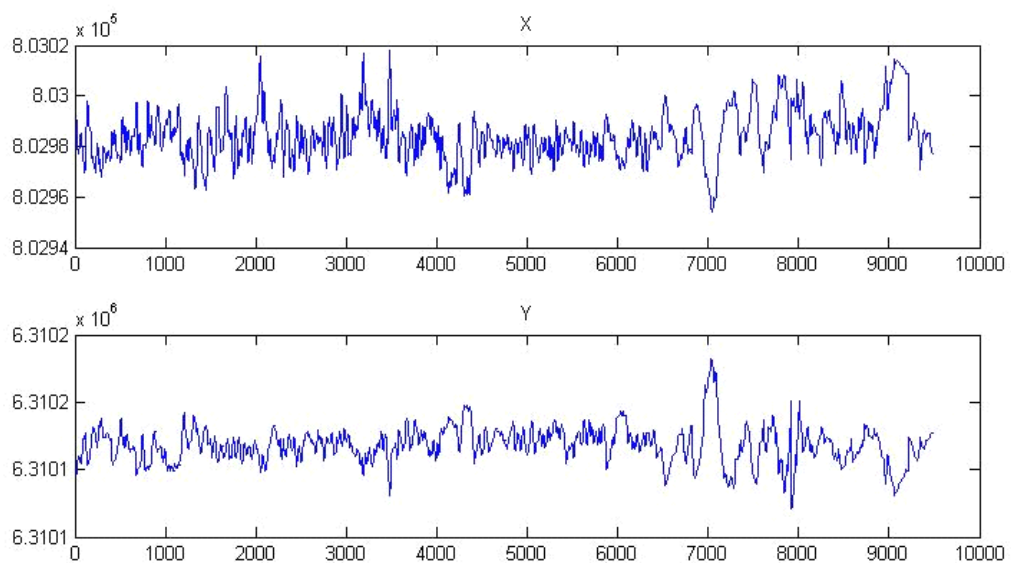


Figure 3.4 error of X,Y position comparison

(2) Comparison of Auto-correlation

Auto-correlation is a kind of mathematical tool for finding repeating patterns. Use the function 'xcorr()' to calculate the auto-correlation of the error in x,y position, the result is shown in figure 3.5, where blue line donate as auto-correlation of random signal, red line donate as auto-correlation of error in x position, green line donate as auto-correlation of error in y position.

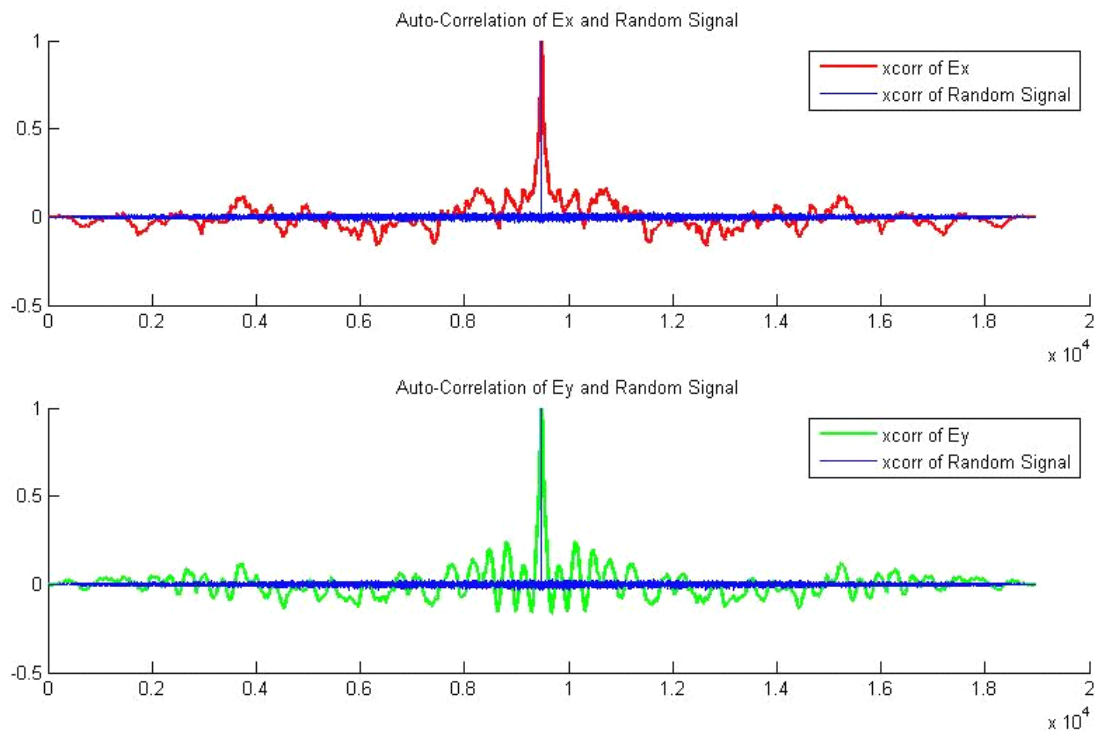


Figure 3.5 Auto-correlation of Ex and Ey compare to random signal

The result is obvious that the auto-correlation of error in x,y position shows the error has more similarity to itself than random signal. The Gps error is correlated, and it's arrive the peak around in 950 of y axis. So there may have more error effects on certain time shift when the error correlation reaches it's peaks. Compare to random signal, the error signal very similar to itself within a time shift, random signal create in random, so it will not repeat itself in all time shift.

3.4 Plot the data of Mobile GPS receiver

The data of mobile GPS receiver is a serial of trace by a car drive in the east side of Halmstad. Use the method in 3.1 to transform data to meter form. Figure 3.6 shows the path. Figure 3.7 shows the combination of the path figure and legend of east part of Halmstad city.

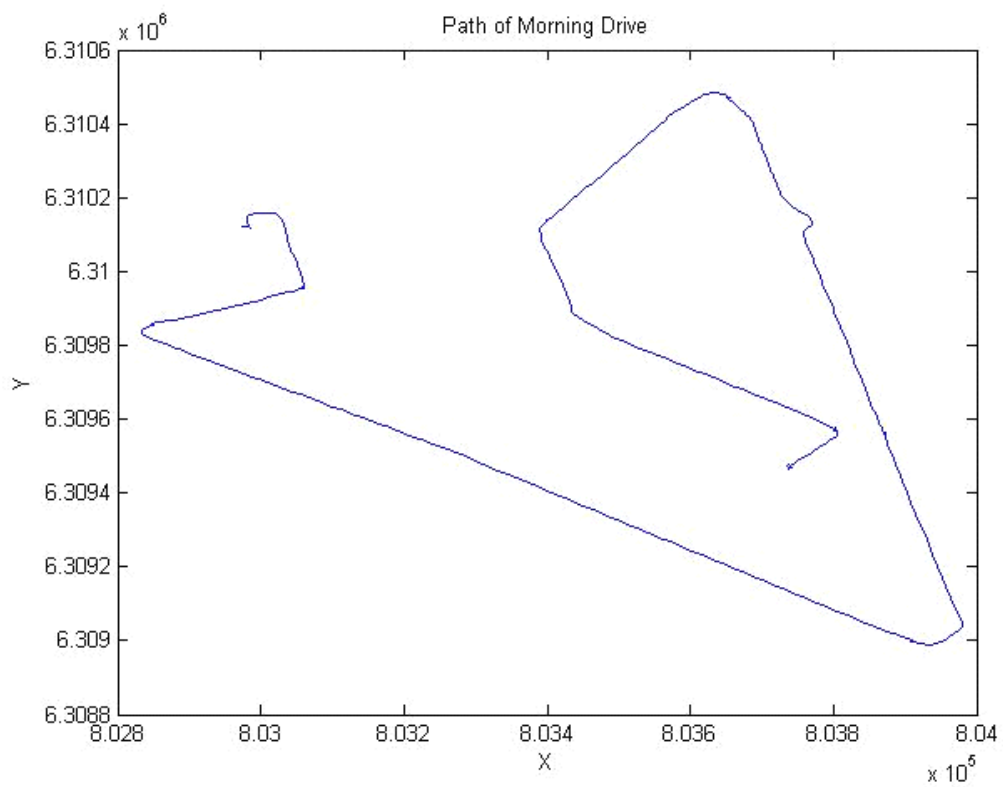


Figure 3.6 Mobile GPS's path



Figure 3.7 Path of driving in east part of Halmstad city

By using zoom, rotate, hyalinize command, the path plotted provided by mobile GPS data fit the shape of legend, thus it's enough to trace back the car driving routing.

3.5 Speed and Headings of the Vehicle

The sample rate of GPS data receive is 1hz, so the time slot of data is 1seconds. Therefore the speed of the car V can be calculate using the formula below:

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\left[\frac{X(n) - X(n+1)}{1}\right]^2 + \left[\frac{Y(n) - Y(n+1)}{1}\right]^2}$$

Where V_x donate the speed of direction x and V_y donate y -axis speed, the speed can be calculated in separate direction at first than combine then. For headings, use the function 'atan2(Dy,Dx)' in matlab. Figure 3.8 plots the speed and headings of the vehicle against time.

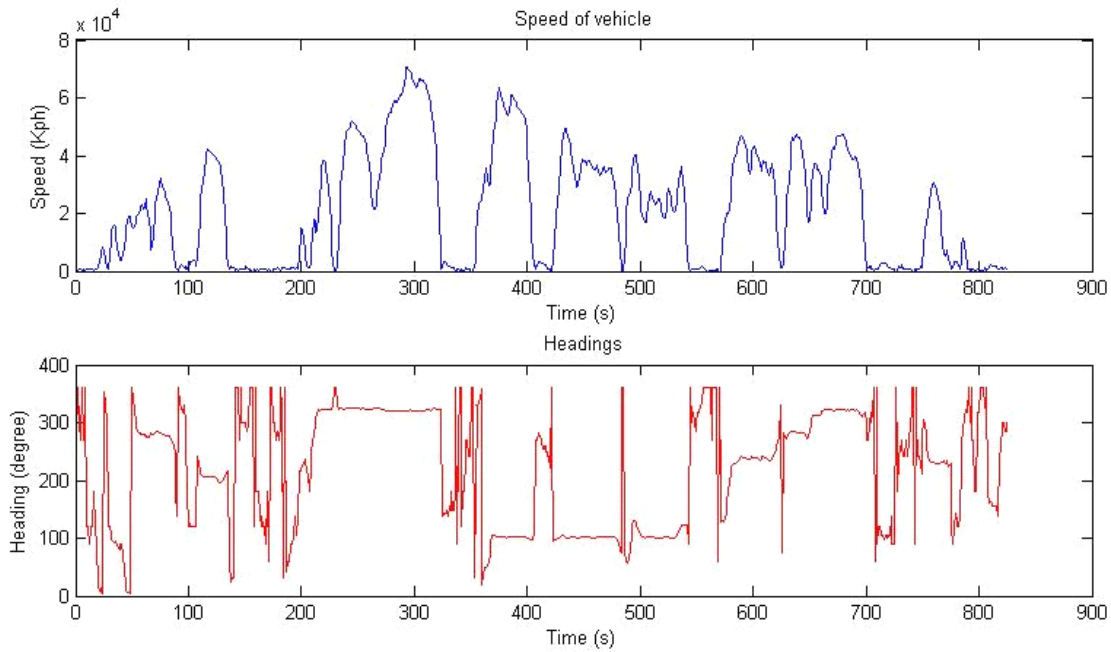


Figure 3.8 speed and heading of the vehicle

According to the figure, the speed is quite accurate because the maximum speed is the same to the speed meter in the vehicle. But the estimation in position is not accurate like this, Because the speed is calculated from the subtraction of these data, the subtraction also cancel the error from two operate number. Thus it's quite accurate for speed measurement.

The error of the vehicle must be calculated separately, in other words, the error should be divide into segments according to every period of car. For example, the time of vehicle drive during Laholmsvägen is approximately 240 to 310, the variance is 2.5701, But while driving near Kyrka in very low speed, the variance is 9.2047e+003, quite large. the error comparison is shown in figure 3.9.

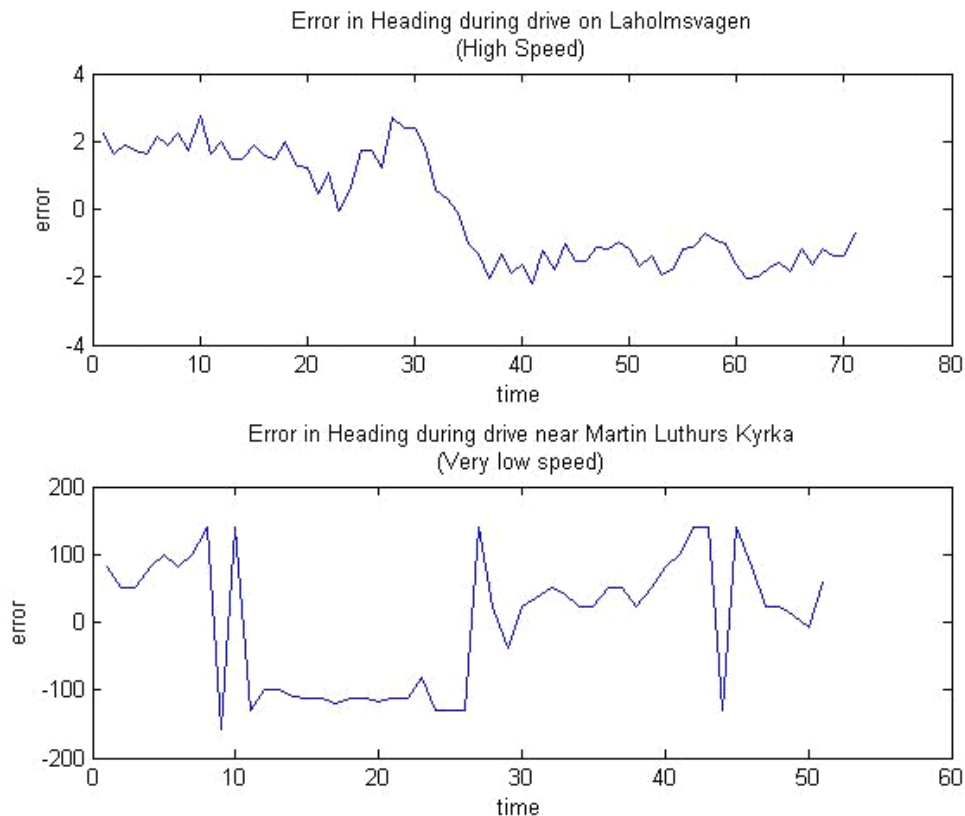


Figure3.9 Error of Heading drive during Laholmsvägen

Because there is errors in static measurement, it's like a uncertain point vibrating, thus when calculating the headings, even every small error can make two adjacent value different and cause large difference in headings. While driving at a high speed, it's like a super large scale compare with the GPS error, thus the error comes less important. The heading will not effect largely by GPS error.

4. Conclusions and further improvements

This exercises is mainly about GPS receiver data conversion and analysis. Longitude and Latitude Conversion is presented. And Error analysis shows the relation of covariance and error in position, the error has a pattern in time thus can be observed by calculate auto-correlation of itself, which is different from the random signal that never repeat itself. Driving path, speed and headings of the vehicle are calculated and plotted, the result shows that the speed measurement is quite accurate and the headings error will be large when in low speed, this is because the subtraction we take in speed calculation also cancel the the error part of two adjacent point and even small error in position will effect largely in headings, which is an angle of two direction.

Further improvements should be made for step forward, for example, more material should be referenced and detail of theory and method should be provide in section 2 and it would be more practical if can access to the data receive section of

GPS, in other word, get data on our own and thus will be more data type to analysis.

5. Reference

- [1] Global Positioning System Data Processing. http://bse.unl.edu/adamchuk/web_ssm/web_GPS.html
- [2] Longitude and Latitude Conversion Table. http://bse.unl.edu/adamchuk/web_ssm/web_GPS_tb.html
- [3] Covariance. <http://en.wikipedia.org/wiki/Covariance>
- [4] Covariance matrix. http://en.wikipedia.org/wiki/Covariance_matrix
- [5] Auto-correlation <http://en.wikipedia.org/wiki/Auto-correlation>