PID control - Simple tuning methods

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PID control

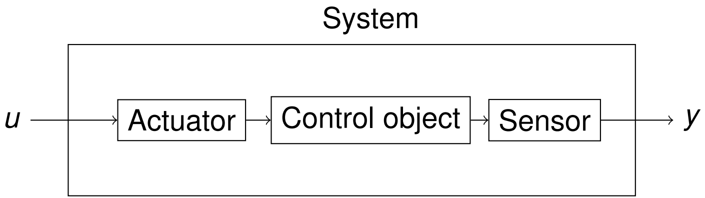
PID control

Tank process

- Tank with level control
- Two connected tanks
- Pump for in-flow of water
- Level measurements
- Valve for out-flow (disturbance)

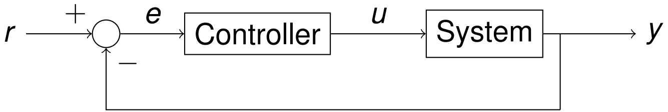
PID control

Open-loop system



- u control signal (pump voltage)
- Actuator (tubes to pump, power amplifier , in-flow)
- Control object (level in Tank with in- and out-flow)
- Sensors (pressure sensors, tubes, elektronics)
- y output (measurement of water level)

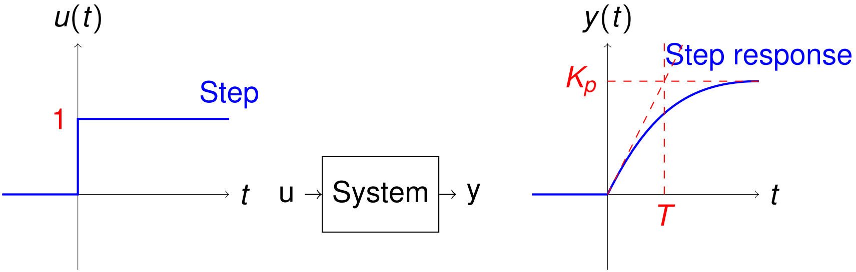
Closed-loop system



Give reference (set-point of water level in tank) r to controller in stead of pump voltage

PID control

Time constant and stationary gain

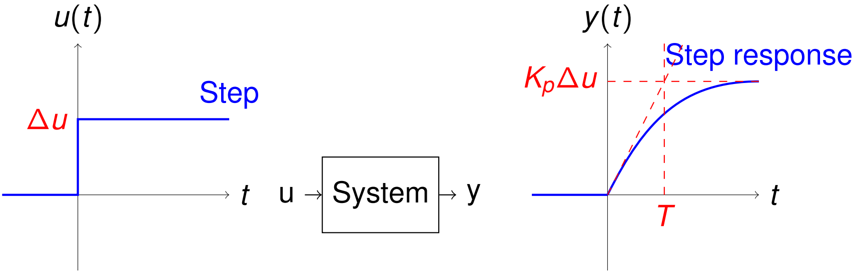


Example–Stove plate

- $u(t)$ power to stove plate
- $y(t)$ temperature on plate

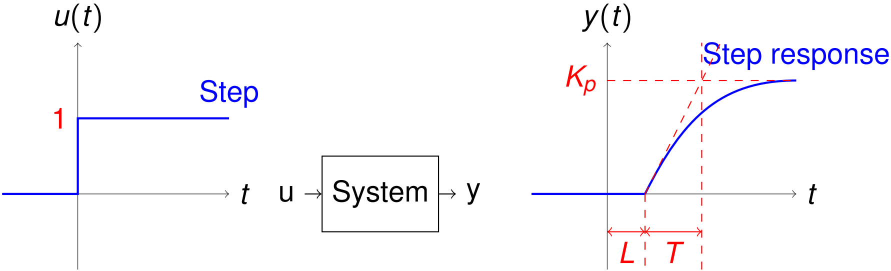
- Time constant T
- Stationary gain K_p

Time constant and stationary gain

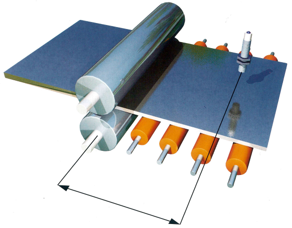


- Step response size proportional to step size
- Start step from equilibrium

Dead-time (time delay)

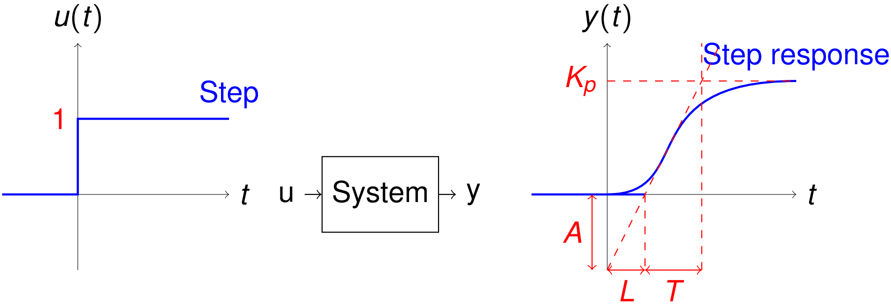


Example – roll transport time



Dead-time (time delay, lag) L

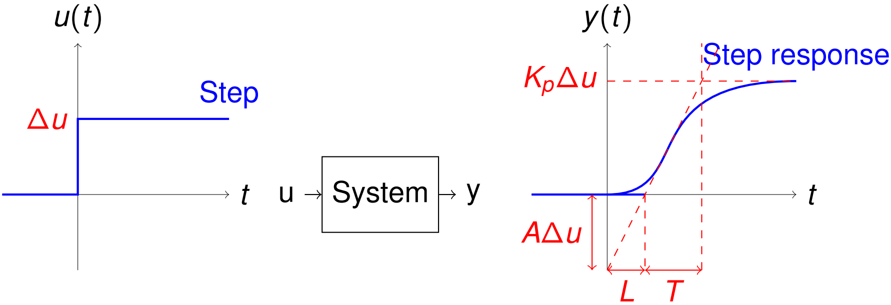
Step response model



Step response model from unit step:

- T Time constant
- K_p Stationary gain
- L Dead-time
- A see measure above

Step response model

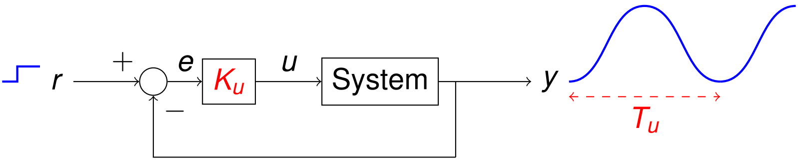


Step response model from experiment:

- T Time constant
- K_p Stationary gain
- L Dead-time
- A see figure

PID control

Self-oscillation model



Experiment

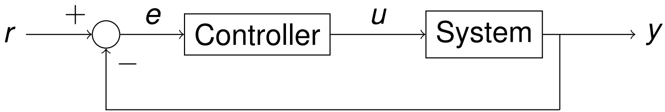
- P-control (closed-loop system!)
- Crank up gain to self-oscillation
- Reference-step starts oscillation

Self-oscillation model

- Ultimate gain K_u
- Ultimate period T_u

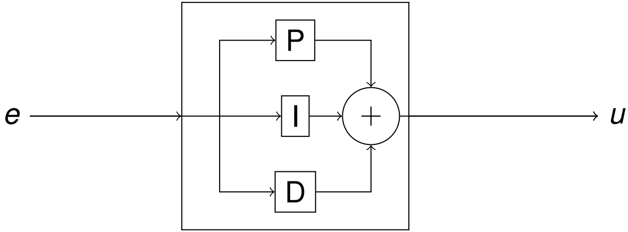
PID control

PID structure

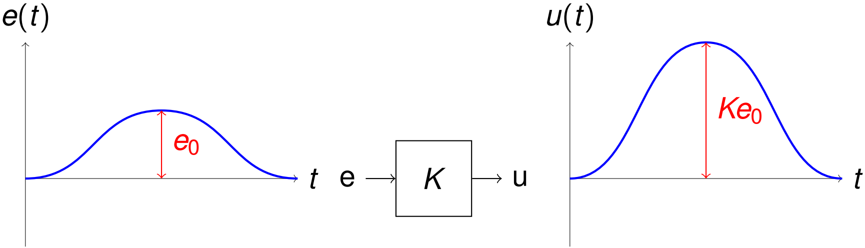


- r reference, set-point (SP)
- y output, measured signal to be regulated
- $e = r - y$ control error

PID-controller:

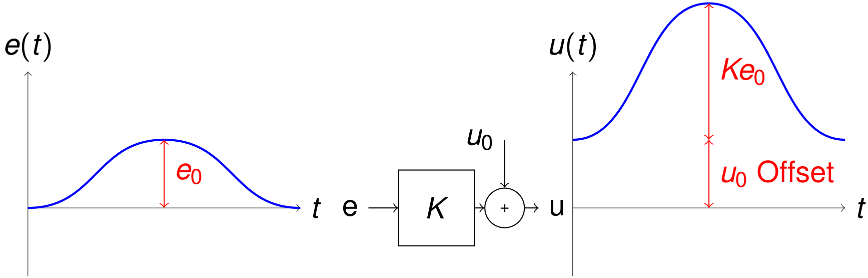


P-controller



Control signal Proportional to control error

P-controller with offset

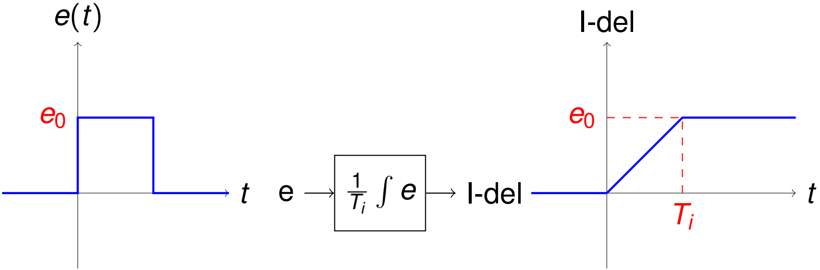


Control signal Proportional to error plus 'offset'

Example: speed control of car

u_0 given gas at control-start

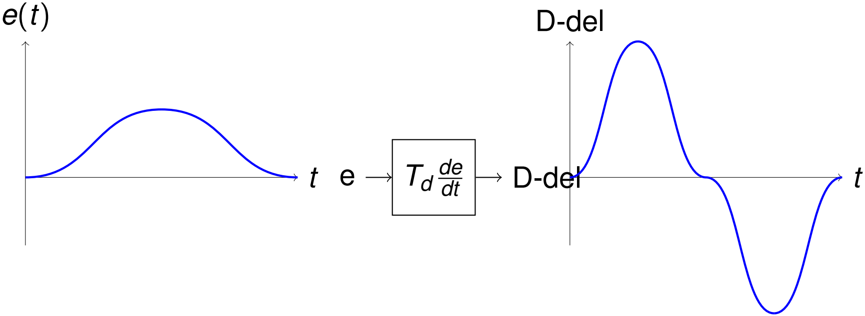
I-part (integrator)



I-part

- $\propto$ surface under $e(t)$ so far
- becomes as big as constant error e_0 in time T_i
- used to eliminate remaining error

D-part (derivator)



D-part

- $\propto$ slope on $e(t)$
- used to damp oscillations

PID-controller structure

Control signal = P + I + D

$$u(t) = K[e(t) + \frac{1}{T_i}\int^t e(s)ds + T_d\frac{de(t)}{dt}]$$

Three parameters to tune K, T_i and T_d

PID control

Ziegler-Nichols step response method

- Measure A and L from step response
- T_p expected time constant for closed-loop system

Controller	K	T_i	T_d	T_p
P	1/A			4L
PI	0.9/A	3L		5.7L
PID	1.2/A	2L	L/2	3.4L

PID control

Ziegler-Nichols self-oscillation method

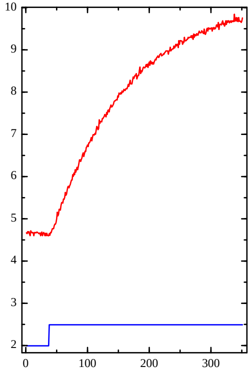
- Measure ultimate gain K_u and period T_u from experiment
- T_p expected time constant for closed-loop system

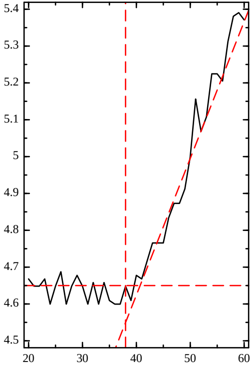
Controller	K	T_i	T_d	T_p
P	$0.5K_u$			T_u
PI	$0.4K_u$	$0.8T_u$		$1.4T_u$
PID	$0.6K_u$	$0.5T_u$	$0.125T_u$	$0.85T_u$

PID control

Step response

Step response model for the left tank

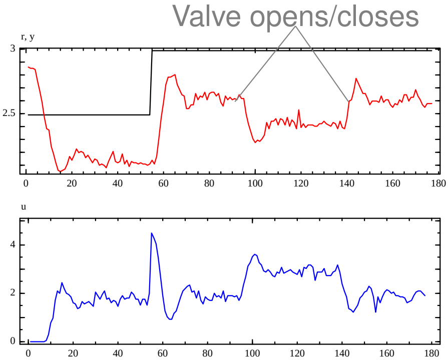




Estimated from figure:

- $A\Delta u = 0.1$, $\Delta u = 0.5$ $\Rightarrow A = 1/5$
- $L = 3$

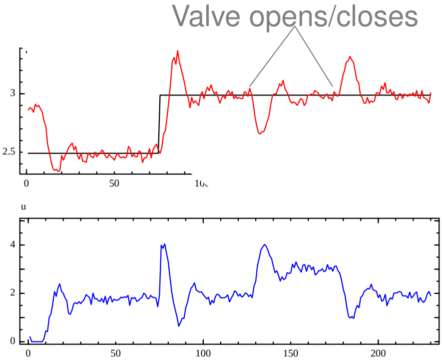
P-control of the left tank



Ziegler-Nichols P-control

- $K = 1/A = 5$
- Stationary error after valve opening even if offset is used

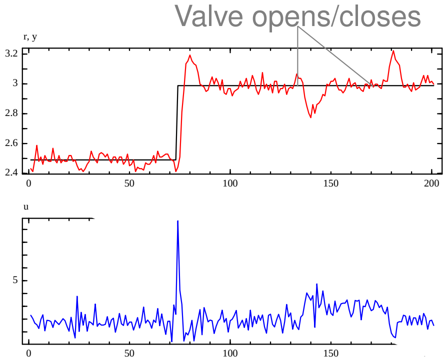
PI-control of the left tank



Ziegler-Nichols PI-control

- $K = 0.9/A = 4.5$
- $T_i = 3L = 9$
- No stationary error after valve disturbance

PID-control of the left tank

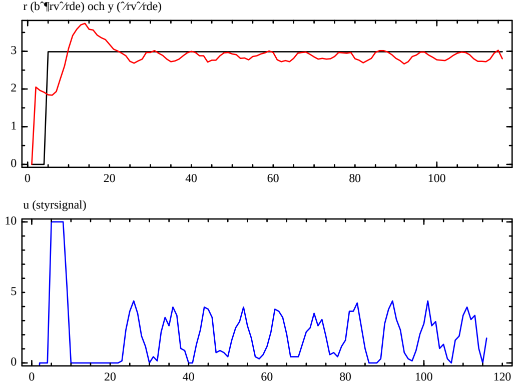


Ziegler-Nichols PID-control

- $K = 1.2/A = 6$
- $T_i = 2L = 6$
- $T_d = L/2 = 1.5$
- No stationary error after valve disturbance
- Fast and damped
- Noisier control signal

Self-oscillation experiment

Self-oscillation model for the left tank



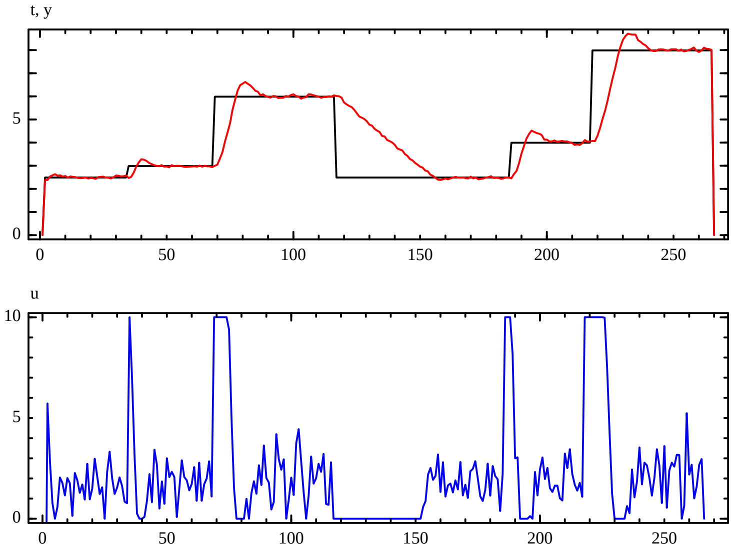
Ultimate gain and period

- $K_u = 15$ (from tuning)
- $T_u = 10$ (from figure)

Z-N PID-tuning:

$$K = 0.6K_u = 8$$
$$T_i = 0.5T_u = 5$$
$$T_d = 0.125T_u = 1.25$$

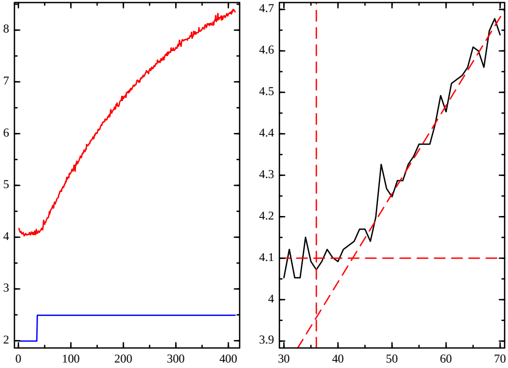
Ziegler-Nichols PID from self-oscillation model



PID control

Step response

Step response model for the right tank

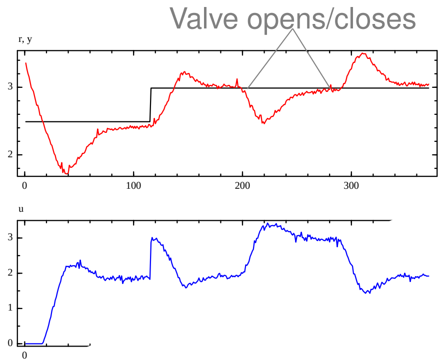


Roughly estimated (bad precision!):

- $A\Delta u \approx 0.2$, $\Delta u = 0.5$ $\Rightarrow A = 2/5$
- $L \approx 10$

From larger figure: $A\Delta u = 0.15$, $L = 8$

PI-control of the right tank

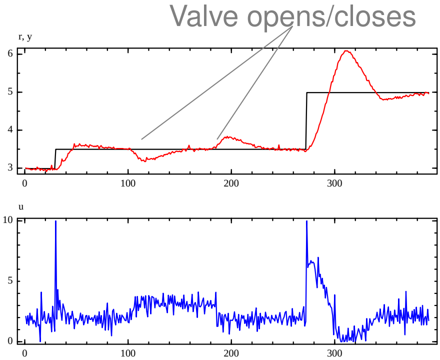


Ziegler-Nichols PI-control

- $K = 0.9/A = 2.25$
- $T_i = 3L = 30$

Disturbance eliminated in 60s
(Compare $T_p = 5.7L = 57s$)

PID-control of the right tank (step response tuning)



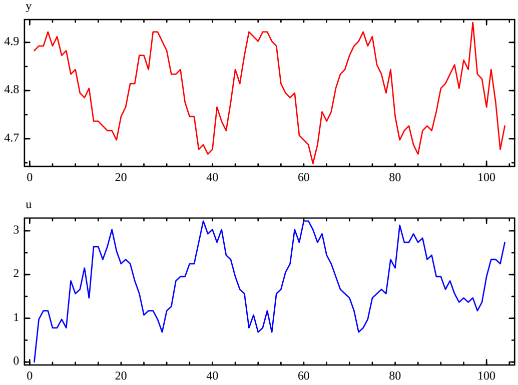
Ziegler-Nichols PID-control

- $K = 1.2/A = 3$
- $T_i = 2L = 20$
- $T_d = L/2 = 5$

Disturbance eliminated i 40s
(Compare $T_p = 3.4L = 34s$)

Self-oscillation experiment

Self-oscillation model for the right tank



Ultimate gain and period

- $K_u = 10$ (from tuning)
- $T_u = 25$ (from figure)

Z-N PID-tuning:

$$K = 0.6K_u = 6$$
$$T_i = 0.5T_u \approx 13$$
$$T_d = 0.125T_u \approx 3$$

PID-control of the right tank (self-oscillation tuning)