(1)

$f = \frac{3}{8}$; $\omega = \frac{3\pi}{4}$

a) $x_1(n) = \cos\left(2\pi \frac{3}{8} \cdot n\right) \qquad -\infty \le n \le \infty$

infinite duration, periodic signal; period $N=8$

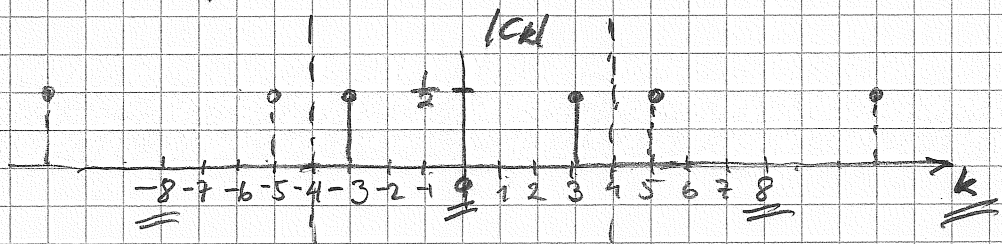
Fourier series expansion when $N=8$:

$$x_1(n) = \sum_{k=0}^{7} c_k e^{j\frac{2\pi}{8}\cdot k\cdot n} \quad ; \quad \{c_k\}_{k=0}^{7} \text{ is the freq. descr.}$$

$$= c_0 + c_1 e^{j\frac{2\pi}{8}\cdot 1\cdot n} + \dots + c_7 e^{j\frac{2\pi}{8}\cdot 7\cdot n} \qquad (1)$$

$$x_1(n) = \cos\left(2\pi\frac{3}{8}n\right) \underset{\text{Euler id}}{=} \frac{1}{2}\left[e^{j\frac{2\pi}{8}\cdot 3\cdot n} + e^{-j\frac{2\pi}{8}\cdot 3\cdot n}\right] \qquad (2)$$

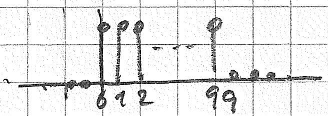
Compare (1) and (2) $\Rightarrow$ $c_3 = \frac{1}{2}$ and $c_{-3} = \frac{1}{2}$



$c_{-3} = c_5$ because $c_{k+8} = c_k$

b) $x_2(n) = w(n) \cdot \cos\left(2\pi\frac{3}{8}\cdot n\right) \longleftrightarrow \frac{1}{2}\left[W\left(f-\frac{3}{8}\right) + W\left(f+\frac{3}{8}\right)\right] = X_2(f)$

- finite duration
- aperiodic

$w(n) =$ 

"pulse"

$= u(n) - u(n-100)$

$\mathcal{Z}$

$$W(z) = \frac{1}{1-z^{-1}} - \frac{z^{-100}}{1-z^{-1}} = \frac{1-z^{-100}}{1-z^{-1}} \qquad |z|>1$$

$$W(\omega) = W(z)\big|_{z=e^{j\omega}} = \frac{1-e^{-j100\omega}}{1-e^{-j\omega}} = \frac{e^{-j50\omega}\left(e^{j50\omega}-e^{-j50\omega}\right)}{e^{-j\omega/2}\left(e^{j\omega/2}-e^{-j\omega/2}\right)}$$

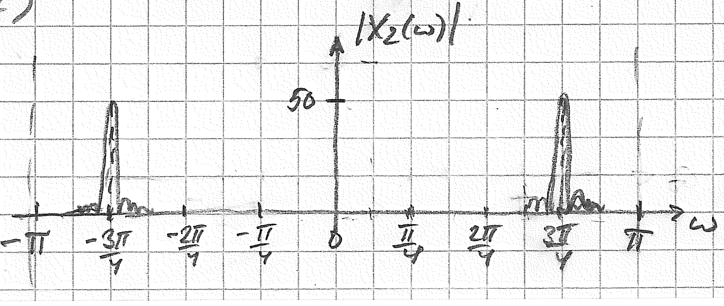
$$= e^{-j\frac{99}{2}\omega}\frac{\sin(50\omega)}{\sin(\omega/2)} \qquad \omega \neq 0$$

$$W(0) = \sum_{n=0}^{99} 1 = 100$$

zero-crossings?

$50\omega = k\cdot\pi \qquad k = \pm 1, \pm 2$

$\omega = \frac{\pi}{50}\cdot k$



②

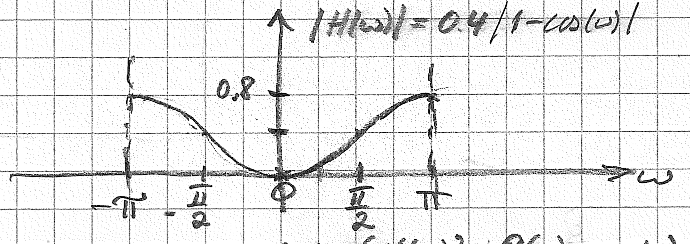
a) $h_{lp}(n) = \{\overset{-1}{0.2}, \overset{0}{\underset{\uparrow}{0.4}}, \overset{1}{0.2}\}$

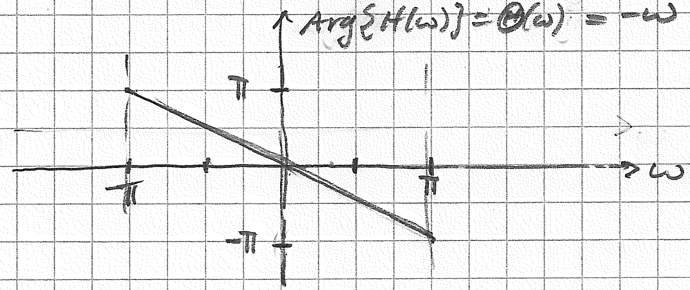
$h_{hp}(n) = (-1)^n h_{lp}(n) = \{-0.2, \underset{\uparrow}{0.4}, -0.2\}$

$$h(n) = h_{hp}(n-1)$$
$$\Downarrow \mathcal{F}$$
$$H(\omega) = e^{-j\omega} H_{hp}(\omega) \quad \text{where} \quad H_{hp}(\omega) = \sum_{n=-1}^{1} h_{hp}(n) e^{-j\omega n}$$

so $H(\omega) = e^{-j\omega}\left(-0.2 e^{j\omega} + 0.4 - 0.2 e^{-j\omega}\right)$

$= e^{-j\omega}\left(0.4 - 0.4\cos(\omega)\right) = 0.4\left(1-\cos(\omega)\right)\cdot e^{-j\omega}$





b) $x(n) = \underset{\omega_1 = 0}{1.2} + 0.8 \sin(\underset{\omega_2 = \pi/2}{\tfrac{\pi}{2} \cdot n}) \qquad -\infty < n < \infty$

$y(n) = H(0)\cdot 1.2 + |H(\tfrac{\pi}{2})| \cdot 0.8 \sin\left(\tfrac{\pi}{2}\cdot n + \Theta(\tfrac{\pi}{2})\right)$

$H(0) = 0$

$H(\tfrac{\pi}{2}) = 0.4\left(1-\cos(\tfrac{\pi}{2})\right) e^{-j\pi/2} = 0.4 e^{-j\pi/2}$

$$\begin{cases} |H(\tfrac{\pi}{2})| = 0.4 \\ \Theta(\tfrac{\pi}{2}) = -\tfrac{\pi}{2} \end{cases}$$

So $y(n) = 0.4 \cdot 0.8 \sin\left(\tfrac{\pi}{2} n - \tfrac{\pi}{2}\right) = 0.32 \sin\left(\tfrac{\pi}{2}(n-1)\right)$.

(3)

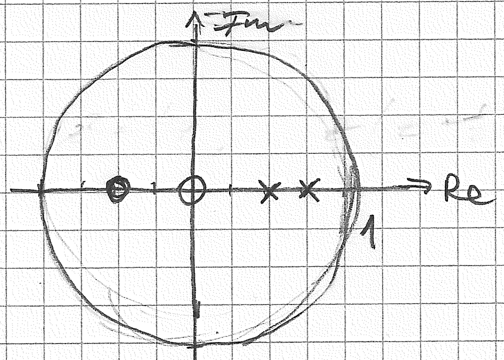
a) $h(n) = \left[5\left(\frac{3}{4}\right)^n - 4\left(\frac{1}{2}\right)^n\right] u(n)$

$$\Big\downarrow \mathcal{Z}$$

$$H(z) = 5\,\frac{1}{(1-\frac{3}{4}z^{-1})} - 4\cdot\frac{1}{(1-\frac{1}{2}z^{-1})}$$

$$= \frac{5(1-\frac{1}{2}z^{-1}) - 4(1-\frac{3}{4}z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \underline{\frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})}} \begin{matrix} \cdot z^2 \\ \cdot z^2 \end{matrix}$$

$$= \frac{z\,(z+\frac{1}{2})}{(z-\frac{3}{4})(z-\frac{1}{2})} \Rightarrow \begin{cases} \text{zeros} & z_1 = 0 \quad z_2 = -\frac{1}{2} \\ \text{poles} & p_1 = \frac{3}{4} \quad p_2 = \frac{1}{2} \end{cases}$$



The system is stable!
causal system
and $|p_k| < 1 \quad k = 1, 2$

b) $H(z) = \frac{Y(z)}{X(z)} = \frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})}$

$$Y(z)(1-\tfrac{5}{4}z^{-1}+\tfrac{3}{8}z^{-1}) = X(z)(1+\tfrac{1}{2}z^{-1})$$

$$\Big\downarrow \mathcal{Z}^{-1}$$

$$\boxed{y(n) - \tfrac{5}{4}y(n-1) + \tfrac{3}{8}y(n-2) = x(n) + \tfrac{1}{2}x(n-1)}$$ diff eq.

$$\Big\downarrow \mathcal{Z}^{+}$$

$$Y^+(z) - \tfrac{5}{4}\left(z^{-1}Y^+(z) + y(-1)\right) + \tfrac{3}{8}\left(z^{-2}Y^+(z) + y(-2) + y(-1)\cdot z^{-1}\right) = X(z)(1+\tfrac{1}{2}z^{-1})$$

$$Y^+(z) = \frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})}X(z) + \frac{\frac{5}{4}y(-1) - \frac{3}{8}(y(-2)+y(-1)z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})}$$

$$x(n) = u(n) \longrightarrow X(z) = \frac{1}{1-z^{-1}}$$

$$y(-1) = 0\ ;\ y(-2) = 1$$

$$Y^+(z) = \frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})(1-z^{-1})} + \frac{-\frac{3}{8}}{(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2})}$$

③ kont.

$$Y^+(z) = Y_1^+(z) + Y_2^+(z)$$

$$Y_1^+(z) = \frac{\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2}\right)\left(1-z^{-1}\right)} \begin{matrix} \times z^3 \\ \times z^3 \end{matrix} = \frac{z^2\left(z+\frac{1}{2}\right)}{\left(z^2-\frac{5}{4}z+\frac{3}{8}\right)(z-1)}$$

$$\frac{Y_1^+(z)}{z} = \frac{z\left(z+\frac{1}{2}\right)}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{2}\right)(z-1)} = \frac{A_1}{(z-3/4)} + \frac{B_1}{\left(z-\frac{1}{2}\right)} + \frac{C_1}{(z-1)}$$

$$\begin{cases} A_1 = \dfrac{\frac{3}{4}\left(\frac{3}{4}+\frac{1}{2}\right)}{\left(\frac{3}{4}-\frac{1}{2}\right)\left(\frac{3}{4}-1\right)} = \dfrac{\frac{3}{4}\cdot\frac{5}{4}}{\frac{1}{4}\cdot\left(-\frac{1}{4}\right)} = -\dfrac{15\cdot 16}{16\cdot 1} = -15 \\[2ex] B_1 = \dfrac{\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\right)}{\left(\frac{1}{2}-\frac{3}{4}\right)\left(\frac{1}{2}-1\right)} = \dfrac{\frac{1}{2}}{\left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right)} = \dfrac{1\cdot 8}{2\cdot 1} = 4 \\[2ex] C_1 = \dfrac{1\left(1+\frac{1}{2}\right)}{\left(1-\frac{3}{4}\right)\left(1-\frac{1}{2}\right)} = \dfrac{\frac{3}{2}}{\frac{1}{4}\cdot\frac{1}{2}} = \dfrac{3\cdot 8}{2\cdot 1} = 12 \end{cases}$$

$\downarrow \mathcal{Z}^{-1}$

$$y_1(n) = \left[-15\left(\frac{3}{4}\right)^n + 4\left(\frac{1}{2}\right)^n + 12\right]u(n)$$

$$Y_2^+(z) = \frac{-\frac{3}{8}}{\left(1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2}\right)} \begin{matrix} \times z^2 \\ \times z^2 \end{matrix} = \frac{-\frac{3}{8}z^2}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{2}\right)}$$

$$\frac{Y_2^+(z)}{z} = \frac{-\frac{3}{8}z}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{2}\right)} = \frac{A_2}{(z-3/4)} + \frac{B_2}{\left(z-\frac{1}{2}\right)}$$

$$\begin{cases} A_2 = \dfrac{-\frac{3}{8}\cdot\frac{3}{4}}{\frac{3}{4}-\frac{1}{2}} = \dfrac{-\frac{9}{32}}{\frac{1}{4}} = -\dfrac{9\cdot 4}{32\cdot 1} = -\dfrac{9}{8} \\[2ex] B_2 = \dfrac{-\frac{3}{8}\cdot\frac{1}{2}}{\frac{1}{2}-\frac{3}{4}} = \dfrac{-\frac{3}{16}}{-\frac{1}{4}} = \dfrac{3\cdot 4}{16\cdot 1} = \dfrac{3}{4} \end{cases}$$

$\downarrow \mathcal{Z}^{-1}$

$$y_2(n) = \left(-\frac{9}{8}\left(\frac{3}{4}\right)^n + \frac{3}{4}\left(\frac{1}{2}\right)^n\right)u(n)$$

$$y(n) = y_1(n) + y_2(n)$$

4

$$h(n) = \{\underset{\uparrow}{\overset{0}{1}}, \overset{1}{0.3}, \overset{2}{0}, \overset{3}{0.1}\}$$

a)

$$x(n) = \{\underset{\uparrow}{\overset{0}{3}}, \overset{1}{1}, \overset{2}{2}, \overset{3}{1}\}$$

$y(n) = x(n) * h(n)$ by graphical solution:

3	1	2	1	$x(k)$

0.1	0	0.3	1 $\rightarrow$	$h(n-k)$

3	1.9	2.3	1.9	0.4	0.2	0.1	$y(n)$

$$y(n) = \{\underset{\uparrow}{3}, 1.9, 2.3, 1.9, 0.4, 0.2, 0.1\}$$

b) $N \geq 7$ to achieve $y'(n) = y(n)$

```
X = fft(x, 7);
H = fft(h, 7);
Y = X.*H;
y = ifft(Y, 7);
```

c) $Y'(k) = \underset{N}{\mathrm{DFT}}[y(n)] \longleftrightarrow y'(n) = \sum_{l=-\infty}^{\infty} y(n - l \cdot N)$ $\quad n = 0, 1, 2, \dots N-1$

$k = 0, 1, 2, \dots, N-1$

periodic extension of $y(n)$ with a period of N.

$$N = 4 \Rightarrow \dots + y(n+4) + y(n) + y(n-4) + \dots$$

Graphical solution:

-4	-3	-2	-1	0	1	2	3	4	5	6
				3	1.9	2.3	1.9	0.4	0.2	0.1
3	1.9	2.3	1.9	0.4	0.2	0.1				
								3	1.9	2.3	1.9	0.4	0.2	0.1

3.4	2.1	2.4	1.9

$$y'(n) = \{\underset{\uparrow}{3.4}, 2.1, 2.4, 1.9\}$$