Digital Control Exercise 6a

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Problem 1 Sampling

a) Down

The continuous transfer function is:

$$G(s) = \frac{-s}{s^2 + ds + g_l} = -\frac{s}{s^2 + 0.1s + 0.981}$$

The transfer function can be converted into:

$$G(s) = \frac{-s}{[s - (-0.05 + 0.9872i)][s - (-0.05 - 0.9872i)]}$$

$$= \frac{-0.5 - 0.0253i}{s - (-0.05 + 0.9872i)} + \frac{-0.5 + 0.0253i}{s - (-0.05 - 0.9872i)}$$

Assume $G_1(s) = \frac{1}{s-(-0.05+0.9872i)}$, $G_2(s) = \frac{1}{s-(-0.05-0.9872i)}$

$$H_1(q^{-1}) = \frac{\frac{e^{-0.05+0.9872i} - 1}{-0.05 + 0.9872i} q^{-1}}{1 - e^{-0.05+0.9872i} q^{-1}} = \frac{(0.8258 + 0.4409i)q^{-1}}{1 - (0.5226 + 0.7948i)q^{-1}}$$

$$H_2(q^{-1}) = \frac{\frac{e^{-0.05-0.9872i} - 1}{-0.05 - 0.9872i} q^{-1}}{1 - e^{-0.05-0.9872i} q^{-1}} = \frac{(0.8258 - 0.4409i)q^{-1}}{1 - (0.5226 - 0.7948i)q^{-1}}$$

$$H(q^{-1}) = (-0.5 - 0.0253i)H_1(q^{-1}) + (-0.5 + 0.0253i)H_2(q^{-1})$$

$$= (-0.5 - 0.0253i)\frac{(0.8258 + 0.4409i)q^{-1}}{1 - (0.5226 + 0.7948i)q^{-1}} + (-0.5 + 0.0253i)\frac{(0.8258 - 0.4409i)q^{-1}}{1 - (0.5226 - 0.7948i)q^{-1}}$$

$$= \frac{(-0.4017 - 0.2413i)q^{-1}}{1 - (0.5226 + 0.7948i)q^{-1}} + \frac{(-0.4017 + 0.2413i)q^{-1}}{1 - (0.5226 - 0.7948i)q^{-1}}$$

$$= \frac{-0.8035q^{-1} + 0.8035q^{-2}}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$

b) Up

The continuous transfer function is:

$$G(s) = \frac{s}{s^2 + ds - g_l} = \frac{s}{s^2 + 0.1s - 0.981}$$

The transfer function can be converted into:

$$G(s) = \frac{s}{(s - 0.9417)(s + 1.0417)} = \frac{0.4748}{s - 0.9417} + \frac{0.5252}{s + 1.0417}$$

Assume $G_1(s) = \frac{0.4748}{s-0.9417}$, $G_2(s) = \frac{0.5252}{s+1.0417}$

$$H_1(q^{-1}) = \frac{\frac{e^{0.9417} - 1}{0.9417} q^{-1}}{1 - e^{0.9417} q^{-1}} = \frac{1.6612q^{-1}}{1 - 2.5643q^{-1}}$$

$$H_2(q^{-1}) = \frac{\frac{e^{-1.0417} - 1}{-1.0417} q^{-1}}{1 - e^{-1.0417} q^{-1}} = \frac{0.6212q^{-1}}{1 - 0.3529q^{-1}}$$

$$H(q^{-1}) = 0.4748H_1(q^{-1}) + 0.5252H_2(q^{-1})$$

$$= 0.4748\frac{1.6612q^{-1}}{1 - 2.5643q^{-1}} + 0.5252\frac{0.6212q^{-1}}{1 - 0.3529q^{-1}}$$

$$= \frac{0.7887q^{-1}}{1 - 2.5643q^{-1}} + \frac{0.3263q^{-1}}{1 - 0.3529q^{-1}}$$
$$= \frac{1.1150q^{-1} - 1.1150q^{-2}}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$

Problem 2 Estimation

a)

Here is the variable `samples` I got:

60823.7250	-0.0896	-0.3283	60879.8850	0.0112	0.3122
60824.8480	-0.0672	-0.2814	60881.0080	0.1792	0.3858
60825.9720	0.1120	0.0549	60882.1320	-0.0784	-0.0914
60827.0950	0.1344	0.1506	60883.2550	-0.0896	-0.2283
60828.2180	-0.1008	0.0437	60884.3780	0.1232	-0.0797
60829.3410	-0.1008	0.0906	60885.5010	0.2128	-0.0341
60830.4640	0.0784	0.0580	60886.6240	-0.0112	-0.0489
60831.5880	0.1120	-0.1775	60887.7480	-0.1792	0.1629
60832.7110	-0.0896	-0.2444	60888.8710	0.1008	0.3356
60833.8340	-0.1008	0.1082	60889.9940	0.1792	-0.0608
60834.9570	0.1456	0.3368	60891.1170	-0.0784	-0.4393
60836.0800	0.0224	0.0016	60892.2400	-0.1680	-0.1411
60837.2040	-0.1680	-0.2206	60893.3640	0.1120	0.3656
60838.3270	-0.0224	-0.0257	60894.4870	0.0560	0.2355
60839.4500	0.1680	0.0742	60895.6100	-0.1344	-0.1047
60840.5730	0.0896	-0.0818	60896.7330	0.0000	-0.1405
60841.6960	-0.1568	-0.0887	60897.8570	0.1232	-0.1156
60842.8200	-0.0560	0.2050	60898.9800	0.1344	-0.0827
60843.9430	0.1680	0.1944	60900.1030	-0.1008	0.0139
60845.0660	0.0224	-0.2067	60901.2260	-0.0672	0.2752
60846.1890	-0.1344	-0.2536	60902.3490	0.0784	0.2161
60847.3120	-0.1568	0.1060	60903.4730	0.1568	-0.1819
60848.4360	0.1120	0.3484	60904.5960	-0.0784	-0.4231
60849.5590	0.1232	-0.0020	60905.7190	-0.1680	-0.0327
60850.6820	-0.0672	-0.3450	60906.8420	0.1456	0.4425
60851.8050	-0.1792	-0.1480	60907.9650	0.0896	0.1888
60852.9280	0.0784	0.2889	60909.0890	-0.0896	-0.2147
60854.0520	0.1904	0.1880	60910.2120	-0.1904	-0.2037
60855.1750	-0.1008	-0.2105	60911.3350	0.0672	0.1155
60856.2980	-0.0896	-0.1255	60912.4580	0.1904	0.0810
60857.4210	0.1232	0.0987	60913.5810	0.0000	-0.1577
60858.5440	0.2016	0.0225	60914.7050	-0.1792	-0.0636
60859.6680	-0.0896	-0.1519	60915.8280	0.0000	0.2494
60860.7910	-0.1232	0.0789	60916.9510	0.1680	0.1433
60861.9140	0.0672	0.2552	60918.0740	-0.0224	-0.2525
60863.0370	0.2128	-0.0014	60919.1970	-0.1120	-0.2083
60864.1600	-0.0448	-0.3641	60920.3210	0.1232	0.1374
60865.2840	-0.1904	-0.1269	60921.4440	0.1904	0.1210
60866.4070	0.1008	0.3567	60922.5670	0.0000	-0.0959
60867.5300	0.0448	0.2108	60923.6900	-0.1680	-0.0412
60868.6530	-0.1568	-0.1189	60924.8130	0.0336	0.2042
60869.7760	-0.0672	-0.1215	60925.9370	0.2016	0.0638
60870.9000	0.0896	-0.0501	60927.0600	0.0112	-0.2825
60872.0230	0.1904	-0.0683	60928.1830	-0.1792	-0.1638
60873.1460	-0.0224	-0.1080	60929.3060	0.0112	0.2831
60874.2690	-0.1904	0.1330	60930.4290	0.1904	0.2568
60875.3920	0.0448	0.3629	60931.5530	-0.0112	-0.1928
60876.5160	0.1904	0.0261	60932.6760	-0.0784	-0.2485
60877.6390	0.0000	-0.4402	60933.7990	0.0112	0.0324
60878.7620	-0.1792	-0.2771	60934.9220	0.0112	0.1826

Next is to use the extracted data samples for least-square estimation. As is known from the physical background of the system, the model should be a second-order system. Assume:

$$y(k) = \theta_1 y(k-2) + \theta_2 y(k-1) + \theta_3 u(k-2) + \theta_4 u(k-1)$$

Write it into the matrix form:

$$\begin{bmatrix} y(1) & y(2) & u(1) & u(2) \\ y(2) & y(3) & u(2) & u(3) \\ \vdots & \vdots & \vdots & \vdots \\ y(98) & y(99) & u(98) & u(99) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} y(3) \\ y(4) \\ \vdots \\ y(100) \end{bmatrix} \quad (W\hat{\theta} = Y)$$

Using the least-square method, the optimal solution should be:

$$\hat{\theta} = (W^T W)^{-1} W^T Y$$

Code is shown below:

```
s=samples;
y=s(3:100,3); y1=s(1:98,3); y2=s(2:99,3); u1=s(1:98,2); u2=s(2:99,2);
y=y-mean(y); y1=y1-mean(y1); y2=y2-mean(y2); u1=u1-mean(u1); u2=u2-mean(u2);
W = [y1 y2 u1 u2]; th = (W'*W)\W'*y
```

The result I got is: $th = \begin{bmatrix} -0.9065 \\ 0.8704 \\ 0.7224 \\ -0.8763 \end{bmatrix}$, which means the approximate linear model for this system is:

$$y(k) = -0.9065y(k-2) + 0.8704y(k-1) + 0.7224u(k-2) - 0.8763u(k-1)$$

Write it into the transfer function form:

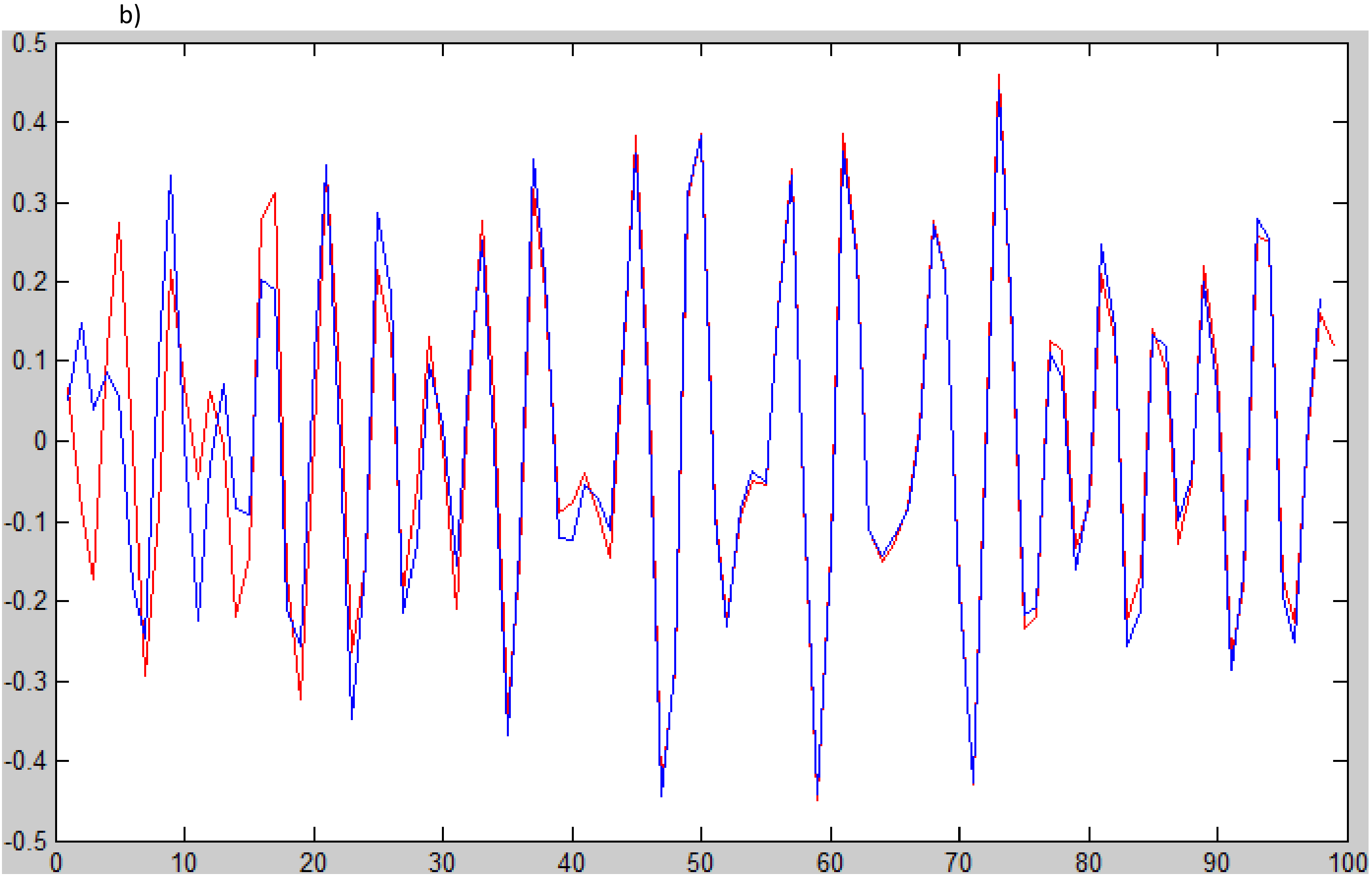
$$G(q^{-1}) = \frac{-0.8763q^{-1} + 0.7224q^{-2}}{1 - 0.8704q^{-1} + 0.9065q^{-2}}$$

The transfer function I calculated in Problem 1a is:

$$H(q^{-1}) = \frac{-0.8035q^{-1} + 0.8035q^{-2}}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$

Comparing with those two, we can see that the difference between them is not small except the coefficient of q^{-2} in A-polynomial.

b)



The blue curve is the real response and the red one the estimated model response. We can see at first, the difference between the two curves is great but later the two curves are quite close with each other.

Problem 3 Control design

Down a)

In problem 1a, the transfer function we got is:

$$H(q^{-1}) = \frac{-0.8035q^{-1} + 0.8035q^{-2}}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$

So $A = [1, -1.0451, 0.9048], B = [0, -0.8035, 0.8035]$. Enter the system to `polp.sq` and move all the poles to the origin point to get $A_c = 1$. The result we get is:

$$R = [1 \quad 0.1632], \quad S = [-1.0976 \quad -0.1838]$$

Down b)

In problem 2a, the transfer function we got is:

$$G(q^{-1}) = \frac{-0.8763q^{-1} + 0.7224q^{-2}}{1 - 0.8704q^{-1} + 0.9065q^{-2}}$$

So $A = [1, -0.8704, 0.9065], B = [0, -0.8763, 0.7224]$. Enter the system to `polp.sq` and move all the poles to the origin point to get $A_c = 1$. The result we get is:

$$R = [1 \quad 0.1632], \quad S = [-1.0976 \quad -0.1838]$$

Up c)

In problem 1b, the transfer function we got is:

$$H(q^{-1}) = \frac{1.1150q^{-1} - 1.1150q^{-2}}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$

So $A = [1, -2.9172, 0.9048], B = [0, 1.1150, -1.1150]$. Enter the system to `polp.sq` and move all the poles to the origin point to get $A_c = 1$. The result we get is:

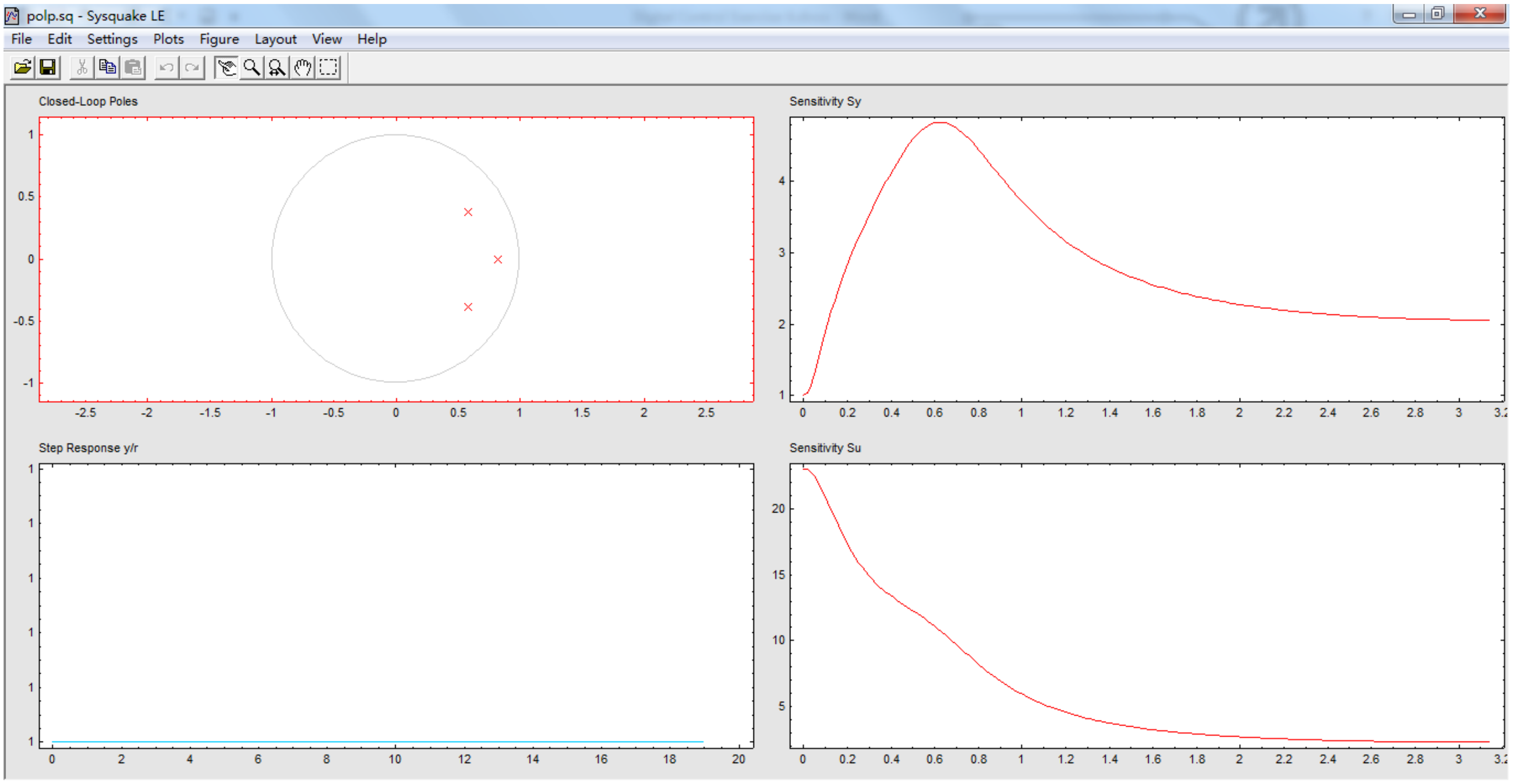
$$R = [1 \quad -1.9878], \quad S = [4.3991 \quad -1.6131]$$

Up d)

In problem 1b, the transfer function we got is:

$$H(q^{-1}) = \frac{1.1150q^{-1} - 1.1150q^{-2}}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$

So $A = [1, -2.9172, 0.9048], B = [0, 1.1150, -1.1150]$. Enter the system to `polp.sq` and move all the poles so that minimizes $\|S_y\|_\infty$.



The result we get is:

$$R = [1 \quad -1.0558], \quad S = [1.7867 \quad -0.5017]$$

Problem 4 Implementation and evaluation

Down a)

From the problem 3, we know

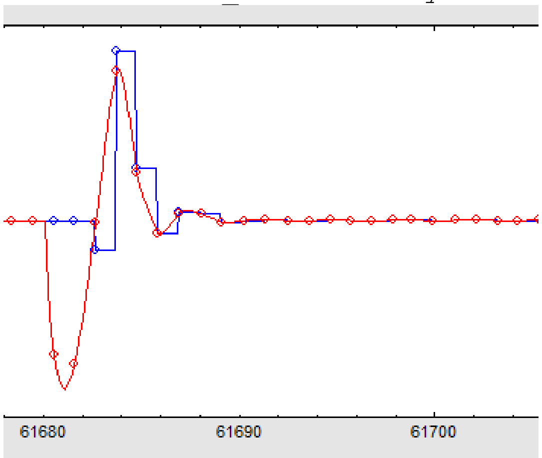
$$R = [1 \quad 0.1632], \ \ S = [-1.0976 \quad -0.1838]$$

So the feedback control part should be:

$$(1 + 0.1632q^{-1})u(k) = -(-1.0976 - 0.1838q^{-1})y(k)$$
$$u(k) = -0.1632u(k-1) + 1.0976y(k) + 0.1838y(k-1)$$

So the code should be:

```
u = -0.1632*u_o + 1.0976*y + 0.1838*y_o;
```



(You can check by running the file PendulumDamp_3ab.sq)

Down b)

This is almost the same with Down a).

Down c)

From the problem 3, we know

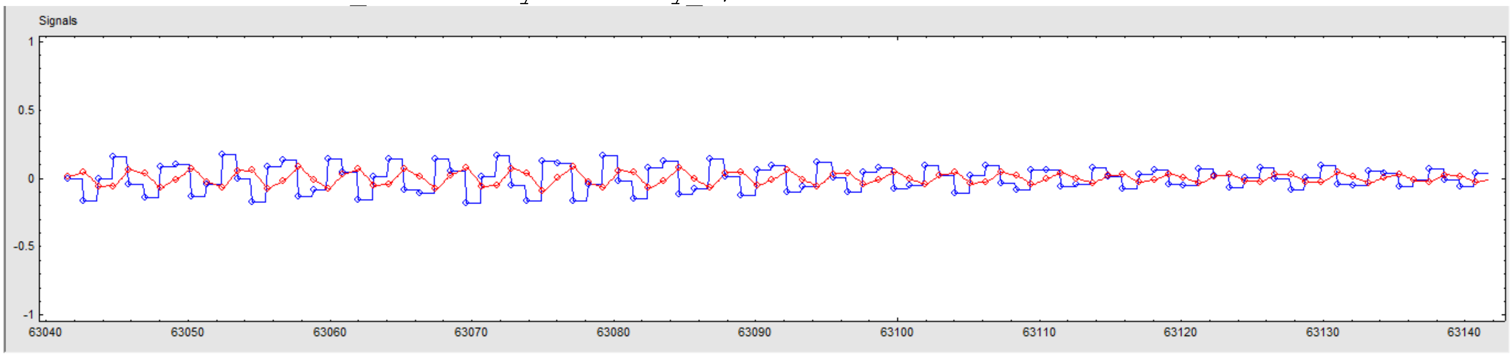
$$R = [1 \quad -1.9878], \ \ S = [4.3991 \quad -1.6131]$$

So the feedback control part should be:

$$(1 - 1.9878q^{-1})u(k) = -(4.3991 - 1.6131q^{-1})y(k)$$
$$u(k) = 1.9878u(k-1) - 4.3991y(k) + 1.6131y(k-1)$$

So the code should be:

```
u=1.9878*u_o-4.3991*y+1.6131*y_o;
```



(You can check by running the file PendulumDamp_3c.sq)

Down d)

From the problem 3, we know

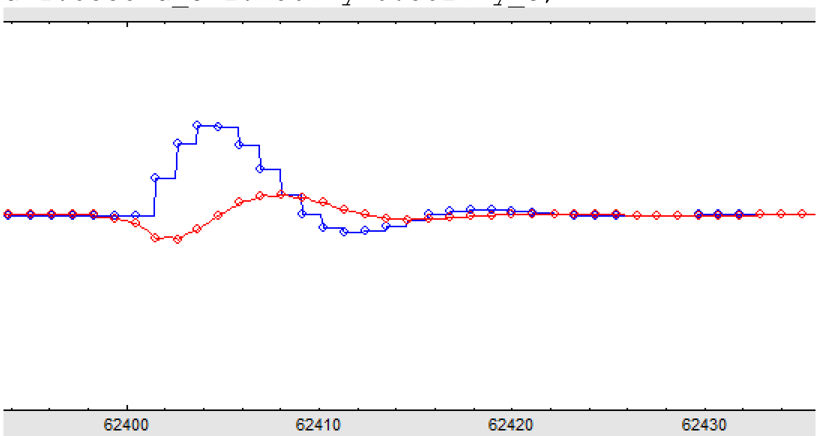
$$R = [1 \quad -1.0558], \quad S = [1.7867 \quad -0.5017]$$

So the feedback control part should be:

$$(1 - 1.0558q^{-1})u(k) = -(1.7867 - 0.5017q^{-1})y(k)$$
$$u(k) = 1.0558u(k-1) - 1.7867y(k) + 0.5017y(k-1)$$

So the code should be:

```
u=1.0558*u_o-1.7867*y+0.5017*y_o;
```



(You can check by running the file PendulumDamp_3d.sq)

We can see that the Dead-beat tuning controller for 'Down' works quite well. But the Dead-beat tuning controller for 'Up' works not well, because the system oscillates seriously and it will take a very long time for the pendulum to reach a balance state. Comparing with the Dead-beat tuning method, the tuning method based on the sensitivity function of Sy performs much better. We can see that the system can reach the balance state much faster without so long time of oscillation.

Digital Control Exercise 6b

Problem 1 Control design

Down a)

From the previous exercise, we know the sampled model is:

$$H(q^{-1}) = \frac{-0.8035q^{-1} + 0.8035q^{-2}}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$

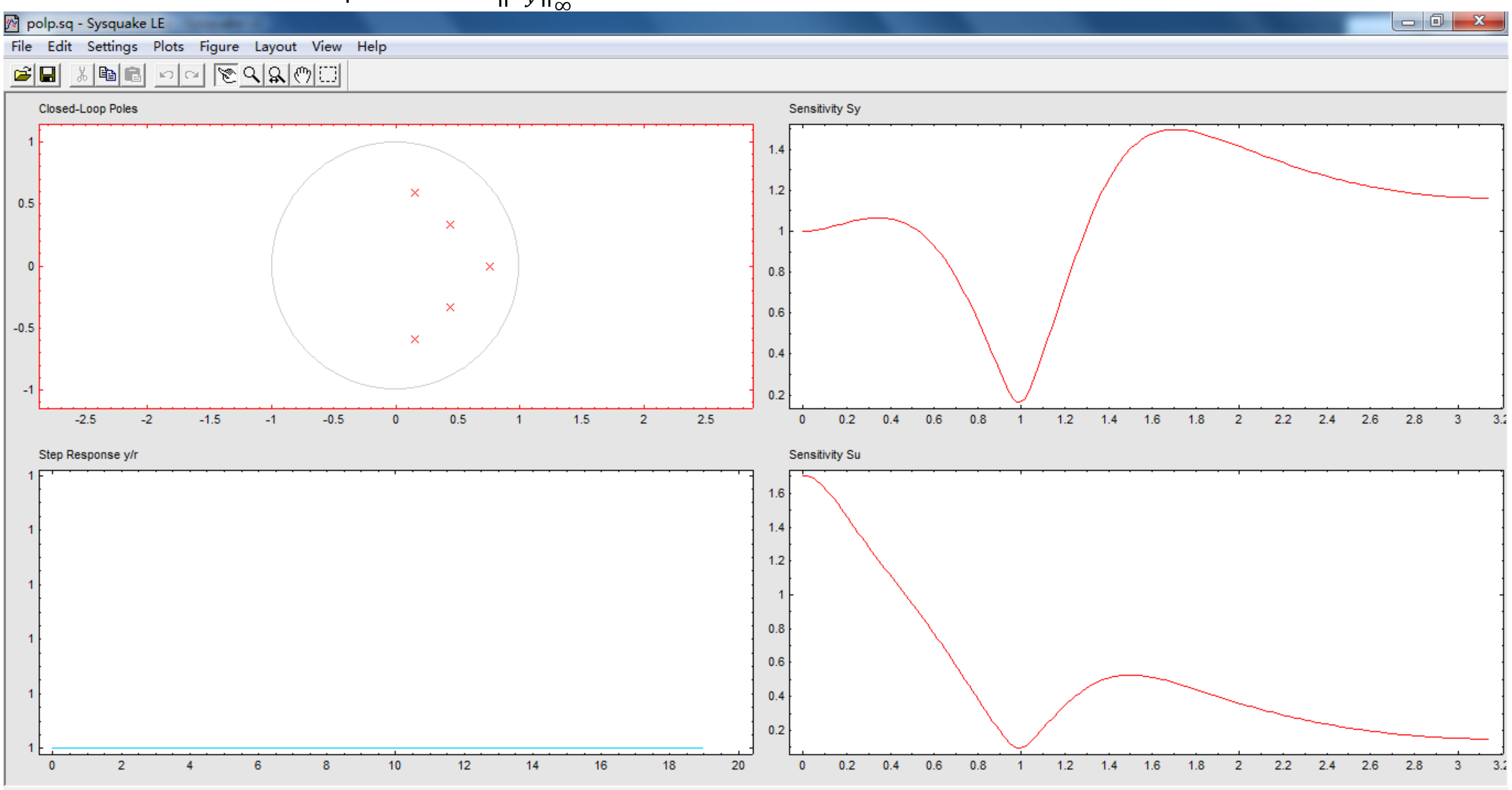
Now since the pendulum should follow the reference position

$$u = (1 - q^{-1})v$$
$$Ay = Bu = B(1 - q^{-1})v$$

So the new sampled model should be:

$$G(q^{-1}) = \frac{B(1 - q^{-1})}{A} = \frac{(-0.8035q^{-1} + 0.8035q^{-2})(1 - q^{-1})}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$
$$G(q^{-1}) = \frac{-0.8035q^{-1} + 1.6070q^{-2} - 0.8035q^{-3}}{1 - 1.0451q^{-1} + 0.9048q^{-2}}$$

So $A = [1, -1.0451, 0.9048, 0]$, $B = [0, -0.8035, 1.6070, -0.8035]$. Enter the system to `polp.sq` and move the poles so that $\|S_y\|_\infty < 2$.



The result I get is:

```
R = [1, -1.205227573785277, 0.333523029149839];
S = [-0.379683429745841, 5.377649837140578e-2, 0.107155478091893];
```

Then the T-polynomial is:

$$T = R(1) = 1 - 1.2052 + 0.3335 = 0.1283$$

Up b)

From the previous exercise, we know the sampled model is:

$$H(q^{-1}) = \frac{1.1150q^{-1} - 1.1150q^{-2}}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$

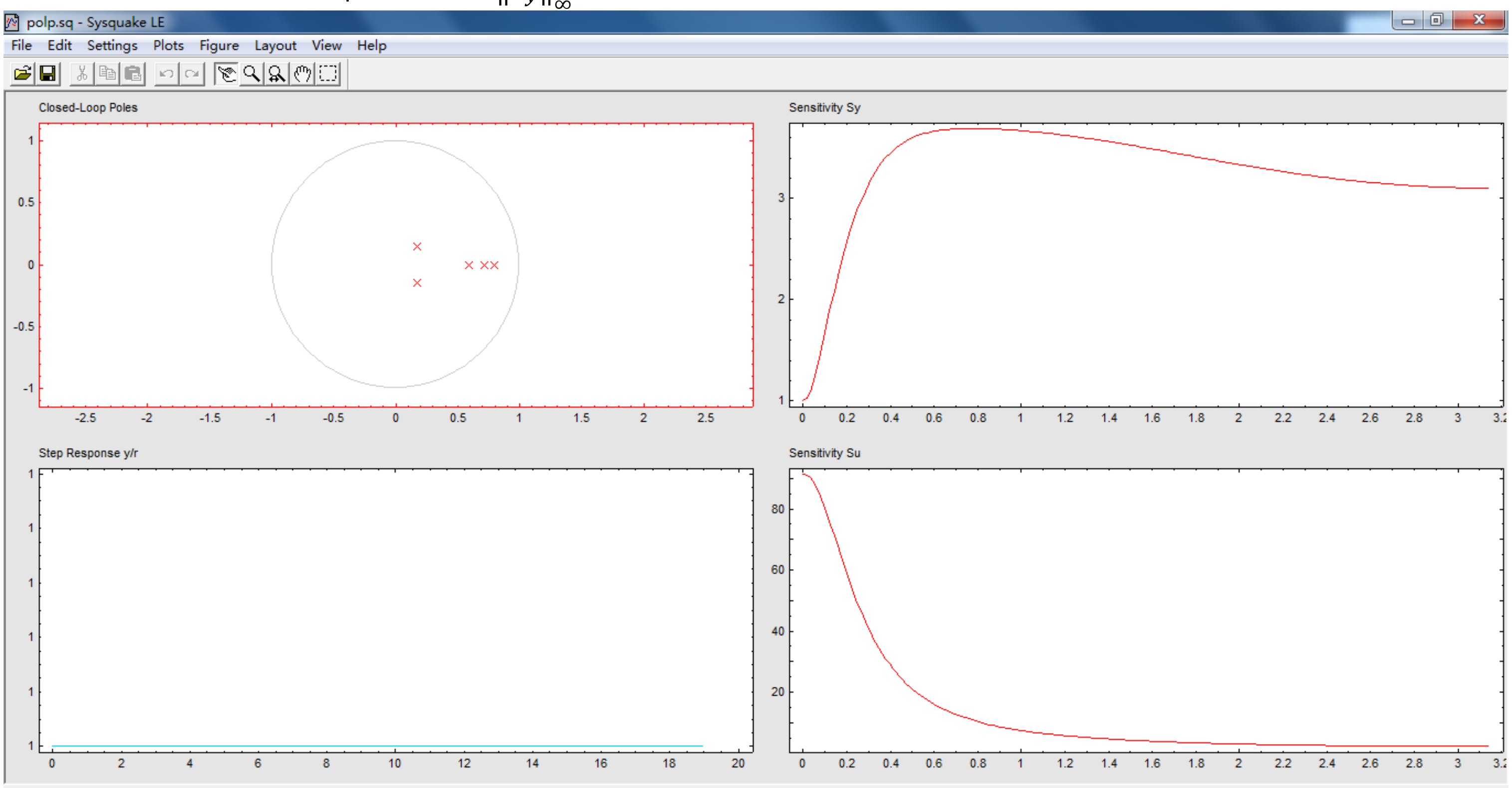
Now since the pendulum should follow the reference position

$$u = (1 - q^{-1})v$$
$$Ay = Bu = B(1 - q^{-1})v$$

So the new sampled model should be:

$$G(q^{-1}) = \frac{B(1 - q^{-1})}{A} = \frac{(1.1150q^{-1} - 1.1150q^{-2})(1 - q^{-1})}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$
$$G(q^{-1}) = \frac{1.1150q^{-1} - 2.2300q^{-2} + 1.1150q^{-3}}{1 - 2.9172q^{-1} + 0.9048q^{-2}}$$

So $A = [1, -2.9172, 0.9048, 0]$, $B = [0, 1.1150, -2.2300, 1.1150]$. Enter the system to `polp.sq` and move the poles so that $\|S_y\|_\infty < 4$.



The result we get is:

```
R = [1, -2.191611659843037, 1.174471268501992];
S = [2.399126574105173, -0.815918501363259, -1.533614335334607e-2];
```

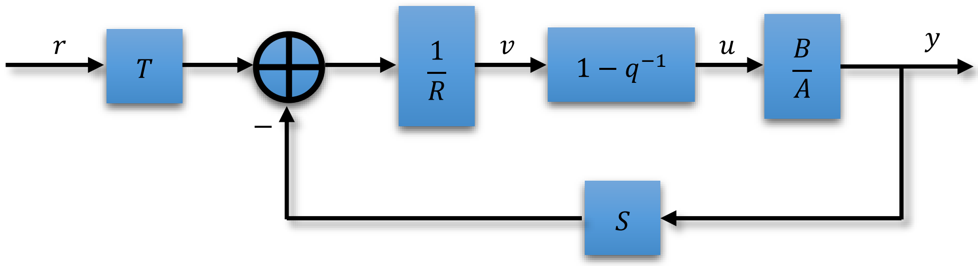
So $A_c = AR + BS = 1 - 2.434q^{-1} + 2.213q^{-2} - 0.9318q^{-3} + 0.1872q^{-4} - 0.0171q^{-5}$
$= [1 - 0.7904q^{-1}][1 - 0.7200q^{-1}][1 - 0.5857q^{-1}][1 - (0.1689 + 0.1509i)q^{-1}][1 - (0.1689 - 0.1509)q^{-1}]$

Obviously, the slowest pole in A_c is $\lambda_s = 0.7904$. So the T-polynomial is:

$$T = R(1)\frac{1 - \lambda_s q^{-1}}{1 - \lambda_s} = (1 - 2.1916 + 1.1745)\frac{1 - 0.7904q^{-1}}{1 - 0.7904} = -0.0818 + 0.0646q^{-1}$$

Problem 2 Implementation and verification

The structure of the controller in this exercise should be:



So the signal v can be calculated like this:

$$v = (rT - Sy)\frac{1}{R}$$

Down a)

In problem 1, the result we get is:

```
R = [1, -1.205227573785277, 0.333523029149839];
S = [-0.379683429745841, 5.377649837140578e-2, 0.107155478091893];
```

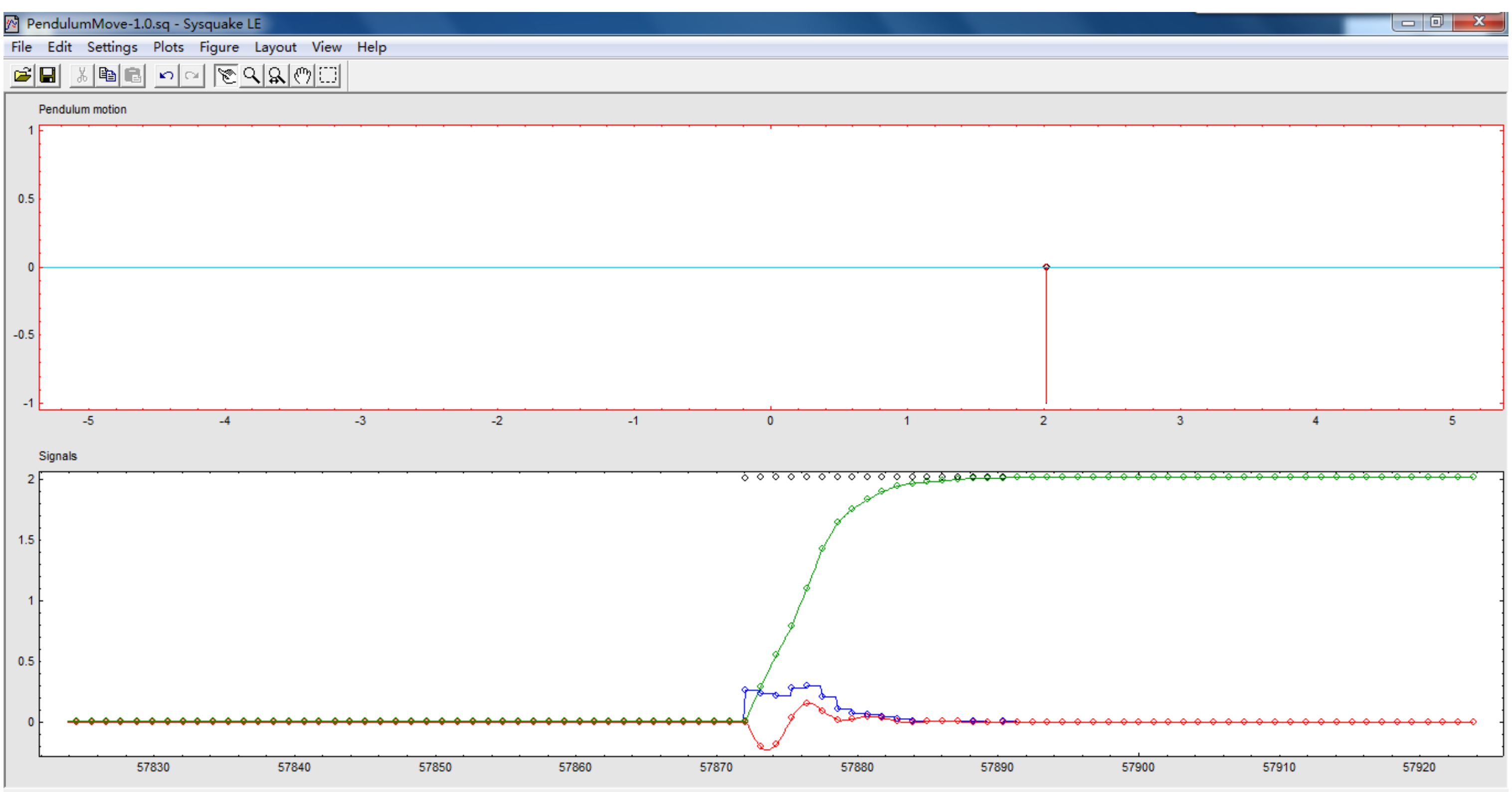
$T = R(1) = 0.1283$

$$v = [0.1283r - (-0.3797 + 0.0538q^{-1} + 0.1072q^{-2})y]\frac{1}{1 - 1.2052q^{-1} + 0.3335q^{-2}}$$

$$v(k) = 0.1283r(k) + 0.3797y(k) - 0.0538y(k-1) - 0.1072y(k-2) + 1.2052v(k-1) - 0.3335v(k-2)$$

So the code should be:

```
v=0.1283*r+0.37968*y-0.053776*samples(100,3)-0.10716*samples(99,3)+1.2052*x-0.33352*samples(100,5);
```



Up b)

In problem 1, the result we get is:

```
R = [1, -2.191611659843037, 1.174471268501992];
S = [2.399126574105173, -0.815918501363259, -1.533614335334607e-2];
```

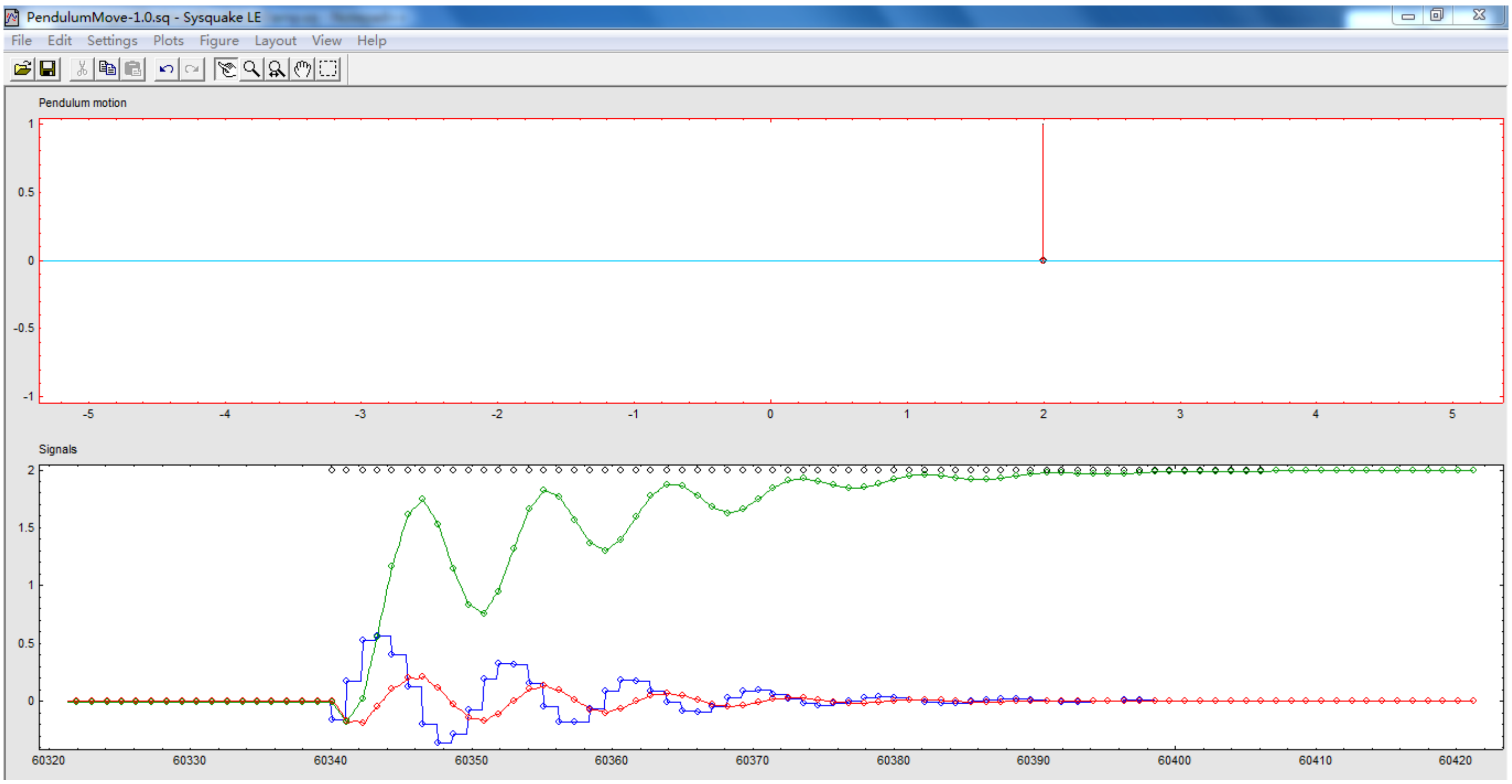
$T = -0.0818 + 0.0646q^{-1}$

$$v = [(-0.0818 + 0.0646q^{-1})r - (-2.3991 - 0.8159q^{-1} - 0.0534q^{-2})y]\frac{1}{1 - 2.1916q^{-1} + 1.1745q^{-2}}$$

$$v(k) = 0.0818r(k) + 0.0646r(k-1) - 2.3991y(k) + 0.8159y(k-1) + 0.0153y(k-2) + 2.1916v(k-1) - 1.1745v(k-2)$$

So the code should be:

```
v=-0.081768*r+0.064628*r_o-2.3991*y+0.81592*samples(100,3)+0.015336*samples(99,3)+2.1916*x-1.1745*samples(100,5);
```



(You can check by running the file PendulumMove-sungao.sq*)*

We can see that both the 'Down' controller and the 'Up' controller works well. It takes not very long time for the both pendulums to follow the reference position and keep balance at the same time.