Digital Control Exercise 5

Gao Sun sungao12@student.hh.se

Problem 1 Tank models at different operating points

a)

$$\frac{d\Delta h}{dt} = f(h, V) \approx \frac{\partial f}{\partial h}(h_0, V_0)\Delta h + \frac{\partial f}{\partial V}(h_0, V_0)\Delta V$$

Let $y = \Delta h = h - h_0$, and $u = \Delta V = V - V_0$,

$$\frac{dy}{dt} = py + du, \quad \begin{cases} p = -\dfrac{\alpha}{2\sqrt{h_0}} \\ d = 1 \end{cases}$$

As in this condition, y varies with mean close to $h_0 = 20$, so $p = -\frac{1}{2\sqrt{20}} = -\frac{\sqrt{5}}{20}$.

Zero-order-hold sampling with sampling period $h_s = 1$ gives:

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^{p} = e^{-\frac{\sqrt{5}}{20}} \approx 0.8942 \\ b = \dfrac{\lambda - 1}{p} = \dfrac{e^{-\frac{\sqrt{5}}{20}} - 1}{-\dfrac{\sqrt{5}}{20}} \approx 0.9461 \end{cases}$$

After these preparation work, we manually adjust u_0 to make y varies with mean close to $h_0 = 20$. Here is the variable of `samples` we got:

73083.151	2.6358	20.3296	0	73116.972	4.2833	24.2362	0
73084.272	2.4711	18.3573	0	73118.095	3.6243	23.5574	0
73085.279	4.6127	16.6361	0	73119.216	2.9653	22.2567	0
73086.394	5.2717	17.1928	0	73120.277	4.448	20.4995	0
73087.507	5.6012	18.3657	0	73121.369	5.6012	20.4175	0
73088.609	5.9307	19.7273	0	73122.443	5.7659	21.5144	0
73089.693	3.9538	21.248	0	73123.522	3.1301	22.6635	0
73090.763	3.2948	20.5857	0	73124.584	4.1185	21.0261	0
73091.854	2.8006	19.3095	0	73125.676	5.2717	20.5455	0
73092.978	3.6243	17.6296	0	73126.768	5.6012	21.3061	0
73094.041	4.7775	17.0562	0	73127.891	5.9307	22.3484	0
73095.162	5.7659	17.7353	0	73128.982	3.6243	23.5889	0
73096.284	5.7659	19.37	0	73130.109	3.1301	22.2781	0
73097.376	4.2833	20.7726	0	73131.197	2.9653	20.6455	0
73098.467	3.6243	20.4905	0	73132.289	4.7775	19.0227	0
73099.593	2.8006	19.5355	0	73133.414	5.2717	19.4619	0
73100.653	5.107	17.9194	0	73134.477	5.7659	20.3236	0
73101.748	5.4364	18.8174	0	73135.598	5.7659	21.6503	0
73102.868	5.7659	19.9724	0	73136.661	4.9422	22.7688	0
73103.992	5.9307	21.3431	0	73137.749	3.4596	22.9442	0
73105.045	3.4596	22.6484	0	73138.872	3.1301	21.5352	0
73106.112	3.1301	21.3375	0	73139.873	2.9653	20.1027	0
73107.235	2.9653	19.764	0	73140.974	4.1185	18.531	0
73108.296	5.4364	18.2846	0	73142.086	4.7775	18.3367	0
73109.424	5.6012	19.5117	0	73143.211	5.107	18.8591	0
73110.545	5.9307	20.759	0	73144.301	5.6012	19.6423	0
73111.603	3.2948	22.1328	0	73145.303	5.7659	20.7506	0
73112.634	4.9422	20.757	0	73146.425	5.4364	22.0294	0
73113.726	5.6012	21.1545	0	73147.454	2.8006	22.7537	0
73114.835	5.6012	22.2016	0	73148.575	3.9538	20.6726	0
73115.909	5.9307	23.1047	0	73149.693	5.4364	20.049	0

73150.761	5.107	21.0148	0	73171.476	5.7659	18.8424	0
73151.853	3.2948	21.5531	0	73172.599	5.9307	20.3449	0
73152.913	4.448	20.2039	0	73173.692	5.4364	21.8078	0
73154.005	5.6012	20.1557	0	73174.785	3.9538	22.5988	0
73155.13	5.9307	21.3318	0	73175.877	3.2948	21.7737	0
73156.188	3.6243	22.6442	0	73176.996	3.1301	20.3285	0
73157.28	2.9653	21.4745	0	73178.002	2.8006	19.017	0
73158.403	5.6012	19.7111	0	73179.029	5.2717	17.5065	0
73159.527	5.9307	20.9381	0	73180.09	5.9307	18.591	0
73160.589	3.789	22.2975	0	73181.183	5.2717	20.2545	0
73161.711	3.6243	21.3109	0	73182.241	3.9538	21.0246	0
73162.712	3.4596	20.3701	0	73183.366	3.4596	20.3562	0
73163.833	2.8006	19.2599	0	73184.481	3.2948	19.2532	0
73164.894	5.6012	17.6742	0	73185.551	4.2833	18.1528	0
73166.017	5.9307	19.1442	0	73186.671	4.6127	18.1766	0
73167.079	4.1185	20.7009	0	73187.798	3.4596	18.5454	0
73168.172	3.4596	20.2567	0	73188.859	4.7775	17.7004	0
73169.292	2.9653	19.1609	0	73189.978	4.2833	18.2982	0
73170.364	5.2717	17.7373	0	73191.102	4.448	18.3041	0

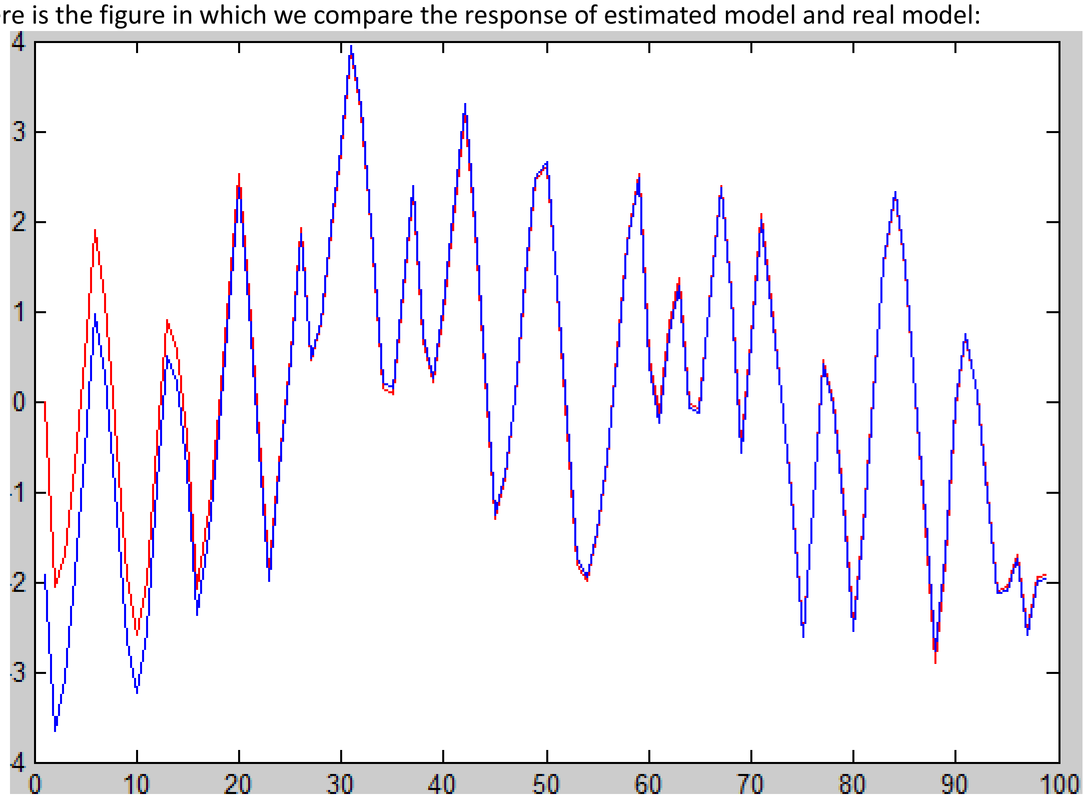
With this set of data, we use the Sysquake in least-squares method and we got the analytical solution:

$$\theta = \begin{bmatrix}\lambda \\ b\end{bmatrix} \approx \begin{bmatrix}0.8792 \\ 1.0194\end{bmatrix}$$

Comparing with the theoretical λ and b, λ is quite closed while b is not so closed.

b)

Here is the figure in which we compare the response of estimated model and real model:



The blue curve is the real response and the red one the estimated model response. We can see that at first, the difference between the two curves is great but later the two curves are quite close with each other.

c)

As in this condition, y varies with mean close to $h_0 = 350$, so $p = -\frac{1}{2\sqrt{350}} = -\frac{\sqrt{14}}{140}$.
Zero-order-hold sampling with sampling period $h_s = 1$ gives:

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^{p} = e^{-\frac{\sqrt{14}}{140}} \approx 0.9736 \\ b = \dfrac{\lambda - 1}{p} = \dfrac{e^{-\frac{\sqrt{14}}{140}} - 1}{-\dfrac{\sqrt{14}}{140}} \approx 0.9868 \end{cases}$$

After these preparation work, we manually adjust u_0 to make y varies with mean close to $h_0 = 350$.
Here is the variable of `samples` we got:

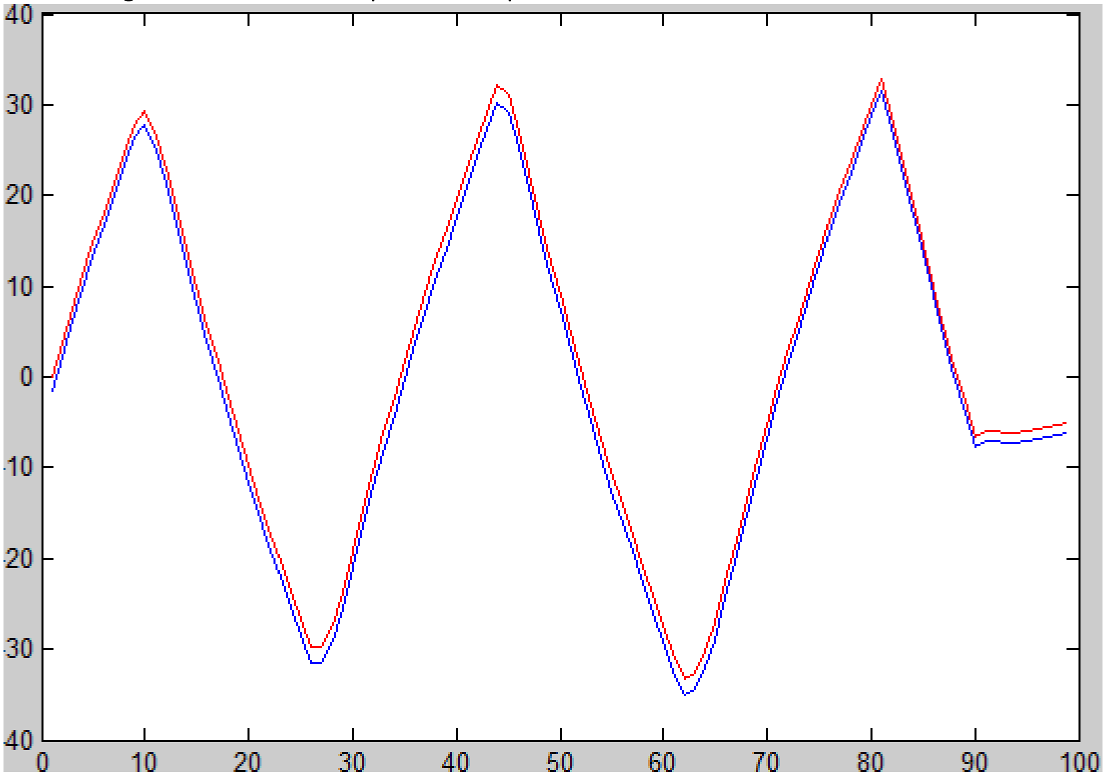
81152.681	22.24	346.3123	350.0686	81207.843	15.1561	359.2174	350.0686
81153.804	22.24	350.3285	350.0686	81208.967	15.1561	355.0124	350.0686
81154.927	22.24	354.2261	350.0686	81210.087	15.1561	350.9454	350.0686
81156.049	22.24	358.0058	350.0686	81211.211	15.1561	346.9842	350.0686
81157.171	22.24	361.6751	350.0686	81212.338	15.1561	343.1307	350.0686
81158.296	22.24	365.2471	350.0686	81213.461	15.1561	339.4056	350.0686
81159.418	22.24	368.7064	350.0686	81214.58	15.1561	335.8051	350.0686
81160.541	22.24	372.069	350.0686	81215.581	15.1561	332.6761	350.0686
81161.665	22.24	375.338	350.0686	81216.703	15.1561	329.2694	350.0686
81162.712	20.9221	378.299	350.0686	81217.825	15.1561	325.9664	350.0686
81163.724	17.4625	379.7696	350.0686	81218.949	15.1561	322.7589	350.0686
81164.816	15.6504	377.5888	350.0686	81220.073	15.1561	319.6501	350.0686
81165.942	15.3209	373.3923	350.0686	81221.101	18.4509	316.8912	350.0686
81167.065	15.1561	368.9623	350.0686	81222.224	19.9336	317.6092	350.0686
81168.187	15.1561	364.4815	350.0686	81223.348	20.7573	319.9461	350.0686
81169.308	15.1561	360.1344	350.0686	81224.408	22.24	322.944	350.0686
81170.435	15.1561	355.8915	350.0686	81225.532	22.24	327.6687	350.0686
81171.554	15.1561	351.8022	350.0686	81226.656	22.24	332.2489	350.0686
81172.679	15.1561	347.8122	350.0686	81227.781	22.24	336.6939	350.0686
81173.802	15.1561	343.9474	350.0686	81228.902	22.24	340.9897	350.0686
81174.924	15.1561	340.2013	350.0686	81230.027	22.24	345.1716	350.0686
81175.925	15.1561	336.9539	350.0686	81231.15	22.24	349.2217	350.0686
81177.047	15.1561	333.4177	350.0686	81232.271	22.24	353.1451	350.0686
81178.171	15.1561	329.9825	350.0686	81233.396	22.24	356.9665	350.0686
81179.294	15.1561	326.6549	350.0686	81234.517	22.24	360.6628	350.0686
81180.417	15.1561	323.429	350.0686	81235.641	22.24	364.2611	350.0686
81181.481	17.9567	320.4641	350.0686	81236.766	22.24	367.7581	350.0686
81182.55	20.0984	320.5223	350.0686	81237.888	22.24	371.145	350.0686
81183.632	21.2515	322.862	350.0686	81239.009	22.24	374.4319	350.0686
81184.752	22.24	326.4826	350.0686	81240.133	22.24	377.6333	350.0686
81185.876	22.24	331.099	350.0686	81241.257	22.24	380.7434	350.0686
81187	22.24	335.5749	350.0686	81242.293	15.9798	383.5318	350.0686
81188.122	22.24	339.9081	350.0686	81243.409	15.1561	379.5665	350.0686
81189.124	22.24	343.6678	350.0686	81244.5	14.8267	374.912	350.0686
81190.247	22.24	347.7626	350.0686	81245.624	14.8267	369.887	350.0686
81191.368	22.24	351.729	350.0686	81246.625	14.9914	365.5338	350.0686
81192.49	22.24	355.582	350.0686	81247.746	14.9914	360.9742	350.0686
81193.492	22.24	358.9273	350.0686	81248.871	14.9914	356.5315	350.0686
81194.612	22.24	362.5633	350.0686	81249.992	14.9914	352.2343	350.0686
81195.736	22.24	366.1064	350.0686	81251.117	14.9914	348.0489	350.0686
81196.86	22.24	369.5469	350.0686	81252.176	19.2746	344.2226	350.0686
81197.983	22.24	372.8854	350.0686	81253.269	18.4509	344.9996	350.0686
81199.109	22.24	376.1367	350.0686	81254.392	18.4509	344.8633	350.0686
81200.229	22.24	379.2785	350.0686	81255.394	18.4509	344.7452	350.0686
81201.245	18.6157	382.0513	350.0686	81256.485	18.7804	344.62	350.0686
81202.319	15.9798	381.0656	350.0686	81257.605	18.7804	344.8589	350.0686
81203.412	15.1561	377.249	350.0686	81258.728	18.7804	345.0912	350.0686
81204.471	14.9914	372.7917	350.0686	81259.852	18.7804	345.3169	350.0686
81205.596	15.1561	368.0059	350.0686	81260.977	18.7804	345.536	350.0686
81206.717	15.1561	363.5567	350.0686	81262.1	18.7804	345.7482	350.068

With this set of data, we use the Sysquake in least-squares method and we got the analytical solution:

$$\theta = \begin{bmatrix} \lambda \\ b \end{bmatrix} \approx \begin{bmatrix} 0.9703 \\ 1.0916 \end{bmatrix}$$

Comparing with the theoretical λ and b, λ is quite closed while b is not so closed.

Here is the figure in which we compare the response of estimated model and real model:



The blue curve is the real response and the red one the estimated model response. We can see that at first, the difference between the two curves is great but later the two curves are quite close with each other.

Problem 2 Changing model when opening the valve

Now the valve is open, which means $\alpha = 4$ now.

As in this condition, y varies with mean close to $h_0 = 20$, so $p = -\frac{4}{2\sqrt{20}} = -\frac{\sqrt{5}}{5}$, $d = 1$.

Zero-order-hold sampling with sampling period $h_s = 1$ gives:

$$y(k) = \lambda y(k-1) + bu(k-1), \quad \begin{cases} \lambda = e^{ph_s} = e^{p} = e^{-\frac{\sqrt{5}}{5}} \approx 0.6394 \\ b = \dfrac{\lambda - 1}{p} = \dfrac{e^{-\frac{\sqrt{5}}{5}} - 1}{-\dfrac{\sqrt{5}}{5}} \approx 0.8063 \end{cases}$$

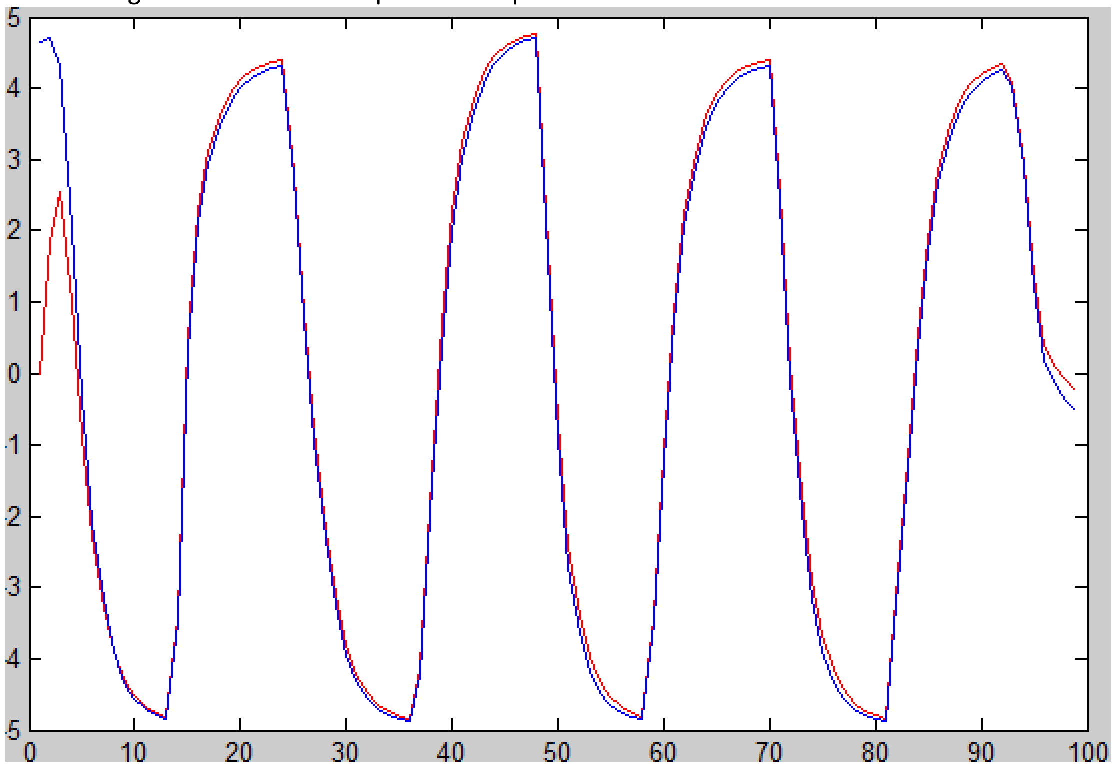
After these preparation work, we manually adjust u_0 to make y varies with mean close to $h_0 = 20$. Here is the variable of `samples` we got:

83425.336	20.2631	25.4393	0
83426.456	20.2631	25.519	0
83427.579	19.7689	25.5703	0
83428.665	17.4625	25.1662	0
83429.733	16.4741	22.9089	0
83430.854	16.1446	20.528	0
83431.977	15.9798	18.7824	0
83432.979	15.9798	17.6973	0
83434.101	15.9798	16.9618	0
83435.224	15.9798	16.5346	0
83436.349	15.9798	16.2883	0
83437.471	15.9798	16.1475	0
83438.597	15.9798	16.0667	0
83439.706	17.6272	16.0212	0
83440.777	21.0868	17.3668	0
83441.87	20.4278	21.1567	0
83442.992	20.0984	22.9645	0
83444.118	20.0984	23.801	0
83445.239	20.0984	24.3262	0
83446.24	20.0984	24.6305	0
83447.36	20.0984	24.8529	0
83448.483	20.0984	24.9951	0
83449.485	20.0984	25.078	0
83450.606	20.0984	25.1387	0
83451.69	18.2862	25.1765	0
83452.76	17.133	23.6257	0
83453.819	16.6388	21.6494	0
83454.911	16.1446	19.9355	0
83455.912	16.1446	18.5556	0
83457.033	15.9798	17.6087	0
83458.155	15.9798	16.9102	0
83459.157	15.9798	16.5386	0
83460.278	15.9798	16.2912	0
83461.401	15.9798	16.1491	0
83462.524	15.9798	16.0678	0
83463.646	15.9798	16.0214	0
83464.739	16.8035	15.9954	0
83465.77	19.4394	16.6457	0
83466.86	19.9336	19.2975	0
83467.861	20.2631	21.2032	0
83468.985	20.2631	22.8484	0
83470.105	20.2631	23.8723	0
83471.229	20.2631	24.5211	0
83472.351	20.2631	24.9323	0
83473.474	20.2631	25.1949	0
83474.598	20.2631	25.3628	0
83475.724	20.2631	25.4704	0
83476.845	20.2631	25.539	0
83477.937	16.8035	25.5821	0
83479.059	16.1446	22.4903	0
83480.186	15.8151	19.9679	0
83481.309	15.9798	18.1575	0
83482.43	15.9798	17.2314	0
83483.521	15.9798	16.7023	0
83484.644	15.9798	16.385	0
83485.768	15.9798	16.2027	0
83486.891	15.9798	16.0984	0
83488.014	15.9798	16.0388	0
83489.129	17.6272	16.005	0
83490.231	19.2746	17.3869	0
83491.324	19.9336	19.6064	0
83492.446	20.0984	21.5701	0
83493.569	20.0984	22.9277	0
83494.694	20.0984	23.7773	0
83495.814	20.0984	24.3108	0
83496.938	20.0984	24.6503	0
83498.062	20.0984	24.8662	0
83499.185	20.0984	25.0036	0
83500.31	20.0984	25.0914	0
83501.432	20.0984	25.1473	0
83502.495	17.9567	25.1816	0
83503.616	16.4741	23.2743	0
83504.709	16.1446	20.8032	0
83505.831	15.9798	18.9495	0
83506.952	15.9798	17.698	0
83508.076	15.9798	16.9612	0
83509.201	15.9798	16.5337	0
83510.324	15.9798	16.2881	0
83511.446	15.9798	16.1474	0
83512.571	15.9798	16.0667	0
83513.691	15.9798	16.0208	0
83514.754	17.9567	15.9956	0
83515.813	18.9452	17.613	0
83516.938	19.6041	19.5008	0
83518.059	19.9336	21.2125	0
83519.185	20.0984	22.5614	0
83520.307	20.0984	23.5456	0
83521.429	20.0984	24.165	0
83522.552	20.0984	24.5575	0
83523.555	20.0984	24.7854	0
83524.675	20.0984	24.9517	0
83525.797	20.0984	25.0581	0
83526.911	19.7689	25.1256	0
83527.95	18.6157	24.8864	0
83528.951	17.6272	23.7842	0
83530.072	17.6272	22.0991	0
83531.194	17.9567	21.0513	0
83532.318	17.9567	20.6997	0
83533.443	17.9567	20.485	0
83534.564	17.9567	20.3547	0

With this set of data, we use the Sysquake in least-squares method and we got the analytical solution:

$$\theta = \begin{bmatrix} \lambda \\ b \end{bmatrix} \approx \begin{bmatrix} 0.6163 \\ 0.8702 \end{bmatrix}$$

Comparing with the theoretical λ and b, λ is quite closed while b is not so closed.
Here is the figure in which we compare the response of estimated model and real model:



The blue curve is the real response and the red one the estimated model response. We can see that at first, the difference between the two curves is great but later the two curves are quite close with each other.