# Digital Control: Exercise 2

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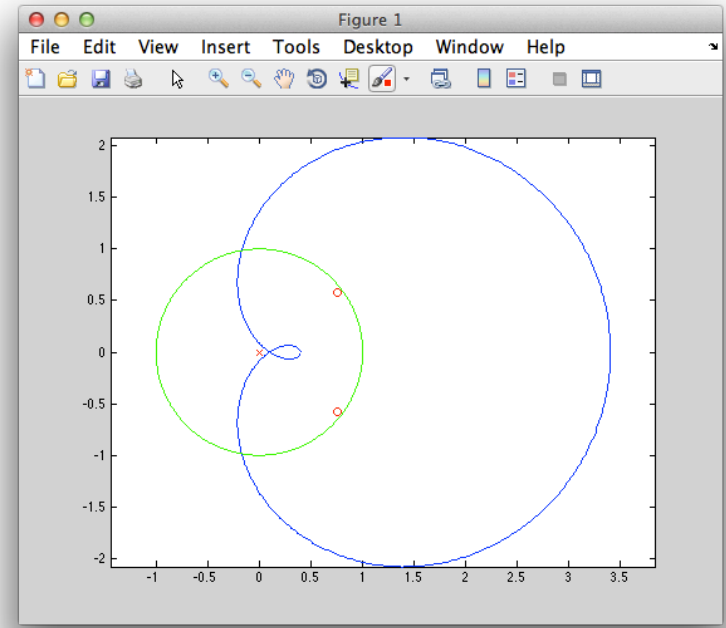
**1.** Determine stability/instability for the following characteristic polynomials by evaluating $\boldsymbol{Ae^{(-i\omega)}}$, $\boldsymbol{\omega = 0 \rightarrow 2\pi}$ as well as calculating the poles.

A. $Aq^{-1} = 1 - 1.5q^{-1} + 0.9q^{-2}$

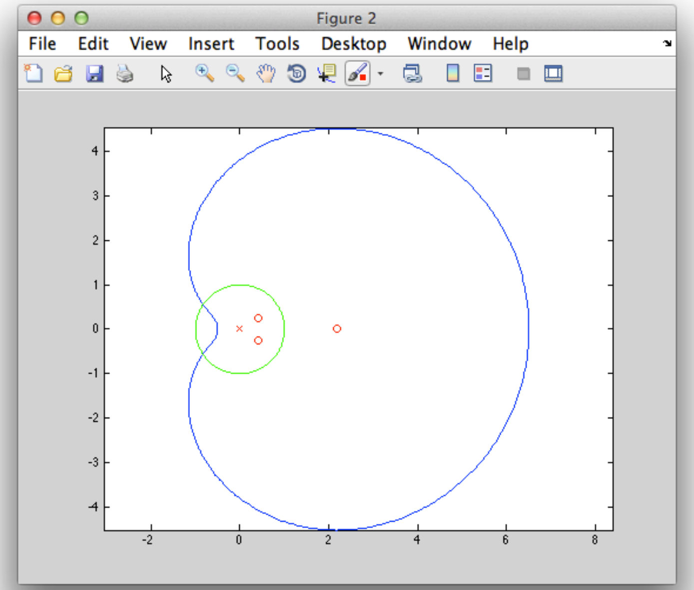
The Nyquist Curve is shown in the following figure.




In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the Nyquist curve doesn't enclose the original point, so there's no unstable pole. To confirm this, we can calculate the polynomial roots, here we can get the $roots = 0.7500 \mp 0.5809i$, which are all inside the unit circle. So, we can say the system is stable.

B. $Aq^{-1} = 1 - 3q^{-1} + 2q^{-2} - 0.5q^{-3}$

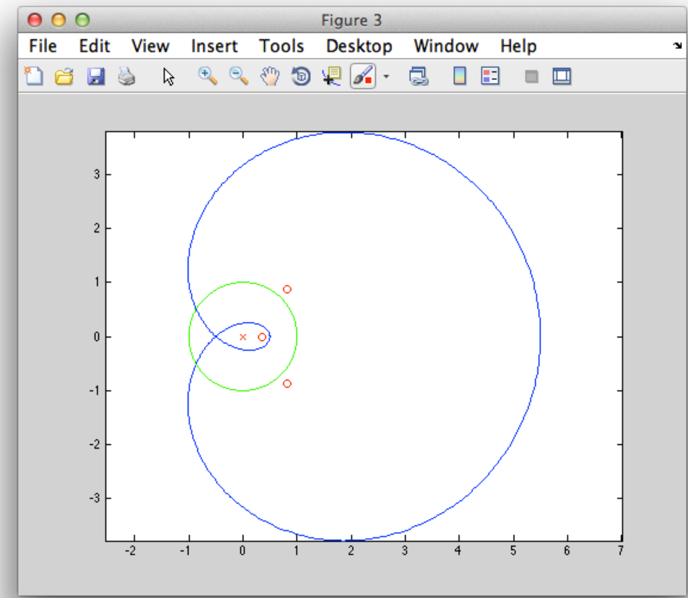
The Nyquist Curve is shown in the following figure.




In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the Nyquist curve enclose the original point, so there's unstable pole. To confirm this, we can calculate the polynomial roots, here I can get the $roots = 2.1914, 0.4043 \mp 0.2544i$, which 2.1914 is outside the unit circle. So, we can say the system is unstable.

C. $Aq^{-1} = 1 - 2q^{-1} + 2q^{-2} - 0.5q^{-3}$

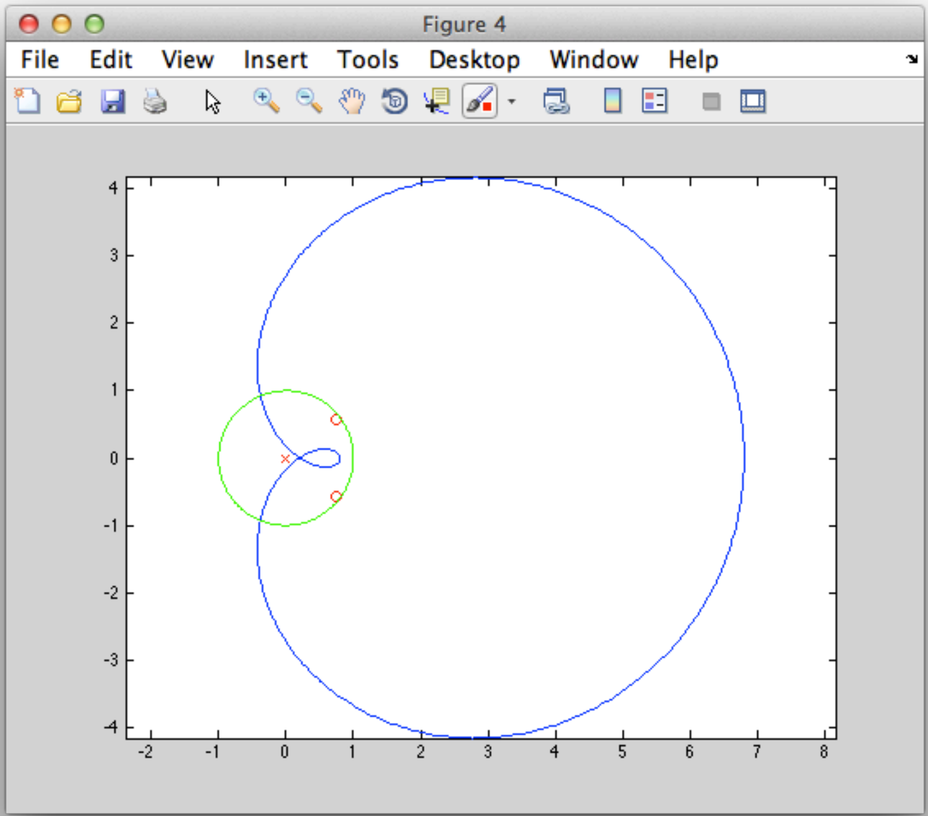
The Nyquist Curve is shown in the following figure.



In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the Nyquist curve enclose the original point twice, so there are unstable poles. To confirm this, we can calculate the polynomial roots, here I can get the $roots = 0.3522, 0.8239 \mp 0.8607i$, which$0.8239 \mp 0.8607i$ are outside the unit circle. So, we can say the system is unstable.

D. $Aq^{-1} = 2 - 3q^{-1} + 1.8q^{-2}$

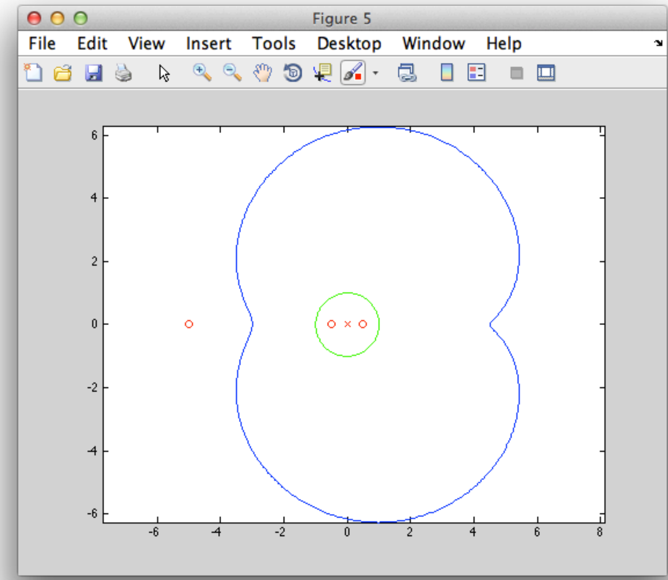
The Nyquist Curve is shown in the following figure.




In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the Nyquist curve doesn't enclose the original point, so there's no unstable pole. To confirm this, we can calculate the polynomial roots, here I can get the $roots = 0.7500 \mp 0.5809i$, which are all inside the unit circle. So, we can say the system is stable.

E. $Aq^{-1} = 1 + 5q^{-1} - 0.25q^{-2} - 1.25q^{-3}$

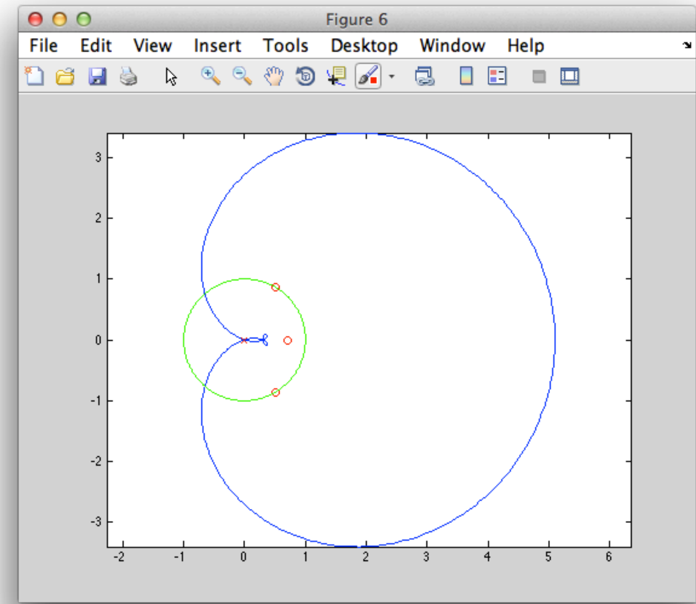
The Nyquist Curve is shown in the following figure.




In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the Nyquist curve enclose the original point, so there's unstable pole. To confirm this, we can calculate the polynomial roots, here I can get the $roots = -5, \mp 0.5$, which -5 is outside the unit circle. So, we can say the system is unstable.

F. $Aq^{-1} = 1 - 1.7q^{-1} + 1.7q^{-2} - 0.7q^{-3}$

The Nyquist Curve is shown in the following figure.



In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle. So in this figure, we can clearly see the original point is just on Nyquist curve, so there are two unstable poles. To confirm this, we can calculate the polynomial roots, here I can get the $roots = -7, 0.5 \mp 0.866i$, which $0.5 \mp 0.866i$ are just on the unit circle. So, we can say the system is unstable.

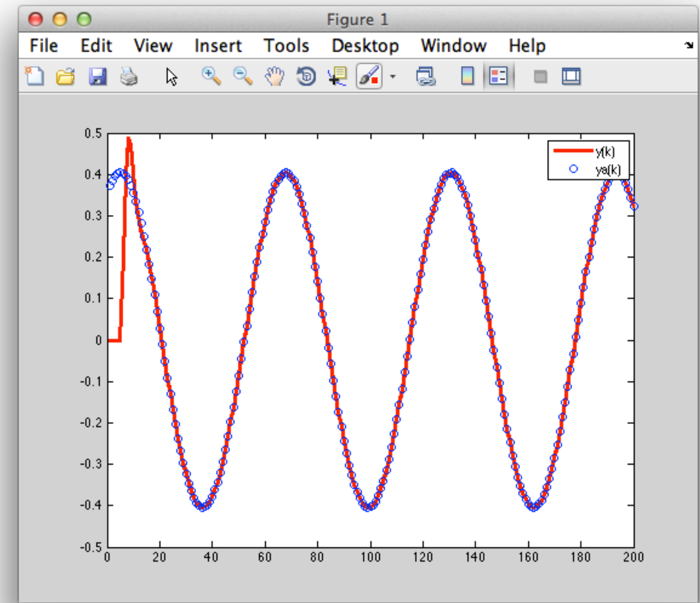
## 2. Consider the process

$$y(k) = G(q^{-1})u(k), G(q^{-1}) = \frac{0.2q^{-5}}{1 - q^{-1} + 0.52q^{-2}}$$

A. The code is as following:

```
clear all; close all;
k=(1:200);
u=[0,0,0,0,0,cos(0.1*k)];
y=filter(0.2,[1,-1,0.5,0,0,0],u);
figure;
plot(y(1:200),'r','LineWidth',2);hold on;
z=exp(-0.1*i);
G=0.2*z^5/(1-z+0.5*z^2);
ya=abs(G)*cos(0.1*k+angle(G));
plot(ya(1:200),'bo');
legend('y(k)','ya(k)');
```

The result we get is as following:

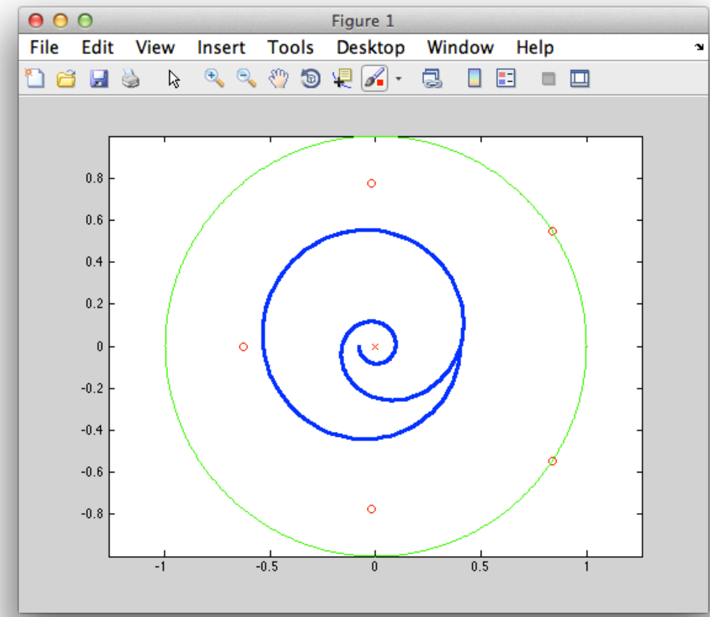


From the figure, we can easily found that $y(k) \rightarrow y_a(k)$.

## B. The code is as following:

```
clear all;close all;
alpha = [0:0.01:pi];
z = exp(-i*alpha);
G=0.2*z.^5./(1-z+0.5*z.^2);
neg_imag_G = [];
account = 1;
figure(1);
plot(real(G),imag(G),'b','LineWidth',2);axis equal;
hold on;
plot(0,0,'rx');
hold off;
x = inline('imag(0.2*exp(-5*i*w)/(1-exp(-i*w)+0.5*exp(-2*i*w)))','w');
w = fzero(x,0.5);
nag_z = real(0.2*exp(-5*i*w)/(1-exp(-i*w)+0.5*exp(-2*i*w)));
K_max = abs(1/nag_z);
G_closed=0.2*z.^5./(1-z+0.5*z.^2+K_max*0.2*z.^5);
zero_p=[0.2];
pole_p=[1 -1 0.5 0 0 K_max*0.2];
figure(2),zplane(zero_p,pole_p);
sys = tf(zero_p,pole_p,-1);
figure(3),plot(impulse(sys));
axis([0 100 -0.2 0.2]);
```

The Nyquist Curve is shown in the following figure.

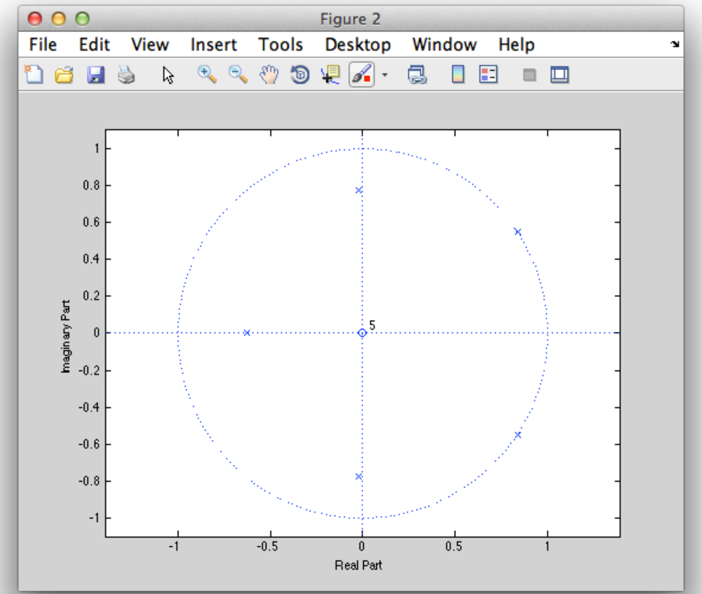


In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles are the roots, the green circle area is the unit circle.

Here, I use `fzero()` function to found the point on the curve which cross the negative axis, and we get the position of this point is (-0.5346,0i), then we get the $K_{max} = \left|\frac{1}{-0.5346}\right| \approx 1.8706$ for the closed loop system. Then we can check the closed loop system is stable or not.

$$G_{closed} = \frac{0.2q^{-5}}{1 - q^{-1} + 0.52q^{-2} + K_{max}\times 0.2q^{-5}} = \frac{0.2q^{-5}}{1 - q^{-1} + 0.52q^{-2} + 0.3741q^{-5}}$$

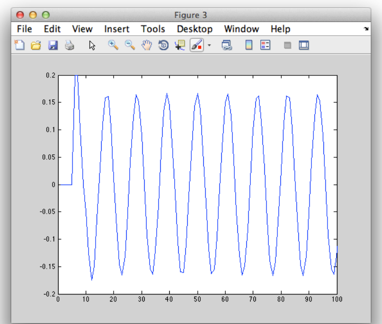
The zplane figure is shown as following.




Here we can get the $roots = 0.8365 \mp 0.5480i, -0.0231 \mp 0.7723, -0.6268$, the two poles that are just on the stability margin are $0.8365 \mp 0.5480i$.

By the `fzero()` function, we can also get when the curve cross the negative axis, $\omega = 0.5799$ and $K_{max} = 1.8706$, if this is correct, the oscillation period should be $osc_p = \frac{2\pi}{0.5799} \approx 10.8349$, to confirm this, I have plot the impulse response out.

The impulse response signal is shown in the following figure.




So, from this figure, we can found that the period is almost 10.835 as we expect. The question is verified.

## 3. Consider the process

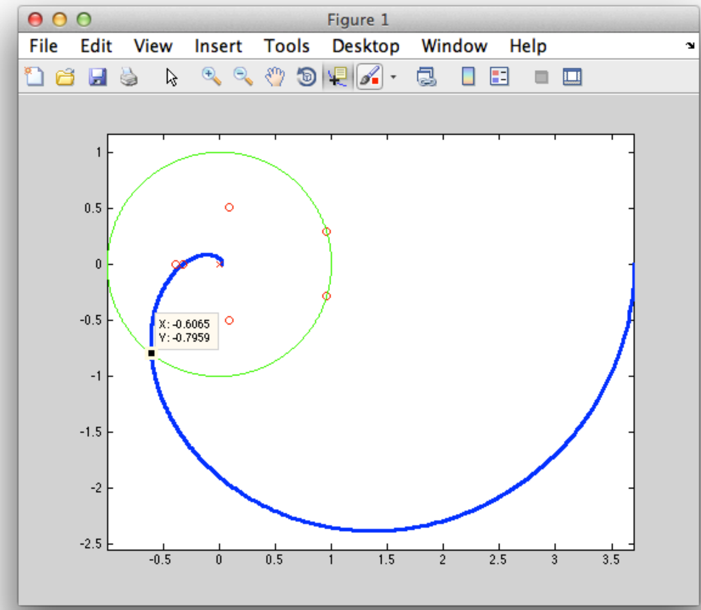
$$G(q^{-1}) = \frac{0.1q^{-2}}{(1-0.1q^{-1})(1-0.7q^{-1})(1-0.9q^{-1})}$$

A. The code is as following:

```
clear all;close all;
alpha = [0:0.01:pi];
angle = [0:0.01:2*pi];
z = exp(-i*alpha);
G=0.1*z.^2./((1-0.1*z).*(1-0.7*z).*(1-0.9*z));
neg_imag_G = [];
account = 1;
figure(1);
plot(real(G),imag(G),'b','LineWidth',2);axis equal;
hold on;
plot(0,0,'rx');
```

…

The Nyquist Curve is shown in the following figure.




In this figure, the blue curve is the Nyquist Curve, and red cross is the original point (0,0), the red circles is the cross point on the negative axis, the green circle area is the unit circle. The black point is the cross point on unit circle.

The point we can get the gain margin is the red circle point which position is (-0.3194,0i) with $\omega = 0.6120$, $K_{max} = \left|\frac{1}{-0.3194}\right| \approx 3.1309$.

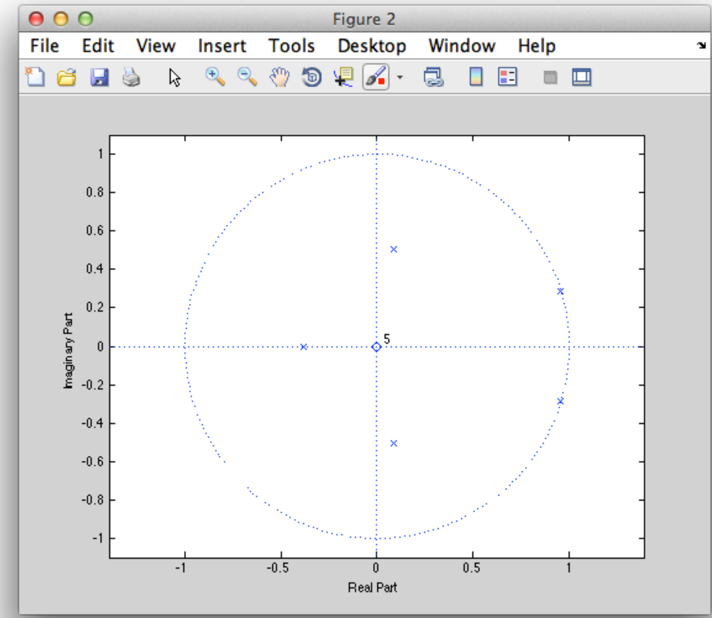
The point I can get the phase margin is the black point which position is (-0.6065-0.7959i), with $\omega = 0.2860$. So we can get the phase margin is $\varphi_{pm} = arctan\frac{-0.7959}{-0.6065} \approx 0.9196$.

B.

From the result that we got from question A, the delay margin can be calculated as $\tau_{pm} = \frac{\varphi_{pm}}{\omega} = \frac{0.9196}{0.2860} = 3.2154$. So three sample delays can be used before the closed-loop will get unstable. To confirm this, we can get $G_{closed}$ as following:

$$G_{closed} = \frac{0.1q^{-2}}{(1-0.1q^{-1})(1-0.7q^{-1})(1-0.9q^{-1})+0.1q^{-5}}$$
$$= \frac{0.1q^{-2}}{1-1.7q^{-1}+0.79q^{-2}-0.063q^{-3}+0.1q^{-5}}$$

The zplane figure is shown as following.




Here we can get the $roots = 0.9538 \mp 0.2871i, 0.0886 \mp 0.5041, -0.3847$, the two poles that are just on the stability margin are $0.8365 \mp 0.5480i$.

The same way as question 2.1.B, the oscillation period should be $osc_p = \frac{2\pi}{0.2860} \approx 21.9692$, to confirm this, I have plot the impulse response out.
The impulse response signal is shown in the following figure.
So, from this figure, we can found that the period is almost 21.9692 as we expect. The question is verified.

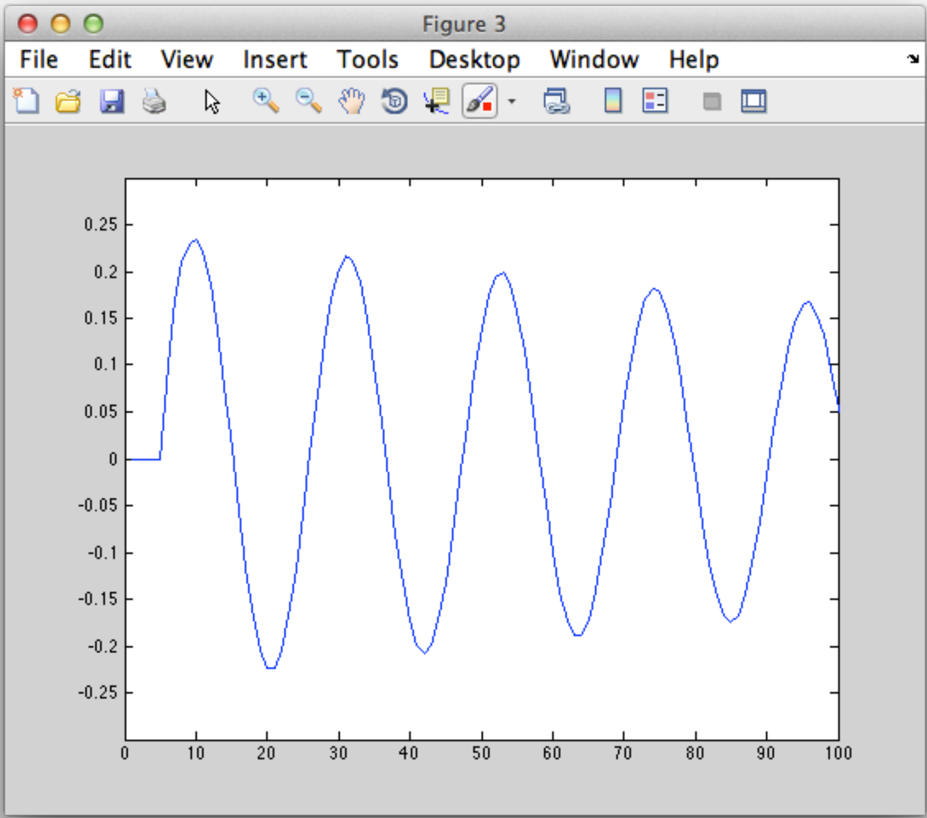


Figure 3
File Edit View Insert Tools Desktop Window Help
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
10
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50
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70
80
90
100